8 Price and Volume Measures

Price and volume measures in the QNA should be derived from observed price and volume data and be consistent with corresponding annual measures. This chapter examines specific aspects of price and volume measures derived at the quarterly frequency. In particular, it shows how to aggregate quarterly price and volume measures at the elementary level using Laspeyres and Fisher index formulas, how to derive quarterly chain volume series using alternative linking techniques, and how to handle the lack of additivity of quarterly chain volume series.

Introduction

1 A primary objective in compiling quarterly national accounts (QNA) is to obtain an accurate price and volume decomposition of quarterly transactions in goods and services. This decomposition provides the basis for measuring growth and inflation in macroeconomic aggregates, such as gross domestic product (GDP) in volume terms or household consumption deflator. To meet this objective, quarterly changes of transactions in goods and services at current prices need to be factored into two components: quarterly price changes and quarterly volume changes. As general principles, QNA price and volume measures should reflect the movements in quarterly price and volume indicators[1] and be temporally consistent with the corresponding price and volume measures derived from the annual national accounts (ANA).

2 The *2008 SNA* (chapter 15) defines basic principles for deriving price and volume measures within the system of national accounts in accord with index number theory and international standards of price statistics.[2] A key recommendation in the *2008 SNA*, also present in the *1993 SNA*, is to move away from the traditional national accounts measures "at constant prices"[3] toward chain-linked measures. Annual chain indices are superior to fixed-base indices, because weights are updated every year to reflect the current economic conditions. Chaining also avoids the need for re-weighting price and volume series when the base year is updated every five or ten years, which usually generates large revisions in the history of price and volume developments.[4] The *2008 SNA* recommends superlative index number formulas such as the Fisher and Tornquist formulas; however, a national accounts system based on Laspeyres volume indices (and the associated implicit Paasche price indices) is considered an acceptable alternative for practical reasons. A summary of the main recommendations of the *2008 SNA* is given in Box 8.1.

3 The *2008 SNA* also contains specific guidance on the compilation of quarterly price and volume measures. Although the same principles apply to both QNA and ANA, some complications derive from the different frequency of observation and the overarching requirement that quarterly and annual figures (when derived from independent compilation systems) should be made consistent with each other. The *2008 SNA* suggests that a sound approach to derive quarterly volume estimates is to calculate annually chained Laspeyres-type quarterly volume measures from quarterly data that are consistent with annual supply and use tables (SUT) expressed in current prices and in the prices of the previous year. Using

[1] See Chapter 3 for an overview of price and volume indicators for gross domestic product by economic activities and by expenditure components.

[2] Main references of international standards of price statistics are the *Consumer Price Index Manual: Theory and Practice* (ILO and others, 2004a), *Producer Price Index Manual: Theory and Practice* (ILO and others, 2004b), and *Export and Import Price Index Manual: Theory and Practice* (ILO and others, 2009).

[3] Constant price measures are based on fixed-base Laspeyres volume indices (i.e., weights taken from a fixed-base year) and corresponding current period-weighted Paasche price indices.

[4] Chain-linked series are still subject to benchmark revisions (based on comprehensive data sources available every five or ten years) and methodological revisions (due to changes in national accounting principles) to current price data, which may generate difference in the aggregate price and volume indices.

Box 8.1 Main Recommendations on Price and Volume Measures in the *2008 SNA*

This box quotes the main recommendations of the *2008 SNA* on expressing national accounts in volume terms (*2008 SNA*, paragraph 15.180):

(a) Volume estimates of transactions in goods and services are best compiled in a supply and use framework, preferably in conjunction with, and at the same time as, the current value estimates. This implies working at as detailed a level of products as resources permit.

(b) In general, but not always, it is best to derive volume estimates by deflating the current value with an appropriate price index, rather than constructing the volume estimates directly. It is therefore very important to have a comprehensive suite of price indices available.

(c) The price indices used as deflators should match the values being deflated as closely as possible in terms of scope, valuation, and timing.

(d) If it is not practical to derive estimates of value added in real terms from a supply and use framework and either the volume estimates of output and intermediate consumption are not robust or the latter are not available, then satisfactory estimates can often be obtained using an indicator of output, at least in the short term. For quarterly data, this is the preferred approach, albeit with the estimates benchmarked to annual data. An output indicator derived by deflation is generally preferred to one derived by quantity extrapolation.

(e) Estimates of output and value added in volume and real terms should only be derived using inputs as a last resort, since they do not reflect any productivity change.

(f) The preferred measure of year-to-year movements of gross domestic product (GDP) volume is a Fisher volume index; price changes over longer periods being obtained by chaining: that is, by cumulating the year-to-year movements.

(g) The preferred measure of year-to-year inflation for GDP and other aggregates is, therefore, a Fisher price index; price changes over long periods being obtained by chaining the year-to-year price movements, or implicitly by dividing the Fisher chain volume index into an index of the current value series.

(h) Chain indices that use Laspeyres volume indices to measure year-to-year movements in the volume of GDP and the associated implicit Paasche price indices to measure year-to-year inflation provide acceptable alternatives to Fisher indices.

(i) Chain indices for aggregates cannot be additively consistent with their components whichever formula is used, but this need not prevent time series of values being compiled by extrapolating base year values by the appropriate chain indices.

(j) A sound approach to deriving quarterly current value and volume estimates is to benchmark them to annual estimates compiled in a supply and use framework. This approach lends itself to the construction of annually chained quarterly volume measures using either the Fisher or Laspeyres formula.

annual weights increases consistency with the annual estimates and makes the quarterly indices less subject to volatility due to seasonal effects and short-term irregularities present in quarterly data.

4 The ideal way of producing volume estimates of QNA aggregates is to work at a very detailed level. The next section discusses some basic principles to derive volume estimates in the national accounts at the elementary aggregation level, adapted to the quarterly context. For each individual transaction, the same estimation method should be used to derive volume estimates in both ANA and QNA. As discussed in Chapter 3, for most market transactions, the best results are generally obtained by deflating current price values using appropriate price indices. Volume extrapolation should be employed where appropriate price data are not available or are not observable (e.g., nonmarket output), while the application of quantity revaluation in the QNA can be considered for those transactions where detailed quantities are available on a quarterly basis.

5 When detailed quarterly data on output and intermediate consumption are available, volume estimates of value added should be derived using a double indicator method. Volume estimates of output and intermediate consumption should be derived independently using appropriate price or volume indices. However, quarterly data on detailed intermediate inputs may not be available or may be so with a long time lag. In these cases, the calculation of quarterly value added in volume should be based on single indicator methods. Typically, a fixed relationship between output and value added in volume terms is assumed. The next section elaborates further on using alternative single indicator methods to best approximate the double indicator approach.

6 Strict consistency between QNA and direct ANA price and volume measures is only guaranteed when annual and quarterly changes are aggregated using the same system of weights. Coherently, with the *2008 SNA*, the preferred solution to achieve fully consistent QNA and ANA price and volume measures is to calculate Laspeyres-type volume indices with annual weights from the previous year. When the annual overlap (AO) technique is used for chain-linking quarterly indices,[5] annually chained Laspeyres-type quarterly volume measures are also consistent with the corresponding annual Laspeyres volume measures. Quarterly indices based on other index formulas, including Paasche and Fisher, or linked with other techniques (e.g., the one-quarter overlap [QO] technique) do not aggregate exactly to their corresponding direct annual indices. In such cases, consistency between QNA and ANA price and volume measures requires either that the ANA measures are derived as the annual sum of QNA measures or that consistency is forced on the QNA data using benchmarking techniques.

7 Notwithstanding the practical advantages of Laspeyres-type volume indices, a price and volume decomposition based on superlative indices (like Fisher) remains a theoretically superior solution for both ANA and QNA. The Fisher formula is a symmetric one, one in which price and quantity relatives are aggregated using weights from both the base period and the current period, and provides a better aggregation of elementary price and quantity relatives between the two periods than the Laspeyres formula (which uses the base period) and the Paasche formula (which uses the current period). This chapter illustrates a solution to develop a Fisher-based price and volume estimation system in the QNA based on (true) quarterly and annual Fisher indices.

8 Price and volume series should guarantee time-series characteristics: that is, data from different periods should be comparable in a consistent manner. Sequence of price and volume indices having different weight periods (e.g., volume series at previous year's prices) are not comparable over time and should not be presented in the form of time series. Chain-linking is a necessary operation to transform annual and quarterly links from the previous year (or from the previous quarter, in the case of quarterly Fisher indices) into consistent time series. This chapter provides guidance on how to calculate quarterly chain volume series using alternative linking techniques. Furthermore, it discusses how to resolve some practical issues arising from the lack of additivity of chain-linked measures, including the calculation of additive contributions to percent changes from nonadditive chain volume series based on the Laspeyres and Fisher formulas.

9 Strictly adhering to the *2008 SNA* principles, this chapter emphasizes the advantages of compiling chain-linked measures. Many countries, however, are still compiling traditional constant price estimates in both the ANA and QNA and are far from implementing chain-linked measures. These countries will find useful to examine the specific QNA methodological issues presented in the first three sections (basic principles, temporal consistency of price and volume measures, and choice of index formula for QNA volume measures), because these issues apply equally to constant price estimates. On the other hand, the discussion on chain-linking presented in the remainder of the chapter is more relevant for those countries that have already implemented chain-linking in the QNA or that plan to implement it soon.

Basic Principles for Deriving Volume Measures at the Elementary Aggregation Level

10 Volume measurement relates to decomposition of transaction values at current prices into their price and volume components. The aim of this decomposition is to analyze how much of the change is due to

[5] As mentioned in this chapter, the annual overlap technique may introduce a break in the chain volume series between one year and the next. However, this happens only if there are strong changes in quantity weights within the year (see Annex 8.1).

price movements and how much to volume changes.[6] This decomposition is admissible for transactions in goods and services for which it is possible to assume that the current value is composed of a price and a quantity component. In addition to pure transactions in goods and services, volume measures can be compiled for transactions such as taxes and subsidies on products, trade margins, consumption of fixed capital, and stocks of inventories and produced fixed assets. The accounting framework makes it possible to define and construct volume measures for value added, although value added does not represent any observable flow of goods and services that can be factored into a price and volume component directly. This section discusses some basic principles for deriving volume measures at the elementary aggregation level in the national accounts and how they should be implemented in the QNA context.

11 Volume estimates of national accounts should start from a very detailed level.[7] The most disaggregated level in the national accounts defines the level at which transactions in current values are deflated or extrapolated using available price or volume indices. To obtain accurate results, it is desirable for the price and volume indices to be as homogeneous as possible. The more detailed are the indices, the more homogeneous are the product groups measured by the indices. In the national accounts, these indices are considered elementary price indices, even though they are already aggregations of more detailed price indices. When the type of products of the index is homogeneous, the different underlying weighting methodologies can be assumed to be irrelevant and the price and volume changes from the indices can be used as price deflator or volume extrapolator for an elementary transaction of QNA.

12 In the QNA, the elementary level of aggregation should be decided on the basis of the ANA detail and the scope of price and volume indicators available on a quarterly basis. The ANA classification (by product, by industry, by expenditure function, etc.) generally defines the finest level of disaggregation possible for the QNA. Ideally, QNA price and volume measures should be derived at the same detail level used in the ANA. More often, the QNA detail is more aggregated than the ANA detail due to the reduced set of information available at the quarterly level. It is unnecessary and inefficient to keep the same ANA disaggregation in the QNA when the quarterly information set does not permit to distinguish nominal price and volume measures at that detail.

13 Prices and volumes are intrinsic components of nominal values. Denote with $c^{(s,y)}$ the value at current prices of an elementary QNA transaction for quarter s of year y, with $s=1,2,3,4$ and $y=1,2,\ldots$.[8] At the micro level, this transaction can be thought of as the sum of a (finite) number of individual "price × volume" transactions:

$$c^{(s,y)} = \sum\nolimits_j c_j^{(s,y)} = \sum\nolimits_j p_j^{(s,y)} q_j^{(s,y)}, \qquad (1)$$

where

j is an index for transactions included in the aggregate $c^{(s,y)}$,

$p_j^{(s,y)}$ is the price of transaction j in quarter s of year y, and

$q_j^{(s,y)}$ is the volume (quantity plus quality effects) of transaction j in quarter s of year y.

The entire set of individual transactions $c_j^{(s,y)}$, including their price and quantity details, are rarely directly observable. In the QNA, the quarterly value $c^{(s,y)}$ is derived using some quarterly value indicator (directly in nominal terms or derived as the combination of price–quantity indices). For any given year, the quarterly figures (equation (1)) are made consistent with the corresponding (generally more comprehensive) annual observation C_j^y through benchmarking.

14 As noted in this chapter, the most frequent solution adopted by countries for calculating consistent

[6] The expression "volume change" in the national accounts includes both quantity changes and quality changes. Changes in quality over time should be recorded as changes in volume and not as changes in price. Compositional changes should also be recorded as changes in volume, such as those resulting from a shift from or to higher quality products.

[7] Working at a detailed level means that, for example, volume estimates of gross domestic product (GDP) by industry should be derived from volume estimates of detailed economic activities, or that volume estimates of GDP by expenditure be derived from volume estimates of detailed categories of demand aggregates.

[8] Differently from previous chapters, notation in this chapter shows the time dimension in superscript and the item dimension in subscript. This notation is used in many price index theory textbooks and adopted by the *2008 SNA* (chapter 15). Lower-case letters denote quarterly observations, with quarter and year indicated in brackets. Upper-case letters denote annual observations.

price and volume measures in both ANA and QNA is to use annual weights.[9] This approach should be followed for both annual and quarterly data. For chain-linked measures, the weights should be updated every year. The volume measure associated with equation (1), denoted with $k^{y-1\to(s,y)}$, is expressed as the quantities of quarter s of year y valued at the prices of the previous year $y-1$:

$$k^{y-1\to(s,y)} = \sum_j k_j^{y-1\to(s,y)} = \sum_j P_j^{y-1} q_j^{(s,y)}, \quad (2)$$

where

P_j^{y-1} is a weighted average price of transaction j in year $y-1$ (for a discussion on how best to calculate weighted averages of quarterly price indices, see "Main Principles of Seasonal Adjustment" of this chapter). Equation (2) provides the quarterly volume measure at the (weighted average) prices of the previous year (or at previous year's prices) of the elementary transaction j for quarter s of year y.

15 By contrast, a constant price measure is expressed as follows:

$$k^{b\to(s,y)} = \sum_j k_j^{b\to(s,y)} = \sum_j P_j^{b} q_j^{(s,y)}, \quad (3)$$

where the quarterly quantities of quarter s of year y are valued at the average prices of a base year b. The advantage of using the volume estimate at previous year's prices in equation (2) instead of the constant price measure in equation (3) is that the weights are updated every year and are not taken from a fixed (and often distant) base year.

16 When detailed quantities in the current quarter and prices of the previous year are available, the volume measure $k^{y-1\to(s,y)}$ can be obtained by quantity revaluation. This method may provide accurate price and volume decomposition, as long as quality changes are incorporated in the quantities observed. This approach lends itself very well for homogeneous products, where quality changes are less likely to occur. Quantity revaluation finds some applications for agricultural products, whose quarterly quantities may be derived from work-in-progress models based on detailed crop forecasts, or for highly concentrated industries, such as oil-producing industries which often provide detailed data on their quarterly production through oil-related business associations.

17 More commonly, volume measures $k^{y-1\to(s,y)}$ are calculated using one of two alternative methods: price deflation and volume extrapolation.[10]

Price Deflation

18 The volume estimate $k^{y-1\to(s,y)}$ is derived by dividing the current price value $c^{(s,y)}$ by an appropriate price index. Ideally, the volume estimate $k^{y-1\to(s,y)}$ should be derived using a quarterly Paasche-type price index[11]:

$$PP^{y-1\to(s,y)} = \frac{\sum_j p_j^{(s,y)} q_j^{(s,y)}}{\sum_j P_j^{y-1} q_j^{(s,y)}}. \quad (4)$$

In effect, it is easily shown that

$$\frac{c^{(s,y)}}{PP^{y-1\to(s,y)}} = \frac{\sum_j p_j^{(s,y)} q_j^{(s,y)}}{\dfrac{\sum_j p_j^{(s,y)} q_j^{(s,y)}}{\sum_j P_j^{y-1} q_j^{(s,y)}}} = \sum_j P_j^{y-1} q_j^{(s,y)} = k^{y-1\to(s,y)}. \quad (5)$$

Paasche-type price indices are rarely available for national accounts purposes.[12] They require weights from every period and are difficult to calculate in practice. Price indices are usually calculated using the Laspeyres formula with a fixed-base year, with weights taken from a survey conducted in that year.[13] Denoting with $LP^{b\to(s,y)}$ a Laspeyres-type price index with a fixed-base year b, it is possible to calculate a price relative of quarter s of year y from the previous year $y-1$ as follows:

$$LP^{y-1\to(s,y)} = \frac{LP^{b\to(s,y)}}{LP^{b\to y-1}}, \quad (6)$$

[9] The following discussion can easily be adapted to calculating indices from the previous quarter, as required to derive quarterly Fisher indices.

[10] Chapter 3 identifies whether price deflation or volume extrapolation are most suitable for gross domestic product components by economic activity and by expenditure categories.

[11] A quarterly Paasche-type price index is a weighted harmonic average of price relatives with weights from the current quarter.

[12] A notable exception of Paasche-type aggregation is unit value indices in merchandise trade statistics.

[13] In practice, statistical offices do not calculate Laspeyres-type indices but Lowe indices, where the weight period precedes the base period. On the relationship between Lowe, Laspeyres, and Paasche price indices, see the *Consumer Price Index Manual: Theory and Practice* (ILO and others, 2004a).

that is, the ratio between the fixed-base index for quarter s of year y and the fixed-base index for year $y-1$. Replacing $PP^{y-1\to(s,y)}$ with the (fixed-base) Laspeyres-type price index $LP^{y-1\to(s,y)}$ in equation (6) will provide an approximate volume measure $k^{y-1\to(s,y)}$.[14] Working at a detailed elementary level is crucial for assuming that a fixed-base Laspeyres price index is close to the ideal current period-weighted Paasche price index.

Volume Extrapolation

19 This method requires an annually weighted Laspeyres-type quarterly volume index, which is defined as follows:

$$LQ^{y-1\to(s,y)} = \frac{\sum_j P_j^{y-1} q_j^{(s,y)}}{\frac{1}{4}\sum_j P_j^{y-1} Q_j^{y-1}}, \tag{7}$$

where Q_j^{y-1} is the annual quantity of transaction j in year $y-1$.

