Timing of International Bailouts

Se-Jik Kim
This paper proposes that international rescue financing should not be provided to a country where a crisis first occurs, but rather to any country that suffers a subsequent crisis. Such a timing-based lending facility can be Pareto-superior to both laissez-faire and existing international crisis lending facilities, when domestic governments have more information on their own economies than does the international lender of last resort. The new facility mitigates moral hazard owing to information asymmetry by not rescuing the first-hit country. At the same time, it limits crisis contagion by rescuing countries in subsequent crises. Even in the presence of common shocks, the timing-based facility can reduce global risks of crisis because it induces countries to undertake greater crisis-prevention efforts so as not to become the first country hit.

JEL Classification Numbers: F33, F34

Keywords: International lender of last resort, crisis contagion, moral hazard, information asymmetry

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1 I am very grateful to Olivier Jeanne, Simon Johnson, Allan Meltzer, Ashoka Mody, Jaewoo Lee, and Jeromin Zettelmeyer for valuable comments.
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I. INTRODUCTION

This paper proposes a new international rescue lending facility based on the timing order of crises. It also shows that under information asymmetry, the new timing-based facility can function more effectively than both laissez-faire and existing international crisis-lending facilities in simultaneously resolving the problems of moral hazard and crisis contagion (and even in resolving a coordination problem arising in the case of common shocks and vulnerabilities).

When financial crises swept over Asia in 1997 and 1998, following the Mexican crisis of 1994–95, the international community came to recognize that forestalling new crises was now more important than resolving the ones already taking place. In particular, traditional lending facilities, such as the Stand-By Arrangement of the International Monetary Fund (IMF), were blamed by some for focusing on bailing out countries that were already in the midst of crises rather than preventing crises and, hence, raising moral hazard despite the IMF’s role in abating contagion of crises across countries (see, e.g., Meltzer, 1997; Calomiris, 1998; Fischer, 1998 and 1999; Eichengreen, 2000; Jeanne and Wyplosz, 2001; Krueger, 2002; and IFIAC, 2000). Accordingly, there was a marked growth in demand for a new lending facility aimed at crisis prevention to obviate future threats to the stability of the international financial system as compared with such traditional lending facilities for crisis resolution.2

Given these circumstances, the IMF adopted the Contingent Credit Lines (CCL) as a lending facility for crisis prevention in 1999, but the CCL turned out to be ineffective owing to information problems. Under the CCL, only the countries that met certain prequalifications were eligible for financial assistance in the event of a crisis. So countries were expected to pursue sound economic policies and financial systems to meet the prequalifications; that would help to contain the contagion of crises and, at the same time, abate moral hazard, thus minimizing global risks. However, not one member country applied for the CCL, which was terminated in 2003, suggesting that the initial demand for the facility was being eclipsed by its potential problems. Particularly notable was the problem owing to information asymmetry between countries and the international crisis.

2 New ways of debt restructuring have also been in high demand. Collective-action clauses and a sovereign debt restructuring mechanism have been proposed as solutions to the restructuring problem of collective representation (see Eichengreen and Mody, 2000a and 2000b; Eichengreen, 2000; Krueger, 2002; and Haldane, Penalver, Saporta, and Shin, 2003). Lerrick and Meltzer (2003) also propose that the IMF provide a cash-support bid to insolvent sovereign borrowers at a discount from the government’s minimum offer on restructured value of debt (while providing liquidity support, tied to preconditions, at a penalty rate to solvent but illiquid borrowers).
Given that only the countries that met certain prequalifications were eligible to use the CCL, a critical condition had to be satisfied for the CCL to effectively prevent both moral hazard and crisis contagion; that is, accurate, near-perfect monitoring of the international crisis lender on the countries. In reality, however, substantial information asymmetry exists between residents and foreigners, which may hinder perfect monitoring by international organizations. In such a situation, the CCL may have been effective in containing crisis contagion but ineffective in deterring moral hazard.

Since the CCL turned out to be ineffective, a question arises: is there any other way to forestall new crises and, at the same time, resolve the ones already occurring? Is there any way to resolve both moral hazard and crisis contagion at the same time, particularly in the real world of information asymmetry?

This paper explores an approach that would simultaneously resolve the problems of moral hazard and crisis contagion, and, hence, effectively both prevent new crises and resolve the ones already taking place. More specifically, it proposes a new lending facility that would differentiate rescue financing to different countries by the order in which crises occur. Under this facility, which focuses on the timing of crises, rescue financing would not be provided to the country where the crisis first began but rather to any country faced with a crisis within a short period after the initial crisis occurred.

The key ideas underlying this new facility, particularly for the case of crisis contagion, are twofold. First, the facility differentiates rescue financing to different countries based on whether a country is more likely to be the one that originates the crisis or not; it aims at not rescuing the crisis originator but the victims of contagion. So it can be likened to holding the house that initiated the fire responsible but rescuing the houses that caught fire from the other houses. By doing so, the facility provides solid incentives for countries to prevent crises and stops a crisis, if one breaks out, from spreading further.

Second, the facility distinguishes between the crisis originator and the victims of contagion based on the timing order of crises. Regarding the point in time, the country where the

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3 There were some other problems in implementing the CCL. For example, applying for the CCL could have been interpreted as a signal of weakness of the economy, and so no country would have wanted to apply (see, for example, IFIAC, 2000; and De Gregorio, Eichengreen, Ito, and Wyplosz, 1999).

4 Throughout this paper, I use the terms “international lender of last resort” and “international crisis lender” interchangeably. Jeanne and Wyplosz (2001) use the term “lender of last resort” only for a situation of a self-fulfilling crisis, while many other authors (for example, Fischer, 1999; Calomiris, 1998; and Lerrick and Meltzer, 2003) use the term more broadly and suggest that the IMF could play the role of an international lender of last resort even without providing unlimited liquidity.
crisis erupted first is very likely to be the crisis originator, and the ones having crises after
the first are more likely to be the infected. This facility uses the potential time lag between
the crises in the crisis originator and the infected countries. In this regard, it can be
likened to identifying the house that originated the fire and the houses that caught fire
from neighboring houses by determining the timing order of the fire. The intuition behind
this method of identification is straightforward. If one house was on fire and then, half an
hour later, a neighboring house started burning, the chance that fire spread from the latter
to the former is almost nil, while the chance of the fire having spread the other way around
is very high.

The strength of this proposed facility is that the identification depends on the simple and
objective rule that is based only on the timing of crises, which is observable and verifiable
by anybody. Therefore, the identification under this new facility, unlike existing
international crisis lending facilities, does not have to depend on the monitoring capability
or the discretional judgment of the international crisis lender. 5

To develop the theory more formally, this paper proposes a simple model of crisis contagion
and moral hazard (owing to rescue financing) under information asymmetry. In particular,
the model assumes crisis contagion, broadly defined as the transmission of crises across
countries, following a large literature on crisis contagion (see, for example, Eichengreen,
Rose, and Wyplosz, 1996; Glick and Rose, 1998; Kaminsky and Reinhart, 2000; Allen and
Gale, 2000; Jeanne, 2000; Calvo and Mendoza, 2000; Goodhart and Huang, 2000; Forbes
and Rigobon, 2002; and Kodres and Pritsker, 2002). This assumption captures the fact
that the bulk of the empirical literature suggests that crisis contagion is systematically
present in the data (Kaminsky, Reinhart, and Végh, 2002). For example, using data for
two different currency crises during the 1970s through the 1990s, Glick and Rose (1998)
find that currency crises affect countries tied together by international trade. Based on an
analysis of Argentina after the Mexican crisis and of Indonesia after the crisis in Thailand,
Kaminsky and Reinhart (2000) suggest that crisis contagion transmitted through financial
linkages is important. 6

In addition, the model assumes that there are two types of crises depending on the origin

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5 Gai, Hayes, and Shin (2001) show how critically the efficacy of crisis management by
the IMF depends on the quality of its monitoring of debtor countries. Under imperfect
monitoring, “rules” offer an advantage over “discretion” by protecting against policymakers’
arbitrary decision making and resulting policy blunders (see, for example, Stokey, 2002).

