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Optimal Fiscal and Monetary Policy with Nominal and Indexed Debt

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Abstract

This paper highlights the importance of debt composition in setting optimal fiscal and monetary policy over short-run business cycles and in the long run. Nominal debt as state-contingent debt can be a significant policy tool to reduce the volatility of distortionary government policy, thereby reducing macroeconomic volatility while increasing equilibrium output and consumption. The welfare gain from using nominal debt to hedge against shocks to the government budget is as large as the welfare gain from the ability to issue debt.

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I. INTRODUCTION

This paper analyzes the implications of debt composition in setting optimal fiscal and monetary policy over the business cycle and in the long run. Examination of sovereign debt markets indicates that some governments have debt structures with predominantly nominal or fixed-rate debt while others primarily issue debt indexed to the exchange rate, interest rate, or general price index. Many economists like Bach and Musgrave (1941), Lucas and Stokey (1983), and McCallum (1984) have long advocated the use of inflation-indexed debt as a means to eliminate the incentive for governments to inflate away existing nominal liabilities. However, more recently Goldstein (2002) and Reinhart, Rogoff, and Savastano (2003) have argued that an indexed debt structure can be destabilizing if large shocks are transmitted into additional debt costs at inopportune times, as seen recently in several emerging market crises. This paper builds primarily on the work of Bohn (1988) and Chari, Christiano, and Kehoe (1991)[2] to assess the value of different debt structures in a stochastic monetary economy with distortionary fiscal and monetary policy.

The paper combines the traditional general equilibrium framework of macroeconomics with the public finance approach from Ramsey (1927) to calibrate and simulate a stochastic monetary model under various debt-to-income ratios and differing compositions of nominal and indexed debt. Since this analysis assumes a closed economy, all indexed debt is modeled as inflation-indexed debt. The model employed is a combination of a cash-in-advance model and a stochastic growth model, similar to models employed in Cooley and Hansen (1995), Chari et al. (1991), and Lucas and Stokey (1983). The household derives utility from leisure and consumption while the government raises revenue to finance its exogenous stochastic spending through distortionary means: a tax on labor income and the ability to print money.[3] The government also has the ability to issue debt and can choose between two types of debt: nominal (fixed-rate) or indexed (inflation-indexed) debt.

The model captures the loss from distortionary government policy within the nonlinearity of the labor supply equation since the contemporaneous tax on labor income and money growth are determinants of optimal household labor supply in equilibrium. Therefore, the labor supply equation formalizes the assumption of a loss function over distortionary taxes and inflation as discussed in Bohn (1988) and Barro and Gordon (1981). Shocks that cause variations in government policy are transmitted through optimal labor supply to output, remaining household allocations, and the equilibrium price system while feeding back into the government budget constraint through tax revenue. Equilibrium decisions by

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[2]Hereafter refered to as Chari et al.

[3]See Ljungqvist and Sargent (2000) for a recent formulation of optimal monetary and fiscal policy.

households, firms, and the government are then passed into future periods through the price level and interest rate equations. When choosing a combination of fiscal and monetary policy, the government must take into account the relationship between this policy mix and household labor supply to minimize distortions. Optimal policies, or Ramsey policies, maximize consumer welfare while minimizing distortions within the system.

The presence of nonlinear distortions to labor requires the use of a simulation procedure which captures these effects. The projection method of Judd (1998, 1992) is used to solve the Euler conditions for the optimal Ramsey policy for money growth, taxes, debt and shadow price of debt. The use of nonlinear simulation methods significantly changes the policy implications of debt policy. Using only linear-quadratic methods, there is little impact of debt composition on welfare. However, the gain in welfare from moving from real to nominal debt is equivalent to increasing consumption growth by 0.6% for a model economy calibrated with U.S. data. This change in welfare is nearly as large as the increase in welfare from the ability to issue debt which was first identified by Barro (1979, 1987). This larger change in welfare is a reflection of Jensen's inequality which generates the hedging effects of nominal debt.

The Ramsey problem was solved in economies with and without debt and under various combinations of nominal and indexed debt. While the baseline economy contains no debt, the low-debt economy uses the prevailing debt-to-income ratio in the United States and the high-debt economy is calibrated to twice this level. When debt is present, the solution was derived under different combinations of nominal and indexed debt ranging from a majority nominal debt policy (95 percent nominal, 5 percent indexed) which matches the current U.S. debt composition to a majority indexed debt policy (5 percent nominal, 95 percent indexed). Then using the optimal Ramsey plan that defines policy choices of the government, allocations by the household, and the resulting price system, each economy was simulated under the effects of technology and government spending shocks in order to examine the effects of debt structure on optimal policies and activity.

In terms of the properties of fiscal and monetary policy, the findings are generally consistent with those in Chari et al. (1991) and elsewhere, but with some differences. Tax rates on labor are relatively constant over the business cycle, more so in the economies with debt. Regardless of the presence of debt or debt structure, government policy attempts to follow the Friedman rule which results in an expected gross nominal interest rate equal to 1.0. In enacting this monetary policy rule, the government equates the real gross rate of return across the three assets (money balances, nominal debt, and indexed debt) in expectation, satisfying Euler conditions. As discussed in Chari, et al. (1991, 1996), the so-called Friedman rule is optimal in a variety of monetary economies with distorting taxes and this paper extends this result to include a variety of different debt specifications. Within the business cycle, monetary policy is countercyclical with respect to technology shocks and procyclical with respect to government consumption, but only under economies with mostly indexed debt. The result is reversed for economies with mostly nominal debt. Monetary policy is procyclical for economies with mostly nominal debt.

The existence of debt provides the government with an additional degree of policy freedom which allows for a smoother path of distortionary taxes and money growth over time, thereby affording the household a smoother stream of consumption and leisure. As shocks to technology and government spending affect the government budget constraint, optimal policy responds by smoothing the impact of distortionary taxes and money growth with debt issuance. In this manner, the existence of debt allows households to behave in a fashion consistent with Friedman's permanent income hypothesis, whereby households consume based on permanent income and save and borrow in response to transitory changes in income. These results confirm the tax smoothing role of government debt as discussed in Barro (1979, 1987), regardless of the breakdown between nominal and indexed debt. Even if an indexed debt structure is needed to create capital market access in sufficient quantities, as evidenced by the existing debt circulating in international and domestic capital markets, this analysis suggests that this outcome is preferable to no capital market access.

However, the main contribution of this paper is that the role of nominal debt as state-contingent debt can be a significant policy tool to reduce the volatility of distortionary government policy since the gain in welfare from using nominal debt as opposed to indexed debt is as large as the gain in welfare from the ability to issue debt. The simulated economies with higher ratios of nominal debt are less volatile than economies with higher ratios of indexed debt since nominal debt acts as a hedge against unexpected shocks to the government budget. Economies with higher ratios of nominal debt also have higher steady-state levels of output and consumption. As discussed in Bohn (1988) and Chari et al. (1991) and quantified here, unexpected shocks to the economy that call for an increase in distortionary government revenue also correspond to states with higher-than-expected inflation which reduces the value of existing nominal debt and counterbalances the need to increase government revenue. In states of the world with positive shocks to government spending and negative shocks to technology, government responds by increasing distortionary revenue policy and the shadow value of reducing debt is increased, creating a positive correlation between inflation and the multiplier on the government budget constraint. In this context, nominal debt acts as a hedge against shocks to the government budget by providing a non-distortionary source of revenue, permitting a smoother path for fiscal and monetary policy and household consumption. The greater the level of debt and larger the percentage of nominal debt, the larger the hedging contribution. These results suggest that sovereign debt management strategies need to broaden the concept of the cost of the debt as more than purely first-order financing costs to include a second-order concept of variance of stock adjustments. Depending on the type and magnitude of economic shocks that prevail, optimal debt management should strive over time to create a debt structure that includes nominal debt in sufficient quantities to further reduce macroeconomic volatility and minimize costs associated with the business cycle.

The conclusions of this paper also lend additional credence to the argument that economic growth and macroeconomic volatility are negatively related as discussed in Ramey and Ramey (1995) and that reductions in macroeconomic volatility and minimization of the cost of business cycles can entail significant increases in overall welfare. The idea that government policy uncertainty could have negative effects on growth has been examined by Aizenman and Marion (1993) who find that the magnitude and persistence of tax policy fluctuations jointly determine the pattern of investment and growth with negative correlation. The welfare gains estimated here from reducing policy and business cycle volatility are much larger than those in Lucas (1987) and are attributable to the nonlinearities and convexities within the model.

The conclusions of this paper also highlight more generally the importance of time consistent government policy. Without a commitment mechanism in place, the government is unable to credibly commit to a future series of policy actions beforehand, leaving policy discretionary. As discussed in Lucas and Stokey (1983), the government will have an incentive to inflate away fixed-rate nominal liabilities unless all prices are predetermined or distortionary taxes can be avoided. If taxes are distortionary, the incentive to inflate away nominal liabilities is diminished, but not entirely removed. The lack of a commitment mechanism may be what leads to highly indexed debt structures to begin with suggesting that consistency of government policy may be a necessary precondition to enable full implementation of debt management strategies that yield the reductions in macroeconomic volatility discussed here. Consequently, moving from an indexed debt structure to a nominal debt structure may require governments to focus on commitment strategies sustained by reputational mechanisms as discussed in Chari, Kehoe, and Prescott (1989), Chari and Kehoe (1990, 1993), and Stokey (1991).

The paper proceeds as follows. Section II outlines the stochastic monetary economy and describes the basic theoretical framework underlying the analysis. Section III defines the Ramsey equilibrium, discusses the calibration and solution procedure, and details the results under different levels and compositions of debt. Section IV provides concluding remarks.

II. A STOCHASTIC MONETARY ECONOMY

The properties of debt composition and its relation to optimal policies and allocations are examined in a stochastic monetary economy. The model is a combination of a cash-in-advance model and a stochastic growth model, similar to models employed in

Cooley and Hansen (1995), Chari et al. (1991), and Lucas and Stokey (1983). The economy is populated by a representative household, a representative firm, and a government. The household derives utility from leisure and two consumption goods, a cash good and a credit good where previously accumulated cash balances are needed to purchase units of the cash good. Output is produced according to a production function that combines capital, labor, and technology, where the process governing technology is assumed to be exogenous and stochastic. The government raises revenue with distortionary effects to finance its exogenous stochastic spending through a tax on labor income, printing money, or financing debt. In addition to the level of debt, the government also has the ability to choose between two types of debt: nominal (fixed-rate) or indexed (inflation-indexed) debt.

