Interest Rates, Credit Rationing, and Investment in Developing Countries

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Abstract

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This paper examines the impact of interest rates and inflation on bank loans and investment within a framework that mimics the financial sectors prevailing in most low-income developing countries. The paper emphasizes the importance of treating the lending and deposit rates of interest as distinct parameters in investment equations. The spread between the two rates is indicative of default risk and has a negative impact on incremental loan amounts associated with higher lending rates, in particular in economies with flawed institutions. The model presented in the paper highlights the importance of promoting macroeconomic stability and upgrading institutions and informational infrastructure.

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I. INTRODUCTION

This paper examines the impact of interest rates and inflation on bank loans and investment in a theoretical framework that mimics the financial sectors prevailing in most low-income developing countries. It also investigates whether the proposition that, for developing countries, interest rates are irrelevant in investment equations in the presence of a credit constraint is always justified.

The usefulness of monetary policy as a tool for carrying out economic goals is based on the notion that there is a relationship between the quantity of money and income. Banks play an important role in the money-creation process, as well as in the mobilization and allocation of financial savings. In the United States, the severity of business downturns during the Great Depression has partially been attributed to the collapse of the banking system. In developing countries, following the early 1970s work of McKinnon (1973) and Shaw (1973), the sluggishness of investment and economic growth was attributed in part to poor financial intermediation associated with financial repression, including interest rate ceilings.

McKinnon (1973) and Shaw (1973) contend that financial repression through controlled interest rates and the ensuing credit rationing impedes economic growth by discouraging financial savings and fostering low, inefficient investment. The kind of credit rationing McKinnon (1973) and Shaw (1973) draw attention to is identified as disequilibrium rationing, in contrast to another type of rationing, called equilibrium rationing.² The idea of disequilibrium rationing is predicated on the notion that, in competitive markets where information is perfect, interest rates clear the market for savings and investment. In reality, credit markets are different from standard markets in that excess demand can exist as an equilibrium situation, characterized as equilibrium credit rationing. Two strands of literature explain the equilibrium credit rationing phenomenon: the "earlier" literature, represented by Hodgman (1960) and Freimer and Gordon (1965), and the "current" literature, represented by Jaffee and Russell (1976) and Stiglitz and Weiss (1981). The earlier literature explains credit rationing by special characteristics of loan markets, such as default risk (Freimer and Gordon, 1965) and customer relationships (Kane and Malkiel, 1965). The current literature attributes credit rationing to imperfect information, which leads to adverse selection and moral hazard. This theory explains why a rational bank may choose to keep the lending rate at a certain level.

² The literature also makes a distinction between equilibrium rationing and dynamic rationing based on whether interest rates are at their long-run equilibrium level or have departed from it. Rationing that exists when interest rates are at their long-run equilibrium level is referred to as equilibrium rationing. When some disturbance in market conditions makes interest rates deviate from their long-run levels, the return to those long-run levels is not instantaneous, because of the "stickiness" emphasized by Keynesian economists. Under the circumstances, the credit rationing that occurs is called dynamic rationing. (Jaffee and Modigliani, 1969). Moreover, a distinction is made between rationing by number of loans and rationing by loan size (Keeton, 1979).
and limit the amount it lends, or why higher lending rates would not necessarily induce higher credit.

Advocates of financial liberalization contend that removing interest rate ceilings would increase financial intermediation and foster growth through higher and more efficient investment. However, while repressing the financial system with administrative ceilings on interest rates may harm capital formation, liberalizing by removing the ceilings has not been firmly proven to be growth enhancing. In some instances, even after financial liberalization, the incentive for fostering high financial savings, as well as for increasing credit in the formal financial system is lacking. In other instances, lending at high interest rates has undermined economic growth by reducing borrowing firms' ability to service their debt, thereby weakening banks' balance sheet positions. On the empirical front, conclusions on the relationships between interest rates and investment are not unanimous. Rittenberg (1991) suggests that investment will be positively correlated with below-equilibrium interest rates and negatively correlated with above-equilibrium rates; Greene and Villanueva (1991) find a negative relationship between real interest rates and investment; and Gelb (1989) indicates that real interest rates have no significant impact on investment. In line with findings such as Gelb's, a number of empirical studies on investment in least-developed countries (LDCs) exclude interest rates from the regressions (Cardoso, 1993; Ramirez, 2000).

This paper presents a general equilibrium framework for analyzing how interest rates and inflation affect bank loans and investment. Specifically, the relationships among these variables are derived from the optimizing behavior of a representative household and a representative bank. An important contribution of the paper is that the model presented is consistent with the financial systems prevalent in most LDCs. In particular, in order to capture the notion of thin financial markets, the paper incorporates in a cash-in-advance (CIA) model three features that characterize most LDCs: (i) their surplus spending units can hold only three assets, namely, currency, bank deposits, and physical assets (capital); (ii) the payment system is characterized by a low usage of bank deposits as a means of payment, making the size of cash transactions an important parameter of the economic environment; and (iii) owing to the difficulty of selling financial claims, deficit spending units rely mostly on accumulated savings or bank credit. Furthermore, bank credit is rationed. When the deficit spending unit is the

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3 Stiglitz (1993) lists some advantages of low deposit rates of interest. He argues that low deposit rates can be beneficial for an economy whether or not they are passed to firms through low lending rates. Another argument in favor of low deposit rates comes from van Wijnbergen (1983b) and the neostructuralists. They contend that informal credit markets provide more complete intermediation than banks because banks are subject to reserve requirements that reduce credit availability. This assumption supports their argument that, when higher deposit rates induce portfolio shifts from assets in the informal credit markets to bank deposits, the rate of economic growth may be reduced as the overall amount of credit available to businesses contracts. This paper focuses on the formal banking sector.

4 In developing countries cash in advance is more than an assumption—it is the way transactions are carried out. Hellwig (1993, p.223) states that "the imposition of the CIA constraint is usually justified on descriptive grounds, by an appeal to realism. The problem
government, the likelihood of the deficit being monetized is high because most of the time the banking system is the main purchaser of government securities.

The paper goes beyond most of the existing literature on the determinants of investment by treating the lending rate and the deposit rate as two distinct parameters. The distinction is critical particularly where large interest rate spreads prevail. An important premise of the analysis is that, for given deposit and lending rates, the representative bank determines the reserve ratio by choosing credit ceilings endogenously. Owing to credit ceilings, the household may not be able to borrow the amount it would like to at the prevailing rates. The interest rates the paper refers to are nominal; inflation is included as a separate parameter.

The analysis introduces the level of development of institutions and information structure into a risk-of-default-based “earlier” model of equilibrium credit rationing. The level of development of institutions and information structure is very important to the lending process because it has a bearing on the severity of ex-ante asymmetric information associated with the borrower’s type as in Stiglitz and Weiss (1981) or ex post asymmetric information with regard to the realized return on the borrower’s investment as in Williamson (1987). To a certain extent, the analysis reconciles some of the earlier and current literature on equilibrium credit rationing. The paper argues that the difference between disequilibrium—à la McKinnon and Shaw—and equilibrium rationing may not necessarily be clear-cut. Credit rationing at low lending rates can be the optimal choice of a bank even in the absence of administrative ceilings on lending rates. If the likely overall effect of a higher lending rate is to diminish the lender’s expected profit, banks, in particular in economies characterized by flawed institutions and poor informational infrastructures, can choose to keep lending rates low and ration credit rather than raise rates. The paper underscores the importance of institutions and informational infrastructure as a setting within which asymmetric information has a bearing on how bank loans and investment respond to higher interest rates.

