Inflation Targeting with NAIRU Uncertainty and Endogenous Policy Credibility

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Abstract

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

Stochastic simulations are employed to compare performances of monetary policy rules in linear and nonlinear variants of a small macro model with NAIRU uncertainty under different assumptions about the way inflation expectations are formed. Cases in which policy credibility is ignored or treated as exogenous are distinguished from cases in which credibility and inflation expectations respond endogenously to the monetary authorities' track record in delivering low inflation. It is shown that endogenous policy credibility strengthens the case for forward-looking inflation forecast based rules relative to backward-looking Taylor rules.

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I. INTRODUCTION

Inflation targeting has become a popular monetary policy strategy for industrial countries. Quantitative targets or target ranges for inflation have been announced by New Zealand (in 1990), Canada (1991), Israel (1991), the United Kingdom (1992), Australia (1993), and Sweden (1993), while a number of other countries have implemented strategies that emphasize informal objectives for inflation. In describing the attractiveness of such strategies, proponents have emphasized that an inflation targeting framework can make the objectives of monetary policy more transparent and, over time, may result in an increase in policy credibility that in turn has desirable implications for macroeconomic performance.²

The increasing popularity of inflation targeting has been accompanied by significant strides in the use of simple macro models to advance the conceptual framework for formulating such strategies.⁴ The formal literature has adopted a broad interpretation of inflation targeting, defining it to imply that policy is characterized by a reaction function in which the monetary policy instrument is adjusted in response to, but not necessarily only in response to, deviations of the inflation rate from an explicit target.⁵ Research by economists at central banks and elsewhere has demonstrated, within the context of linear models, that relatively attractive macroeconomic performances can be delivered by adopting policy rules in which the monetary authorities adjust short-term interest rates in response to both deviations of a recently observed inflation rate from target and deviations of a recently observed level of output from potential.⁶ Such strategies are often referred to as Taylor rules, based on a formulation advanced by Taylor (1993). Under some model specifications,

² Finland and Spain also operated with quantitative inflation targets for several years prior to relinquishing monetary policymaking to the European Central Bank.

³ See, for example, Fischer (196), Freedman (1996), Svensson (1997), Bernanke and Mishkin (1997), Debelle (1997).


⁵ Rudebush and Svensson (1999) and Svensson (1999) distinguish between reaction functions that are essentially derived as first-order conditions for minimizing policy loss functions and reaction functions that are simply postulated, suggest that the term “targeting rule” should only be applied to the former class of reaction functions.

⁶ See, for example, Levin, Wieland, and Williams (1999) and Taylor (1999).
analysis has suggested that Taylor rules can be outperformed by rules in which interest rates are adjusted in response to both the output gap and the deviation from target of an inflation forecast (rather than the deviation from target of the current inflation rate).\(^7\) We follow the terminology of Batini and Haldane (1999), Amano, Coletti, and Macklem (1999) and Clark, Laxton and Rose (2000) in referring to the latter type of reaction functions as inflation-forecast-based (IFB) rules. Such rules have the feature of inducing the authorities to base their interest rate settings on the determinants of (future) inflation, given their information/assumptions about the macroeconomic model.

Economists confront a fundamental difficulty when attempting to analyze how monetary policy strategies should be formulated. On the one hand, the challenge of addressing the relevant issues with coherence and depth requires a formal model that specifies the relationships between the instrument variables that the monetary authorities control and the target variables that are used to evaluate macroeconomic performance. Analysis of the hypothetical performances of mechanical rules within simple but fairly realistic macroeconomic models can provide valuable guidance about the types of policy reactions that are likely to be relatively effective for achieving and maintaining macroeconomic stability in the real world. On the other hand, the "true model" of the relationships between policy instruments and targets is much more complex than economists can hope to formalize, not only because it is difficult to model the aggregate behavior of large numbers of heterogeneous economic agents, but also because the behavior of individual agents evolves over time with innovations in information technologies, the introduction of new markets and products, changes in fiscal and structural policies, and so forth.

The fact that the true model of macroeconomic behavior is unknowable has led to the view that "simple algebraic formulations of ...[policy] rules cannot and should not be mechanically followed by policymakers."\(^8\) Accordingly, while it can be constructive to use simple macro models with appealing specifications to analyze how different types of monetary policy reaction functions compare in delivering relatively attractive outcomes for the means and variances of policy target variables, it would be dangerous to take the additional step of suggesting that a particular calibration of a monetary policy rule is optimal and should be adhered to indefinitely.\(^9\)

\(^7\) For example, Batini and Haldane (1999), Amano, Coletti, and Macklem (1999), and Rudebusch and Svensson (1999).

\(^8\) Taylor (1993), p. 213.

\(^9\) Related to this point, the issue of rules versus discretion for monetary policy has become less actively debated during the 1990s. This may reflect, first, a general acceptance of the premise that a fully-state-contingent rule for monetary policy is not a relevant possibility in a world in which knowledge about the macroeconomic structure and the nature of disturbances is incomplete, and second, a general awareness of the fact that simple (or partially-state-contingent) rules and discretion cannot be unambiguously ranked. In this context, Flood and Isard (1989, 1990) suggested that monetary authorities should be given incentives to follow simple rules with "escape clauses," in recognition of the tradeoff between the various

(continued...)
The unknowability of the true macroeconomic model has also led to the view that the evaluation of policy rules should focus on the robustness of their performances across different classes of plausible models.\textsuperscript{10} Our point of departure for this paper is the observation that progress in the search for robustness has been deficient in several dimensions. In particular, most tests of monetary policy rules have been performed using models that abstract from endogenous policy credibility,\textsuperscript{11} that make unrealistically strong assumptions about the monetary authority's knowledge of the NAIRU, and that ignore possible convexity of the short-run Phillips curve. These deficiencies require attention. Most central bankers regard endogenous policy credibility and NAIRU uncertainty as fundamental characteristics of the worlds in which monetary policy must be conducted, and we regard convex short-run Phillips curves as considerably more plausible than linear Phillips curves and as a strategically more appropriate assumption for policy analysis.\textsuperscript{12}

In light of the above considerations, this paper compares the relative performances of different calibrations of Taylor rules and IFB rules\textsuperscript{13} within several different variants of a model of the inflation-unemployment process. The basic model, taken from Callen and

benefits of the rules and the social costs of failing to modify reaction functions in certain unforeseeable circumstances. Amano, Coletti, and Macklem (1999), among others, emphasize that credibility is a two-edge sword, and that the credibility benefits of monetary policy rules cannot be effectively reaped unless the monetary authorities are prepared to change their reaction function over time in response to new information about macroeconomic structure.

\textsuperscript{10} McCallum (1988).

\textsuperscript{11} Inflation expectations are often modeled as a weighted sum of backward- and forward-looking components, and in this context a number of studies have followed Freedman (1996) in defining the forward-looking component as the announced inflation target and in interpreting the weight on this component as a measure of policy credibility. Within this framework, Amano, Coletti, and Macklem (1999) have analyzed the implications for monetary policy of "exogenous" changes in credibility. In addition, Tetlow, von zur Muehlen, and Finan (1999) have analyzed the optimal form of simple rules when private agents have to learn about the rule. To our knowledge, however, few if any evaluations of policy reaction functions have modeled credibility as a variable that responds endogenously to the monetary authority's performance in delivering macroeconomic stability.

\textsuperscript{12} See, for example, Isard and Laxton (1996), Clark and Laxton (1997), and Laxton, Rose, and Tambakis (1999). See also Summers (1988), who questioned the value of basing policy analysis on models in which monetary policy is incapable of influencing the average rates of inflation and unemployment.

\textsuperscript{13} We report simulations for two calibrations of each class of rules. The first corresponds to the calibration suggested in Taylor (1993); the second is based on a calibration suggested by our analysis in Isard and Laxton (1999).
Laxton (1998), was estimated using quarterly data for the Australian economy.\textsuperscript{14} The variants of the model include different combinations of linear and nonlinear short-run Phillips curves with different assumptions about the way the public forms its inflation expectations. We distinguish three cases of inflation expectations: backward-looking expectations under which credibility is ignored; forward-looking expectations that treat credibility as exogenous; and a forward-looking case in which credibility and inflation expectations respond endogenously to the policy track record in delivering low inflation.

In practice, monetary policy operates under considerable uncertainty about model parameters, including imprecise estimates of the NAIRU.\textsuperscript{15} Our comparisons of monetary policy rules attempt to capture this element of reality by treating the NAIRU as an uncertain and time varying parameter.\textsuperscript{16} The authorities are assumed to continually update their estimates of the NAIRU based on their continuing observations of unemployment and inflation and their partial knowledge of the true macroeconomic model. But like all other economists, they inevitably make serially-correlated errors in estimating the NAIRU ex ante, so it is important to evaluate monetary policy rules in the realistic context of ongoing policy errors.

