Monetary Policy in a Financial Crisis*

Preliminary, Comments Welcome

Lawrence J. Christiano†   Christopher Gust‡   Jorge Roldos§


Abstract

We model a financial crisis as a time when collateral constraints are binding. We ask whether a cut in interest rates will, under such circumstances, cause an expansion or a recession. We describe model economies that can rationalize either view. The difference between them has to do with flexibility in production. If there are substantial substitution possibilities among factors of production, and diminishing returns are not too great, then an interest rate cut will produce an expansion. If there are substantial inflexibilities in the economy, then an interest rate cut will produce a recession.

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‡ Northwestern University
§ Federal Reserve Board of Governors.
"International Monetary Fund."
1. Introduction

In recent years there has been considerable controversy over the appropriate monetary policy in the aftermath of a financial crisis. Some argue that the central bank should raise domestic interest rates to defend the currency and halt the flight of capital. Others argue that interest rate reductions are called for. They note that a country that has just experienced a financial crisis is typically sliding into a steep recession. They appeal to the widespread view that in developed economies like the US, central banks typically respond to situations like this by reducing interest rates. These authors urge the same medicine for emerging market economies in the wake of a financial crisis. They argue that to raise interest rates at such a time is a mistake, and is likely to make a bad situation even worse. One expositor of this view, Paul Krugman (1999, pp.103-105), puts it this way:

“But when financial disaster struck Asia, the policies those countries followed in response were almost exactly the reverse of what the United States does in the face of a slump. Fiscal austerity was the order of the day; interest rates were increased, often to punitive levels....Why did these extremely clever men advocate policies for emerging market economies that would have been regarded as completely perverse if applied at home?”

We describe a framework that allows us to articulate the two views just described. The framework has two fundamental building blocks. First, we assume that to carry out production, firms require domestic working capital to hire labor and international working capital to purchase an imported intermediate input. Second, we adopt the asset market frictions formalized in the limited participation model as analyzed in Lucas (1990), Fuerst (1992) and Christiano and Eichenbaum (1992, 1995). The limited participation assumption has the consequence that an expansionary monetary action makes the domestic banking system relatively liquid and induces firms to hire more labor. To the extent that the imported intermediate input complements labor, the interest rate drop leads to the increased use of this factor too. This is in the spirit of the traditional liquidity channel emphasized in the closed economy literature, which stresses the positive effects of an interest rate cut on output.

In our open economy, an interest rate cut may also lead to a real exchange rate depreciation. Under normal conditions, this does not mitigate the positive output effects associated with the traditional liquidity channel. However, during a crisis the real exchange rate depreciation may lead to very different effects in our model.

We model a crisis as a time when international loans must be collateralized by physical assets such as land and capital, and that this restriction is binding. We suppose that under normal conditions, collateral constraints either do not exist or are not binding. This can occur, for example, because output in addition to land and capital is acceptable as collateral.
in such times. Alternatively, a crisis may be a time when the fraction of domestic assets accepted as collateral by foreigners declines. In any case, factors that cause the collateral constraint to become binding are treated as exogenous in our analysis.

In the model framework we adopt, the monetary transmission mechanism can be very different during a crisis, and the following scenario is possible. When the central bank engineers a fall in the domestic rate of interest, the resulting real depreciation leads to a reduction in the value (expressed in units of tradeable goods) of domestic collateralizable assets. When collateral constraints are already binding, this produces a further reduction in the supply of international credit needed to sustain imports of the intermediate good. Given a sufficient lack of substitutability between the imported good and domestic factors of production, the fall in imports can exacerbate the fall in output associated with the financial crisis. We display a model which articulates this recession view about an interest rate cut.

Although the scenario just described is a possibility, the opposite can occur too. In particular, an interest rate cut may lead to an expansion in output and in imports of the intermediate goods. This can happen if the interest rate cut produces a relaxation of collateral constraints by raising asset values. There is a variety of channels through which this can occur in the model. Asset values may rise because the marginal physical product of capital increases with increased utilization of variable factors of production. They may rise if the real interest rate used to discount future income flows from assets falls with the domestic rate of interest. Finally, asset values may also rise if the shadow value of capital in relaxing the collateral constraint increases. We display a second model which articulates this expansion view about an interest rate cut.

We use the two models just described to help us identify the features of the environment that are key to determining whether an interest rate cut will be expansionary or contractionary in the aftermath of a financial crisis or credit crunch.

The organization of the paper is as follows. The first section presents an empirical discussion of collateral constraints. The second section presents our general model. The third section presents a version of the model simplified by the assumption that the current account is held identically equal to zero. Various versions of this model can be studied analytically, and we exploit this to gain insights into the nature of the monetary transmission mechanism in the presence of a binding collateral constraint. The fourth section extends the analysis by allowing the current account to be endogenous. The final section concludes.
2. Collateral Constraints and Related Literature

The assumption that the land and capital of the firm is collateral for international loans plays an important role in our analysis. This section presents a brief discussion of how we interpret this assumption. We recognize the enforcement problems and other financial market imperfections that are pervasive in emerging markets hence do not restrict ourselves to a narrow (legal or contractual) view of collateral. Rather, we see collateral constraints as capturing the tightening of credit conditions and associated balance sheet problems seen in the aftermath of a financial crisis. They do not just reflect the insistence by creditors that collateral be written explicitly into loan contracts, but also the possibility that regulators induce banks to invest only in ‘secure’ loans, loans to companies that have ample assets in the event that things go wrong. We now briefly provide some empirical evidence on the use of collateral in loan markets.

The use of collateral in loan markets is widespread. Berger and Udell (1990) document that around 70 percent of commercial and industrial loans in the US are secured and Black, De Meza and Jeffrey report similar evidence for the UK. In emerging markets, Gelos and Werner (1999) report that around 60 percent of loans are collateralized in Mexico, while survey evidence from the Bank of Thailand put the figure at more than 80 percent for that country. A review of financial conditions of the Asian crises countries (IMF 1999) notes that lending against collateral was a widespread practice also in these countries.

There is also ample evidence that collateral practices are procyclical. Asea and Blomberg (1998) provide evidence that bank lending standards vary over the US business cycle: in an average contraction, the degree of loan collateralization and spreads charged over Treasuries increase over the year before the trough. In an interesting paper on the recent emerging market crises, Edison, Luangaram and Miller (2000) report that Thai banks that used to lend up to 70-80 percent of the value of pledged collateral before the crisis, moved to lend up to just 50-60 percent after the crisis. More important for our paper, the fraction of syndicated loans in international markets that is collateralized reached a peak (at 42 percent of the total; see Table 1) in the year of the Asian crises. This is reinforced by the fact that the level of syndicated loans also peaked in 1997, in line with the finding in Chadha and Folkerts-Landau (2000) that suggests that commercial banks appear to be lenders of “second-to-last resort”.

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Table 1: Syndicated Loans to Emerging Mark

(in billions of U.S. dollars)

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Secured</th>
<th>Secured as % of Total</th>
</tr>
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<tbody>
<tr>
<td>1993</td>
<td>47.5</td>
<td>7.9</td>
<td>16.5</td>
</tr>
<tr>
<td>1994</td>
<td>64.9</td>
<td>11.5</td>
<td>17.7</td>
</tr>
<tr>
<td>1995</td>
<td>93.0</td>
<td>16.1</td>
<td>17.3</td>
</tr>
<tr>
<td>1996</td>
<td>104.3</td>
<td>22.0</td>
<td>21.1</td>
</tr>
<tr>
<td>1997</td>
<td>143.7</td>
<td>61.4</td>
<td>42.7</td>
</tr>
<tr>
<td>1998</td>
<td>77.3</td>
<td>25.9</td>
<td>33.5</td>
</tr>
<tr>
<td>1999</td>
<td>73.1</td>
<td>26.3</td>
<td>35.9</td>
</tr>
</tbody>
</table>

Source: Capital Data, Loanware

There is, however, substantial debate on what constitutes international collateral. In a recent paper, Caballero and Krishnamurthy (1999) assume (not without caveats) that only a fraction of the assets used in the tradable sector would be accepted as collateral by foreign creditors, whereas the totality of domestic assets would be accepted as domestic collateral. Inadequate amounts of international collateral and imperfect aggregation of domestic collateral are shown to lead to fire sales of domestic assets and financial crises.

In recognition of the difficulties defining what constitutes international collateral and the different institutional arrangements that characterize emerging financial markets, we consider two approaches to modeling collateral constraints.

In the first, collateral is the physical assets of the borrower. This is the collateral that would be written explicitly into debt contracts, if collateral were included at all. It is also what would comfort regulators concerned about the riskiness of bank loan portfolios. Under this specification of the collateral constraint, borrowing cannot exceed the value of the individual firm’s assets.

Under an alternative model of the collateral constraint, individual firms would face no collateral constraint at all. Instead, market interest rates would adjust until the amount of borrowing by firms matches the value of the aggregate collateral of the economy. This way of modeling the collateral constraint has potentially different implications for the effects of interest rate cuts. Under the individual collateral constraint, a general shortage of collateral
has a tendency to raise asset prices endogenously. This happens because in bidding for capital, individual firms recognize the value of capital as collateral. Under the aggregate collateral constraint, this mechanism would be absent. As a result, shortages of collateral would not have the same tendency to generate an increase in collateral endogenously.

One interpretation of the aggregate collateral model is that enforcement problems prevent the inclusion of standard collateral in contracts. Instead, it is a system of government guarantees that back up loans. As a result, it is the government's collateral that ultimately matters. Under this model of collateral, the government's collateral is related to the economy-wide value of assets. This model seems consistent with the literature on external debt (e.g., Eaton, Gersowitz and Stiglitz, 1986), where the decision to default and/or renegotiate external debt is modeled as a country-wide phenomenon, dependent on aggregate variables. This model also seems consistent with the behavior of foreign creditors, who usually set "country exposure limits" and spreads that are related to aggregate variables.

The imposition of both types of collateral constraints is also consistent with recent models of financial crises that focus on the role of guarantees and moral hazard as causes of emerging markets crises (such as Krugman, 1999, Corsetti, Pessenti and Roubini, 1998, Dooley, 1998, and Bumside, Eichenbaum and Rebele, 2000). In fact, we should stress that our model does not attempt to model the causes of the financial crises but its effects on credit conditions. Now after a crisis happens, i.e., after the guarantees are exercised, the only way in which foreigners would extend further credit to a country would be under further guarantees or additional collateral. This additional collateral has been seen in explicit private (syndicated loan) contracts, in the tightening of bank regulations that impose directly or indirectly collateral requirements, and in governments' intervention and sales of distressed assets.

At this point, this paper only includes our work analyzing the role of individual collateral constraints, the first way of modeling collateral discussed above. We have begun to study the effects of incorporating the aggregate collateral model. However, the initial results suggest that there is little difference between the two models from the standpoint of our fundamental interest in this paper: understanding the economic effects of interest rate changes in times of crisis. In later drafts we plan to include our analysis of this alternative way of modeling collateral to document whether it does or does not make a difference.

The collateral constraint in this paper provides a financial friction that captures in a natural way the "balance sheet" effects frequently mentioned in the discussion of recent crises. A number of recent papers (Krugman (1999), Aghion, Bacchetta and Banerjee (2000), Cespedes, Chang and Velasco (2000), Gertler, Gilchrist and Natalucci (2000), Mendoza (2000)) introduce credit constraints that capture some aspects of the balance sheet channel. However, while these papers incorporate the adverse effects derived from the existence of foreign currency denominated debts, the credit frictions in those papers constrain borrowing
to be a fraction of current income—rather than the current value of physical assets.\footnote{A notable exception is Gertler, Gilchrist and Natalucci (2000), but the nature of the credit constraint is different from the one in this paper. Moreover, the current account is always zero in that paper, hence missing the connection between capital flows and asset prices observed in the crises episodes.} The mismatch between assets and liabilities that underscored the financial vulnerability of the crises countries is captured in our model by the fact that a large fraction of domestic assets are in the nontraded sector while a relatively large fraction of liabilities are denominated in terms of the traded good. Hence, a large and persistent fall in the relative price of nontraded goods (i.e., a real exchange rate depreciation) results in a fall in the value of assets relative to liabilities that endogenously magnifies the tightening of credit conditions derived from the initial imposition of collateral constraints in the aftermath of the crisis. In other words, our economy is capable of displaying a “sudden stop” in capital inflows (Calvo, 1998) that is associated with large and persistent falls in output and external debt.

3. The Model

We adopt a standard traded good-non traded good small open economy model. The model has households, firms, a financial intermediary, and a domestic monetary authority.

3.1. Households

There is a representative household, which derives utility from consumption, $c_t$, and leisure as follows:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, L_t),$$

(3.1)

where $L_t$ denotes labor. We adopt the following specification of utility:

$$u(c, L) = \frac{\left[ c - \frac{\psi_n}{1+\psi} L^{1+\psi} \right]^{1-\sigma}}{1-\sigma}.$$  

(3.2)

The household begins the period with a stock of liquid assets, $\hat{M}_t$. Of this, it deposits $D_t$ with the financial intermediary, and the rest, $\hat{M}_t - D_t$, is allocated to consumption expenditures. The cash constraint that the household faces on its consumption expenditures is:

$$P_t c_t \leq W_t L_t + \hat{M}_t - D_t,$$

(3.3)
where $W_t$ denotes the money wage rate and $P_t$ denotes the price level.

The household also faces a flow budget constraint governing the evolution of its assets:

$$\hat{M}_{t+1} = R_t (D_t + X_t) + P_t^T \pi_t + \left[ W_t I_t + \hat{M}_t - D_t - P_t c_t \right]. \tag{3.4}$$

Here, $R_t$ denotes the gross domestic rate of interest, $\pi_t$ is profits which derive from household’s ownership of firms, and $X_t$ is a liquidity injection from the monetary authority. Here, $\pi_t$ is measured in units of traded goods, and $P_t^T$ is the domestic currency price of traded goods. The term on the right of the equality reflects the household’s sources of liquid assets at the beginning of period $t + 1$: interest earnings on deposits and on the liquidity injection, profits and any cash that may be left unspent in the period $t$ goods market.

The household maximizes (3.1) subject to (3.3)-(3.4), and the following timing constraint. A given period’s deposit decision is made before that period’s liquidity injection is realized, while all other decisions are made afterward. The household’s Euler equation for labor is:

$$\psi_0 L_t^0 = \frac{W_t}{P_t}, \tag{3.5}$$

which is the labor supply equation. The intertemporal Euler is:

$$u_{c,t} = \beta R_t u_{c,t+1} \frac{P_t}{P_{t+1}}. \tag{3.6}$$

### 3.2. Firms

There are two types of representative, competitive, firms. The first produces the final consumption good, $c$, purchased by households. Final goods production requires intermediate goods which are produced by the second type of representative firm. We now discuss these two types of firms.

#### 3.2.1. Final Good Firms

The production function of the final good firms is:

$$c = \left\{ \left[ (1 - \gamma) c^T \right]^{\frac{n-1}{\eta}} + \left[ \gamma c^N \right]^{\frac{n-1}{\eta}} \right\}^{\frac{1}{n-1}}, \quad 1 \geq \eta \geq 0, \quad 0 < \gamma < 1,$$
where $c^T$ and $c^N$ denote quantities of tradeable and non-tradeable intermediate inputs, respectively. In effect, these firms are retailers that package traded and nontraded intermediate goods into a final consumption good. Here, $\eta$ denotes the elasticity of substitution in production between the two intermediate inputs. As $\eta \to 0$,

$$ c = \min \left\{ (1 - \eta) c^T, \gamma c^N \right\}. \quad (3.7) $$

We exclude $\eta \geq 1$ from consideration on empirical grounds. As noted above, the price of $c$ is denoted by $P$, while $P^T$ and $P^N$ denote the money prices of the traded and nontraded inputs, respectively. The firm takes these prices parametrically.

Zero profits and efficiency imply the following relation between prices:

$$ p = \left[ \left( \frac{1}{1 - \gamma} \right)^{1-\eta} \left( \frac{p^N}{\gamma} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad p = \frac{P}{P^T}. \quad (3.8) $$

For $\eta \neq 1$, efficiency also dictates:

$$ p^N = \frac{\gamma}{1 - \gamma} \left( \frac{1 - \gamma}{\gamma c^N} \right)^{\frac{1}{\eta}}, \quad p^N = \frac{P^N}{P^T}. \quad (3.9) $$

When $\eta = 0$, this expression is replaced by $(1 - \gamma) c^T = \gamma c^N$, as implied by (3.7). The object, $P$, in the model corresponds to the model’s ‘consumer price index’, denominated in units of the domestic currency. The object, $p$, is the consumer price index denominated in units of the traded good.