The volume measure $k^{y-1\to(s,y)}$ can be derived ideally by extrapolating the (rescaled) current price value of the previous year using the index $LQ^{y-1\to(s,y)}$: that is,

$$\frac{1}{4}C^{y-1}\cdot LQ^{y-1\to(s,y)} = \sum_j P_j^{y-1} Q_j^{y-1}\cdot$$
$$\sum_j \frac{P_j^{y-1} q_j^{(s,y)}}{P_j^{y-1} Q_j^{y-1}} = \sum_j P_j^{y-1} q_j^{(s,y)} = k^{y-1\to(s,y)}. \tag{8}$$

Likewise prices, the available volume indices are normally fixed-base Laspeyres-type indices. Similar to the price relative calculated in equation (6), a (fixed-base) quantity relative from the previous year can be calculated as follows:

$$LQ^{y-1\to(s,y)} = \frac{LQ^{b\to(s,y)}}{LQ^{b\to y-1}} \tag{9}$$

and used in equation (8) to extrapolate the volume change from the previous year. At constant prices, the volume index $LQ^{b\to(s,y)}$ can be used directly to extrapolate the current price data in the base year.

20 In most countries, quarterly GDP is derived from the production approach. This fact results from a greater availability of quarterly data by economic activity compared with expenditure and income transactions. Therefore, it assumes particular relevance how volume estimates of quarterly value added are calculated. As discussed in Chapter 3, the best method to derive volume measures of value added is to use double indicator methods—a volume measure of value added as the difference between a direct estimate of output in volume and a direct estimate of intermediate consumption in volume (each of which can be derived either by direct revaluation, deflation, or volume extrapolation). However, in practice, the information needed for obtaining independent and reliable volume estimates of output and intermediate consumption may not be available or may not be of sufficient quality. In particular, to derive a proper deflator for intermediate consumption for each activity, detailed data on intermediate consumption by product in the current quarter is needed.

21 In the QNA, simplified approximation methods sometimes need to be used.[15] One such simplified method is to use volume indicators to extrapolate value added. This is called the *single extrapolation technique*. The single extrapolation technique, using a volume estimate of output[16] to extrapolate value added, is based on an underlying assumption of a constant relationship between output, intermediate consumption, and value added in volume terms. This assumption usually holds true in the short run for many industries in periods of economic stability, while it is a highly questionable assumption in the long run and for countries with rapid structural changes. The fixed-ratio assumption in volume terms should be checked continuously looking at the annual benchmarks of national accounts, making sure that there are no sudden changes in the output-to-intermediate consumption ratio between one year and the next.

[14] Constant price data can be derived directly by dividing the current price data $c^{(s,y)}$ with a fixed-base Laspeyres price index $LP^{b\to(s,y)}$ or by extrapolating the current price data in the base year $\frac{1}{4}C^b$ with a fixed-based Laspeyres volume index $LQ^{b\to(s,y)}$.

[15] For an empirical assessment of the differences between double deflation and single indicator methods, see Alexander and others (2017).

[16] As noted in Chapter 3, input-related volume indicators (such as deflated wages or employment data) may be considered to extrapolate value added when information on output is absent or less reliable (one example is nonmarket output).

22 An alternative, less satisfactory, approximation is to use a price indicator (e.g., the price deflator for output, intermediate consumption, or a wage index) to deflate value added directly. This is known as the *single deflation technique*. The single deflation technique, using the price deflator for output as the deflator for value added, is based on an underlying assumption of a constant relationship between the price deflators for output, intermediate consumption, and value added. While there is reason to expect the relationship between output, intermediate consumption, and value added in volume terms to change only gradually, there is no reason to expect a stable relationship between the price deflators for output, intermediate consumption, and value added. This is a highly questionable assumption to rely on because price relatives may change abruptly, even in the short term. For this reason, the single deflation technique should be avoided.

23 When simplified methods such as the single extrapolation technique are used, it is strongly recommended to estimate all the components of the production account in volume terms, and not only value added. Furthermore, it is recommended to derive estimates based on more than one estimation technique and to assess the estimates and the validity of the underlying assumptions by inspecting and comparing the implicit deflators for output, intermediate consumption, and value added, or by assessing the intermediate consumption shares at the quarterly frequency.

Aggregating Price and Volume Measures Over Time

24 Aggregation over time means deriving less frequent data (e.g., annual) from more frequent data (e.g., quarterly). Incorrect aggregation of prices, or price indices, over time to derive annual deflators can introduce errors in independently compiled annual estimates and thus can cause inconsistency between QNA and ANA estimates, even when they are derived from the same underlying data. When deriving annual volume estimates by deflating annual current price data, a common practice is to compute the annual price deflators as a simple unweighted average of monthly or quarterly price indices. This practice may introduce substantial errors in the derived annual volume estimates, even when inflation is low. This may happen when

a. there are seasonal or other within-year variations in prices or quantities and

b. the within-year pattern of variation in either prices or quantities is unstable.

25 Volume measures for aggregated periods of time should conceptually be constructed from period-total quantities for each individual homogenous product. The corresponding implicit price measures would be quantity-weighted period-average price measures. For example, annual volume measures for single homogenous products[17] should be constructed as sums of the quantities in each subperiod. The corresponding implicit annual average price, derived as the annual current price value divided by the annual quantity, would therefore be a quantity-weighted average of the prices in each quarter. As shown in Example 8.1, the quantity-weighted average price will generally differ, sometimes significantly, from the unweighted average price. Similarly, for groups of products, conceptually, annual volume measures can be constructed as a weighted aggregate of the annual quantities for each individual product. The corresponding implicit annual price deflator for the group would be a weighted aggregate of the quantity-weighted annual average prices for the individual products. This annual price deflator for the group based on the quantity-weighted annual average prices would generally differ, sometimes significantly, from the annual price deflators derived as a simple unweighted average of monthly or quarterly price indices often used in ANA systems—deflation by the latter may introduce substantial errors in the derived annual volume estimates.

26 Consequently, to obtain correct volume measures for aggregated periods of time, deflators should take into account variations in quantities as well as prices within the period. For example, annual deflators could be derived implicitly from annual volume measures derived from the sum of quarterly volume

[17] Homogenous products are identical in physical and economic terms to other items in that product group and over time. In contrast, when there are significant variations among items or over time in the physical or economic characteristic of the product group, each version should be treated as a separate product (e.g., out-of-season fruit and vegetables such as old potatoes may be regarded as different products than in-season fruit and vegetables such as new potatoes).

Example 8.1 Weighted and Unweighted Annual Averages of Prices (or Price Indices) When Sales and Price Patterns Through the Year Are Uneven

Quarter	Quantity (1)	Price (2)	Current Price Value (3)	Unweighted Average Price (4)	Unit Value Weighted Average Price (5) = (3)/(1)	Volume estimates: At Unweighted Average 2010 Prices (6) = (4)*(1)	Volume estimates: At Weighted Average 2010 Prices (7) = (5)*(1)
q1	0	80	0			0	0
q2	150	50	7,500			7,500	6,750
q3	50	30	1,500			2,500	2,250
q4	0	40	0			0	0
2010	**200**		**9,000**	**50**	**45**	**10,000**	**9,000**
q1	0	40	0			0	0
q2	180	50	9,000			9,000	8,100
q3	20	30	600			1,000	900
q4	0	40	0			0	0
2011	**200**		**9,600**	**40**	**48**	**10,000**	**9,000**
Change from 2010 to 2011 (%)	0.00		6.67	–20.00	6.67	0.00	0.00
Direct Deflation of Annual Current Price Data							
2011 at 2010 prices	9,600/(40/50) = 9,600/0.8 = 12,000						
Change from 2010	(12,000/9,000 – 1) × 100 = 33.3%						

This example highlights the case of an unweighted annual average of prices (or price indices) being misleading when sales and price patterns through the year are uneven for a single homogenous product. The products sold in the different quarters are assumed to be identical in all economic aspects.

In the example, the annual quantities and the quarterly prices in quarters with nonzero sales are the same in both years, but the pattern of sales shifts toward the second quarter of 2011. As a result, the total annual current price value increases by 6.67 percent.

If the annual deflator is based on a simple average of quarterly prices, then the deflator appears to have dropped by 20 percent. As a result, the annual constant price estimates will wrongly show an increase in volume of 33.3 percent.

Consistent with the quantity data, the annual sum of the quarterly volume estimates for 2010 and 2011, derived by valuing the quantities using their quantity-weighted average 2010 price, shows no increase in volumes (column 7). The change in annual current price value shows up as an increase in the implicit annual deflator, which would be implicitly weighted by each quarter's proportion of annual sales in volume terms.

Price indices typically use unweighted averages as the price base, which corresponds to valuing the quantities using their unweighted average price. As shown in column 6, this results in an annual sum of the quarterly volume estimates in the base year (2010) that differs from the current price data, which it should not. As explained above and in this chapter, quarterly weighted prices should be used to derive annual prices. The difference between unweighted and weighted annual prices in the base year, however, can easily be removed by a multiplicative adjustment of the complete constant price time series, leaving the period-to-period rate of change unchanged. The adjustment factor is the ratio between the annual current price data and sum of the quarterly volume data in the base year (9,000/10,000).

estimates obtained using the following three-step procedure:

a. benchmark the quarterly current price data/indicator(s) to the corresponding annual current price data,

b. construct quarterly volume data by dividing the benchmarked quarterly current price data by the quarterly price index, and

c. derive the annual volume data as sum of the quarterly volume data.

Equivalently, the annual volume measure could be obtained by deflating, using an annual deflator that weights the quarterly price indices by the volume values of that transaction for each quarter. Either way of calculation achieves annual deflators that are quantity-weighted average annual price measures.

27 The procedure described above guarantees the best results of deflation if it is possible to obtain a reliable measurement of the quarterly pattern at current prices. If the current price indicator used to decompose the annual value is deemed to provide an inaccurate quarterly decomposition of the year (e.g., seasonal effects which are not fully representative of the transaction), the annual volume data could be affected by a distorted allocation of weights to quarterly prices. When it is not possible to derive accurate quarterly decomposition of current price data, unweighted averages of sub-annual indices represent a feasible choice for the ANA.

28 A more difficult case occurs when the annual estimates are based on more detailed price and value information than is available quarterly. In those cases, if seasonal volatility is significant, it would be possible to approximate the correct procedure using weights derived from more aggregated, but closely related, quarterly data.

29 The issue of price and quantity variations also applies within quarters. Accordingly, when monthly data are available, quarterly data will better take into account variations within the period if they are built up from the monthly data.

30 In many cases, variation in prices and quantities within years and quarters will be so insignificant that it will not substantially affect the estimates. Comparing weighted and unweighted averages can help identify the products for which the distinction is most relevant. Primary products and high-inflation countries are cases where the variation can be particularly significant. Of course, there are many cases in which there are no data to measure variations within the period.

31 A related problem that can be observed in quarterly data at constant prices of a fixed-base year is the annual sum of the quarterly volume estimates in the base year differing from the annual sum of the current price data, which should not be the case. This difference can be caused by the use of unweighted annual average prices as the price base when constructing monthly and quarterly price indices. Deflating quarterly data with deflators constructed with unweighted average prices as the price base corresponds to valuing the quantities using their unweighted annual average price rather than their weighted annual average price. This difference in the base year between the annual sum of the quarterly volume estimates and the annual sum of the current price data can easily be removed by a multiplicative adjustment of the complete volume series, leaving the period-to-period rate of change unchanged. The adjustment factor is the ratio between the annual current price data and the sum of the initial quarterly volume data based on the unweighted annual average prices in the base year, which, for a single product, is identical to the ratio of the weighted and unweighted average price.

Index Formula for QNA Volume Measures

32 Using the same notation introduced earlier, the application of revaluation, deflation, or volume extrapolation methods at the most detailed level in the QNA generates a set of elementary volume indices:

$$q_j^{y-1\to(s,y)} = \frac{k_j^{y-1\to(s,y)}}{C_j^{y-1}/4}, \qquad (10)$$

where

j denotes a generic QNA transaction,

$q_j^{y-1\to(s,y)}$ is a volume index from year $y-1$ to quarter s of year y for the j-th transaction,

$k_j^{y-1\to(s,y)}$ is the volume estimate of quarter s of year y at previous year's prices, and

$C_j^{y-1}/4$ is the (rescaled) annual value at current prices in the previous year.

Because numerator and denominator are valued using the same set of prices, the ratio measures a volume movement from year $y-1$ to quarter s of year y. The formula is additive within the year and coincides with the annual volume index. It is also additive across QNA transactions: the same formula can be used to extrapolate higher-level aggregates. Equation (1) provides the links to form chain-linked volume series, which is discussed in section "Chain-Linking in the QNA."

33 In a constant price system, equation (1) is modified as follows:

$$q_j^{b\to(s,y)} = \frac{k_j^{b\to(s,y)}}{K_j^{b\to y-1}/4}, \qquad (11)$$

where

$q_j^{b\to(s,y)}$ is a fixed-base volume index of quarter s of year y for transaction j,

$k_j^{b \to (s,y)}$ is the estimate of quarter s of year y at constant prices of a (fixed) base year b, and

$K_j^{b \to y-1} / 4$ is the (rescaled) constant price data in the previous year.

Because equation (11) derives fixed-base indices (i.e., indices expressed with a common base year), there is no need for using linking techniques between different years. Linking, however, is still necessary when the base year changes and the rebased series need to be linked to the series in the old base year. The techniques introduced in section "Chain-Linking in the QNA" are also relevant for linking constant prices series with different base years.

34 Elementary volume indices (equation (1) or (11)) need to be aggregated to derive QNA volume estimates. This section discusses how to aggregate elementary indices using the Laspeyres and Fisher formulas.

Laspeyres-Type Formula

35 A Laspeyres-type index aggregates elementary indices using weights from the base period. The base period for the QNA elementary volume indices shown in equation (1) is the previous year $y-1$.[18] An annually weighted Laspeyres-type quarterly volume index $LQ^{y-1 \to (s,y)}$ can be calculated as the weighted average of elementary volume indices of quarter s of year y with weights from year $y-1$:

$$LQ^{y-1 \to (s,y)} = \sum_{j=1}^{n} q_j^{y-1 \to (s,y)} \cdot W_j^{y-1} = \sum_{j=1}^{n} q_j^{y-1 \to (s,y)} \cdot \frac{C_j^{y-1}}{\sum_{j=1}^{n} C_j^{y-1}} \quad (12)$$

where

j is the index for transactions in the aggregate,

n is the number of transactions in the aggregate,

$q_j^{y-1 \to (s,y)}$ is the elementary volume index of transaction j from year $y-1$ to quarter s of year y as shown in equation (1),

C_j^{y-1} is the annual value at current prices of transaction j for year $y-1$,

$\sum_j C_j^{y-1}$ is the sum of all the annual values in the aggregate at current prices for year $y-1$, and

W_j^{y-1} is the share of C_j^{y-1} in the aggregate for year $y-1$.

Calculation of annually weighted Laspeyres-type volume measures from elementary volume indices is shown in Example 8.2.

36 Combining equation (1) and equations (2)–(9), equation (12) can be rewritten as follows:

$$LQ^{y-1 \to (s,y)} = \frac{\sum_{j=1}^{n} P_j^{y-1} q_j^{(s,y)}}{\frac{1}{4} \sum_{j=1}^{n} P_j^{y-1} Q^{y-1}}, \quad (13)$$

where

$q_j^{(s,y)}$ is the quantity of transaction j in quarter s of year y,

P_j^{y-1} is the price of transaction j in year $y-1$, and

Q_j^{y-1} is the quantity of transaction j in year $y-1$.

Equation (13) shows that a Laspeyres-type index is the ratio between the quantities of the current quarter valued at the (average) prices of the previous year and the rescaled annual value of the previous year at current prices. This notation is commonly found in the presentation of index numbers; however, it is difficult to apply in practice because, as noted before, price and quantities of QNA transactions are not available in most situations. For this reason, equation (12) is used in practice and is applied in the examples throughout this chapter.

37 As discussed earlier, annual weights for Laspeyres-type volume indices are generally preferable over quarterly weights. Use of the prices of one particular quarter, the prices of the corresponding quarter of the previous year, the prices of the corresponding quarter of a "fixed-base year," or the prices of the previous quarter are not appropriate for time series of Laspeyres-type volume measures in the national accounts for the following reasons:

- Consistency between directly derived ANA and QNA Laspeyres-type volume measures requires that the same price weights are used in the ANA

[18] For the sake of clarity, the following notation is based on volume indices at previous year's prices. However, any index aggregation formulas presented in this section apply equally to fixed-base indices.

Example 8.2 Deriving Annual and Quarterly Volume Measures Using Laspeyres-Type Formula

	Current Prices			Elementary Price Indices (Previous Year = 100)		Elementary Volume Measures (in Monetary Terms)			Elementary Volume Indices (Previous Year = 100)		Laspeyres Volume Index (Previous Year = 100)	Laspeyres Volume Measure (in Monetary Terms)
	(1)			(2)		(3) = (1)/(2) × 100			(4)		(5)	(6)
	A	B	Total	A	B	A	B	Sum	A	B	Total	Total
2010	600.0	900.0	1,500.0			600.0	900.0	*1,500.0*	100.00	100.00	100.00	*1,500.0*
2011	660.0	854.9	1,514.9	102.63	98.50	643.1	867.9	*1,511.0*	107.18	96.43	100.73	*1,511.0*
2012	759.0	769.5	1,528.5	101.72	98.34	746.2	782.5	*1,528.7*	113.05	91.53	100.91	*1,528.7*
2013	948.8	615.6	1,564.4	99.34	101.08	955.1	609.0	*1,564.1*	125.83	79.14	102.33	*1,564.1*
q1 2011	159.7	218.9	378.6	102.00	99.00	156.6	221.1	*377.7*	104.38	98.27	100.71	*377.7*
q2 2011	163.2	213.7	376.9	102.50	98.00	159.2	218.1	*377.3*	106.15	96.92	100.61	*377.3*
q3 2011	167.4	210.6	378.0	103.00	98.00	162.5	214.9	*377.4*	108.35	95.51	100.65	*377.4*
q4 2011	169.7	211.7	381.4	103.00	99.00	164.8	213.8	*378.6*	109.84	95.04	100.96	*378.6*
Sum 2011	**660.0**	**854.9**	**1,514.9**			**643.1**	**867.9**	***1,511.0***	**107.18**	**96.43**	**100.73**	***1,511.0***
q1 2012	174.2	204.1	378.3	102.50	97.00	170.0	210.4	*380.4*	103.00	98.45	100.43	*380.4*
q2 2012	180.4	201.4	381.8	102.00	99.00	176.9	203.4	*380.3*	107.19	95.19	100.42	*380.3*
q3 2012	188.9	192.3	381.2	101.00	98.50	187.0	195.2	*382.3*	113.35	91.35	100.93	*382.3*
q4 2012	215.5	171.7	387.2	101.50	99.00	212.3	173.4	*385.7*	128.68	81.15	101.85	*385.7*
Sum 2012	**759.0**	**769.5**	**1,528.5**			**746.2**	**782.5**	***1,528.7***	**113.05**	**91.53**	**100.91**	***1,528.7***
q1 2013	224.7	166.0	390.7	100.50	100.00	223.6	166.0	*389.6*	117.83	86.29	101.95	*389.6*
q2 2013	235.8	156.3	392.1	99.50	101.00	237.0	154.8	*391.7*	124.89	80.44	102.52	*391.7*
q3 2013	242.9	148.5	391.4	99.00	101.50	245.4	146.3	*391.7*	129.30	76.05	102.49	*391.7*
q4 2013	245.4	144.8	390.2	98.50	102.00	249.1	142.0	*391.1*	131.30	73.79	102.35	*391.1*
Sum 2013	**948.8**	**615.6**	**1,564.4**			**955.1**	**609.0**	***1,564.1***	**125.83**	**79.14**	**102.33**	***1,564.1***

(Rounding errors in the table may occur.)

