A Debt Overhang Model for Low-Income Countries

JUNKO KOEDA

This paper presents a theoretical model to explain how debt overhang is generated in low-income countries and discusses its implications for aid design and debt relief. It finds that the extent of debt overhang and the effectiveness of debt relief depend on a recipient country’s initial economic conditions and level of total factor productivity. [JEL E21, F34, F35, F43, O16, O21]

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Debt overhang—the relationship between heavy debt and low growth—is a fundamental concept in the literature that argues in favor of debt relief. Unfortunately, the majority of existing theoretical models are designed for middle-income countries suffering from heavy nonconcessional debt burdens. These models do not apply to low-income countries (LICs) where external loans are highly concessional and comprise a large share of debt. In addition, existing debt overhang models typically provide no explanation as to why the debtor country has excess debt in the first place.¹

To fill these gaps, this paper formulates Cohen and Sachs’s (1986) sovereign debt model as a concessional lending problem and numerically demonstrates how a link between large debt and low growth may be

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¹For theoretical models that explain debt accumulation in low-income countries, see for example, Easterly (1999); he shows that impatience could lead to overborrowing.
generated in LICs. The model focuses on the effect of a cutoff, which is an income level above which the country loses its eligibility for aid assistance. Such cutoffs exist with multilateral concessional lending from the World Bank’s International Development Association (IDA) and by the IMF with loans under the Poverty Reduction and Growth Facility (PRGF).2

This paper shows that an LIC—when it has no effective tools to raise the country’s total factor productivity (TFP)—may have an incentive to accumulate a significant amount of concessional debt and allocate resources to consumption rather than investment. Such a country would manage its large debt at a very low cost by allowing the economy to stagnate around the cutoff, and thus would become permanently aid dependent. This is more than just a theoretical possibility; this paper provides empirical evidence of growth stagnation around the cutoff.

There are two types of agents in the model: an official creditor and an LIC debtor. The creditor lends at a fixed subsidized interest rate if the debtor country lies at or below the cutoff. Above the cutoff, the creditor lends at the world interest rate. The creditor can commit to the contracts but the debtor country cannot. The creditor thus imposes a participation constraint to prevent the debtor country from defaulting. Imposing a participation constraint is equivalent to imposing an endogenous debt ceiling constraint. The LIC debtor maximizes the representative agent’s welfare subject to this lending rule. Some researchers argue that the focus should be on “bad” governments that care about their own welfare rather than that of households. This paper shows, however, that a debt overhang problem may occur even with a benevolent government.

Last, this paper proposes policy implications for aid design and debt relief. For aid design, it finds that the existing eligibility criteria and graduation policies may be improved to provide stronger growth incentives and to reduce the cost of aid assistance. For debt relief, a one-time stock treatment can promote growth, given certain initial conditions and TFP.

I. Theoretical Literature

This paper is related to the sovereign debt and debt overhang literature. First, the model endogenizes debt sustainability3 by incorporating enforcement mechanisms—an important topic in the sovereign debt literature. There are two main types of models that explain enforcement mechanisms in the literature: reputation and sanction models. In reputation models, debtors find it painful to be excluded from future credit markets. One classic

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2Note, however, that in practice there are other criteria, such as the country’s creditworthiness and performance, that affect the determination of IDA loan eligibility in addition to per capita income eligibility criteria. This means that there may be some countries below the cutoff that are disqualified for IDA loans and some that are above the cutoff but are qualified for IDA loans, or both.

reputation model is that of Eaton and Gersovitz (1981). They assume a concave utility function so that the country has an incentive to smooth consumption over time. The output path takes two values in turn, high and low. In this environment, the country does not want to be excluded from the international capital markets, because in financial autarky it cannot smooth consumption. Bulow and Rogoff (1989), on the other hand, show conditions under which reputation does not provide sufficient repayment incentives. Other aspects of reputation have been studied by, for example, Atkeson (1991) and Cole and Kehoe (1998).

In sanction models, debtors are penalized on default. A common way of introducing the default penalty is to assume a loss of a fraction of output on default. This can be, for example, the loss of access to short-term trade credits. Some researchers argue that sanction models fail to consider possible renegotiation processes and analyze the processes in the context of dynamic bargaining games. Yet debt renegotiation itself can be costly—Rose (2005) finds that debt renegotiation is associated with an economically and statistically significant decline in bilateral trade between a debtor and its creditors.

Cohen and Sachs (1986) incorporate components of both reputation and sanction models into their enforcement mechanisms. Their model is a neoclassical growth model in which the initial capital stock lies below the steady state. The country can borrow from abroad at the given world interest rate. As long as the country’s capital remains scarce, the world interest rate is lower than the initial marginal product of capital, so the country finds it painful to be excluded from external borrowing. The marginal product of capital decreases as the country accumulates capital, eventually converging to the world interest rate. In the steady state, the country’s default cost is merely the one that comes from sanctions. An important contribution of Cohen and Sachs is that they analyze sovereign debt dynamics in the context of growth. Thus, their model may be useful when thinking about development problems of an LIC with good growth prospects.

The theoretical literature on debt overhang that explains the relationship between large debt and low growth in LICs lags behind the empirical literature. The existing debt overhang models typically consider a case in which initial debt is so large that the country would be insolvent unless it received some form of debt relief (Krugman, 1988; and Sachs, 1989). In these models, excess debt reduces the supply of new loans by scaring off creditors; it also reduces the demand for new investment and discourages policy efforts to reform by acting like a distortionary tax where a fraction of future output is assumed to be used for repayments of the initial debt. This discourages domestic investment, resulting in low growth.