### 3.2.2. Intermediate Inputs

A single representative firm produces the traded and non-traded intermediate inputs. That firm manages three types of debt, two of which are short-term. The firm borrows at the beginning of the period to finance its wage bill and to purchase a foreign input, and repays these loans at the end of the period. In addition, the firm holds the outstanding stock of external (net) indebtedness, $B_t$. 
The firm’s optimization problem is:

$$\max \sum_{t=0}^{\infty} \beta^t \Lambda_{t+1} \pi_t,$$

(3.10)

where

$$\pi_t = p_t^N y_t^N + y_t^T - w_t R_t L_t - R^* z_t - r^* B_t + (B_{t+1} - B_t),$$

(3.11)
denotes dividends, denominated in units of traded goods. Also, $B_t$ is the stock of external debt at the beginning of period $t$, denominated in units of the traded good; $R^*$ is the gross rate of interest (fixed in units of the traded good) on loans for the purpose of purchasing $z_t$; and $r^*$ is the net rate of interest (again, fixed in terms of the traded good) on the outstanding stock of external debt. The price, $\Lambda_{t+1}$, is taken parametrically by firms. In equilibrium, it is the multiplier on $\pi_t$ in the (Lagrangian representation of the) household problem:

$$\Lambda_{t+1} = \frac{u_{c,t+1} P_t^T}{p_{t+1} P_{t+1}^T \beta} \frac{p_t^T}{p_{t+1} P_{t+1}^T} \frac{1}{1 + x_t \beta},$$

(3.12)

where

$$p_t^T = \frac{p_t^T}{M_t}.$$

Here, $M_t$ is the aggregate stock of money at the beginning of period $t$, which is assumed to evolve according to:

$$\frac{M_{t+1}}{M_t} = 1 + x_t.$$

(3.13)

Note that under our notational convention, all lower case prices except one, expresses that price in units of the traded good. The exception, $p_t^T$, is the domestic currency price of traded goods, scaled by the beginning of period stock of money. Alternatively, $p_t^T$ is the inverse of a measure of real balances.

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2The intuition underlying (3.12) is straightforward. The object $\Lambda_{t+1}$ in (3.12), is the marginal utility of one unit of dividends, denominated in traded goods, transferred by the firm to the household at the end of period $t$. This corresponds to $P_t^T \pi_t$ units of domestic currency. The households can use this currency in period $t + 1$ to purchase $P_t^T \pi_t / P_{t+1}$ units of the consumption good. The value, in period $t$, of these units of consumption goods is $\beta u_{c,t+1} P_t^T \pi_t / P_{t+1}$, or $\beta u_{c,t+1} P_t^T \pi_t / (p_{t+1} P_{t+1}^T)$, where $u_{c,t}$ is the marginal utility of consumption. This is the first expression in (3.12).
The firm production functions are:

\[ y^T = \left\{ \theta \left[ \mu_1 V \right]^{\frac{\mu_2}{\theta}} + (1 - \theta) \left[ \mu_2 \right]^{\frac{\mu_2}{1 - \theta}} \right\}^{\frac{1}{\theta}}, \]  
\[ V = A \left( K^T \right)^{\nu} \left( I^T \right)^{1 - \nu}, \]  
\[ y^N = \left( K^N \right)^{\alpha} \left( I^N \right)^{1 - \alpha}, \]  

where \( \xi \) is the elasticity of substitution between value-added in the traded good sector, \( V_t \), and the imported intermediate good, \( z_t \). In the production functions, \( K^T \) and \( K^N \) denote capital in the traded and non-traded good sectors, respectively. They are owned by the representative intermediate input firm. We keep the stock of capital fixed throughout the analysis. It does not depreciate and there exists no technology for making it bigger.

Our specification of technology is designed to encompass a variety of cases. Two are of particular interest because they correspond to different ways of formalizing the notion that \( z \) is essential in the production of traded goods. In the first, there is no substitutability between \( z \) and \( V \) in production, i.e., \( \xi = 0 \), so that

\[ y^T = \min \left\{ \mu_1 V, \mu_2 z \right\}. \]  

(3.15)

In this case, if for some reason \( z \) is reduced, then adjustments in \( V \) cannot prevent a fall in \( y^T \). If in addition, there is no substitution between \( y^T \) and \( y^N \) in the production of the consumption good (i.e., \( \eta = 0 \)) then a fall in \( y^T \) will also bring down \( y^N \). This is what we had in mind in the introduction, when we said that with lack of substitution, a collateral constraint-enforced reduction in \( z \) could bring down the whole economy. The second case where \( z \) is essential, occurs when it is the only variable factor of production. This corresponds to \( \xi = \mu_1 = \mu_2 = \nu = 1 \), when

\[ y^T = \left( AK^T \right)^{\theta} z^{1 - \theta}. \]  

(3.16)

Again, if something (say a tightening of the collateral constraint) induces a fall in \( z \), then \( y^T \) must fall and \( y^N \) must therefore fall too, if \( \eta \) is small enough.

We impose the following restriction on borrowing:

\[ \frac{B_{t+1}}{(1 + r^*)^t} \to 0, \text{ as } t \to \infty. \]  

(3.17)

We suppose that international financial markets impose that this limit cannot be positive. That it cannot be negative is an implication of firm optimality.
The firm’s problem at time \( t \) is to maximize (3.10) by choice of \( B_{t+j+1}, y^N_{t+j}, y^T_{t+j}, z_{t+j}, L^T_{t+j} \) and \( L^N_{t+j} \), \( j = 0, 1, 2, \ldots \), and the indicated technology. In addition, the firm takes all prices and rates of return as given and beyond its control. The firm also takes the initial stock of debt, \( B_t \), as given. This completes the description of the firm problem in the pre-crisis version of the model, when collateral constraints are ignored.

The crisis brings on the imposition of the following collateral constraint:

\[
\tau^N q^N_t K^N + \tau^T q^T_t K^T \geq R^* z_t + (1 + r^*) B_t + u_t R_t L_t, \tag{3.18}
\]

where \( L_t \equiv L^T_t + L^N_t \). Here, \( q^i, i = N, T \) denote the value (in units of the traded good) of a unit of capital in the nontraded and traded goods sectors, respectively. Also, \( \tau^i \) denotes the fraction of these stocks accepted as collateral by international creditors. The left side of (3.18) is the total value of collateral, and the right side is the payout value of the firm’s debt. It is the total amount that the firm would have to pay, to completely eliminate all its debt by the end of period \( t \). Before the crisis, firms ignore (3.18), and assign a zero probability that it will be implemented. With the coming of the crisis, firms believe that (3.18) must be satisfied in every period henceforth, and do not entertain the possibility that it will be removed.

The equilibrium value of the asset prices, \( q^i_t, i = N, T \), is the amount that a potential firm would be willing to pay in period \( t \), in units of the traded good, to acquire a unit of capital and start production in period \( t \). We let \( \lambda_t \geq 0 \) denote the multiplier on the collateral constraint (= 0 in the pre-crisis period) in firm problem. Then, \( q^i_t \) is the derivative of the Lagrangian representation of the firm’s problem with respect to \( K^i_t \):

\[
q^i_t = VM P^i_{k,t} + \lambda_t \tau^i_t q^i_t + \frac{\beta}{\lambda_{t+1}} \sum_{j=1}^{\infty} \beta^{j-1} \lambda_{t+1+j} \left\{ VM P^i_{k,t+j} + \lambda_{t+j} \tau^i_t q^i_{t+j} \right\} \tag{3.19}
\]

or,

\[
q^i_t = \frac{VM P^i_{k,t} + \beta \lambda_{t+2} q^i_{t+1}}{1 - \lambda_t \tau^i_t}, \quad i = N, T. \tag{3.20}
\]

Here, \( VM P^i_{k,t} \) denotes the period \( t \) value (in terms of traded goods) marginal product of capital in sector \( i \). With our assumptions on technology, these are:

\[
VM P^N_{k,t} = \alpha^N_{\bar{p}_t} \frac{y^N_t}{K^N},
\]

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VMP\textsubscript{t,t} = \begin{cases} 
\left( \frac{\mu_r}{m \nu V_t} \right)^{\frac{\xi}{\epsilon}} \theta \nu \mu_1 V_t, & \xi \neq 0 \\
\nu \frac{\mu_1 V_t}{K \nu} \left[ 1 - \frac{(1 + \lambda_1^\text{r}) r^*}{\mu_2} \right], & \xi = 0 
\end{cases}.

When \( \lambda_t \equiv 0 \), (3.19) is just the standard asset pricing equation. It is the present discounted value of the value of the marginal physical product of capital. When the collateral constraint is binding, so that \( \lambda_t \) is positive, then \( q_t^* \) is greater than this. This reflects that in this case capital is not only useful in production, but also for relieving the collateral constraint. In our model capital is never actually traded since all firms are identical. However, if there were trade, then the price of capital would be \( q_t^* \). If a firm were to default on its credit obligations, the notion is that foreign creditors could compel the sale of its physical assets in a domestic market for capital. The price, \( q_t^* \), is how much traded goods a domestic resident is willing to pay for a unit of capital. Foreign creditors would receive those goods in the event of a default. We assume that with these consequences for default, default never occurs in equilibrium.

We now derive the Euler equations of the firm. Differentiating the date 0 Lagrangian representation of the firm problem with respect to \( B_{t+1} \):

\[ 1 = \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} (1 + r^*) (1 + \lambda_{t+1}), \quad t = 0, 1, 2, \ldots \]  

(3.21)

Following standard practice in the small open economy literature, we assume \( \beta (1 + r^*) = 1 \), so that

\[ \Lambda_{t+1} = \Lambda_{t+2} (1 + \lambda_{t+1}), \quad t = 0, 1, 2, \ldots \]  

(3.22)

A high value for \( \lambda \), which occurs when the collateral constraint is binding, raises the effective rate of interest on debt. The interpretation is that when \( \lambda \) is large, then the debt has an additional cost, beyond the direct interest cost. This cost reflects that when the firm raises \( B_{t+1} \) in period \( t \), it not only incurs an additional interest charge in period \( t + 1 \), but it is also further tightens its collateral constraint in that period. This has a cost because, via the collateral constraint, the extra debt inhibits the firm’s ability to acquire working capital in period \( t + 1 \). Thus, when \( \lambda \) is high, there is an additional incentive for firms to reduce \( \pi \) and ‘save’ by paying down the external debt. Although the firm’s actual interest rate on external debt taken on in period \( t \) is \( 1 + r^* \), it’s ‘effective’ interest rate is \( (1 + r^*) (1 + \lambda_{t+1}) \).

The firm’s first order conditions for labor in the nontraded and traded sectors, and for \( z \)

\footnote{See, for example, Obstfeld and Rogoff (1997).}
are, when $\xi \neq 0$:

\[
(1 - \alpha) P_t^N y_t^N \frac{Y_t}{L_t} = w_t (1 + \lambda_t) R_t \\
\left( \frac{y_t^T}{\mu_1 V_t} \right)^{\frac{1}{\xi}} \theta (1 - \nu) \frac{\mu_1 V_t}{L_t} = w_t (1 + \lambda_t) R_t \\
\left( \frac{y_t^T}{\mu_2 z_t} \right)^{\frac{1}{\xi}} (1 - \theta) \mu_2 = (1 + \lambda_t) R^* 
\]

(3.23) (3.24) (3.25)

The presence of $R_t$ on the right side of (3.23)-(3.24) reflects that to hire labor, firms must borrow cash in advance in the domestic money market, at the gross interest rate, $R_t$. When the collateral constraint is binding, then the effective interest rate is higher than $R_t$. The gross interest rate on short term foreign loans, $R^*$, appears on the right of (3.25) because firms must borrow foreign funds in advance to acquire $z_t$. Note that the effective foreign interest rate is higher than the actual interest rate when the collateral constraint is binding.

When $\xi = 0$, then of course (3.23) still holds, but (3.24) and (3.25) are replaced by:

\[
(1 - \nu) \frac{\mu_1 V_t}{L_t} \left[ 1 - \frac{(1 + \lambda_t) R^*}{\mu_2} \right] = w_t (1 + \lambda_t) R_t \\
\mu_1 V_t = \mu_2 z_t 
\]

(3.26) (3.27)

Ignoring the term in square brackets in (3.26), this is just the marginal product of $L_t^T$ in producing $\mu_1 V_t$. The term in square brackets reflects that expansions in $y^T$ also requires an increase in $z$.

3.3. Financial Intermediary and Monetary Authority

The financial intermediary takes domestic currency deposits, $D_t$, from the household at the beginning of period $t$. In addition, it receives the liquidity transfer, $X_t = x_t M_t$, from the monetary authority.\footnote{In practice, injections of liquidity do not occur in the form of lump sum transfers, as they do here. It is easy to show that our formulation is equivalent to an alternative, in which the injection occurs as a result of an open market purchase of government bonds which are owned by the household, but held by the financial intermediary. We do not adopt this interpretation in our formal model in order to conserve on notation.} It then lends all its domestic funds to firms who use it to finance their employment working capital requirements, $W_t L_t$. Clearing in the money market requires
\[ d_t + x_t = W_t L_t, \] or, after scaling by the aggregate money stock,

\[ d_t + x_t = \frac{D_t}{M_t} L_t. \]  

(3.28)

where \( d_t = D_t / M_t \).

The monetary authority in our model simply injects funds into the financial intermediary. Its period \( t \) decision is taken after the household has selected a value for \( D_t \), and before all other variables in the economy are determined. This is the standard assumption in the limited participation literature. It is interpreted as reflecting a sluggishness in the response of household portfolio decisions to changes in market variables. With this assumption, a value of \( x_t \) that deviates from what households expected at the time \( D_t \) was set produces an immediate reaction by firms and the financial intermediary but not, in the first instance, by households. The name, 'limited participation', derives from this feature, namely that not all agents react immediately to (or, 'participate in') a monetary shock. As a result of this timing assumption, many models exhibit the following behavior in equilibrium. An unexpectedly high value of \( x_t \) swells the supply of funds in the financial sector (\( D_t \) on the left side of (3.28) cannot fall in response to a positive \( x_t \) shock). To get firms to absorb the increase in funds, a fall in the equilibrium rate of interest is required. When that fall does occur, they borrow the increased funds and use them to hire more labor and produce more output.

We abstract from all other aspects of government finance. The only policy variable of the government is \( x_t \).

3.4. Equilibrium

We consider a perfect foresight, sequence-of-market equilibrium concept. In particular, it is a sequence of prices and quantities having the properties: (i) for each date, the quantities solve the household and firm problems, given the prices, and (ii) the labor, goods and domestic money markets clear.

Clearing in the money market requires that (3.28) hold and that actual money balances, \( M_t \), equal desired money balances, \( \hat{M}_t \). Combining this with the household’s cash constraint, (3.3), we obtain the equilibrium cash constraint:

\[ p_t^T p_t c_t = 1 + x_t. \]  

(3.29)

According to this, the total, end of period stock of money must equal the value of final
output, $c_t$. Market clearing in the traded good sector requires:

$$y^T_t - R^* z_t - r^* B_t - c^T_t = - (B_{t+1} - B_t).$$  \hspace{1cm} (3.30)

The left side of this expression is the current account of the balance of payments, i.e., total production of traded goods, net of foreign interest payments, net of domestic consumption. The right side of (3.30) is the change in net foreign assets. Equation (3.30) reflects our assumption that external borrowing to finance the intermediate good, $z_t$, is fully paid back at the end of the period. That is, this borrowing resembles short-term trade credit. Note, however, that this is not a binding constraint on the firm, since our setup permits the firm to finance these repayments using long term debt. Market clearing in the nontraded good sector requires:

$$y^N_t = c^N_t.$$  \hspace{1cm} (3.31)

It is instructive to study this model’s implications for interest parity. Combining the household and firm intertemporal conditions, (3.6) and (3.21), with (3.12), we obtain

$$R_{t+1} = (1 + r^*) \frac{P^T_{t+1}}{P^T_t} (1 + \lambda_{t+1}), \; t = 0, 1, 2, ...$$  \hspace{1cm} (3.32)

On the right hand side, of this expression, $(1 + r^*) \frac{P^T_{t+1}}{P^T_t}$ is the rate of interest on external debt, expressed in domestic currency units. Expression (3.32) with $\lambda = 0$ is the usual interest rate parity relation. When $\lambda > 0$, there is a collateral premium on the domestic rate of interest. Expression (3.32) highlights our implicit assumption that foreign and domestic markets for loanable funds are isolated, at least in times when the collateral constraint is binding. When $\lambda > 0$, so that the domestic interest rate exceeds the foreign rate, lenders of foreign currency would prefer to exchange their currency for domestic currency and lend in the domestic currency market. Similarly, firms borrowing domestic funds for the purpose of paying their wage bill would prefer to borrow in foreign currency market and convert the proceeds into domestic currency. That $\lambda > 0$ is possible in equilibrium reflects that we rule out this type of cross-border borrowing and lending.\textsuperscript{5}

As an empirical proposition, interest rate parity does poorly. In response to this, re-

\textsuperscript{5}Our market-segmentation assumption may capture what actually happens in the aftermath of a financial crisis. Domestic residents may be fearful of borrowing in foreign markets because of concerns about exchange risk (hedging markets tend to become very illiquid at times like this). Similarly, foreign residents may not want to lend in domestic markets. While our market-segmentation assumption may be plausible, the factors that justify it are not present in our model.
searchers often introduce exogenously a term like our $\lambda$ in (3.32). In conventional practice, $\lambda$ is interpreted as reflecting a risk premium. Our setup may provide an alternative interpretation.