The paper shows that the spread between the lending and the deposit rates is indicative of default risk and has a negative impact on incremental loan amounts associated with higher lending rates. Moreover, it demonstrates that even in the presence of a credit constraint, the deposit rate has a positive impact on investment, while the lending rate can have a negative impact. Furthermore, the paper shows that inflation is negatively related to both bank loans and investment. The paper suggests that, in estimating investment equations for developing

with this piecemeal introduction of realism into an otherwise highly idealized model begs the question of what is the function of this constraint, and how does this function fit into the conceptual structure of the overall model.”

Brock and Suarez (2000) suggest that the persistence of high interest rate spreads has been a disturbing outcome of market-oriented reforms in some Latin American countries. In sub-Saharan Africa, during 1996-99, deposit rates averaged 9 percent in Tanzania, 22 percent in Malawi, and 27 percent in Zimbabwe. During the same period, spreads between lending and deposit rates averaged 17 percent, 19 percent, and 14 in Tanzania, Malawi, and Zimbabwe, respectively.
countries, focusing on only one interest rate variable, as is often done in the literature, may generate misleading results.

The remainder of the paper is organized as follows: Section II describes the theoretical model and Section III contains concluding remarks.

II. THE MODEL

A. Overview of the Model

The economy is made up of a producer-consumer representative household, a representative bank, and the government. On the household's side, the model bears some resemblance to Stockman (1981), Englund and Svensson (1988), Mosseči (1990), Palivos et al. (1993), and Hutchison (1995).

On the representative bank's side, the model borrows from Keeton (1979) and builds on the credit-rationing model developed by Freimer and Gordon (1965), whose shortcomings are explained by Jaffee and Modigliani (1969) and Jaffee and Stiglitz (1990), to derive the loan supply schedule.6

There is one good in the economy that can be purchased with either currency or deposits.7 The distinction is motivated by the existence of transaction and information costs.8 Although there is only one good, for convenience, the terms "currency good" and "deposit good" are used throughout the paper to refer to transactions paid for with currency and with bank deposits, respectively. While the currency good can be purchased only with currency, the deposit good can be purchased with either currency or deposits. Nonetheless, since deposits pay interest, a rational agent would prefer to use deposits to purchase the deposit good. The "two" goods are used for consumption, as well as for investment purposes. Let \( \nu \) be the level of development of institutional factors and informational infrastructure, \( i \) the nominal lending rate, \( r \) the nominal deposit rate, and \( \Pi \) the rate of inflation. Let \( \theta \) be the fraction of investment financed with currency. Assume \( \theta = \theta(\nu, i, r, \Pi) \). In reality, country-specific factors not captured by \( \nu \) would also influence \( \theta \). The representative household operates in a time-discrete infinite horizon. It divides its wealth among consumption (of the currency good and the deposit good), capital acquisition, and currency and deposit holding. Assume that capital depreciates totally after one period. Hence, \( k_{t+1} = inv_t \), where \( k_{t+1} \) is capital in period \( t + 1 \) and \( inv_t \) is real investment in period \( t \). The production function is \( y = Af(k) \), where

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6 The derived loan offer is nonmonotonic in the lending rate. Although based on risk of default, the model does not exclude asymmetric information.

7 A list of symbols used is given in section I of the appendix.

8 As in most LDCs, a check is a valid means of payment only when the seller knows and trusts the buyer; otherwise, currency is required.
$\tilde{A}$ is the product of a technology parameter, $A$, and $\nu$, which is a composite index of the level of macroeconomic stability and that of the development of informational infrastructure and institutional factors, including contract enforcement mechanisms. $k$ is the amount of capital used. The index, $\nu$, could be considered as part of $A$. It is singled out to ease the understanding of the part of the analysis where the importance of information and institutional factors will be emphasized. Nonetheless, throughout the analysis, where there is no need to separate $\nu$ from $A$, $\tilde{A}$ is used for their product.

B. The Government

The government creates fiat money and can influence nominal lending and deposit rates of interest. At the beginning of each period, the government issues fiat money as transfer a to the representative household. The amount of transfer, $\tau_t$, is a fraction, $\sigma_t$, of the existing monetary base. The monetary base evolves as follows:

$$H_t = (1 + \sigma_t) H_{t-1}$$

(1)

The real monetary base in the beginning of period $t$ is defined as

$$h_t = \frac{H_t}{P_{t-1}}$$

(2)

Assuming that there are no taxes and that the government does not consume goods, its budget constraint in real terms can be written as

$$\tau_t = \frac{\sigma_t h_{t-1}}{1 + \Pi_{t-1}}$$

(3)

where $\Pi_{t-1}$ is the rate of inflation between $t-2$ and $t-1$; $\tau_t$ is the amount of real money transfers to the representative household at time $t$ that can be written as:

$$\tau_t = \tau_t^h + \tau_t^c$$

(4)

where $\tau_t^h$ and $\tau_t^c$ represent, respectively, the real currency and real value of the deposit transfers.

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9 From a conceptual standpoint, this index is close to the bureaucratic efficiency index (BEI) that can be computed from indices published by Business International.

10 Other real variables, like, for instance, $z_t$, $b_t$, and $k_t$, are derived in the same manner.

11 The government provides the representative bank with reserves (in the form of currency), to back the increase in the bank’s deposit liabilities.
The government determines the monetary base. However, the distribution of the monetary base between currency and bank reserves is determined by the optimizing behavior of the representative household and the representative bank.

C. The Banking System

Activities

The representative bank issues deposits and makes loans at given interest rates. Loans made in period $t$ are expected to be fully repaid in period $t+1$. The bank's assets consist of reserves and loans. Its liabilities are solely deposits.

At given interest rates, the representative bank optimally determines how much it would lend, and then the amount becomes a binding constraint for the borrower.

The Loan Supply

At time $t$, the household borrows an amount to carry into $t+1$, $B_{t+1}$. This amount, with real value $b_{t+1}$, is invested in a project whose uncertain outcome is given by $Q(k, X) = \tilde{A}f(k_{t+1})X$, where $\tilde{A}$ and $k$ are as defined above and $X$ is a random variable with probability density $g(x)$. $Q(k, X)$ satisfies the following conditions: $Q_s > 0$, $Q_x > 0$, and $Q_{kk} < 0$.