However, because some key features of LICs are not incorporated into these models, their applicability to this context is questionable. In particular,
the majority of loans to LICs are highly concessional and are provided by official creditors who are neither profit maximizers nor risk neutral. This may generate a unique lending pattern—for example, contrary to the existing models, large debt may not discourage new official lending, as argued by Easterly (2002).

The model presented below formulates some specific LIC characteristics by considering the case in which an LIC debtor has no access to foreign private loans but has access to subsidized loans provided by a benevolent creditor.

II. Empirical Motivation

This work begins with empirical documentation showing that there is some economic stagnation around the cutoff. I run growth regressions using an unbalanced panel of 94 countries, of which 33 are LICs. The data set is taken from the Penn World Table Version 6.1 (Heston, Summers, and Aten, 2002), the World Bank’s World Development Indicators, and the Barro-Lee (1993) data set.

As for data on the cutoff, I use the operational cutoff, which was formally recognized by IDA donors in IDA8 in 1987. Prior to this date, a higher cutoff, known as the historical cutoff—initially set at $250 in 1964—was used for the IDA cutoff. The operational cutoff was introduced in the early 1980s because of the limited availability of IDA resources and the attention to poor performance in LICs. Both cutoffs are updated annually according to the world inflation rate using the SDR deflator.

The dependent variable is the percentage annual growth rate of real GDP per capita. The explanatory variables are those typically included in a standard growth regression: the percentage of population growth (GPO), the percentage investment share of real GDP per capita (INV), the initial secondary schooling attained as a percentage of the total population in 1985 (INIT_EDU), and the initial level of real GDP per capita in 1988 (INIT_RGDP). In addition to these variables, I include the variable of interest, a measure of proximity to the cutoff in the form of a Bartlett kernel:

\[
PROX_{it} = \begin{cases} 
1 - |z_{it}| & \text{for } |z_{it}| \leq 1 \\
0 & \text{for } |z_{it}| > 1 
\end{cases}
\]

where \( z_{it} = \frac{\ln y_{it} - \ln \bar{y}_t}{\ln(1 + b)} \),

where \( y_{it} \) is country i’s GNI per capita in year t, \( \bar{y}_t \) is the cutoff in year t, and \( b \) is a scaling factor that controls the width of the kernel band. Note that a

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5I exclude observations on Rwanda for 1994 from the sample where Cook’s distance, leverage, and studentized residuals exceed the conventional cutoffs. For conventional cutoffs and methodology, see Chen and others (2003, Chapter 2, 2.1).

6See World Bank (2001) for a detailed description of IDA eligibility criteria.

7The corresponding 1988 data are not available, so the 1985 data are used.

8GNI is commonly denoted as GNP. GNI is the new terminology under the 1993 System of National Accounts (SNA), replacing the old terminology—GNP—under the 1968 SNA.
negative coefficient for PROX implies that there is a negative relationship between the country’s growth rate and its proximity to the cutoff. The scaling factor, $b$, is set equal to $\frac{1}{2}$, but I obtained similar results in the cases where $b = \frac{1}{3}$ and $b = \frac{1}{4}$. Appendix IV reports the distribution of PROX and partial regression of the growth rate on PROX.

Table 1 shows the estimation results based on a pooled sample for 1988–2000. The ordinary least square (OLS) coefficient for PROX is negative and is statistically significant at the 5 percent level. To take into account endogeneity of PROX, INV, and INIT_RGDP, I also run two-stage least square (2SLS) regressions. Here, one-year lags of these variables are used as the instruments for PROX and INV, and the log of real GDP per capita in 1985 is used as the instrument for INIT_RGDP. The corresponding first-stage regressions are reported in Appendix V. The coefficient of PROX remains negative and significant at 5 percent.\(^9\) When the sample is restricted to those countries that lie below the cutoff, the significance level improves. This empirical evidence is consistent with the paper’s theoretical result—the existence of a cutoff could result in economic stagnation at or around it.\(^10\)

\(^9\)This significance level corresponds to the one-sided test.

\(^10\)I also carried out regressions using specifications similar to the basic regression discussed in Chapter 12 of Barro and Sala-i-Martin (2004). The data I used here contain 79 countries, 13 of which are low-income countries. The dependent variable is the average real GDP per capita growth for 1990–2000. The explanatory variables are similar to those of Barro and Sala-i-Martin plus the average of PROX for 1990–2000. The 2SLS coefficient for the average of PROX becomes negative and insignificant but the $t$-values of all other regressors are considerably lower than in Barro and Sala-i-Martin (Table 12.3). This is probably because the sample size is only 79—one-third fewer than that of Barro and Sala-i-Martin—because the period covered (1990–2000) is much shorter.
III. The Model

Official creditors typically fix their concessional interest rates; for example, the rates of the World Bank’s IDA and the IMF’s PRGF are 0.75 and 0.5 percent, respectively. I thus consider the following concessional lending rule: the lender who has full access to the world financial markets loans out funds at a fixed subsidized interest rate ($\tilde{r}$) as long as the country’s output per capita ($y$) is below the cutoff ($\bar{y}$).\(^\text{12}\) The interest rates for concessional lending are thus set according to the following rule:

\[
\tilde{r}_{t+1} = \begin{cases} 
\tilde{r} & \text{if } y_t \leq \bar{y}, \\
 r & \text{otherwise},
\end{cases}
\]

(1)

where $r$ is the world interest rate. I assume that the borrower country has no access to foreign private financing, given that in practice, the majority of loans to LICs are offered by official lenders.