Assumptions of a fixed capital stock and logarithmic preferences enable computation of closed form equilibrium solutions for the private sector given a particular government policy. The Ramsey equilibrium solves for optimal fiscal and monetary policy given the equilibrium behavior of the private sector. This Ramsey equilibrium may be reduced to four conditions for money growth, taxes, the shadow price of debt and labor given the equilibrium behavior of interest and prices. The projection method is then applied to solve for the four policy functions and conduct simulations. If the private sector is made more complex, these four conditions would need to be augmented with equilibrium conditions for interest rates and prices. These additional conditions would limit the accuracy of the projection method since additional equations would limit the number of nodes the computer can solve. In order to focus on the distortionary effects of labor taxes, a fixed capital stock is assumed. Thus, the optimal government policy will account for its impact on interest rates and prices as well as the optimal behavior of the household and firms.

A. Production

Aggregate output, Y_t, is produced according to the following constant returns-to-scale production function,

$$Y_t = \exp(\theta_t) H_t^{\alpha} K_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where K_t and H_t are the aggregate capital stock and labor supply, respectively, and θ_t represents the available technology. Technology is assumed to be the realization of an exogenous stochastic process and evolves according to the following law of motion,

$$\theta_t = \rho_\theta \theta_{t-1} + \epsilon_{\theta,t}, \quad 0 < \rho_\theta < 1. \tag{2}$$

The random variable, $\epsilon_{\theta,t}$, is normally distributed with mean zero and standard deviation $\sigma_{\theta,t}$ and the realization of $\epsilon_{\theta,t}$ is known to all agents at the beginning of period t. The capital stock is assumed to remain fixed throughout the analysis so that $K_t = K$ and all returns to capital are distributed to the household as income. A constant capital stock can be justified, in part, by the well established result from the literature on optimal taxation that tax rates on capital should be close to zero on average.[4] Given a fixed capital stock, the representative firm seeks to maximize profit, equal to $Y_t - w_t H_t$, by choosing labor demand. The first-order condition for the firm's labor decision is,

$$w\left(\theta_t, H_t\right) = \alpha \exp(\theta_t) H_t^{\alpha-1} K_t^{1-\alpha}. \tag{3}$$

The firm equates the real wage rate to the marginal product of labor.

B. Households

The representative household obtains utility from consumption and leisure. Preferences are summarized by the following utility function,

$$E_t \sum_{t=0}^{\infty} \beta^t \left[a \log C_{1t} + (1-a) \log C_{2t} - \gamma H_t\right], \tag{4}$$

where C_1 is the cash good, C_2 is the credit good, γ is a positive constant and $0 < \beta, a < 1$. The specification of linear labor supply is discussed in Hansen (1985) and Rogerson (1988) and is derived from the assumptions that labor is indivisible and allocation of labor is determined by employment lotteries.

The household enters period t with previously accumulated assets equal to the stock of money holdings, M_t, and gross returns from nominal government bonds, $B_t^N R_{t-1}^N$, and indexed government bonds, $B_t^L R_{t-1}^L$.[5] These assets augment the income received from capital and the after-tax income from labor and are used to finance household expenditures during the period. Entering the period, the current shocks to technology and government spending are revealed. As a result of this specification, households know the past and current realization of technology and government spending and form expectations over future possible values. After the shocks are revealed and expectations are formed, the household then chooses labor supply, consumption of the cash and credit goods, the amount of money to be carried into the next period, and stocks of nominal and indexed government debt. Overall, household allocations must satisfy the following budget constraint,

$$C_{1t} + C_{2t} + \frac{M_{t+1}^d}{P_t} + \frac{B_{t+1}^N}{P_t} + B_{t+1}^L \leq \left(1 - \alpha\tau_t\right) Y_t + \frac{M_t}{P_t} + \frac{B_t^N}{P_t} R_{t-1}^N + B_t^L R_{t-1}^L, \tag{5}$$

where P_t equals the price level and τ_t is the tax applied to labor income, αY_t. The term

[4] For earlier work on optimal capital income taxes, see Atkinson (1971), Diamond (1973), Pestieu (1974), Atkinson and Sandmo (1980), Judd (1985), Chamley (1986), and Chari et al. (1991, 1994).

[5] The superscript "L" is used to denote that the return is indexed, or "linked", to the price level.

$M^d_{t+1}$is the demand for money balances by the representative household to be used in the next period and is aggregated across households in relation to money supply in equilibrium. Previously accumulated money balances are used to purchase the cash good in the current period and must also satisfy the cash-in-advance constraint,

$$P_t C_{1t} \leq M_t. \tag{6}$$

The representative household maximizes 4 subject to 5 and 6. The resulting Euler equations are,[6]

$$M^d_{t+1} : \frac{1-a}{C_{2t}} = \beta E_t \left\{ \frac{a}{C_{1t+1}} \frac{P_t}{P_{t+1}} \right\}, \tag{7}$$

$$B^N_{t+1} : \frac{1-a}{C_{2t}} = \beta E_t \left\{ \frac{1-a}{C_{2t+1}} \frac{P_t}{P_{t+1}} R^N_t \right\}, \tag{8}$$

$$B^L_{t+1} : \frac{1-a}{C_{2t}} = \beta E_t \left\{ \frac{1-a}{C_{2t+1}} R^L_t \right\}, \tag{9}$$

$$H_t : \gamma = \frac{1-a}{C_{2t}} \left(1 - \alpha \tau_t\right) \alpha \frac{Y_t}{H_t}. \tag{10}$$

The Euler conditions on nominal and indexed bonds can be used to derive the conditions on the two interest rates as,

$$R^L_t = \frac{1}{\beta C_{2t}} \left[\frac{1}{E_t \left[\frac{1}{C_{2t+1}} \right]} \right], \tag{11}$$

$$R^N_t = \frac{1}{\beta C_{2t}} \left[\frac{1}{E_t \left[\frac{1}{C_{2t+1}} \frac{P_t}{P_{t+1}} \right]} \right]. \tag{12}$$

Maximization of expression 4 is subject to $M^d \geq 0$ for all $t \geq 0$, given the initial stock of money, M_0. There is no similar restriction on debt since negative stocks of government bonds would indicate household indebtedness to the government, although transversality conditions will prevent debt from growing without bound in either direction. Households would not find it optimal to violate these conditions because alternative allocations exist that would afford higher levels of consumption and higher lifetime utility.

The specification of log preferences causes income and substitution effects to cancel, allowing equilibrium household allocations to be characterized for a given set of

[6] See Cosimano and Gapen (2003) for details.

government policy. Since the capital stock is fixed, eliminating investment, output can either be consumed by households or used by the government resulting in the economy-wide resource constraint,

$$C_{1t} + C_{2t} + G_t = Y_t. \tag{13}$$

The resource constraint can be used with 6 and 20 in the Euler condition for money balances to yield closed-form solutions for consumption, prices, and interest rates. Assuming money supply equals money demand in equilibrium, or $M_{t+1} = M^d_{t+1}$, the closed-form solutions for consumption are as follows,

$$C_{1t} = \frac{(Y_t - G_t)\,\beta\left(\frac{a}{1-a}\right)\exp(-\mu_{t+1})}{1+\beta\left(\frac{a}{1-a}\right)\exp(-\mu_{t+1})}, \tag{14}$$

$$C_{2t} = \frac{(Y_t - G_t)}{1+\beta\left(\frac{a}{1-a}\right)\exp(-\mu_{t+1})}. \tag{15}$$

Inserting the solution for the cash good in 14 into the cash-in-advance constraint in 6, which holds with equality in equilibrium, produces the following closed-form equation for the price level,

$$P_t = \frac{M_t}{(Y_t - G_t)}\left[\frac{1+\beta\left(\frac{a}{1-a}\right)\exp(-\mu_{t+1})}{\beta\left(\frac{a}{1-a}\right)\exp(-\mu_{t+1})}\right]. \tag{16}$$

The closed-form solutions for the real and nominal interest rate are found by inserting the solution for the credit good in 15 and the price level in 16 at time t and $t+1$ into the Euler conditions in 11 and 12.

The solution for the credit good in 15 along with the wage rate in 3 can be substituted into the Euler condition for labor in 10 to solve for the equilibrium quantity of labor. Doing so, and noting the specification for output in 1, defines an implicit function,

$$h\left(H_t, g_t, \theta_t, \mu_{t+1}, \tau_t\right) = 0. \tag{17}$$

This equation cannot be solved for H_t explicitly, but the implicit function theorem allows for the construction of an implicit function which defines the explicit function. Dfined derivatives can be obtained as long as an implicit function is known to exist under the implicit function theorem. Since an implicit function for equilibrium labor can be constructed,[7] optimal household allocations and the equilibrium price system are all functions of contemporaneous government policy and the exogenous shocks to government spending and technology.

[7] See Cosimano and Gapen (2003) for additional details.

However, the equilibrium price system is also dependent on past policy and expectations of future policy and uncertainty. The price level is dependent on the choice of money balances during the previous period which is a result of the cash-in-advance specification. Consequently, the choice of money growth in period t by the government affects the price level in period t and in period $t+1$. The real interest rate in period t is also a function of the expectation over future government policy and labor supply decisions in period $t+1$ since the real interest rate applied to the stock of real debt chosen by the household in period t will not be available for use again until period $t+1$. The same is true for the nominal interest rate, but in equilibrium the nominal interest rate only depends on the rate of time preference and the rate of money growth expected to prevail in the upcoming period.

The equilibrium labor equation in 17 formalizes the assumption of a loss function over distortionary taxes and inflation as discussed in Bohn (1988), Barro and Gordon (1981), and elsewhere. These authors use a quadratic loss function to capture the excess burden of taxes and allocative distortions of inflation. The stochastic monetary economy presented here incorporates the idea of a loss function within the nonlinearity of the equilibrium labor equation since the contemporaneous tax on labor income and money growth are determinants of optimal household labor supply. While debt is not explicitly present in the equilibrium labor function, it is indirectly present since the choices of taxes and money determine the level of debt as a residual in the government budget constraint. Variations in government policy directly affect labor supply, output, remaining household allocations, and the equilibrium price system while feeding back into the government budget constraint through tax revenue. In addition, the shocks to technology and government spending also induce responses by both households and the government. Equilibrium decisions by households, firms, and the government are then transmitted across time through the price level and interest rates.

III. THE RAMSEY EQUILIBRIUM

Real government consumption, G_t, is assumed to follow an exogenous stochastic process. Government policy includes sequences of labor taxes and supplies of money, nominal bonds, and indexed bonds which must satisfy the following budget constraint,

$$\frac{M_t}{P_t} + \frac{B_t^N}{P_t} R_{t-1}^N + B_t^L R_{t-1}^L = \tau_t \alpha Y_t - G_t + \frac{B_{t+1}^N}{P_t} + B_{t+1}^L + \frac{M_{t+1}}{P_t}, \tag{18}$$

where the initial stocks of money, M_0, nominal bonds, B_0^N, and indexed bonds, B_0^L, are given. The money supply and government spending in period t are assumed to grow at the rate $\exp(g_t) - 1$ and $\exp(\mu_{t+1}) - 1$, respectively. Thus, the level of government spending and money stock are defined as,

$$G_t = \exp(g_t)G_{t-1}, \tag{19}$$

$$M_{t+1} = \exp(\mu_{t+1})M_t. \tag{20}$$

The random variable g_t is assumed to evolve according to the following autoregressive process,

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t}, \tag{21}$$

where $\epsilon_{g,t}$, is normally distributed with mean zero and standard deviation $\sigma_{g,t}$. Like the shock to technology, the realization of $\epsilon_{g,t}$ is known to all at the beginning of period t.