Assume that the investor's liability is limited to the proceeds of the investment. Considering that the investor is a residual claimant on the investment's returns, the amount of repayment to the bank is $\min[Q(k, X), b_{t+1}(1+i_t)]$. Assume further that the investor has alternative means of finance, which can be used to complement bank loans. It is further assumed that the random variable $X$ takes values in the interval $[0, \varepsilon]$, and that $\varepsilon$ is a function of institutional factors, macroeconomic stability, and the informational infrastructure, that is, $\varepsilon = \varepsilon(\nu)$. The inclusion of the index $\nu$ in the analysis brings to light asymmetric information in the Freimer-Gordon model. The limited liability feature of the loan contract, together with agency problems associated with credit, makes the level of development of institutions and informational infrastructure a very important element of the lending process. Note that $\varepsilon(\nu) > 0$, and $\varepsilon_{\nu}(\nu) < 0$. At time $t$, the bank's expected profit from its lending to be carried, $E_t(\Psi)$, into $t+1$ is given by the following:

---

12 Each bank assumes that it is too small for its decisions to affect market interest rates.

13 The existing informational infrastructure has an impact on the selection of potential borrowers. For instance, when specialized credit reference agencies provide fast and reliable information on the creditworthiness of loan applicants, the lending risk is somewhat reduced. As stated by Mishkin (1992, p. 183), adverse selection takes place before the transaction when bad credit risks are the ones most likely to receive loans. After a loan has been made, good institutions and informational infrastructures help in curbing the cost of contract enforcement and that of monitoring the return of the borrower's investment project referred to by Williamson (1987).
\[ E_i(\Psi) = \tilde{A}f(k_{i+1}) \int_{x_0}^x xg(x)dx + \int_{x_0}^x b_{i+1}(1 + i_i)g(x)dx - b_{i+1}(1 + r_i). \] \hspace{1cm} (5)

In equation (5), \( x \) is the value of the random variable \( X \) from which the investment return can repay the loan (principal and interest); \( x = \frac{(1 + i_i)b_{i+1}}{\tilde{A}f(k_{i+1})} \). The higher \( i_i \) or \( b_{i+1} \), the higher \( x \). The higher \( x \), the higher the probability that the representative household will default.

The bank's expected profit-maximizing loan is such that\(^{14}\)

\[ \int_{x_0}^x g(x)dx = \frac{i_i - r_i}{1 + i_i}. \] \hspace{1cm} (6)

Since the integral on the left-hand side is the probability of default, it justifies the interpretation that the optimal loan is such that the probability of default is equal to the excess of the loan rate over the opportunity cost, normalized by the loan rate factor \( 1 + i_i \).

The banking system chooses the amount of loans, \( b_{i+1} \), that maximizes its expected profit in period \( t \), \( E_i(\Psi) \), given in expression (5). Assuming that \( g(x) \) is uniformly distributed in the interval \([0, \varepsilon]\), the derived optimal \( b_{i+1} \) can be written as

\[ \bar{b}_{i+1} = \left[ \frac{\varepsilon(\cdot)(i_i - r_i)}{(1 + i_i)^2} \right] \tilde{A}f(k_{i+1}). \] \hspace{1cm} (7)

The loan offer is consistent with the observation that, for a given expected return, the representative bank does not monotonically increase the amount of credit as the lending rate increases. There is an amount the bank would not go beyond regardless of the lending rate.

**D. The Household: Decision Problem and Optimization**

The household enters period \( t \) with a predetermined amount of real currency balances, deposits, and debt—respectively, \( h_{m,t} \), \( z_t \), and \( b_t \). At the beginning of the period, its bank account is credited by the amount of a bank loan, \( b_{i+1} \), and by government deposit transfers, \( \tau_{i}^g \).\(^{15}\) The household receives also another part of government transfers, \( \tau_{i}^h \), in currency. It allocates its wealth among consumption of the currency good, \( c_{it} \), consumption of the deposit good, \( c_{2t} \), currency and deposit holding to carry into the next period, \( h_{m_{t+1}} \) and \( z_{t+1} \), respectively, and investment, \( k_{t+1} \). The output produced at the beginning of period \( t —

\(^{14}\) The derivation of (6) is presented in the appendix, Section II.

\(^{15}\) Bank loans granted in period \( t \) and carried into period \( t + 1 \).
\[ y = \tilde{A}f(k_t) \] — results from production decisions made at the beginning of period \( t-1 \). The function \( f() \) is increasing, strictly concave, continuous and twice differentiable, and it satisfies the Inada conditions.

The household's optimization problem is presented below. In period \( t \), the household chooses \( \{c_{1t}, c_{2t}, h_{1t}, z_{t+1}, k_{t+1}\} \) to maximize its lifetime utility subject to an intertemporal budget constraint (equation 9), a credit constraint (equation 12), and two liquidity constraints. Accordingly, there is a liquidity constraint for the currency good (equation 10), and another for the deposit good (equation 11). The deposit good is also purchased with credit money.

The household's decision problem can be written as a recursive equation:

\[
V(h_{ht}, z_t, k_t) = \max \{ U(c_{1t}, c_{2t}) + \beta V(h_{ht+1}, z_{t+1}, k_{t+1}) \}
\]

where \( \beta \in (0,1) \), \( U_1(c_{1t}, c_{2t}) > 0 \), \( U_2(c_{1t}, c_{2t}) < 0 \)

\[
U_{11}(c_{1t}, c_{2t}) < 0, \ U_{22}(c_{1t}, c_{2t}) > 0
\]

subject to

\[
c_{1t} + c_{2t} + h_{ht-1} + z_{t+1} + k_{t+1} = \tilde{A}f(k_t) + \frac{(1 + r_{t-1})z_t}{1 + \Pi_t} + \tau_t^h + \frac{h_{ht}}{1 + \Pi_t} + \tau_t^s - \frac{(1 - \theta_t)k_{t+1}}{1 + \Pi_t} + b_{t+1}
\]

(9)

\[
c_{1t} + \theta_t k_{t+1} \leq \frac{h_{ht}}{1 + \Pi_t} + \tau_t^h
\]

(10)

\[
c_{2t} + (1 - \theta_t)k_{t+1} \leq \frac{z_t}{1 + \Pi_t} + \tau_t^s + b_{t+1}
\]

(11)

\[
b_{t+1} \leq \bar{b}_{t+1}.
\]

(12)

Note that \( b_{t+1} \) is not a choice variable. In (12), \( b_{t+1} \) is the effective loan amount, which, added to the household's wealth, allows it to determine its consumption and assets holding.\textsuperscript{16}

\textsuperscript{16} A caveat to the model is the suggestion that rationing is built into the effective loan amount, limiting the comparative statics analysis to considerations of whether rationing is stiffer or not, depending on the direction the aggregate loan amount takes. The environment in which banks operates in low income developing countries makes it very likely that they almost always ration credit through either the number of loans or their size.
Let $\gamma_t$, $\lambda_t$, $\lambda_{2t}$, and $\lambda_{3t}$ be the Kuhn-Tucker multipliers for constraints (9), (10), (11), and (12) respectively.\footnote{The interpretation of the multipliers is the same as in Englund and Svensson (1988).} The first-order conditions and other derivations are specified in section III of the Appendix.

The following derived relationships are worthy of note:

$$\frac{\beta \gamma_t U_i(t_{t+2})}{1 + \Pi_{t+2}} + U_z(t_{t+1}) = U_i(t_{t+1}) \quad (13)$$

and

$$\beta^2 A \frac{f_k(t_{t+1})}{I + \Pi_{t+2}} U_i(t_{t+2}) = \theta_i(\cdot) U_i(t_{t+1}) + (1 - \theta_i(\cdot)) U_j(t_{t+1}) \quad (14)$$

where $U_j(t_{t+s})$ represents the marginal utility of good $j$ evaluated at time $t+s$; $j = \{1,2\}$, $s = \{0,1,2\}$.