In addition, I impose a participation constraint to motivate the borrower to adhere to the contract.\(^\text{13}\) With this constraint, the borrower’s value function under repayment is required to be greater than or equal to its value function under default. The borrower country solves the following problem:

\[
\max_{\{c_t, k_{t+1}, X_{t+1}\}} \sum_{j=1}^{\infty} \beta^{j-1} u(c_t),
\]

(2)

subject to

\[
v^D(k_t) \leq u(c_t) + \beta \sum_{j=1}^{\infty} \beta^{j-1} u(c_{t+j}) \quad \forall t,
\]

(3)

\[
c_t = f(k_t) - x_t + X_{t+1}/(1 + \tilde{r}_{t+1}) - X_t,
\]

(4)

\[
k_{t+1} = (1 - \delta)k_t + x_t,
\]

(5)

\[k_1 \text{ and } X_1 \text{ are given,}
\]

(6)

\[\tilde{r}_{t+1} \text{ follows the rule given by Equation (1) with } y_t = f(k_t),
\]

(7)

where $c$, $x$, $k$, and $X$ denote consumption, investment, capital, and repayment obligation, respectively. The repayment obligation in period $t$, $X_t$, is defined

\(^{11}\)More precisely, this is the service charge that the World Bank currently imposes on the loans.

\(^{12}\)The model assumes that the cutoff is expressed in terms of GDP per capita. In practice, however, it is defined in terms of GNI per capita, which excludes the interest payments on external debt from GDP. Appendix I presents the borrower’s problem where the cutoff is expressed in terms of GNI per capita. The main conclusions hold true for the GNI case.

\(^{13}\)The cases with no participation constraints are discussed in Appendix II.
by \( X_t = (1 + r_t)D_t \), where \( D_t \) is concessional debt due in period \( t \).\(^{14}\) \( \beta \) is the discount factor where \( r \equiv 1/\beta - 1 \) is assumed in order for consumption in the steady state to be flat. The participation constraint is given by Equation (3). The LIC’s flow budget constraint is given by Equation (4). The production function is given by \( f(k_t) \). The transition equation for capital is given by Equation (5), where \( \delta \) is the rate of capital depreciation. The value function under default, \( v^D(k) \), is the value function in autarky with penalties for violating the participation constraint:

\[
v^D(k) = \max_{k'} \left\{ u((1 - \lambda)f(k) - k' + (1 - \delta)k) + \beta v^D(k') \right\},
\]

where \( \lambda \) is the fraction of output lost. I assume that such a violation incurs two types of costs: the exclusion of the violator from future concessional lending and the loss of a fraction of the violator’s output. I also assume that when the participation constraint is binding, the LIC adheres to the borrowing contract.

In each period, the borrower country compares the value function under repayment, \( v^R(k, X) \), with that under default, \( v^D(k) \). When \( v^R(k, X) \geq v^D(k) \) the country repays; otherwise it defaults. The value function under repayment, \( v^R(...) \), is given by:

\[
v^R(k, X) = \max_{k', X'} \left\{ u(f(k) - k' + (1 - \delta)k \\
+ \frac{X'}{(1 + \bar{r}(k)} - X) + \beta v^R(k', X') \right\},
\]

subject to \( v^R(k', X') \geq v^D(k') \),

where \( v^R(...) \) is increasing in \( k \) and is decreasing in \( X \). Appendix III interprets the first-order conditions (FOCs) of the Bellman equation (equations (9) and (10)).

To make a connection with Cohen and Sachs’s model (1986), the participation constraint can be replaced with a debt capacity function \( h(k) \), which is defined implicitly by \( v^R(k, h) = v^D(k) \), where \( \partial v^R(k, X)/\partial X \) is strictly negative. In other words, given \( k \), \( h(k) \) is uniquely determined and thus the case where \( h(k) \) is backward bending in \( k \) can be excluded. Thus the model is expressed with \( X \) rather than \( D \), because otherwise the interest rates would be a function of the previous period’s capital (call this \( k_{-1} \)), and thus \( k_{-1} \) would need to be treated as an additional state variable in the model’s recursive equation.

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debt capacity function is well defined. The original value function under repayment can be rewritten as

\[ v^R(k, X) = \max_{k', X'} \{ u(f(k) - k' + (1 - \delta)k
\]

\[ + \min \left\{ \frac{X'}{(1 + \bar{r}(k))^\eta}, \frac{h(k')}{(1 + \bar{r}(k))} \right\} - X \} + \beta v^R(k', X'). \]  \tag{11}

This formulation is the same as that of Cohen and Sachs (1986),\(^{15}\) except that in this paper I numerically derive the value functions and the implied debt capacity function using the value function iteration method.\(^{16}\) I also extend their model to analyze the dynamics of concessional loans to LICs.