The goal of the government is to maximize the welfare of the household subject to raising revenues through distortionary means. After the shocks to the system are revealed, the government selects a policy profile and households respond with a set of allocations. The resulting equilibrium determines the state variables for the next period. Therefore, when choosing an optimal policy mix of taxes, money supply, and debt, the government must take into account the equilibrium reactions by households and firms to the chosen policy mix. The government is also constrained in its policy choices since it must choose a policy mix to maximize household utility while satisfying the government budget constraint. The following definition describes the Ramsey equilibrium.

Definition 1. *A Ramsey equilibrium is defined as:*

1. *A feasible allocation is a sequence of* $\{C_{1t}\}_{t=1}^{\infty}$, $\{C_{2t}\}_{t=1}^{\infty}$, $\{H_t\}_{t=1}^{\infty}$, $\{G_t\}_{t=1}^{\infty}$ *that satisfy the resource constraint in 13.*

2. *A price system is a set of nonnegative bounded sequences* $\{P_t\}_{t=1}^{\infty}$, $\{w_t\}_{t=1}^{\infty}$, $\left\{R_t^N\right\}_{t=1}^{\infty}$, $\left\{R_t^L\right\}_{t=1}^{\infty}$.

3. *A government policy is a set of sequences* $\{\tau_t\}_{t=1}^{\infty}$, $\{M_{t+1}\}_{t=1}^{\infty}$, $\left\{B_{t+1}^N\right\}_{t=1}^{\infty}$, $\left\{B_{t+1}^L\right\}_{t=1}^{\infty}$.

4. *Given the exogenous sequences* $\{g_t\}_{t=1}^{\infty}$ *and* $\{\theta_t\}_{t=1}^{\infty}$*; initial stocks of money, nominal bonds, and indexed bonds; and* $M_0 = M_0^d$*; a competitive equilibrium is a feasible allocation, a price system, and a government policy such that (a) given the price system and government policy, the allocation solves both the firm's problem and the household's problem; and (b) given the allocation and price system, the government policy satisfies the sequence of government budget constraints.*

5. *The Ramsey Problem is to choose a competitive equilibrium that maximizes household utility in 4. The competitive allocation that solves the Ramsey Problem is called the Ramsey plan or Ramsey equilibrium.*

The Ramsey problem in the general equilibrium dynamic programming setting incorporates many of the reputational mechanisms for credible government policies as discussed in Ljungqvist and Sargent (2000).[8] In general, the government would find it optimal to deviate from its original set of policies if allowed and some mechanism, reputational or otherwise, is needed to ensure credibility of government policy. Given this mechanism the government solves for the commitment equilibrium.[9] Under the assumption that an institution or commitment technology exists through which the government can bind itself to a particular sequence of policies, the government attempts to maximize 4 subject to 18 while taking into account the equilibrium specification for the price system in equations 11, 12, and 16 and the optimal response by households and firms in equations 1, 14, 15, and 17. Before the shocks to government spending and technology are observed, state variables include bonds and money balances issued the previous period. In order to allow for comparison across Ramsey equilibriums with different compositions of nominal and indexed debt, the mix of debt is specified exogenously, leaving the government to choose labor taxes and money growth with the level of debt as the residual.

After the shocks to spending and technology are realized, optimal policy is a mapping of state variables to labor taxes, money supply, and the amount of debt so that the government's budget constraint is satisfied. Like the household maximization problem, the government's problem can be set up as a dynamic programming problem whereby the government seeks to maximize,

$$V\left(s_t\right) = \underset{\Delta_t}{Max}\left\{ \begin{array}{c} a\log C_{1t} + (1-a)C_{2t} - \gamma H_t + \\ \lambda_{gt}\left(\begin{array}{c} \tau_t \alpha Y_t - G_t + \frac{B_{t+1}^N}{P_t} + B_{t+1}^L + \\ \left(\exp(\mu_{t+1}) - 1\right)\frac{M_t}{P_t} - \frac{B_t^N}{P_t}R_{t-1}^N - B_t^L R_{t-1}^L \end{array}\right) + \beta E_t V\left(s_{t+1}\right) \end{array}\right\} \tag{22}$$

where $\Delta_t = (\tau_t,\, \mu_{t+1},\, B_{t+1}^N,\, B_{t+1}^L)$ is the set of choice variables and s_t represents the set of state variables $\left(\frac{B_t^N}{P_{t-1}},\, B_t^L,\, \frac{M_t^d}{P_{t-1}},\, \theta_{t-1},\, g_{t-1},\, \tau_{t-1},\, R_{t-1}^N,\, R_{t-1}^L\right)$. Here the government takes consumption of the cash good as given by 14, consumption of the credit good by 15, labor is given by 17, output satisfies the production function in 1, the price level is given by 16, the interest rate on indexed debt is 11, and the interest rate on nominal debt is 12. λ_{gt} is the Lagrange multiplier on the government budget constraint. The first-order conditions

[8] They use the analysis of Abreu, Pearce and Stacchetti (1990) to represent the government policy game as a dynamic programming problem.

[9] The solution was also solved and simulated under discretionary policy. Given that sources of government revenue are distortionary in this economy, optimal policy does not fully inflate away nominal liabilities. This result conforms to that in Lucas and Stokey (1983) which shows that the government has the incentive to inflate away nominal liabilities unless all prices are predetermined or distortionary taxes can be avoided.

for the Ramsey problem are,[10]

$$\tau_t \quad : \quad \left\{ \lambda_{gt} \left[\begin{array}{c} \frac{a}{C_{1t}} \frac{\partial C_{1t}}{\partial \tau_t} + \frac{1-a}{C_{2t}} \frac{\partial C_{2t}}{\partial \tau_t} - \gamma \frac{\partial H_t}{\partial \tau_t} + \\ \alpha Y_t \left(\frac{\tau_t}{H_t} \frac{\partial H_t}{\partial \tau_t} + 1 \right) - B_t^L \frac{\partial R_{t-1}^L}{\partial \tau_t} + \\ \left(\frac{B_t^N}{P_t} R_{t-1}^N - \frac{B_{t+1}^N}{P_t} - \left(\exp(\mu_{t+1}) - 1 \right) \frac{M_t}{P_t} \right) \frac{1}{P_t} \frac{\partial P_t}{\partial \tau_t} \end{array} \right] \right\} =$$
$$\beta E_t \left\{ \lambda_{gt+1} B_{t+1}^L \frac{\partial R_t^L}{\partial \tau_t} \right\}, \tag{23}$$

$$\mu_{t+1} \quad : \quad \left\{ \lambda_{gt} \left[\begin{array}{c} \frac{a}{C_{1t}} \frac{\partial C_{1t}}{\partial \mu_{t+1}} + \frac{1-a}{C_{2t}} \frac{\partial C_{2t}}{\partial \mu_{t+1}} - \gamma \frac{\partial H_t}{\partial \mu_{t+1}} + \\ \alpha \tau_t \frac{Y_t}{\partial \mu_{t+1}} - \frac{M_{t+1}}{P_t} \exp(\mu_{t+1}) - \frac{B_t^N}{P_t} \frac{\partial R_{t-1}^N}{\partial \mu_{t+1}} - B_t^L \frac{\partial R_{t-1}^L}{\partial \mu_{t+1}} + \\ \left(\frac{B_t^N}{P_t} R_{t-1}^N - \frac{B_{t+1}^N}{P_t} - \left(\exp(\mu_{t+1}) - 1 \right) \frac{M_t}{P_t} \right) \frac{1}{P_t} \frac{\partial P_t}{\partial \mu_{t+1}} \end{array} \right] \right\} =$$
$$\beta E_t \left\{ \lambda_{gt+1} \left[\left(\frac{B_{t+1}^N}{P_{t+1}} R_t^N - \frac{B_{t+2}^N}{P_{t+1}} \right) \exp(\mu_{t+1}) + B_{t+1}^L \frac{\partial R_t^L}{\partial \mu_{t+1}} \right] \right\}, \tag{24}$$

$$B_{t+1}^N : \frac{\lambda_{gt}}{P_t} = \beta E_t \left\{ \frac{\lambda_{gt+1}}{P_{t+1}} R_t^N \right\}, \tag{25}$$

$$B_{t+1}^L : \lambda_{gt} = \beta E_t \left\{ \lambda_{gt+1} R_t^L \right\}, \tag{26}$$

where λ_{gt} represents the marginal utility of relaxing the government budget constraint by one unit or, as suggested by Bohn (1988), the value that households place on the ability of the government to raise revenue from a source "outside" the economy. Such an ability would be equivalent to collection of a lump-sum tax. The multiplier is, therefore, the shadow value on reducing debt.

The distortionary effects of money and tax policy are evident through examination of the first-order conditions to the government's problem. For example, the Euler condition in 23 describes the trade-off between taxation and issuing debt. The first terms on the left-hand side reflect the change in consumption of the cash and credit goods and provision of labor by the household from a change in taxes. A change in the tax rate enters consumption of the cash and credit good indirectly via the equilibrium labor condition. The partial derivative of the equilibrium labor condition carries a negative sign, so that $\partial H_t/\partial \tau_t$, $\partial C_{1t}/\partial \tau_t$, $\partial C_{2t}/\partial \tau_t$ are all negative. Higher labor income taxes discourage labor supply, reduce output, and decrease consumption which negatively impacts household welfare

[10] Under the discretionary and commitment cases, the first-order condition for money shown here are actually $\partial/\partial \left(\exp(-\mu_{t+1})\right)$. This was done for simplicity of computation. The optimal government policy for money balances can then be found by taking the $-\log(x)$ of the result.

through the utility function offset by additional leisure. The bracketed term in 23 describes the change in the government budget constraint from a change in taxes scaled by the shadow value of debt. The first term inside the bracket represents the direct change in tax revenue from a change in tax policy, the sign of which depends on the nonlinear response of labor supply to a change in taxes. At low levels of tax burden, increases in the tax rate will result in an overall increase in tax revenue since the negative response of labor supply will be minimal. As the tax burden increases, the response of labor supply becomes more pronounced and reduces the increase in tax revenue for the government on the margin. The next term results from the commitment technology and details the change in the interest rate on indexed debt during the previous period, respectively, from a change in the one-period ahead tax rate. As discussed in the previous section, the equilibrium nominal interest rate only depends on the rate of time preference and expected future money growth and does not enter the Euler condition for taxes. The remaining terms inside the bracket describe the price effect on nominal resources chosen in the previous and current period. In particular, an increase in taxes increases the price level today since consumption of the cash good falls, reducing the real value of the payoff on nominal bonds and money balances chosen during the previous period. These combined effects in the left-hand side of 23 must be equal to the alternative policy of issuing additional indexed debt carried into the next period.