Details about computing equilibrium for this model are reported in the appendix.

4. Qualitative Analysis of the Equilibrium

Our full model is not analytically tractable and so to understand its implications for the questions we ask requires numerical simulation. However, in special cases of our model it is possible to obtain analytic results, at least locally. We report these results here. We find that the implications of a model for the consequences of an interest rate cut depend sensitively on the assumptions. For example, we describe a scenario in which the presence of a collateral constraint has the effect of amplifying the expansionary effects of an interest rate cut that would occur in the absence of the constraint. We also show how the collateral constraint can cause the economy to contract in response to an interest rate cut. In this case, the contraction is marked by a reduction in foreign capital inflows.

We identify a set of assumptions under which a cut in the domestic rate of interest can produce the latter effect. Under these assumptions, $x$ is the only variable factor of production in the production of traded goods and is subject to diminishing returns; traded and non-traded goods are not very substitutable in the production of final goods; and the size of the external debt is small. The assumptions that the elasticity of substitution between traded and non-traded goods is low and that the debt is low appears crucial to the result. That is, it is possible to construct examples where a combination of the other assumptions does not hold and where an interest rate cut still produces a recession. However, in the examples considered below, a modest degree of substitution between traded and non-traded goods and a modest amount of external debt always has the consequence that an interest rate cut produces an expansion. In future drafts we plan a more systematic set of experiments in the space of model economies considered in this paper.

In the first subsection below, we describe the nature of the monetary experiments analyzed here. The second subsection identifies a particular version of our model for which we have analytic results. That section also explains why our strategy of characterizing monetary policy in terms of the interest rate simplifies the technical analysis of the model, while entailing no loss of generality. The third subsection investigates the properties of that model, and of deviations from that model.
4.1. The Nature of the Policy Experiment

In our analysis, we compare two equilibria, for $t = 0, 1, 2, \ldots$. In both, the collateral constraint is binding in each date. In each case, we characterize monetary policy by the choice of the nominal interest rate, $R_t$, in the domestic money market. In the baseline equilibrium, $R_t$ is held constant, $R_t = R_s$, in each period. Our restriction that the current account is always zero guarantees that the relative prices and quantities in this equilibrium are time-invariant. In the policy intervention equilibrium, the monetary authority unexpectedly implements a one-time drop in the interest rate in $t = 0$, i.e., $R_0 < R_s$, $R_t = R_s$ for $t \geq 1$. This drop has a non-neutral impact on allocations because of our assumption - taken from the literature on the limited participation models of money - about the timing of actions by different agents during the period. At the beginning of the period, the household makes a deposit decision. Then, the monetary authority takes its action and after that all the other period $t$ variables are determined. We assume that at the beginning of period $t = 0$, when the household makes its deposit decision, it expects $R_t = R_s$ for $t \geq 0$. At the beginning of period $t = 1, 2, \ldots$ the household expects $R_t = R_s$ despite the fact that its expectation was violated in period $t = 0$.

Given the assumptions of our model, the relative prices and quantities in the baseline and policy intervention equilibria are identical in $t \geq 1$, but they differ in $t = 0$. Our analysis focuses on this difference in period 0. In particular, we investigate what conditions guarantee that output and employment in $t = 0$ for the policy intervention equilibrium are lower than they are in the baseline equilibrium. Because they are time invariant, we refer to values of relative prices and quantities in $t \geq 0$ in the baseline equilibrium, and $t > 0$ in the policy intervention equilibrium as their steady state values. Because of the simplicity of these equilibria, the analysis has a static flavor. It only involves comparing the steady state relative prices and quantities with the $t = 0$ values of the variables in the policy intervention equilibrium.

4.2. A Simplified Model

Throughout this section, we assume $B_{t+1} = B_t$. In addition, we assume that $z$ is essential in production of the traded good, and that labor cannot be adjusted in that sector. We capture this with the specification, $\xi = \nu = \mu_1 = \mu_2 = 1$, so that the traded goods production function is given by (3.16). For simplicity, we also exclude the wage bill from the collateral constraint:

$$\tau^N_q^N K^N + \tau^T q^T K^T \geq R^\ast z + (1 + \tau^\ast)B. \quad (4.1)$$

With these simplifications, we show that we can analyze the response of the $t = 0$ values
of the variables in our model to the $t = 0$ cut in the interest rate as the intersection of two curves, each one involving the endogenous variables, $p^N$ and $L$, and the exogenous policy variable, $R_0$ (when there is no risk of confusion, we drop time subscripts). The first curve summarizes equilibrium in the labor market, and so we refer to it as the LM (‘Labor Market’) curve. The other curve, because it incorporates restrictions from the asset market, is called the AM (‘Asset Market’) curve. We now discuss these in turn. The simplicity of the analysis reflects in part the fact that we characterize policy in terms of the interest rate, rather than the money supply. The last subsection below shows that this involves no loss of generality, since there is always a money growth rate that can support any interest rate policy, as long as $R > 1$.

4.2.1. Labor Market

Equating the household and nontraded good firm Euler equations for labor, (3.5) and (3.23), we obtain:

$$RL^{\psi + \alpha} = \frac{p^N(1 - \alpha)(K)^\alpha}{\psi \eta p}.$$  

(4.2)

In this expression, it is understood that $p$ is the simple function of $p^N$ given in (3.8). As noted above, we think of $R$ as an exogenous variable. So, this expression characterizes the relationship between $L$ and $p^N$ imposed by equilibrium in the labor market. It is easy to see that this LM equation is positively sloped when graphed with $p^N$ on the vertical axis and $L$ on the horizontal. A higher $p^N$ is consistent with a higher $L$ because it shifts the labor demand curve to the right, while leaving the location of labor supply unchanged.\footnote{Following convention, we think of labor supply and demand as corresponding to the Euler equations, (3.5) and (3.23). We think of these relationships in a diagram with $W/P$ on the vertical axis and $L$ on the horizontal.} It is also easy to see that a fall in $R$ shifts the LM equation to the right. This reflects that a fall in $R$ shifts labor demand to the right and this results in an increase in equilibrium $L$ for a fixed level of $p^N$.

4.2.2. Asset Market

We now turn to the AM equation. This is constructed by combining the production functions in both sectors, (3.14), the first order condition for the intermediate input, (3.25), the $p^N$ equation, (3.9), and the collateral constraint, (4.1), under the assumption that it is binding. Substitute the expression for asset prices, (3.19), into the collateral constraint, (4.1),

---

\*\*\*The absence of a multiplier in (4.2) reflects that we now drop the wage bill from the collateral constraint.\*\*\*
evaluated with an equality and assume that \( \tau^N = \tau^T = \tau \) to obtain:

\[
\frac{T}{1 - \lambda_T} \left[ \theta y^T + \alpha p^N y^N + \Omega p_c \right] = R^* z + (1 + r^*) B, \quad (4.3)
\]

where \( \Omega = \frac{\beta}{p_c c_s} (q^N K^N + q^T K^T) \) is a constant. Absence of a time subscript indicates \( t = 0 \), and the subscript, \( s \), denotes steady state. Here, we have used the fact, \( \Lambda_2/\Lambda_1 = pc/p_s c_s \). The first two terms in the left hand side of the collateral constraint are the \( t = 0 \) \( VM P_k^t \) multiplied by the respective capital stocks and the third is the present discounted value of future cash flows. Using the zero profit condition on final consumption good firms, \( pc = c^T + p^N c^N \), we can write current spending in terms of tradables as

\[
pc = \left\{ 1 + \left[ \frac{(1 - \gamma) p^N}{\gamma} \right]^{\eta - 1} \right\} p^N c^N. \quad (4.4)
\]

Substituting this into (4.3), our expression for the collateral constraint reduces to:

\[
\frac{T}{1 - \lambda_T} \left\{ \theta y^T + \alpha \left[ 1 + \left[ \frac{(1 - \gamma) p^N}{\gamma} \right]^{\eta - 1} \right] p^N y^N \right\} = R^* z + (1 + r^*) B \quad (4.5)
\]

Equilibrium in the goods market yields the following expression for \( p^N \):

\[
p^N = \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{1}{\eta}} \left( \frac{c^T}{c^N} \right)^{\frac{1}{\eta}} = \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{1}{\eta}} \left( \frac{A \left( K^T \right)^\theta z^{1 - \theta} - R^* z - r^* B}{(K^N)^{\alpha} L^1 - \alpha} \right)^{\frac{1}{\eta}}. \quad (4.6)
\]

Finally, take into account the first order condition for \( z \):

\[
(1 - \theta) A \left( K^T \right)^\theta z^{-\theta} = (1 + \lambda) R^*. \quad (4.7)
\]

Equations (4.5), (4.6), and (4.7) represent three equations in the four unknowns, \( \lambda, z, p^N \) and \( L \). The third defines \( \lambda \) as a function of \( z \) and the second defines \( z \) as a function of \( p^N \) (it is single-valued as long as \( \lambda \geq 0 \)) and \( L \). So, the three equations can be used to define a
relationship between \( p^N \) and \( L \) alone. This relationship is what we call the AM curve.

It is clear that the slope of the AM curve is essential in determining whether an interest rate cut is expansionary or contractionary. For example, if it is downward sloped, then a shift right in the LM curve induced by a cut in the interest rate drives \( L \) up and \( p^N \) down. The contractionary case results when the AM curve is positively sloped and cuts the LM curve from below. In general, it is not possible to say what the slope of the AM curve is. We shall see in the next subsection that for particular parameter configurations, it is possible to determine the slope.

Finally, we find it useful to define the version of the AM curve that holds when the collateral constraint is not binding.\(^8\) In this case, finite \( z \) requires \( \theta > 0 \). When the collateral constraint is not binding, we lose one equation, (4.5), and one variable, \( \lambda \), from our system. As a result, the AM curve is defined simply by (4.6) and (4.7) with \( \lambda = 0 \). It is trivial to see that in this case, the AM curve is definitely downward sloped.

### 4.2.3. Equilibrium

As the previous discussion indicates, to construct the AM curve it is necessary to first compute the values of the variables in the baseline equilibrium (i.e., the steady state values of the variables). This is a straightforward exercise, which is discussed in the appendix. In the numerical experiments reported in this paper, we always found that the steady state of the model is unique.

In the remainder of this subsection we verify that for a given period 0 interest rate, \( R \), values of \( p^N \) and \( L \) defined by the intersection of the AM and LM curves correspond to a policy intervention equilibrium. By this we mean that, given such values of \( p^N \) and \( L \), values for \( p, c^N, c^T, c, w, \lambda, z, y^T, y^N, q^T, q^N, p^T, x \) can be found which satisfy all the equilibrium conditions for \( t = 0 \). Verifying that this is true for all but the last two variables is straightforward. For example, \( p \) can be constructed from \( p^N \) using (3.8), \( c^N \) can be constructed from the nontraded good production function, and so on.

We now briefly discuss the construction of \( p^T \) and \( x \). Divide the money market clearing condition, (3.28), by the equilibrium cash constraint, (3.29), to obtain:

\[
\frac{d + x}{1 + x} = \frac{w L}{p c} = \frac{p^N (1 - \alpha)}{p R L} \frac{c^N L}{c} = \frac{1 - \alpha}{R} \frac{p^N c^N}{p c} = \frac{1 - \alpha}{R} \frac{1}{\left[ 1 + \left( \frac{(1 - \gamma) p^N}{\gamma} \right)^{\eta - 1} \right]}.
\]

\(^8\)The AM curve in this case is a bit of a misnomer, since asset prices do not appear.
after using (3.23) and (4.4). Since \( d \) is predetermined at its steady state value, this expression can be used to deduce \( x \). Obviously, there is always an \( x \) that satisfies this expression, for any \( R > 1 \).\(^9\) Whether a cut in \( R \) requires that the monetary authority increases or decreases \( x \) depends upon the response of \( p^N \). We can determine \( p^T \) from (3.29).

Finally, we use a standard argument to deduce the nominal exchange rate from \( p^T \). We assume purchasing power parity in foreign and domestic traded goods. Then, taking the initial stock of money and the foreign price level as predetermined, we can interpret variations in \( p^T \) as reflecting movements in the nominal exchange rate.

### 4.3. Effects of an Interest Rate Cut

In this section, we examine the response of equilibrium outcomes at \( t = 0 \) to an interest rate cut. Consider first the case when the collateral constraint is not binding. As noted above, in this case the AM curve is downward sloping. From this we conclude:

**Proposition 1** If the collateral constraint is not binding and \( A > R^* \), then a cut in \( R \) produces a rise in \( L \), a fall in \( p^N \), and no change in \( z \).

The monetary transmission mechanism underlying this result corresponds to the standard mechanism emphasized in the literature on the limited participation model of money. A cut in \( R \) reduces the cost of hiring labor, and so results in an expansion in employment and a rise in the production of nontraded goods. The cut in the interest rate produces a fall in the marginal cost of producing nontraded goods, relative to the marginal cost of producing traded goods, and this results in the fall in \( p^N \). The central bank engineers the cut in \( R \) by producing a suitable move in \( x \).

We now turn to the case when the collateral constraint is binding in both the baseline and policy intervention equilibria. We begin with the case, \( \theta = 0 \), when \( z \) is the only factor of production in the traded good sector. In this case, a cut in \( R \) is always expansionary. When \( \theta = 0 \), substitution of (4.6) and (4.7) into (4.5) results in the following analytic representation of the AM curve:

\[
\left[ \left( \frac{\lambda_T}{1 - \lambda_T} \right) \Omega - 1 \right] \left( \frac{1 - \gamma}{\gamma} \right)^{\eta - 1}
\]

\[\text{(4.8)}\]

\(^9\)We exclude \( R \leq 1 \). For \( R < 1 \) it can be verified that there is no equilibrium. With a negative interest rate, firms could make infinite profits by borrowing infinite amounts of money. For example, a firm that borrows \( Y \) units of domestic currency at the beginning of the period returns \( RY \) units of currency to the financial intermediary at the end of the period and pays \( (1 - R)Y > 0 \) units of currency out as dividends. Dividends are made infinite by making \( Y \) infinite. No equilibrium could exist in this case, since the supply of funds in the money market at any date is bounded above. We also exclude \( R = 1 \) because our calculations assume that the equilibrium cash in advance constraint, (3.29), holds as a strict equality.
\[
\begin{align*}
\frac{\partial p^N}{\partial L} &= -\frac{\gamma y^N}{\eta[\gamma + \lambda(1 + r^*)]} \left( \frac{1}{1 - \lambda} \right) \frac{B}{B^N} y^N (1 - \eta) y^N \lambda (\alpha + \Omega) (1 - \lambda) \frac{y^N}{L}.
\end{align*}
\]

In addition, it is evident from (4.6) that when \( \theta = 0 \), \( z \) is an increasing function of \( (p^N)^{\eta} y^N \). Finally, as long as \( A > R^* \), \( \lambda \) is a positive constant.