Deflation at the Elementary Level

This example explains how to derive volume estimates of two transactions at the most detailed level (A and B) and how to derive a volume index using an annually weighted Laspeyres-type formula. Annual and quarterly data at current prices of the two transactions from 2010 to 2013 are presented in column 1, with the quarterly split available from q1 2011. On average, transaction A shows a 16.5 percent increase a year, while transaction B declines at a 11.9 percent annual rate: total increase is 1.4 percent a year. The relative size of transactions A and B is reverted after three years. Column 2 contains the elementary price indices for A and B of each quarter compared with the previous year, as explained in equations (1)–(9). Volume estimates for A and B are obtained by price deflation in column 3. For instance, volume estimates of A for the quarters of 2011 are calculated as follows:

q1 2011: (159.7/102.0) × 100 = 156.6
q2 2011: (163.2/102.5) × 100 = 159.2
q3 2011: (167.4/103.0) × 100 = 162.5
q4 2011: (169.7/103.0) × 100 = 164.8.

The same operations are done using the annual data. As explained in this chapter, annual price changes are derived as weighted average of the quarterly indices with weights given by the quarterly volume estimates in column 3. Note that because annual indices are weighted average of quarterly indices, the sum of the quarterly volume estimates corresponds to the independently calculated annual volume figure. This condition is also met for the total aggregate.

Elementary Volume Indices Elementary volume indices are shown in column 4. For the annual data, they are derived implicitly by dividing the annual volume measures in column 3 by the current price value in the previous year. For instance, the annual index for 2011 for transaction A is 643.1/600 = 107.18. For the quarterly data, the elementary volume indices are derived by dividing the quarterly volume measures in column 3 by the rescaled current price value in the previous year (see equation (9)). The quarterly index for q1 2011 for transaction A is 156.6/(600/4) = 104.38.

Laspeyres-Type Volume Indices and Laspeyres-Type Volume Measures in Monetary Terms

The annually weighted Laspeyres-type volume indices in column 5 are calculated as a weighted average of the elementary volume indices in columns 4. The weights are the share at current prices from the previous year. The annual indices are calculated as follows:

2011: 107.18 × (600/1,500) + 96.43 × (900/1,500) = 100.73
2012: 113.05 × (660/1,514.9) + 91.53 × (854.9/1,514.9) = 100.91
2013: 125.83 × (759/1,528.6) + 79.14 × (769.5/1,528.6) = 102.33.

Similar to the annual indices, the quarterly indices are calculated using weights from the previous year. For the quarters of 2011,

q1 2011:	104.38 × (600/1,500) + 98.27 × (900/1,500) = 100.71
q2 2011:	106.15 × (600/1,500) + 96.92 × (900/1,500) = 100.61
q3 2011:	108.35 × (600/1,500) + 95.51 × (900/1,500) = 100.65
q4 2011:	109.84 × (600/1,500) + 95.04 × (900/1,500) = 100.96.

For the quarters of 2012,

q1 2012:	103.00 × (660/1,514.9) + 98.45 × (854.9/1,514.9) = 100.43
q2 2012:	107.19 × (660/1,514.9) + 95.19 × (854.9/1,514.9) = 100.42
q3 2012:	113.35 × (660/1,514.9) + 91.35 × (854.9/1,514.9) = 100.93
q4 2012:	128.68 × (660/1,514.9) + 81.15 × (854.9/1,514.9) = 101.85.

Volume estimates in monetary terms are derived by multiplying the Laspeyres volume indices by the total current price value in the previous year. For 2011 and 2012,

2011:	100.73 × 1,500 = 1,511.0	2012:	100.91 × 1,514.9 = 1,528.7
q1 2011:	100.71 × (1,500/4) = 377.7	q1 2012:	100.43 × (1,514.9/4) = 380.4
q2 2011:	100.61 × (1,500/4) = 377.3	q2 2012:	100.42 × (1,514.9/4) = 380.3
q3 2011:	100.65 × (1,500/4) = 377.4	q3 2012:	100.93 × (1,514.9/4) = 382.3
q4 2011:	100.96 × (1,500/4) = 378.6	q4 2012:	101.85 × (1,514.9/4) = 385.7.

It is easily shown that the sum of the quarterly volume measures in monetary terms corresponds to the corresponding annual volume measure. This condition is verified within each link using the Laspeyres-type formula. In addition, note that the quarterly volume measures in monetary terms are equal to the sum of the deflated elementary transactions shown in column 3 at both annual and quarterly levels.

and the QNA, and that the same price weights are used for all quarters of the year.

- The prices of one particular quarter are not suitable as price weights for volume measures in the ANA, and thus in the QNA, because of seasonal fluctuations and other short-term volatilities in relative prices. Use of weighted annual average prices reduces these effects. Therefore, weighted annual average prices are more representative for the other quarters of the year as well as for the year as a whole.
- The prices of the corresponding quarter of the previous year or the corresponding quarter of a "fixed-base year" are not suitable as price weights for volume measures in the QNA because the derived volume measures only allow the current quarter to be compared with the same quarter of the previous year or years. Series of year-to-year changes do not constitute time series that allow different periods to be compared and cannot be linked together to form such time series. In particular, because they involve using different prices for each quarter of the year, they do not allow different quarters within the same year to be compared. For the same reason, they do not allow the quarters within the same year to be aggregated and compared with their corresponding direct annual estimates. Furthermore, as shown in Chapter 1, changes from the same period in the previous year can introduce significant lags in identifying the current trend in economic activity.
- The prices of the previous quarter are not suitable as price weights for Laspeyres-type volume measures for two reasons:
  a. The use of different price weights for each quarter of the year does not allow the quarters within the same year to be aggregated and compared with their corresponding direct annual estimates.
  b. If the quarter-to-quarter changes are linked together to form a time series, short-term volatility in relative prices may cause the quarterly chain-linked measures to show substantial drift compared to corresponding direct measures.

38 In sum, the Laspeyres formula offers a very convenient solution to achieve consistency between ANA and QNA volume measures. As shown in Example 8.2, the sum of annually weighted Laspeyres-type quarterly volume measures (i.e., the quarterly volume estimates at previous year's prices) matches the independently derived Laspeyres-type annual volume measures (i.e., the annual volume estimate at previous year's prices). Moreover, the quarterly volume estimates at previous year's prices are additive within each link (quarter or year). Laspeyres-type indices have these properties because annual and quarterly indices use the same set of weights. Fisher indices, as explained in paragraph 8.76, do not have these properties and need to be reconciled when they are calculated at different frequencies.

39 Because Laspeyres-type volume estimates in monetary terms are additive in each period, volume estimates of aggregates can simply be derived as the sum of the elementary volume components (see Example 8.2). As noted at the beginning of this subsection, equation (12) can be used to calculate Laspeyres-type volume indices from both elementary items and aggregates. They can be derived by dividing the sum of elementary volume components for a particular quarter by the (rescaled) aggregate estimate at current prices of the previous year (i.e., by applying equation (1) on the aggregate estimates).

Fisher-Type Formula

40 A Fisher index is the geometric mean of the Laspeyres and Paasche indices. A Fisher index is a symmetric index, one that makes equal use of the prices and quantities in both the periods compared and treat them symmetrically. Symmetric indices satisfy a set of desirable properties in index number theory (like the time reversal test) and are to be preferred for economic reasons because they assign equal weight to the two situations being compared.[19]

41 Calculation of annually weighted quarterly Fisher-type indices is complicated. They should be derived as symmetric annually weighted Laspeyres-type and Paasche-type quarterly volume indices. However, the (implicit) Paasche-type quarterly index corresponding to the annually weighted Laspeyres-type quarterly index shown in equation (12) has weights from the current quarter (i.e., the current period). This would make the geometric average of Laspeyres and Paasche indices (i.e., the Fisher index) temporally asymmetric, because the weight structure would be taken from the previous year and the current quarter.

42 The *2008 SNA* illustrates a solution to calculate symmetric annually weighted quarterly Fisher-type indices (paragraphs 15.53–55). For each pair of consecutive years, Laspeyres-type and Paasche-type quarterly indices are constructed for the last two quarters of the first year and the first two quarters of the second year. The annual value shares are taken from the two years to construct Laspeyres-type and Paasche-type quarterly indices. The annually chained Fisher-type indices are derived as the geometric mean of these two indices. The resulting quarterly Fisher indices need to be benchmarked to annual chain Fisher indices. At the end of the series (when Paasche indices using annual weights from the current year are impossible to calculate), true quarterly Fisher indices can be used to extrapolate the annually chained Fisher-type indices.

43 True quarterly Fisher indices provide results that are not exactly consistent with corresponding annual Fisher indices; nevertheless, they are usually close enough when quantity and price weights are relatively stable within the year. When the Fisher formula is chosen in the ANA, the preferred solution for the QNA is to calculate true quarterly Fisher indices (with quarterly weights) and benchmark them to the corresponding annual Fisher indices.[20] The benchmarking process forces the quarterly volume measures to be consistent with the annual ones. Before benchmarking, the difference between the annual and quarterly indices should be investigated carefully to detect possible drifts in the chain quarterly series (see the drift problem in the section "Frequency of Chain-Linking").

44 To calculate quarterly Fisher volume indices, quarterly Laspeyres volume indices and quarterly Paasche volume indices[21] are necessary. They can be calculated as follows:

$$LQ^{t-1\to t} = \sum_{j=1}^{n} q_j^{t-1\to t} \cdot \frac{c_j^{t-1}}{\sum_{j=1}^{n} c_j^{t-1}} \tag{14}$$

$$PQ^{t-1\to t} = \left(\sum_{j=1}^{n} \left(q_j^{t-1\to t} \right)^{-1} \cdot \frac{c_j^{t}}{\sum_{j=1}^{n} c_j^{t}} \right)^{-1}, \tag{15}$$

[19] Other symmetric (and superlative) indices are the Walsh and Törnqvist indices. Details on the theory of symmetric and superlative indices can be found in the *Consumer Price Index Manual: Theory and Practice* (ILO and others, 2004a).

[20] The United States adopts this solution to calculate consistent annual and quarterly Fisher price and volume indices in the national accounts (see Parker and Seskin, 1997).

[21] Quarterly Paasche volume indices adopt as weights the current price data for the most recent quarter. Because data for the last quarter may be subject to large revisions, Paasche indices could be more volatile over time than the corresponding Laspeyres indices.

where

t is a generic index for quarters,

$q_j^{t-1\to t}$ is an elementary volume index for transaction j from quarter $t-1$ to t (e.g., the usual quarterly percent change), and

c_j^t is the current price data of transaction j in quarter t.

Defining $q_j^{t-1\to t} = q_j^t / q_j^{t-1}$ and $c_j^t = p_j^t q_j^t$, equations (14) and (15) can be rewritten in the usual notation:

$$LQ^{t-1\to t} = \frac{\sum_j p_j^{t-1} q_j^t}{\sum_j p_j^{t-1} q_j^{t-1}}$$

$$PQ^{t-1\to t} = \frac{\sum_j p_j^t q_j^t}{\sum_j p_j^t q_j^{t-1}},$$

which shows clearly that a Laspeyres volume index weights the quantities from the two periods compared with prices from the previous quarter $t-1$ and a Paasche volume index uses prices from the current quarter t.

45 The quarterly Fisher volume index is the geometric mean of the Laspeyres index (equation (14)) and the Paasche index (equation (15)):

$$FQ^{t-1\to t} = \sqrt{LQ^{t-1\to t} \cdot PQ^{t-1\to t}}. \qquad (16)$$

Differently from the Laspeyres and Paasche indices (but not their combination), a Fisher index satisfies the value decomposition test. The product of a Fisher price index and a Fisher volume index reproduces the change in the value aggregate for any given period (year or quarter). The Fisher price index can therefore be derived implicitly by dividing the current price data with the Fisher volume index (equation (16)).

46 The procedure described above applies to annual data as well, replacing quarters with annual observations in equations (14) and (15). However, as mentioned before, the quarterly Fisher indices will not be consistent with the annual ones. The best solution is to benchmark the quarterly chain Fisher indices to the annual chain Fisher indices using a benchmarking technique that preserves the original movements in the quarterly indices, such as the Denton proportional benchmarking method (see Chapter 6 for details). For the most recent quarters, the quarterly Fisher indices can be used to extrapolate the benchmarked quarterly indices.

Calculation of annual and quarterly Fisher indices is given in Examples 8.3 and 8.4.

Chain-Linking in the QNA

General

47 The *2008 SNA* recommends moving away from the traditional fixed-base year constant price estimates to chain-linked volume measures. Constant price estimates use the average prices of a particular year (the base period) to weight together the corresponding quantities. Constant price data have the advantage for the users of the component series being additive, unlike alternative volume measures. The pattern of relative prices in the base year, however, is less representative of economic conditions for periods farther away from the base year. Therefore, from time to time, it is necessary to update the base period to adopt weights that better reflect the current conditions (i.e., with respect to production technology and user preferences). Different base periods, and thus different sets of price weights, give different perspectives. When the base period is changed, data for the distant past should not be recalculated (rebased). Instead, to form a consistent time series, data on the old base should be linked to data on the new base.[22] Change of base period and chain-linking can be done with different frequencies: every ten years, every five years, every year, or every quarter/month. The *2008 SNA* recommends changing the base period, and thus conducting the chain-linking, annually.

48 The concepts of base, weight, and reference period should be distinguished clearly. In particular, the term "base period" is sometimes used for different concepts. Similarly, the terms "base period," "weight period," and "reference period" are sometimes used interchangeably. In this manual, following the *2008*

[22] This should be done for each series, aggregates as well as subcomponents of the aggregates, independently of any aggregation or accounting relationship between the series. As a consequence, the chain-linked components will not aggregate to the corresponding aggregates. No attempts should be made to remove this "chain discrepancy," because any such attempt implies distorting the movements in one or several of the series.

Example 8.3 Deriving Annual Volume Measures Using Fisher Formula

	Current Prices			Elementary Price Indices (Previous Year = 100)		Elementary Level Deflation		Elementary Volume Indices (Previous Year = 100)		Laspeyres Volume Index (Previous Year = 100)	Paasche Volume Index (Previous Year = 100)	Fisher Volume Index (Previous Year = 100)
	(1)			(2)		(3) = (1)/(2) × 100		(4)		(5)	(6)	(7)
Year	A	B	Total	A	B	A	B	A	B	Total	Total	Total
2010	600.0	900.0	1,500.0							100.00	100.00	100.00
2011	660.0	854.9	1,514.9	102.63	98.50	643.1	867.9	107.18	96.43	100.73	100.84	100.79
2012	759.0	769.5	1,528.5	101.72	98.34	746.2	782.5	113.05	91.53	100.91	101.09	101.00
2013	948.8	615.6	1,564.4	99.34	101.08	955.1	609.0	125.83	79.14	102.33	102.13	102.23

(Rounding errors in the table may occur.)

This example shows the calculation of Fisher indices with annual data. The elementary volume indices in column 4 are aggregated using the Laspeyres and Paasche formulas in columns 5 and 6. The annual Laspeyres indices are the same calculated in Example 8.2. The Paasche indices are calculated as follows:

2011: 1/[(1/107.18) × (660/1,514.9) + (1/96.43) × (854.9/1,514.9)] = 100.84
2012: 1/[(1/113.05) × (759/1,528.6) + (1/91.53) × (769.6/1,528.6)] = 101.09
2013: 1/[(1/125.83) × (948.8/1,564.4) + (1/79.14) × (615.6/1,564.4)] = 102.13,

which is a harmonic average of quantity indices with weights from the current year. The Fisher indices are derived as geometric average of the Laspeyres and Paasche indices in each year:

2011: $\sqrt{100.73 \cdot 100.84} = 100.79$

2012: $\sqrt{100.91 \cdot 101.09} = 101.00$

2013: $\sqrt{102.33 \cdot 102.13} = 102.23$ ·

SNA and the current dominant national accounts practice, the following terminology is used:

- *Base period* for (i) the base of the price or quantity ratios being weighted together (e.g., period 0 is the base for the quantity ratio q_j^t / q_j^0) and (ii) the pricing year (the base year) for the constant price data.
- *Weight period* for the period(s) from which the weights are taken. The weight period is equal to the base period for a Laspeyres index and to the current period for a Paasche index. Symmetric index formulas like Fisher and Tornquist have two weight periods—the base period and the current period.
- *Reference period* for the period for which the index series is expressed as equal to 100. The reference period can be changed by simply dividing the index series with its level in any period chosen as the new reference period.

49 Chain-linking means constructing long-run price or volume measures by cumulating movements in short-term indices with different base periods. For example, a period-to-period chain-linked index measuring the changes from period 0 to t (i.e., $CI^{0\to t}$) can be constructed by multiplying a series of short-term indices measuring the change from one period to the next as follows:

$$CI^{0\to n} = I^{0\to 1} \cdot I^{1\to 2} \cdot \ldots \cdot I^{t-1\to t} \cdot \ldots \cdot I^{n-1\to n} = \prod_{t=1}^{n} I^{t-1\to t}, \tag{17}$$

where $I^{t-1\to t}$represents a price or volume index measuring the change from period $t-1$ to t, with period $t-1$ as base and reference period.