The trade-off between issuing money and debt is described in 24 and is more complicated since money enters 24 directly through the money growth term and indirectly through the equilibrium labor condition. The first terms on the left-hand side detail the effects of money growth on consumption and labor supply. Increases in money growth decrease equilibrium labor and the so that $\partial H_t/\partial\mu_{t+1}$carries a negative sign. Combined with the direct effects of money on consumption mean $\partial C_{1t}/\partial\mu_{t+1}$and $\partial C_{2t}/\partial\mu_{t+1}$also carry negative signs. The bracketed term, as in the tax derivative, details the impact of changes in money on the government budget constraint scaled by the multiplier. The first term describes the change in labor tax revenue based on the change in equilibrium labor from changes in money growth. Increases in money growth that decrease equilibrium labor result in lower output and reduced labor tax revenue. The second term relates to seigniorage revenues. The next two terms arise from the commitment technology and the remaining terms describe the price effect on nominal resources chosen in the previous and current period. Increases in money growth result in a higher price level, reducing the real value of nominal bonds and money balances chosen during the previous period. These combined effects on the left-hand side must be equal to the alternative policy of issuing debt which matures during the next period. The right-hand side of 24 also contains B^N_{t+2}/P_{t+1} since changes in the money stock during the current period influence the price level in the next period through the cash-in-advance constraint.

A. Calibration and Solution Procedure

The characterization of the policies that generate the Ramsey equilibrium theoretically is difficult since the system is nonlinear. Therefore, the system is characterized quantitatively. Following the process in Cooley and Hansen (1995), Hansen and Wright (1992), Christiano and Eichenbaum (1992), Avery (1987, 1986) and Hansen (1985), the model is calibrated to match the general features of the U.S. economy. Parameter values are chosen such that elements of the non-stochastic steady-state match the average values from the post-Korean War U.S. time series and are summarized in Table 1. This specification is then used as a baseline for different combinations of nominal and indexed debt as well as overall debt-to-income ratios.

The initial ratios and stocks of nominal and indexed debt were calibrated using additional data from the Bureau of the Public Debt and the FRED database. Non-marketable government debt was excluded from the sample. Based on this data, a debt-to-income ratio of 0.349 is used to simulate the U.S. debt-to-income economy or "low" debt economy. This debt-to-income ratio was also doubled in order to simulate of an economy with "high" debt. The ratio of inflation-indexed debt to total U.S. debt was about 5 percent, leaving the remaining 95 percent as nominal (fixed-rate) debt.[11] For comparison purposes, the composition of the debt stock was allowed to vary from mostly nominal debt to mostly indexed debt in order to examine how debt composition influences Ramsey policies.

The computational solution procedure is based on the projection approach as described in Judd (1998, 1992).[12] The set of Euler conditions from the Ramsey problem, the labor equation from the household's problem, and the government budget constraint can be generalized to a set of four operator equations $N(f)$ that define equilibrium. The solution procedure will solve for the optimal set of policies $(H_t,\, \mu_{t+1},\, \tau_t,\, \lambda_{gt})$ as functions of the exogenous shocks $(g_t,\, \theta_t)$ and initial state variables $s_t = \left(\frac{B_t^N}{P_{t-1}},\, B_t^L,\, \frac{M_t^d}{P_{t-1}},\, \theta_{t-1},\, g_{t-1},\, \tau_{t-1},\, R_{t-1}^N,\, R_{t-1}^L\right)$ that set $N(f) = 0$ simultaneously and satisfy the Ramsey equilibrium. One advantage to this approach is that the shadow value of reducing debt from the Ramsey problem, λ_g, is optimally solved for as an endogenous policy variable.[13]

[11] The ratio of indexed debt to total debt was calculated as the sum of inflation-indexed notes and bonds relative to to total marketable government debt held by the public.

[12] Additional information regarding implementation of the solution prodecure is available from the authors upon request.

[13] This approach differs from that take in Chari et al. (1994) who begin by fixing the value of the multiplier and iterate across candidate equilibrium solutions. However, the multiplier in their model is the Lagrange multiplier on the implementability constraint in the primal apprach to the Ramsey problem and is not strictly equivalent to the multiplier in this paper.

Since the set of operator equations is nonlinear, the projection approach begins by defining the policy functions in terms of Chebyshev polynomials. The orthogonal properties of Chebyshev polynomials allow an optimal search for the coefficients necessary to set the system of projection equations equal to zero simultaneously. Each polynomial is a function of the two random elements in the system, government spending and technology. After deciding on the type of polynomial, the Galerkin approach then forms the projections by specifying the number of coefficients in each polynomial. Higher orders of polynomial increase the capacity to model nonlinearities and generally improve the accuracy the solution. However, this numerical approach is computationally constrained and settles for the smallest number of coefficients such that additional coefficients yield relatively little in terms of approximation. Chebyshev collocation methods then divide the state space over θ and g into discrete grid points, where higher numbers of points produces a more defined grid space for which the system is solved over. Since the special properties of Chebyshev polynomials apply their restrictions over $[-1, 1]$, a linear transformation is applied to the state space of θ and g in order to permit their use. While collocation methods determine the fineness of the grid space, the boundaries of the space defining technology and government spending shocks are calibrated from actual U.S. data as described above. The interval for each is taken as a multiple of the standard deviation of the error process.

The set of residual functions also contain conditional expectations which must be evaluated. Since the processes that govern the shocks to technology and government spending are assumed to be distributed $N(0, \sigma^2_{\theta,g})$, expectations can be evaluated using Gauss-Hermite Quadrature. In this procedure, the form of the policy function is assumed to be independent of the realization of the shocks and expectations are found by integrating over the possible realizations of θ and g while treating the policy function as a constant. The value of the function at each node is based on the standard deviation of the error process and the optimally defined weight. While the choice of the number of nodes the function is integrated over determines the accuracy of the approximation, the weighting function in the Gauss-Hermite Quadrature routine is such that most of the density is captured at a relatively low order. Once properly specified, the system is solved using a nonlinear equation optimizer in Matlab. Based on an initial guess of the polynomial coefficients, the program computes the residual functions and uses standard computational techniques to iterate to the Ramsey equilibrium. The characteristics of this equilibrium are described in the next section.

To derive the results in this paper, second-order polynomials are used so that $n = 2$ where n defines the polynomial order. This is often a good starting point for complex systems since low order polynomials will capture a large portion of the nonlinearity within the system without increasing dimension of the system too rapidly. Since each polynomial is a function of two random variables, technology and government spending, each policy function will have a total of $n_\theta \; x \; n_g = 4$ coefficients. In order to ensure that the system is properly identified, the Chebyshev collocation methods define $m_\theta = 2$ technology levels and $m_g = 2$ spending levels for $m_\theta \ x \ m_g = 4$ possible combinations of spending and technology shocks. Consequently, the Galerkin approach defines 16 projection equations and the procedure will optimally solve for four policy functions, $(H_t,\ \mu_{t+1},\ \tau_t,\ \lambda_{gt})$ comprised of 16 total coefficients. The evaluation of expectations operators is done using Gauss-Hermite Quadrature. The number of nodes, r, determines how many points will be used to approximate the integral. Setting $r_\theta = r_g = 11$ requires the computation of 121 possible future combinations of technology and government spending values in order to evaluate each expectation.

The solution procedure begins with an initial guess for each coefficient and the residual functions are computed. Since the entire set of projection equations must equal zero simultaneously, the set of residual functions is simply the stacked vector of each individual Euler condition. The solution program evaluates whether the system is sufficiently "close" to zero by comparing the norm of the residual vector to some predefined criterion. In this low-order case, a norm of $1.0x10^{-9}$ is used as a minimum convergence criterion. The norm of the residual vector must be less than this before the solution procedure quits. If the norm of the residual vector is greater than this stopping criterion, the nonlinear solution program computes the Jacobian and uses the gradient information to adjust the coefficients further. Given the stopping criteria of $1x10^{-9}$, a mistake in the optimal policy function would cost the government less than $1 per billion in nominal GDP.

Given the complexity of the system, a move to higher order polynomials greatly increases the dimensionality of the problem and increases the computational time required. For example, moving from second to third order polynomials means each policy function would have $n_\theta \ x \ n_g = 9$ coefficients. Given the four policy functions, this creates the need to solve for 36 coefficients across 36 projection equations. Under this specification it becomes difficult to drive the norm of the residual vector "close" to zero, meaning that the nonlinear solution procedure may require more iterations to reach the stopping criterion used above or necessitate an increase in the stopping criterion to facilitate the higher order, decreasing the accuracy of the overall solution.

B. Results

The Ramsey problem was solved in economies with and without debt and when debt is present, under various combinations of nominal and indexed debt. The baseline economy contains no debt, the low-debt economy uses the prevailing debt-to-income ratio in the United States, and the high-debt economy is calibrated to twice the U.S. debt-to-income ratio. When debt is present, the solution was derived under various different combinations of nominal and indexed debt, ranging from a majority nominal debt policy (95 percent nominal, 5 percent indexed) which matches the current U.S. debt composition and a majority indexed debt policy (5 percent nominal, 95 percent indexed). Then using the optimal coefficients of the polynomial approximations that describe the Ramsey plan, each economy was simulated under the effects of technology and government spending shocks. Statistics were computed by running multiple simulations of 5000 periods in length, taking logarithms, and filtering each simulated time series using the H-P filter as described in Hodrick and Prescott (1997).

The Steady State

The upper panel in Table 2 represents the steady-state Ramsey equilibrium in levels or growth rates. Optimal household allocations smooth consumption and labor supply with the constant a, the relative importance of the cash good to the credit good in the utility function, determining the split between the two consumption goods. In each model economy, optimal government policy sets money growth equal to the rate of time preference as described in Friedman (1969). According to Friedman, optimal monetary policy satiates the economy with real balances to the extent that it is possible to do so. Government policy that follows the Friedman rule results in an expected gross nominal interest rate equal to 1.0 and expected real return on nominal debt and money balances equal to the inverse of time preference in the steady-state. In enacting this monetary policy rule, the government equates the real gross rate of return across the three assets (money balances, nominal debt, and indexed debt) in expectation, satisfying Euler conditions. As discussed in Chari, et al. (1991, 1996), the so-called Friedman rule turns out to be optimal in a variety of monetary economies with distorting taxes and this paper extends this result to a variety of different debt specifications and stands in contrast to Phelps (1973) who argues that optimality would require taxing the liquidity services from holding money balances.