6 Most empirical studies suggest evidence of excess comovement in asset returns but indi-
cate more limited consensus on whether this comovement increases after a country is hit by
crisis. Forbes and Rigobon (2002) find no significant increase in cross-country linkages after
a shock to one country, while Kaminsky and Schmukler (1999) find asymmetries between
the effects on asset prices of good and bad news from elsewhere.
of the crisis: one is autonomous crisis that occurs internally because of weaknesses in the domestic economy, and the other is imported crisis, which is the outcome of crisis contagion from foreign countries. The chances of a country’s having an autonomous crisis can be significantly lowered by its own voluntary crisis-prevention efforts (for example, implementation of structural reforms in financial and corporate sectors and sound macroeconomic policies), though the chance is not fully eliminated. The chance of a country’s having an imported crisis after another country has already experienced a crisis can be reduced when the international crisis lender bails out the crisis country. In this situation, the crisis lending by the international crisis lender may contain crisis contagion but raise borrower countries’ moral hazard (by reducing the country’s crisis-prevention efforts).

Finally, the model assumes information asymmetries between the government of a country and the international crisis lender. The international crisis lender cannot accurately observe each country’s crisis prevention efforts. Nor can it accurately tell whether a crisis that hit a country originated there or was imported.

Within the model described above, several important conclusions follow. First, both laissez-faire and existing lending facilities could be ineffective in simultaneously resolving the problems of moral hazard and crisis contagion. The laissez-faire approach, in which the international crisis lender does not provide rescue financing to any country, could be effective in containing moral hazard. Nevertheless, it would be ineffective in abating crisis contagion. Meanwhile, traditional crisis lending facilities of the IMF such as Stand-By Arrangements could effectively prevent crisis contagion. However, the traditional lending facilities tend to induce a moral hazard problem, since countries, which expect to receive rescue financing in the event of crisis, would not exert much effort to prevent crises. Furthermore, given information asymmetries, the CCL may be as ineffective as other...
traditional IMF crisis financing facilities in dealing with the problem of moral hazard.

Second, the new lending facility can effectively abate crisis contagion without inducing moral hazard under information asymmetry. The new facility based on the timing order of crises allows the international crisis lender to distinguish an autonomous crisis from an imported crisis even under information asymmetry. By not rescuing the country hit first, the timing-based crisis-lending facility can penalize the country in an autonomous crisis (which has a high chance of not having made adequate crisis-prevention efforts) and, hence, deter the moral hazard problem as effectively as the laissez-faire approach. By rescuing the countries hit later, the new facility could assist the countries that are more likely to have been hit by imported crises and deter crisis contagion as effectively as the existing lending facilities of the IMF. In this way, the new facility could overcome the shortcomings of laissez-faire and existing lending facilities.

Third, when both problems of moral hazard and crisis contagion are serious, the new lending facility can be Pareto-superior to both laissez-faire and existing crisis lending facilities. If there is no crisis contagion, laissez-faire could be the best approach. If the moral hazard problem is negligible, the existing lending facilities of the IMF would induce the optimal outcome. However, when both problems of moral hazard and crisis contagion are serious, both laissez-faire and existing lending facilities can be suboptimal: laissez-faire is not effective in deterring crisis contagion and existing lending facilities are not effective in containing moral hazard. Therefore, a new timing-based facility that would overcome the shortcomings of the other approaches can provide the highest world welfare.

The model used in this study can be adapted to accommodate three varied situations: a case in which a string of crises are generated by high common vulnerabilities to a common external shock, rather than by crisis contagion; a case in which countries may reverse the timing order between autonomous and imported crises; and a case in which timing-based lending induces greater crisis-prevention efforts than laissez-faire. These variations, however, do not alter the results of this study on the effectiveness of the timing-based lending facility. For example, a variant model shows that even for a common shock, the timing-based lending facility can be Pareto-superior to both laissez-faire and existing lending facilities. The intuition behind the result is straightforward. Suppose that the common vulnerability (or the risk of global or regional crises in the event of a common shock) could be substantially reduced only if all the countries simultaneously made their own efforts to prevent crises. Given the situation, a coordination problem may arise under both laissez-faire and existing facilities: since all the crisis countries are treated the same regardless of their crisis-prevention efforts, individual countries will not make strong efforts to prevent a crisis. In contrast, the timing-based lending facility, by penalizing the first

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9 Mody and Taylor (2003) suggest that country-specific movements of the exchange market pressure index (EMPI) can be well explained by common vulnerability indicators.
crisis country, can promote competition among borrower countries not to become the first one. The competition might then induce all the countries to voluntarily upgrade their prevention efforts to the optimal level for the world (or the region) as a whole, which would help resolve the coordination problem.

This paper is organized as follows. Section II proposes a new crisis-prevention facility. Section III presents a simple model as an analytical framework. Sections IV and V analyze the cases of the laissez-faire and existing crisis-lending facilities, respectively. Section VI explores the new timing-based facility. Section VII discusses some extensions of the model. Section VIII concludes.

II. PROPOSAL FOR A NEW CRISIS-PREVENTION FACILITY

In this section, I propose a new lending facility for crisis prevention, entirely different from existing international lending facilities including the Stand-By and the CCL of the IMF and their basic frameworks.

In concrete terms, the paper proposes a two-step crisis-resolving lending facility that differentiates rescue financing by the timing order of crises. More specifically, the new facility operates based on the following rules:

(1) no country where a crisis first occurs (after a considerable period of time, say more than a year, from the previous international financial crisis) \(^{10}\) shall receive rescue financing; and

(2) all countries where crises erupt (within a short period of time, say less than a year) after the initial crisis in (1) shall receive rescue financing.

Such rules use the information derived from the timing of crises; the country hit by crisis first is very likely to have the crisis because of its own problems, while the country facing a crisis shortly after the first crisis has a high chance that contagion causes the crisis. Given the information, the new lending facility aims at punishing the country of the first crisis to abate moral hazard but saving the country of later crisis to prevent contagion. \(^{11}\)

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\(^{10}\) In implementing this new lending facility, the international crisis lender may set a time lag that is needed to make an operational definition of the “first” crisis, based on empirical studies on timing patterns of previous crises. For example, based on the time lags among the ERM crisis in 1992–93, the Mexican crisis in 1994–95, and the East Asian crisis in 1997–98, one may suggest that one year or so would be a reasonable criterion to identify the “first” crisis.

\(^{11}\) Here, for simplicity, the new facility assumes that the country of the first crisis shall not
To speak figuratively, this timing-based rescue lending facility can be considered as a facility that distinguishes between the house where the fire first started and the houses that caught fire from the first house. What the facility does is to put out the fire on houses that later caught fire in order to stop further spreading, that is, crisis contagion; but not to save the house that first started the fire to deter moral hazard.

An important strength of the timing-based facility is that it can fulfill its role of obviating moral hazard under information asymmetry better than existing crisis lending facilities. It can again be likened to a fire fighter who does not know the story of each and every house: instead of monitoring all those houses one by one to prevent fire (which could be almost impossible), she can induce their awareness of the dangers of fire and voluntary preventive efforts, which are more effective for fire prevention.

In the following sections, I present a simple model to more clearly demonstrate the difference between existing international rescue lending facilities and the new timing-based lending facility, which can deter both moral hazard and contagion effect at the same time.

III. The Basic Model

The desirability of a crisis lending facility may hinge on its ability to resolve the problems of crisis contagion and moral hazard. To see how effectively the newly proposed lending facility can resolve these problems, I propose a simple model of crisis that focuses on crisis contagion and moral hazard.

A. Two Types of Crises

There are two periods, $t = 1, 2$. The world consists of a continuum of countries defined along the interval $[0, 2N]$. Each country, indexed by $i$, is populated with a continuum of consumers whose measure is one. There are two types of countries: borrowing country ($i \in [0, N]$) and lender country ($i \in (N, 2N]$). Each lender country has $k$ amount of capital but does not have production technology, so that it has to lend its capital to borrower countries. Each borrower country has $k$ amount of initial capital and production technology, and borrows $k^{i,f}$ amount of capital from lender countries at the beginning of the first period. The loan contract specifies the amount of repayment a borrower

get any rescue financing. But we may instead assume that the country of the first crisis shall receive a part of what the country of later crisis would receive as rescue financing. This does not alter the main results of the paper.

The current model focuses on borrower-side moral hazard problems. But we may explicitly introduce the moral hazard problem of lender countries (or international investors) as well. In this variant model, however, the main result would not change. More specifically, we
country has to make at the end of the final period as \( r k^{i:F} \). If the borrower country cannot repay \( r k^{i:F} \), it is in default.