In equation (13), the left hand side (LHS) represents the gain in utility from consumption of the deposit good in period $t+1$ and $t+2$, and the right hand side (RHS) represents the utility forgone owing to the reduction by one unit of the consumption of the currency good in period $t+1$. Suppose that the household decides to substitute consumption of one unit of the currency good in period $t+1$ with consumption of the deposit good in the same period. The household increases by one unit its deposits holding, which allows consumption of not only one unit of the deposit good in period $t+1$, but also a fraction $r$ of the same good in $t+2$ from the interest earned on deposits.

Equation (14) is the arbitrage condition between investment and consumption along the optimal path. Suppose that the household reduces its investment by one unit in period $t$. Its money holding increases by one unit, of which $\theta$ and $(1-\theta)$ units can be used for consumption of the currency good and the deposit good, respectively. The left hand side represents the loss in utility owing to forgone consumption in period $t+2$ that the proceeds from the investment would have made possible, while the RHS represents the gain in utility from consumption in period $t$.

**E. The Steady State Equilibrium**

A steady state has the following characteristics:

- constancy of all multipliers over time;
- $k_t = k$, $b_t = b$, $h_t = h$, $h_{t+1} = h_{t+1}$, $c_t = c$, $c_{2t} = c_2$, $z_t = z$, for all $t$;
- a constant growth rate of nominal high-powered money, that is, $H_{t+1} = (1 + \sigma)H_t$, for all $t$.

The constancy of $\sigma$ suggests that, since $h_{t+1} = \frac{(1+\sigma)h_t}{(1+\Pi)}$ and $h_{t+1} = h_t$,

$$\sigma = \Pi \quad (15)$$
and the money transfer to the household, \( \tau \), can be expressed as:

\[
\tau = \frac{\Pi}{1+\Pi} h_{h\epsilon} + \frac{\alpha \Pi}{1+\Pi} z.
\]  

The first term on the RHS is transfer in the form of currency while the second term represents transfer in the form of deposit.

Other equilibrium equations are derived in Section IV of the appendix.

Equations (13) and (14) can be rewritten as:

\[
\frac{U_2(c_1, c_2)}{U_1(c_1, c_2)} = \frac{1+\Pi - r\beta}{1+\Pi} \quad \text{and} \quad \beta^2 A_{f_k} = 1+\Pi - (1-\ell)\rho \beta.
\]  

Equations (17) and (18) represent, respectively, the marginal rate of substitution between the deposit and currency goods, and the optimal capital schedule. The latter is part of the equilibrium system. The values of the variables that constitute the macroeconomic equilibrium should solve the household’s problem and the banking system’s problem, and satisfy the government budget constraint and the market-clearing conditions. Details of the derivation, as well as of the equations of the steady state equilibrium, are in Section IV of the appendix.

**Comparative Statics—Investment**

**Results**

As a baseline (thereafter baseline scenario), the analysis neglects bank loans and considers the case where the fraction of investment financed with currency, \( \theta \), is a constant. Comparative statics results under the baseline scenario are derived from the equation of the marginal product of capital, equation (18).

Next, under scenario 1, the paper takes into account the assumption that \( \theta \) is not a constant, that is, \( \theta = \theta \nu, i, r, \Pi \). This scenario combines the marginal product of capital, equation (18), and the representative bank’s loan offer, equation (7), in a system of two endogenous variables, \( k \) and \( b \). Section V of the appendix presents the linearization of the two equations around the steady state, as well as the verification of the stability of the equilibrium. The coefficients of \( dk \) and \( db \) in the linearized equations are grouped into a matrix \( G \), whose determinant, \( \det G \) is negative. The following comparative statics results can be derived:

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\(^{18}\) Equation (18), which is key to the analysis carried out subsequently, holds even if \( b \) is a choice variable in the household’s decision problem.
Result 1—Capital is negatively related to the rate of inflation.

\[
\frac{dk}{d\Pi} = \frac{1}{\beta^2 \tilde{A} f_{kk}} \prec 0 \quad \text{and} \quad \frac{dk}{d\Pi} = \frac{1 + \theta \tilde{A} r}{\det \mathcal{G}} \prec 0.
\]

Expressions (19) and (20) are the outcomes under the baseline scenario and scenario 1, respectively. This result is consistent with Greene and Villanueva (1991), as well as Stockman (1981) and the like.

Result 2—Capital is negatively related to the share of investment to be financed with currency.

\[
\frac{dk}{d\theta} = \frac{r}{\beta \tilde{A} f_{kk}} \prec 0.
\]

Expression (21) is derived under the baseline scenario. To the extent that a high \( \theta \) is indicative of poor financial intermediation, this result is consistent with the notion that poor financial intermediation has a negative impact on investment. This result is in line with King and Levine (1993b).

Result 3—Capital is positively related to the deposit rate.

\[
\frac{dk}{d\rho} = \frac{(1 - \theta)}{\beta \tilde{A} f_{kk}} \succ 0, \quad \text{and} \quad \frac{dk}{d\rho} = -\beta (1 - \theta - \theta, r) \succ 0.
\]

Expressions (22) and (23) relate to the baseline and scenario 1, respectively. This result lends support to the McKinnon and Shaw hypothesis.

Result 4—The relationship between capital and the lending rate is ambiguous.

\[
\frac{dk}{d\iota} = \frac{br \theta}{\det \mathcal{G}}
\]

This expression, derived from scenario 1, suggests that changes in the lending rate influences changes in capital through their impact on the cash economy. If an increase in the lending rate boosts the cash economy through disintermediation, capital decreases. This result is in line with the nonmonotonic nature of the loan offer, emphasized in the modern literature on credit rationing.
Result 5—Capital is positively related to technological improvement and to improvement in the index of macroeconomic stability, institutions, and informational infrastructure.

\[
\frac{dk}{dA} = -\beta^2 \frac{Af_k}{\det G} > 0, \quad (25)
\]

\[
\frac{dk}{d\nu} = -\beta^2 \frac{Af_k}{\det G} > 0, \quad \text{and} \quad (26)
\]

\[
\frac{dk}{d\nu} = -\beta^2 \frac{Af_k + \theta \nu \beta}{\det G} > 0. \quad (27)
\]

Expression (25) is common to the two scenarios. Expression (26) is derived under the baseline scenario, while expression (27) relates to scenario 1.

Discussion

Under both the baseline scenario and scenario 1, investment and the rate of inflation are negatively related, as in most CIA models. This is the "Tobin effect" reversed. Intuitively, the negative impact of inflation on investment is explained by the increase in the cost of holding money, whose prior accumulation is required to purchase goods. This cost is exacerbated by the higher rate of currency holding the household needs to compensate for a reduction in the size of bank credit and curb a possible underinvestment. Indeed, when the rate of inflation increases, the representative bank reduces the amount it is willing to lend. The reduction in the size of bank loans props up the share of cash transactions.\(^\text{19}\)

The deposit rate, \(r\), has a positive impact on investment while the impact of the lending rate, \(i\), is ambiguous. It depends on the bank's loan offer. The necessary condition for an increase in the lending rate to have a positive impact on investment is a reduction in \(\theta\).