The analysis is based on stochastic simulations, with each rule evaluated on the basis of the same sequences of randomly drawn shocks. We employ a fairly standard quadratic loss function to provide a summary statistic for evaluating the rules, and we also report the associated means and standard deviations of inflation and unemployment. Each of the rules embodies the same target rate of inflation, and the stochastic simulations all start with the inflation rate initially at its target level. In the linear models without endogenous credibility, monetary policy does not have first-order welfare effects in the sense that the means of unemployment and inflation are essentially independent of the policy rule. For these linear cases, the IFB rules on which we focus slightly outperform the analogous Taylor rules by delivering lower standard deviations of the unemployment rate with comparable standard deviations of inflation rates, but the differences in performance are not very striking.\textsuperscript{17} For the nonlinear cases with endogenous policy credibility and/or convex Phillips curves—where

\textsuperscript{14} For most of the equations the sample period runs from the early 1980s through the mid-1990s.

\textsuperscript{15} Orphanides (1999) constructs a data base of the information available to U.S. policymakers in real time from 1965 to 1993 and suggests that misperception of the economy’s productive capacity was the primary underlying cause of the inflation of the 1970s.

\textsuperscript{16} See Smets (1999) and Drew, Hunt, and Scott (1999) for other analyses of the implications of uncertainty about the NAIRU (or the output gap).

\textsuperscript{17} In the linear variants of our model, the degree to which IFB rules outperform Taylor rules appears to be slightly greater under forward-looking expectations than under backward-looking expectations. This contrasts with results from other models, in which the optimal degree of forward-lookingness in expectations, which is often associated with the length of contract lags in wage and price setting; see, for example, Batini and Haldane (1999).
monetary policy does have first-order welfare effects—the forward-looking IFB rules outperform Taylor rules, and the choice among different calibrations of IFB rules or Taylor rules can have substantial implications for the means of unemployment and inflation. One line of intuition is that forward-looking and relatively forceful policy reactions can be particularly important when credibility can be lost much more quickly than it can be regained. In this connection we show that a popular calibration suggested by Taylor (1993) for the U.S. economy can be improved upon significantly in our model. Additional intuition for the poor performance of Taylor rules relative to IFB rules comes from noting that the performance of a simple linear rule in a nonlinear model can be improved by specifying the rule in terms of a model-consistent forecast that reflects the nonlinearities in the model.

The paper is organized as follows. Section II motivates the treatment of endogenous policy credibility and presents the basic model. Section III describes the design of our stochastic simulation experiments and discusses the different policy reaction functions and model variants that we consider. Section IV assesses the simulation results. Section V concludes.

II. MODEL OF THE INFLATION-UNEMPLOYMENT PROCESS

A. Background

The analytical frameworks that have been developed for addressing monetary policy issues in an open economy traditionally exhibit the monetary policy transmission mechanism depicted in Figure 1. The authorities control a short-term nominal interest rate ($r_s$) with the objective of influencing the rates of inflation ($\pi$) and unemployment ($u$). As shown by the arrows, changes in the policy instrument are transmitted to the policy target variables through several channels. Adjustments in the nominal interest rate can trigger movements in the nominal exchange rate ($s$), which are transmitted fairly directly to tradable goods prices and inflation and indirectly to unemployment through their effects on the real exchange rate ($z$) and the gap ($y$) between actual and potential domestic output. Changes in the nominal interest rate also affect the real interest rate ($r_s - \pi$), both directly and through the response of inflation expectations ($\pi^e$); changes in the real interest rate in turn influence unemployment through their effects on aggregate demand and the domestic output gap; and changes in the output gap and unemployment rate influence the inflation rate through channels summarized by the Phillips curve. In addition, an important two-way feedback mechanism is superimposed on the transmission process, with inflation expectations responding (inter alia) to the history of inflation, and with inflation influenced in turn by changes in inflation expectations.

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18 In Isard and Laxton (1999) we focus on a variant of IFB rules in which the authorities react to a weighted average of the deviation from target to their own inflation forecast and the deviation from target to the public's inflation forecast (as reflected in survey measures of inflation expectations). Monetary rules that give weight to the latter deviation—that is, to the bias in the public's inflation expectations—would appear to establish a channel for policy to respond directly to changes in credibility.
Figure 1. The Monetary Policy Transmission Mechanism

rs : Nominal short-term interest
s : Nominal exchange
z : Real exchange
y : Output

π : Inflation
π^e : Inflation
u : Unemployment
Figure 1 does not show any feedback from the policy target variables to the policy instrument. The tasks of identifying and implementing a mechanism for reacting to economic developments in a manner that is conducive to macroeconomic stability is the responsibility of the monetary authorities. In particular, the role of monetary policy is to adjust the policy instrument variable (in this case the nominal interest rate) in reaction to observed and anticipated changes in unemployment, inflation, and other macroeconomic variables, taking account of the behavioral relationships among these variables.

In reality, the operation of monetary policy is greatly complicated by two types of uncertainties: imperfect information about the magnitudes of the various transmission effects shown in the diagram, and difficulties in identifying the effects on macroeconomic variables of various types of economic shocks. In principle, there can be exogenous shocks that directly affect the exchange rate, the observed inflation rate, or the expected inflation rate; and there can be exogenous shifts in the output gap associated with shocks to either aggregate demand or potential aggregate supply.

The operation of monetary policy is also complicated by the fact that policy credibility is imperfect and can vary with the effectiveness of the monetary authorities in achieving desirable outcomes for policy target variables. The endogenous behavior of policy credibility and its role in the monetary policy transmission mechanism has not yet been adequately incorporated into the models that have been used to analyze monetary policy issues.

To motivate our interest in exploring the relevance of endogenous policy credibility for the design of monetary policy strategies, Figure 2 plots quarterly data on selected economic indicators for Australia and the United States. The top two panels show recorded inflation rates and survey measures of inflation expectations. In the United States, inflation has been generally subdued and trendless since the early 1980s, after declining sharply during 1980–82 in the context of a relatively tight monetary policy. In Australia, inflation declined more gradually from the levels experienced during the 1970s and has been generally subdued and trendless only since the early 1990s. Survey measures of Australian inflation expectations have remained persistently above recorded inflation rates during the past decade, while survey measures of U.S. inflation expectations have tracked recorded inflation rates fairly closely. Moreover, with similar recorded inflation rates in Australia and the United States during the 1990s, long-term Australian government bonds have required an interest premium relative to the yield on U.S. government bonds, as shown in the third panel; this is consistent with the expected inflation differential. And finally, despite experiencing similar and fairly stable rates of inflation during the 1990s, the two countries have significantly different unemployment rates, as indicated in the bottom panel.

What can explain why these variables have behaved differently in Australia and the United States? One plausible explanation is that the relatively slow decline and small-sample bias of inflation expectations in Australia, compared with the rapid decline and relative unbiasedness of inflation expectations in the United States, is a reflection of imperfect policy credibility that differs across countries. Imperfect policy credibility may not be the entire
Figure 2. Selected Economic Indicators for Australia and the United States

**Inflation and Inflation Expectations (In percent)**

- Australia
- Underlying Inflation
- Inflation Expectations derived from Indexed Bonds
- Melbourne Institute Inflation Expectations

**United States**

- CPI Inflation
- Michigan Survey Inflation Expectations

**10 Year Government Bond Yields (In percent)**

- Australia
- United States

**Unemployment Rates (In percent)**

- Australia
- United States
explanation, but it is a leading candidate that warrants explicit consideration in modeling the monetary policy transmission mechanism.

**B. The Basic Model, Part I**

For purposes of this paper, it is convenient to use the small open-economy model developed by Callen and Laxton (1998), which was estimated using quarterly data for Australia.\(^\text{19}\) Table 1 summarizes the assumed behavior of domestic demand, exports, imports, the unemployment rate, the nominal exchange rate, and the rate of inflation. Table 2 describes the treatment of inflation expectations and policy credibility. The variables in the equations are measured quarterly.

The conceptual underpinnings of the equations in Table 1 are relatively familiar; here we review them briefly and refer readers to Callen and Laxton (1998) for additional discussion and econometric details. The top three equations focus on components of the national income accounts, where domestic demand represents the aggregate domestic expenditures of consumers, investors, and governmental units. The dependent variables refer to detrended measures of domestic demand, exports, and imports.\(^\text{20}\) The estimated equations suggest that aggregate (detrended) domestic demand \((d)\) exhibits a high degree of persistence (a weight of nearly unity on the lagged dependent variable); it also depends negatively on the (two-quarter lagged) level of the real interest rate \((r)\) and positively on a relevant measure of national wealth \((a)\)—namely, the stock of net claims on the rest of the world (as a percent of GDP). Exports \((x)\) exhibit a substantial degree of persistence (a coefficient of 0.48 on the lagged dependent variable) and also depend importantly on the real exchange rate \((e)\), an increase in which represents a real depreciation). Imports \((m)\) likewise exhibit substantial persistence and depend in a conventional manner on domestic demand, exports, and the real exchange rate. Together, the equations for detrended domestic demand, exports, and imports describe the behavior of aggregate output relative to trend, which is used as a measure of the output gap \((y)\), defined as actual output minus potential output). An important property of these equations is that there are significant lags between changes in real monetary conditions—that is, the real interest rate and the real exchange rate—and the output gap.