**IV. The Numerical Results**

Because one cannot solve this problem analytically unless the participation constraint is absent, I solve it numerically using the value function iteration method. I specify the functional forms of the utility and production functions as

\[ u(c) = c^{1-1/\sigma}/(1-1/\sigma) \] and \[ f(k) = Ak^n. \]

**Calibration**

Table 2 lists calibrated parameter values. Depreciation of capital is set at 0.1—a reasonable number in the real business cycle literature (for example, see Kydland and Prescott, 1982). The concessional interest rate is set at 0.75 percent, in line with existing official concessional lending practice. The value of the capital elasticity of the production function (\( \eta = 0.33 \)) is based on the findings in Gollin (2002) and is consistent with the existing real business cycle literature. The paper’s main results are robust to alternative values of \( \eta = 0.3 \) and 0.4; a poverty trap arises with \( \eta = 0.2 \) if there is a slight change in other parameter values.

The discount factor is set at 0.95 and the world interest rate at \( \frac{1}{\beta-1} = 0.0526 \) in order to obtain flat consumption in the steady state. Debt overhang emerges more easily with a smaller value of \( \beta \) (or a higher \( r \)) because the borrower country will have a stronger incentive to consume in earlier periods (or because the benefits from concessional loans are larger). With a higher value of \( \beta \), a poverty trap arises if there is a slight change in other parameter values.

The value for the elasticity of intertemporal substitution (\( \sigma = 0.45 \)) is in line with the calibration results of Ogaki, Ostry, and Reinhart (1996). Using

\(^{15}\)An extension of Cohen and Sachs (1986) can be seen in Borenszstein and Ghosh’s (1989) mode.

\(^{16}\)Here, I use a two-period utility function (that is, \( u(c) + \beta u(c') \)) and an arbitrary debt capacity function (that is, \( h(k, \bar{r}) = \text{constant} \)) as the starting functions; if the participation constraint is violated, the utility function is penalized. I obtain the same fixed point in the functional space.
Cooley and Ogaki’s (1996) two-step procedure, they estimate the lower and upper bounds of the intertemporal elasticity of substitution assuming that the elasticity is an increasing function of the level of wealth. The paper’s main conclusions are robust to alternative values of $s = \frac{1}{3}$ and $\frac{2}{3}$ if there is a slight change in other parameter values.

The fraction of output lost on default ($\lambda$) is set at 0.05. This value needs to be sufficiently positive to maintain the paper’s main conclusions. If it is zero, then there will be no default cost in the steady state and thus the borrower country will have an incentive to default; as a result, no loans will be made.

The cutoff level ($\bar{y}$) is set as a fraction of steady-state output in the United States ($y^{US}$). I use 0.15, because the purchasing-power-parity-adjusted real outputs per capita in most lower-middle-income countries are above this level. I calculate $y^{US}$ by omitting the participation constraint and by assuming that U.S. TFP is 30 (note: this number is just a scaling factor) and the LIC-U.S. TFP ratio is $\frac{1}{3}$. The paper’s main conclusions are sensitive to the LIC’s steady-state output level relative to the cutoff. More details are discussed at the end of this section.

Benchmark Economy

Consider the benchmark economy with initial income ($y_1$) equal to 90 percent of the cutoff (or about 70 percent of steady-state output ($y_{ss}$)), and initial debt repayment obligation ($X_1$) equal to 90 percent of the cutoff (or 100 percent of $y_1$).

17Following this two-step procedure, they first estimate the intratemporal parameters in a two-good model of tradable and non-tradable goods. Given these intratemporal parameters, they then estimate the intertemporal elasticity of substitution by applying the generalized method of moments to the Euler equation.

18Atkeson and Ogaki (1996) show that the intertemporal elasticity of substitution rises with the level of wealth.

19A cursory glance at the TFP ratio of 40 LICs relative to the United States between 1960 and 2000 shows that about one-fourth of LICs have TFP levels that are less than one-third the U.S. level and are stable.
Figures 1 and 2 summarize the results from the numerical solution. To better understand the dynamics of concessional lending, these results are displayed along with those for nonconcessional lending. The only difference between these two loans is the interest rate level, where \( r_{\tilde{r}} = r \) for all \( t \) under nonconcessional loans. Thus, the nonconcessional debt capacity function \( h_N(k) \) is defined implicitly by \( v^R(k, h_N) = v^D(k) \), where \( \partial v^R(k, X)/\partial X \) is strictly negative and \( r_{\tilde{r}} = r \) for all \( t \).

This discontinuity of the concessional debt capacity function \( (h(k))^{20} \) implies that the recipient country must drastically reduce its external debt precisely when it surpasses the cutoff. Such debt reduction is possible only through a steep decline in consumption, which the country may find too

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20This discontinuity emerges in the amount of resources that can be borrowed \((h(k)/(1 + r_{\tilde{r}}))\) as a result of the jump in the interest rate that takes place as soon as the country’s income exceeds the cutoff.
painful. For the benchmark case, this one-time cost of consumption reduction outweighs the long-run benefit of achieving the steady state where output is much higher than the cutoff (call this “high” steady state). Thus the country decides not to cross the line and converges to the cutoff (call this “low” steady state, that is, point a in Figure 2.) On the other hand, under nonconcessional lending, this perverse incentive is absent, and the country steadily grows to reach the high steady state, yet consumption in the short run is lower than in the case of concessional lending (Figure 1).

This result captures the paper’s debt overhang mechanism. Because the cost of servicing debt is kept artificially low, the country is motivated to carry a large amount of debt by consuming excessively and thus does not grow. Figure 3 shows that there exists a debt overhang threshold above which the country is trapped with large debt and no growth.