50 The corresponding run, or time series, of chain-linked index numbers where the links are chained

Example 8.4 Deriving Quarterly Volume Measures Using Fisher Formula

Quarter	Current Prices			Elementary Price Indices (Previous Quarter = 100)		Elementary Level Deflation		Elementary Volume Indices (Previous Quarter = 100)		Laspeyres Volume Index (Previous Quarter = 100)	Paasche Volume Index (Previous Quarter = 100)	Fisher Volume Index (Previous Quarter = 100)
	(1)			(2)		(3) = (1)/(2) × 100		(4)		(5)	(6)	(7)
	A	B	Total	A	B	A	B	A	B	Total	Total	Total
2010	150.0	225.0	375.0							100.00	100.00	100.00
q1 2011	159.7	218.9	378.6	102.00	99.00	156.6	221.1	104.38	98.27	100.71	100.76	100.74
q2 2011	163.2	213.7	376.9	100.49	98.99	162.4	215.9	101.69	98.62	99.92	99.93	99.92
q3 2011	167.4	210.6	378.0	100.49	100.00	166.6	210.6	102.08	98.55	100.08	100.08	100.08
q4 2011	169.7	211.7	381.4	100.00	101.02	169.7	209.6	101.37	99.51	100.33	100.33	100.33
q1 2012	174.2	204.1	378.3	102.13	96.51	170.6	211.5	100.51	99.90	100.17	100.18	100.18
q2 2012	180.4	201.4	381.8	99.51	102.06	181.3	197.3	104.07	96.68	100.08	100.04	100.06
q3 2012	188.9	192.3	381.2	99.02	99.49	190.8	193.3	105.75	95.97	100.59	100.58	100.58
q4 2012	215.5	171.7	387.2	100.50	100.51	214.4	170.8	113.52	88.84	101.07	101.07	101.07
q1 2013	224.7	166.0	390.7	100.75	99.37	223.0	167.1	103.50	97.29	100.75	100.77	100.76
q2 2013	235.8	156.3	392.1	99.00	101.00	238.2	154.8	105.99	93.22	100.57	100.51	100.54
q3 2013	242.9	148.5	391.4	99.50	100.50	244.1	147.8	103.53	94.54	99.95	99.93	99.94
q4 2013	245.4	144.8	390.2	99.49	100.49	246.6	144.1	101.54	97.03	99.83	99.82	99.83

(Rounding errors in the table may occur.)

Quarterly Fisher indices are calculated in this example. They are derived as aggregation of quarter-to-quarter elementary volume indices using quarterly weights from the previous quarter and the current quarter. Quarter-to-quarter elementary price indices are shown in column 2. These indices are consistent with the elementary price indices from the previous year used for the annually weighted Laspeyres-type indices calculated in Example 8.2 (the q1 2011 link is compared with the average level of 2010). The elementary volume indices from the previous quarter are derived in column 4.

As for the annual Fisher indices derived in Example 8.3, the first step is to derive quarterly Laspeyres volume indices and quarterly Paasche volume indices. Taking 2011 as an example, the Laspeyres volume indices are calculated as follows:

q1 2011: [104.38 × (150/375) + 98.27 × (225.0/375)] = 100.71
q2 2011: [101.69 × (159.7/378.6) + 98.62 × (218.9/378.6)] = 99.92
q3 2011: [102.08 × (163.2/376.9) + 98.55 × (213.7/376.9)] = 100.08
q4 2011: [101.37 × (167.4/378.0) + 99.51 × (210.6/378.0)] = 100.33.

Note that these indices are different from the annually weighted Laspeyres-type indices derived in Example 8.2, which use weights from the previous year. The Paasche volume indices for 2011 are derived using equation (15):

q1 2011: 1/[(1/104.37) × (159.7/378.6) + (1/98.27) × (218.9/378.6)] = 100.76
q2 2011: 1/[(1/101.69) × (163.2/376.9) + (1/98.62) × (213.7/376.9)] = 99.93
q3 2011: 1/[(1/102.08) × (167.4/378.0) + (1/98.55) × (210.6/378.0)] = 100.08
q4 2011: 1/[(1/101.37) × (169.7/381.4) + (1/99.51) × (211.7/381.4)] = 100.33.

As evident, the spread between the Laspeyres and Paasche aggregations is very small because relative shares moves slowly between one quarter and the next. The quarterly Fisher indices for 2011 are derived as follows:

q1 2011: $\sqrt{100.71 \cdot 100.76} = 100.74$

q2 2011: $\sqrt{99.92 \cdot 99.93} = 99.92$

q3 2011: $\sqrt{100.08 \cdot 100.08} = 100.08$

q4 2011: $\sqrt{100.33 \cdot 100.33} = 100.33$.

Annual and quarterly Fisher indices derived in Examples 8.3 and 8.4 are not directly comparable until they are chain-linked. See Example 8.8 for their comparison.

together so as to express the full time series on a fixed reference period is given by

$$\begin{cases} CI^{0\to 0} = 1 \\ CI^{0\to 1} = I^{0\to 1} \\ CI^{0\to 2} = I^{0\to 1}\cdot I^{1\to 2} \\ CI^{0\to 3} = I^{0\to 1}\cdot I^{1\to 2}\cdot I^{2\to 3} \\ \\ CI^{0\to n} = \prod_{t=1}^{n} I^{t-1\to t}. \end{cases} \quad (18)$$

51 Chain-linked indices do not have a particular base or weight period. Each link $I^{t-1\to t}$ of the chain-linked index in equation (18) has a base period and one or two weight periods, and the base and weight periods are changing from link to link. By the same token, the full run of index numbers in equation (18) derived by chaining each link together does not have a particular base period—it has a fixed reference period.

52 The reference period can be chosen freely without altering the rates of change in the series. For the chain-linked index time series in equation (18), period 0 is referred to as the index's reference period and is conventionally expressed as equal to 100. The reference period can be changed simply by dividing the index series with its level in any period chosen as a new reference period. For instance, the reference period for the run of index numbers in equation (18) can be changed from period 0 to period 2 by dividing all elements of the run by $CI^{0\to 2}$ as follows:

$$\begin{cases} CI^{2\to 0} = CI^{0\to 1}/CI^{0\to 2} = 1/I^{0\to 1}I^{1\to 2} \\ CI^{2\to 1} = CI^{0\to 1}/CI^{0\to 2} = 1/I^{1\to 2} \\ CI^{2\to 2} = CI^{0\to 2}/CI^{0\to 2} = 1 \\ CI^{2\to 3} = CI^{0\to 3}/CI^{0\to 2} = I^{2\to 3} \\ \\ CI^{2\to n} = CI^{0\to t}/CI^{0\to 2} = \prod_{t=3}^{n} I^{t-1\to t}. \end{cases} \quad (19)$$

53 The chain-linked index series in equation (17) and equations (18) and (19) will constitute a period-to-period chain-linked Laspeyres volume index series if, for each link, the short-term indices $I^{t-1\to t}$ are constructed as Laspeyres volume indices with the previous period as base and reference period: that is, if

$$LQ^{t-1\to t} = \sum_i \frac{q_i^t}{q_i^{t-1}}\cdot w_i^{t-1} = \frac{\sum_i p_i^{t-1}\cdot q_i^t}{\sum_i p_i^{t-1}\cdot q_i^{t-1}} = \frac{\sum_i p_i^{t-1}\cdot q_i^t}{C^{t-1}}, \quad (20)$$

where

$LQ^{t-1\to t}$ represents a Laspeyres volume index measuring the volume change from period t –1 to t, with period t – 1 as base and reference period;

p_i^{t-1} is the price of transaction i in period t–1 (the "price weights");

q_i^t is the quantity of transaction i in period t;

w_i^{t-1} is the base period "share weight": that is, the transaction's share in the total value of period t – 1; and

C^{t-1} is the total value at current prices in period t – 1.

54 Similarly, the chain-linked index series in equation (17) will constitute a period-to-period chain-linked Fisher volume index series if, for each link, the short-term indices $I^{t-1\to t}$ are constructed as Fisher volume indices with the previous period as base and reference period as in equation (16).

55 Any two index series with different base and reference periods can be linked to measure the change from the first year to the last year as follows:

$$CI^{0\to t} = I^{0\to t-h}\cdot I^{t-h\to t}. \quad (21)$$

That is, each link may cover any number of periods. For instance, if in equation (21) t = 10 and h = 5, the resulting linked index ($CI^{0\to 10}$) constitutes a five-year chain-linked annual index measuring the change from year 0 to year 10.

56 Growth rates and index numbers computed for series that can take positive, negative, and zero values—such as changes in inventories and crop harvest data—generally are misleading and meaningless. For instance, consider a series for changes in inventories that is –10 in period one and +20 in period two at the average prices of period one. The corresponding volume growth rate between these two periods is –300 percent (= [(20/–10) – 1] • 100), which obviously is both misleading and meaningless. As a consequence, chain volume measures cannot be calculated for these series. The preferred solution to analyze price and volume effects for such series is to calculate their contribution to percent change, as explained later in this section.

57 As an alternative, the *2008 SNA* provides a solution to calculate pseudo chain volume series from variables that change sign[23]:

a. identify two associated time series that take only positive values and are such that the difference yield the target series,
b. apply chain-linking to the two series separately, and
c. derive the chain volume series as a difference.

58 The chain volume series is called pseudo chain because it is derived as the difference of two chained components, which are not additive by construction. Possible examples are a chain volume series of changes in inventories as a chain volume series of closing inventories less a chain volume series of opening inventories, or a chain volume series of external trade balance as a difference between chain volume series of exports and imports.

Frequency of Chain-Linking

59 The *2008 SNA* recommends that chain-linking should not be done more frequently than annually. This is mainly because short-term volatility in relative prices (e.g., caused by sampling errors and seasonal effects) can cause volume measures that are chain-linked more frequently than annually to show substantial drift—particularly so for nonsuperlative index formulas like Laspeyres and Paasche. Similarly, short-term volatility in relative quantities can cause price measures that are chain-linked more frequently than annually to show substantial drift. The purpose of chain-linking is to take into account long-term trends in changes in relative prices, not temporary short-term variations.

60 Superlative index formulas, such as the Fisher index formula, are more robust against the drift problem than the other index formulas—as illustrated in Example 8.5. For this reason, a quarterly chain-linked Fisher index may be a feasible alternative to annually chain-linked Laspeyres indices for quarterly data that show little or no short-term volatility. The quarterly chain-linked Fisher index does not aggregate exactly to the corresponding direct annual Fisher index.[24] For chain-linked Fisher indices, consistency between QNA and ANA price and volume measures can only be achieved by deriving the ANA measures from the quarterly measures or by forcing consistency on the data with the help of benchmarking techniques. There is no reason to believe that for nonvolatile series the average of an annually chain-linked Fisher will be closer to a direct annual Fisher index than the average of a quarterly chain-linked Fisher.

61 When quarterly weights are preferred, chain-linking should only be applied to Fisher-type indices. Because seasonally adjusted data are less subject to volatility in relative prices and volumes than unadjusted data, quarterly chain Fisher indices of seasonally adjusted data can be expected to produce satisfactory results in most circumstances. On the other hand, quarterly Fisher indices of unadjusted data should always be benchmarked to corresponding annual Fisher indices to avoid possible drifts.

62 For Laspeyres-type volume measures, consistency between QNA and ANA provides an additional reason for not chain-linking more frequently than annually. Consistency between quarterly data and corresponding direct annual indices requires that the same price weights are used in the ANA and the QNA, and consequently that the QNA should follow the same change of base year/chain-linking practice as in the ANA. Under those circumstances, the AO linking technique presented in the next section ensures that the quarterly data aggregate exactly to the corresponding direct index. Moreover, under the same circumstances, any difference between the average of the quarterly data and the direct annual index caused by the QO technique can be resolved through benchmarking.

63 Thus, when the Laspeyres formula is used in the ANA, chain-linked Laspeyres-type quarterly volume measures can be derived consistently by compiling quarterly estimates at the average prices of the

[23] See *2008 SNA* (paragraph 15.62).

[24] Neither does the annually linked, nor the fixed-based, Fisher index.

Example 8.5 Frequency of Chain-Linking and the Problem of "Drift" in the Case of Price and Quantity Oscillation

Observation/Quarter	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Price item A	2	3	4	2
Price item B	5	4	2	5
Quantities item A	50	40	60	50
Quantities item B	60	70	30	60
Total value	400	400	300	400
Volume Indices	**q1**	**q2**	**q3**	**q4**
Fixed-based Laspeyres (q1-based)	100.0	107.5	67.5	100.0
Fixed-based Paasche (q1-based)	100.0	102.6	93.8	100.0
Fixed-based Fisher (q1-based)	100.0	105.0	79.5	100.0
Quarterly chain-linked Laspeyres	100.0	107.5	80.6	86.0
Quarterly chain-linked Paasche	100.0	102.6	102.6	151.9
Quarterly chain-linked Fisher	100.0	105.0	90.9	114.3

(Rounding errors in the table may occur.)

Fixed-Based Laspeyres Index:

$$ {}_{I}{}_{L}^{q1\to t} = \frac{\sum_j p_j^{q1} q_j^{t}}{\sum_j p_j^{q1} q_j^{q1}} \equiv \frac{\sum_j p_j^{q1} q_j^{t}}{c^{q1}} $$

$$ {}_{I}{}_{L}^{q1\to q2} = [(2\times 40+5\times 70)/400]\times 100 = 107.5 $$
$$ {}_{I}{}_{L}^{q1\to q3} = [(2\times 60+5\times 30)/400]\times 100 = 67.5 $$
$$ {}_{I}{}_{L}^{q1\to q4} = [(2\times 50+5\times 60)/400]\times 100 = 100.0 $$

Fixed-Based Paasche Index:

$$ {}_{I}{}_{P}^{q1\to t} = \frac{\sum_j p_j^{t} q_j^{t}}{\sum_j p_j^{t} q_j^{q1}} \equiv \frac{c^{t}}{\sum_j p_j^{t} q_j^{q1}} $$

$$ {}_{I}{}_{P}^{q1\to q2} = [400/(3\times 50+4\times 60)] \times 100 = 102.6 $$
$$ {}_{I}{}_{P}^{q1\to q3} = [300/(4\times 50+2\times 60)] \times 100 = 93.8 $$
$$ {}_{I}{}_{P}^{q1\to q4} = [400/(2\times 50+5\times 60)] \times 100 = 100.0 $$

Quarterly Chain-Linked Laspeyres Index:

$$ {}_{CL}{}_{L}^{q1\to t} = \prod_{\tau=1}^{t} {}_{I}{}_{L}^{(\tau-1)\to\tau} = \prod_{\tau=1}^{t} \frac{\sum_j p_j^{\tau-1} q_j^{\tau}}{\sum_j p_j^{\tau-1} q_j^{\tau-1}} $$

$$ {}_{CL}{}_{L}^{q1\to q2} = {}_{I}{}_{L}^{q1\to q2} = 107.5 $$
$$ {}_{CL}{}_{L}^{q1\to q3} = {}_{CL}{}_{L}^{q1\to q2} \cdot [(3\times 60+4\times 30)/400] = 80.6 $$
$$ {}_{CL}{}_{L}^{q1\to q4} = {}_{CL}{}_{L}^{q1\to q3} \cdot [(4\times 50+2\times 60)/300] = 86.0 $$

Quarterly Chain-Linked Paasche Index:

$$ {}_{CL}{}_{P}^{q1\to t} = \prod_{\tau=1}^{t} {}_{I}{}_{P}^{(\tau-1)\to\tau} = \prod_{\tau=1}^{t} \frac{\sum_j p_j^{\tau} q_j^{\tau}}{\sum_j p_j^{\tau} q_j^{\tau-1}} $$

$$ {}_{CL}{}_{P}^{q1\to q2} = {}_{I}{}_{P}^{q1\to q2} = 102.6 $$
$$ {}_{CL}{}_{P}^{q1\to q3} = {}_{CL}{}_{P}^{q1\to q2} \cdot [300/(4\times 40+2\times 70)] = 102.6 $$
$$ {}_{CL}{}_{P}^{q1\to q4} = {}_{CL}{}_{P}^{q1\to q3} \cdot [400/(2\times 60+5\times 30)] = 151.9 $$

In this example, the prices and quantities in quarter 4 are the same as those in quarter 1: that is, the prices and quantities oscillate rather than move as a trend. The fixed-base indices correspondingly show identical values for q1 and q4, but the chain-linked indices show completely different values. This problem can also occur in annual data if prices and quantities oscillate and may make annual chaining inappropriate in some cases. It is more likely to occur in data for shorter periods, however, because seasonal and irregular effects cause those data to be more volatile.

Furthermore, observe that the differences between the q1 and q4 data for the quarterly chain-linked Laspeyres and the quarterly chain-linked Paasche indices are in opposite directions; and, correspondingly, that the quarterly chain-linked Fisher index drifts less. This is a universal result. This example is based on Szultc (1983).

previous year. These quarterly volume measures for each year should then be linked to form long, consistent time series—the result constitutes an annually chained quarterly Laspeyres index. Alternative linking techniques for such series are discussed section "Chain-Linking Techniques for Quarterly Data."

64 When relative prices are subject to large swings, the quality of chain-linking deteriorates. This may happen due to the effects of oil shocks or in high-inflation situations. In such cases, updating the weight period every year may also produce drift effects like the one described in Example 8.5, and produce inaccurate volume estimates. In such cases, constant price data based on a regular update of the base year (e.g., every five years) are preferable over chain-linking.

Choice of Index Number Formulas for Chain-Linking

65 The *2008 SNA* recommends compiling annually chain-linked price and volume measures, preferably using superlative index number formulas such as the Fisher and Tornquist formulas. The rationale for this recommendation is that index number theory shows that annually chain-linked Fisher and Tornquist indices will most closely approximate the theoretically ideal index. Fisher and Tornquist indices will, in practice, yield almost the same results, and Fisher—being the geometric average of a Laspeyres index and a Paasche index—will be within the upper and lower bounds provided by those two index formulas. Most countries that have implemented chain-linking in their national accounts, however, have adopted the annually chain-linked Laspeyres formula for volume measures.[25]

66 Annual chain-linking of quarterly data implies that each link in the chain is constructed using the chosen index number formula with the average of the previous year $(y-1)$ as base and reference period. The resulting short-term quarterly indices must subsequently be linked to form long, consistent time series expressed on a fixed reference period. Alternative annual linking techniques for such series are discussed in section "Chain-Linking Techniques for Quarterly Data." The annually weighted Laspeyres-type quarterly volume index formula for each short-term link is given in equation (12). While the discussion here focuses on Laspeyres indices, the techniques illustrated and the issues discussed are applicable to all annually chain-linked index formulas.