As expected, government policy choices are interrelated. Monetary policy that follows the Friedman rule requires the government to run a gross-of-interest surplus by setting equilibrium labor income taxes high enough to cover government spending, interest on the debt, and the withdrawal of money balances from the economy. As the debt-to-income ratio rises, the equilibrium tax rate increases to produce a gross-of-interest surplus necessary to cover the associated higher interest costs with the higher stock of government debt. As a result, the distortionary effects of taxation on utility increase as the equilibrium tax rate rises with the level of outstanding debt. A higher debt stock requires higher equilibrium labor taxes, which result in higher welfare costs. This is reflected in the equilibrium value of the multiplier on the government budget constraint which increases when moving from the economy with no debt to the economies with higher debt-to-income ratios. Therefore, as suggested in Bohn (1988) and confirmed here, the shadow value of reducing debt is higher as debt loads increase since distortionary revenue policy imposes additional welfare costs.

The Role of Debt and Debt Composition

The existence of debt provides the government with an additional degree of policy freedom which allows for a smoother path of distortionary taxes and money growth over time, thereby affording the household a smoother stream of consumption and leisure. The simulations of the model economies displayed in the bottom panel of Table 2 indicate that volatility of the economies with debt, as measured by standard deviation in percent, are lower than the baseline economy without debt, regardless of the debt composition. Since the government is required to raise revenue from distortionary means, Ramsey policies smooth the response of fiscal and monetary policy to various shocks that affect the economy and the government budget constraint. Initially, the government sets tax and monetary policy based on generating an expected level of revenues to cover expected government expenditures. As shocks to technology and government spending affect the government budget constraint, optimal policy responds by smoothing the impact of distortionary taxes and money growth with debt. With respect to the ability to issue debt, the results confirm the tax smoothing role as discussed in Barro (1979, 1987). The existence of debt allows households to behave in a manner consistent with Friedman's permanent income hypothesis, whereby households consume based on permanent income and save and borrow in response to transitory changes in income.

The properties of each variable under each of the model simulations match some of the general characteristics of the overall U.S. economy as discussed in Stock and Watson (1999) and Hodrick and Prescott (1997). The economies with mostly nominal and mostly indexed debt under both debt-to-income ratios are reported in Tables 3 and 4. The models only generate about half of the standard deviation of output as found in the U.S. economy, a common shortcoming of most real business cycle models which is magnified here because of the fixed capital stock. However, the autocorrelation of output is slightly higher than found in other studies. One major difference between the models employed here and actual data is the negative correlation between labor and output. This is due to the assumption of a fixed capital stock which eliminates the complementary inputs characteristic of the production function. The existence of capital in the production function combined with the fact that technology augments both capital and labor allows households to balance consumption across both leisure and the two consumption goods, leading to increases in leisure in states of nature that also lead to higher consumption of the cash and credit good. Therefore, positive technology shocks that increase output also increase leisure and vice versa, causing a negative correlation between labor and output. Furthermore, while volatility of labor increases with the debt-to-income ratio and level of nominal debt, it still remains well below that of actual total U.S. employment in hours.[14]

[14] For example, using data from the U.S. from 1953-1996, Stock and Watson (1999, p. 31) report a standard

In addition to the optimality of the Friedman rule, a similar feature of each model is the low volatility of money growth. Almost all of the volatility in distortionary revenue generating policy is accounted for through the volatility of labor taxes, suggesting that preservation of the Friedman rule may take priority over distortionary impacts of labor taxes. This result is the opposite of Chari et al. (1991), who find that money growth should be more variable to preserve smoother taxes on labor income. The model economies with mostly indexed debt also produce negative correlations between money growth and output, while the economies with more nominal debt display a positive correlation. The economies with higher levels of nominal debt, therefore, generate a sort of liquidity effect through a negative correlation between money growth and real interest rates. Finally, the volatility of prices and inflation more closely match features of U.S. data. Since the price level and the rate of inflation are determined by the cash-in-advance constraint in equilibrium, volatility of the cash good imparts volatility into prices and compensates for the lack of volatility in money growth. The correlations of inflation with the shocks to government spending and technology have the expected opposite signs, leading to low correlations between inflation and output.

While this analysis confirms the benefit of the ability to issue debt through policy smoothing, it also suggests that the composition of debt is equally as important. The simulations as displayed in Table 2 and graphically displayed in Figure 1 indicate that business cycle volatility is reduced further and the steady-state levels of output and consumption are higher in economies with higher percentages of nominal debt. As the volatility of distortionary policy is reduced further, the negative welfare effects on activity are minimized for a given debt-to-income ratio. The end result is that households become more willing to supply additional labor in economies with higher ratios of nominal debt, increasing output and consumption of the cash and credit goods. For example, under the U.S. debt-to-income ratio and mostly nominal debt (95 percent nominal, 5 percent indexed), output is 0.29 percent higher, consumption of the cash good is 0.33 percent higher, and consumption of the credit good is 0.44 percent higher relative to an economy with mostly indexed debt. Although leisure falls as labor supply increases to support higher output and consumption, leisure is only reduced by 0.14 percent and the loss of utility from supplying additional labor is more than offset by additional utility of consumption.

To place these figures in context, the increase in output by moving from an indexed debt structure to a nominal debt structure with the U.S. debt-to-income ratio is roughly equal to around US$29 billion based on the current size of U.S. economic output, or slightly larger than the level of personal income for the state of Delaware in 2002. Under the high debt-to-income ratio, the increase in steady-state values is nearly doubled. Output is increased by 0.57 percent, consumption of the cash good by 0.67 percent, and consumption

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deviation of 1.61 for total employment hours.

of the credit good by 0.88 percent by moving from a predominantly indexed debt structure to a nominal debt structure. This represents an increase in output of around US$57 billion based on the current size of the U.S. economy, or roughly equal to the level of personal income for the state of Utah in 2002.

The Hedging Role of Nominal Debt

Economies with higher ratios of nominal debt are less volatile than economies with higher ratios of indexed debt since nominal debt acts as a hedge against shocks to the government budget. The idea of nominal debt as a form of state-contingent debt stems from Bohn (1988) and Chari et al. (1991) and is quantified here in a general equilibrium setting. Shocks to government spending and technology affect allocations and the price system through two primary paths: via the optimal labor decisions of households and the real value of debt issued in the previous period. Unexpected shocks to the economy that call for an increase in government revenue (i.e. higher labor taxes or money growth) also correspond to states with higher inflation. This higher-than-expected inflation reduces the value of existing nominal debt and counterbalances the need to increase government revenue. In this context, nominal debt acts as a hedge against shocks to the government budget by providing a non-distortionary source of revenue, permitting a smoother path for distorting revenue generating policies and consumption. The greater the level of debt and larger the percentage of nominal debt, the larger the hedging contribution. In contrast, the unexpected component of inflation does not diminish the real value of indexed debt which is designed to provide a constant real rate of return over the period.

A positive shock to government spending causes optimal government policy to respond with an increase in labor taxes and money growth, as shown by the positive correlation between government spending shocks and the tax rate in each of the economies with debt in Tables 3 and 4. However, government spending shocks are also positively correlated with prices. In equilibrium, the price level is determined by the cash-in-advance constraint and since money balances for use this period were chosen during the previous period, the change in the price level depends on the response of consumption of the cash good to the shock in government spending. As the government consumes more resources, households respond by increasing labor supply, but the increase in output is not enough to offset the increase in government spending and consumption of the cash good falls. In each of the model economies with debt, government spending shocks are positively correlated with labor supply and negatively correlated with the cash good. Since they reduce consumption of the cash good, positive shocks to government spending result in a higher price level and higher-than-expected inflation, which reduces the real value of nominal debt issued during the previous period. This effect counterbalances the need for increases in taxes and money growth. As reported in Tables 3 and 4 and graphically in Figure 2, the correlation between shocks to government spending and these government policy variables is always positive, but is lowest when the percentage of nominal debt is highest.

Beginning in economies with mostly indexed debt, a negative one-period shock to technology causes output to decrease through the production function even though the household responds by increasing labor supply. The decrease in output creates a need for higher labor taxes and money growth to finance the same level of government spending. Higher money growth and lower consumption of the credit good increase the real and nominal interest rates on indexed and nominal debt, respectively, resulting in higher interest costs in the government budget constraint for next period and reinforcing the need for distortionary revenue policy going forward. Overall, in the economies with mostly indexed debt, government policy is forced to respond with higher labor taxes, money growth, and debt in response to the negative technology shock.

However, as was the case with positive government spending shocks, the negative technology shock is positively correlated with inflation, indicating a role for nominal debt as a hedge. The hedging role for nominal debt is present since the decrease in consumption of the cash good results in higher inflation through the cash-in-advance constraint. As was the case with the positive government spending shock above, the increase in unexpected inflation reduces the real value of existing nominal debt and offsets the need to increase distortionary revenue policy. The value of nominal debt as a hedge declines as the percentage of nominal debt increases. Additional nominal debt lessens the need for higher distortionary revenue policy, eventually creating a situation where negative technology shocks cause lower money and technology growth, spurring labor and output, and lowering the correlation between output and consumption and technology shocks.

Nominal debt is a valuable hedge in the two scenarios described above since the unexpected inflation occurs in states of the economy that would otherwise call for increases in distortionary revenue policy.[15] In these states of the world the shadow value of reducing debt is increased, causing a positive correlation between inflation and the multiplier. The reduction in the real value of existing nominal debt provides a non-distortionary channel of revenue in the government budget constraint, easing the welfare costs of distortionary policy. Debt as a shock absorber, however, works in the opposite direction under positive technology shocks and negative government spending shocks. These states of the world produce decreases in prices which increase the real value of existing nominal debt, yet these increased debt service costs occur when additional resources become available, either through additional output or less government consumption, allowing the household to absorb these costs more easily. In the case of a positive technology shock and high levels of

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[15] While this result confirms the original conjecture by Bohn (1988), the analysis in this chapter is done in a fully-specified general equilibrium setting.

nominal debt, government policy calls for higher labor taxes and money growth, but sufficient economy wide resources remain available from higher output to increase consumption and leisure while simultaneously decreasing the stock of debt. Under negative government spending shocks, government policy reduces labor taxes and money growth, allowing for a similar pattern across consumption, leisure, and debt, but in lower magnitudes. Thus, even though the multiplier and inflation are negatively correlated in these circumstances, sufficient resources are available to more than offset any negative welfare implications. Overall, the value of nominal debt as a hedge occurs when needed most.[16]

While designed primarily to analyze the effects of debt composition as opposed to sustainability, the model also provides insight about the optimal quantity of debt. The optimal level of debt, as derived from its role as a hedge against shocks to the government budget, is directly dependent on the size of the shocks that hit the economy and the amount to which the household values stability of policy. Since the household prefers smooth consumption and leisure, the optimal amount of debt would counteract the direct effect of a shock to the government budget and mute out any distortionary policy response. However, as the amount of nominal debt increases, the marginal value of the gain in hedge falls since the absolute volatility of the system is reduced. As indicated above, unexpected shocks to the economy that call for an increase in government revenue also correspond to states with higher inflation, producing the opening for nominal debt as a hedging device. Thus, nominal debt will still have value as a hedge as long as the correlation between the multiplier, the shadow value of reducing debt, and inflation is positive. The simulations show that the correlation between the multiplier and inflation is highest under the economies with mostly real debt, indicating that existing hedge opportunities are not being filled. As the percentage of nominal debt is increased, the correlation begins to decline, until under the high debt-to-income ratio and mostly nominal debt, the correlation reaches zero. Beyond this level, adjustments in the value of the debt stock begin to outweigh the direct effects of the shocks on the government budget constraint. These results suggest that the optimal level of nominal debt for hedging purposes in the U.S. economy is much higher than current levels.