Each borrower-producer country can fall into an economic crisis that depresses productivity, and there are two kinds of economic crisis depending on the origin of the crisis. One is autonomous crisis that occurs internally as a result of domestic factors, and the other is imported crisis, which is the outcome of crisis contagion from foreign countries. The possibility of having an autonomous crisis can be lowered by voluntary efforts of crisis prevention on the part of each country. For simplicity, assume that the probability of having an imported crisis can hardly be reduced by such voluntary efforts. \(^{13}\)

To discuss the two types of crises more formally, assume that each borrower-producer country produces by the following linear production function \(^{14}\)

\[
y^i = A^i k^i, \quad i \in [0, N],
\]

where \( A^i \) represents the productivity shock that takes \( A \) or \( B(< A) \), and \( k^i \) is capital input employed by the \( i \)-th country. Total capital employed by a borrower country is the sum of domestic capital \( (k^{i:d}) \) and foreign borrowings, that is, \( k^i = k^{i:d} + k^{i:f} \). Capital depreciates 100 percent at the end of \( t = 2 \). The initial capital endowment of each country \( (k) \) is normalized at \( k = 1/2 \), which yields \( k^{i:d} = k^{i:f} = k = 1/2 \) and \( k^i = 2k = 1 \).

Completion of production, defined in Eq.(1), takes two periods, as production goes through two stages. The first stage occurs in the first period \( (t = 1) \), when an autonomous crisis can occur. The second stage happens in the second period \( (t = 2) \), when a bad productivity

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\(^{13}\) We may instead assume that the chance of having an imported crisis is affected by crisis-prevention efforts. However, this modification would not change the results of the paper, as long as crisis-prevention efforts reduce the chance of imported crisis less effectively than that of autonomous crisis.

\(^{14}\) International crisis lending can be justified when there is externality, notably due to crisis contagion, or in the presence of self-fulfilling equilibrium. The current model does not explicitly introduce the element of self-fulfilling crises. But we may introduce such crises, for example, by explicitly introducing complementarity between productivity shock \( (A_t) \) and capital \( (k_t) \) in the production function, together with the possibility of outflow of foreign capital in the second period.
shock due to contagion can be created. During the second period, crisis contagion could take place in two rounds: the crisis in the country having an autonomous crisis in the first period is spread to some countries (the first round of contagion), and the crisis is further spread from the infected countries to other countries (the second round of crisis contagion).

For simplicity, the basic model assumes that autonomous crises occur only in the first period while imported crises occur only in the second period. This assumption formalizes the idea that an imported crisis is very unlikely to occur without the preceding occurrence of a crisis from which it is infected. To be more general, however, Section 7.2 allows autonomous crises to occur in the second period as well and imported crises to occur in the first period as well, and derives similar results.

Given the above production process, the productivity shock is revealed in each period. Let \( A_i^t \) denote the productivity shock (which takes \( A \) or \( B \)) in \( t (= 1, 2) \). Assume that the productivity that a country has for the whole production process, \( A^i \), is determined as

\[
A^i = \min[A_i^1, A_i^2].
\]  
(2)

So if the production process goes without a bad shock in any of the two stages, the productivity is determined to be \( A \). But if a country experiences a bad shock in one of the two production stages, the productivity is given by \( B \). Whether a country has a good productivity shock (\( A \)) or a bad productivity shock (\( B \)) in each period \( (t = 1, 2) \) is observable by anybody (including the international crisis lender).

Let \( \theta_1^i \) be the probability of a country’s having a crisis in the first period, and \( \theta_2^i \) be the probability that the country does not have a crisis in the first period but in the second period. Let \( \theta_A^i \) be the probability of a country’s having an autonomous crisis, and \( \theta_C^i \) be the probability that the country is hit by an imported crisis. Then the crisis probability \( \theta^i \) of each country is given by

\[
\theta^i = \theta_1^i + \theta_2^i = \theta_A^i + (1 - \theta_A^i)\theta_C^i.
\]  
(3)

Then the probability distribution of productivity, \( A^i \), can be summarized by

\[
A^i = \begin{cases} 
A & \text{with probability } (1 - \theta^i) \\
B(< A) & \text{with probability } \theta^i,
\end{cases}
\]  
(4)

which tells us that a country has productivity \( B \) with probability \( \theta^i \), or productivity \( A \), which is bigger than \( B \), with probability \( (1 - \theta^i) \). As a result, the country’s final output, \( y^i \), is determined to be \( Bk^i \) with probability \( \theta^i \), and \( Ak^i \) with probability \( (1 - \theta^i) \) at the end of the final period \( (t = 2) \). Suppose that \( r \) is given as \( B < r < A \).

The above assumptions imply that the output of a country in crisis is equal to \( B \),
regardless of the origin of the crisis. Therefore, in this model it is impossible to distinguish between autonomous crisis and imported crisis by observing the amount of output. It is possible, however, to distinguish between the two types of crises by observing the timing order of crises, given that an imported crisis is unlikely to occur in the first period.

**B. Probability of Autonomous Crisis**

A borrower-producer country may have an autonomous crisis in the first period. The probability that a borrower country is hit by an autonomous crisis, \( \theta_A \), can change according to how much the country strives to prevent crisis by resolving its own macroeconomic or structural problems at the beginning of the first period (but after foreign borrowings are made). That is, the probability of autonomous crisis occurring depends on each country’s efforts for crisis prevention. We denote the efforts of each country for crisis prevention as \( e^i \), defined on the interval \( \in [0, 1] \). The probability of a country’s having an autonomous crisis, \( \theta_A \), is given by

\[
\theta_A = \theta_A(e^i),
\]

which suggests that \( \theta_A \) is affected only by \( e^i \).

Here we assume the relationship between \( e^i \) and \( \theta_A \) as

\[
\frac{\partial \theta_A}{\partial e^i} < 0,
\]

which tells us that the more efforts for crisis prevention, the lower the possibility of having an autonomous crisis.

Meanwhile, a crisis does not always break out for want of efforts by a country. That is, \( \theta_A(0) = 1 \) will not be the case, and we have

\[
\theta_A(0) = \theta_A^{max} < 1,
\]

where \( \theta_A^{max} \) is the maximum of \( \theta_A(e^i) \).

In addition, a country may have a crisis even in the case where it makes its maximum effort to prevent crisis. That is,

\[
\theta_A(1) = \theta_A^{min} > 0,
\]

where \( \theta_A^{min} \) is the minimum of \( \theta_A(e^i) \).

Under the above assumptions, rescue financing provided by the international crisis lender can create a moral hazard problem. Particularly if the international community resolves any country's financial crisis through bailouts, the countries would not put as much effort into crisis prevention as they would have without bailouts, which in turn creates moral hazard and raises \( \theta_A \).
C. Probability of Crisis Contagion

Borrower countries may have crisis contagion in the second period. During the second period, countries may be infected by the crisis of a country having autonomous crisis in the first period or by the crises of the countries that were already infected in the second period. For simplicity, here I do not specify the specific contagion mechanism, while it could be through trade links or capital market links, or something else.\textsuperscript{15}

The chance of crisis contagion in the second period ($\theta^*_C$) rises with the number of countries hit by crisis in the first period. In addition, the chance of crisis contagion is reduced by rescue financing in the first and second period, partly because the adverse effect that a financial distress in a crisis country could have on the production possibility of the other countries with close trade or financial links can be reduced by rescue financing (for simplicity we here do not explicitly introduce the mechanism through which rescue financings abate contagion effects).

To be more specific, let $n_t$ denote the measure of the countries that have a crisis in period $t (= 1, 2)$,\textsuperscript{16} and $\phi_t$ be the fraction of the crisis-hit countries that receive rescue financing from the international crisis lender among all the crisis countries in period $t$. Now assume that the probability of crisis (due to contagion) is given by

\begin{equation}
\theta^*_C = \gamma(a - \phi_1)(b - \phi_2)n_1, \quad a, b > 1,
\end{equation}

where $\gamma(> 0)$ captures the strength of crisis contagion, and $a(> 1)$ and $b(> 1)$ represent the part of crisis contagion that is not affected by the international crisis lender’s rescue financing in the first and second period, respectively.\textsuperscript{17}


\textsuperscript{16}In the current setup with a continuum of (accordingly, an infinite number of) countries, $n_1$ and $N$ represents the “measure” of producer countries in autonomous crisis and total producer countries, respectively. But we will use the terms “number” and “measure” interchangeably throughout the paper.