The fourth equation in Table 1 describes the behavior of a labor market tightness measure—defined as the amount by which the unemployment rate \((u)\) falls below the NAIRU \((\bar{u})\).\(^\text{21}\) The estimated relationship shows a high degree of persistence along with positive dependence on the output gap. The coefficient on the output gap is significantly less than one, which is consistent with casual empiricism: firms hoard labor over the business cycle and, because of hiring and firing costs, adjust labor inputs slowly in response to changes in the demand for their products.

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\(^{19}\) For most equations the sample period runs from the early 1980s through the mid-1990s.

\(^{20}\) The variables were detrended using the HP filter with a smoothing parameter of 1600; see Hodrick and Prescott (1981).

\(^{21}\) The specification explicitly treats \(u - \bar{u}\) as a zero mean process.
Table 1. A Small Model of the Inflation-Unemployment Process, Part 1

*Domestic demand:*

\[ d_t = -0.33 \ r_{t-2} + 8.40 \ a_t + 0.95 \ d_{t-1} + \varepsilon_i^d \]  \hspace{1cm} (1)

*Exports:*

\[ x_t = 0.06 \ z_t + 0.48 \ x_{t-1} + \varepsilon_i^x \]  \hspace{1cm} (2)

*Imports:*

\[ m_t = 0.33 \ x_t + 1.35 \ d_t - 0.09 \ z_t + 0.41 \ m_{t-1} + \varepsilon_i^m \]  \hspace{1cm} (3)

*Labor market tightness:*

\[ (\bar{u}_t - u_t) = 0.20 \ y_t + 0.77 \ (\bar{u}_{t-1} - u_{t-1}) + \varepsilon_i^{(\bar{u}-u)} \]  \hspace{1cm} (4)

*Exchange rate:*

\[ s_t \left[ 1 + s_t - s_t^{US} \right]^{2S} = 0.38 \ s_{t+1}^{MC} + (1 - 0.38) \ s_{t-1} \left[ 1 + E_t \pi_{t+4} - E_t \pi_{t+4}^{US} \right]^{5} + \varepsilon_i^s \]  \hspace{1cm} (5)

*Phillips curve:*

\[ \pi_t = \pi_t^c + 2.14 \left[ \frac{u_t^* - u_t}{u_t - \Phi_{t-1}} \right] + \varepsilon_i^x \]  \hspace{1cm} (6)

where

\[ \pi_t^c = 0.73 \pi_t^e + (1 - 0.73 - 0.04 - 0.02) \pi_{t-1} + 0.04 \pi_t^m + 0.02 \pi_{t-1}^m \]  \hspace{1cm} (7)

and

\[ \pi_t^e = 0.25 \left[ E_{t-1} \pi_{t+3} + E_{t-2} \pi_{t+2} + E_{t-3} \pi_{t+1} + E_{t-4} \pi_{t} \right] \]  \hspace{1cm} (8)
Table 2. A Small Model of the Inflation-Unemployment Process: Part II, Inflation Expectations and Policy Credibility

Inflation expectations 1 year ahead:

\[ E_t \pi_{t+4} = 0.13 \pi_{t+4}^{mc} + (1 - 0.13) E_{t-1} \pi_{t+3} + b_t + \varepsilon_e \tag{9} \]

Low inflation forecasting rule:

\[ \pi_{t}^{L} = 2.5 (1 - 0.34) + 0.34 \pi_{t-1} + \varepsilon_{t}^{L} \tag{10} \]

High inflation forecasting rule:

\[ \pi_{t}^{H} = 8.4 (1 - 0.81) + 0.81 \pi_{t-1} + \varepsilon_{t}^{H} \tag{11} \]

Expectations bias attributable to imperfect policy credibility:

\[ b_t = 0.04 \left[ c_t E_t \{ \pi_{t+4}^L \pi_{t} \} + (1 - c_t) E_t \{ \pi_{t+4}^H \pi_{t} \} - \pi_{TAR} \right] \tag{12} \]

Policy credibility

\[ c_t = 0.83 c_{t-1} + 0.17 \Psi_{t-1} + \varepsilon_{t}^{c} \tag{13a} \]

where

\[ \Psi_t = \frac{[\varepsilon_{t}^{H}]^2}{[\varepsilon_{t}^{H}]^2 + [\varepsilon_{t}^{L}]^2} \tag{13b} \]
In the exchange rate equation, the left-hand side variable represents the one-period forward exchange rate, which, under covered interest parity, equals the prevailing spot exchange rate appropriately adjusted for the interest rate differential.\textsuperscript{22} The right-hand side of the equation is a standard backward- and forward-looking components model of the expected future spot exchange rate ($s_{t+1}^e$), where $s_{t+1}^{mc}$ represents the model-consistent solution and the backward-looking component is simply the lagged spot rate adjusted for the expected inflation differential.\textsuperscript{23} The measures of inflation expectations are based on surveys of the rates of inflation expected in Australia and the United States over the year ahead. We use the notation $\pi_4$ to refer to inflation over four quarters and $\pi$ to refer to annualized rates of inflation over one quarter; $E_{t}\pi_{4,t+1}$ is the public's expectation in quarter $t$ of the rate of inflation over the year through quarter $t+4$. In estimating equation (5), Callen and Laxton (1998) found a large weight on the backward-looking component, consistent with other evidence that high proportions of the short-run behavior of exchange rates (and other asset prices) cannot be explained by fundamentals, but rather are conditioned by the recent behavior of exchange rates (or other asset prices).\textsuperscript{24}

The Phillips curve described by equation (6) embodies the convex specification analyzed by Debelle and Laxton (1997) and Debelle and Vickery (1997).\textsuperscript{25} In the spirit of analyzing the robustness of our simple policy rules, we give equal consideration to linear variants of the model, in which the convex term in the unemployment rate is replaced by a linear approximation, as described in Section IV below.\textsuperscript{26} For expository convenience,

\textsuperscript{22} The appropriate interest factor corresponds to one plus the per-annum interest rate differential expressed as a quarterly rate.

\textsuperscript{23} Adjustment for the expected inflation differential is a necessary condition for ensuring that the behavior of the real exchange rate is independent of the target rate of inflation.

\textsuperscript{24} See Isard (1995). Although we do not investigate the issue in this paper, we have the impression that the choice between full model consistency and a backward- and forward-looking components specification of the expected future spot rate can make a considerable difference in using stochastic simulations to evaluate policy rules when the shocks that drive the simulations are drawn from distribution that reflect the variance of estimated residuals for the historical period over which the model is estimated.

\textsuperscript{25} As described in Appendix I, the Phillips curve is combined with an equation that describes a time-varying DNAIRU to provide a nonlinear estimation problem that can be solved using the Kalman filter technique. Kuttner (1991, 1992, 1994) has applied this idea to measuring potential output.

\textsuperscript{26} Econometric tests generally do not have sufficient power to reject with the hypothesis that the Phillips curve is linear or the hypothesis that the Phillips curve is convex. For discussions of potential pitfalls associated with conventional tests for asymmetries in the Phillips curve, see Clark, Laxton, and Rose (1996) and Laxton, Rose, and Tambakis (1999).
equation (6) consolidates several terms into a composite "core" rate of inflation ($\pi^c$), which is spelled out in equation (7). The first two terms on the right hand side of equation (7) describe the rate of inflation expected over the next quarter. Expectations are assumed to influence inflation dynamics through a wage-and-price setting framework in which the standard contract has a four-quarter horizon, with one-fourth of the contracts respecified every quarter. This is reflected in the $\bar{\pi}^e$ term, which is defined in equation (8) as the average of the one-year-ahead inflation expectations that economic agents held during the four quarters in which currently-prevailing contracts were written. Expected inflation is also assumed to depend on the lagged inflation rate (the second term on the right hand side of equation (7)), which can be viewed as a summary indicator of the incentives to incur the costs of revising price or wage contracts before their specified expiration dates. The third and fourth terms on the right hand side capture the influence on consumer price inflation of the contemporaneous and one-quarter-lagged rates of change in import prices ($\pi^m$).

Note that the coefficients in equation (7) sum to unity, consistent with the long-run natural rate hypothesis. Figure 3 plots the difference between observed inflation and core inflation (vertical axis) against the unemployment rate (horizontal axis). For purposes of the discussion here, we make the simplifying assumption that core inflation and expected inflation coincide, so that the figure can be viewed as an expectations-augmented Phillips curve. Consistent with the specification in equation (6), the short-run Phillips curve is convex with vertical asymptote at $u = \Phi$ and horizontal asymptote at $\pi - \pi^e = -\gamma$.\footnote{In equation (6) the estimated value of $\gamma$ is 2.14. The estimation and stochastic simulations are based on the assumption that $\Phi_t = \max(0, u^{*}_t - 4)$, and it turns out that $u^{*}_t$ is always strictly greater than 4 in the actual and hypothetical data we address.} The magnitude of $u^*$ corresponds to the unemployment rate at which actual inflation and expected inflation coincide, such that there would be no systematic pressure for inflation to rise or fall in the absence of stochastic shocks. This corresponds to the non-accelerating-inflation rate of unemployment in a deterministic world. We refer to $u^*$ as the DNAIRU.