Measuring the Gains

A certainty equivalence framework is used to measure the gain to households from the ability to issue debt and from the hedging role of nominal debt. Two certainty equivalent

[16] The idea that the degree of indexation rests on the type and direction of shocks is similar to that found by Gray (1978) in examining optimal indexation and length of wage contracts. In particular, Gray finds that the optimal degree of indexation is positively related to monetary disturbances, but negatively related to real disturbances.

values of consumption are computed to evaluate any gain: a lump-sum present discounted value and a constant per-period value. Since the household has three variables that enter into the utility function, the certainty equivalent measures are computed in terms of the cash good to simplify the exposition. Values are taken from an average across a set of simulations for each calibrated economy and are displayed in Table 5 and Figure 3. The upper half of the table shows the lump-sum discounted present value increase in consumption of the cash good that will leave the household indifferent between the economy with no debt and the various economies with debt. For example, the gain in lifetime utility to the household from moving from an economy with no debt to an economy with the U.S. debt-to-income ratio and mostly indexed debt (5 percent nominal, 95 percent indexed) is equivalent to one-time payment of an additional 1.38 units of consumption of the cash good in the current period. This value is equal to 115 percent of the steady-state value of one-period consumption. The gain in utility from moving from one debt composition to another under the same debt-to-income ratio (i.e. the value of nominal debt as a hedge), is the difference between each value under the same debt-to-income ratio. Therefore, the gain in utility from changing the debt composition from mostly indexed debt to mostly nominal debt under the U.S. debt-to-income ratio is equal to $2.11 - 1.38 = 0.73$ units of the cash good in present value terms, or 61 percent of one-period consumption of the cash good.

The bottom half of Table 5 displays the certainty equivalent measure in per period consumption of the cash good. As opposed to a lump-sum payment, the data represent the constant per period increase in consumption of the cash good necessary to make the household indifferent between an economy without debt and an economy with debt. The gain in household utility from moving from no debt to an economy with the U.S. debt-to-income ratio and primarily indexed debt is equal to 1 percent of the cash good per period. Changing the composition of the debt from an indexed debt structure to a nominal debt structure yields an additional gain of 0.6 percent of the cash good per period. Under the high debt-to-income ratio, changing the debt structure yields an increase of 2.0 percent of the cash good per period. Overall, the value of nominal debt as a hedge against shocks to the government budget is roughly equal to the ability to issue debt.

These figures suggest that optimal debt policy should look beyond debt levels and first-order financing costs and give equal attention to the debt composition and second-order volatility costs. Sustainability issues aside, it may be sound policy for countries to use additional covenants such as indexation clauses that tie the value of the principal or coupon payments to interest rates, exchange rates, or in this case the price level to gain or expand market access. The ability to issue debt or deepen access, even if indexed, should entail important gains through the additional degree of freedom for government policymakers especially if the alternative is little or no ability to issue debt. However, this analysis also suggests that a predominantly indexed debt structure is a second-best solution and policymakers should strive to include sufficient amounts of nominal debt to further smooth distortionary government policy and reduce macroeconomic and business cycle volatility.

Furthermore, this paper lends additional credence to the argument that economic growth and macroeconomic volatility are negatively related, and that reductions in macroeconomic volatility and minimization of the cost of business cycles can entail large increases in overall welfare. The idea that government policy uncertainty could have negative effects on growth was examined by Aizenman and Marion (1993) who find that the magnitude and persistence of tax policy fluctuations jointly determine the pattern of investment and growth with negative correlation. The welfare gains estimated here from reducing policy and business cycle volatility are much larger than those in Lucas (1987), who estimated the cost of business cycles as equivalent to less than one-tenth of one percent of consumption, and are attributable to the nonlinearity and convexities within the model. As discussed in Kim and Kim (2003), the size of welfare gains is heavily dependent on preserving nonlinearities within the model as opposed to alternative methods, such as loglinearization, which may significantly underestimate gains or introduce errors in the estimation procedure.

While this model examines convexities from distortionary taxation and money growth on labor supply preferences, many other examples of convexities can be found in existing studies. Analysis by Bernanke (1983) on irreversible investment and by Ramey and Ramey (1991) on rigidities in the production process suggest that increased volatility results in lower investment and, therefore, lower growth. Black (1987) examined whether countries face a choice between a high-growth, high-variance economy and a low-growth, low-variance economy depending on the available technology, suggesting the growth and volatility may be positively correlated. More recently, Ramey and Ramey (1995) examine cross-country data and find that reductions in the volatility of output growth equal to one standard deviation of its cross-country variation equate to an increased growth rate of one-third of one percent in OECD countries, roughly equal to the increases in steady-state output and consumption reported here. For examples of nonlinearities in preferences, see Galí, Gertler, and López-Salido (2002) and Woodford (2001). In particular, Galí, Gertler, and López-Salido (2002) examine business cycle fluctuations under imperfect competition and find similar increases in welfare.

IV. CONCLUSION

This paper focuses on the importance of debt composition in the setting of optimal fiscal and monetary policy over both short-run business cycles and the long-run. The main conclusion is that the role of nominal debt as state-contingent debt can be a significant policy tool for reducing volatility of distortionary government policy. Reductions in the volatility of fiscal and monetary policy lead to a reduction in macroeconomic volatility and increases in equilibrium output and consumption. Overall, the gain in welfare from using nominal debt to hedge against shocks to the government budget is as large as the gain in welfare from the ability to issue debt. The ability to issue debt is worthwhile even if additional covenants (i.e. indexation) are needed in order to access capital markets in sufficient quantities to smooth distortionary government policy. However, these results suggest that sovereign debt management strategies need to view the true cost of the debt as more than purely first-order financing costs and include a second-order concept of variance of stock adjustments. Depending on the type and magnitude of economic shocks that prevail, optimal debt management should strive over time to create a debt structure that includes nominal debt in sufficient quantities to further reduce macroeconomic volatility and minimize costs associated with the business cycle.

In addressing this issue, the paper combines the traditional general equilibrium framework of macroeconomics with the Ramsey approach in public finance to calibrate and simulate a stochastic monetary model under various debt-to-income ratios and differing compositions of nominal and indexed debt. The stochastic monetary economy presented here incorporates the idea of a loss function within the nonlinearity of the labor supply equation since the contemporaneous tax on labor income and money growth are determinants of optimal household labor supply. Shocks that cause variations in government policy are transmitted to labor supply, output, remaining household allocations, and the equilibrium price system, eventually feeding back into the government budget constraint through tax revenue. Equilibrium decisions by households, firms, and the government, are then transferred across time through the price level and interest rates.

Economies with higher ratios of nominal debt are less volatile than economies with higher ratios of indexed debt since nominal debt acts as a hedge against unexpected shocks to the government budget. Shocks to government spending and technology affect allocations and the price system through two primary paths: via the optimal labor decisions of households and the real value of debt issued in the previous period. Unexpected shocks to the economy that call for an increase in distortionary government revenue also correspond to states with higher inflation. This higher-than-expected inflation reduces the value of existing nominal debt and works against the need to increase government revenue. In states of the world with positive shocks to government spending and negative shocks to technology, the shadow value of reducing debt is increased, creating a positive correlation between inflation and the multiplier on the government budget constraint. In this manner, nominal debt acts as a hedge against shocks to the government budget by providing a non-distortionary source of revenue, permitting a more smooth path for distorting revenue generating policies and consumption.

Table 1: Parameter values corresponding to U.S. economy.

Parameter Values							
α	β	a	γ	ρ_θ	σ_θ	ρ_g	σ_g
0.4	0.991	0.84	1.877	0.95	0.007	0.96	0.021

Table 2: Selected Simulations: Steady State Values and Standard Deviations.

Steady State Values in Levels

	Baseline	*U.S. Debt-to-Income*					*High Debt-to-Income*				
	No	Percent Nominal Debt					Percent Nominal Debt				
Variable	Debt	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%
Output	1.740	1.740	1.741	1.742	1.744	1.745	1.740	1.742	1.745	1.748	1.750
Cash Good	1.198	1.199	1.200	1.201	1.202	1.203	1.199	1.201	1.202	1.205	1.207
Credit Good	0.228	0.228	0.228	0.229	0.229	0.229	0.228	0.229	0.229	0.230	0.230
Labor	0.310	0.310	0.310	0.311	0.311	0.311	0.310	0.311	0.312	0.312	0.313
Multiplier	0.096	0.098	0.098	0.098	0.098	0.098	0.100	0.100	0.100	0.099	0.099
Gov. Spending	0.313	0.313	0.313	0.313	0.313	0.313	0.313	0.313	0.313	0.313	0.313
Inflation	0.991	0.991	0.991	0.991	0.991	0.991	0.991	0.991	0.991	0.991	0.991
Nominal Interest Rate	-	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Real Interest Rate	-	1.009	1.009	1.009	1.009	1.009	1.009	1.009	1.009	1.009	1.009
Money Growth Rate	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009
Tax Rate [1/]	0.186	0.190	0.189	0.189	0.189	0.189	0.193	0.193	0.192	0.192	0.192
Tax Rate [2/]	0.311	0.316	0.316	0.315	0.315	0.315	0.321	0.321	0.321	0.320	0.320

1/ In percent of total income.