67 Countries have opted for an annually chained Laspeyres formula instead of an annually chained Fisher formula[26] for volume measures mainly for several practical reasons:

a. Experience and theoretical studies indicate that annual chain-linking tends to reduce index number spread to the degree that the exact choice of index number formula assumes less significance (see, e.g., *2008 SNA*, paragraph 15.41).

b. The Laspeyres formula is simpler to work with and to explain to users than the Fisher index. For instance, time series of annually chained Laspeyres indices can be converted easily into series of data valued at the average prices of the previous year that are additive if corresponding current price data are made available. This feature makes it easy for users to construct their own aggregates from published data.

c. The annually chained quarterly Fisher index does not aggregate to the corresponding direct annual index.[27] The annually chained Laspeyres index, linked using the AO technique discussed in the next subsection, does.[28]

d. The Fisher formula is not consistent in aggregation within each link; it is only approximately

[25] Currently, only the United States and Canada have opted for a chain-linked Fisher index. The United States adopted an annually chain-linked quarterly Fisher-type formula in 1996, using annual weights in both the Laspeyres and the Paasche part of the index. In 1999, the United States moved to a standard quarterly chain Fisher index that is benchmarked to the corresponding annual Fisher index. In 2001, Canada implemented a quarterly chain Fisher volume index as the official volume measure of the expenditure-based gross domestic product (see Chevalier, 2003).

[26] For example, the European Union's statistical office (Eurostat) requires member states to provide annually chain-linked volume measures using the Laspeyres formula.

[27] Neither does the quarterly chain linked, nor the fixed-based, quarterly Fisher index.

[28] However, this may not be a decisive argument for two reasons. First, simulations indicate that, in practice, the difference between a direct annual Fisher and the average of a quarterly Fisher may often not be significant and may easily be removed using benchmarking techniques (see Example 8.8). Second, the one-quarter overlap technique for Laspeyres indices also introduces differences between direct annual indices and the average of quarterly indices.

consistent in aggregation (i.e., the sum of volume estimates of two components in monetary terms is not equal to the volume estimate of their sum).

e. The formulas for computing contribution to percent change are easier for data based on the annually chained Laspeyres formula than for data based on the Fisher index (see section "Contributions to Percent Change from Chain-Linked Measures").

f. The Laspeyres formula, in contrast, is additive within each link (prior to chain-linking). This makes it easier to combine chain-linking with compilation analytical tools like SUT and input–output (IO) tables that require additivity of components.

g. Chain volume measures in monetary terms[29] based on the annually chained Laspeyres formula will be additive in the reference year and the subsequent year,[30] while volume measures based on the Fisher index will not.

68 When the Fisher formula is chosen, true Fisher indices should be calculated in both ANA and QNA, and the quarterly indices should be benchmarked to the annual indices. By constraining the quarterly indices to the annual ones, the benchmarking process makes sure that the Fisher-based QNA volume measures are free from possible drifts generated by seasonality or short-term volatility in the quarterly data.

Chain-Linking Techniques for Quarterly Data

69 Two alternative techniques for chain-linking of annually weighted quarterly data are usually applied: the annual overlap (AO) technique and the one-quarter overlap (QO) technique. While standard price statistics compilation exclusively uses the QO technique, the AO technique may be more practical for Laspeyres-type volume measures in the national accounts because it results in data that aggregate exactly to the corresponding direct annual index. In contrast, the QO technique does not result in data that aggregate exactly to the corresponding direct annual index. The QO technique, however, provides the smoothest transition between each link, while the AO technique may introduce a step between each link. The two linking techniques are presented below.[31]

70 In addition to these two conventional chain-linking techniques, a third technique sometimes is used based on changes from the same period in the previous year (the "over-the-year" technique). The over-the-year technique corresponds to the QO technique applied to each individual quarter of the year. In situations with strong changes in relative quantities and relative prices, the over-the-year technique can result in distorted seasonal patterns in the linked series. For this reason, the over-the-year technique should be avoided in the QNA.

The Annual Overlap Technique

71 The AO technique implies compiling estimates for each quarter at the weighted annual average prices of the previous year. The annual data at previous year's prices provide the linking factors to scale the quarterly data upward or downward. The AO technique requires quarterly volume measures at previous year's prices and annual current price data. It consists of the following three steps:

Step 1: Calculate quarterly volume indices from the previous year

Quarterly volume indices for a given quarter are derived as relative change between the volume estimate at previous year's prices for the quarter and the (rescaled) current price data in the previous year. In mathematical terms,

$$q^{y-1\to(s,y)} = \frac{k^{y-1\to(s,y)}}{C^{y-1}/4} \text{ for } y = 2,3,\ldots \text{ and } s = 1,\ldots,4, \tag{22}$$

where

$k^{y-1\to(s,y)}$ is the volume measure in quarter s of year y at the prices of the previous year and C^{y-1} is the current price data for year $y-1$.

[29] See discussion in section "Presentation of Chain-Linked Measures" on presenting chain volume measures in monetary terms.

[30] See Example 8.4 for an illustration of the nonadditivity property of most index number formulas besides the fixed-based Laspeyres formula.

[31] Annex 6.1 compares the annual overlap (AO) and one-quarter overlap techniques formally and provides an interpretation of the possible step in the AO technique.

Step 2: Link the quarterly volume indices using annual overlaps

The quarterly chain indices $q^{1\rightarrow(s,y)}$ are derived using the recursion

$$q^{1\rightarrow(s,y)} = Q^{1\rightarrow 2} \cdot Q^{2\rightarrow 3} \cdot \ldots \cdot Q^{t-1\rightarrow t} \cdot \ldots \cdot Q^{y-2\rightarrow y-1} \cdot q^{y-1\rightarrow(s,y)} \cdot 100\,, \qquad (23)$$

where

$$Q^{t-1\rightarrow t} = \frac{K^{t-1\rightarrow t}}{C^{t-1}} \qquad (24)$$

are the annual links (i.e., the annual growth rates), with

$K^{t-1\rightarrow t}$ being the volume measure of year t at the prices of year $t-1$ and

C^{y-1} is the current price data for year $y-1$.

Step 3: Re-reference the quarterly chain series to a chosen year

By construction, the reference year of the quarterly chain indices $q^{1\rightarrow(s,y)}$ is year 1. It is possible to re-reference the chain series to any other year, denoted by r, by dividing the chain series with the corresponding annual chain index: that is,

$$q^{r\rightarrow(s,y)} = \frac{q^{1\rightarrow(s,y)}}{Q^{1\rightarrow r}} \cdot 100 \text{ for } y = 2, 3, \ldots, \quad s = 1, \ldots, 4, \text{ and } 1 \leq r \leq y\,, \qquad (25)$$

where

$Q^{1\rightarrow r} = Q^{1\rightarrow 2} \cdot Q^{2\rightarrow 3} \cdot \ldots \cdot Q^{r-1\rightarrow r}$ is the annual chain index for year r.

The chain indices $q^{r\rightarrow(s,y)}$ can be expressed in monetary terms by multiplying the entire series by the (rescaled) annual current price data of the reference year.

Example 8.6 provides an illustration of the AO technique.

The One-Quarter Overlap Technique

72 The QO technique requires compiling estimates for the fourth quarter of each year (e.g., the overlap quarter) at the weighted annual average prices of the current year in addition to estimates at the average prices of the same year. The ratio between the estimates for the fourth quarter at the average prices of the previous year and at the average prices of the current year provides the linking factor to scale the quarterly data up or down. Similar to the AO technique, the QO technique is calculated in three steps:

Step 1: Calculate quarterly volume indices from the fourth quarter of the previous year

Quarterly volume indices for a given quarter are derived as relative change between the volume estimate at previous year's prices of that quarter and the estimate of the fourth quarter in the previous year at the average prices of the same year. In mathematical terms,

$$q^{(4,y-1)\rightarrow(s,y)} = \frac{k^{y-1\rightarrow(s,y)}}{cy^{(4,y-1)}} \text{ for } y = 3, 4, \ldots, \quad s = 1, \ldots, 4, \qquad (26)$$

with

$$cy^{(4,y-1)} = \sum_j P_j^{y-1} q_j^{(4,y-1)}$$

aggregating the quantities of the fourth quarter of year $y-1$ using the average prices of the whole year $y-1$, which differs from the current price data $c^{(4,y-1)}$ where the quarterly quantities are valued at the prices of the fourth quarter.[32]

Step 2: Link the quarterly volume indices using quarterly overlaps

The quarterly chain indices $q^{1\rightarrow(s,y)}$ using the QO technique are derived using the recursion

$$q^{1\rightarrow(s,y)} = q^{1\rightarrow(4,2)} \cdot q^{(4,2)\rightarrow(4,3)} \cdot \ldots \cdot q^{(4,t-1)\rightarrow(4,t)} \cdot \ldots \cdot q^{(4,y-1)\rightarrow(s,y)} \cdot 100\,, \qquad (27)$$

[32] Usually, there is no information on the price and volume development for the first year of the series (i.e., volume estimates for year 1 at the prices of year 0 are unavailable). As a consequence, it is not possible to derive a quarterly link from the fourth quarter of year 1. By convention, the one-quarter overlap technique uses the same links used in the annual overlap approach for year 2 (see formula (22)).

Example 8.6 Chain-Linking Annually Weighted Laspeyres-Type Indices: Annual Overlap Technique

Year/ Quarter	Current Prices			Previous Year's Prices			Volume Measures (Previous Year = 100)			Chain-Indices with Annual Overlap (2010 =100)			Chain Volume Measures with Annual Overlap in Monetary Terms			
	(1)			(2)			Step 1			Step 2			Step 3			
	A	B	Sum	A	B	Sum	A	B	Sum	A	B	Sum	A	B	Sum	Discrepancies
2010	600.0	900.0	1,500.0							100.00	100.00	100.00	600.0	900.0	1,500.0	*0.0*
2011	660.0	854.9	1,514.9	643.1	867.9	1,511.0	107.18	96.43	100.73	107.18	96.43	100.73	643.1	867.9	1,511.0	*0.0*
2012	759.0	769.5	1,528.5	746.2	782.5	1,528.7	113.05	91.53	100.91	121.17	88.27	101.65	727.0	794.4	1,524.7	*–3.3*
2013	948.8	615.6	1,564.4	955.1	609.0	1,564.1	125.83	79.14	102.33	152.47	69.86	104.01	914.8	628.7	1,560.2	*–16.6*
q1 2011	159.7	218.9	378.6	156.6	221.1	377.7	104.38	98.27	100.71	104.38	98.27	100.71	156.6	221.1	377.7	*0.0*
q2 2011	163.2	213.7	376.9	159.2	218.1	377.3	106.15	96.92	100.61	106.15	96.92	100.61	159.2	218.1	377.3	*0.0*
q3 2011	167.4	210.6	378.0	162.5	214.9	377.4	108.35	95.51	100.65	108.35	95.51	100.65	162.5	214.9	377.4	*0.0*
q4 2011	169.7	211.7	381.4	164.8	213.8	378.6	109.84	95.04	100.96	109.84	95.04	100.96	164.8	213.8	378.6	*0.0*
q1 2012	174.2	204.1	378.3	170.0	210.4	380.4	103.00	98.45	100.43	110.39	94.94	101.17	165.6	213.6	379.4	*–0.2*
q2 2012	180.4	201.4	381.8	176.9	203.4	380.3	107.19	95.19	100.42	114.88	91.79	101.15	172.3	206.5	379.3	*–0.5*
q3 2012	188.9	192.3	381.2	187.0	195.2	382.3	113.35	91.35	100.93	121.49	88.09	101.67	182.2	198.2	381.3	*–0.8*
q4 2012	215.5	171.7	387.2	212.3	173.4	385.7	128.68	81.15	101.85	137.91	78.25	102.60	206.9	176.1	384.8	*–1.8*
q1 2013	224.7	166.0	390.7	223.6	166.0	389.6	117.83	86.29	101.95	142.77	76.17	103.63	214.2	171.4	388.6	*–3.1*
q2 2013	235.8	156.3	392.1	237.0	154.8	391.7	124.89	80.44	102.52	151.33	71.01	104.20	227.0	159.8	390.8	*–4.0*
q3 2013	242.9	148.5	391.4	245.4	146.3	391.7	129.30	76.05	102.49	156.68	67.13	104.18	235.0	151.0	390.7	*–4.6*
q4 2013	245.4	144.8	390.2	249.1	142.0	391.1	131.30	73.79	102.35	159.09	65.14	104.03	238.6	146.6	390.1	*–4.9*
Sum of Quarterly Values																
2011	660.0	854.9	1,514.9	643.1	867.9	1,511.0	107.18	96.43	100.73	107.18	96.43	100.73	643.1	867.9	1,511.0	*0.0*
2012	759.0	769.5	1,528.5	746.2	782.5	1,528.7	113.05	91.53	100.91	121.17	88.27	101.65	727.0	794.4	1,524.7	*–3.3*
2013	948.8	615.6	1,564.4	955.1	609.0	1,564.1	125.83	79.14	102.33	152.47	69.86	104.01	914.8	628.7	1,560.2	*–16.6*

(Rounding errors in the table may occur.)

This example shows how to calculate chain Laspeyres-type volume indices and chain Laspeyres-type volume measures expressed in monetary terms using the annual overlap technique. Calculations are done for annual and quarterly data separately. Columns 1 and 2 display the data at current prices and at the average prices of the previous year derived in Example 8.2. The annual overlap technique consists of three steps.

Step 1. Derive volume indices from the previous year

For each year and quarter, compile volume indices with the previous year as base period. These are the links of the chain volume series. They are obtained by dividing the estimate at previous year's prices (column 2) by the estimate at current prices in the previous year (column 1). For quarterly data, the current price data from the previous year is divided by 4 to rescale the value to be comparable on a quarterly basis. For the total,

2011:	(1,511.0/1,500.0) × 100 = 100.73	2012:	(1,528.7/1,514.9) × 100 = 100.91
…		…	
q1 2011:	[377.7/(1,500.0/4)] × 100 = 100.71	q1 2012:	[80.36/(1,514.9/4)] × 100 = 100.43
q2 2011:	[377.3/(1,500.0/4)] × 100 = 100.61	q2 2012:	[380.30/(1,514.9/4)] × 100 = 100.42
…		…	

Step 2. Chain-link volume indices with the annual overlap technique

The volume indices obtained at Step 1 are chain-linked using the annual overlap technique. Each volume index is multiplied by the average of the chain-linked index in the previous year. Note that quarterly data are linked through annual data, which is the distinctive feature of the annual overlap approach. For 2011, the chain volume indices remain the same (the previous year's index is 100). Calculations for 2012 and 2013 are as follows:

2012:	(100.91 × 100.73)/100 = 101.65	2013:	(102.33 × 101.65)/100 = 104.01
q1 2012:	(100.43 × 100.73)/100 = 101.17	q1 2013:	(101.95 × 101.65)/100 = 103.63
q2 2012:	(100.42 × 100.73)/100 = 101.15	q2 2013:	(102.52 × 101.65)/100 = 104.20
…			…

Step 3. Calculate the chain volume series in monetary terms

For annual data, the chain-linked indices are rescaled using the annual value at current prices in the reference year. For quarterly data, the annual value of the reference year is divided by 4. In this example, the reference year is 2010.

2011:	(100.73 × 1,500.0)/100 = 1,511.0
2012:	(101.65 × 1,500.0)/100 = 1,524.7
2013:	(104.01 × 1,500.0)/100 = 1,560.2
q1 2011:	[100.71 × (1,500.0/4)]/100 = 377.7
q2 2011:	[100.61 × (1,500.0/4)]/100 = 377.3
…	
q1 2012:	[101.17 × (1,500.0/4)]/100 = 379.4
q2 2012:	[101.15 × (1,500.0/4)]/100 = 379.3
…	
q1 2013:	[103.63 × (1,500.0/4)]/100 = 388.6
q2 2013:	[104.20 × (1,500.0/4)]/100 = 390.8
…	

Note that the sum of the quarterly chain volume data for each year is equal to the annual chain volume data. This property is only guaranteed by using annually weighted Laspeyres-type quarterly volume indices that are chained with the annual overlap technique. However, the sum of chain-linked data of transactions A and B does not match the chain-linked total data (except in the year following the reference period). Discrepancies are shown in the last column of the table. Chain-linked components never add up to chain-linked aggregates, as discussed in the section on lack of additivity.

where

$$q^{(4,t-1)\to(4,t)} = \frac{k^{(4,t)}}{cy^{(4,t-1)}} \tag{28}$$

are the quarterly links from the fourth quarter of consecutive years and

$q^{1\to(4,2)}$ is the quarterly link from the first year, as derived in equation (23).

Step 3: Re-reference the quarterly chain series to a chosen year

This step is equal to Step 3 presented above for the AO technique. For comparison with the annual data, the same reference year is usually chosen. Example 8.7 provides a numerical illustration of the QO technique.

73 The QO technique preserves better the time-series properties of the chain volume series. In using quarterly overlaps, it provides the smoothest transition between the fourth quarter of one year and the first quarter of the next year. However, when Laspeyres-type volume measures are implemented, compilers and users of QNA may prefer the use of the AO technique for several practical reasons:

a. The QO technique requires the calculation of quarterly data at the prices of the current year and at the prices of the previous year, while the AO technique requires only estimates at the prices of the previous year.

b. Estimates at the prices of the current year are usually not published, and therefore users are unable to replicate the calculation of chain volume measures using the QO technique or, more importantly, calculate chain-linked estimates of different aggregations.

c. To preserve consistency with the annual data, the QO technique requires an additional step of benchmarking. Benchmarking may also be necessary to remove a possible drift introduced by linking to the fourth quarter of each year. Furthermore, by using benchmarking, the original changes of q1–q3 derived from the QO technique are all adjusted to fit the given annual totals. The benchmarking step may affect the statistical properties of the chained series, with possible impact on the measurement of business-cycle peaks and troughs.

d. The AO technique may give similar results to the QO technique in many circumstances. It can be shown that the two techniques differ for an annual factor that depends on the difference between the quantity shares in the fourth quarter and the quantity shares of the whole year (see Annex 8.1). Relative quantity weights of macroeconomic aggregates tend to be stable within a year, especially when they are expressed in seasonally adjusted form.

e. Following a general principle of consistency of the system of national accounts, it is preferable to use the same methodology to derive annual and quarterly volume estimates. When Laspeyres-type indices are used in the national accounts, the AO technique for quarterly data is the only method for chain-linking annual data.

74 Quarterly Fisher indices should always be chain-linked using the QO technique. Differently from annually weighted Laspeyres indices, quarterly and annual Fisher indices are never consistent and there is no reason to adopt the AO approach for the sake of consistency. The quarterly chain Fisher indices should be benchmarked to annual chain Fisher indices to avoid possible drifts in the quarterly data, especially when the data include seasonal effects or short-term volatility. Example 8.8 provides a numerical illustration of benchmarking quarterly chain Fisher indices to annual ones.

75 To conclude, the QO technique with benchmarking to remove any discrepancies with the annual data provides the best results for chain-linking. However, when Laspeyres-type volume measures are implemented in both ANA and QNA (i.e., when a system of annual and quarterly volume estimates at previous year's prices is implemented), the AO technique can be used to obtain quarterly chain-linked data that are automatically consistent with their annual counterparts. Experimental tests (on a continuous basis) should be performed to verify that the AO technique does not introduce artificial steps between years in the chain-linked series.