An important point is that the DNAIRU is not a feasible stable equilibrium in a stochastic world with a convex Phillips curve. The average rate of unemployment consistent with non-accelerating-inflation in a stochastic world, denoted by $\bar{u}$ and referred to as the NAIRU, must be greater than the DNAIRU when the Phillips curve is convex. This can be illustrated in Figure 3 by assuming that actual inflation turned out to be uniformly distributed between plus and minus one percentage point of core (or expected) inflation, which would imply an average rate of unemployment of $\bar{u} = 0.5 (u_1 + u_2)$. It can easily be seen that with a wider distribution of the actual inflation rate around core inflation, the average rate of unemployment would be even greater. The fact that the difference between the NAIRU and DNAIRU—and hence the average rate of unemployment—depends, in a nonlinear world, on the degree to which the authorities succeed in mitigating the variance of inflation has important implications for monetary policy, as discussed below.
Figure 3. The Convex Phillips Curve
For the most part, the simulation experiments performed with the model start with the economy in a hypothetical equilibrium state. Given the model equations, the simulation results do not depend on historical data, which we generate somewhat arbitrarily. The following points may be noted, however, about the historical data used by Callen and Laxton (1998) in estimating the model equations. The data on inflation expectations for Australia reflect median responses taken from the Westpac bank Melbourne Institute Survey, which asks households for their expectations of inflation one year ahead. 28 The analogous data for the United States represent median responses from the Michigan consumer survey of one-year ahead changes in the U.S. consumer price index. And the data on the NAIRU and DNAIRU were constructed by Callen and Laxton (1998) in a manner consistent with the structure of the Phillips curve and historical movements in inflation, inflation expectations, unemployment, and import prices. 29

C. Inflation Expectations and Policy Credibility

Table 2 presents the equations that describe the behavior of inflation expectations with endogenous policy credibility. 30 The top equation focuses on explaining survey measures of inflation expectations. As noted above, a four-quarter average of these survey measures was assumed in specifying the Phillips curve relation described in Table 1, and the simulation experiments described below use forecasts from equation (9) as hypothetical survey measures of the public’s inflation expectations. The first two terms in the equation are intended to represent a weighted average of a forward-looking model-consistent inflation measure and the one-quarter lag of the survey measure of inflation expectations. For purposes of this paper, the former measure is derived from a proxy for the model—namely, as the fitted values of an auxiliary equation that predicts observed inflation over the year ahead using four lagged values each of the unemployment rate, a long-term interest rate, the survey measure of inflation expectations, and the inflation rate. The third term in equation (9) is intended to capture the expectations bias (b) attributable to imperfect policy credibility.

In modeling imperfect policy credibility and quantifying the expectations bias term, we start from the notion that agents who have experienced high inflation in the past are likely to attach a time-varying probability to the prospect that the monetary authorities will remain truly committed and capable of delivering low inflation in the future. To develop this notion in a simple way, we adopt the following paradigm. It is assumed that economic agents, in forming their inflation expectations, distinguish between two scenarios. Under one scenario, monetary policy is successful in achieving and maintaining price stability, and the inflation

28 The data are based on monthly surveys of 1200 randomly selected adults. Our time series was constructed by averaging the median responses over the three months of each quarter.

29 The methodology employed to develop the historical date for $u^*$ and $\bar{u}$ involves using a Kalman filter to solve a system of equations that includes two Phillips curves and an unemployment equation; see Callen and Laxton (1998).

30 The model variants without endogenous credibility are described in Section IV below.
Figure 4. Historical Perspectives on Policy Credibility
rate either remains at, or gravitates toward, the authorities' target rate of inflation. Under the second scenario, policy is less successful in maintaining price stability and inflation gravitates toward a higher steady-state rate. In each scenario the expected inflation rate (i.e., the value of the forecasting rule with an expected error of zero) is assumed to reflect a weighted average of the steady-state inflation rate and the most recently observed inflation rate; this implies that the expected inflation rate exhibits persistence, adjusting each quarter in proportion to, but by a smaller amount than, the observed change in the (lagged) inflation rate. For an inflation target of 2.5 percent, the inflation rate expected under the low-inflation scenario can be described by equation (10), where 0.34 is an estimated parameter. Analogously, under the assumption that the steady-state inflation rate for the high-inflation scenario is 8.4 percent, the inflation rate expected under the high-inflation scenario can be described by equation (11).³¹

Equation (12) defines the measure of expectations bias, where the terms in square brackets measure the discrepancy between the inflation target (π⁰) and a weighted average of the inflation forecasts under the two scenarios. As reflected in equation (13a), the weight on the low-inflation scenario (c) is treated as a time-varying parameter that exhibits a high degree of persistence (as implied by the model estimates), but changes from period to period by an amount that reflects the extent to which inflation outcomes are more consistent with the low inflation scenario than with the high inflation scenario. The term ψ, as defined in equation (13b), provides our measure of the extent to which inflation outcomes are consistent with the low-inflation scenario.³² Note that if inflation outcomes are repeatedly completely consistent with the low-inflation scenario, c = 0 and ψ = 1, so c converges to unity. Likewise, if inflation outcomes are repeatedly completely consistent with the high inflation scenario, c = 0, ψ = 0, and c converges to zero.

The above two-scenario paradigm bears a resemblance to a two-state regime-switching model with time-varying transition probabilities.³³ Note that the inflation target coincides with the steady-state outcome under the low inflation scenario (i.e., the model is estimated and simulated with π⁰ = 2.5); that c, can be interpreted as the subjective

³¹ The model for the high-inflation state is based on work by Tarditi (1996); see the discussion in Callen and Laxton (1998). Note also that the 8.4 percent steady-state inflation rate used in the high inflation forecasting rule corresponds roughly to the average rate of inflation in Australia during the 1980s (recall Figure 2).

³² The measures of ε⁰ and ε¹ are constructed by substituting the realized inflation outcomes into the left-hand-sides of equations (10) and (11), respectively. Such measures correspond to the ex-post errors associated with interpreting the realized inflation outcomes as ex-ante forecasts of inflation under each scenario.

³³ Flood and Garber (1983) presented an early model of stochastic switching in policy regimes, and Hamilton (1988, 1989) also contributed importantly to catalyzing the use of regime-switching models. See Laxton, Ricketts, and Rose (1994) for an application to the analysis of learning and endogenous monetary policy credibility in Canada; see Kaminsky and Leiderman (1998) for an application to developing countries.
probability attached to the low inflation state; and that in the limiting case of $c_i = 1$, long-run inflation expectations coincide with the inflation target. Based on the latter two properties, we regard $c_t$ as a measure of the stock of credibility at time $t$. Although the associated measure of bias is derived from a relatively simple model of learning\textsuperscript{34} and might not remain reasonable if the monetary authorities shifted their inflation target significantly away from 2.5, it has the attractive features of behaving in a stable manner (not subject to multiple equilibria) and converging to zero as $c$ converges to one. Furthermore, the estimated behavior of $c_t$ and $\psi$ from the mid-1970s through the mid-1990s seems plausible; see Figure 4. The paradigm thus seems to provide a reasonable way of introducing endogenous policy credibility into a formal model for analyzing monetary policy issues.

III. THE STOCHASTIC SIMULATION FRAMEWORK AND SELECTED POLICY RULES

A. The Monte Carlo Experiments

The assumptions underlying our Monte Carlo experiments are as follows:

1. The "true model" of macroeconomic behavior consists of the equations in Tables 1 and 2 (with different versions of equations (6) and (9) under different model variants, as described below), together with the monetary policy rule and a process (described in Appendix I) that generates the evolution of the DNAIRU and NAIRU over time.

2. The DNAIRU follows a bounded random walk\textsuperscript{35} with floor at 4 percent and ceiling at 10 percent. Conditional on not hitting either bound, the DNAIRU changes from one quarter to the next by a random amount drawn from a normal distribution with mean zero and standard deviation corresponding to that of the distribution of estimated DNAIRUs during the historical sample period;\textsuperscript{36} for cases in which the DNAIRU could not change by the full amount of the random draw without moving below 4 percent or above 10 percent, the DNAIRU moves to its floor or ceiling, respectively.

3. In each period $t$, the monetary authorities update their estimate of the DNAIRU and set the period-$t$ interest rate based on an information set $\Omega_t$ that includes: the complete specification of the true model except for the process that generates the DNAIRU and

\textsuperscript{34} In a more sophisticated model of learning, the measure of $\bar{H}$ and $\bar{L}$ would be adjusted for the public’s estimates of the authorities’ control errors. However, the development of such a learning model is beyond the scope of this paper, which simply aims to illustrate that endogenous policy credibility is a relevant consideration in the design of monetary policy rules.

\textsuperscript{35} The assumption of boundedness seems conceptually appropriate. However, the results we report in this paper are based on simulations that are all initialized with the DNAIRU at 7.0, and in which the DNAIRU rarely, if ever, hits its floor or ceiling.