\textsuperscript{17}To explicitly distinguish between the first and the second round of contagion, we may introduce a three period version of the model. Then, the probabilities of crisis contagion in the second and third period (denoted by $\theta^*_C^2$ and $\theta^*_C^3$) can be given by

\begin{equation*}
\begin{aligned}
\theta^*_C^2 &= \gamma_2(a - \phi_1)n_1, \\
\theta^*_C^3 &= \gamma_3(a - \phi_2)n_2, \quad a > 1,
\end{aligned}
\end{equation*}

where $\gamma_2(> 0)$ captures the strength of the first round of crisis contagion (the effect of the crisis in the first period on the chance of the crisis in the second period), $\gamma_3(> 0)$ represents that of the second round of contagion, and $a$ represents the part of crisis contagion that is
This assumption suggests that $\frac{\partial \theta_i^C}{\partial n_1} > 0$ and $\frac{\partial \theta_i^C}{\partial \phi_1}, \frac{\partial \theta_i^C}{\partial \phi_2} < 0$. That is, the more countries in the world experience a crisis in the first period or the less crisis-countries are bailed out by the international crisis lender, the more countries have a crisis (due to contagion) in the second period.

The assumption of $a, b > 1$ implies that $\theta_i^C = \gamma(a - 1)(b - 1)n_1 > 0$ when $\phi_1 = \phi_2 = 1$. That is, even in the case where all crisis countries receive rescue financing, crises due to contagion will occur (though its chance will be reduced given $n_1$).

For notational convenience, define $\delta \equiv 1 - \theta_i^C$, while the range of $\delta$ is given by: $\delta \in [\delta^l, \delta^u]$ where $\delta^l \equiv 1 - \gamma ab N \theta_A^{\max}$ and $\delta^u \equiv 1 - \gamma(a - 1)(b - 1)N \theta_A^{\min}$.

### D. International Crisis Lender

There is the international crisis lender (or the international lender of last resort) which may provide rescue financing to crisis countries. If it decides to rescue a country in crisis, it provides financing which amounts to $\rho = r k^{i,f}$. This amount of rescue financing is just large enough for the country in crisis to make repayments. As $r k^{i,f}$ amount of crisis lending is provided, the country in crisis makes the required repayment $r k^{i,f}$ to the lender countries (therefore it is no longer in default).

For simplicity, assume that the reserves for rescue financing are raised from contributions made by all the countries. So each country’s contribution to the reserves can be seen as insurance premium, and rescue financing can be thought of as insurance payments. As a result, the countries that receive rescue financing here will not have to repay $\rho$ to the international crisis lender later. Regarding the timing of making contributions, assume that contributions were already made in the past (before $t = 1$). Therefore, the international crisis lender does not have to raise the reserves in the initial period ($t = 1$). Of course, we may instead assume that the international crisis lender raises the reserves at the beginning of the initial period, rather than before the initial period. Or we may assume $\rho$ to be just a pure lending that should be repaid later. But these variations do not change the main results (see Appendix B for the analysis on these cases).

Furthermore, there is information asymmetry between the international crisis lender and the countries. The international crisis lender cannot accurately observe nor verify the effort made by each country for crisis prevention, $e^i$, possibly reflecting the situation with huge

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Not affected by the international crisis lender’s rescue financing. By adding $\theta_i^C$ and $\theta_i^C$, we can then derive Eq.(9), which is for the two period version.

Without losing generality, we may consider the cases of partial financing (i.e., $\rho < r k^{i,f}$) or different $\rho$s (for example, $\rho = r k^{i,f}$ for some countries but $\rho < r k^{i,f}$ for other countries).
state verification costs (Townsend, 1979).

E. Government

The government of each borrower-producer country can reduce the chance of crisis by exerting efforts to prevent autonomous crisis at the beginning of the first period. Crisis prevention indubitably calls for expenses, time and efforts. The cost (in terms of the utility of the representative consumer) that a country has to entail for crisis prevention, denoted by \( v(e^i) \), is given by

\[
v(e^i) = \psi e^i, \tag{10}\]

where \( \psi > 0 \) captures the cost per unit of crisis prevention effort.

The government seeks to maximize the expected utility of the representative consumer by choosing \( e^i \):

\[
\max_{e^i} E(u(c^i, e^i)) = E(u(c^i)) - v(e^i)
= E(\ln c^i) - \psi e^i, \tag{11}\]

which suggests that more consumption or less efforts to prevent crisis induce higher utility, while consumption raises utility following a logarithmic function.\(^{19}\)

F. Costs of Debt Repudiation

At the end of the second period when production is completed, borrower countries are supposed to repay \( r k^i f \) to lender countries. Assume that any borrower country repays to lender countries in two cases: where it is not hit by crisis, or where it is hit by crisis but receives rescue financing from the international crisis lender. For simplicity, however, the borrower country does not repay in the case where it is hit by crisis and the international crisis lender does not provide rescue financing.\(^{20}\)

\(^{19}\) The logarithmic utility function substantially simplifies the calculations in later sections, in addition to assuring risk aversion, while assuming more general utility functions, for example CRRA utility function, would not alter the main results of the paper.

\(^{20}\) This assumption can be derived as a result of the optimal choice of borrower countries. To illustrate this, assume that the cost due to nonpayment, denoted by \( \lambda \), satisfies

\[
\ln A - \ln(A - r/2) < \lambda < \ln B - \ln(B - r/2).
\]

Now suppose that there is no international crisis lender, and consider a borrower country which decides whether or not to repay \( r k^i f \) after the productivity shock is revealed. If the borrower country does not repay \( r k^i f \), the consumption of the representative consumer is given by \( A k^i = A(2k) = A \) in the case of no crisis, and \( B k^i = B(2k) = B \) in the case of crisis (recall that \( k = 1/2 \)). So, given the cost due to nonpayment, the utility from
If a borrower country repudiates its foreign debts, lenders would impose sanctions. As a result, the borrower country would not be able to borrow foreign capital nor have access to trade credits which are imperative for the next production cycle (while the current model does not explicitly introduce the next production cycle by assuming that there is only one production cycle that takes two periods). So defaulting on foreign debts would be very costly. Reflecting such costs, I assume that if a country does not repay $rk^{i,f}$, it incurs costs which amounts to $\lambda$ (in terms of the utility of the representative consumer).

A country has the highest welfare (or utility of the representative consumer) when it is not hit by crisis. It has the second highest welfare when it is hit by crisis but receives rescue financing, and the lowest welfare when it is hit by crisis and does not receive rescue financing. More specifically, we have $\ln(A - r/2) > \ln B > \ln B - \lambda$.  

### IV. LAISSEZ-FAIRE

Under the environment described in the basic model above, what would be the best way to minimize both moral hazard and contagion? This section examines how the equilibrium levels of moral hazard and crisis contagion are determined under laissez-faire.

In this case of laissez-faire, no rescue financing is provided to the countries in crisis, that is, $\phi_1 = \phi_2 = 0$. Given Eq.(9), it then follows that $\theta^i_C = \gamma ab n_1$. So crisis contagion occurs to its maximum given $n$. Then, if a borrower country does not have a bad productivity shock in any of the two periods, the consumption of the representative consumer is given by $Ak^i - rk^{i,f} = (A - r/2)$ (recall that $k^i = 1$ and $k^{i,f} = 1/2$). If the country was hit by a bad productivity shock in one of the two periods, it does not repay $rk^{i,f}$. So the consumption is given by $Bk^i = B$, while the borrower country incurs the cost due to debt repudiation, $\lambda$.

consumption ($u(c^i)$) is given by $(\ln A - \lambda)$ in the case of no crisis, and $(\ln B - \lambda)$ in the case of crisis. Meanwhile, if the borrower country repays $rk^{i,f}$, the consumption of its representative consumer is given by $Ak^i - rk^{i,f} = (A - r/2)$ in the case of no crisis, and $Bk^i - rk^{i,f} = (B - r/2)$ in the case of crisis. So the utility from consumption is given by $\ln(A - r/2)$ and $\ln(B - r/2)$. Then given the above assumption on $\lambda$, we have $\ln(B - r/2) < \ln B - \lambda$ so that a defaulting country would not repay. We also have $\ln(A - r/2) > \ln A - \lambda$ so that the country which did not have a crisis would make the repayment.  