76 On the other hand, quarterly Fisher indices are never automatically consistent with their annual counterparts and should always be linked with the QO technique to preserve the best quality time-series characteristics of such series. When consistency is required with the annual data, benchmarking should be used to remove any resulting discrepancies between quarterly and annual Fisher indices. Quarterly Fisher indices may contain nonnegligible drifts when the formula is applied to quarterly data containing seasonal effects and short-term volatility.

Example 8.7 Chain-Linking Annually Weighted Laspeyres-Type Indices: The One-Quarter Overlap Technique

Year/ Quarter	Current Year's Prices			Previous Year's Prices			Volume Measures (q4 Previous Year = 100)			Chain-Indices with One-Quarter Overlap (2010 = 100)			Chain Volume Measures with One-Quarter Overlap in Monetary Terms			
	(1)			(2)			Step 1			Step 2			Step 3			
	A	B	Sum	A	B	Sum	A	B	Sum	A	B	Sum	A	B	Sum	*Discrepancies*
2010	600.0	900.0	1,500.0													
2011	660.0	854.9	1,514.9	643.1	867.9	1,511.0										
2012	759.0	769.5	1,528.5	746.2	782.5	1,528.7										
2013	948.8	615.6	1,564.4	955.1	609.0	1,564.1										
q1 2011	160.7	217.8	378.5	156.6	221.1	377.7	104.38	98.27	100.71	104.38	98.27	100.71	156.6	221.1	377.7	*0.0*
q2 2011	163.4	214.8	378.2	159.2	218.1	377.3	106.15	96.92	100.61	106.15	96.92	100.61	159.2	218.1	377.3	*0.0*
q3 2011	166.8	211.7	378.5	162.5	214.9	377.4	108.35	95.51	100.65	108.35	95.51	100.65	162.5	214.9	377.4	*0.0*
q4 2011	169.1	210.6	379.7	164.8	213.8	378.6	109.84	95.04	100.96	109.84	95.04	100.96	164.8	213.8	378.6	*0.0*
q1 2012	172.9	206.9	379.8	170.0	210.4	380.4	100.51	99.90	100.17	110.39	94.94	101.13	165.6	213.6	379.2	*0.0*
q2 2012	179.9	200.1	380.0	176.9	203.4	380.3	104.59	96.58	100.15	114.88	91.79	101.11	172.3	206.5	379.2	*–0.3*
q3 2012	190.2	192.0	382.2	187.0	195.2	382.3	110.61	92.69	100.67	121.49	88.09	101.63	182.2	198.2	381.1	*–0.7*
q4 2012	216.0	170.6	386.5	212.3	173.4	385.7	125.56	82.34	101.59	137.91	78.25	102.56	206.9	176.1	384.6	*–1.7*
q1 2013	222.1	167.8	389.9	223.6	166.0	389.6	103.53	97.33	100.79	142.77	76.17	103.37	214.2	171.4	387.6	*–2.1*
q2 2013	235.4	156.4	391.9	237.0	154.8	391.7	109.73	90.74	101.35	151.33	71.01	103.94	227.0	159.8	389.8	*–3.0*
q3 2013	243.7	147.9	391.6	245.4	146.3	391.7	113.61	85.78	101.33	156.68	67.13	103.92	235.0	151.0	389.7	*–3.7*
q4 2013	247.5	143.5	391.0	249.1	142.0	391.1	115.36	83.24	101.18	159.09	65.14	103.77	238.6	146.6	389.2	*–4.0*
Sum of Quarterly Values																
2011	660.0	854.9	1,514.9	643.1	867.9	1,511.0	107.18	96.43	100.73	107.18	96.43	100.73	643.1	867.9	1,511.0	*0.0*
2012	759.0	769.5	1,528.5	746.2	782.5	1,528.7	110.32	92.88	100.64	121.17	88.27	101.61	727.0	794.4	1,524.1	*–2.7*
2013	948.8	615.6	1,564.4	955.1	609.0	1,564.1	110.55	89.27	101.16	152.47	69.86	103.75	914.8	628.7	1,556.3	*–12.7*

(Rounding errors in the table may occur.)

This example shows how to calculate chain Laspeyres-type volume indices and chain Laspeyres-type volume measures expressed in monetary terms using the one-quarter overlap technique. Because the method uses quarterly links, this method applies only to quarterly data. Column 1 displays the data at the average prices of the same year. At the annual level, they are equal to the current price data. Quarterly data are different because quantities are valued at the average prices of the whole year and not at the prices of each quarter. Column 2 shows the estimates at previous year's prices shown in Example 8.6.

Step 1. Derive volume indices from the fourth quarter in the previous year

For each quarter, compile volume indices with the fourth quarter in the previous year as base period. These are the links of the chain volume series. They are obtained dividing the estimate at previous year's prices (column 2) by the estimate of the fourth quarter in the previous year valued at the average prices of the previous year (column 1). Because there is no quarterly data for 2010, the 2011 link is derived using the annual overlap technique as in the previous example. Instead, for 2012 and 2013, the volume indices for the total are calculated as follows:

q1 2012:	$(380.4/379.7) \times 100 = 100.17$	q1 2013:	$(389.6/386.5) \times 100 = 100.79$
q2 2012:	$(380.3/379.7) \times 100 = 100.15$	q2 2013:	$(391.7/386.5) \times 100 = 101.35$
…		…	

Step 2. Chain-link volume indices with the one-quarter overlap technique

The volume indices obtained at Step 1 are chain-linked using the one-quarter overlap technique. Each volume index is multiplied by the chain-linked index of the fourth quarter in the previous year. Differently from the annual overlap approach, the link is given by the fourth quarter of each year (and not the whole year). Calculations for 2012 and 2013 are as follows:

q1 2012:	(100.17 × 100.96)/100 = 101.13	q1 2013:	(100.79 × 102.56)/100 = 103.37
q2 2012:	(100.15 × 100.96)/100 = 101.11	q1 2013:	(101.35 × 102.56)/100 = 103.94
…			

Step 3. Calculate the chain volume series in monetary terms

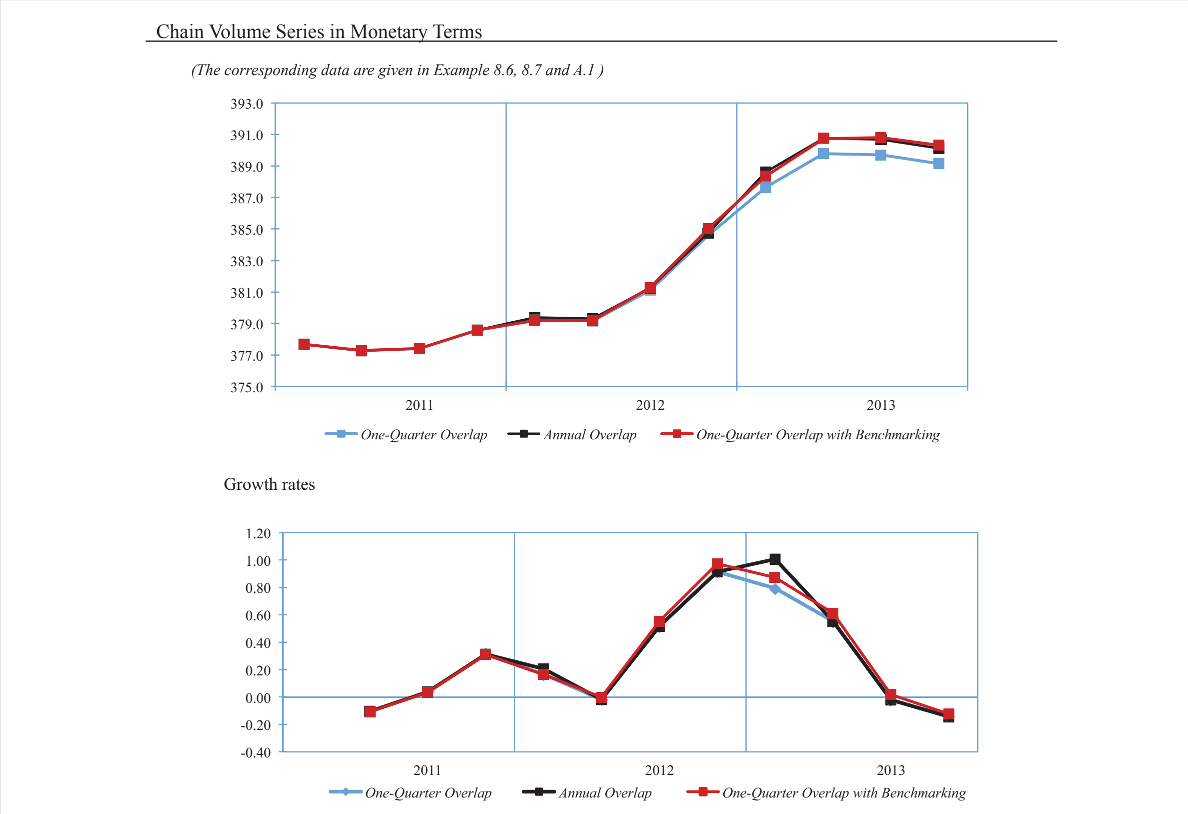
For consistency with the annual overlap approach, the quarterly chain indices are rescaled using the annual value at current prices in 2010 (i.e., the reference year is 2010).

q1 2011:	[100.71 × (1,500.0/4)]/100 = 377.7
q2 2011:	[100.61 × (1,500.0/4)]/100 = 377.3
…	
q1 2012:	[101.13 × (1,500.0/4)]/100 = 379.2
q2 2012:	[101.11 × (1,500.0/4)]/100 = 379.2
…	
q1 2013:	[103.37 × (1,500.0/4)]/100 = 387.6
q2 2013:	[103.94 × (1,500.0/4)]/100 = 389.8
…	

Using the quarterly overlap technique, the sum of the quarterly chain volume data for each year does not match the annual chain volume data. In fact, the quarterly sum for 2012 and 2013 (1,524.1 and 1,556.3, respectively) are different from the annual chain volume data in monetary terms (1,524.7 and 1,560.2 from Example 8.6). However, quarterly chain indices derived with the one-quarter overlap technique can be made consistent with annual chain indices using benchmarking (see Example A8.1). It should be noted that this example has been designed to emphasize the difference between the annual and quarterly overlap techniques. Differences between the approaches are generally smaller in real-life series.

The chain volume series derived with the annual overlap technique, the one-quarter overlap technique, and the one-quarter overlap technique with benchmarking are plotted in Figure 8.1.

Figure 8.1 Annually Weighted Laspeyres Indices: Annual Overlap and One-Quarter Overlap Techniques



Lack of Additivity of Chain-Linked Measures

77 In contrast to constant price data, chain-linked volume measures are not additive. To preserve the correct volume changes, related series should be linked independently of any aggregation or accounting relationships that exist between them; as a result, additivity is lost. Additivity is a specific version of the consistency in aggregation property for index numbers. Consistency in aggregation means that an aggregate can be constructed both directly by aggregating the detailed components and indirectly by aggregating sub-aggregates using the same aggregation formula. Lack of additivity is an intrinsic characteristic of a chain-linking system and should be communicated clearly to users.

78 Before the application of any chain-linking techniques, however, annually weighted Laspeyres-type indices are consistent in aggregation within each link—both across variables and between different frequencies. The corresponding volume estimates at previous year's prices (expressed in monetary terms) are additive. This formula makes it possible to calculate volume estimates at previous year's prices of an aggregate as the sum of volume estimates at previous year's prices of its components, as well as deriving annual volume estimates as the sum of the corresponding quarterly volume estimates. Additivity is maintained because the weight period (the previous year) coincides with the base period and the system of weights (the current price data from the previous year) is additive. Additivity of these estimates is crucial to compile SUT in volume terms and to calculate additive contributions to percent change. All other indices in common use are not additive within each link.[33]

79 Chain volume series derived by chaining annually weighted Laspeyres-type indices using the AO technique are also additive in the reference year and the subsequent year, as shown in Example 8.6.

[33]The reason for non-additivity is that different weights are used for different annual periods, and therefore, will not yield the same results unless there have been no shifts in the weights.

Example 8.8 Chain-Linking and Benchmarking Quarterly Fisher Indices

	Quarterly		Annual		Difference	Quarterly Benchmarked
	Fisher Volume Index (Previous Quarter = 100)	Chain Fisher Volume Index (2010 = 100)	Fisher Volume Index (Previous Year = 100)	Chain Fisher Volume Index (2010 = 100)	Quarterly Chain Fisher – Annual Chain Fisher	Benchmarked Chain Fisher Volume Index (2010 =100)
	(1)	(2)	(3)	(4)	(5) = (4) – (2)	(6)
2010		100.00	100.00	100.00	0.00	100.00
2011		100.80	100.79	100.79	0.01	100.79
2012		101.86	101.00	101.79	0.07	101.79
2013		104.11	102.23	104.06	0.05	104.06
q1 2011	100.74	100.74				100.73
q2 2011	99.92	100.66				100.65
q3 2011	100.08	100.74				100.72
q4 2011	100.33	101.07				101.04
q1 2012	100.18	101.25				101.19
q2 2012	100.06	101.31				101.24
q3 2012	100.58	101.90				101.82
q4 2012	101.07	102.99				102.91
q1 2013	100.76	103.77				103.71
q2 2013	100.54	104.32				104.27
q3 2013	99.94	104.26				104.21
q4 2013	99.83	104.08				104.04

(Rounding errors in the table may occur.)

This example calculates the annual chain Fisher indices and the quarterly chain Fisher indices from the data obtained in Examples 8.3 and 8.4, and uses the Denton proportional method to benchmark the quarterly chain indices to the annual ones.

The quarterly Fisher links are reported in column 1. They are chain-linked using the one-quarter overlap technique, that is, by chaining recursively the indices from the previous quarter shown in column 1:

q2 2011: (99.92 × 100.74)/100.0 = 100.66
q3 2011: (100.08 × 100.66)/100.0 = 100.74
q4 2011: (100.33 × 100.74)/100.0 = 101.07
q1 2012: (100.18 × 101.07)/100.0 = 101.25
…
q4 2013: (99.83 × 104.26)/100.0 = 104.08.

The annual average of the quarterly chain indices are shown at the top of column 2:

2011: (100.74+100.66+100.74+101.07)/4 = 100.80
2012: (101.25+101.31+101.90+102.99)/4 = 101.86
2013: (103.77+104.32+104.26+104.08)/4 = 104.11.

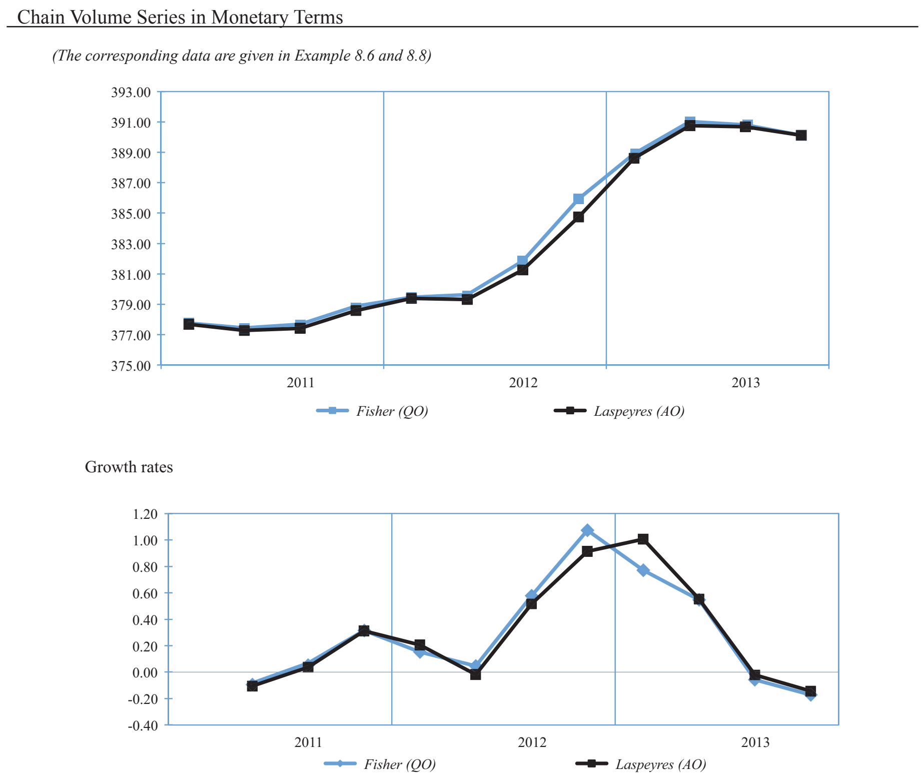
The chaining procedure is applied to the annual data shown in column 3, with the results shown in column 4:

2012: (101.00 × 100.79)/100 = 101.79
2013: (102.23 × 101.79)/100 = 104.06.

Column 5 shows small differences between the annual averages of the quarterly chain Fisher indices and the annual chain Fisher indices. Column 6 shows the quarterly benchmarked chain Fisher indices using the Denton proportional method. It can be noted that the small discrepancies of 2012 and 2013 are distributed smoothly over the quarters.

Figure 8.2 compares the quarterly benchmarked chain Fisher volume series shown in column 6 and the quarterly chain Laspeyres volume series derived with the annual overlap technique (column 3 of Example 8.6). Both series are expressed in monetary terms with reference year 2010.

Figure 8.2 Chain Laspeyres Volume Series and Chain Fisher Volume Series



Chain-Linking, Benchmarking, and Seasonal Adjustment

80 Benchmarking and seasonal adjustment require consistent time series with a fixed reference period at a detailed level, while many standard national accounts compilation methods require additive data. Examples of national accounts compilation methods requiring additive data include estimating value added as the difference between output and intermediate consumption, commodity flow techniques, and use of SUT as an integrating framework. Both requirements may appear inconsistent with chain-linking. This section explains how to address the lack of additivity of chained series for benchmarking and seasonal adjustment purposes.

81 Benchmarking and seasonal adjustment should be applied to chain-linked volume data (expressed either in index form or monetary terms). On the contrary, sequences of Laspeyres-type volume indices at previous year's prices in equation (12) or Fisher volume indices at previous quarter's prices in equation (16) do not have time-series properties and should not be benchmarked or seasonally adjusted directly. These indices can be derived indirectly from benchmarked and seasonally adjusted data at current prices and in chain-linked form using the inverse process of chain-linking ("unchaining"). The Laspeyres formula is additive within each link, therefore it can be used to derive any required aggregations from benchmarked and seasonally adjusted components.