\textsuperscript{36} Callen and Laxton (1998) describe the methodology employed to develop the historical estimate of the DNAIRU and NAIRU.
NAIRU (i.e., complete information about equations (1)-(3) and (5)-(14)); the history of all observable variables (including the survey measures of inflation expectations) through period \( t-1 \), as well as the inflation rate for period \( t \); and the probability distributions (but not the realizations) of the shocks for period \( t \) and all future periods. This implicitly assumes that the inflation rate is the first relevant period-\( t \) statistic that becomes known to the authorities, and that the period-\( t \) interest rate is set immediately following the arrival of information about the period-\( t \) inflation rate.\(^37\) Although the authorities do not know the true processes that generate the DNAIRU and NAIRU, they update their estimates of these parameters each period on the basis of their information about the structure of the model and the history of unemployment and inflation.\(^38\)

4. The exogenous shocks are drawn from independent normal distributions with zero means and standard deviations that reflect the unexplained variances of the dependent variables during the historical periods over which the equations were estimated.

5. The authorities use a prespecified policy rule, along with the assumption that the realizations of random shocks beyond period \( t \) will coincide with their expected values of zero, to determine the interest rate setting for period \( t \) and to generate forecasts of the complete future timepaths of all the macroeconomic variables in the model, including interest rates and the DNAIRU.

Given these assumptions, for each candidate policy rule we simulate a hypothetical path of the economy over a horizon of 100 quarters. Starting in period \( t \), the monetary authorities observe data through period \( t-1 \), update their forecasts for all variables through the end of a 50-quarter horizon,\(^39\) including their forecasts for the period-\( t \) values of the variables that enter the policy rule (with perfect foresight or advanced knowledge of the period-\( t \) inflation rate), and determine the period-\( t \) interest rate setting. After the forecasts are generated and the interest rate is set, the shocks for period \( t \) are drawn randomly (but consistently with the prespecified probability distributions) and the period-\( t \) values of relevant variables are determined from the true model. The determination of the period-\( t \) solution is based on the assumptions that the (expected) future path of the real interest rate coincides with the authorities' forecast, and that the NAIRU follows the process described by the true model. After the period-\( t \) solution is added to the hypothetical history of the

\(^{37}\) From technical and strategic perspectives, giving the authorities knowledge of the period-\( t \) inflation rate reduces the incidence of unstable stochastic simulations for the Taylor rule cases and appears to act in the direction of understating the extent to which IFB rules dominate Taylor rules in our simulation experiments.

\(^{38}\) The procedure that the authorities are assumed to use to update their estimates of the DNAIRU and NAIRU is described in Appendix I.

\(^{39}\) A value of 50 quarters is sufficiently long to insure that errors in the terminal conditions will not induce errors in the variables of interest. Under inflation targeting, the price level has a unit root, and the procedure for period-to-period updating of the authorities' forecasts also involves period-to-period updating of the terminal conditions.
economy, the authorities generate updated forecasts (including updated estimates of the NAIRU) and set the interest rate for period $t+1$. And so forth.

The process of generating 100-quarter simulations is conducted 100 times for the given policy rule, starting each time from the same initial position of the economy. Together the 100 simulations provide 10,000 observations for each variable in the model. The process is then replicated for the other policy rules, using the same 100 sequences of randomly drawn shocks.\textsuperscript{40} For purposes of evaluating the relative attractiveness of the different rules and rule calibrations, we define the loss function

$$L_t = (\pi_t - \pi^{TAR})^2 + \theta \left[ u_t - (u_t^* - \beta) \right]^2 + \nu (r_{st} - r_{st-1})^2$$

(14)

where $\theta$, $\beta$, and $\nu$ are parameters and $u_t^*$ is the DNAIRU (deterministic NAIRU). For $\beta = 0$ this corresponds to the specification that has been used in other recent simulation studies of monetary policy rules.\textsuperscript{41} More generally, it also allows us, somewhat in the spirit of Barro and Gordon (1983a, 1983b) and Rogoff (1985), to consider cases in which the authorities' preferences with regard to unemployment are not symmetric around the DNAIRU but center on an unemployment rate below the DNAIRU (i.e., cases with $\beta > 0$).\textsuperscript{42} For each of the alternative rules and rule calibrations we compare the simulated sample means and standard deviations of the inflation rate and the unemployment rate and calculate, as a summary statistic, the sum of the 10,000 observations on $L_t$ for the base-case parameter settings ($\beta, \theta, \nu$) = (1, 1, 0.5).\textsuperscript{43}

\textsuperscript{40} For the model variants with endogenous policy credibility, a few draws of the random shocks led to explosive simulations under rule calibrations that placed relatively high weights on the unemployment gap—specifically, the $(\alpha, \gamma) = (0.5, 1)$ calibrations; see below. We actually performed somewhat more than 100 sets of simulations and then discarded the results for the draws in which convergence failure was experienced under any one of the rule calibrations.

\textsuperscript{41} For example, Rudebusch and Svensson (1999). Somewhat analogously, Levin, Wieland, and Williams (1999) focus on minimizing the first two terms of (14) subject to an upper bound in the third term.

\textsuperscript{42} Faust and Svensson (1999) focus on a similar loss function with $\beta > 0$ in analyzing the pros and cons of central bank transparency.

\textsuperscript{43} The setting $\nu = 0.5$ corresponds to the base-case value used by Rudebusch and Svensson (1999).
B. The Model Variants

The relative performance of the policy rules were evaluated within each of six variants of the basic model. In the first three variants, the convex Phillips curve (equation (6) in Table 1) was replaced with the linear approximation\textsuperscript{44}

\[ \pi_t = \pi_t^c + 0.535 (u^*_t - u_t) + \varepsilon_t^\pi \]  \hspace{1cm} (6a)

For the first of these cases, as well as the first case with a convex Phillips curve, we assume that the public forms its inflation expectations in a backward-looking manner; thus, we replace equation (9) with

\[ E_t \pi_{t+4} = \pi_t \]  \hspace{1cm} (9a)

Under the second case for each class of Phillips curves, it is assumed that the public forms its inflation expectations in a forward-looking manner but treats policy credibility as exogenous. For these cases the simulations retain equation (9) but set \( b_\pi = 0 \) for all \( t \). The third model variant for each class of Phillips curve is forward-looking with endogenous policy credibility.

C. The Policy Rules

For each of the six model variants we used our stochastic simulation experiments to evaluate macroeconomic performance under two forms of policy rules: Taylor rules and analogous inflation forecast based (IFB) rules.\textsuperscript{45} Part of the motivation for focusing on these linear forms of policy rules is pragmatic; particularly in the nonlinear variants of our model, the task of deriving the optimal functional form would be horrendous. In addition, simple classes of rules are transparent and relatively appealing to policymakers, Taylor rules have been popular in the literature since the early 1990s, and IFB rules have been shown to deliver reasonable economic performances over a wide range of disturbances.\textsuperscript{46} It may be noted that linear IFB rules, by focusing on the deviation from target of the authorities' inflation forecast,

\textsuperscript{44} The simulation set \( \Phi_t = u_t^* - 4 \), so the convex term in the unemployment rate in equation (6) can be expressed as \( 2.14 F(g) \), where \( g = u^*_t - u \). The linear approximation in equation (6a) replaces \( F(g) \) with \( F'(0)(u^*_t - u) = (2.14/4)(u^*_t - u) = 0.535 (u^*_t - u) \).

\textsuperscript{45} Forward-looking IFB rules have been used for almost a decade at the Bank of Canada to solve nonlinear macroeconomic models designed for policy analysis. With the development of more robust and efficient solution methods, these rules are now starting to be used in other policymaking institutions. See Armstrong and others (1998) and Julliard and others (1998) for a discussion of the algorithms that can be used to solve these types of models.

\textsuperscript{46} On IFB rules see Amano, Coletti, and Macklem (1999) and Batini and Haldane (1999).
have the appealing feature (in comparison with Taylor rules) of taking account of any nonlinearities in the macroeconomic model insofar as the inflation forecast reflects the structure of the model.

The two classes of rules were specified in the general forms:

\[
    r_t = r^* + \tilde{E}_t \{ \alpha (\pi_t - \pi^{TAR}) + \gamma (\bar{u}_t - u_t) \mid \Omega_t \} 
\]

and

\[
    r_t = r^* + \tilde{E}_t \{ \alpha (\pi_{t+1} - \pi^{TAR}) + \gamma (\bar{u}_t - u_t) \mid \Omega_t \} 
\]

where

\[
    r_t = r_{st} - \tilde{E}_t \{ E_t \pi_{t+4} \mid \Omega_t \} 
\]

Here \( r_{st} \) is the nominal interest rate setting at time \( t \); \( r_t \) is the concept of the real interest rate on which aggregate demand depends; \( E_t \pi_{t+4} \) denotes the public's expectations at time \( t \) of the inflation rate over the year ahead; \( r^* \) is a constant corresponding to the equilibrium real interest rate in a deterministic world under the prespecified initial conditions of the economy; \( \pi^{TAR} \) denotes the target rate of inflation; \( \tilde{E}_t \{ \mid \Omega_t \} \) denotes a model-consistent forecast at time \( t \) based on the authorities' information set \( \Omega_t \), which includes information about the model along with the observed values of the inflation rate through period \( t \) and all other economic variables through quarter \( t-1 \); \( \pi_t \) and \( u_t \) represent the rates of inflation and unemployment; and \( \bar{u}_t \) is the authorities' estimate of the NAIRU based on observed data through period \( t-1 \) (see Appendix I). The bracketed terms in equations (15) and (16) are relatively traditional components of policy reaction functions discussed in the literature, corresponding to the deviation of inflation (or the authorities' inflation forecast) from target and the deviation of the unemployment rate from the NAIRU. Experimentation suggested that specifying the second class of rules in terms of an inflation forecast that looks ahead three quarters was capable of producing reasonable macroeconomic stability.