21 See Bulow and Rogoff (1989) for a model which explicitly introduces sovereign default penalties, and Rose (2002) for empirical support for the penalties.

22 Note that without losing generality, we may instead assume that borrower countries in crisis repay to lenders even when the international crisis lender does not provide rescue financing, and that $\ln(A - r/2) > \ln B > \ln(B - r/2)$.  

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21 See Bulow and Rogoff (1989) for a model which explicitly introduces sovereign default penalties, and Rose (2002) for empirical support for the penalties.

22 Note that without losing generality, we may instead assume that borrower countries in crisis repay to lenders even when the international crisis lender does not provide rescue financing, and that $\ln(A - r/2) > \ln B > \ln(B - r/2)$.  

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Using these, the expected utility in the absence of international crisis lending is given by
\[ E(u(c^i, e^i)) = E(\ln c^i) - \psi e^i \]
\[ = (1 - \theta^i) \ln(A - r/2) + \theta^i \ln B - \theta^i \lambda - \psi e^i. \]  
(12)

For notational convenience, define \( w^i \equiv E(u(c^i, e^i)) \). Then the government of each borrower country \((i \in [0, N])\) will solve the following optimization problem \([P1]\):

\[ [P1] \max_{e^i} w^i = [1 - \theta_A(e^i) - (1 - \theta_A(e^i))(\gamma abn_1)] \ln(A - r/2) \]
\[ + [\theta_A(e^i) + (1 - \theta_A(e^i))(\gamma abn_1)](\ln B - \lambda) - \psi e^i. \]  
(13)

In solving \([P1]\), I focus on the case where in the absence of the international crisis lender, each borrower country makes positive efforts for crisis prevention. For this, I assume that the following condition holds.

**Condition (i)**

*For any \( e^i \in [0, 1] \) and \( \delta \in [\delta^l, \delta^u] \), it holds that*

\[ \psi + \frac{\partial \theta_A(e^i)}{\partial e^i} \delta \lambda < \frac{\partial \theta_A(e^i)}{\partial e^i} \delta(- \ln(A - r/2) + \ln B) < \psi \]  
(14)

which tells us that the cost of crisis prevention (\( \psi \)) and the cost of debt repudiation (\( \lambda \)) are not negligible.

Under Condition (i), differentiating \( w^i \) with respect to \( e^i \) gives

\[ \frac{\partial w^i}{\partial e^i} = \left[ \frac{\partial \theta_A^i}{\partial e^i} (1 - \gamma abn_1) + (1 - \theta_A^i) \gamma ab \frac{\partial n_1}{\partial \theta_A^i} \frac{\partial \theta_A^i}{\partial e^i} \right] (- \ln(A - r/2) + \ln B - \lambda) - \psi. \]  
(15)

Note that given the assumption of a continuum of borrower countries, the measure of the countries having an autonomous crisis \((n_1)\) is given by \( n_1 = \int_0^N \theta_A^i dj \). It then follows that

\[ \frac{\partial n_1}{\partial \theta_A^i} = 0, \]  
(16)

which suggests that the measure of the countries having an autonomous crisis is not affected by the change in the \( i \)-th country’s probability of having an autonomous crisis, \( \theta_A^i \).
Using Eq.(16), the derivative in Eq.(15), regardless of the other governments’ effort for crisis prevention, becomes
\[
\frac{\partial w_i}{\partial e_i} = \frac{\partial \theta_A^i}{\partial e_i}(1 - \gamma ab n_1)(-\ln(A - r/2) + \ln B - \lambda) - \psi.
\] (17)
which, given Condition (i), is positive for all \(e_i \in [0, 1]\).

So the optimal solution of the \(i\)-th government for crisis prevention in this case (denoted by \(e_i^L\)) is given by
\[
e_i^L = 1,
\] (18)
regardless of the other countries’ efforts to prevent their own autonomous crises.

This holds for all the borrower-producer countries, and hence the Nash equilibrium is determined as a case where all the countries make maximum efforts to prevent crisis: \(e_i^L = 1\), for all \(i \in [0, N]\). So in this case, there is no moral hazard problem.

As a result, the probability of a country’s having an autonomous crisis is at its minimum: \(\theta_A(1) = \theta_A^{min} > 0\). Note that though all the borrower countries make efforts for crisis prevention to its maximum, some of them experience autonomous crises (Eq.(8)). The measure of those countries is given by \(n_1 = \int_0^N \theta_A^{min} dj = N \theta_A^{min}\).\(^{23}\) Given a positive \(n_1\), the chance that a country has a crisis due to contagion, \(\theta_C^i\), is given by \(\theta_C^i = \gamma ab n_1 = \gamma ab N^{\theta_A^{min}}\).
This suggests that in equilibrium contagion occurs, particularly at its maximum for a given \(n_1\).

In this case, lender countries \((i \in (N, 2N])\) receive \(r k^i f\) from the borrower countries that did not have a crisis, and nothing from the countries that had a crisis. So each lender country, which invests evenly across borrower countries for risk pooling, will receive \(r k^i f (1 - \theta^i) = (r/2)(1 - \theta_A^{min} - (1 - \theta_A^{min})\gamma ab N\theta_A^{min})\).

Now define the world’s welfare to be the sum of the utility of the representative consumer over all the borrower and lender countries (that is, \(\int_0^N [E(\ln c^i) - \psi e^i] di + \int_N^{2N} E(\ln r k^i f) di\)). Then the world’s welfare in the absence of the international crisis lender, denoted by \(W^L\), is given by
\[
W^L = N[(1 - \theta_A^{min} - (1 - \theta_A^{min})\gamma ab N\theta_A^{min}) \ln(A - r/2)
\]
\(^{23}\) Given the assumption of a continuum of countries, there are many “first” crisis countries (or countries that have autonomous crisis in the first period). We may interpret this result as representing that there are many regions each of which has its own first crisis country. Or we may explicitly introduce a variant model in which there are a continuum of different regions of borrower countries with measure one, each of which has \(N\) countries, and crisis contagion occurs only within each region. Of course, the modification does not alter the results of the paper.
\[
+ (\theta_A^{\min} + (1 - \theta_A^{\min}) \gamma ab N \theta_A^{\min}) (\ln B - \lambda) - \psi \\
+ N [\ln(r/2) + \ln(1 - \theta_A^{\min} - (1 - \theta_A^{\min}) \gamma ab N \theta_A^{\min})].
\]

Note that in the absence of the international crisis lender, moral hazard can be prevented. However, crisis contagion cannot be effectively deterred under the laissez-faire approach. As a result, the welfare of the entire world may not attain its maximum possible value. The reason is that externalities due to crisis contagion cannot be resolved by each individual country under laissez-faire. Hence there may arise the need for an international coordinator or crisis lender.

V. EXISTING CRISIS-LENDING FACILITIES

How can the international crisis lender or the international lender of last resort resolve the problem of externality due to crisis contagion? What would be the effect of a contagion-preventing rescue policy on the problem of moral hazard?

A. Traditional Crisis Lending Facilities

First we examine how traditional crisis lending facilities of the IMF such as the Stand-By Arrangement affect each of contagion and moral hazard problems. Traditional lending facilities are often offered to countries in crisis, regardless of whether the countries are hit by autonomous crisis or imported crisis.

Suppose that under such lending facilities, all the crisis-hit countries receive rescue financing, and hence we have \( \phi_1 = \phi_2 = 1 \). Given Eq.(9), it then follows that \( \theta_C = \gamma(a - 1)(b - 1)n_1 \). So crisis contagion could be minimized given \( n_1 \).