82 Annually chained Laspeyres-type quarterly volume measures with the annual overlap technique are automatically consistent with corresponding annual chain Laspeyres measures and do not require benchmarking. However, when the annual price indices used to deflate ANA variables are derived as simple average of quarterly price indices, benchmarking is still necessary to eliminate the (usually small) inconsistencies between annual and quarterly measures. In theory, annual Laspeyres-type volume measures could be derived as the sum of quarterly Laspeyres-type volume measures.

83 Seasonal adjustment can be applied either to price and volume indicators (i.e., the input data) or to chain QNA price and volume series (i.e., the output data). In the former case, seasonally adjusted price and volume indices are used to deflate and extrapolate seasonally adjusted QNA data at current prices. An advantage of this approach is that seasonal effects are detected (and removed) from series showing a seasonal pattern that is observed from actual data. The deflation/extrapolation methodology in the QNA can introduce spurious seasonality in the unadjusted QNA volume series (like, e.g., a possible step in the first quarter using the AO technique), and this may hamper the quality of the seasonal adjustment results. On the other hand, applying seasonal adjustment to the QNA volume series allows a better control process of the seasonal profile of QNA components and aggregates (especially when aggregates are derived using the direct approach).

84 The sequence of benchmarking, seasonal adjustment, and chain-linking in the QNA can be configured in different ways. The following procedure is an example of a well-designed combination of the three steps:

- Derive seasonally adjusted price and volume indices (fixed-weighted or chain-linked) at the most detailed level of aggregation.
- Calculate QNA volume series at the elementary level by deflating or extrapolating benchmarked QNA current price data using both unadjusted and seasonally adjusted price and volume indices, following the procedures discussed in paragraphs 8.10–23 to calculate elementary price and volume indices.
- Derive QNA volume indices at every detail level using the preferred index formula (Laspeyres or Fisher). When using the Laspeyres formula, aggregate volume data in monetary terms can be derived simply as the sum of elementary volume estimates.
- Chain-link the QNA volume series (with the preferred linking technique) in both unadjusted and seasonally adjusted forms.
- Verify that seasonally adjusted chain QNA volume series do not contain spurious seasonality (following the indications given in Chapter 7). Residual seasonality may remain from the seasonal adjustment process or introduced artificially by chain-linking with the AO technique. In the latter case, the QO technique with benchmarking should be used.
- Benchmark the chain QNA volume series to the corresponding chain ANA volume series (if they are inconsistent).
- As discussed above, a possible variant of this approach is to apply seasonal adjustment to the chain-linked unadjusted QNA volume series. If consistency with ANA is required for seasonally adjusted data, benchmarking will be necessary to force the seasonally adjusted data to comply with the relevant annual values.

Contributions to Percent Change from Chain-Linked Measures

85 The inconvenience for users of chain-linked measures being nonadditive can be reduced somewhat by presenting measures of the components' contribution to percent change in the aggregate. Contributions to percent change measures are additive and thus allow cross-sectional analysis, such as explaining the relative importance of GDP components to overall GDP volume growth. The exact formula for calculating contribution to percent change depends on the aggregation formula used in constructing the aggregate series considered and the time span the percent change covers. This section illustrates solutions to calculate additive contributions from annually chained Laspeyres-type indices and quarterly Fisher indices.

86 Additive contributions to percent change can be calculated from annually chained Laspeyres-type quarterly volume measures when the AO technique is used.[34] The data required are the quarterly chain (Laspeyres-type) volume series expressed in monetary terms and the corresponding annual chain (implicit) Paasche deflators. This solution uses a different formula for the first quarter, where an adjustment factor is needed to make the contributions exactly additive.

87 Assuming that the AO technique is used for chain-linking,[35] exact quarterly contributions for q2–q4 can be derived using the following formula:

$$cL_{x,z}^{(s-1,y)\to(s,y)} = 100 \cdot \left(\frac{xch^{(s,y)} - xch^{(s-1,y)}}{zch^{(s-1,y)}} \right) \left(\frac{DX^{y-1}}{DZ^{y-1}} \right),$$

$$\text{for } s = 2,3,4\,, \tag{29}$$

[34] For more details on the methodology to calculate additive contributions from annually chained Laspeyres-type volume series, refer to a technical note by INSEE (2007).

[35] Formula can be used to calculate contributions from chain-linked Laspeyres-type measures derived with the one-quarter overlap technique, but the contributions are not exactly additive.

where

$xch^{(s,y)}$ is the annually chained Laspeyres-type quarterly volume measure of component x in quarter s of year y,

$zch^{(s-1,y)}$ is the annually chained Laspeyres-type quarterly volume measure of aggregate z in quarter $s-1$ of year y,

DX^{y-1} is the annual chain deflator[36] for component X in year $y-1$, and

DZ^{y-1} is the annual chain deflator for aggregate Z in year $y-1$.

For the first quarter ($s=1$), the formula for additive contributions requires an additional term:

$$cL_{x,z}^{(4,y-1)\to(1,y)} = 100 \cdot \left[\left(\frac{xch^{(1,y)} - xch^{(4,y-1)}}{zch^{(4,y-1)}}\right)\left(\frac{DX^{y-1}}{DZ^{y-1}}\right) + \left(\frac{xch^{(4,y-1)}}{zch^{(4,y-1)}} - \frac{XCH^{y-1}}{ZCH^{y-1}}\right)\left(\frac{DX^{y-1}}{DZ^{y-1}} - \frac{DX^{y-2}}{DZ^{y-2}}\right)\right], \qquad (30)$$

where

XCH^{y-1} is the annual chain Laspeyres-type volume measure for component X in year $y-1$ and

ZCH^{y-1} is the annual chain Laspeyres-type volume measure for aggregate Z in year $y-1$.[37]

An example of contributions to percent change from annually chained Laspeyres-type quarterly volume measures is given in Example 8.9. The example shows that equation (29) also applies to annual data.

88 Equation (30) can be modified to derive additive contributions for year-on-year percent changes:

$$cL_{x,z}^{(s,y-1)\to(s,y)} = 100 \cdot \left[\left(\frac{xch^{(s,y)} - xch^{(s,y-1)}}{zch^{(s,y-1)}}\right)\left(\frac{DX^{y-1}}{DZ^{y-1}}\right) + \left(\frac{xch^{(s,y-1)}}{zch^{(s,y-1)}} - \frac{XCH^{y-1}}{ZCH^{y-1}}\right)\left(\frac{DX^{y-1}}{DZ^{y-1}} - \frac{DX^{y-2}}{DZ^{y-2}}\right)\right]. \qquad (31)$$

These contributions are very helpful to analyze the development of chain-linked volume series unadjusted for seasonal effects.

89 When quarterly Fisher indices are used, contributions to percent change from quarter t–1 to quarter t can be calculated using the following formula[38]:

$$cF_{x,z}^{t-1\to t} = 100 \cdot \left[\frac{\frac{z^{t-1}}{z^t}\left(x^t \frac{p_x^{t-1}}{p_x^t} - x^{t-1}\right) + FQ_z^t\left(x^t - x^{t-1}\frac{p_x^{t-1}}{p_x^t}\right)}{z^t + FQ_z^t \sum_j c_j^{t-1}\frac{p_j^t}{p_j^{t-1}}}\right], \qquad (32)$$

where

FQ_z^t is the Fisher volume index for the aggregate z in quarter t with quarter t – 1 as base and reference period,

z^t is the current price data of aggregate z in quarter t,

x^t is the current price data of component x in quarter t,

c_j^t is the current price data of a generic component j of aggregate z in quarter t, and

p_j^t is the price for component j (including x) in quarter t.

Contributions $cF_{x,z}^{t-1\to t}$ provide an exact decomposition of the aggregate percent change of a quarterly Fisher volume index.[39]

90 Contributions of changes in inventories (and any other variables that can take negative, zero, or positive values) should be calculated residually using formula (29) or (32). For example, contribution of changes in inventories can be derived as the difference between the contributions of gross capital formation and gross fixed capital formation to GDP growth.

Presentation of Chain-Linked Measures

91 There are some important aspects to consider in presenting chain-linked measures in publications:

- whether to present measures of percent change or time series with a fixed reference period,
- whether to present time series as index numbers or in monetary terms,

[36] Annual chain deflators can be calculated implicitly as the annual current price series divided by the annual chain volume series.

[37] The adjustment factor (i.e., the second addend of equation) is usually very small. Formula can be used to provide an approximate decomposition of the quarter-to-quarter change in the first quarter.

[38] Formula is drawn from Chevalier (2003, Appendix II). This formula is currently used by the United States and Canada to derive contributions from chain Fisher indices of national accounts (from both annual and quarterly data). However, quarterly contributions are adjusted to offset (i) the effects of benchmarking quarterly Fisher indices to the annual ones and (ii) the use of percent change expressed at annual rates.

[39] More details on the property of this formula are given in Ehemann, Katz, and Moulton (2002) and Marshall (2002).

Example 8.9 Contributions to Percent Change from Annually Chained Laspeyres-Type Volume Measures

Quarter/ Year	Current Prices			Chain Volume Measures (Laspeyres formula, Annual Overlap, and Monetary Terms)			Implicit Chain Deflator			Contribution to Percent Change			Percent Change
	(1)			(2)			(3) = (1)/(2) × 100			(4)			(5)
	A	B	Total	A	B	Total	A	B	Total	A	B	Sum	Total
2010	600.00	900.00	1,500.00	600.00	900.00	1,500.00	100.00	100.00	100.00				
2011	660.00	854.90	1,514.90	643.07	867.91	1,510.98	102.63	98.50	100.26	2.87	–2.14	*0.73*	0.73
2012	759.00	769.50	1,528.50	727.02	794.42	1,524.71	104.40	96.86	100.25	5.69	–4.78	*0.91*	0.91
2013	948.80	615.60	1,564.40	914.81	628.74	1,560.20	103.71	97.91	100.27	12.83	–10.50	*2.33*	2.33
q1 2011	159.70	218.90	378.60	156.57	221.11	377.68							
q2 2011	163.20	213.70	376.90	159.22	218.06	377.28				0.70	–0.81	*–0.11*	–0.11
q3 2011	167.40	210.60	378.00	162.52	214.90	377.42				0.88	–0.84	*0.04*	0.04
q4 2011	169.70	211.70	381.40	164.76	213.84	378.60				0.59	–0.28	*0.31*	0.31
q1 2012	174.20	204.10	378.30	165.59	213.61	379.38				0.25	–0.04	*0.21*	0.21
q2 2012	180.40	201.40	381.80	172.33	206.53	379.31				1.82	–1.84	*–0.02*	–0.02
q3 2012	188.90	192.30	381.20	182.23	198.20	381.27				2.67	–2.15	*0.52*	0.52
q4 2012	215.50	171.70	387.20	206.87	176.07	384.75				6.61	–5.70	*0.91*	0.91
q1 2013	224.70	166.00	390.70	214.16	171.38	388.62				2.08	–1.08	*1.00*	1.00
q2 2013	235.80	156.30	392.10	227.00	159.76	390.77				3.44	–2.89	*0.55*	0.55
q3 2013	242.90	148.50	391.40	235.02	151.04	390.69				2.14	–2.16	*–0.02*	–0.02
q4 2013	245.40	144.80	390.20	238.64	146.56	390.13				0.97	–1.11	*–0.14*	–0.14

(Rounding errors in the table may occur.)

This example shows how to derive additive contributions to percent change from annually chained Lasperyres-type volume measures expressed in monetary terms. Current price data in column 1 and chain volume series in column 2 are taken from Example 8.6. In this table, figures are shown with two decimal places to reduce rounding errors in the contributions. As shown by equations (29) and (30), the annual chain (implicit) deflator is needed in the calculations. The chain deflator is derived as the current price data divided by the chain volume data. For the total, the annual chain deflators are calculated as follows:

2011: 1,514.90/1,510.98 = 100.26
2012: 1,528.50/1,524.71 = 100.25
2013: 1,564.40/1,560.20 = 100.27.

To calculate contributions using equations (29) and (30), the data required are the quarterly chain volume series in column 2 and the annual chain deflator in column 3. Annual contributions for transaction A are calculated as follows:

2011: [(643.07 – 600)/1,500.0] × (100.0/100.0) × 100 = 2.87
2012: [(727.02 – 643.07)/1,510.98] × (102.63/100.26) × 100 = 5.69
2013: [(914.81 – 727.02)/1,524.71] × (104.40/100.25) × 100 = 12.83.

For transaction B,

2011: [(867.91 – 900)/1,500.0] × (100.0/100.0) × 100 = –2.14
2012: [(794.42 – 867.91)/1,510.98] × (98.50/100.26) × 100 = –4.78
2013: [(628.74 – 794.42)/1,524.71] × (96.86/100.25) × 100 = –10.50.

The sum of contributions for transactions A and B returns the annual percent changes in the chain volume aggregate, shown in column 5:

2011: 2.87 + (–2.14) = 0.73
2012: 5.69 + (–4.78) = 0.91
2013: 12.83 + (–10.50) = 2.33.

For quarterly data, equation (29) applies for q2–q4. For example, contribution of transaction A in q2 2012 is given as follows:

q2 2012: [(172.33 – 165.59)/379.38] × (102.63/100.26) × 100 = 1.82.

For q1, equation (30) should be used to derive contributions that are exactly additive. The formula incorporates an adjustment factor that modifies the contribution calculated with equation (29). As an example, contribution for transaction A in q1 2012 is calculated as follows:

q1 2012: [(165.59 – 164.76)/378.60] × (102.63/100.26) × 100 + [(164.76/378.60) – (643.07/1,510.98)] × [(102.63/100.26) – (100.0/100.0)] × 100 = 0.25,

where the adjustment factor is shown in the second row.

- terminology to avoid confusing chain-linked measures in monetary terms for constant price data (fixed-based measures),
- choice of reference year and frequency of reference year change—among others, as a means to reduce the inconvenience of nonadditivity associated with chain-linked measures, and
- whether to present supplementary measures of contribution of components to percent change in aggregates.

92 Chain-linked price and volume measures must, at the minimum, be made available as time series with a fixed reference period. The main reason is that data presented with a fixed reference period allow different periods and periods of different duration to be compared and provide measures of long-run changes. Thus, presentation of price and volume measures should not be restricted to presenting only tables with period-to-period or year-on-year percent change nor tables with each quarter presented as a percentage of a previous quarter. For users, tables with percent changes derived from the time series may represent a useful supplement to the time series with a fixed reference period and may be best suited for presentation of headline measures. Tables with such data cannot replace the time-series data with a fixed reference period, however, because such tables do not provide the same user flexibility. Tables with each quarter presented as a percentage of a previous quarter (e.g., the previous quarter or the same quarter in the previous year) should be avoided, because they are less useful and can result in users confusing the original index with the derived changes. Restricting the presentation of price and volume measures to presenting changes only runs counter to the core idea behind chain-linking, which is to construct long-run measures of change by cumulating a chain of short-term measures.

93 Chain-linked volume measures can be presented either as index numbers or in monetary terms. The difference between the two presentations is in how the reference period is expressed. As explained in paragraph 8.44, the reference period and level can be chosen freely without altering the rates of change in the series. The index number presentation shows the series with a fixed reference period that is set to 100, as shown in Examples 8.6–8.8. The presentation is in line with usual index practice. It emphasizes that volume measures fundamentally are measures of relative change and that the choice and form of the reference point, and thus the level of the series, is arbitrary. It also highlights the differences of chain-linked measures from constant price estimates and prevents users from treating components as additive. Alternatively, the time series of chain-linked volume measures can be presented in monetary terms by multiplying the series by a constant to equal the constant price value in a particular reference period, usually a recent year. While this presentation has the advantage of showing the relative importance of the series, the indication of relative importance can be highly sensitive to the choice of reference year and may thus be misleading.[40] Because relative prices are changing over time, different reference years may give very different measures of relative importance. In addition, volume data expressed in monetary terms may wrongly suggest additivity to users who are not aware of the nature of chain-linked measures. On the other hand, they make it easier for users to gauge the extent of nonadditivity. Both presentations show the same underlying growth rates and both are used in practice.

94 Annually chain-linked Laspeyres volume measures in monetary terms are additive in the reference period. The nonadditivity inconvenience of chain volume measures in monetary terms may further be reduced by simultaneously doing the following:

- using the average of a year and not the level of a particular quarter as reference period,
- choosing the last complete year as reference year, and
- moving the reference year forward annually.

This procedure may give chain volume measures presented in monetary terms that are approximately additive for the last two years of the series. As illustrated in Example 8.6, the chain discrepancy increases (unless the weight changes are cyclical or noise) the more distant the reference year is. Thus, moving the reference year forward can reduce the chain discrepancies significantly for the most recent section of the time series (at the expense of increased nonadditivity at the beginning of the series). For most users, additivity at

[40] For the same reason, measuring relative importance from chain-linked data can be grossly misleading. For most purposes, it is better to make comparisons of relative importance based on data at current prices—these are the prices that are most relevant for the period for which the comparisons are done, and restating the aggregates relative to prices for a different period detracts from the comparison.

the end of the series is more important than additivity at the beginning of the series.

95 To avoid chain discrepancies completely for the last two years of the series, some countries have adopted a practice of compiling and presenting data for the quarters of the last two years as the weighted annual average prices of the first of these two years. That second-to-last year of the series is also used as reference year for the complete time series. Again the reference year is moved forward annually. This approach has the advantage of providing absolute additivity for the last two years (provided a Laspeyres formula with annual weights is used).

96 Chain-linked volume measures presented in monetary terms are not constant price measures and should not be labeled as measures at "Constant xxxx Prices." Constant prices mean estimates based on fixed-price weights, and thus the term should not be used for anything other than true constant price data based on fixed-price weights. Instead, chain-volume measures presented in monetary terms can be referred to as "chain-volume measures referenced to their nominal level in xxxx."

97 The nonadditivity inconvenience of chain-linking often can be circumvented by simply noting that chain Laspeyres volume measures are additive within each link. For that reason, chain-linked Laspeyres volume measures, for instance, can be combined with analytical tools like volume SUT and IO tables/models that require additivity.