Whether or not the country is trapped in a debt overhang depends on the country’s initial conditions: initial debt and capital (Figure 3). The intuition is as follows: First, the higher the initial debt level, the more likely the country is to converge to the low steady state, because this allows the country to manage heavier debt at a low interest rate. Second, the lower the country’s initial capital, the larger the impact of short-run growth. As a result, the country tries to borrow a larger quantity of concessional loans to raise both investment and consumption in the short run, and thus it is more likely to be

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Figure 2. Endogenous Debt Capacity Ceilings: The Benchmark Case

Note: This figure shows the endogenous debt capacity function under concessional loans ($h(k)$), solid line) and under nonconcessional loans ($h^N(k)$, dashed line). They are reported as a fraction of $y_s$. The y-axis is the repayment obligation ($X$) as a fraction of the steady-state output level ($y_s$). The x-axis is the level of capital per capita ($K$) as a fraction of the steady-state capital level ($K_{ss}$). The vertical dotted line is the cutoff.
trapped in the low steady state. In short, the country converges to the high steady state only if initial debt is low enough, initial income is high enough, or both conditions hold.

Whether or not the country is trapped in a debt overhang is also conditional on the country’s TFP level (Figure 4). The higher the TFP, the higher the steady-state output level relative to the cutoff, and therefore the greater the long-run benefit of achieving the high steady state. With a higher TFP, the country thus finds it more costly to be trapped in the low steady state. Figure 4 shows that the debt overhang threshold shifts up with a higher TFP level. With a higher TFP level, the country is more likely to lie below the debt overhang threshold. Note that here the initial income level is kept the same across different TFP levels; initial capital levels are adjusted accordingly.

V. Policy Implications and Conclusions

Implications for Aid Design

Even though the paper does not solve for the most efficient form of aid,\textsuperscript{21} it does have implications for aid design. First, the model implies that the

\textsuperscript{21}For more discussion towards an optimal debt relief proposal see, for example, Rajan (2005).
perverse incentive arising from per capita income eligibility exists under other forms of aid as well. For example, if a country repeatedly receives sufficiently large grants with a similar income per capita eligibility criterion, it may have an incentive to stagnate around the cutoff in order to maintain future grant eligibility.

Second, the model suggests that the existing eligibility and allocation rules could take into account other aspects, in addition to income per capita, in order to avoid the perverse incentive. In practice, some measures of TFP are already taken into account in the aid allocation formula of the multilateral development banks. For example, the IDA Country Policy and Institutional Assessment takes into account policy and structural indicators.

Third, the model implies that certain forms of graduation policies may provide stronger growth incentives as well as reduce the cost of aid assistance. For example, the model’s debt overhang may disappear if the concessional interest rates are allowed to increase with income levels. This implies that a more gradual move from concessional to nonconcessional lending provides the right incentive for growth. Indeed, the existence of “blend countries” suggests that something is at work in IDA and other multilateral development banks’ operational rules.

Implications for Debt Relief

The model implies that a one-time-debt-relief stock treatment may be effective in helping a country get out of the poverty trap and achieve growth.
For example, suppose that the benchmark economy receives one-time relief that enables the country to move below the debt overhang threshold (solid line in Figure 3). The country now converges to the high steady state.

One-time debt relief may also be effective even if the country initially lies above the cutoff. Consider a country that has relatively high initial debt and lies in region B in Figure 3. Note that this country has an incentive to go back to the cutoff because the benefit of raising the debt ceiling by reducing capital is greater than the cost of lowering output. The country is thus better off reducing output until it eventually falls to the cutoff. Here upfront debt relief that moves the country from B to D is effective in achieving growth.

Note that if such a stock treatment is accompanied by factors that can raise TFP, such as productivity growth and an improvement in institutional quality, then the debt overhang threshold itself will shift upward (Figure 4) resulting in a larger number of countries that lie below the threshold. This means that if debt relief resources are used for development purposes that also directly raise TFP, more countries will be able to achieve growth given the same amount of debt relief. The above arguments provide some justification for the recent one-time-debt-relief stock treatment, known as the Multilateral Debt Relief Initiative (MDRI). The MDRI is a 100 percent debt stock cancellation by the IMF, the IDA, and the African Development Bank for a group of LICs; its goal is to free up resources to help countries achieve the United Nations’ Millennium Development Goals.

However, there are some caveats to this argument. First, for this type of stock treatment to work, it is important that recipient countries view it as a one-time event. If they do not, the poverty trap may reemerge if countries receive repeated debt relief with similar income per capita eligibility criteria. Second, the theoretical environment may be too efficient; that is, the model assumes that the country can efficiently reallocate freed resources from debt relief to productive activities. In reality, however, it may be quite difficult to handle a sudden increase in resources in the presence of weak institutions.

Conclusions

Whether an LIC is trapped in a debt overhang depends on its initial conditions and its TFP. The larger the initial debt, the stronger the incentives an LIC has to manage its debt at a low interest rate by becoming permanently aid dependent. The lower an LIC’s initial income, the more it tries to borrow a larger quantity of concessional loans to raise both investment and consumption in the short run and thus becomes more likely to be trapped in the low steady state. Last, the lower the level of TFP, the more likely it becomes that the benefit of remaining at the cutoff exceeds the long-run benefit of achieving the high steady state.
APPENDIX I

Per Capita Income Cutoff: GDP vs. GNI

The model defines the cutoff in terms of GDP per capita. However, in practice, it is defined in terms of GNI per capita, which excludes the interest payments on external debt from GDP. Because a high level of external debt alters the level of capital stock above which the country loses eligibility, this new feature introduces an additional incentive for debt accumulation. However, as shown below, the paper’s key conclusion—a poverty trap occurs depending on the country’s TFP and initial conditions—still holds under the GNI case. This appendix solves a borrower’s problem in which the country’s output per capita is expressed in terms of GNI per capita—more specifically, it solves the problem of Equations (2)–(7) with \( y \) now defined as \( y = f(k) - rX/(1 + r) \).