Note that the policy reaction functions are specified in the form of rules for real interest rate adjustment. Although monetary policy operates by setting the nominal interest

\[47\] Taylor's (1993) version of the Taylor rule used a backward-locking measure of inflation expectations to measure the real interest rate.

\[48\] We have not undertaken an extensive search for the optimal inflation forecast horizon. It may be noted that three quarters is roughly half the time that is generally believed to be required for interest rates to have their full effects on the economy.
rate, in our model (and most others) the extent to which monetary policy adjustment stimulates or restrains aggregate demand depends on the change in the real interest rate. It would thus make no sense to propose that policy be guided by a nominal interest rate rule that could not be explicitly translated into an economically reasonable rule for the real interest rate on which aggregate demand depends.

Note also that in four of the six model variants, the Taylor rule defined by equations (15) and (17) embodies a forward-looking measure of the real interest rate. By contrast, the conventional form of the Taylor rule embodies a backward-looking measure of the real interest rate, as in the two model variants that impose condition (9a). In general, a conventional completely-backward-looking Taylor rule performs worse than the Taylor rule defined by equations (15) and (17).49

The simulation results we report below focus first on two specific calibrations of the \((a, \gamma)\) parameters for each class of rules. This allows us to assess robustness by comparing the performances of specific rules across the six model variants. The first calibration, \((a, \gamma) = (.5, 1)\), corresponds roughly to the weights originally suggested for the U.S. case by Taylor (1993), after adjusting \(\gamma\) for the fact that Taylor specifies his rule in terms of the output gap rather than the unemployment gap, and that the unemployment gap tends to vary about half as much over business cycles as the output gap. The second parameter setting, \((a, \gamma) = (2, 1)\), is based on a calibration of the inflation forecast based (IFB) rule that we found to perform relatively well in another study.50 We later report, and discuss the limited relevance of, simulation results for the (approximately) optimal Taylor rule calibrations for two of the model variants.

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49 See Isard, Laxton, and Eliasson (1999) for stability analysis of conventional Taylor rules in linear and nonlinear variants of a closed-economy model of the U.S. economy. In that study, simulations with conventional Taylor rules in the nonlinear model variants with forward-looking expectations generated explosive behavior under most drawings of the random shocks.

50 See Isard and Laxton (1999). We also experimented with a third calibration of the Taylor rule, namely \((a, \gamma) = (.5, 2)\), corresponding to a suggestion in Taylor (1999). Our results suggested that moving from \((.5, 1)\) to \((.5, 2)\) tends to worsen macroeconomic performance under conditions of NAIRU uncertainty and endogenous policy credibility. Consistently, the results reported below suggest that performance under a Taylor rule can be improved by moving from \((.5, 1)\) to \((2, 1)\).
IV. SIMULATION RESULTS

Table 3 reports simulation results for the means and standard deviations of unemployment and inflation, along with the cumulative welfare losses, under each of the six different model variants and four different policy rules. For the linear model variants without endogenous policy credibility (columns 1 and 2), the choice of monetary policy rule (after conditioning on a prespecified inflation target) has almost no effects on the mean rates of inflation and unemployment—a rather implausible proposition in reality, but one of the logical implications of linear models. A second result for these cases is that there are only minor differences between the performances of the Taylor rules and the inflation forecast based rules; for each calibration, the two types of rules deliver similar standard deviations of inflation, with the IFB rule generating a slightly lower standard deviation of unemployment and a slightly lower cumulative welfare loss. A third result, hardly surprising in a linear model, is that the rules with relatively stronger reactions to deviations of inflation (or the inflation forecast) from target (i.e., the (2, 1) calibrations) deliver relatively lower standard deviations of inflation and relatively higher standard deviations of unemployment.

For the two cases of convex Phillips curves without endogenous policy credibility (columns 4 and 5), the IFB rules produce slightly lower means for both unemployment and inflation, as well as lower standard deviations for the case with forward-looking expectations, in comparison with the corresponding Taylor rules. Consistently, the IFB rules generate somewhat lower values for the summary measure of the cumulative welfare loss. With regard to the two alternative calibrations, we again find a hardly surprising result; within each class of reaction functions, the rule with relatively stronger reaction to the unemployment gap (i.e., the (.5, 1) calibration) generates a lower mean for the unemployment rate and a higher average inflation rate.

The results are qualitatively different and quantitatively more striking in the cases with endogenous policy credibility. For the case with a linear Phillips curve and endogenous credibility (column 3), the sensitivity of performance to the choice of weights is significantly greater than in the linear cases without endogenous credibility, and the means of both unemployment and inflation are lower under the (2, 1) calibrations, which are more responsive to deviations from equilibrium in an absolute sense but relatively less responsive to the unemployment gap. This result suggests that in seeking to improve upon the policy rule calibration that Taylor (1993) suggested for the United States, a calibration that gives

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51 It may be noted that the sample means and standard deviations of the DNAIRU are identical for all combinations of model variants and policy rules, reflecting the fact that in each case the stochastic simulations are based on identical initial positions and sequences of random shocks.

52 Other papers that have employed stochastic simulations to evaluate the performances of monetary policy rules have focused almost exclusively on linear models and have often summarized the relative performances of different rules by plotting the associated standard deviations of unemployment and inflation on a two-dimensional graph.
Table 3. Simulated Means and Standard Deviations of Unemployment and Inflation, and Cumulative Welfare Losses

<table>
<thead>
<tr>
<th>Rule</th>
<th>LB (1)</th>
<th>LFX (2)</th>
<th>LFE (3)</th>
<th>NB (4)</th>
<th>NFX (5)</th>
<th>NFE (6)</th>
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<tbody>
<tr>
<td>Mean of Unemployment Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor (.5, 1)</td>
<td>7.00</td>
<td>7.00</td>
<td>7.45</td>
<td>7.19</td>
<td>7.19</td>
<td>7.91</td>
</tr>
<tr>
<td>Taylor (.2, 1)</td>
<td>7.01</td>
<td>7.01</td>
<td>7.15</td>
<td>7.24</td>
<td>7.32</td>
<td>7.55</td>
</tr>
<tr>
<td>IFB (.5, 1)</td>
<td>7.01</td>
<td>7.00</td>
<td>7.41</td>
<td>7.17</td>
<td>7.16</td>
<td>7.84</td>
</tr>
<tr>
<td>IFB (.2, 1)</td>
<td>7.01</td>
<td>7.01</td>
<td>7.13</td>
<td>7.19</td>
<td>7.25</td>
<td>7.45</td>
</tr>
<tr>
<td>Mean of Inflation Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>2.55</td>
<td>3.55</td>
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<td>2.79</td>
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<td>2.53</td>
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<td>2.61</td>
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<td>2.73</td>
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<td>3.83</td>
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<tr>
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<td>2.56</td>
<td>2.59</td>
<td>2.59</td>
<td>2.64</td>
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<td>Standard Deviation of Unemployment Rate</td>
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</tr>
<tr>
<td>Taylor (.5, 1)</td>
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<td>1.09</td>
<td>0.98</td>
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<td>1.17</td>
<td>1.05</td>
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<tr>
<td>IFB (.5, 1)</td>
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<td>0.94</td>
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<tr>
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<td>1.07</td>
<td>0.95</td>
<td>1.05</td>
<td>1.11</td>
</tr>
<tr>
<td>Standard Deviation of Inflation Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor (.5, 1)</td>
<td>0.68</td>
<td>1.07</td>
<td>1.29</td>
<td>0.81</td>
<td>1.14</td>
<td>1.63</td>
</tr>
<tr>
<td>Taylor (.2, 1)</td>
<td>0.55</td>
<td>0.72</td>
<td>0.73</td>
<td>0.61</td>
<td>0.84</td>
<td>0.81</td>
</tr>
<tr>
<td>IFB (.5, 1)</td>
<td>0.70</td>
<td>1.06</td>
<td>1.17</td>
<td>0.80</td>
<td>1.09</td>
<td>1.39</td>
</tr>
<tr>
<td>IFB (.2, 1)</td>
<td>0.57</td>
<td>0.71</td>
<td>0.71</td>
<td>0.62</td>
<td>0.75</td>
<td>0.74</td>
</tr>
<tr>
<td>Cumulative Welfare Loss 2/</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor (.5, 1)</td>
<td>10.35</td>
<td>11.74</td>
<td>20.54</td>
<td>11.90</td>
<td>13.15</td>
<td>30.44</td>
</tr>
<tr>
<td>Taylor (.2, 1)</td>
<td>10.83</td>
<td>12.06</td>
<td>12.50</td>
<td>11.41</td>
<td>13.33</td>
<td>14.04</td>
</tr>
<tr>
<td>IFB (.5, 1)</td>
<td>10.20</td>
<td>11.44</td>
<td>17.34</td>
<td>11.56</td>
<td>12.46</td>
<td>25.02</td>
</tr>
<tr>
<td>IFB (.2, 1)</td>
<td>10.14</td>
<td>11.19</td>
<td>11.39</td>
<td>10.52</td>
<td>11.68</td>
<td>12.37</td>
</tr>
</tbody>
</table>

1/ The LB, LFX, and LFE variants have linear Phillips curves; NB, NFX, NFE have nonlinear Phillips curves. The LB, NB variants have backward looking expectations; LFX, NFX have forward-looking expectations but treat policy credibility as exogenous; LFE, NFE have forward-looking expectations and treat policy credibility as endogenous.