A positive impact will result only if the lending rate remains in the region of the loan supply where the latter is increasing in \(i\), that is, \(\forall i < i^*\), where \(i^* = \frac{Af(k)X}{b} - 1\). Note that \(i^*\) is the lending rate that maximizes the bank's expected profit.

Investment and technological change (as well changes in institutions and informational infrastructure) are positively related. A technological (institutional) improvement undeniably allows the marginal product of capital and the steady state level of capital to be higher. This

\(^{19}\) Palivos et al. (1993) consider the possibility of reinstatement of the Tobin effect. They suggest that an increase in the inflation (money growth) rate can cause a "sufficient" increase in the fraction of investment purchased on credit that would induce a decrease in the shadow cost of capital. The model presented in this paper does not allow such possibility since higher inflation is unambiguously associated with a reduction in bank credit.
result is in line with Mauro (1995) who find a robust positive relationship between a measure of the quality of institutions (bureaucratic efficiency) and investment and Grogan and Moers (2001) who find a positive relationship between institutional quality and both growth and investment.

**Comparative Statics—Bank Loans**

**Results**

The comparative statics results of the impact on bank loans of technology, inflation, deposit and lending rates, and information and macroeconomic stability are as shown below. These results pertain to scenario 1 only.

**Result 6**—Bank loans and inflation are negatively related.

\[
\frac{db}{d\pi} = \frac{\Lambda \tilde{A}f_r (1 + \theta \beta r)}{\det G} \leq 0.
\]  

(28)

**Result 7**—The relationship between bank loans and interest rates is ambiguous.

\[
\frac{db}{dr} = -\frac{\tilde{A}f(k)}{(1+i)^2} - b_k \left[ \frac{\beta (1 - \theta - \theta, r)}{\det G} \right] \quad \text{and}
\]

\[
\frac{db}{di} = \left[ \frac{\epsilon(1 - i + 2r)}{(1+i)^3} \right] + \left[ \frac{\Lambda \tilde{A}f_r \beta r \theta_i}{\det G} \right].
\]

(29) (30)

**Result 8**—Bank loans are positively related to macroeconomic stability, informational infrastructure, and institutions.

\[
\frac{db}{d\nu} = Af(k) \left[ \frac{\epsilon_v (i - r) \nu}{2(1+i)^3} + \Lambda \right] \geq 0.
\]

(31)

**Result 9**—Bank loans are positively related to technological improvement.

\[
\frac{db}{dA} = \Lambda v f(k) - \frac{A \beta^2 \Lambda f_k^2}{\det G} \geq 0.
\]

(32)

**Discussion**

Equation (28) shows that inflation has a negative impact on bank loans. A rise in the rate of inflation lowers the real return on loans, leading to reduced lending.
The relationship between the deposit rate and bank loans is ambiguous. It depends on many elements, including the subjective discount factor, $\beta$; the fraction of investment spending paid for with currency, $\theta$; and, through $\epsilon$, the index of macroeconomic stability and informational infrastructure, $\nu$. After some transformation, one finds that the deposit rate would have a positive impact on bank lending if the following relationship is satisfied:

$$\left|1 - \theta \right| > \frac{\beta \sigma f_{w} b}{(i - r) + \theta, r}.$$  \hspace{1cm} (33)

Expression (33) suggests that the lower $\theta$, the greater the likelihood that an increase in the deposit rate would have a positive impact on bank lending. Accordingly, the size of the cash economy may be too high for an increase in the deposit rate to induce more lending.

The impact of an increase in the lending rate on bank loans is ambiguous. As the loan offer reveals, the relationship between bank loans and the lending rate is nonmonotonic. The representative bank determines the maximum amount, $\bar{b}$, it would lend at a given lending rate. This maximum amount is determined in relation to the deposit rate and the range of the random part of the distribution of investment’s proceeds. The figures derived from the calibration of the model (Appendix, Section IV) illustrate how $\bar{b}$ varies with the lending rate, $i$, for given levels of the deposit rate, $r$, and development of informational infrastructure and macroeconomic stability, $\nu$.

As one would expect, technology and bank loans are positively related. Intuitively, a technological improvement increases the marginal product of capital and thereby the expected return on investment. As a result, the default risk is lowered, thus encouraging the representative bank to increase the amount it is willing to lend. Improvement in the index $\nu$ has an unambiguously positive impact on bank lending.

**Calibration of the Model**

Results of the calibration of the model are presented in section VI of the appendix. The calibration is somewhat naïve: it assumes that interest rates and inflation do not affect the rate of time preference. The primary purpose of the exercise is to illustrate that (i) the risk of default increases with the widening of the spread between the lending and the deposit rates; (ii) for a given level of informational infrastructure and macroeconomic stability, the wider the spread, the smaller the incremental loan amount induced by a 1 percentage point increase in the lending rate; and (iii) improvements in informational infrastructure play the role of positive productivity shocks and boost bank loans significantly.

Comparisons between the cases presented in the table in section VI of the appendix, indicate that for a given $\nu$, the wider the interest rate spread, the smaller the incremental loan amount induced by a 1 percentage point increase in the lending rate (figures in column 12; cases 1 and 2). Also, for a given interest rate spread, the higher $\nu$, the larger the incremental loan amount induced by a 1 percentage point increase in the lending rate (figures in column 12; cases 1 and 3).
Proposition 1. When lending rates increase in an environment where little or no effort has been made to improve macroeconomic stability, informational infrastructure, and institutional factors, optimal bank lending may improve only marginally or it may even decrease.

When the lending rate increases in the portion of the loan offer where \( b \) is increasing in \( i \), \( b \) is positive, \( \theta \) is negative, and \( \frac{db}{di} \) is positive. Nonetheless, if informational infrastructure and institutional factors have not improved enough, that is, \( \varepsilon \) remains too low, \( \frac{db}{di} \) may be marginal. The following observations provide insights into the proposition:

- Considering that the spread between the lending and the deposit rates is indicative of the risk of default (see equation (6)), in an economy characterized by a low \( \varepsilon \) and a wide spread, a rational representative bank would not increase very much the amount it lends at higher lending rates.

- From expression \( \bar{x} = \frac{(1+i)b}{Af(k)} \) where \( \bar{x} \) is the threshold value of the random variable \( X \) below which the household defaults— it is clear that the higher \( i \), the higher \( \bar{x} \). When the lending rate increases from \( i_1 \) to \( i_2 \), the threshold increases as well, from \( x_1 \) to \( x_2 \). For a given level of institutions and informational infrastructure, \( \nu \), there is a high likelihood that the new threshold falls beyond \( \varepsilon = \varepsilon(\nu) \) — the actual range of the random variable \( X \). The implication is that \( \nu \), as well as the associated \( \varepsilon \), could be unsuitable for high lending rates.

- When \( \varepsilon \) is too low, the loan offer could be quite flat; movements along the loan offer associated with higher lending rates will not bring about significant increases in loan amounts from a rational bank. However, when there is a shift in the loan offer resulting from a higher \( \varepsilon \), increased lending rates will be compatible with significantly higher levels of bank loans.