The Bellman equation now has three state variables because \( r_{t+1} \) becomes a function of \( k_t \) and \( X_t \):

\[
v^R(k, X, r) = \max_{k', X'} \left\{ u \left( f(k) - k' + (1 - \delta)k + \frac{X'}{(1 + r')} - X \right) 
+ \beta v^R(k', X', r') \right\},
\]

(A.1)

Figure A1. The Benchmark Economy with a GNI per Capita Cutoff: Numerical Results

Note: Panels (a)–(d) show how the benchmark economy will respond under the concessional lending (solid line) and nonconcessional lending (dotted line) schemes. The paths of output, consumption, investment, and repayment obligation are shown as a fraction of the steady-state output level (\( y_{ss} \)).
subject to $v_R(k, \tilde{r}, X_0, r_\sim_0)$, where $r_\sim_0$ follows the rule given by Equation (1) and is equal to $r$ if $f(k)/C_0 r_\sim X/(1 + r_\sim)$, and otherwise. The debt capacity function $h(k, r)$ is defined implicitly by $v_R(k, h, r_\sim) = h(k, r)/y_{ss}$, where $h$ is strictly negative.

I numerically solve for two value functions, $v_R(k, X, r)$ and $v_R(k, r)$, because $r_\sim$ can take only two values, $r$ and $\tilde{r}$. I solve these functions in the same manner as in Section IV using the same parameter values and functional forms. The numerical results for the benchmark economy (see Appendix Figures A1 and A2) are very similar to those presented in Section IV.

In the GNI case, however, the corresponding endogenous debt capacity function is more complicated than Figure 2 in the text. Lines $F_r$ and $F_\tilde{r}$ are the graphs of $X = ((1 + \tilde{r})/\tilde{r})f(k) - \tilde{y}$, where $\tilde{r}$ is equal to $r$ and $\tilde{r}$, respectively. These lines determine the interest rate in the next period; for example, if $\tilde{r}_t = \tilde{r}$, then $\tilde{r}_{t+1} = r$ if the country lies on the right-hand side of the line $F_{\tilde{r}}$, and $\tilde{r}_{t+1} = \tilde{r}$ otherwise. $k_r$ and $k_{\tilde{r}}$ are the levels of capital at the points of discontinuity for $h(k, r)$ and $h(k, \tilde{r})$, respectively. $k$ is the level of capital that satisfies $\tilde{y} = Ak^\eta$.

Could a poverty trap still occur in the GNI case? Yes. Suppose the country initially lies at point (a) in Appendix Figure A2 with $\tilde{r}_t = \tilde{r}$. This implies that the interest rate in period 2 is also $\tilde{r}$, because the point $(k_1, X_1)$ lies on the left-hand side of the line $F_{\tilde{r}}$. If the country chooses to cross $k_r$ in period 2, the point $(k_2, X_2)$ will lie on the right-hand side of the line $F_r$. Here, consumption in period 2 must be very low, because the country needs to reduce borrowing. This is the same story discussed in Section IV.
The new feature of the GNI case can be demonstrated as follows. Suppose the country initially lies on the right-hand side of the line \( F_r \), say at point (b), with \( \bar{r}_1 = r \). This implies that the interest rate in period 2 is also \( r \). Here, the country could significantly increase its borrowing and consumption in period 2—for example, by moving to point (c)—as long as the point \((k_2, X_2)\) remains at or below \( h(k, r) \). However, this implies that the country would need to move back below \( h(k, \bar{r}) \) in period 3, which requires consumption in that period to be very low. As a result, the country would not choose to converge to point \((kr, h(kr, r))\), that is, the point of discontinuity of \( h(k, r) \).

Oscillating solutions, in which the country chooses to move back and forth between the right- and left-hand sides of the line \( F_r \), could also occur, depending on the initial conditions. This is because under the GNI cutoff rule, the problem has multiple discontinuities, as shown in the figure.

**APPENDIX II**

**No Participation Constraints**

This appendix considers the environment in which the LIC fully precommits to honoring the conditions of the concessional lending scheme that is imposed by the creditor so that there is no need to impose a participation constraint. In practice, though, this is an unrealistic assumption because it allows the LIC unlimited access to the donor’s funds. Analyzing this nonparticipation constraint environment, however, is nonetheless useful to understand the role of a debt ceiling constraint.