Recall that the international crisis lender is assumed to provide rescue financing which amounts to \( \rho = r k^i_f \) to each country in crisis. Furthermore, if a borrower country does not have a bad productivity shock, the consumption of its representative consumer is given by \( (A - r/2) \), as under laissez-faire. If the country is hit by a bad productivity shock, the representative consumer consumes \( B \), while the country does not incur the cost due to debt repudiation, \( \lambda \). Using these, the maximization problem of each borrower country can be expressed as

\[
\begin{align*}
\text{[P2]} \quad \max_{e^i} w_i &= [1 - \theta_A(e^i) - (1 - \theta_A(e^i))(\gamma(a - 1)(b - 1)n_1)] \ln(A - r/2) \\
&+ [\theta_A(e^i) + (1 - \theta_A(e^i))(\gamma(a - 1)(b - 1)n_1)] \ln B - \psi e^i. \tag{20}
\end{align*}
\]

Using Eq.(16), the derivative of \( w_i \) with respect to \( e^i \) is given by (regardless of the other
countries’ efforts for crisis prevention)

\[
\frac{\partial w_i}{\partial e^i} = \frac{\partial \theta_A^i}{\partial e^i}(1 - \gamma(a - 1)(b - 1)n_1)(-\ln(A - r/2) + \ln B) - \psi,
\]

which, under Condition (i), is negative for all \(e^i \in [0, 1]\), suggesting that a country’s raising efforts for crisis prevention reduces its welfare.

Therefore the optimal solution of \(e^i\) for any borrower country in this case, denoted by \(e^{i,T}\), is given by

\[
e^{i,T} = 0,
\]
regardless of the other countries’ decisions. Therefore, in Nash equilibrium, we have \(e^{i,T} = 0\) for all the counties. This suggests that traditional crisis lending facilities, which provide countries with an insurance against the happening of a bad productivity shock, create moral hazard.

In the equilibrium, the probability that a country has an autonomous crisis is at its maximum: \(\theta_A(0) = \theta_A^{\text{max}}\). Meanwhile, the chance that the country has contagion, \(\theta^i_C\), is given by \(\theta^i_C = \gamma(a - 1)(b - 1)N\theta_A^{\text{max}}\).

In this case, lender countries receive \(rk^{i,f}(= r/2)\) from the borrower countries regardless of the occurrence of crisis as the international crisis lender fully bails out defaulting countries. The world’s welfare in this case (denoted by \(W^T\)) is then given by

\[
W^T = N[(1 - \theta_A^{\text{max}} - (1 - \theta_A^{\text{max}})\gamma(a - 1)(b - 1)N\theta_A^{\text{max}})\ln(A - r/2) \\
+ \theta_A^{\text{max}} + (1 - \theta_A^{\text{max}})\gamma(a - 1)(b - 1)N\theta_A^{\text{max}})\ln B] \\
+ N[\ln(r/2)].
\]

In sum, traditional crisis-lending facilities are able to contain the problem of contagion but begot the problem of moral hazard. From the standpoint of the world as a whole, however, the optimal level of efforts to prevent crisis may be positive. In this regard, traditional lending facilities could be sup-optimal.

**B. Contingent Credit Lines (CCL)**

Now we explore how the Contingent Credit Lines (CCL), which was introduced by the IMF in 1999, would function under information asymmetry. As discussed earlier, a key idea of the CCL is prequalification. When countries that satisfy some prequalification are hit by a bad productivity shock (either in the first or second period), the CCL facility provides rescue financing.

The key to efficient operation of the CCL is how accurately the international crisis lender can observe and verify \(e^i\). If there is no information asymmetry between the international
crisis lender and each country, the CCL would work as follows. The international crisis lender would set a threshold for the country’s efforts to prevent crisis (denoted by \( e^{th} \in (0, 1] \)), and it would allow only the countries that meet the threshold level to use the CCL facility in the event of economic crisis. More specifically, when the country’s actual effort for crisis prevention, \( e^i \), is less than the threshold level, \( e^{th} \), the rescue financing would be set to be zero \( (\rho = 0) \), and when it is higher than or equal to the threshold, then the rescue financing would be set positive at \( \rho = rk^{i,f} = r/2 \).

However, due to informational asymmetries, it is often very difficult for the international crisis lender to accurately observe and verify each country’s efforts to prevent crisis, \( e^i \). Recall that the basic model assumes the case of information asymmetry where \( e^i \) cannot be observed nor verified accurately by the international crisis lender. In this case, its rescue financing \( (\rho) \) has to rely on the observed efforts, denoted by \( \tilde{e}^i \), not the actual efforts \( e^i \). More specifically, the rule for rescue financing under the CCL is set as

\[
\rho = \begin{cases} 
0 & \text{if } \tilde{e}^i < e^{th} \\
 rk^{i,f} & \text{if } \tilde{e}^i \geq e^{th}.
\end{cases}
\]

That is, when the observed effort for crisis prevention \( (\tilde{e}^i) \) is lower than the threshold level, no rescue financing is provided, and when it is higher than or equal to the threshold, then the amount of rescue financing that is just enough to satisfy the repayment requirement \( rk^{i,f} \) is provided.

To highlight the role of information asymmetry, suppose that the international crisis lender’s observation \( (\tilde{e}^i) \) on the actual effort by the government of each country \( (e^i) \) is inaccurate. Particularly assume that

\[
\tilde{e}^i = \begin{cases} 
0 & \text{if } e^i < \alpha \\
1 & \text{if } e^i \geq \alpha,
\end{cases}
\]

which suggests that the observed effort \( (\tilde{e}^i) \) takes one if the true effort is not lower than \( \alpha \), and zero otherwise.

Then a country will receive a positive rescue financing in the event of crisis as long as its true effort \( (e^i) \) is greater than or equal to \( \alpha \). It is because in this case the country’s effort observed by the international crisis lender will be \( \tilde{e}^i = 1 \). But the country will receive no rescue financing in the event of crisis if its true effort \( (e^i) \) is lower than \( \alpha \).

Given these, the government of each borrower country with private information solves the following optimization problem \([P3]\):
\[ [P3] \quad \max_{e_i} w_i = \Phi(e_i)[(1 - \theta_A(e_i) - (1 - \theta_A(e_i))\gamma(a - 1)(b - 1)n_1) \ln(A - r/2)
+ (\theta_A(e_i) + (1 - \theta_A(e_i))\gamma(a - 1)(b - 1)n_1) \ln B - \psi e_i]
+ (1 - \Phi(e_i))(1 - \theta_A(e_i))\gamma abn_1) \ln(A - r/2)
+ (\theta_A(e_i) + (1 - \theta_A(e_i))\gamma abn_1)(\ln B - \lambda) - \psi e_i], \] 

(26)

where \( \Phi(e_i) \) is an index number which takes one if \( e_i \geq \alpha \) and zero otherwise.

To solve the optimization problem, consider first the case where \( e_i < \alpha \). If a borrower country makes crisis prevention efforts within this range, the international crisis lender observes \( \tilde{e}_i = 0 \). Therefore, there is no rescue financing (\( \rho = 0 \)). In addition, \( \Phi(e_i) = 0 \), and hence the optimization problem [P3] is reduced to [P1], which is for the case of laissez-faire. Recall that given Condition (i), it holds \( \frac{\partial w_i}{\partial e_i} > 0 \). Therefore, it would be better for the government to raise \( e_i \) as high as possible within the range of \( e_i(< \alpha) \).

Now consider the case where \( e_i \geq \alpha \). For this range of \( e_i \), the international crisis lender observes \( \tilde{e}_i = 1 \). So there will be full bailouts (\( \rho = rk_{i,f} \)) regardless of the amount of \( e_i \). We also have \( \Phi(e_i) = 1 \), and hence the optimization problem [P3] is reduced to [P2], which is for the case of traditional lending facilities. Under Condition (i), it holds \( \frac{\partial w_i}{\partial e_i} < 0 \) for this range of \( e_i \). Hence the government would reduce \( e_i \) as low as possible within the range of \( e_i \geq \alpha \).

For simplicity, assume that the expected utility evaluated at \( e_i = \alpha \) (that is, \( E(u(e_i, \alpha)) \)), is not greater under laissez-faire than under traditional lending facilities. It then follows that the optimal level of crisis prevention effort in this case (\( e_i^{C} \)) is given by

\[ e_i^{C} = \alpha, \]

(27)

which suggests that each borrower country sets \( e_i \) at the lowest level among the \( e_i \)'s which satisfy the prequalification (that is, \( \tilde{e}_i \geq e_i^{th} \)).

Under the CCL, any country satisfying prequalification receives rescue financing when it faces a bad productivity shock. Therefore in this case, contagion can be contained by rescue financing, as under traditional lending facilities. In addition, the possibility of an autonomous crisis is determined to be \( \theta_A(\alpha) \), which is higher than under laissez-faire but lower than under traditional lending facilities. This suggests that the CCL may improve over traditional lending facilities by reducing the moral hazard problem.