Note first that when \( B = 0 \), (4.8) pins down a unique value for \( p^N \), so that the AM equation is horizontal. In this case, a cut in \( R \) produces a rise in \( L \) and no change in \( p^N \) or \( z \). The intuition for this is simple, and can be seen by inspecting (4.5) and (4.6). Note that, when \( B = \theta = 0 \) two things happen. First, an equiproportional rise in \( z \) and \( y^N \) produces no change in \( p^N \). This is because with \( B = \theta = 0 \) there are no diminishing returns as an increase in \( z \) produces an increase in \( c^T \). Second, for fixed \( p^N \), an equiproportional increases in \( y^N \) and \( z \) produces equiproportional increases in the left and right side of the collateral constraint. Under these circumstances, the collateral constraint simply does not get in the way of the type of expansion in output associated with an interest rate cut when the collateral constraint is nonbinding. On the contrary, the collateral constraint amplifies the response of employment to an interest rate shock by preventing the decline in \( p^N \) that would occur in the absence of that constraint, according to Proposition 1.

When \( B > 0 \) then both proportionality results cited in the previous paragraph fail, and the AM curve is no longer horizontal. For example, there are now diminishing returns in transforming additional \( z \) into extra \( c^T \). With \( B > 0 \) the AM curve has a negative slope, according to (4.8). Loosey, a rise in \( B \) produces a clockwise rotation in the AM curve. As a result, a cut in \( R \) generates a rise in \( L \) and a fall in \( p^N \) when \( B > 0 \). Equation (4.8) also shows that \( (p^N)^{\eta} y^N \) rises with the cut in \( R \) for \( 0 \leq \eta < 1 \). This implies that the cut in \( R \) generates a rise in \( z \). We summarize these findings in a proposition:

**Proposition 2** (i) When \( \theta = B = 0 \), \( A > R^* \), a cut in \( R \) produces a rise in \( L \) and \( z \), and no change in \( p^N \).

(ii) When \( \theta = 0 \), \( B > 0 \) and \( A > R^* \), a cut in \( R \) produces a rise in \( L \) and \( z \), and a fall in \( p^N \).

---

\[\text{Equation (4.8) suggests the possibility that when } \eta > 1 \text{ and large enough, then the AM curve may be positively sloped with } B > 0, \text{ perhaps even steeper than the LM curve. The latter case is the one that is required for a cut in } R \text{ to generate a recession. We have not considered this case because we view the case, } \eta > 1, \text{ as empirically implausible. Still, analysis of this case may yield insights into the nature of our model, and we plan to do this in future drafts.}

\[\text{The slope of the AM curve is given by:}
\]

\[\frac{dp^N}{dL} = -\frac{[\gamma + \lambda(1 + r^*)]B}{\eta[\gamma + \lambda(1 + r^*)]B/p^N + (1 - \eta) y^N \lambda (\alpha + \Omega) (1 - \lambda) \frac{y^N}{L}}.
\]
We conclude from this discussion that when \( \theta = 0 \), our simple environment cannot rationalize the notion that an interest rate cut produces a recession.

We now turn to the case, \( \theta > 0 \). Suppose first that \( \eta = 1 \). From (4.6), we see that \( z \) can be expressed as a function of \( p^N y^N \). According to (4.7) \( \lambda \) is a function of \( z \), and, hence of \( p^N y^N \). Substituting these results into (4.5), we conclude that when \( \theta > 0 \) and \( \eta = 1 \), the AM curve pins down \( p^N y^N \). In particular, the curve is downward-sloping. As a result, a cut in \( R \) produces a rise in \( L \) and a fall in \( p^N \). Because \( p^N y^N \) remains unchanged, it follows that \( z \) does not change. The AM curve and the LM curves before and after the cut in the interest rate are displayed in Figure 1. We summarize this finding as follows:

**Proposition 3** When \( \theta > 0 \) and \( \eta = 1 \), then a cut in \( R \) produces a rise in \( L \), a fall in \( p^N \), and no change in \( z \).

We have not been able to obtain analytic results for \( 0 \leq \eta < 1 \), when \( \theta > 0 \). However, when we linearize the AM curve about steady state we find, for \( \eta = 0 \):\(^{13}\)

\[
\frac{dp^N}{dL} = \frac{p^N}{L} \left\{ \partial y^T - (1 - \lambda \tau) \left[ r^* + \lambda (1 + r^*) \right] B \right\} (1 - \alpha).
\]

Note that when \( B = 0 \), this expression is definitely positive. If, in addition, the slope is steeper than the slope of the LM curve, we know that with a small cut in the interest rate, there is a period 0 set of equilibrium allocations in which \( L \) and \( p^N \) are both lower. In this case, \( z \) must fall too. We have constructed numerical examples with \( B = \eta = 0 \), in which the AM curve indeed does cut the LM curve from below and a cut in the interest rate does generate a drop in \( L \) and \( z \). In these examples, we verified numerically that there is a unique intersection to the LM and AM curves. Figure 2 displays the AM and LM curves for one example with \( \eta = 0 \).\(^{14}\)

So far, we have found the following. We have an example with diminishing returns in the production of traded goods, zero elasticity of substitution between traded and nontraded goods, and low external debt, in which a cut in \( R \) induces a fall in output. However, we find that substantial deviation from any one of these assumptions reverses the result.

\(^{12}\)This requires that the function mapping \( z \) into \( A(K^T)^{\theta} z^{1-\theta} - R^* z - r^* B \) be invertible. It is invertible, given that we restrict \( z \) to those values that satisfy (4.7) with \( \lambda \geq 0 \).

\(^{13}\)See the appendix for a derivation.

\(^{14}\)The parameter values underlying this example are: \( \beta = 1/1.05, \alpha = 0.25, \theta = 0.6, x_c = 0.06, \psi = 1, \psi_0 = 0.3, K_N = K^T = 1, A = 1.9, R^* = 1 + r^* = 1.05, \tau = 0.01, \gamma = 0.5, B = 0 \). When, \( \eta = 0 \), we obtain the following steady state properties for this model: \( L_s = 0.604, p_s^N c_s^N / c_s^T = 1.459, \lambda = 0.796 \). We defined GDP as \( p_s^N c_s^N + c_s^T + r^* B \). In this example, we found that this quantity drops 6 percent with a 4 percentage point cut in \( R \).
Consider, for example, the parameter, \( \eta \). We found that when it was increased to about 0.2, then an interest rate cut leads to an expansion in \( L \). However, \( z \) still falls in this case. It falls enough so that \( GDP \) falls too, when measured in base year prices. When \( \eta \) was increased to 0.3, then \( GDP \) actually rises.\(^{15}\)

Consider the effect of raising \( B \). The preceding discussion suggests the possibility that increasing the debt could rotate the AM curve clockwise from a position with positive slope to one with a negative slope. In numerical experiments we have found that this is indeed the case. Figure 3 displays the results of one such experiment. It corresponds to the model economy underlying Figure 2, except that \( B \) has been increased to 0.1, or 27 percent of \( GDP \).\(^{16}\) We find it intriguing that the addition of substantial amount of external debt can convert a situation from one in which an interest rate cut results in a contraction, into one in which it results in an expansion. The economic interpretation of this finding deserves further exploration.

We have also explored more basic perturbations on the production function, by changing \( \mu_1, \mu_2, \nu, \xi \) from their values of unity in the above examples. One consistent result we found is that reductions in \( \nu \), which opens up a role for variable labor in the production of \( y^T \), moves the system in the direction of the result that a drop in \( R \) produces an expansion in the economy.\(^{17}\) By reducing the costs associated with diminishing labor productivity of reallocating labor across sectors, dropping \( \nu \) seems to help support assets values and prevent a tightening of the collateral constraint in the wake of a cut in \( R \). This seems to operate in two ways. First, a reallocation of resources away from the nontraded good sector and towards the traded good sector limits the fall in \( p^N \) after a cut in \( R \). Other things the same, this supports asset values in that sector. Second, the allocation of labor towards the traded good sector pushes up asset values there by raising the productivity of capital in that sector. Although we did find values for \( \mu_1, \mu_2, \nu, \xi \) that imply a large reduction in output and employment after an interest rate cut, the reduction in output and employment was converted into an expansion with the introduction of a modest amount of substitutability between \( c^N \) and \( c^T \) and a modest amount of external debt.\(^{18}\)

---

\(^{15}\)When the example of the previous footnote was modified so that \( \eta = 0.2, 0.3 \) the percent change in base year \( GDP \) induced by a 4 percentage point drop in \( R \) is –0.12 and 0.3 respectively.

\(^{16}\)We did an experiment using the parameter values from the previous footnote. We set \( \eta = 0 \) and \( B = 0.4 \). In the steady state, this implies a debt to GDP ratio of 0.60, or 60 percent. We found that \( L \) and \( z \) rise 1.5 and 3.1 percent, respectively, with a 4 percentage point drop in \( R \). With \( B = 0 \), \( L \) and \( z \) both drop by 7.9 and 23 percent, respectively with the same drop in \( R \).

\(^{17}\)For example, \( \nu \) was reduced from unity to 0.85, then a four percentage point cut in \( R \) produces an 0.04 percent jump in \( GDP \) and an 0.87 percent jump in total employment. Recall from a previous footnote that when \( \nu = 1 \), then there is a 7.94 percent drop in employment and a 6 percent drop in \( GDP \).

\(^{18}\)For example, with \( \mu_1 = 1, \mu_2 = 2.1, \xi = 0.7, \nu = 0.85, B = 0 \), a four percentage point cut in \( R \) produces a 15 percent drop in total employment and a 14 percent drop in real \( GDP \). When \( B \) is then raised to 0.1 (so that the debt to GDP ratio in the steady state is 0.18) and \( \eta \) is increased to 0.3, then total employment
5. Quantitative Analysis

In this section we study versions of our model in which the external debt is endogenous. We saw in the previous section how the implications of a model for the effects of a domestic interest rate cut are sensitive to assumptions. To further clarify the nature of this sensitivity, this section analyzes two versions of our model: one that rationalizes the view that an interest rate cut reduces output and utility and another that rationalizes the opposite view.

The nature of the monetary experiment is similar to the one studied in the previous section. There is a benchmark analysis, in which monetary policy is treated as constant and the economy is confronted with a binding collateral constraint. When the debt is endogenous, the economy responds to this situation by running a current account surplus until the debt is reduced to the point where the collateral constraint is marginally non-binding. At this point, the current account drops to zero and the economy is in a steady state. We analyze the effects of a cut in the nominal rate of interest, as the economy transits to this new steady state. As in the previous section, the policy intervention has a non-neutral impact because it can affect the degree of liquidity in domestic financial markets. This in turn reflects our specification that monetary actions occur at a point in time when the household’s deposit decision is a predetermined variable.19

The impact on the transition path of the interest rate cut is very different for the two economies that we consider. The two economies differ in the way they model production in the traded good sector. In one, labor plays no role and output is the Cobb-Douglas function of $z$ and $K^T$ only given in (3.16). In the other, labor is used in the traded good sector. In this case, the production function is given by the Leontief specification in (3.15), with value-added, $V$, given by the specification in (3.14). In each model economy, the production function in the nontraded sector corresponds to the specification in (3.14). Also, in each model production of the consumption good involves zero elasticity of substitution between traded and non-traded goods, as in (3.7). Finally, preferences in the two economies are the same.

Consistent with the analysis of the previous section, we find that the model without labor in the traded good production function has the implication that output contracts, foreign capital inflows dry up and welfare falls with a cut in the domestic rate of interest. The other model implies that an economic expansion follows a cut in the interest rate.

The following section discusses the parameter values used for the two models. The section after that presents and discusses the numerical simulation results.

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19We hasten to add that it is only predetermined for one period. After that, the deposit decision is free to respond.
5.1. Parameter Values and Steady State

The parameter values for our two models are displayed in Table 2. Consider first the parameter values for the model in which labor enters in the production of tradables (the left side of Table 2). These were chosen to replicate several stylized features of emerging market economies, in particular those of some of the recent crises countries. The share of tradables in total production for Korea, assuming that tradables correspond to the non-service sectors, was approximately one third before the crisis. Combining this assumption with estimates of labor shares from A. Young (1995), we estimate shares of capital for the tradable and non-tradable sector in Korea to be respectively 0.48 and 0.21. Based on figures for Argentina, Uribe (1995) and Rebelo and Vegh (1995) estimate the same shares to be 0.52 and 0.37. We take an intermediate point between these estimates by specifying \( \nu = 0.50 \) and \( \alpha = 0.35 \). Reinhart and Vegh (1995) estimate the elasticity of intertemporal substitution in consumption for Argentina to be equal to 0.2. We adopt a somewhat higher elasticity by setting \( \sigma = 2 \). We take the foreign interest rate to be equal to 6 percent and we assume a rate of money growth of 6 percent to obtain a nominal domestic interest rate of 12.3 percent, roughly in line with the experience of Korea and Thailand in the years before the crises. We set \( \psi = 3 \), implying a labor supply elasticity of \( 1/3 \). This is low by comparison to that used in standard business cycle models. Our choice of a low labor supply elasticity is conservative. The result based on this model that we stress is that a cut in \( R \) generates and expansion. We presume that a higher labor supply elasticity would have simply resulted in a greater expansion.

As mentioned above, the share of tradable goods in production is roughly one third, so we calibrate the remaining parameters of the model to produce a ratio of consumption of nontradables to tradables of approximately 2. In addition, we chose \( \tau \) and the stock of debt in the initial steady state equilibrium so that the initial and final debt to output ratio correspond roughly to the experience of Korea and Thailand. Korea's (Thailand's) external debt started at 33% of GDP by end-1997 (60.3%) and is forecasted to be at 26.8% of GDP (51% of GDP) and the end of the year 2000. Based on these observations, we aimed to parameterize the models so that the model economy starts in the range, 30-60%, and then dropped in the range of 8-10 percentage points.

We chose comparable parameter values for the model in which labor does not appear in the traded good production function (right side of Table 2). One difference is that we chose a higher labor supply elasticity for this model. In the case of this model, we stress its implication that a cut in \( R \) generates a fall in output. We presume that a lower labor supply elasticity would only have made this contraction worse.\(^{20}\)

\(^{20}\)This conjecture is still to be verified.
Table 2: Parameter Values, Two Models Used in Analysis

<table>
<thead>
<tr>
<th></th>
<th>Labor in Traded Good Sector</th>
<th>No Labor In Traded Good Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.333</td>
<td>0.33</td>
</tr>
<tr>
<td>$\psi$</td>
<td>3.00</td>
<td>1</td>
</tr>
<tr>
<td>$R$</td>
<td>1.12</td>
<td>1.12</td>
</tr>
<tr>
<td>$R^*$</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td>$r^*$</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.36 $K^N$</td>
<td>0.36 $K^N$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.50 $K^T$</td>
<td>1</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>0.95 $\mu_2$</td>
<td>1 $\mu_2$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.16 $\theta$</td>
<td>NA</td>
</tr>
<tr>
<td>$\psi_0$</td>
<td>0.0036 $\sigma$</td>
<td>2 $\psi_0$ 0.0036 $\sigma$</td>
</tr>
<tr>
<td>$A$</td>
<td>2.40 $\xi$</td>
<td>0</td>
</tr>
<tr>
<td>$A$</td>
<td>3.2 $\xi$</td>
<td>1</td>
</tr>
</tbody>
</table>

The steady state properties of the version of the model meant to capture the pre-crisis situation, the one that ignores the collateral constraint, is reported in Table 3. The collateral constraint is imposed in period 0, and the economy eventually converges to the new steady state, one in which the collateral constraint is not binding. The properties of that steady state are reported in Table 4.
Table 3: Steady State for Two Models, Ignoring Collateral Constraint

<table>
<thead>
<tr>
<th>Labor in Traded Good Sector</th>
<th>No Labor In Traded Good Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>3.5  $z$</td>
</tr>
<tr>
<td>$L^T$</td>
<td>0.99 $L^N$</td>
</tr>
<tr>
<td>$c^T$</td>
<td>1.48 $c^N$</td>
</tr>
<tr>
<td>$w$</td>
<td>0.74 $V$</td>
</tr>
<tr>
<td>$\frac{p^Nc^N}{c^T}$</td>
<td>2.19 $y^T$</td>
</tr>
<tr>
<td>$p^N$</td>
<td>1.10 $p^T$</td>
</tr>
<tr>
<td>$q^T$</td>
<td>14.4 $q^N$</td>
</tr>
<tr>
<td>$B$</td>
<td>2.55 $\frac{B}{p^Nc^N+y^T-R^z}$</td>
</tr>
</tbody>
</table>