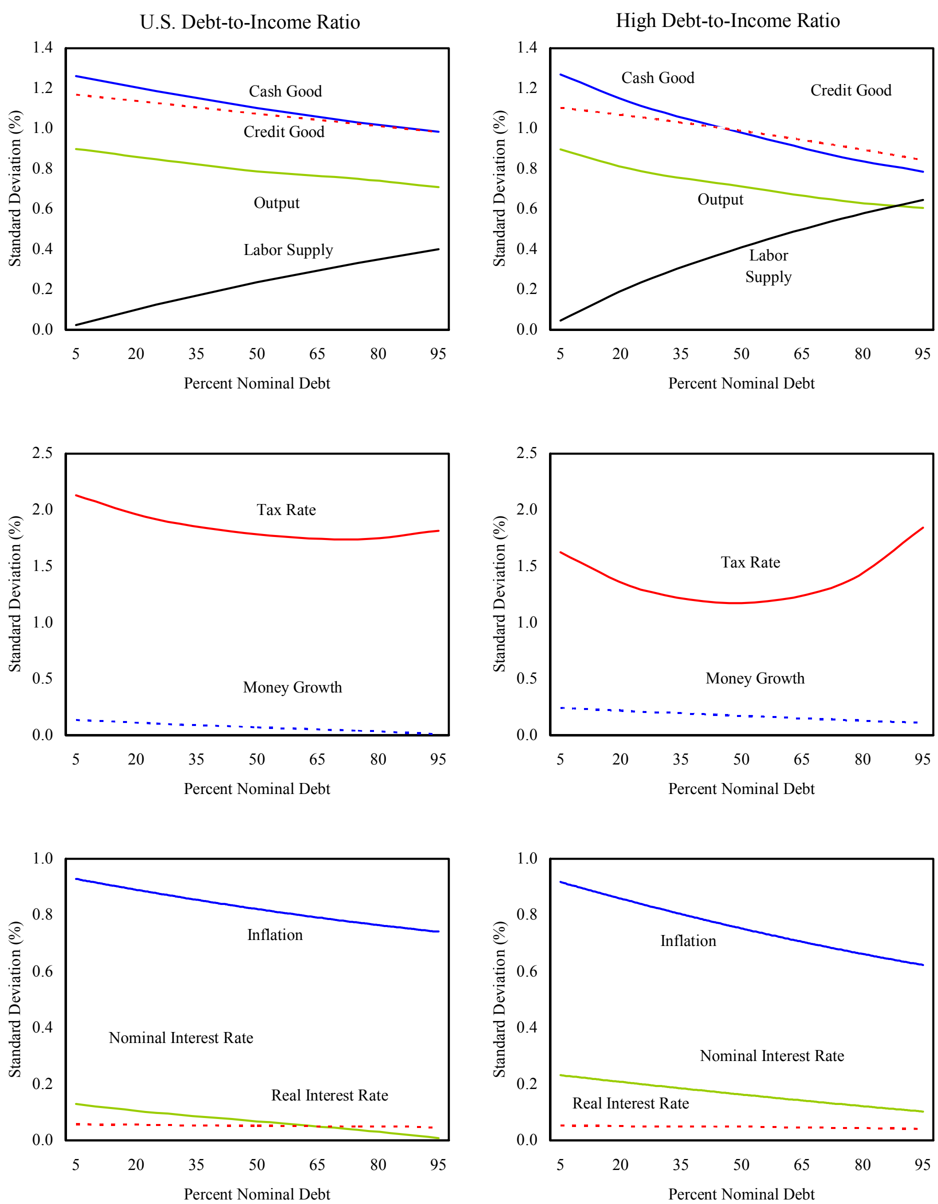
2/ In percent of labor income.

Standard Deviation in Percent

	Baseline	*U.S. Debt-to-Income*					*High Debt-to-Income*				
	No	Percent Nominal Debt					Percent Nominal Debt				
Variable	Debt	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%
Output	0.92	0.90	0.85	0.79	0.75	0.71	0.90	0.79	0.71	0.64	0.61
Cash Good	1.28	1.26	1.19	1.10	1.03	0.98	1.27	1.11	0.98	0.86	0.79
Credit Good	1.28	1.17	1.13	1.07	1.02	0.98	1.11	1.04	0.99	0.92	0.84
Labor	0.00	0.02	0.12	0.24	0.33	0.40	0.05	0.23	0.41	0.55	0.65
Multiplier	4.54	4.30	3.72	3.18	2.72	2.51	4.22	3.15	2.40	2.12	2.25
Price Level	1.28	1.30	1.22	1.11	1.04	0.98	1.48	1.27	1.15	1.04	0.88
Inflation	0.98	0.93	0.88	0.82	0.78	0.74	0.92	0.83	0.75	0.68	0.62
Nominal Interest Rate	-	0.13	0.10	0.07	0.04	0.01	0.24	0.19	0.16	0.15	0.09
Real Interest Rate	-	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.04
Debt	-	0.12	0.34	0.58	0.79	0.94	0.12	0.32	0.52	0.66	0.76
Money Growth Rate	0.00	0.13	0.10	0.07	0.04	0.01	0.25	0.19	0.17	0.16	0.10
Tax Rate [1/]	2.83	2.13	1.92	1.78	1.74	1.82	1.62	1.29	1.17	1.35	1.84

1/ In percent of total income.

Figure 1: Model Simulations: Steady-State Values and Standard Deviations.



Note: Standard deviation of household allocations, government policy, and the price system under varrying combinations of nominal and indexed debt are reported for both the U.S. and high debt-to-income ratios. As the percentage of nominal debt increases, the overall volatility of each economy declines, increasing household welfare by reducing volatility of consumption.

Table 3: Simulated Economy with U.S. Debt-to-Income Ratio

Majority Indexed Debt (5% Nominal, 95% Indexed)

	Cross-Correlation of Output with:							Cross-Correlation				
								Money	Tax		Shocks	
Variable	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	Growth	Rate	Mult.	Tech.	Gov.
Output	0.272	0.470	0.712	1.000	0.712	0.470	0.272	-0.278	-0.282	-0.472	1.000	0.010
Cash Good	0.232	0.406	0.617	0.870	0.622	0.413	0.240	-0.713	-0.716	-0.841	0.874	-0.481
Credit Good	0.243	0.423	0.643	0.907	0.648	0.429	0.250	-0.655	-0.659	-0.797	0.910	-0.410
Labor	-0.230	-0.401	-0.611	-0.862	-0.616	-0.409	-0.238	0.725	0.727	0.851	-0.865	0.495
Multiplier	-0.120	-0.216	-0.331	-0.472	-0.342	-0.232	-0.138	0.966	0.978	1.000	-0.478	0.875
Gov. Spending	0.011	0.012	0.013	0.010	0.002	-0.003	-0.005	0.945	0.956	0.875	0.003	1.000
Price Level	-0.159	-0.339	-0.562	-0.822	-0.610	-0.428	-0.276	0.639	0.644	0.764	-0.825	0.420
Inflation	-0.195	-0.245	-0.305	-0.372	0.296	0.255	0.214	0.377	0.377	0.424	-0.374	0.280
Nom. Int. Rate	-0.066	-0.123	-0.191	-0.276	-0.202	-0.137	-0.083	1.000	0.989	0.966	-0.283	0.947
Real Int. Rate	-0.231	-0.402	-0.609	-0.854	-0.611	-0.407	-0.237	0.599	0.600	0.747	-0.939	0.340
Debt	-0.086	-0.222	-0.388	-0.587	-0.823	-0.589	-0.392	0.481	0.479	0.567	-0.636	0.307
Money Growth	-0.067	-0.124	-0.193	-0.278	-0.203	-0.138	-0.083	1.000	0.988	0.966	-0.285	0.945
Tax Rate	-0.069	-0.126	-0.196	-0.282	-0.206	-0.140	-0.084	0.988	1.000	0.978	-0.290	0.956

Majority Nominal Debt (95% Nominal, 5% Indexed)

	Cross-Correlation of Output with:							Cross-Correlation				
								Money	Tax		Shocks	
Variable	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	Growth	Rate	Mult.	Tech.	Gov.
Output	0.262	0.465	0.710	1.000	0.710	0.465	0.262	0.576	0.576	0.213	0.987	0.144
Cash Good	0.212	0.372	0.565	0.789	0.562	0.368	0.209	-0.043	-0.046	-0.429	0.878	-0.491
Credit Good	0.214	0.375	0.569	0.795	0.566	0.370	0.210	-0.034	-0.037	-0.421	0.883	-0.483
Labor	-0.212	-0.373	-0.565	-0.790	-0.563	-0.368	-0.209	0.039	0.044	0.428	-0.879	0.489
Multiplier	0.047	0.091	0.145	0.213	0.149	0.098	0.053	0.912	0.921	1.000	0.053	0.996
Gov. Spending	0.029	0.059	0.096	0.144	0.100	0.066	0.035	0.886	0.890	0.996	-0.017	1.000
Price Level	-0.222	-0.381	-0.571	-0.792	-0.559	-0.361	-0.200	0.038	0.041	0.425	-0.880	0.487
Inflation	-0.172	-0.211	-0.252	-0.293	0.309	0.263	0.214	0.023	0.024	0.168	-0.328	0.191
Nom. Int. Rate	0.144	0.260	0.402	0.573	0.405	0.265	0.147	1.000	0.992	0.914	0.434	0.889
Real Int. Rate	-0.225	-0.396	-0.602	-0.842	-0.599	-0.392	-0.222	-0.049	-0.047	0.343	-0.917	0.406
Debt	-0.082	-0.212	-0.372	-0.565	-0.790	-0.562	-0.368	0.031	0.031	0.306	-0.629	0.352
Money Growth	0.145	0.261	0.404	0.576	0.407	0.267	0.148	1.000	0.992	0.912	0.438	0.886
Tax Rate	0.144	0.260	0.403	0.576	0.407	0.267	0.149	0.992	1.000	0.921	0.436	0.890

Table 4: Simulated Economy with High Debt-to-Income Ratio

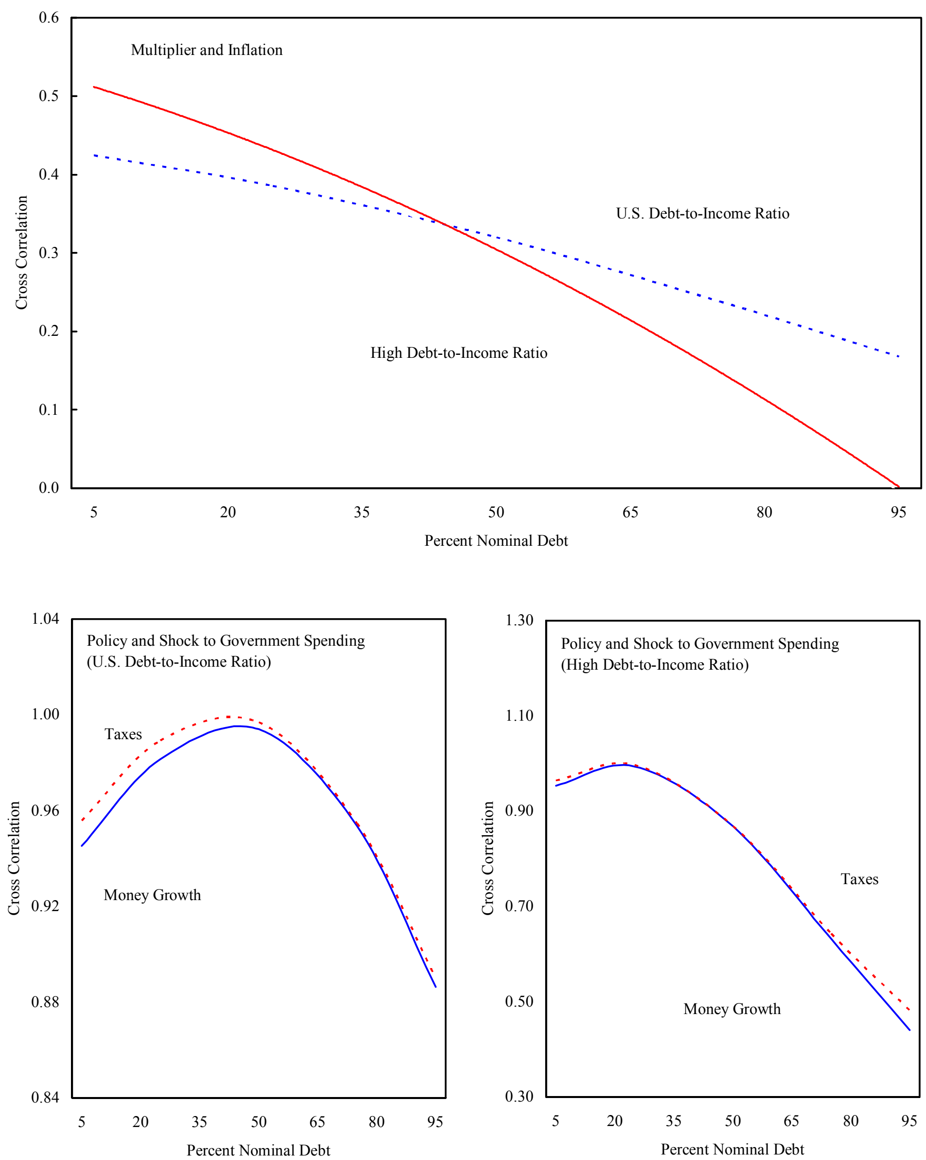
Majority Indexed Debt (5% Nominal, 95% Indexed)