However, the extent to which it abates the moral hazard problem depends on the degree of information asymmetry. Particularly when the problem of information asymmetry is substantial, the CCL might not operate effectively.
To illustrate this, consider the case where $\alpha$ is close to zero. Then a country will receive rescue financing in the event of crisis, even when it made only a little effort for crisis prevention. In this case, the CCL would become almost the same as in the Stand-By facility case, and the world’s welfare under the CCL could be approximated by that under traditional rescue financing facilities (that is, Eq.(23)).

The above discussions suggest that under information asymmetry, the CCL may not be effective in reducing moral hazard. When $e^i$ cannot be observed and verified accurately, setting the threshold level of $e^i$ and determining the prequalification based on that threshold level can lead to such critical problem.

VI. TIMING-BASED CRISIS-LENDING FACILITY

In the previous sections, we have shown that existing international lending facilities may be ineffective in reducing moral hazard, particularly under information asymmetry. This section will examine how the new lending facility proposed in Section 2 works in our model, and show that it can effectively resolve both problems of moral hazard and crisis contagion.

A. Timing-Based Crisis-Lending Facility

Within the model, the new facility can be formulated as follows:

(i) $\phi_1 = 0$, that is, in the first period, no country receives rescue financing; and

(ii) $\phi_2 = 1$, that is, in the second period, any country with a bad productivity shock receives rescue financing.

So rescue financing ($\rho = rk^{i,f}$) is provided to any crisis country in the second period, but to none in the first period. Given Eq.(9), we then have: $\theta^{i,c} = \gamma a(b - 1)n_1$. In addition, a country, if hit by crisis in the first period, produces $Bk^i = B$, and does not repay $rk^{i,f}$, while it incurs the cost due to nonrepayment, $\lambda$. So the utility from consumption in the event of crisis in the first period is $\ln B - \lambda$. Meanwhile, the country, if hit by crisis in the second period, produces $B$, but receives rescue financing and repays $\rho = rk^{i,f}$. So the utility from consumption is $\ln B$. If the country does not have a bad productivity shock, the utility from consumption is $\ln(A - r/2)$.

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24 I implicitly assume that the policy on rescue financing adopted by the international crisis lender ($\phi_t$) is credible. Of course, we may explicitly introduce the cost due to the loss of reputation large enough to induce the international crisis lender to commit to its policy on $\phi_t$. 
Given these, the government of each borrower country solves the following optimization problem \([P4]\):

\[
[P4] \max_{\epsilon_i} w_i = [1 - \theta_A(e^i) - (1 - \theta_A(e^i))(\gamma a(b-1)n_1)] \ln(A - r/2) \\
+ \theta_A(e^i)(\ln B - \lambda) + (1 - \theta_A(e^i))(\gamma a(b-1)n_1) \ln B - \psi e^i. \tag{28}
\]

Given \(\frac{\partial w_i}{\partial \theta_A} = 0\), differentiating \(w_i\) with respect to \(e^i\) gives

\[
\frac{\partial w_i}{\partial e^i} = \frac{\partial \theta_A}{\partial e^i}[(1 - \gamma a(b-1)n_1)(-\ln(A - r/2) + \ln B) - \lambda] - \psi, \tag{29}
\]

which, given Condition (i), is positive for any \(e^i\).

So the optimal solution for the government of any borrower country in this case, denoted by \(e^{i,N}\), is given by

\[
e^{i,N} = 1,
\]

and therefore we have \(e^{i,N} = 1\) for all \(i \in [0, N]\) in Nash equilibrium.

This suggests that the timing-based crisis prevention facility raises the countries’ effort for crisis prevention to the maximum under information asymmetry, in sharp contrast with existing international lending facilities. By exerting maximum efforts for crisis prevention \((e^{i,N} = 1)\), each country reduces its probability of having an autonomous crisis to \(\theta_A^{\min}\).

Note that given \(e^i\) is not greater than one, the new lending facility and the laissez-faire approach induce the same amount of crisis prevention efforts: \(e^{i,N} = e^{i,L} = 1\). However, if we allow \(e^i\) to be unbounded from the above, the timing-based lending facility can induce higher crisis prevention efforts than laissez-faire (see Section 7.3).

In addition to raising crisis prevention efforts, this timing-based facility may help contain crisis contagion. Note that it does not provide rescue financing to the countries experiencing autonomous crises in the first period \((\phi_1 = 0)\). Therefore, once autonomous crises occurred, it cannot prevent the first round of crisis contagion. However, it lowers the chance of crisis contagion to \(\gamma a(b-1)\theta_A^{\min}N\) by reducing the number of countries in autonomous crisis to \(\theta_A^{\min}N\) compared to existing lending facilities. Furthermore, the new facility helps contain the second round of crisis contagion by providing a full rescue financing to the countries having crises owing to contagion during the second period \((\phi_2 = 1)\).

The above discussion suggests that the new lending facility can effectively resolve both problems of moral hazard and contagion under information asymmetry. The new facility
based on the timing order of crises allows the international crisis lender to distinguish between autonomous crisis and imported crisis under information asymmetry. By not rescuing the country hit first, the timing-based lending facility can punish the country in autonomous crisis and hence deter the moral hazard problem as effectively as laissez-faire. By rescuing the countries hit later, the new facility can assist the countries hit by imported crises and, therefore, deter crisis contagion as effectively as existing lending facilities of the IMF. In this way, the new facility may overcome the shortcomings of laissez-faire and existing lending facilities.

Now we examine the world’s welfare. In this case, lender countries receive nothing from the countries that had a crisis in the first period, but \( rk^{i,f} \) from the countries that did not have a bad shock in any period and those that had a bad shock in the second period. So each lender country will receive \( rk^{i,f} (1 - \theta_A^{\text{min}}) = (r/2)(1 - \theta_A^{\text{min}}) \). Then the world’s welfare in this case is given by

\[
W^N = N[(1 - \theta_A^{\text{min}} - (1 - \theta_A^{\text{min}})\gamma a(b - 1)N\theta_A^{\text{min}}) \ln(A - r/2) + \theta_A^{\text{min}}(\ln B - \lambda) \\
+ (1 - \theta_A^{\text{min}})(\gamma a(b - 1)N\theta_A^{\text{min}}) \ln B - \psi] \\
+ N[\ln(r/2) + \ln(1 - \theta_A^{\text{min}})].
\]

(31)

In the following subsections, I compare the world’s welfare under the timing-based lending with those under laissez-faire and existing international lending facilities.

**B. Welfare Comparison with Laissez-Faire**

I first compare the world’s welfare under the timing-based lending facility (Eq.(31)) with that under laissez-faire (Eq.(19)). The difference in welfare, denoted by \( \Delta W^{N-L} (\equiv W^N - W^L) \), is given by

\[
\Delta W^{N-L} = N[(1 - \theta_A^{\text{min}})\gamma aN\theta_A^{\text{min}}(\ln(A - r/2) - \ln B) + (1 - \theta_A^{\text{min}})\gamma abN\theta_A^{\text{min}}\lambda] \\
+ N[- \ln(1 - \gamma abN\theta_A^{\text{min}})],
\]

(32)

which suggests that an introduction of the new lending facility in an laissez-faire economy reduces the chance of contagion without causing a moral hazard problem (and hence without raising the chance of autonomous crisis).

From Eq.(32), it is obvious that

\[
\Delta W^{N-L} > 0,
\]

(33)

that is, \( W^N \) is greater than \( W^L \). This result suggests that the timing-based lending facility can bring higher welfare than laissez-faire in the presence of crisis contagion. The intuition behind this result is clear. The timing-based lending facility is as effective as laissez-faire in preventing moral hazard, because it induces countries to put as much voluntary crisis prevention efforts as laissez-faire. Moreover, the new facility generates additional benefits of reducing crisis contagion, which the laissez-faire approach cannot.
C. Welfare Comparison with Existing Facilities

Next, I compare the world’s welfare between the timing-based facility and existing international rescue lending facilities. The difference in welfare, denoted by $\Delta W^{N-T} (\equiv W^N - W^T)$, is given by

$$\Delta W^{N-T} = N[((\theta^\text{max}_A - \theta^\text{min}_A) + ((a - 1)\theta^\text{max}_A(1 - \theta^\text{max}_A)$$

$$- a\theta^\text{min}_A(1 - \theta^\text{min}_A))\gamma N(b - 1)(\ln(A - r/2) - \ln B)]$$

$$+ N[-\theta^\text{min}_A \lambda - \psi + \ln(1 - \theta^\text{min}_A)].$$  \(34\)

The terms inside the first bracket of Eq.(34) represent the welfare gain of the timing-based facility compared to existing lending facilities, and the terms inside the second bracket capture its welfare costs. The welfare gain depends on how effectively the new facility can reduce the chances of crises by abating the moral hazard problem that arises under traditional lending facilities. As Eq.(34) indicates, the welfare gain due to the crisis prevention effect critically depends on $\theta^\text{min}_A$ for a given $\theta^\text{max}_A$. The smaller $\theta^\text{min}_A$ is, the larger the welfare gain is.