In the absence of participation constraints, capital overshoots in period 1 as a result of a subsidized interest rate. The problem is given by

\[
\max_{c_t, k_{t+1}} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t),
\]

Subject to the intertemporal budget constraint

\[
f(k_1) + (1 - \delta)k_1 + \sum_{i=2}^{\infty} \left( \prod_{s=2}^{i} \left( 1 + \bar{r}_s \right) \right) \left( f(k_i) + (1 - \delta)k_i \right) \\
= (1 + \bar{r}_1)D_1 + c_1 + k_2 + \sum_{i=2}^{\infty} \left( \prod_{s=2}^{i} \left( 1 + \bar{r}_s \right) \right) (c_t + k_{t+1}),
\]

where \( K_1, D_1, \) and \( \bar{r}_1 \) are given. FOCs are given by

\[
u'(c_t) = \mu \quad \text{for } t = 1, \tag{A.2}
\]

\[
\beta^{t-1} u'(c_t) = \mu \prod_{s=2}^{t} \left( \frac{1}{1 + \bar{r}_s} \right) \quad \text{for } t \geq 2, \tag{A.3}
\]

\[
\bar{r}_{t+1} = f'(k_{t+1}) - \delta \quad \text{for } t \geq 1, \tag{A.4}
\]

where \( \mu \) is the shadow price. Initially, the country can borrow at the concessional interest rate (that is, \( \bar{r}_2 = \bar{r} \)) because I assume that initial output lies below the cutoff. At any level of capital above the cutoff, the country can borrow only at the world interest rate. The capital levels in period 2 and in the steady state, \( k_2 \) and \( k_{ss} \), are pinned down by \( \bar{r} = f'(k_2) - \delta \) and \( r = f'(k_{ss}) - \delta \) (by equation (A.4)). These equations imply that \( k_2 \) is greater than \( k_{ss} \) because the concessional interest rate is lower than the world interest rate \( (\bar{r} < r) \).
Thus capital overshoots the steady state in period 1. However, from period 3 onward, capital is at its steady-state level (that is, \( k_j = k_{ss} \) for \( j \geq 3 \), because as of period 2, the country no longer has access to concessional loans. Its capital level exceeds the cutoff, and the capital level is \( k_{ss} \) (from equation (A.4)). Consumption, too, overshoots in period 1 (\( c_1 > c_2 = c_{ss} \)). This is implied by the following Euler equations: \( u'(c_1) = \beta (1+r)u'(c_3) \) and \( u'(c_2) = \beta (1+r)u'(c_3) \), because \( \beta (1+r) < 1, \beta (1+r) = 1, \) and \( u'(c) \) is decreasing in \( c \).

Once \( \{k_2, k_{ss}\} \) and \( \{c_1, c_{ss}\} \) are pinned down, the path of debt, \( \{D_2, D_{ss}\} \) can be derived via the budget constraint. The dynamics of concessional loans without a participation constraint are thus characterized by the overshooting of capital and consumption in period 1 as a result of the low concessional interest rate. The donor’s budget, \( \alpha \), is determined by \( \alpha = D_2 (r-\tilde{r})/(1+r) \).

**APPENDIX III**

The FOCs of the Bellman Equation

The Bellman equation (equations (9) and (10)) can be rewritten as Equations (A.5) and (A.6)

\[
v^R(k, X) = \max_{k', X'} \left\{ u(f(k) - k' + (1 - \delta)k + \frac{X'}{(1 + \tilde{r}(k))} - X) + \beta v^R(k', X') \right\},
\]

subject to \( X' \leq h(k') \), (A.6)

because \( v^R(k', X') \geq v^D(k') \) and \( X' \leq h(k') \) are equivalent by construction, given the debt capacity function. In the following, I consider two cases: \( X' < h(k') \) and \( X' = h(k') \), and interpret the corresponding FOCs.

CASE 1: \( X' < h(k') \)

The FOC with respect to \( k' \) is given by \( u_c(c) = \beta v^R(k', X') \), and by the envelope theorem, \( v^R(k, X) = u_c(c)(f_c(k) + 1 - \delta) \).

The Euler equation is given by

\[
u_c(c) = \beta u_c(c')(f_c(k') + 1) + (1 - \delta).
\]

The FOC with respect to \( X' \) is given by \( -u_c(c)/(1 + \tilde{r}(k, X)) = \beta v^R(k', X') \), and by the envelope theorem, \( v^R(k, X) = -u_c(c) \). Combining the above two equations we get

\[
u_c(c) = \beta (1 + \tilde{r}(k, X)) u_c(c').
\] (A.8)

Equation (A.8) can be rewritten as \( u_c(c) = \beta (1 + \tilde{r}) u_c(c') \) if the country lies strictly below the cutoff, and \( u_c(c) = \beta (1 + r) u_c(c') \) if the country lies strictly above the cutoff.

Because \( \beta (1 + \tilde{r}) < \beta (1 + r) = 1, \) the country has an incentive to overconsume in earlier periods under the concessional lending scheme.

CASE 2: \( X' = h(k') \)

The Bellman equation (equations (A.5) and (A.6)) can be rewritten as

\[
v^R(k, X) = \max_{k', X'} \left\{ u(f(k) - k' + (1 - \delta)k + \frac{h(k')}{(1 + \tilde{r}(k))} - X) + \beta v^R(k', h(k')) \right\}.
\]
The FOC with respect to $k'$ is given by

$$u_c(c) \left(1 - \frac{h(k')}{1 + \delta(k)} \right) = \beta [v^R_k(k', h(k')) + v^R_k(k', h(k')) h_k(k')]$$

and by the envelope theorem, $v^R_k(k, X) = u_c(c) (f_k(k) + 1 - \delta)$, and $v^R_k(k, X) = -u_c(c)$. Thus the Euler equation is given by

$$u_c(c) \left(1 - \frac{h_k(k')}{1 + \delta(k)} \right) = \beta u_c(c') (f_k(k') - h_k(k') + 1 - \delta). \quad (A.9)$$