Table 4: Steady State for Two Models, Respecting Collateral Constraint

<table>
<thead>
<tr>
<th>Labor in Traded Good Sector</th>
<th>No Labor In Traded Good Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>3.5  $z$</td>
</tr>
<tr>
<td>$L^T$</td>
<td>0.96 $L^N$</td>
</tr>
<tr>
<td>$c^T$</td>
<td>1.49 $c^N$</td>
</tr>
<tr>
<td>$w$</td>
<td>0.74 $V$</td>
</tr>
<tr>
<td>$\frac{p^Nc^N}{c^T}$</td>
<td>2.23 $y^T$</td>
</tr>
<tr>
<td>$p^N$</td>
<td>1.11 $p^T$</td>
</tr>
<tr>
<td>$q^T$</td>
<td>14.2 $q^N$</td>
</tr>
<tr>
<td>$B$</td>
<td>1.99 $\frac{B}{p^Nc^N+y^T-R^z}$</td>
</tr>
</tbody>
</table>

5.2. Baseline Scenario

We now consider the dynamic effects of the imposition of the collateral constraint, when monetary policy takes the form of a constant money growth rate throughout the transition
to the new steady state. Figures 4a-4b shows the variables of the model in equilibrium, as the economy transits from the high initial debt to the lower level of debt in the steady state where the collateral constraint is marginally non-binding.\footnote{Here are some notes on Table 5. In each case, the top of the column corresponds to \( t = 0 \), and the bottom to \( t = 12 \). The \( t = 0 \) period corresponds to the date in which the collateral constraint is imposed. In the results in Table 5, money growth is kept constant \( x_t = x \) throughout. The correspondence between the notation in the table and the notation in the paper is as follows. B \( \sim R_t, q_n \sim q_t^N, Ln \sim L_t^N, L_t \sim L_t^T, \) pt \( \sim p_t^N, z \sim z_t, cu \sim c_t^u, \) \( c_t^L, \) lam \( \sim \lambda_t, \) yt \( \sim y_t^T, \) ca/(y pre-crisis): current account divided by pre-crisis gross steady state output, \( L_t, R_t, d \sim D_t/M_t, \) pt \( \sim p_t^T, \) p_t/pts \( \sim p_t^T/(p_t^T \) in pre-crisis steady state), gross output (in traded good units) \( p_t^N c_t^N + y_t^T, w \sim w_t, \) VMPkt \( \sim VMP_t^N, \) VMPkn \( \sim VMP_t^K, \) Value of total assets \( \tau(q_t^N K^N + q_t^T K^T), \) foreign debt \( \sim (1 + \rho) B_t + w_t R_t L_t + z_t R_t^N, \) bigam(\( i = 1 \))bigam(\( ii = 1 \)) \( \tau^{-1} \Lambda_{t-1}/\Lambda_t - 1, \) dividends \( \sim \pi_t \) form (??), \( pt = P(t)/P(t-1) \) \( \sim \pi_t = P_t/P_{t-1}, R(t)/p(t+1) \) \( \sim R_t/\pi_{t+1}, \) mrs(t,t+1) \( \sim u_{c_t,t}/(\beta \pi_{c_t+1}). \)} The results in Figures 4a-4b pertain to the economy in which labor is used in the traded good sector. Note that firms respond to the enormous 87 percentage point jump in the shadow cost of foreign borrowing by paying off the external debt \( (\lambda_0 = 0.87). \) The current account goes from zero before the collateral constraint is imposed to 3.8 percent of output in the period that it occurs. The current account remains higher than roughly one percent of output for about four years. The imposition of the collateral constraint generates a cutback in imports of \( z \) and this - because of lack of substitutability across sectors (we have \( \eta = 0 \)) - leads to a general slowdown in economic activity across the entire economy. Total employment drops roughly 20 percent, but almost all of this drop comes from the non-traded good sector. Proportionally, there is more employment in the traded good sector after the imposition of the collateral constraint. This reflects the shifting of resources towards that sector to help produce the traded goods sent abroad to pay down the external debt. The overall slowdown in economic activity contributes to a fall in asset values, as the marginal physical product of assets decline. The domestic rate of interest, \( R, \) remains high in the period of the shock, and also for several years after wards. Finally, the nominal exchange rate shows an immediate, substantial depreciation followed by a period of appreciation to correct for the initial overshooting. Similarly, there is a sharp real exchange rate depreciation (a fall in \( p^N \)), followed by a gradual appreciation.

Figures 5a-5b display the corresponding results for the model in which labor cannot be used in the traded sector. The effects are all qualitatively similar, although they tend to be larger because the debt in this model is much higher in the initial steady state than it is in the model discussed in the previous paragraph. Thus, the shadow cost of borrowing jumps 648 percent in the period of the imposition of the collateral constraint. Associated with this there is a major reduction in employment and imports of the intermediate good. Capital inflows (to finance \( z \)) display the ‘sudden stop’ feature emphasized by Calvo (1998). Finally, as in the previous paragraph, the collateral constraint triggers an immediate real
and nominal depreciation that overshoots.

We take it that these characteristics of our models correspond reasonably well, at least qualitatively, with what actually happened after the 1997 financial crises in several Asian countries (see Boom, et. al. (2000).) On this basis, we feel justified in using these models to study the effects of a cut in domestic interest rates in the wake of a financial crisis. We turn to this now.

5.3. The Effect of an Interest Rate Cut

We now suppose that in period 0, after the collateral constraint has been imposed, the monetary authority temporarily deviates from its constant money growth path. It does so by doing whatever it takes to the money supply to obtain a given reduction in the period 0 rate of interest. This policy action, which is unanticipated, is executed after the household has made its deposit decision. Agents expect, correctly, that the monetary authority will revert to its constant money growth path in \( t \geq 1 \). This one-time change policy has no impact on the ultimate steady state to which the economy is headed. It only affects the nature of the transition path. We ask what it does to the economic variables along that path, and whether things are made better or worse in a welfare sense.

The results are reported in Table 5. The cut in the interest rate is 9-10 percentage points in these two economies. It is accomplished by a one-time change in money growth in period 0, after which the steady state money growth rate of 6 percent is resumed. In explaining what happens in the two models, it is useful to center the discussion on the collateral constraint, (4.1), which we reproduce here:

\[
\tau q^N K^N + \tau q^T K^T = R^* z + (1 + r^*) B.
\]

Here, we have not included the wage bill, which we abstract from temporarily. In both model economies, the cut in the interest rate generates a nominal depreciation (see Table 5). Other things the same, this makes the left side, the asset side, of the collateral constraint fall. To see this, recall that the collateral constraint is measured in units of traded goods. So if only \( P^T \) rose, and no other price - when measured in domestic currency units - or quantity

---

\( ^{22} \)We have not been able to establish formally the uniqueness properties of the baseline equilibrium and the policy intervention equilibrium. We compute the equilibrium by solving for multipliers that satisfy a particular high dimensional equation. To build confidence in uniqueness we search for alternative solutions to this equation by initiating the numerical calculations from different initial conditions.

\( ^{23} \)This corresponds to the traditional view of monetary policy in open economies. See Musa (2000) for evidence from episodes of emerging market crises that seem to provide support for the traditional view.
changed, the asset side would fall, requiring a fall in $z$. There is another price effect that may have a similar impact on the collateral constraint. In particular, the cut in the domestic rate of interest, by having a relatively large impact on marginal costs in the non-traded good sector, may cause the relative price of goods produced in that sector, $p^N$, to fall. This has a further depressive effect on the asset side of the collateral constraint, because $p^N$ is used to value the productivity of the assets in the nontraded good sector.\footnote{Recall that $p^N$ enters in the value of the marginal product of capital in the non-traded good sector (see (3.19).)} If this were the whole story, then the interest rate cut leaves us with a mismatch between the asset and liability sides of the collateral constraint, which could only be resolved by reducing capital inflows through a cut in $z$.

But, this is not the whole story. Inspection of the collateral constraint reveals another option: one could in principle increase $z$. Of course, this has the wrong effect on the liability side of the collateral constraint. However, this problem is somewhat alleviated if the external debt is very large.\footnote{In this context, our analysis of the previous section is relevant. There we presented evidence that suggests: (i) if a model is to rationalize the notion that an interest rate cut generates a recession, then the AM curve must be positively sloped and cut the LM curve from below, and (ii) increasing the external debt rotates the AM curve clockwise. Conditions (i) and (ii) suggest that if an economy with low debt produces a recession with an interest rate cut, then the recession will be smaller or it may even turn into a boom for a model in which the external debt is higher.} In this case, the percentage increase in the liability side of the collateral constraint associated with a given rise in $z$ is small. What about the impact of a rise in $z$ on the asset side of the collateral constraint? In general - and in our models specifically - one expects the increased use of $z$ to raise the value of the economy’s assets by raising their marginal physical product. However, this channel is not strong enough if $z$ is subject to strongly diminishing returns. In this case, the rise in asset values associated with a rise in $z$ is small and likely to be dominated by the rise in liabilities.\footnote{There is another channel that could in principle be operative. A cut in the domestic interest rate, by increasing the supply of nontraded goods (recall, the interest rate cut reduces the marginal cost of those goods), raises the demand for traded goods, to the extent that these complement with nontraded goods. If so, then the shadow cost of the collateral constraint is likely to increase and this can have the effect of raising asset prices. This effect does not appear to be strong in the particular numerical examples displayed here.}

This is the situation in the model where labor does not enter the traded good sector. In that model, the equilibrium response of $z$ involves a reduction in $z$, not an increase. This reduction in $z$ sets into motion additional forces in our model which keep it falling. In particular, the lack of substitutability between traded and nontraded goods in the production of final consumption goods has the consequence that a fall in $z$ reduces demand for the nontraded good, so that employment there falls. This has the effect of further reducing asset values, aggravating the assets and liability mismatch in the collateral constraint. The effects on asset prices can be seen in Table 5, which shows that $q^T$ rises by a very small amount,
and is dominated in the collateral constraint by the fall in $q^N$. Hence, the value of assets as a whole falls.

The situation is different in the model where labor does play a role in the traded good sector. Now the option of restoring equality to the collateral constraint by increasing $z$ is a greater possibility. This is because an infusion of labor into the traded good sector can work against the diminishing returns associated with an increase in $z$. As a result, a rise in $z$ could in principle raise the asset side of the collateral constraint by more than the liability side. The tables indicate that this is precisely what happens in the model in which labor enters the traded good sector.
Table 5: Effect of Cut in $R$ at Date 0 (Relative to constant x path)

<table>
<thead>
<tr>
<th>Change in:</th>
<th>Labor in Traded Good Sector</th>
<th>No Labor In Traded Good Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>-0.10</td>
<td>-0.09</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-0.038</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Percent Change in:

<table>
<thead>
<tr>
<th>Change in:</th>
<th>Labor in Traded Good Sector</th>
<th>No Labor In Traded Good Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>3.62</td>
<td>-0.43</td>
</tr>
<tr>
<td>$z$</td>
<td>2.32</td>
<td>-2.04</td>
</tr>
<tr>
<td>$L^T$</td>
<td>4.70</td>
<td>NA</td>
</tr>
<tr>
<td>$L^N$</td>
<td>3.08</td>
<td>-0.43</td>
</tr>
<tr>
<td>$c^T$, $c^N$</td>
<td>1.96</td>
<td>-0.27</td>
</tr>
<tr>
<td>$W/P^T$</td>
<td>11.9</td>
<td>-4.53</td>
</tr>
<tr>
<td>$W/P$</td>
<td>13.62</td>
<td>4.13</td>
</tr>
<tr>
<td>Current Account</td>
<td>6.09</td>
<td>-3.89</td>
</tr>
<tr>
<td>$p^T$</td>
<td>2.13</td>
<td>4.57</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>0.99</td>
<td>1.04</td>
</tr>
<tr>
<td>$p^N$</td>
<td>0.98</td>
<td>-8.16</td>
</tr>
<tr>
<td>$q^T$</td>
<td>1.50</td>
<td>0.10</td>
</tr>
<tr>
<td>$q^N$</td>
<td>1.48</td>
<td>-0.40</td>
</tr>
<tr>
<td>$VMP^T_k$</td>
<td>4.58</td>
<td>-0.82</td>
</tr>
<tr>
<td>$VMP^N_k$</td>
<td>2.96</td>
<td>-8.41</td>
</tr>
</tbody>
</table>

We calculated the present discounted value of utility from period 0 on, for our baseline scenario and for the scenario in which the monetary authority responds by cutting the rate of interest. We did this for each of our two models. Note that utility in the steady state to which the economy converges after the collateral constraint is imposed is higher than utility.
in the pre-crisis steady state. This reflects the wealth effects of the reduced level of debt in the collateral-constrained steady state. In the case of the model with labor in the traded good sector, the present discounted utility in the equilibrium with the interest rate cut is higher than what it is when the monetary authority does not react. Utility falls with the interest rate cut in the other model.

The welfare calculation in the model with labor in the traded good sector is perhaps less interesting than it is for the other model. In both models there are forces that make the Friedman rule, $R = 1$, optimal. That cutting the interest rate might seem a good idea in a model is therefore not surprising. What is noteworthy is our finding that cutting the interest rate in the model without labor in the traded good sector generates a fall in utility. This shows concretely how counterproductive a policy like this is in such an economy.

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Present Discounted Value of Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Labor in</td>
</tr>
<tr>
<td>Pre-crisis steady state</td>
<td>-20.68759</td>
</tr>
<tr>
<td>Collateral-constrained steady state</td>
<td>-20.51759</td>
</tr>
<tr>
<td>Baseline transition path, $t \geq 0$</td>
<td>-20.81189</td>
</tr>
<tr>
<td>Path, $t \geq 0$, with interest rate cut</td>
<td>-20.79200</td>
</tr>
</tbody>
</table>

|                                                 | No Labor in                        |
| Traded Good Sector                              | -3.39104                           |
| Traded Good Sector                              | -3.33955                           |
| Baseline transition path, $t \geq 0$             | -3.94652                           |
| Path, $t \geq 0$, with interest rate cut         | -3.95739                           |

6. Conclusion

We analyzed a small open economy model in which firms require two types of working capital: domestic currency to hire domestic inputs and foreign currency to finance imports of an intermediate input. We adopt a reduced form model of a financial crisis, and ask what is the economic impact of a cut in the domestic rate of interest at such a time. We model a financial crisis as a time when collateral constraints on borrowing are imposed and are binding. Our notion of a ‘financial crisis’ corresponds to what some might think of as a ‘credit crunch’.

In our model, application of binding collateral constraints causes the economy to run a current account surplus and brings its debt down to the point where the collateral constraint is marginally non-binding. During the transition, the collateral constraint limits how much
of the intermediate good can be imported, and so output and employment are low. In addition, asset values fall with the slowdown in activity, and real and nominal exchange rates depreciate and overshoot with the onset of the crisis. These features of the transition dynamics in our model correspond - at least qualitatively - with what we observed in the Asian crisis that began in late 1997. We hope that this justifies taking our reduced form model of a financial crisis seriously, as a laboratory for studying the economic effects of a cut in the domestic interest rate in the aftermath of a financial crisis.

To understand our analysis of the effects of the interest rate cut, it is sufficient to keep in mind firms' collateral constraint: the requirement that the value of their assets be no less than the value of their liabilities. We model the former as consisting of productive assets such as land and capital in the domestic economy. Also, most of firms' liabilities take the form of international debt. Our framework captures the tensions emphasized in the literature that are created by operation of this collateral constraint.

First, an interest rate cut is likely to produce a nominal exchange rate depreciation. Other things the same, this tightens the collateral constraint by producing a fall in the value of the domestic assets of the firm, while not affecting the value of international liabilities. This effect arises from the widely discussed mismatch in the currency denomination of assets and liabilities. This effect could be compounded if in addition to a nominal depreciation, there is also a real depreciation.

We argued that unless there is something in the environment that can offset diminishing returns to the intermediate good, then the economy is likely to respond to the mismatch just described by cutting back on the purchase of the foreign intermediate good. If that good is not very substitutable in production processes in the economy, then the implication is that a general contraction must follow. We displayed a model economy with precisely this property. However, we also displayed a model economy having the property that an interest rate cut produces an expansion. In that model, there is another factor of production, labor, which can be used to resist diminishing marginal productivity of the intermediate good. The key to the expansion is that the labor is used for that purpose.