Summary of Key Recommendations

- *For consistency reasons, ANA and QNA volume data should be derived using the same formula index. A superlative index, such as the Fisher index, is the preferred formula for aggregating elementary price and volume indices in the QNA. An acceptable alternative is to use a Laspeyres formula for volumes with the implicit Paasche formula for prices.*
- *Quarterly Fisher indices should be calculated using quarterly weights. The Fisher formula is more robust against the drift problem than other index formulas. Quarterly Fisher indices should be chain-linked using the one-quarter overlap technique. The quarterly chain Fisher series should be benchmarked to the corresponding annual chain Fisher series to preserve consistency and eliminate possible drifts from the quarterly indices (especially when quarterly data contain seasonal effects and short-term volatility).*
- *When the Laspeyres volume index is chosen, quarterly volume measures should be derived using annual weights from the previous year. Quarterly volume measures based on the Laspeyres formula can be chain-linked using either the one-quarter overlap (QO) technique or the annual overlap (AO) technique. The QO technique is the best choice to preserve the time-series properties of the volume series, but should always be used in conjunction with benchmarking to remove inconsistencies with the annual chain-linked data. Instead, the AO technique can be used to derive quarterly volume measures that are automatically consistent with the corresponding annual ones. When the AO technique is preferred, tests should be run to verify that there are no artificial steps between years in the chain-linked series.*
- *Because chain volume data in monetary terms are never additive, the discrepancy between chain-linked components and chain-linked aggregates should not be removed.*
- *To reduce the inconvenience of nonadditivity, chain-linked measures should be presented as contributions to percent change in the aggregates. Formulas that calculate additive contributions from annually chained Laspeyres indices and chain Fisher indices should be preferred. Additive volume data at previous year's prices should also be made available to users.*

Annex 8.1 Interpreting the Difference between the Annual Overlap and One-Quarter Overlap Techniques

1 Annually weighted Laspeyres-type quarterly volume measures can be chain-linked using two alternative techniques: the annual overlap (AO) technique and the one-quarter overlap (QO) technique. As discussed in this chapter, the AO technique has the advantage of producing quarterly indices that are consistent with the corresponding annual chain indices; however, it may introduce a step between one year and the next. For this reason, the QO technique preserves better the time-series properties of the quarterly indices. When consistency with the annual data is strictly required, the chain series obtained with the QO technique can be benchmarked to the corresponding annual chain indices. This annex clarifies and interprets the factor explaining the difference between the chain series derived with the AO and QO techniques and highlights the effects of benchmarking on the QO chain-linked series.

2 The following algebra shows that chain volume series derived with the AO and QO linking techniques differ for a constant factor in each linking year. This factor is defined as the ratio between a price index with quantity weights from the fourth quarter and a price index with quantity weights from the whole year.

3 The AO linking technique is defined by equations (22)–(25) of this chapter. Assuming no quarterly price and volume decomposition in the first year, the quarterly links for the AO and QO techniques are equal for second year. The two techniques provide different results from third year onwards. The quarterly chain indices for the quarters of the third year with reference to the first year are calculated as follows:

$$q_{AO}^{1\to(s,3)} = Q^{1\to 2} \cdot q^{2\to(s,3)} \cdot 100\,, \tag{A1}$$

where

$$s = 1,2,3,4\,,$$

$$q^{2\to(s,3)} = \frac{k^{2\to(s,3)}}{C^2/4}\,, \tag{A2}$$

$$Q^{1\to 2} = \frac{K^{1\to 2}}{C^1} \tag{A3}$$

with

$k^{2\to(s,3)}$ the volume estimate of quarter s of year 3 at the prices of year 2,

$K^{1\to 2}$ the volume estimate of year 2 at the prices of year 1, and

C^1 and C^2 the annual current price data for years 1 and 2.

Replacing the above expressions in equation (A1), the annual links for year 3 become

$$q_{AO}^{1\to(s,3)} = \left(\frac{K^{1\to 2}}{C^1}\right)\cdot\left(\frac{k^{2\to(s,3)}}{\frac{1}{4}C^2}\right)\cdot 100. \tag{A4}$$

4 The recursion formula of the QO technique is defined by equations (26)–(28). The quarterly chain indices for the quarters of the third year with reference to the first year are calculated as follows:

$$q_{QO}^{1\to(s,3)} = q^{1\to(4,2)} \cdot q^{(4,2)\to(s,3)} \cdot 100\,. \tag{A5}$$

Differently from the AO technique, equation (A5) uses a quarterly linking factor from the fourth quarter of the second year ($q^{1\to(4,2)}$) and not the annual linking factor of the second year ($Q^{1\to 2}$). In addition, the QO technique carries forward the movement of the current quarter from the fourth quarter of the previous year ($q^{(4,2)\to(s,3)}$) and not from the previous year ($q^{2\to(s,3)}$).

Using equation (A2) for $q^{1\to(4,2)}$ and equation (26) for $q^{(4,2)\to(s,3)}$, the linking formula in equation (A5) can be expressed as follows:

$$q_{QO}^{1\to(s,3)} = \left(\frac{k^{1\to(4,2)}}{\frac{1}{4}C^1}\right)\cdot\left(\frac{k^{2\to(s,3)}}{cy^{(4,2)}}\right)\cdot 100\,, \tag{A6}$$

where

$k^{1\to(4,2)}$ is the quarterly volume estimate at previous year's prices of quarter 4 of year 2 and

$cy^{(4,2)}$ is the quarterly estimate at the average prices of year 2 of quarter 4, year 2.

5 The ratio between equations (A4) and (A6) explains the differences between the AO and QO techniques. For the third year, the ratio is equal to

$$d^{2\rightarrow 3} = \frac{\left[\frac{K^{1\rightarrow 2}}{C^1}\cdot\frac{k^{2\rightarrow(s,3)}}{\frac{1}{4}C^2}\right]}{\left[\frac{k^{1\rightarrow(4,2)}}{\frac{1}{4}C^1}\cdot\frac{k^{2\rightarrow(s,3)}}{cy^{(4,2)}}\right]}. \quad \text{(A7)}$$

Factor $d^{2\rightarrow 3}$ explains the difference between the AO and QO approaches when the quarterly indices of the third year 3 are linked to the second year. This ratio also formalizes the step problem of the AO technique.

6 After rearranging the terms and doing simple algebra operations on equation (A7), ratio $d^{2\rightarrow 3}$ can be expressed as follows:

$$d^{2\rightarrow 3} = \frac{\left[\frac{cy^{(4,2)}}{k^{1\rightarrow(4,2)}}\right]}{\left[\frac{C^2}{K^{1\rightarrow 2}}\right]}. \quad \text{(A8)}$$

Each term of equation (A8) can be expressed as a "price × volume" expression as follows:

$$cy^{(4,2)} = \sum_j P_j^2 q_j^{(4,2)},$$

$$k^{1\rightarrow(4,2)} = \sum_j P_j^1 q_j^{(4,2)},$$

$$C^2 = \sum_j P_j^2 Q_j^2, \text{ and}$$

$$K^{1\rightarrow 2} = \sum_j P_j^1 Q_j^2 \ .$$

Replacing the above expressions into equation (A8) provides the following ratio:

$$d^{2\rightarrow 3} = \frac{\left[\frac{\sum_j P_j^2 q_j^{(4,2)}}{\sum_j P_j^1 q_j^{(4,2)}}\right]}{\left[\frac{\sum_j P_j^2 Q_j^2}{\sum_j P_j^1 Q_j^2}\right]}, \quad \text{(A9)}$$

which helps in interpreting the difference between the AO and QO techniques. The numerator of equation (A9) is a price index from the first year to the second year, with quantities from the fourth quarter of the second year. The denominator is also a price index from the first year to the second year, but the quantities are those of the second year (the denominator is a true annual Paasche price index). The larger are the differences between these two price indices, the larger are the differences between the chain-linked series calculated with the AO and QO techniques (and the bigger is the risk of introducing a step using the AO approach).

7 Based on expression (A9), the AO and QO techniques provide similar results when the quantity shares in the fourth quarter of a linking year are similar to the quantity shares for the same year as a whole. Large differences between quarterly and annual shares of quantities may arise from data with different seasonal patterns or in periods characterized by strong relative changes. In these situations, the AO technique may introduce an artificial step in the chain volume series. On the contrary, the step problem for the AO technique should be negligible for data that are seasonally adjusted, present relatively stable seasonal patterns, and are characterized by relative stability within the year.

8 Equation (A8) can be generalized for any linking year as follows:

$$d^{t-1\rightarrow t} = \frac{\left[\frac{cy^{(4,t-1)}}{k^{t-2\rightarrow(4,t-1)}}\right]}{\left[\frac{C^{t-1}}{K^{t-2\rightarrow t-1}}\right]} \text{ for } t = 3,4,5,\dots .$$

The chain-linked ratio $d^{2\rightarrow y}$

$$d^{2\rightarrow y} = d^{2\rightarrow 3}\cdot d^{3\rightarrow 4}\cdot\dots\cdot d^{y-1\rightarrow y} \quad \text{(A10)}$$

is equal to the ratio between the chain volume series derived from the AO and QO techniques.

Example A8.1 demonstrates this equivalence using the numerical example used in this chapter.

9 The only disadvantage of the QO technique is that it provides quarterly chain indices that are inconsistent with the corresponding annual chain indices. In monetary terms, this means that the annual

Example A8.1 Annual Overlap, One-Quarter Overlap, and One-Quarter Overlap with Benchmarking

	Chain Volume Series with Annual Overlap (AO)		Chain Volume Series with One-Quarter Overlap (QO)		Ratio AO/QO	Chain Volume Series with One-Quarter Overlap with Benchmarking (QOB)		Ratio QOB/QO	Differences between AO and QOB
	(1)		(2)		(3) = (1)/(2)	(4)		(5) = (4)/(2)	(6) = (4) – (1)
	Level	Percent Change	Level	Percent Change	Level	Level	Percent Change	Level	Percent Change
2011	1,511.0		1,511.0			1,511.0			
2012	1,524.7	0.9	1,524.1	0.9	1.00039	1,524.7	0.9		
2013	1,560.2	2.3	1,556.3	2.1	1.00250	1,560.2	2.3		
q1 2011	377.7		377.7		1.00000	377.7		1.0000	
q2 2011	377.3	–0.1	377.3	–0.1	1.00000	377.3	–0.1	1.0000	0.0
q3 2011	377.4	0.0	377.4	0.0	1.00000	377.4	0.0	1.0000	0.0
q4 2011	378.6	0.3	378.6	0.3	1.00000	378.6	0.3	1.0000	0.0
q1 2012	379.4	0.2	379.2	0.2	1.00039	379.2	0.2	0.9999	0.0
q2 2012	379.3	0.0	379.2	0.0	1.00039	379.2	0.0	1.0001	0.0
q3 2012	381.3	0.5	381.1	0.5	1.00039	381.3	0.6	1.0005	0.0
q4 2012	384.8	0.9	384.6	0.9	1.00039	385.0	1.0	1.0010	0.1
q1 2013	388.6	1.0	387.6	0.8	1.00250	388.4	0.9	1.0018	–0.1
q2 2013	390.8	0.6	389.8	0.6	1.00250	390.7	0.6	1.0024	0.1
q3 2013	390.7	0.0	389.7	0.0	1.00250	390.8	0.0	1.0028	0.0
q4 2013	390.1	–0.1	389.2	–0.1	1.00250	390.3	–0.1	1.0030	0.0

Columns 1 and 2 show level and percent change of the chain Laspeyres-type volume series using the annual overlap (AO) and one-quarter overlap (QO) techniques derived in Examples 8.6 and 8.7, respectively. As shown in column 3, the two series are identical for the 2011 quarters and differ for two constant factors in 2012 and 2013.

The ratio between the AO and QO techniques is explained in formula. Using the figures for 2012 in Examples 8.6 and 8.7,

$$d^{2011\to 2012} = \left[\frac{cy^{(4,2011)}}{K^{2010\to(4,2011)}}\right] \Big/ \left[\frac{C^{2011}}{K^{2010\to 2011}}\right] = \left[\frac{379.73}{378.60}\right] \Big/ \left[\frac{1,514.90}{1,511.00}\right] = 1.00039,$$

which is the ratio between the AO series and the QO series as shown in column 3 in 2012. For 2013,

$$d^{2012\to 2013} = \left[\frac{cy^{(4,2012)}}{K^{2011\to(4,2012)}}\right] \Big/ \left[\frac{C^{2012}}{K^{2011\to 2012}}\right] = \left[\frac{386.52}{385.75}\right] \Big/ \left[\frac{1,528.50}{1,528.67}\right] = 1.00211.$$

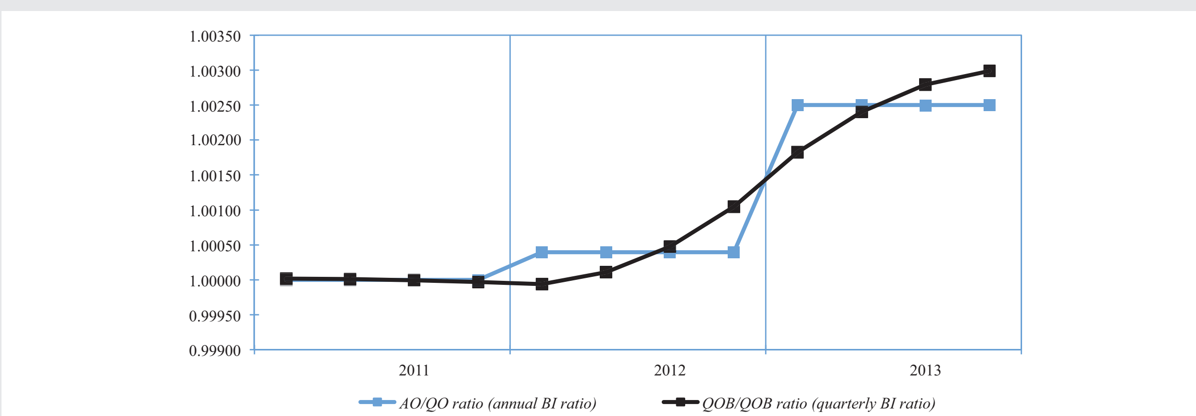
The chain ratio for 2013 is

$$d^{2011\to 2013} = 1.00039 \cdot 1.00211 = 1.00250,$$

which corresponds to the constant factor for 2013 as shown in column 3.

To eliminate discrepancies with the annual data, the QO series should be benchmarked to the annual chain volume series (the AO series does not present such inconsistencies). Column 4 shows the QO benchmarked (QOB) series using the Denton proportional benchmarking method. The differences with the AO series, shown in column 5, are distributed smoothly between 2012 and 2013. Figure A8.1 shows how the Denton method realigns the QO series with the annual benchmarks. Note that the AO/QO ratio can be interpreted as the annual benchmark-to-indicator ratio in the benchmarking process of the QO series. The QOB/QO ratio is the interpolation of the AO/QO ratio based on the proportional benchmarking method.

Figure A8.1 Annually Weighted Laspeyres Indices: Annual Overlap and One-Quarter Overlap Techniques



sum of the chain quarterly volume measures does not add up to the independently chained annual volume measures. To eliminate the inconsistencies, the quarterly chain indices using the QO technique should be benchmarked to the annual chain indices. The benchmarking process should be conducted with a method that preserves the movements in the original QO series and, at the same time, satisfies the annual benchmark indices. As recommended in Chapter 6, the Denton proportional benchmarking method can be used to this purpose. Benchmarking using the Denton method distributes smoothly the discrepancies between the QO series and the annual chain-linked series.

10 Under the benchmarking framework, the chain-linked ratio (equation (A10)) corresponds to the annual benchmark-to-indicator (BI) ratio resulting from benchmarking the quarterly chain volume series derived with the QO technique to annual chain indices. A time-series analysis of the annual BI ratio can be helpful to appreciate the size and direction of the differences between the AO and QO linking techniques. When small variations of equation (10) are noted over time, the AO and QO techniques are expected to produce similar results.

Example A8.1 and Figure A8.1 shows the effects of benchmarking a quarterly chain volume series derived with the QO technique to the corresponding annual chain volume series.

Bibliography

Al, P.G., B. Balk, S. de Boer, and G.P. den Bakker (1985), "The Use of Chain Indices for Deflating the National Accounts," *Statistical Journal of the United Nations Economic Commission for Europe*, 4 (5): 47–68.

Alexander, T., C. Dziobek, M. Marini, E. Metreau, and M. Stanger (2017), *Measure Up: A Better Way to Calculate GDP*, Staff Discussion Notes No. 17/02, Washington, DC: IMF.

Baquite National De Belgique (2010), "Issues Encountered with Quarterly Volume Balances Measured in Chain-linked Euros: Levels and Contributions to Growth—A New Approach for the Quarterly National Accounts," Annex to *Comptes Nationaux—Comptes Trimestriels 2010-II*, pp. 10–21.

Chevalier, M. (2003), *Chain Fisher Volume Index Methodology*, Research Paper, Statistics Canada, Ottawa, November.

Ehemann, C., A.J. Katz, and B.R. Moulton (2002), "The Chain-Additivity Issue and the US National Economic Accounts," *Journal of Economic and Social Measurement*, 28(1/2): 37–49.

Eurostat (2001), *Handbook on Price and Volume Measures in National Accounts*, Luxembourg: Office for Official Publications of the European Commission.

ILO, IMF, OECD, Eurostat, UN, Economic Commission for Europe, and World Bank (2004a), *Consumer Price Index Manual: Theory and Practice*, Geneva: ILO.

ILO, IMF, OECD, UN, Economic Commission for Europe, and World Bank (2004b), *Producer Price Index Manual: Theory and Practice*, Washington, DC: IMF.

ILO, IMF, OECD, UN, Economic Commission for Europe, and World Bank (2009), *Export and Import Price Index Manual: Theory and Practice*, Washington, DC: IMF.

INSEE (2007), "Calcul des Contributions en Volumes Chaînés," Technical note to users, available at http://www.insee.fr/fr/themes/theme.asp?theme=16&sous_theme=8&type=2&nivgeo=0&page=methodologie.html.

Kulshreshtha, A.C. (2000), *Basic Principles and Practices in Rebasing and Linking National Accounts Series*, Paper prepared for the ADB/ESCAP Workshop on Rebasing and Linking National Accounts Series in Selected Developing Member Countries, Bangkok, Thailand, March.

Landefeld, J.S., and R.P. Parker (1997), "BEA's Chain Indexes, Time Series, and Measures of Long-Term Economic Growth," *Survey of Current Business*, 77(5): 58–68.

Marshall, B.R. (2002), "Additive Decompositions for Fisher, Törnqvist and Geometric Mean Indexes," *Journal of Economic and Social Measurement*, 28(1/2): 51–61.

Parker, R.P., and E.P. Seskin (1997), "Annual Revision of the National Income and Product Accounts," *Survey of Current Business*, August: 6–35.

Szultc, B., 1983, "Linking Price Index Numbers," in eds. W.E. Diewert and C. Montmarquette, *Price Level Measurement: Proceedings of a Conference Sponsored by Statistics Canada*, Ottawa: Statistics Canada, pp. 537–566.

United Nations (2010), *International Recommendations for the Index of Industrial Production*, New York: U.N. Statistics Divisions.

United Nations, European Commission, International Monetary Fund, and Organization for Economic Co-operation and Development (2008), *The System of National Accounts, 2008* (New York: United Nations).