2/ Corresponds to the (normalized) sum of the 10,000 observations for $L_t$—as defined by equation (14) for $(\beta, \theta, \nu) = (1, 1, 0.5)$—based in 100 simulations over horizons of 100 quarters.
relatively more weight to the output/unemployment gap, as discussed by Taylor (1999) may be inferior to a rule that places relatively more weight on the deviation of inflation from target.\textsuperscript{53} In this context, it seems important to recognize that in a world with considerable uncertainty about the NAIRU, policymakers are bound to make significant errors in estimating the unemployment gap, and such errors are likely to be positively autocorrelated.\textsuperscript{54} Given that the Phillips curve tends to transmit systematic errors in estimating the unemployment gap into changes in the inflation rate, the greater the uncertainty that surrounds estimates of the unemployment gap, the stronger should be the policy reaction to inflation relative to unemployment, other things equal.\textsuperscript{55}

The costs of not responding to emerging inflation with sufficient speed and force are particularly high in the case with endogenous policy credibility and a nonlinear Phillips curve (column 6). Consider, in particular, the difference between the (.5, 1) Taylor rule, which reacts relatively weakly to inflation, and only after changes in the inflation rate are observed, and the (2, 1) IFB rule, which reacts relatively strongly to the inflation forecast. The mean unemployment rate is nearly 1/2 of a percentage point lower in the latter case, while the mean inflation rate is more than 1 1/2 percentage points lower. For the case with endogenous credibility and a linear Phillips curve (column 3), the choice between the same two rules makes a difference of roughly 1/3 of a percentage point for the average unemployment rate and 1 percentage point for the average inflation rate.

We draw two inferences from the results in Table 3. First, the (.5, 1) calibration suggested by Taylor (1993) is dominated across a range of model variants by a calibration that responds more aggressively to deviations from target, and with a relatively greater weight on inflation than on unemployment. Second, forward-looking IFB rules tend to dominate backward-looking Taylor rules.

For two of the model variants—in particular, the extreme cases of the linear model with backward-looking expectations (LB) and the nonlinear model with forward-looking expectations and endogenous credibility (NFE)—we have extended the analysis somewhat further to determine, via grid searches, the (approximately) optimal calibrations of the Taylor

\textsuperscript{53} Taylor’s (1999) suggestion was that a calibration of (.5, 1) would outperform (.5, .5) in a rule specification analogous to equation (15) but with the output gap in place of the unemployment gap. As noted earlier, based on a rough estimate that the unemployment gap tends to vary about half as widely as the output gap over the business cycle, we view a weight of unity on the unemployment gap as broadly comparable to a weight of 0.5 on the output gap.

\textsuperscript{54} See also Smets (1999) and Drew, Hunt, and Scott (1999).

\textsuperscript{55} This result does not necessarily imply that estimates of the unemployment gap are too imprecise to inform monetary policy in a useful way, even though the authorities’ estimates of the unemployment gap may often be incorrect as well as in magnitude. In particular, in simulations not reported in this paper, we have shown that IFB rules that place no weight on the unemployment gap are dominated by other calibrations.
rule under the base-case loss-function parameters \((\beta, \theta, \nu) = (1, 1, 0.5)\). The top line in each of the two panels of Table 4 describes the optimal calibration and simulated performance for the corresponding model variant. For the LB model variant, the optimal calibration reacts with a weight of 0.7 to deviations of inflation from target and 1.3 to the unemployment gap, generating a policy loss of 10.31. This calibration corresponds fairly closely to the two calibrations suggested by Taylor (1993, 1998)—namely, \((\alpha, \gamma) = (0.5, 1)\) and \((\alpha, \gamma) = (0.5, 2)\)—and achieves a degree of macroeconomic stability only very slightly better than the \((0.5, 1)\) calibration, which generates a policy loss of 10.35 (recall Table 3). For the NFE model variant the optimal calibration reacts with a weight of 1.8 to the deviation of inflation from target and 0.8 to the unemployment gap, generating a policy loss of 13.72—slightly lower than that generated by the \((\alpha, \gamma) = (2, 1)\) calibration shown in Table 3.

The bottom line in each panel of Table 4 describes the macroeconomic performance that results from following the optimal Taylor rule calibration for the other model variant. Note, first, that the optimal calibration for the LB variant performs very poorly in the NFE variant and, second, that relative to the optimal outcomes for each model variant, the optimal calibration for the LB variant performs much more poorly in the NFE variant than the optimal calibration for the NFE variant performs in the LB variant. As can be seen in the bottom line of the table, adopting the optimal LB calibration in the NFE model variant has the stagflationary effect of raising both the average unemployment rate and the average inflation rate relative to the outcomes for the optimal NFE calibration.

Such results illustrate that knowledge of the optimal policy-rule calibration associated with a specific macro model has little relevance for policymaking in the absence of a strong presumption that the model is true; indeed, the optimal calibration for a specific hypothetical model can perform very poorly in a somewhat different model variant. This is the type of finding that has motivated McCallum (1988) and Levin, Wieland, and Williams (1999) to search for rules and rule calibrations that perform well across a variety of plausible macro models.

The results also challenge the traditional practice of relying heavily on linear macro models for evaluating policy rules and rule calibrations. Although Table 4 presents a very limited set of results, it provides a basis for the conjecture that rules and rule calibrations that perform well in nonlinear models with forward-looking expectations are much more robust than rules and rule calibrations that perform well in linear models with backward-looking expectations. As an inference specific to the class of Taylor rules that this paper analyzes, our results suggest that, in general, policymakers should react more aggressively to deviations from inflation/unemployment targets than has been inferred from simulation analysis based on linear macro models.
Table 4. Simulated Results for Optimal and Suboptimal Calibrations of Taylor Rules

<table>
<thead>
<tr>
<th>Calibration of Taylor Rule</th>
<th>Cumulative Welfare Loss</th>
<th>Mean Unemployment Rate</th>
<th>Mean Inflation Rate</th>
<th>Standard Deviation of Unemployment</th>
<th>Standard Deviation of Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Backward-Looking Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal Calibration $(\alpha, \gamma) = (0.7, 1.3)$</td>
<td>10.31</td>
<td>7.01</td>
<td>2.55</td>
<td>0.95</td>
<td>0.62</td>
</tr>
<tr>
<td>Alternative Calibration $(\alpha, \gamma) = (1.8, 0.8)$</td>
<td>10.89</td>
<td>7.01</td>
<td>2.52</td>
<td>1.06</td>
<td>0.58</td>
</tr>
<tr>
<td>Nonlinear Forward-Looking Model with Endogenous Policy Credibility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal Calibration $(\alpha, \gamma) = (1.8, 0.8)$</td>
<td>13.72</td>
<td>7.41</td>
<td>2.67</td>
<td>1.21</td>
<td>0.82</td>
</tr>
<tr>
<td>Alternative Calibration $(\alpha, \gamma) = (0.7, 1.3)$</td>
<td>25.27</td>
<td>7.86</td>
<td>3.86</td>
<td>1.24</td>
<td>1.37</td>
</tr>
</tbody>
</table>
In addition, our analysis has shown that in a (nonlinear) model with forward-looking expectations, NAIRU uncertainty, and endogenous policy credibility, policymakers who (loosely speaking) care about unemployment as much as they care about inflation should react much more forcefully to deviations of inflation from target than to deviations of unemployment from the estimated NAIRU. In contrast with Taylor's (1993, 1998) suggestions, this result recognizes that when policymakers make relatively large and serially-correlated errors in estimating unemployment gaps, it can be costly in a nonlinear world to react very strongly to their estimates of unemployment gaps, even when they attach relatively high policy losses to unemployment.

V. CONCLUSIONS

The economics profession has taken significant strides in recent years in using simple macroeconomic models to analyze the relative performances of different hypothetical policy rules. Although many economists and policymakers recognize that mechanical adherence to a simple policy rule would be a recipe for disaster, such analysis has been helpful in advancing the conceptual framework for formulating inflation targeting strategies, and information derived from simple policy rules is regularly used by some central bankers to help structure thinking about the settings of monetary policy instruments. At the same time, contributors to the recent research on monetary policy rules have emphasized concerns about the validity of inferences drawn from specific macro models and have made efforts to explore the robustness of their conclusions.

This paper has been motivated by the observation that in their search for robust policy rules, economists have not yet adequately explored how their proposed rules perform in models with endogenous policy credibility, NAIRU uncertainty, and convex Phillips curves. Most (if not all) central bank governors in industrial countries regard endogenous policy credibility and NAIRU uncertainty as fundamental characteristics of the worlds in which monetary policy must be conducted, and we regard convex Phillips curves as another important reduced-form feature of the real world.