The numerical simulations suggest that $h(., .)$ is decreasing in $k$ unless the country lies sufficiently above the cutoff. Thus there is upward pressure for $c$, whereas there is downward pressure for $c_0$—the country has an incentive to overconsume around the cutoff in earlier periods. To state this intuition more formally, rewrite Equation (A.9) as

$$u_c(c) = \beta \frac{1 + \delta(k)}{1 + \delta(k) - h_k(k')} u_c(c') [f_k(k') + 1 - \delta - h_k(k')]. \quad (A.10)$$

One can interpret the bracketed term as the return from savings. The term $\beta (1 + \delta(k)) / (1 + \delta(k) - h_k(k'))$ can be interpreted as the effective discounting factor. If $h(., .)$ were constant so that $h_k(., .)$ would be zero, then Equation (A.10) would reduce to the familiar expression with the return from savings given by $f_k(k') + 1 - \delta$ and the discounting factor given by $\beta$. The implication of the downward-sloping $h$ (that is, $h_k(., .) < 0$) is twofold. First, the return from capital, the bracketed term in Equation (A.10), is suppressed. Second, the ratio in Equation (A.10) is less than 1, which effectively lowers the discount rate. Both effects act to depress savings.

**APPENDIX IV**

A Description of PROX

There are more than 1,300 observations on PROX between 1987 and 2000, of which about 4 take positive values (that is, only $\frac{1}{4}$ of the observations lie close to the cutoffs). Appendix Figure A3 shows the histogram of PROX excluding observations with $PROX = 0$. When I carry out a simple OLS regression of the growth rate of real GDP per capita on $PROX$, the coefficient for $PROX$ is negative ($-2.22$) and is statistically significant at 1 percent.

Appendix Figure A4 relates the growth rate with the level of GNI after removing regressors other than $PROX$. Recall that the empirical model is $y = \hat{c} + \hat{\alpha} PROX + \gamma Z + \epsilon$, where $y$ is the percentage per capita annual growth rate of real GDP. $PROX$ is a measure of proximity to the cutoff, and $Z$ represents the other explanatory variables. Denote $\hat{c}$, $\hat{\alpha}$, and $\gamma$ as the OLS coefficients. Appendix Figure A4 plots the OLS residuals of the growth rate excluding $\hat{\alpha} PROX$ (that is, $y - \hat{c} - \hat{\gamma} Z$) and $\hat{\alpha} PROX$ against the percentage deviation of GNI from the threshold for IDA eligibility.

**APPENDIX V**

First-Stage Regressions

See Table A1.
Figure A3. Frequency of PROX

Note: This figure reports the histogram of PROX excluding observations with PROX = 0.

Figure A4. Ordinary Least Square Residuals Excluding alpha hat*PROX

Note: Remember that the paper’s empirical model is $y = c + \alpha PROX + \gamma Z + \varepsilon$, where $y$ is the percent annual growth rate of real GDP per capita, $PROX$ is a measure of proximity to the cutoff, and $Z$ represents the other explanatory variables. Denote $\hat{c}$, $\hat{\alpha}$, and $\hat{\gamma}$ as the OLS coefficients. The figure plots the ordinary least squares (OLS) residuals of the growth excluding $\hat{\alpha}PROX$ (that is, $y - \hat{c} - \hat{\gamma}Z$) and $\hat{\alpha}PROX$ against the deviation of GNI from the threshold for IDA eligibility.
Examples

To provide some examples that support this paper’s key conclusions, this appendix reports a set of countries that have converged to (group A) and diverged from (group B) the cutoff. The group A countries have much higher concessional debt than the group B countries. The bottom two countries in group B were affected by the Asian financial crisis in 1997 (Figure A5).

APPENDIX VII

Data

See Tables A2 and A3.
Figure A5. Examples


Note: The dashed lines report concessional loans as a percent of GNI between 1987 and 2000 for countries that have converged to the cutoff (Group A) and that have diverged from the cutoff (Group B). The solid lines show the percent deviations from the per capita income cutoffs.
### Table A2. Data

<table>
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<th>Variables</th>
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<th>Source</th>
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<td>The growth rate of real GDP per capita (in percent)</td>
<td>Constructed from real GDP per capita, Constant prices: Laspeyres ($RGDPL$) in Summers-Heston data set, version 6.1</td>
</tr>
<tr>
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<td>GNI per capita in current U.S. dollars, Atlas methodology</td>
<td>World Development Indicators</td>
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<tr>
<td>$\bar{y}$</td>
<td>The operational IDA cutoff in terms of GNI per capita in U.S. dollars, Atlas methodology</td>
<td>World Bank GNI/capita operational guidelines</td>
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<td>GPO</td>
<td>Percent population growth (a year)</td>
<td>Constructed from population (POP) in Heston, Summers, and Aten, 2002</td>
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<td>INV</td>
<td>Investment share of real GDP per capita (in percent a year)</td>
<td>Heston, Summers, and Aten, 2002</td>
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<td>The log of real GDP per capita in 1985, constant prices: Laspeyres</td>
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<td>$INIT_{EDU}$</td>
<td>Percent of secondary schooling attained in the total population in 1985</td>
<td>Barro-Lee data set</td>
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Note: IDA = World Bank International Development Association.

### Table A3. Country or Regional Coverage of the Data Set

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Table A3 (concluded)

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REFERENCES


Junko Koeda