This proposition underscores the role that institutions, informational asymmetries, and macroeconomic stability play in optimal bank lending. Given the environment in which banks operate in LDCs, the likelihood of a meaningful positive \( \frac{db}{di} \) is very small. Referring to bank loans in the United States when interest rates were controlled under Regulation Q — as opposed to the post-liberalization period — Allen (1987) suggests that the difference between disequilibrium and equilibrium rationing may not always be clear-cut.\(^{20}\) A rational

\(^{20}\) Regulation Q refers to a legislation under which the Federal Reserve System of the United States had the power to set maximum interest rates that banks could pay on savings deposits.
representative bank is well aware that the borrowers’ risk characteristics and their future activities may be affected by higher lending rates in a way that may undermine profits. Although the imperfections of risk markets are present in all economies, these “imperfections are particularly strong in LDCs, and can have particularly strong effects.”\textsuperscript{21} In LDCs, when higher lending rates in a liberalized environment do not lead to increased optimal lending, policymakers should focus on upgrading institutional factors and informational infrastructure and promoting macroeconomic stability. In this connection, a number of researchers have expressed concerns about the efficacy of increased interest rates in LDCs.\textsuperscript{22}

Following this analysis of the impact of changes in interest rates on bank loans, an examination of the comparative statics results derived earlier would help ascertain whether changes in policy parameters influence investment solely through bank loans.

\textit{Proposition 2. Interest rates may be relevant in investment equations even in the presence of a credit constraint.}

\textbf{Proof.} Let the difference between the impact of an increase in the deposit rate [lending rate] on investment and its impact on bank loans be computed from equations (29) and (24) [(30) and (25)]. Subtracting (24) from (29) yields:

\begin{equation}
\frac{dk}{dr} - \frac{db}{dr} = -\beta(\theta - \theta r)(1 - \Lambda \widetilde{Af}_k) + \frac{\widehat{Af}(k)}{(1 + i)^2} = 0. \tag{34}
\end{equation}

Also, subtracting (30) from (25) yields:

\begin{equation}
\frac{dk}{di} - \frac{db}{di} = \frac{\theta_p \beta r}{\det G} (1 - \Lambda \widetilde{Af}_k) \left[ \frac{s(1 - i + 2r)}{(1 + i)^3} \right]. \tag{35}
\end{equation}

For low values of $\varepsilon$, $\Lambda$ is smaller than 1 and $\Lambda \widetilde{Af}_k$ is likely to be smaller than 1. It can be seen from expressions (34) and (35) that changes in interest rates affect investment not only through bank credit but also through portfolio shifts that induce changes in investment. An important feature of the comparative statics results is that, although the impact of higher deposit rates on bank loans is ambiguous, their impact on investment is unambiguously positive, underscoring the importance of portfolio shifts. With regard to lending rates, when $\varepsilon$ is low, higher lending rates may not improve bank loans significantly in the rising portion of the loan offer; however, associated changes in spreads and ensuing portfolio shifts may have a significant impact on investment. Moreover, when there is a substantial increase in the lending rate that pushes it into the region of the loan offer where the latter is decreasing in interest rates, the adverse impact on both bank loans and investment could be significant. Hence, the assumption that interest rates are irrelevant in investment equations in the presence of a credit constraint is not always be justified.

\textsuperscript{21} Stiglitz (1993, p.75)

\textsuperscript{22} For instance, McKinnon (1989) suggests that raising interest rates above certain limits in “immature” bank-based capital markets can bring about undue adverse selection among borrowers and undue moral hazard in the banks themselves.
III. CONCLUDING REMARKS

The theoretical model presented is a simple general equilibrium model with rationing in the market for bank loans. The relationships among variables are derived from private sector optimal behavior, given interest rates and the government's issuance of fiat money. By including financial intermediation, the model, unlike Stockman's CIA, explicitly gives interest rate explicitly its place in the money demand function. From the model, the impact of monetary policy parameters on bank loans and investment is analyzed. Specifically, the model shows how bank credit and investment respond to inflation and to the lending and deposit rates of interest.

Building on Jaffee and Modigliani (1969), Keeton (1979), Allen (1987), and Jaffee and Stiglitz (1990), this paper contends that the nonmonotonic loan offer derived from models like the Freimer-Gordon can be attributed to asymmetric information, although not specifically modeled as such. The limited liability feature built into the loan contract, together with uncertainty and asymmetric information, makes the level of development of institutions and informational infrastructure a salient aspect of the lending process.

An important finding of the paper is that changes in inflation or interest rates do not affect investment solely through bank loans; there is an extra impact attributable to portfolio shifts. The model confirms some earlier findings in the literature regarding the relationship between inflation and capital expansion. As in Stockman (1981), Abel (1985) and Palivos et al. (1993), it suggests that capital expansion is negatively related to the rate of inflation. Another important finding of the paper is that the lending and deposit rates are two parameters that can have impacts of opposite signs on either investment or bank credit. In this vein, the model unambiguously confirms the McKinnon and Shaw hypothesis as to the positive effects of higher deposit rates on investment. As far as lending rates are concerned, the model lends support to the literature on asymmetric information. It suggests that increased lending rates do not necessarily lead to increased optimal bank credit. The spread between the lending and the deposit rates is indicative of default risk and is negatively related to the incremental loan amounts resulting from higher lending rates. Moreover, the model suggests that the lending rate may be relevant to optimal capital formation. Agents who have access to bank loans do not need to accumulate as much money in currency as those who cannot borrow to finance their spending. By incorporating this reality in the model, the paper finds that the lending rate influences long-run capital through bank loans, as well as the share of capital to be financed with currency. To the extent that a high rate of currency holding is indicative of an underdeveloped financial system, the analysis implies that developing the financial system will promote capital formation.

The implication of the model for empirical analyses is that, since the lending and deposit rates can have impacts of opposite signs on investment, they both may be relevant in investment equations even in the presence of a credit constraint. Alternatively, the spread between these two rates, which is indicative of the risk of default on bank credit, is very likely to be relevant.
From a policy perspective, the results suggest that, although both the lending and deposit rates are determinants of investment, raising them does not necessarily lead to significant increases in investment. It is important first to create an environment in which interest rate policy can contribute to enhancing investment. In particular, the reduction of the magnitude of currency transactions is an important contributing factor to the success of interest rate reforms. Moreover, a sound interest rate policy should be part of a package including macroeconomic stability and actions to mitigate the effects of credit market imperfections. Even though the model hypothesizes the rate of currency holding, $\theta$, to be a function of interest rates, inflation, and the level of development of informational infrastructure and macroeconomic stability, the paper acknowledges that $\theta$ depends on many other factors, which vary from country to country. Policymakers can find ways to encourage the development of information mechanisms, promote sound financial institutions, decrease uncertainty about the convertibility of deposits into currency, and influence many other country-specific factors that promote the cash economy. Without these preconditions, higher interest rates would not significantly increase the share of investment financed through bank credit and may jeopardize the profitability of credit-financed investment. The development of institutional and informational structures and the promotion of macroeconomic stability would encourage banks to finance more and/or innovative investment projects instead of confining their resources in low-risk investment. A corollary policy implication of the model is that it would be useful for policymakers in LDCs to identify the factors that drive the spread between the lending and deposit rates, as a widening of this spread has an adverse impact on the availability of bank credit and on investment.
I. List of Symbols Used in the Model