Much of the formal inflation targeting literature has focused on the distinction between Taylor rules—in which the nominal interest rate setting depends on recently observed rates of inflation and unemployment—and rules under which the authorities react to an inflation forecast, here referred to as inflation forecast based (IFB) rules. Virtually all evaluations and comparisons of such rules have been conducted in linear models. In some

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56 See, for example, Amano, Coletti, and Macklem (1999) and Batini and Haldane (1999) for studies that have influenced thinking at the Bank of Canada and Bank of England, respectively. See Kohn (1999) for perspectives on how information about the implications of monetary rules helps structure thinking by some members of the U.S. Federal Open Market Committee.

57 See, for example, McCallum (1988) and Levin, Wieland, and Williams (1999).
cases inflation expectations are assumed to be backward looking and the issue of endogenous policy credibility is ignored. In other cases, expectations are forward-looking and policy credibility is (implicitly) treated as exogenous. In all of these linear models, moreover, we are asked to believe that monetary policy has no first-order effects on welfare in the sense that, for a given target inflation rate, the choice of policy rule has no influence on the average rates of unemployment or inflation that economies experience over time. Because such studies do not adequately address the commonly-observed phenomenon of prolonged periods of bias between inflation expectations and inflation outcomes, which presumably reflects the nature of monetary policy, their evaluations of policy rules abstract from an important feature of the monetary policy transmission mechanism.

By contrast, the analysis in this paper has focused on the implications of endogenous policy credibility. Our comparisons of how rules perform in several different model variants suggest that the introduction of endogenous credibility leads to some qualitatively different and quantitatively more striking perspectives on the choice between the different calibrations of Taylor rules and IFB rules, and that these perspectives are reinforced by the introduction of convex Phillips curves. More specifically, our simulation results suggest that in a world that embodies the long-run natural rate hypothesis, and in which credibility and inflation expectations respond endogenously to the track record of the authorities in delivering low inflation, with credibility more easily lost than regained, forward-looking IFB rules tend to have a significant advantage over Taylor rules. Moreover, for a given target rate of inflation, the choice between different calibrations of the reaction parameters has significant implications for the means of the unemployment rate and the inflation rate, as well as for the variances of unemployment and inflation. In this connection, the (.5,1) calibration—which broadly corresponds to the parameter values suggested by Taylor (1993) for the U.S. economy—is significantly outperformed by a calibration that reacts more forcefully to deviations from targets, and with relatively stronger reactions to deviations from the inflation target than to unemployment gaps.

It should be emphasized that the Taylor rule specification we have analyzed in this paper essentially assumes that the policymaker adjusts a model-consistent measure of the real interest rate in reaction to the most recent observations of inflation and unemployment. For four of our six model variants this specification gives the Taylor rule significantly stronger stabilizing properties than the conventional specification, which embodies a backward-looking measure of the real interest rate.58

The key policy message we draw from our analysis is that it is very important for monetary policy to be forward looking. Although other studies have found that backward-looking Taylor rules perform well in linear macro models with exogenous policy credibility, this finding begins to break down with the introduction of nonlinearities, such as those

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58 As noted earlier, in simulation analysis of a closed-economy model of the United States, Isard, Laxton, and Eliasson (1999) found that in nonlinear model variants with forward-looking expectations, conventional Taylor rules led to explosive behavior for most drawings of random shocks.
associated with endogenous credibility, convex Phillips curves, floors on nominal interest rates, asymmetric hysteresis in labor markets, and so forth. It may also be noted that—for the specific model variants analyzed in this paper—simple linear policy rules that embody model-consistent inflation measures of real interest rates and react to model-consistent inflation forecasts appear to be reasonably successful in stabilizing the economy.

The conclusions of simulation studies inevitably depend on the particular models that are analyzed as well as on the parameterization of the policy loss function. For that reason we have focused on six different model variants and looked through the summary loss statistics by reporting the means and standard deviations of inflation and unemployment.

Intuitively, the following line of argument provides strong reason to suspect that our conclusions about the relative stabilizing properties of Taylor rules and IFB rules generalize to other plausible nonlinear models and loss function parameterizations. First, in nonlinear models with forward-looking agents, success in preventing an acceleration of inflation generally hinges on the effectiveness of the monetary authorities in avoiding prolonged states of excess demand. Second, in most macro models adjustments in the nominal interest rate are transmitted to aggregate demand primarily through the real interest rate. Third, in reality monetary policymakers confront considerable uncertainty about the behavior of the economy, and economists tend to make serially correlated errors in estimating output and unemployment gaps, so even the best informed policymakers occasionally come to realize that they had been misgauging the strength of the economy in the recent past, and that their policy errors have led to a state of significant excess demand. And finally, when an economy is experiencing significant excess demand, the nominal interest rate adjustments that would be dictated by a backward-looking Taylor rule may be insufficient to raise the level of the real interest rate that is perceived by forward-looking market participants, and might therefore allow excess demand to continue to strengthen, accompanied by a continuing upward spiral in market participants' inflation expectations.\(^{59}\)

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\(^{59}\) This is especially the case for the conventional form of the Taylor rule, which embodies a backward-looking measure of the real interest rate and reacts to a backward-looking measure of inflation.
An Updating Procedure for Estimating the NAIRU

We assume that in each period $t$ the monetary authorities construct estimates of the DNAIRU and NAIRU in a model-consistent manner based on their knowledge of the structure of the model and the histories of both the unemployment rate and the inflation rate. To investigate the implications of uncertainty about the NAIRU, it is assumed that the authorities know the true structure of the Phillips curve

$$\pi_t = \pi_t^c + 2.14 \frac{(u_t^* - \Phi_t)}{(u_t - \Phi_t)} + \varepsilon_t^\pi$$

(A1)

but are unable to observe $u_t^*$, $\Phi_t$ or the error term $\varepsilon_t^\pi$.

Inferences about these unobservable variables can be derived from information about the structure of the Phillips curve as well as historical information about movements in $\pi$, $\pi^c$, and $\Phi$. However, it is well known that because of significant measurement error in the Phillips curve relationship ($\varepsilon_t^\pi$ in equation A1), there can be significant errors in the estimates of $u_t^*$ derived directly from the Phillips curve. For this reason it has been common for researchers in policy making institutions to also base their estimates of the NAIRU on trend movements in unemployment rates.

The analysis in this paper is based on an updating process for the DNAIRU (and the NAIRU) that takes account of historical information about both unemployment and inflation and can be formulated as an explicit Kalman filtering problem in which the monetary authorities are assumed to gradually learn about shifts in the underlying DNAIRU. We assume that the authorities operate under the assumption that the DNAIRU is subjected to permanent shocks, or that the change in the DNAIRU follows a random walk according to

$$u_t^* = u_{t-1}^* + \varepsilon_t^{u*}$$

(A2)

where $\varepsilon_t^{u*}$ is distributed as $N(0, \sigma_u^*)$, with $\sigma_u^*$ measuring the degree of volatility in the underlying DNAIRU. In addition, the monetary authorities are assumed to know that for a given policy rule, there will be a constant difference ($\alpha$) between NAIRU and DNAIRU.

$$\alpha = \bar{u}_t - u_t^*$$

(A3)
Finally, the monetary authorities are assumed to know that the business cycle component of unemployment, $\varepsilon_i^\ast$, is a stationary process with a fixed mean of zero.

$$u_i = \bar{u} + \varepsilon_i^\ast$$ (A4)

As shown in Callen and Laxton (1998), these equations can be solved as a Kalman filtering problem where the monetary authorities continuously update their estimates of the DNAIRU and the NAIRU, as well as other relevant parameters such as the degree of convexity in the Phillips curve. For our purposes here, the estimation process has been simplified by assuming that the monetary authorities know the true parameters of the Phillips curve as well as the "true" degree of volatility in the underlying DNAIRU estimates. This results in an orderly updating process for the DNAIRU and NAIRU, where the monetary authorities make mistakes estimating the NAIRU and gradually improve their historical estimates over time as new data are released on inflation and unemployment developments.

The solution technique in the Monte Carlo experiments involves the following process.

The solution at the beginning of period $t$ provides estimates of the histories of the DNAIRU and NAIRU through period $t-1$, along with forecasts of the DNAIRU and NAIRU through a terminal simulation horizon $T$. The forecasts are based on the assumption that realizations of all future shocks coincide with the zero means of the probability distributions of the shocks. Thus, in period $t$ the DNAIRU and NAIRU are forecast to remain unchanged at their estimated period $t$ values.\textsuperscript{60}

\textsuperscript{60} In the true model we assume that the DNAIRU process is a bounded random walk that ranges between 4 and 10. However, for the purpose of updating the $u^\ast$ and $\bar{u}$ estimates we make a simplifying assumption that the monetary authorities are not able to observe these bounds and therefore act as if the DNAIRU process is unbounded. An alternative would be to relax this assumption, in which case the monetary authorities would always forecast the DNAIRU to gradually return back to a fixed steady state value. This would somewhat (continued...)
References