In our analysis we hope that we have been able to identify the features of the environment that are key to determining whether an interest rate cut will be expansionary or contractionary in the aftermath of a financial crisis or credit crunch. A contraction is likely to occur if the economy is relatively inflexible, if factor substitution possibilities are limited and diminishing returns are strong. If there are substantial substitution possibilities and diminishing returns are not strong, an interest rate cut can generate an expansion, even a strong one. We conclude that resolving the debate over the effects of an interest rate cut in the aftermath of a financial crisis requires understanding how much short-run flexibility there is in the economy.
7. Appendix

In this technical appendix we discuss various issues raised in the text. The first subsection discusses the computation of the steady state in the version of the model of section 4 in which the collateral constraint is binding. The second subsection derives the linearization formulas used in the local analysis in section 4. The third subsection discusses the solution of the version of our model analyzed in section 5, in which the current account is not constrained to be zero.

7.1. Steady State in the Model of Section 4

For convenience, we repeat some of the equations of the model here:

\[ RL^{\psi+\alpha} = \frac{p^N(1 - \alpha) (K^N)^\alpha}{\psi_{\alpha}^D}. \] (7.1)

The collateral constraint is:

\[ \frac{\tau N \alpha p^N (K^N)^\alpha L^{1-\alpha}}{1 - \lambda \tau N - \beta} + \frac{\tau T \theta A (K^T)^\theta z^{1-\theta}}{1 - \lambda \tau T - \beta} = R^* z + (1 + r^*) B. \] (7.2)

The first order necessary condition for \( z \) in the traded good sector is:

\[ (1 - \theta) A (K^T)^\theta z^{1-\theta} = (1 + \lambda) z R^*. \] (7.3)

The price equation is:

\[ p^N = \frac{1 - \gamma}{\gamma} \left( \frac{A (K^T)^\theta z^{1-\theta} - R^* z - r^* B}{(K^N)^\alpha L^{1-\alpha}} \right)^{\frac{1}{\eta}}. \] (7.4)

When \( \eta = 0 \), we replace (7.4) with

\[ (K^N)^\alpha L^{1-\alpha} = A (K^T)^\theta z^{1-\theta} - R^* z - r^* B. \] (7.5)
The unknowns are \( L, p^N, z, \lambda \).

We now discuss how to find the steady state when \( \theta > 0 \) and \( \eta = 0 \). [the case, \( \eta > 0 \) will be added later]. We use the equations, (7.1)-(7.3) and (7.5) to define a mapping from \( z \) to \( z' \), whose fixed point corresponds to an equilibrium. Rewrite (7.2) as follows:

\[
p^N = \frac{(1 - \lambda \tau^N - \beta) \left[ R^*z + (1 + R^*)B - \frac{\tau^T \theta A(k^T)^{\theta - 1}z}{1 - \lambda \tau^T - \beta} \right]}{\tau^N \alpha (K^N)^{\alpha - 1} L^{1 - \alpha} - \frac{R^*z}{1 - \theta} R^* - R^*}, \tag{7.6}
\]

Combining (7.5) and (7.3), we obtain (the discussion below assumes \( B = 0 \), which will be fixed later):

\[
(K^N)^{\alpha} L^{1 - \alpha} = z \left[ A(k^T)^{\theta} z - R^* \right] = \frac{1 + \lambda}{1 - \theta} R^* - R^* = \frac{\lambda + \theta}{1 - \theta} R^*z \tag{7.7}
\]

or,

\[
L = \left[ \frac{\lambda + \theta}{1 - \theta (K^N)^{\alpha}} \right]^{\frac{1}{1 - \alpha}}. \tag{7.8}
\]

Note that (7.3) defines a mapping from \( z \) to \( \lambda \). Taking this and (7.8) into account, (7.6) defines a mapping from \( z \) to \( p^N \):

\[
p^N = f(z), \tag{7.9}
\]

where

\[
f(z) = \frac{1 - \lambda \tau^N - \beta (1 - \theta)}{1 - \lambda \tau^T - \beta (\lambda + \theta)} \left[ (1 - \lambda \tau^T - \beta) - \frac{\tau^T (1 + \lambda \theta) / (1 - \theta)}{\tau^N \alpha} \right]. \tag{7.10}
\]

Here, it is understood that \( \lambda \) is the function of \( z \) implied by (7.3).

Solving (3.1) for \( L \):

\[
L = \left\{ \frac{(1 - \alpha) (K^N)^{\alpha}}{R \psi_0 \left[ \frac{1}{p^*} + 1 \right]} \right\}^{\frac{1}{1 - \alpha}}. \tag{7.11}
\]
Combining this with (7.7), we obtain:

\[
\left[ \frac{\lambda + \theta}{1 - \theta} \right]^{1 - \alpha} \left[ \frac{R^* z}{(K^N)^{\alpha}} \right]^{1 - \alpha} = \left[ \frac{(1 - \alpha) \left( K^N \right)^{\alpha}}{R\psi_0 \left( \frac{1}{\tau^N} + 1 \right)} \right]^{1 - \alpha}.
\]

Solve this for \( z \):

\[
z = \frac{1 - \theta \left( K^N \right)^{\alpha}}{\lambda + \theta} \frac{R^*}{\left[ \frac{(1 - \alpha) \left( K^N \right)^{\alpha}}{R\psi_0 \left( \frac{1}{\tau^N} + 1 \right)} \right]^{1 - \alpha}} = g(p^N, \lambda),
\]

say. We can use this and \( f \) in (7.9) to define a mapping from \( z \) into itself:

\[
z' = g(p^N, \lambda) = g(f(z), \lambda(z)) = h(z),
\]

say, where \( \lambda(z) \) summarizes (7.3). It is easy to see that \( h \) is an increasing function of \( z \) as long as \( \tau^N = \tau^T \).

To actually find the fixed point, if it exists, it is useful to be able to restrict the set of candidate equilibrium values of \( z \). We know that we must have \( \lambda \geq 0 \), \((1 - \lambda r^N - \beta) \geq 0\), \((1 - \lambda r^T - \beta) \geq 0\), \( p^N \geq 0 \). The first of these implies an upper bound on \( z \), and the others imply a lower bound. (These conditions imply \( I \geq 0 \), so we don’t list that separately.) From (7.9) we see that \( p^N \geq 0 \) requires:

\[
\lambda \leq \frac{1 - \theta}{\tau^T} \left[ 1 - \beta - \frac{\theta}{1 - \theta} \right] = \frac{(1 - \theta)(1 - \beta)}{\tau^T} - \theta
\]

This places a lower bound on \( z \):

\[
z \geq \left\{ \frac{R^*}{(1 - \theta) A (K^T)^{\alpha}} \left[ \frac{(1 - \theta)(1 - \beta)}{\tau^T} - \theta + 1 \right] \right\}^{-1}.
\]

We want this lower bound (of course!) to be less than the upper bound on \( z \) implied by
\( \lambda \geq 0 \). This places a restriction on the parameters:

\[
\theta \leq \frac{(1 - \theta)(1 - \beta)}{\tau^T},
\]

or,

\[
\tau^T \leq \frac{(1 - \theta)(1 - \beta)}{\theta}.
\]

This is a pretty tight upper bound on \( \tau^T \).

The value of a unit of capital in the non-traded and traded good sectors is, respectively:

\[
q^N = \frac{\alpha p^N (K^N)^{\alpha-1} L^{1-\alpha}}{1 - \lambda \tau^N - \beta}, \quad q^T = \frac{\theta A (K^T)^{\alpha-1} z^{1-\theta}}{1 - \lambda \tau^T - \beta}.
\]

7.2. Linearization of the Model in Section 4

We derive provide formulas for linearizing the model of section 4 about its steady state. Define the percent deviation of a variable from its steady state value as \( \hat{x} = dx/x \).

Linearizing the \( LM \) curve around the steady state we obtain:

\[
\dot{R} + (\psi + \alpha) \dot{L} = \dot{p}^N - \dot{p}.
\]

or

\[
\dot{R}_t + (\psi + \alpha) \dot{L}_t = \left[ 1 - \left( \frac{p_{t+1}}{p^N_t} \right)^{\eta^{-1}} \right] \dot{p}_t^N,
\]

where we have used the fact that

\[
\dot{p} = \left( \frac{p_{t+1}}{p^N_t} \right)^{\eta^{-1}} \dot{p}^N.
\]

Finally, rearrange to obtain

\[
\dot{p}_t^N = \frac{\dot{R}_t + (\psi + \alpha) \dot{L}_t}{1 - \left( \frac{p_{t+1}}{p^N_t} \right)^{\eta^{-1}}}, \quad (7.12)
\]
where
\[ 1 - \left( \frac{p \gamma}{p^N} \right)^{\eta-1} \geq 0, \]

after evaluating \( p \) in terms of \( p^N \).

To linearize the AM curve, begin with the \( p^N \) equation:

\[ \ddot{p}^N = -\frac{1 - \alpha}{\eta} \dot{L} + \frac{1}{\eta} \dot{c}^T \]

From the resource constraint we obtain:

\[ d \dot{c}^T = \left[ (1 - \theta) A \left( K^T \right)^\theta z^{-\theta} - R^* \right] dz, \]

so that

\[ \dot{c}^T = \left( \frac{(1 - \theta) A \left( K^T \right)^\theta z^{-\theta} - R^*}{\dot{c}^T} \right) z \dot{z}, \]

where \( \dot{z} = dz/z \). Then,

\[ \ddot{p}^N = -\frac{1 - \alpha}{\eta} \dot{L} + \frac{1}{\eta} \left[ (1 - \theta) A \left( K^T \right)^\theta z^{-\theta} - R^* \right] \frac{z}{\dot{c}^T} \dot{z}, \]

or

\[ \ddot{p}^N = -\frac{1 - \alpha}{\eta} \dot{L} + \left( \frac{\lambda R^* z}{\eta \dot{c}^T} \right) \dot{z}. \]

We can now obtain \( \dot{z} \) as a function of \( \ddot{p}^N \) as follows:

\[ \dot{z} = \frac{1 - \alpha}{\eta} \frac{\dot{L} + \ddot{p}^N}{\lambda R^* z} = \frac{\dot{c}^T}{\lambda R^* z} \left[ (1 - \alpha) \dot{L} + \eta \ddot{p}^N \right], \quad (7.13) \]

From the first order condition for labor in the traded goods sector, \( (1 - \theta) A \left( K^T \right)^\theta z^{-\theta} = (1 + \lambda) R^* \), we obtain

\[ -\theta \dot{z} = (1 + \lambda) = \frac{(1 + \lambda t) - (1 + \lambda)}{(1 + \lambda)} = \frac{\lambda t - \lambda}{1 + \lambda}, \quad (7.14) \]
\[
\frac{\tau}{1 - \lambda \tau} R^* z \lambda \dot{\lambda} \\
+ \left( \frac{\tau}{1 - \lambda \tau} \right) \left( \alpha + \Omega \left[ 1 + \left( \frac{(1 - \gamma) p^N}{\gamma} \right)^{\eta - 1} \right] \right) (1 - \alpha) p^N (K^N)^{\alpha} L^{1 - \alpha} \dot{L} \\
- \left( \frac{\tau}{1 - \lambda \tau} \right) \left\{ \alpha p^N (K^N)^{\alpha} L^{1 - \alpha} \right\} \dot{p}^N \\
= \left\{ R^* z - \frac{(1 - \theta) \tau \theta A (K^T)^{\theta} z^{1 - \theta}}{1 - \lambda \tau} \right\} \dot{z}
\]

or,

\[
\frac{\tau}{1 - \lambda \tau} R^* z \lambda \dot{\lambda} \\
+ \left( \frac{\tau}{1 - \lambda \tau} \right) \left( \alpha + \Omega \left[ 1 + \left( \frac{(1 - \gamma) p^N}{\gamma} \right)^{\eta - 1} \right] \right) (1 - \alpha) p^N (K^N)^{\alpha} L^{1 - \alpha} \dot{L} \\
+ \left( \frac{\tau}{1 - \lambda \tau} \right) \left\{ p^N (K^N)^{\alpha} L^{1 - \alpha} \left[ \alpha + \Omega \left[ 1 + \eta \left( \frac{(1 - \gamma) p^N}{\gamma} \right)^{\eta - 1} \right] \right] \right\} \dot{p}^N \\
= \left\{ R^* z - \frac{(1 - \theta) \tau \theta A (K^T)^{\theta} z^{1 - \theta}}{1 - \lambda \tau} \right\} \dot{z}
\]

Let’s simplify the expression on \((1 - \alpha) \dot{L}\). According to the collateral constraint:

\[
\frac{\tau}{1 - \lambda \tau} \left[ \theta A (K^T)^{\theta} z^{1 - \theta} + \left( \alpha + \Omega \left[ 1 + \left( \frac{(1 - \gamma) p^N}{\gamma} \right)^{\eta - 1} \right] \right) p^N (K^N)^{\alpha} L^{1 - \alpha} \right] = R^* z + (1 + r^*) B
\]
so that,

\[
\frac{\tau \lambda}{1 - \lambda T} \left( \alpha + \Omega \left\{ 1 + \left[ \frac{(1 - \gamma)p^N}{\gamma} \right]^\eta - 1 \right\} \right) p^N y^N
\]

\[= \lambda R^* z + \lambda (1 + r^*) B - \frac{\tau \lambda}{1 - \lambda T} \theta y^T \]

\[= (1 - \theta) y^T - R^* z - \frac{\tau \lambda}{1 - \lambda T} \theta y^T + \lambda (1 + r^*) B \]

\[= y^T - \theta y^T \frac{1}{1 - \lambda T} - R^* z + \lambda (1 + r^*) B \]

\[= c^T + r^* B - \frac{\theta y^T}{1 - \lambda T} + \lambda (1 + r^*) B \]

\[= c^T - \frac{\theta y^T}{1 - \lambda T} + [r^* + \lambda (1 + r^*)] B \]

Now consider the term on \(p^N\).

\[
\eta c^T - \frac{\tau \lambda}{1 - \lambda T} \left( \alpha + \Omega \left\{ 1 + \eta \left[ \frac{(1 - \gamma)p^N}{\gamma} \right]^\eta - 1 \right\} \right) p^N y^N
\]

\[= c^T - \frac{\tau \lambda}{1 - \lambda T} \left( \alpha + \Omega \left\{ 1 + \left[ \frac{(1 - \gamma)p^N}{\gamma} \right]^\eta - 1 \right\} \right) p^N y^N \]

\[+(\eta - 1) \left( c^T - \frac{\tau \lambda}{1 - \lambda T} \Omega \left[ \frac{(1 - \gamma)p^N}{\gamma} \right]^\eta - 1 p^N y^N \right) \]

\[= \frac{\theta y^T}{1 - \lambda T} - [r^* + \lambda (1 + r^*)] B + (\eta - 1) \left( c^T - \frac{\tau \lambda}{1 - \lambda T} \Omega \left[ \frac{(1 - \gamma)p^N}{\gamma} \right]^\eta - 1 p^N y^N \right) . \]

But,

\[
c^T - \frac{\tau \lambda}{1 - \lambda T} \left[ \frac{(1 - \gamma)p^N}{\gamma} \right]^\eta - 1 p^N y^N \]

\[= y^T - R^* z - r^* B - \frac{\tau \lambda}{1 - \lambda T} \Omega \left[ \frac{(1 - \gamma)p^N}{\gamma} \right]^\eta - 1 p^N y^N \]

\[= y^T - (1 + \lambda) R^* z + \lambda [R^* z + (1 + r^*) B] \]

\[-\lambda (1 + r^*) B - r^* B - \frac{\tau \lambda}{1 - \lambda T} \Omega \left[ \frac{(1 - \gamma)p^N}{\gamma} \right]^\eta - 1 p^N y^N \]

\[= y^T - (1 + \lambda) R^* z + \lambda [R^* z + (1 + r^*) B] - \lambda (1 + r^*) B - r^* B - \frac{\tau \lambda}{1 - \lambda T} \Omega \left[ \frac{(1 - \gamma)p^N}{\gamma} \right]^\eta - 1 p^N y^N \]
\begin{align*}
&= y^T + \frac{\tau \lambda}{1 - \lambda \tau} \left[ \theta A \left( K^T \right)^\theta z^{1-\theta} + \left( \alpha + \Omega \left\{ \frac{1 + \left( \frac{(1 - \gamma) p^N}{\gamma} \right)^{n-1}}{\gamma} \right\} \right) p^N y^N \right] \\
&\quad - \frac{\tau \lambda}{1 - \lambda \tau} \Omega \left( \frac{(1 - \gamma) p^N}{\gamma} \right)^{n-1} p^N y^N - (1 + \lambda) R^* z - [r^* + \lambda (1 + r^*)] B \\
&= y^T - (1 - \theta) y^T + \frac{\tau \lambda}{1 - \lambda \tau} \left[ \theta A \left( K^T \right)^\theta z^{1-\theta} + (\alpha + \Omega) p^N y^N \right] - [r^* + \lambda (1 + r^*)] B \\
&= \theta y^T + \frac{\tau \lambda}{1 - \lambda \tau} \left[ \theta y^T + (\alpha + \Omega) p^N y^N \right] - [r^* + \lambda (1 + r^*)] B \\
&= \theta y^T + \frac{\tau \lambda}{1 - \lambda \tau} (\alpha + \Omega) p^N y^N - [r^* + \lambda (1 + r^*)] B \\
\end{align*}