Variable	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	Money Growth	Tax Rate	Mult.	Shocks Tech.	Shocks Gov.
	Cross-Correlation of Output with:							Cross-Correlation				
Output	0.271	0.475	0.716	1.000	0.716	0.475	0.271	-0.255	-0.251	-0.443	1.000	0.011
Cash Good	0.233	0.408	0.615	0.861	0.617	0.413	0.240	-0.708	-0.705	-0.854	0.870	-0.496
Credit Good	0.252	0.442	0.666	0.931	0.667	0.445	0.258	-0.586	-0.585	-0.766	0.937	-0.352
Labor	-0.229	-0.401	-0.604	-0.846	-0.607	-0.406	-0.236	0.729	0.724	0.866	-0.855	0.520
Multiplier	-0.124	-0.216	-0.324	-0.452	-0.325	-0.220	-0.133	0.963	0.974	1.000	-0.457	0.883
Gov. Spending	0.006	0.008	0.014	0.015	0.009	-0.001	-0.010	0.953	0.964	0.876	-0.006	1.000
Price Level	-0.114	-0.275	-0.472	-0.712	-0.547	-0.402	-0.276	0.523	0.521	0.653	-0.708	0.349
Inflation	-0.204	-0.260	-0.317	-0.386	0.266	0.232	0.203	0.468	0.467	0.515	-0.393	0.377
Nom. Int. Rate	-0.066	-0.116	-0.175	-0.248	-0.179	-0.125	-0.079	1.000	0.988	0.967	-0.268	0.955
Real Int. Rate	-0.258	-0.452	-0.681	-0.952	-0.682	-0.455	-0.263	0.528	0.530	0.726	-0.958	0.287
Debt	-0.093	-0.244	-0.427	-0.644	-0.903	-0.648	-0.433	0.530	0.456	0.588	-0.643	0.287
Money Growth	-0.066	-0.118	-0.176	-0.250	-0.181	-0.126	-0.080	1.000	0.987	0.963	-0.271	0.953
Tax Rate	-0.066	-0.117	-0.175	-0.248	-0.179	-0.126	-0.081	0.987	1.000	0.974	-0.266	0.964

Majority Nominal Debt (95% Nominal, 5% Indexed)

Variable	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	Money Growth	Tax Rate	Mult.	Shocks Tech.	Shocks Gov.
	Cross-Correlation of Output with:							Cross-Correlation				
Output	0.271	0.469	0.713	1.000	0.713	0.469	0.271	0.871	0.971	0.874	0.952	0.305
Cash Good	0.183	0.320	0.489	0.689	0.494	0.328	0.190	0.471	0.526	0.251	0.877	-0.477
Credit Good	0.199	0.348	0.532	0.749	0.537	0.356	0.206	0.559	0.590	0.331	0.915	-0.397
Labor	-0.183	-0.320	-0.489	-0.688	-0.494	-0.267	-0.190	-0.472	-0.526	-0.256	-0.878	0.477
Multiplier	0.242	0.414	0.626	0.874	0.621	0.407	0.234	0.812	0.954	1.000	0.685	0.723
Gov. Spending	0.086	0.146	0.220	0.305	0.213	0.137	0.077	0.440	0.482	0.724	0.000	1.000
Price Level	-0.353	-0.450	-0.556	-0.667	-0.395	-0.177	-0.009	-0.478	-0.519	-0.274	-0.829	0.399
Inflation	-0.127	-0.137	-0.151	-0.156	0.385	0.310	0.239	-0.063	-0.104	-0.005	-0.234	0.217
Nom. Int. Rate	0.237	0.412	0.626	0.878	0.626	0.412	0.237	1.000	0.830	0.823	0.780	0.448
Real Int. Rate	-0.208	-0.369	-0.563	-0.792	-0.568	-0.376	-0.218	-0.629	-0.638	-0.394	-0.937	0.325
Debt	-0.072	-0.184	-0.321	-0.491	-0.692	-0.496	-0.329	-0.329	-0.378	-0.180	-0.627	0.341
Money Growth	0.235	0.408	0.621	0.871	0.621	0.409	0.236	1.000	0.820	0.812	0.775	0.440
Tax Rate	0.264	0.456	0.693	0.971	0.691	0.454	0.261	0.820	1.000	0.954	0.864	0.482

Figure 2: Selected Cross Correlations.

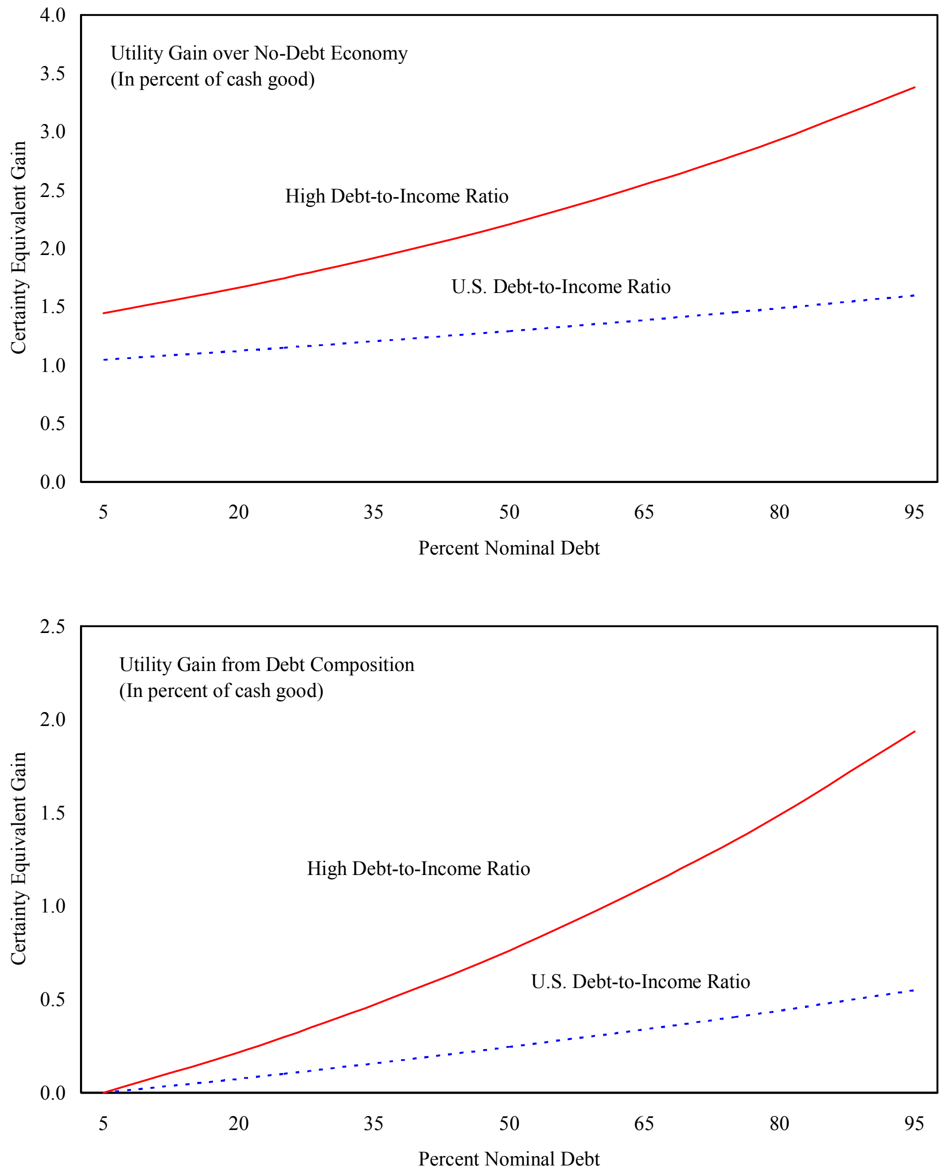


Note: The upper panel plots the cross correlation between the multiplier and inflation. The shadow value of reducing debt is highest under indexed debt compositions and declines as more nominal debt is utilized. The lower panels plot the correlation between distortionary government policy and the shock to government spending. As more nominal debt is utilized, the correlation between distortionary government policy and shocks to government spending is reduced.

Table 5: Certainty Equivalence. Increase in consumption of the cash good necessary to make the household indifferent between no debt and the selected debt composition

Certainty Equivalent Consumption Over No-Debt Economy										
	U.S. Debt-to-Income					High Debt-to-Income				
	Percent Nominal Debt					Percent Nominal Debt				
	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%
	Discounted Present Value									
Cash Good Equivalent	1.38	1.52	1.70	1.92	2.11	1.90	2.30	2.91	3.69	4.46
Percent of Steady State	115	127	142	160	176	159	192	243	308	372
	Per Period Value									
Cash Good Equivalent	0.013	0.014	0.015	0.017	0.019	0.017	0.021	0.026	0.033	0.040
Percent of Steady State	1.0	1.1	1.3	1.5	1.6	1.4	1.7	2.2	2.8	3.4

Figure 3: Certainty Equivalent Gains.



Note: The upper panel lists the certainty equivalent gain in consumption of the cash good necessary to make the household indifferent between the no-debt economy and the selected debt-to-income ratio and debt composition. The lower panel diplays the certainty equivalent gain in consumption of the cash good necessary to make the household indifferent between an economy with an indexed debt structure and the selected debt composition. If measured in terms of total consumption, the percentages would be lower.

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