\( \tilde{Af}(k) \): production function
\( t \): time subscript
\( B_t \): nominal amount of bank loans carried into period \( t \)
\( H_t \): nominal monetary base at \( t \)
\( H_{hs,t} \): nominal household’s currency holding carried into period \( t \)
\( inv_t \): real investment in period \( t \)
\( Z_t \): nominal deposits holding from period \( t - 1 \) into period \( t \)
\( K_t \): nominal capital acquired in period \( t - 1 \) carried in time \( t \)
\( b_t \): real amount of bank loans during period \( t \)
\( c_1 \): real cash good consumption
\( c_2 \): real deposit good consumption
\( \sigma \): rate of outside money growth
\( g \): real government expenditures on goods
\( h \): total real monetary base
\( h_{hs} \): real amount of currency held by the household
\( i \): nominal lending rate
\( r \): nominal deposit rate
\( k \): real capital stock
\( z \): real deposits outstanding
\( P \): price level
\( U \): utility function
\( \alpha \): banks' reserve ratio
\( \beta \): discount factor
\( \delta \): The rate of time preference
\( v \): index of the level of institutional and informational infrastructure and macroeconomic stability.
\( \theta \): share of capital financed with currency
\( \Pi \): inflation rate
\( \tau \): money transfer from the government
\( \Psi \): bank's profit
II. Derivation of Expression (6) Shown in the Text

\[ \frac{\partial E(\Psi)}{\partial b} = (1 + i) \left[ \int_0^\infty g(x)dx \right] - (1 + r) = 0. \]

Adding and subtracting \( \int_0^\infty g(x)dx \) to the term in brackets yields the following:

\[ (1 + i) \left[ \int_0^\infty g(x)dx + \int_0^\infty g(x)dx - \int_0^\infty g(x)dx \right] = 1 + r. \]

The sum of the first two terms in brackets being 1, the expression can be rewritten as

\[ \int_0^\infty g(x)dx = 1 - \frac{1 + r}{1 + i}, \text{ or} \]

\[ \int_0^\infty g(x)dx = \frac{i - r}{1 + i}. \]

III. Optimization of the Household’s Decision Problem

The first-order conditions of the consumer problem are as follows:

\[ U_1(c_{1t}, c_{2t}) = \gamma_{it} + \lambda_{it} + \lambda_{3t}, \]

(A1)

\[ U_2(c_{1t}, c_{2t}) = \gamma_{it} + \lambda_{2t} + \lambda_{3t}, \]

(A2)

\[ \beta V_1(c_{1t}) = \gamma_{it} + \lambda_{3t}, \]

(A3)

\[ \beta V_2(c_{1t}) = \gamma_{it} + \lambda_{3t}, \]

(A4)

\[ \beta V_3(c_{1t}) = \gamma_{it} + \theta_t \lambda_{it} + (1 - \theta_t) \lambda_{2t} + \lambda_{3t}, \]

(A5)

\[ \lambda_{it} \left( \frac{h_{it} + \tau_i}{1 + \Pi_t} - \theta_t k_{it+1} - c_{it} \right) = 0, \quad \lambda_{it} \geq 0, \]

(A6)

\[ \lambda_{2t} \left( \frac{z_t + \tau_t}{1 + \Pi_t} - \theta_t k_{it+1} + b_{it+1} - c_{2t} \right) = 0, \quad \lambda_{2t} \geq 0, \]

(A7)

\[ b_{it+1} - \bar{b} \leq 0, \quad \lambda_{3t} \geq 0, \text{ and } \]

(A8)

the budget constraint.
The combination of the first-order conditions with the indirect utilities of currency outside banks, deposits, and capital yields the following Euler equations:

\[ U_1(c_{1t}, c_{2t}) = \lambda_{3t} + \lambda_{4t} + \gamma_t, \]  
\[ U_2(c_{1t}, c_{2t}) = \lambda_{2t} + \lambda_{3t} + \gamma_t, \]  
\[ \beta \frac{\gamma_{t+1} + \lambda_{4t+1} + \lambda_{3t+1}}{1 + \Pi_{t+1}} = \gamma_t + \lambda_{3t}, \]  
\[ \beta \frac{(1 + r_t)(\gamma_{t+1} + \lambda_{4t+1}) + (\lambda_{2t+1})}{1 + \Pi_{t+1}} = \gamma_t + \lambda_{3t}, \text{ and} \]  
\[ \beta \left( A f_k(k_{t+1}) \right) (\gamma_{t+1} + \lambda_{3t+1}) = \gamma_t + \theta_t \lambda_{4t} + (1 - \theta_t) \lambda_{2t} + \lambda_{3t}. \]  

By analogy to Hutchison (1995), (A9), (A10), and (A11) are asset-pricing equations, respectively, for cash, deposit and capital.

(A1) into (A9) yields the following:

\[ \gamma_{t+1} \lambda_{3t} = \frac{\beta}{1 + \Pi_t} \left( U_1(c_{1t+1}) \right). \]  

(A2), (A12) into (A10) yield

\[ \beta \frac{U_1}{1 + \Pi_t} = \frac{\beta}{1 + \Pi_t} (1 - r_t) + U_2(c_{2t}) - \beta U_1(c_{1t}). \]  

(A13)

(A1), (A2) and (A12) into (A11) yield

\[ \beta^2 \frac{\tilde{A} f_k(k_{t+1})}{1 + \Pi_t} U_1(c_{1t+1}) = \theta_t U_1(c_{1t}) + (1 - \theta_t) U_2(c_{1t}). \]  

\[ \beta^2 \frac{\tilde{A} f_k(k_{t+1})}{1 + \Pi_t} U_1(c_{1t}) = \theta_t U_1(c_{1t}) + (1 - \theta_t) U_2(c_{1t}). \]  

(A14)

**IV. Derivation of the Steady State Equilibrium**

Let derive the macroeconomic equilibrium equations.

In a steady state, expression (A13) becomes

\[ \frac{U_2(c_1, c_2)}{U_1(c_1, c_2)} = \frac{1 + \Pi - r \beta}{1 + \Pi}, \]  

(A15)
which is the marginal rate of substitution between the deposit and the currency good.

Equation (A14) becomes

\[ \beta^2 \frac{\tilde{Af}_c}{1 + \Pi} U_1 = \theta U_1 + (1 - \theta) U_2. \]  

(A16)

Dividing both sides of (A16) by \( \frac{U_1}{1 + \Pi} \), after having dropped the time subscripts, and using (A15) yields the following:

\[ \beta^2 \tilde{Af}_c = 1 + \Pi - (1 - \theta) \beta. \]  

(A17)

Besides the five first-order conditions involved in the determination of (A17), the equilibrium should also take into account the four constraints on the household's side, the government budget constraint, the bank's loan offer, and the market-clearing conditions.