So:

\begin{align*}
&= \frac{\tau \lambda}{1 - \lambda \tau} \left( \alpha + \Omega \left\{ \frac{1 + \eta \left( \frac{(1 - \gamma) p^N}{\gamma} \right)^{n-1}}{\gamma} \right\} \right) p^N y^N \\
&= \theta y^T - \left\{ \frac{\theta y^T}{1 - \lambda \tau} - [r^* + \lambda (1 + r^*)] B + (\eta - 1) \left( c^T - \frac{\tau \lambda}{1 - \lambda \tau} \Omega \left[ \frac{(1 - \gamma) p^N}{\gamma} \right]^{n-1} p^N y^N \right) \right\} \\
&= \theta y^T - \left\{ \frac{\theta y^T}{1 - \lambda \tau} - [r^* + \lambda (1 + r^*)] B + \\
&\quad (\eta - 1) \left( \frac{\theta y^T}{1 - \lambda \tau} + \frac{\tau \lambda}{1 - \lambda \tau} (\alpha + \Omega) p^N y^N - [r^* + \lambda (1 + r^*)] B \right) \right\} \\
&= \theta y^T - \left\{ \frac{\theta y^T}{1 - \lambda \tau} - \eta [r^* + \lambda (1 + r^*)] B + (\eta - 1) \left( \frac{\theta y^T}{1 - \lambda \tau} + \frac{\tau \lambda}{1 - \lambda \tau} (\alpha + \Omega) p^N y^N \right) \right\} \\
\end{align*}

Substituting the above expression into (7.15):

\begin{align*}
&= \frac{\tau}{1 - \lambda \tau} R^* z \hat{\lambda} \\
&+ \frac{1}{\lambda} \left[ c^T - \frac{\theta y^T}{1 - \lambda \tau} + [r^* + \lambda (1 + r^*)] B \right] (1 - \alpha) \hat{L} \\
&+ \frac{1}{\lambda} \left[ \eta c^T - \frac{\theta y^T}{1 - \lambda \tau} - (\eta - 1) \frac{\tau \lambda}{1 - \lambda \tau} (\alpha + \Omega) p^N y^N + \eta [r^* + \lambda (1 + r^*)] B \right] \hat{p}^N \\
&= \left\{ R^* z - \frac{(1 - \theta) \tau \theta A \left( K^T \right)^\theta z^{1-\theta}}{1 - \lambda \tau} \right\} \hat{\varepsilon}
\end{align*}

Substituting out for \( \hat{\lambda} \) in terms of \( \hat{\varepsilon} \) from (7.14) and for \( \hat{\varepsilon} \) in terms of \( \hat{L} \) and \( \hat{p}^N \) from (7.13),
we obtain:

\[-\frac{\tau}{1 - \lambda\tau} R^* z \theta (1 + \lambda) \frac{c^T}{\lambda R^* z} [(1 - \alpha) \dot{L} + \eta \dot{p}^N]\]
\[+ \frac{1}{\lambda} \left[ c^T - \frac{\theta y^T}{1 - \lambda\tau} + \frac{[r^* + \lambda(1 + r^*)] B}{\lambda} \right] (1 - \alpha) \dot{L} \]
\[+ \frac{1}{\lambda} \left[ \eta c^T - \eta \frac{\theta y^T}{1 - \lambda\tau} - (\eta - 1) \tau \frac{\lambda}{1 - \lambda\tau} (\alpha + \Omega) p^N y^N + \eta \left[ r^* + \lambda(1 + r^*) \right] B \right] \dot{p}^N\]
\[= \left\{ R^* z - \frac{(1 - \theta) \tau^{T} \theta A (K^T)^{\theta} Z^{1 - \theta}}{1 - \lambda\tau^T} \right\} \frac{c^T}{\lambda R^* z} [(1 - \alpha) \dot{L} + \eta \dot{p}^N]\]

Collecting terms and simplifying (using the first order necessary condition for z and the collateral constraint):

\[\frac{1}{\lambda} \left[ c^T - \frac{\theta y^T}{1 - \lambda\tau} + \frac{[r^* + \lambda(1 + r^*)] B}{\lambda} \right] (1 - \alpha) \dot{L} \]
\[+ \frac{1}{\lambda} \left[ \eta c^T - \eta \frac{\theta y^T}{1 - \lambda\tau} - (\eta - 1) \tau \frac{\lambda}{1 - \lambda\tau} (\alpha + \Omega) p^N y^N + \eta \left[ r^* + \lambda(1 + r^*) \right] B \right] \dot{p}^N\]
\[= \left\{ R^* z + \frac{\tau}{1 - \lambda\tau} R^* z \theta (1 + \lambda) - \left( \frac{\tau}{1 - \lambda\tau} \right) [\theta (1 + \lambda) R^* z] \right\}
\times \frac{c^T}{\lambda R^* z} [(1 - \alpha) \dot{L} + \eta \dot{p}^N]\]
\[= \frac{c^T}{\lambda} [(1 - \alpha) \dot{L} + \eta \dot{p}^N]\]

or

\[\left[ \frac{1}{\lambda} \left( c^T - \frac{\theta y^T}{1 - \lambda\tau} \right) - \frac{c^T}{\lambda} + \frac{[r^* + \lambda(1 + r^*)] B}{\lambda} \right] (1 - \alpha) \dot{L} \]
\[+ \left[ \frac{1}{\lambda} \left( \eta c^T - \eta \frac{\theta y^T}{1 - \lambda\tau} - (\eta - 1) \tau \frac{\lambda}{1 - \lambda\tau} (\alpha + \Omega) p^N y^N \right) - \eta \frac{c^T}{\lambda} + \frac{[r^* + \lambda(1 + r^*)] B}{\lambda} \right] \dot{p}^N\]
\[= 0\]

Multiply by \(\lambda\), cancel terms, as appropriate, multiply by \(1 - \lambda\tau\), to get:

\[\left( -\eta \theta y^T - (\eta - 1) \tau \lambda (\alpha + \Omega) p^N y^N + (1 - \lambda\tau) \eta \left[ r^* + \lambda(1 + r^*) \right] B \right) \dot{p}^N\]
\[
\{ \theta y^T - (1 - \lambda r) [r^* + \lambda(1 + r^*)] B \} (1 - \alpha) \hat{L}.
\]

or,

\[
\frac{\hat{p}^N}{L} = \frac{\{ \theta y^T - (1 - \lambda r) [r^* + \lambda(1 + r^*)] B \} (1 - \alpha)}{\eta \theta y^T + (\eta - 1) r \lambda (\alpha + \Omega)p^N y^N - (1 - \lambda r) \eta [r^* + \lambda(1 + r^*)] B} = s^{AM},
\]

say, where \( s^{AM} \) is the slope of the AM curve. From this expression we can see that for the Cobb-Douglas case \( \eta = 1 \), the slope is definitely negative:

\[
\frac{\hat{p}^N}{L} = -(1 - \alpha) < 0
\]

So, with perfect substitutability, the AM curve slopes downward, and a cut in \( R \) must produce a fall in \( p^N \) and a rise in \( L \). With \( \eta = 0 \):

\[
\frac{\hat{p}^N}{L} = \frac{\{ \theta y^T - (1 - \lambda r) [r^* + \lambda(1 + r^*)] B \} (1 - \alpha)}{r \lambda (\alpha + \Omega)p^N y^N}.
\]

Note that when \( B = 0 \), this expression is definitely positive. It cannot be signed when \( B > 0 \).

By substituting out for \( \hat{p}^N \) in (7.16) from the LM curve, (7.12), we find:

\[
\frac{\hat{R}}{1 - (\eta N)} + s^{LM} \hat{L} = s^{AM} \hat{L},
\]

where

\[
s^{LM} = \frac{\psi + \alpha}{1 - (\eta N)}.
\]

Then,

\[
\frac{\hat{L}}{\hat{R}} = \frac{1}{(s^{AM} - s^{LM}) \left(1 - (\eta N)\right)}.
\]

So, for a cut in \( R \), \( \hat{R} < 0 \), to produce an equilibrium drop in employment, \( \hat{L} < 0 \), we need \( s^{AM} > s^{LM} \). Since the latter is positive, this requires that \( s^{AM} \) be more than just positive.
It must be big enough. The condition is:

\[
\frac{\theta y^T - (1 - \lambda \tau) [r^* + \lambda (1 + r^*)] B}{\eta \theta y^T + (\eta - 1) \tau \lambda (\alpha + \Omega)p^N y^N - (1 - \lambda \tau) \eta [r^* + \lambda (1 + r^*)] B} > \frac{\psi + \alpha}{1 - \left(\frac{e^N}{\eta p}\right)^{1-\eta}}.
\]

7.3. Solving the Main Model

We stress the case where there is substitution between $c^N$ and $c^T$, and between $z_t$ and $V_t$. The case of no substitutability involves obvious modifications on the discussion below and so is not included. A technical manuscript which covers this case in detail is available on request.

We begin with a discussion of the steady state in which all real variables and relative prices with time subscripts are constant. We consider two steady states. The pre-crisis steady state is one in which the initial level of debt, say $B_0$, violates (3.18). We define the post-crisis steady state as one in which the initial level of debt, say $B_\infty$, has the property that the collateral constraint is satisfied as an equality. We then discuss the computation of the dynamic path taking the economy from first steady state to the second. We do this under two scenarios. In our baseline scenario government policy, defined here in terms of money growth, does not change. In the alternative scenario the money growth rate is changed in the period when the collateral constraint is imposed.

7.3.1. Steady State

To understand how these steady states are computed, it is useful to note that - subject to feasibility - corresponding to any initial level of debt there is a unique steady state when the collateral constraint, (3.18), is ignored. To find the post crisis steady state, we simply alter the initial debt until the collateral constraint is satisfied as an equality. Then, $B_0$ is selected as a number bigger than $B_\infty$, to be consistent with data as discussed in the text.

The following discussion explains how we find the steady state corresponding to an arbitrary initial value of the debt.

The steady state interest rate, $R$, is determined from (3.6), (3.13), and the fact that $p_t = P_t/M_t$ is constant in steady state:

\[
R = \frac{1 + x}{\beta}.
\]

From here on, we treat $R$ as a known quantity.
Rewriting the firm’s first order condition for $z$:

$$y^T = \mu_2 z \left( \frac{R^*}{\mu_2} \right)^\xi.$$

The resource constraint in the traded good sector is:

$$y^T - c^T - R^* z = r^* B.$$

Combining this with the previous expression,

$$z R^* \left[ \left( \frac{R^*}{\mu_2} \right)^{\xi-1} - 1 \right] = c^T + r^* B. \quad (7.17)$$

Here is an algorithm for finding the steady state which involves a nonlinear search in the single variable, $c^T$.

Suppose $c^T$ is given. Then, $z$ can be computed from (7.17). $L^T$ may then be obtained from:

$$z = \frac{\mu_1 V}{\mu_2} \left\{ \left( \frac{R^*}{\mu_2} \right)^{\xi-1} - 1 \right\}^{\frac{1}{\xi-1}}. \quad (7.18)$$

Given $L^T$, $L^N$ may be obtained by combining the price equation:

$$p^N = \gamma \left[ \frac{1 - \gamma}{\gamma c^N} \left( \frac{1 - \gamma}{\gamma c^T} \right)^{\frac{1}{\xi}} \right] \frac{1}{1 - \gamma} \left( \frac{1 - \gamma}{\gamma c^N} \right)^{\frac{1}{\xi}}. \quad (7.19)$$

and equality of $VMP_L$’s across the two sectors:

$$(1 - \alpha)p^N \left( K^N \right)^{\alpha} \left( L^N \right)^{-\alpha} = \left\{ 1 - \left( \frac{R^*}{\mu_2} \right)^{1-\xi} \right\}^{\frac{1}{1-\xi}} (1 - \theta)\mu_1 A \left( K^T \right)^{\theta} \left( L^T \right)^{-\theta}.$$
Substitute the former into the latter, to obtain:

\[
(1 - \alpha) \frac{\gamma}{1 - \gamma} \left( \frac{(1 - \gamma) c^T}{K^N} \right)^{\frac{1}{\gamma}} \left( K^N \right)^\alpha \left( L^N \right)^{-\alpha} = \left\{ 1 - \left( \frac{R^*}{\mu_2} \right)^{1-\xi} \right\} ^{-\frac{1}{\nu_2}} (1-\theta) \mu_1 A \left( K^T \right)^\theta \left( L^T \right)^{-\theta},
\]

or

\[
\left( L^N \right)^{-[\frac{1}{\eta}(1-\alpha)+\alpha]} = D, \tag{7.20}
\]

where

\[
D = \left\{ 1 - \left( \frac{R^*}{\mu_2} \right)^{1-\xi} \right\} ^{-\frac{1}{\nu_2}} (1-\theta) \mu_1 A \left( K^T \right)^\theta \left( L^T \right)^{-\theta}
\]

With \( L^N, c^T \) in hand, compute \( p^N \) from the price equation, (7.19). Finally, assess whether labor supply equals labor demand in the traded good sector:

\[
\frac{1}{R} \left\{ 1 - \left( \frac{R^*}{\mu_2} \right)^{1-\xi} \right\} ^{-\frac{1}{\nu_2}} (1-\theta) \mu_1 A \left( K^T \right)^\theta \left( L^T \right)^{-\theta} = \psi_0 \left( L^T + L^N \right)^\psi.
\]  

(7.21)

Adjust \( c^T \) until (7.21) holds exactly. The five equations, (7.17)-(7.21), can be used in this way to pin down the five variables, \( L^N, L^T, p^N, c^T, z \).

The variables, \( p \) and \( c \), may be obtained from (7.24) and (7.25). Then, obtain \( p^T \) from

\[
pp^T c = 1 + x.
\]

The wage rate comes from

\[
w = \psi_0 L^\psi p, \quad L = L^T + L^N
\]

We now obtain the steady state value of \( d \), the ratio of deposits to the beginning of period money stock. Dividing (3.3) by \( P^T \):

\[
pc = wL + \frac{1}{p^T} - \frac{d}{p^T},
\]
so that
\[ d = p^T [wL - pc] + 1. \]

We require \( 0 \leq d \leq 1. \)

Now we go for the asset values. From (3.20),
\[ q^i = VMP^i_K + \frac{q^i}{1 + r^*}, \]
so that
\[ q^i = \frac{1 + r^*}{r^*} VMP^i_K, \quad i = N, T, \]
where
\[ VMP^N_K = p^N \frac{\partial y^N}{K^N}, \]
\[ VMP^T_K = \left[ \frac{y^T}{\mu_1 V} \right]^{\frac{1}{\theta \mu_1 V}} \frac{1}{K^T}. \]

The value of collateral is, in units of the traded good,
\[ q^N K^N + q^T K^T \]

This completes the discussion of the steady state.

7.3.2. The Transition Path

We imagine that in date 0 the economy has an initial debt level of \( B_0 \). At this level of debt the collateral constraint is binding. In the baseline equilibrium, money growth is kept constant. That is, \( x_t = x \) for \( t = 0, 1, 2, \ldots \). We compute the equilibrium path of the economy to the new steady state where the debt level is \( B_\infty \). In the second equilibrium, \( x_0 \neq x \), but \( x_t = x \) for \( t = 1, 2, \ldots \). In this equilibrium, the monetary adjustment is unanticipated in the sense that when households make their deposit decision in the beginning of period 0 they do so under the assumption that they are in the baseline equilibrium. As noted above, they do not adjust this decision when it turns out that \( x_0 \neq x \).