Taking into account the government's budget constraint, equation (3), and including equation (16) in the household's budget constraint yields the following:

\[ c_1 + c_2 + k = \tilde{Af}(k) \quad \text{and} \]

\[ b = \frac{z(r + c_{1} \Pi - \Pi)}{i - \Pi}. \]  

(A18)

(A19)

The equality between the banking system's assets and liabilities is given by

\[ \bar{b} = (1 - \alpha)z. \]  

(A20)

**Equations of the Steady State Equilibrium**

Equations (A17)–(A20), together with the following equations, describe the steady state equilibrium. For convenience, some expressions referred to earlier are repeated below:

\[ \bar{b} = \left[ \frac{\varepsilon(i - r)}{(1 + i)^2} \right] \tilde{Af}(k), \]

\[ \sigma = \Pi, \]

\[ \tau = \frac{\sigma}{1 + \Pi} (h_{ks} + \alpha \varepsilon)^{23}, \]

23 In line with expression (16) in the text.
\[ b = \frac{z(r + \alpha \rho - \Pi)}{i - \Pi}, \]

\[ b = \bar{b}, \]

\[ c_1 + c_2 + k = \tilde{A} f(k), \]

\[ c_1 + \theta k = h_{kr}, \]

\[ c_2 = \frac{z(1 + \alpha \Pi)}{1 + \Pi} + b - (1 - \theta)k, \text{ and} \]

\[ \varepsilon = \varepsilon(v). \]

The endogenous variables are \( c_1, c_2, k, h_{kr}, b, \) and \( z \) while the exogenous variables are \( \Pi, r, i, \Lambda, \) and \( v. \)

\[ V. \text{ Stability of the Steady State} \]

Totally differentiating equations (18) and (7) in the text yields the following:

\[
\begin{bmatrix}
\beta^2 \tilde{A} f_{ik} & 0 \\
-\Lambda \tilde{A} f_{ik} & 1
\end{bmatrix}
\begin{bmatrix}
dk \\
db
\end{bmatrix} =
\begin{bmatrix}
-\beta^2 -v f_{k} & 1 + \theta_1 \beta r & -\beta(1 - \theta - \theta, r) & \beta r, r & -\beta^2 A f_{k} + \theta, r \beta \\
\Lambda v(f(k)) & 0 & \frac{\tilde{A} f(k)}{(1 + i)^2} & \frac{\tilde{A} f(k)}{(1 + i)^2} & A f(k) \left( \frac{\varepsilon_v (i - r) v}{(1 + i)^2 + \Lambda} \right)
\end{bmatrix}
\begin{bmatrix}
dA \\
d\Pi \\
dr \\
di \\
dv
\end{bmatrix}
\]

where \( \Lambda = \frac{\varepsilon (i - r)}{(1 + i)^2}. \)

Let \( G \) represent the matrix that premultiplies \( [dk \quad db], \) \( DetG \) and \( TrG \) its determinant and trace, respectively:

\[ DetG = \beta^2 \tilde{A} f_{ik} < 0 \text{ and} \]

\[ TrG = 1 + \beta^2 \tilde{A} f_{ik}. \]
Let \( \eta_1 \) and \( \eta_2 \) be the two eigenvalues of the matrix \( G \). \( \eta_1 \) and \( \eta_2 \) are the roots of the polynomial 
\[
P(\eta) = \eta^2 - \text{Tr} G \eta + \text{Det} G = 0
\]
and
\[
\Delta = \left(1 + \beta^2 \widetilde{Af}_{kk}\right)^2 - 4 \beta^2 \widetilde{Af}_{kk} = \left(1 - \beta^2 \widetilde{Af}_{kk}\right)^2.
\]

Since the discriminant \( \Delta \) is positive, the two characteristic roots are real. It is known that \( \text{Det} G = \eta_1 \eta_2 \) and \( \text{Tr} G = \eta_1 + \eta_2 \). Since \( \text{Det} G \) is negative, the characteristic roots have opposite signs. Saddlepath stability requires that \( \text{Tr} G \) be positive and that the negative root lies within the unit circle \((-1,1)\). Given that the absolute value of the positive root is one, a positive \( \text{Tr} G \) entails that the absolute value of the negative characteristic root, \( \beta^2 \widetilde{Af}_{kk} \), is lower than 1. Therefore, the steady state is a saddle, with \( \left| \beta^2 \widetilde{Af}_{kk} \right| < 1 \).

**VI. Calibration of the Model**

The following relations are used in the calibration to derive the values shown in the table presented below:

- \( y = \widetilde{A}k^\nu \) is the non-random part of the production function.
- \( \varepsilon^*y \) in column (6) is the investment outcome—\( Q(k, X) = \widetilde{Af}(k)X \)—assuming that the random variable \( X \) takes its maximum value, \( \varepsilon \). As stated earlier, \( \varepsilon = \varepsilon(\nu) \).
- \( \varepsilon = 0.8 + 0.3\nu \); where \( \nu \) varies between 0 and 5, with 5 representing the best environment.
- \( \theta = 0.7 - 0.083\nu - \frac{\Delta b}{k} \).

The choice of coefficients in the expression \( \theta \) is guided by the observation that, in economies with developed institutions and informational infrastructures, such as the United States and other industrialized countries, roughly 20 percent of households remain liquidity constrained. Accordingly, for \( \nu \) equal 5, \( \theta \) will be in the neighborhood of 0.2. In low-income developing countries, where \( \nu \) is far below 5, \( \theta \) will be higher.

\( k^* \) in column (10) is the representative household’s optimal capital derived from equation (18) in the text.

\[
k^* = \left[ \frac{1 + \Pi - \beta r (1 - \theta)}{\beta^2 a \widetilde{A}} \right]^{\frac{1}{\alpha - 1}}, \text{ while}
\]

\( b \) in column (11) is derived using equation (7) in the text, with

\[
r = 0.06 ; \delta = 0.35 ; \beta = 0.7407 ; \Pi = 0.05 ; a = 0.6 ; A = 1.1
\]

The high rate of time preference and the associated discount factor are consistent with the institutional problems prevailing in most developing countries. The conclusions of the numerical exercise will not be affected by changes in the rate of time preference.
Table A1. Calibration Results

<table>
<thead>
<tr>
<th>$i$</th>
<th>$1+i$</th>
<th>$i-r$</th>
<th>$\frac{i-r}{1+i}$</th>
<th>$e/(e)^*$</th>
<th>$y$</th>
<th>$\nu$</th>
<th>$\bar{A}$</th>
<th>$k^*$</th>
<th>$\bar{b}$</th>
<th>$\Delta \bar{b}$</th>
<th>$\frac{\Delta \bar{b}}{k}$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
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### Case 1: initial $i = 0.15$; $\nu = 1.2$

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<th>$i-r$</th>
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<th>$e/(e)^*$</th>
<th>$y$</th>
<th>$\nu$</th>
<th>$\bar{A}$</th>
<th>$k^*$</th>
<th>$\bar{b}$</th>
<th>$\Delta \bar{b}$</th>
<th>$\frac{\Delta \bar{b}}{k}$</th>
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<th>$\frac{\Delta \bar{b}}{k}$</th>
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<th>$y$</th>
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References