We first consider the computation of the baseline equilibrium. We then discuss the computation of the equilibrium in which there is a monetary intervention. The basic strategy
is based on solving a system of non-linear equations in the Lagrange multipliers on the collateral constraint. For a given set of Lagrange multipliers, we compute a sequence of candidate allocations and prices, imposing the following conditions: (i) quantities and prices eventually end up in the new steady state; (ii) the initial level of debt is \( B_0 \); and (iii) all equilibrium conditions except the collateral constraint are imposed. We then evaluate the collateral constraint at each date. We adjust the Lagrange multipliers until it is satisfied. If the multipliers turn out to violate non-negativity, then we conclude there is no equilibrium.

**Baseline Scenario** At date 0, \( B_0 \) is given. We want to compute an equilibrium set of sequences,

\[
q_t^T, q_t^N, c_t^T, c_t^N, I_t^T, I_t^N, p_t^N, p_t^T, R_t, w_t, z_t, B_{t+1}, \lambda_t, \ t = 0, 1, 2, 3, \ldots
\]

for a given sequence of \( x_t \)'s. These 13 sequences must satisfy 13 equilibrium conditions. These are the two equations defining the \( q \)'s (3.20); the firm’s intertemporal Euler equation (3.22), and its three intra-temporal Euler equations, (3.23), (3.24), (3.25); the marginal condition relating \( p^N \) to the consumption goods, (3.9); the resource constraint in the traded and nontraded good sectors, (3.30) and (3.31); the collateral constraint, (3.18); the household’s intra- and inter-temporal Euler equations, (3.5), (3.6); and finally, the cash in advance constraint, (3.29).\(^{27}\)

We seek an equilibrium which converges asymptotically to the steady state where the debt is \( B_\infty \) and the collateral constraint is marginally non-binding. This means that all the above sequences converge to their values in the steady state equilibrium whose computation is discussed in the previous section.

Here is our strategy for accomplishing this. We assume that the system arrives in a steady state in period \( T + 2 \) (in practice, we found that \( T = 10 \) works well.) We specify exogenously (below, we explain in detail how this is done), a sequence, \( \lambda \equiv (\lambda_0, \lambda_1, \ldots, \lambda_{T+1}) \), with \( \lambda_t = 0 \) for \( t \geq T + 2 \). Also, \( \Lambda_{T+2} = \Lambda_s \), where the subscript, ‘s’, means steady state. Similarly, all the other 13 variables are assumed to be in the new steady state for \( t \geq T + 2 \). The value of \( T \) that we used in the calculations satisfies the property that the economy has for all practical purposes achieved convergence to the new steady state before \( T + 2 \).

\(^{27}\)Equilibrium also requires that the limiting condition, (3.17), be satisfied. We can verify that this is satisfied ex post, when we have found a set of Lagrange multipliers which produce allocations where the collateral constraint is satisfied in each period.
The idea is to vary \( \lambda \) until the collateral constraint,

\[
\tau^N q_t^N K_t^N + \tau^T q_t^T K_t^T - R^* z_t - (1 + r^*) B_t = 0
\]

is satisfied for \( t = 0, 1, \ldots, T + 1 \). (This equation is satisfied by construction for \( t \geq T + 2 \).) To do this, we need to compute a mapping from \( \lambda \) to the \( q_t^i \)'s, the \( z_t \)'s and the \( B_t \)'s.

First, we set up a mapping:

\[
\left( \Lambda_{t+1}, p_t^T \right) \rightarrow \left( c_t^N, c_t^T, p_t^N, w_t, R_t, L_t^N, L_t^T, \Lambda_t, p_{t-1}^T \right),
\]  

(7.22)

starting with \( t = T + 2 \) and ending with \( t = 1 \). We then handle \( t = 0 \) separately.

**Dates, \( t > 1 \)** The object, \( \Lambda_t \) is obtained using (3.22):

\[
\Lambda_t = \Lambda_{t+1}(1 + \lambda_t),
\]

which is an equation that is available for \( t = 1, \ldots, T + 2 \). Then, we make use of (3.12) to solve for \( p_{t-1}^T \):

\[
\Lambda_t = \frac{u_{t,t} p_{t-1}^T}{p_t} \frac{1}{1 + x_{t-1} \beta},
\]

(7.23)

which is available for \( t = 1, \ldots, T + 2 \). To solve this, we require the other variables first. We do this using our equilibrium conditions and the given \( \Lambda_t, p_t^T \). We find these variables by setting up a one-dimensional search for \( L_t^N \). So, suppose that in addition to \( \left( \Lambda_t, \lambda_t, p_t^T \right) \), we have \( L_t^N \). Then, from our assumptions about technology, \( c_t^N = \left( K_t^N \right)^\alpha \left( L_t^N \right)^{1-\alpha} \). Given \( L_t^N \) and \( c_t^N \), the following two equations can be solved for \( p_t^N \) and \( c_t^T \):

\[
p_t^N = \frac{\gamma}{1 - \gamma} \left( \frac{(1 - \gamma) c_t^T}{\gamma c_t^N} \right)^{\frac{1}{\gamma}}
\]

(7.24)

\[
p_t p_t^T c_t = 1 + x_t,
\]
where

\[ p = \left( \frac{1}{1 - \gamma} \right)^{1 - \eta} + \left( \frac{p^N}{\gamma} \right)^{\frac{1}{1 - \eta}} \]  \tag{7.25}

\[ c = \left\{ \left[ (1 - \gamma) c^T \right]^\frac{\eta - 1}{\eta} + \left[ \gamma c^N \right]^\frac{\eta - 1}{\eta} \right\}^{\frac{\eta}{\eta - 1}} \]  \tag{7.26}

This can be treated as a one dimensional search problem in \( c_i^T \) alone.

We now have \( I^N, p^N, p, c^N \) and \( c^T \) in hand. The next step is to find \( z \) and \( L^T \). One equation that is useful for this purpose is the first order condition for \( z \):

\[ \left( \frac{y^T}{\mu_2 z} \right)^{\xi} \mu_2 = (1 + \lambda)R^*. \]

Since

\[ \frac{y^T}{\mu_2 z} = \left\{ \left[ \frac{\mu_1 V^T}{\mu_2 z} \right]^{\frac{\xi - 1}{\xi}} + 1 \right\}^{\frac{\xi}{\xi - 1}}, \quad V = A(K^T)^{\theta} \left( L^T \right)^{1-\theta}, \]

we have

\[ \frac{\mu_1 V}{\mu_2 z} = \left\{ \left[ \frac{(1 + \lambda)R^*}{\mu_2} \right]^{\xi - 1} - 1 \right\}^{\frac{\xi}{\xi - 1}} \]  \tag{7.27}

We require

\[ \frac{(1 + \lambda)R^*}{\mu_2} < 1, \]

which guarantees that, as long as \( 0 \leq \xi \leq 1 \), the object in braces is positive. This is necessary, for \( z \) and \( V \) to be positive. This is a restriction we place on \( \lambda \).

Equation (7.27) involves two unknowns, \( z \) and \( L^T \). We need another equation to pin these two variables down. Before obtaining these, it is useful to work on the expression for the marginal product of labor in the traded good sector:

\[ VMF_{PL}^T = \left( \frac{y^T}{\mu_1 V} \right)^{\frac{1}{\xi}} (1 - \theta) \mu_1 \frac{V_i}{L_i}. \]
or,
\[
VMP_L^T = \left\{ 1 + \left[ \frac{\mu_2 z}{\mu_1 V} \right]^{\xi-1} \right\}^{\frac{1}{1-\xi}} (1 - \theta) \mu_1 \frac{V}{L^T}. \tag{7.28}
\]

From (7.27),
\[
\left[ \frac{\mu_1 V}{\mu_2} \right]^{-\frac{\xi-1}{\xi}} = \left\{ \left[ \frac{(1 + \lambda) R^*}{\mu_2} \right]^{\xi-1} - 1 \right\}^{-1}.
\]

Substituting this into (7.28),
\[
VMP_L^T = \left\{ 1 - \left( \frac{(1 + \lambda) R^*}{\mu_2} \right)^{1-\xi} \right\}^{\frac{1}{1-\xi}} (1 - \theta) \mu_1 \frac{V}{L^T}.
\]

Equating the VMP_L's in each sector:
\[
(1 - \alpha)p^N \frac{y_N^N}{L^N} = \left\{ 1 - \left( \frac{(1 + \lambda) R^*}{\mu_2} \right)^{1-\xi} \right\}^{\frac{1}{1-\xi}} (1 - \theta) \mu_1 A \left( K^T \right)^{\theta} \left( L^T \right)^{-\theta}.
\]

This can be solved for \( L^T \):
\[
L^T = \left( L^N \right)^{\frac{1}{\alpha}} \left\{ \left[ 1 - \left( \frac{(1 + \lambda) R^*}{\mu_2} \right)^{1-\xi} \right]^{\frac{1}{1-\xi}} (1 - \theta) \mu_1 A \left( K^T \right)^{\theta} \right\}^{\frac{1}{\alpha}}. \tag{7.29}
\]

Then, setting \( V = A \left( K^T \right)^{\theta} \left( L^T \right)^{1-\theta} \), we obtain \( z \) from (7.27):
\[
z = \frac{\mu_1 V}{\mu_2} \left\{ \left[ \frac{(1 + \lambda) R^*}{\mu_2} \right]^{\xi-1} - 1 \right\}^{\frac{1}{\xi-1}}. \tag{7.30}
\]

Compute
\[
u_{c,t} = \left( c_t - \frac{\psi_0}{1 + \psi} \left( L^T_t + L^N_t \right)^{1+\psi} \right)^{-\sigma}.
\]

Now, it is possible to solve for \( p_{t+1}^T \) using (7.23). But, we are not done yet, because we
started with a guess for \( I_t^N \).

From the labor supply equation, (3.5),

\[
w = p\psi_0 \left( L^T + I^N \right)^\psi.
\]

(7.32)

From (3.32) we obtain \( R_t \):

\[
\beta R_t = (1 + x_{t-1}) (1 + \lambda_t) \left( p^T_t / p^T_{t-1} \right),
\]

(7.33)

\( t = 1, 2, \ldots \). Evaluating the product of \( R \) obtained from here and \( w \) obtained from (7.32), we can evaluate (3.24):

\[
f(I_t^N; \Lambda_t, \lambda_t, p^T_t) = (1 - \alpha) p^N_t \frac{y^N_t}{I^N_t} - w_t R_t.
\]

(7.34)

The idea is to adjust \( I_t^N \) until \( f(I_t^N; \Lambda_t, \lambda_t, p^T_t) = 0 \).

**Date** \( t = 0 \) We now have in hand,

\[
\left( c^N_t, c^T, p^N_t, w_t, R_t, I^N_t, I^T_t, \Lambda_t, p^T_{t-1} \right), \text{ for } t = 1, \ldots, T + 2.
\]

Next, we seek \( \left( c^N_t, c^T, p^N_t, w_t, R_t, I^N_t, I^T_t \right) \) for \( t = 0 \). Note that we do not have \( \Lambda_0 \), since (3.7) is not available for \( t = 0 \). This means that we cannot find \( I^N_0 \) by setting \( f = 0 \), in (7.34) as we do for \( t = 1, \ldots, T + 1 \). We replace equation (7.33) by

\[
\frac{u_{c,0}}{p_0 \psi_0^T} = \frac{\beta R_0 u_{c,1}}{p_1 p_1^T (1 + x_0)}.
\]

(7.35)

Then, we solve for \( I^N_0 \) as follows. Fix \( I^N_0 \). Solve for \( p^N_0, \psi_0, c^N_0, c^T_0, c_0 \) using the iterative algorithm described around (7.24). Then, compute \( I^T_0 \) using (7.29), \( z_0 \) using (7.30), \( u_{c,0} \) using (7.31), and \( u_0 \) using (7.32). Solve for \( u_0 R_0 \) using

\[
u_0 R_0 = (1 - \alpha) p^N_0 \left( \frac{K^N}{I^N_0} \right)^\alpha.
\]

55
and compute $R_0$ from $u_0R_0/u_0$. Finally, evaluate

$$g(I_0^N) = \frac{u_{c,0}}{p_0p_0^T} - \frac{u_{c,1}}{1 + x_0} \beta R_0.$$ 

Adjust $I_0^N$ until $g(I_0^N) = 0$. Another way to write the $g$ function substitutes based on $pp^T c = 1 + x$,

$$g(I_0^N) = \frac{u_{c,0} c_0}{1 + x_0} - \frac{u_{c,1} c_1}{1 + x_1} \beta R_0,$$

or,

$$g(I_0^N) = u_{c,0} c_0 - u_{c,1} c_1 \beta R_0.$$ 

The next step is to evaluate the $q_i^i$'s and the $B_t$'s. The $q_i^i$'s can be solved recursively from (3.20). The $B_t$'s can be obtained by simulating (3.30) forward, for the fixed $B_0$.

In practice, we found that the following parameterization of the $\lambda$’s works well. We let $\lambda_t$ for $t = 0, 1, ..., N$, $N < T$ be free parameters and we set $\lambda_t$ for $N < t < T + 2$ by linear interpolation and imposing $\lambda_{T+2} = 0$. We chose the free $\lambda$’s to enforce exactly the collateral constraints in periods $t = 0, 1, 2, ... N − 1$ and $T$. The adequacy of this computational strategy can be evaluated ex post by evaluating the collateral constraints for dates with $t \neq T$ and $t \notin \{0, 1, ..., N\}$. We found that this procedure works well for $T = 10$ and $N = 6$.

**Surprise Scenario** We now suppose that $d_0$ is set according to the equilibrium in the previous subsection. In reflection of this, we drop the household's dynamic first order condition, (7.35), from consideration in period 0. The computational strategy for finding the equilibrium in this scenario is identical to what we described in the previous subsection, with the exception of this change for period 0. We review the period 0 calculations now. We suppose that $\lambda_0$ and $p_0^T$ are given and we seek to solve for $(c_t^T, c_t^N, p_t^N, w_t, R_t, L_t^N, L_t^T, z_t)$ for $t = 0$. [to be completed]
References


Figure 1
The Effect of an Interest Cut with Some Elasticity of Substitution and No Debt

$\beta = 1/1.05$, $\alpha = 0.25$, $\theta = 0.6$, $\chi = 0.06$, $\psi = 1$, $\psi_0 = 0.3$,

$K^N = K_T^T = 1$, $A = 1.9$, $R^* = 1 + r^* = 1.05$, $\tau = 0.01$, $B = 0$, $\eta = 0.9$
Figure 2
The Effect of an Interest Cut with Low Elasticity of Substitution and No Debt

\[ p_n, L, \beta = \frac{1}{1.05}, \alpha = 0.25, \theta = 0.6, x_s = 0.06, \psi = 1, \psi_0 = 0.3, \]

\[ K^N = K^T = 1, A = 1.9, R^* = 1 + r^* = 1.05, \tau = 0.01, B = 0, \eta = 0 \]
Figure 3
The Effect of an Interest Cut with Low Elasticity of Substitution and Modest Debt

\[ \beta = 1/1.05, \alpha = .25, \theta = .6, \chi_s = .06, \psi = 1, \psi_0 = .3, \]

\[ K^N = K^T = 1, A = 1.9, R^* = 1 + r^* = 1.05, \tau = .01, B = 0.2, \eta = 0 \]
Figure 4a: Transitional Path to Lower Debt During Crisis, No Policy Response, Labor in Traded Sector

Notes: % dev from ss - Percent Deviation from Pre-Crisis Steady State
% of ss output - Percent of Pre-Crisis Gross Output
Figure 4b: Transitional Path to Lower Debt During Crisis, No Policy Response, Labor in Traded Sector

Note: % dev from ss - Percent Deviation from Pre-Crisis Steady State
Figure 5a: Transitional Path to Lower Debt During Crisis, No Policy Response, No Labor in Traded Sector

Notes: % dev from ss - Percent Deviation from Pre-Crisis Steady State
% of ss output - Percent of Pre-Crisis Gross Output
Figure 5b: Transitional Path to Lower Debt During Crisis, No Policy Response, No Labor in Traded Sector

Note: % dev from ss - Percent Deviation from Pre-Crisis Steady State