Assume that when $\theta^\text{min}_A$ goes to zero (so the crisis prevention effect is at its maximum), the welfare gain due to the crisis prevention effect is greater than the welfare cost. More specifically, assume that

$$\theta^\text{max}_A[1 + (1 - \theta^\text{max}_A)(a - 1)\gamma N(b - 1)](\ln(A - r/2) - \ln B) > \psi.$$  \(35\)

Given (35), it can be shown that there exists a threshold level of $\theta^\text{min}_A$, denoted by $\theta^\text{th}_A$, below which we have

$$\Delta W^{N-T} > 0.$$  \(36\)

This tells us that as long as $\theta^\text{min}_A$ is lower than $\theta^\text{th}_A$ for a given $\theta^\text{max}_A$ (that is, the crisis prevention effect is greater than a certain threshold level), we have $W^N > W^T$ (see Appendix A for the proof).

This result suggests that the new lending facility can induce higher world’s welfare than existing lending facilities in the presence of the moral hazard problem. The timing-based facility, by inducing $e^{i,N} = 1$, can prevent moral hazard more effectively than existing lending facilities. It can also help reduce the chance of crisis contagion by reducing the number of the countries hit by autonomous crisis, let alone by providing rescue financing to the countries hit by crisis in the second period. In contrast, existing lending facilities contain crisis contagion only by providing rescue financing to the countries that have already had a crisis.

Note that the new facility can help resolve a coordination problem that may arise under existing lending facilities. When all the countries make efforts to reduce their own chances
of autonomous crisis, there will be a substantial decline in the number of the countries hit by autonomous crises in the first period. Given crisis contagion and consequent externality, such efforts will also reduce the number of the countries hit by imported crises in the second period. As a result, the world’s welfare gain from the countries’ crisis prevention efforts can far exceed an individual country’s welfare gain. Under existing facilities, however, individual countries, which seek to maximize their own welfare, do not make crisis-prevention efforts up to the optimal level for the world as a whole. In contrast, the new facility induces countries to undertake crisis prevention efforts to the optimal level.

In sum, the above results suggest that the timing-based facility can be Pareto-superior to both laissez-faire and existing lending facilities. In the absence of crisis contagion, the laissez-faire approach could be the best. Without the moral hazard problem, existing crisis lending facilities could be the optimal. In the presence of both problems of moral hazard and crisis contagion, however, the new lending facility can provide the highest welfare to the world by effectively resolving both problems.

VII. EXTENSIONS AND DISCUSSIONS

The basic model allows for some extensions to explore various situations that may potentially affect the effectiveness of the timing-based lending facility. In this section, I address three such extensions: the case of high common vulnerabilities to a common shock; the case where the reversion of the timing order of crises is allowed; and the case where crisis prevention efforts could be without bound. I will show that those extensions do not alter the main results.

A. Common Shocks and Vulnerabilities

Some strings of crises may occur due to crisis contagion while others due to high common vulnerabilities to a common shock. The basic model focuses on the former case given that a vast majority of existing studies find evidence of crisis contagion (Kaminsky, Reinhart and Végh, 2002). However, we can modify the basic model to show that even in the case of common shocks and vulnerabilities (Mody and Taylor, 2003), the timing-based facility would bring an optimal outcome.

Consider a variant model where a series of crises occur due to high common vulnerabilities combined with a common shock. Let $\epsilon$ denote a common shock, and $\chi \in [\chi^{\text{min}}, \chi^{\text{max}}]$ denote common vulnerabilities. Under common vulnerabilities, borrower countries face the same probability of having a crisis. The probability of a country’s experiencing a crisis depends on the common shock and common vulnerabilities as

$$\theta^i = \chi \epsilon.$$  \hspace{1cm} (37)

So the higher $\chi$ or $\epsilon$ is, the more likely the country is to suffer a crisis.
The productivity shock in any of the two periods here is the result of a common shock. It is not an autonomous nor an imported shock. For simplicity, assume that a country may have a bad productivity shock in any of the two periods, but not in both periods. Let $\tilde{\theta}_i^1$ be the chance of having a crisis due to the common shock in the first period, and $\tilde{\theta}_i^2$ be the chance of having a crisis due to the common shock in the second period. Then we have: $\theta^i = \tilde{\theta}_i^1 + \tilde{\theta}_i^2 = \chi \epsilon$.

The possibility of a country’s having a crisis in any of the two periods, $\theta^i$, cannot be lowered only by its own voluntary efforts to reduce it, given $\theta^i = \chi \epsilon$. However, if a country strives to prevent crisis by solving its own structural or macro problems (at the beginning of the first period), it may delay the occurrence of crisis. More specifically, assume that $\frac{\partial \tilde{\theta}_i^i}{\partial e^i} = -\frac{\partial \tilde{\theta}_i^i}{\partial e^i} < -\psi / \lambda$. (38)

which tells us that a country’s crisis prevention efforts cannot lower its chance of crisis throughout the two periods ($\theta^i$), but its chance of having crisis earlier ($\tilde{\theta}_i^1$). The more efforts for crisis prevention, the lower the possibility of having a crisis in the first period. Eq.(38) also indicates that the reduction in the chance of crisis in the first period, induced by a country’s crisis prevention efforts, will be offset by the same increase in the chance of crisis in the second period.

The magnitude of common vulnerabilities ($\chi$) depends on how many countries make efforts to prevent crisis. It rises with the number of the countries that make crisis prevention efforts. To be more specific, assume that

$$\chi = \chi \left( \int_0^N e^i di \right), \quad \frac{\partial \chi}{\partial (\int_0^N e^i di)} < 0. \quad (39)$$

Given the assumption of a continuum of borrower countries, we have $\frac{\partial \chi}{\partial e^i} = 0$, suggesting that a single country’s crisis prevention efforts hardly affect the common vulnerability. However, if all the countries put efforts to prevent crisis at the same time, it is lowered.

Under the variant of the model, now examine how the equilibrium levels of global financial risks are determined under laissez-faire, existing crisis lending facilities, and the timing-based lending facility. First, consider the laissez-faire case. In this case, no crisis-hit country receives rescue financing. As a result, the government of each borrower country

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25 Without losing generality, we may assume that countries may have autonomous crisis as well and that the voluntary crisis prevention efforts can also reduce the chance of autonomous crisis. For example, we may assume that $\theta^i = \chi \epsilon + \theta_A^i = \tilde{\theta}_i^1 + \tilde{\theta}_i^2 + \theta_A^i$ and $\frac{\partial \theta_A^i}{\partial e^i} < 0$. The main results, however, will not alter as long as the crisis delay effect (i.e., $\frac{\partial \tilde{\theta}_i^1}{\partial e^i}$) is large compared to the crisis prevention effect (i.e., $\frac{\partial \theta_A^i}{\partial e^i}$).
solves the following optimization problem (at the very beginning of the first period):

$$
\max_{e^i} w_i = [1 - \tilde{\theta}_1^i - \tilde{\theta}_2^i] \ln(A - r/2) + [\tilde{\theta}_1^i + \tilde{\theta}_2^i] \ln (B - \lambda) - \psi e^i. \tag{40}
$$

Under Eq. (38), together with the assumption of a continuum of borrower countries, we have \(\frac{\partial w_i}{\partial e^i} = -\psi < 0\), regardless of the size of the common vulnerability, \(\chi\). So the optimal solution of a borrower country is given by

$$
e^{i,L} = 0, \tag{41}
$$

regardless of the other countries’ effort to prevent crisis. As this holds for all the borrower countries, the Nash equilibrium is determined as a case where all the borrower countries make no efforts to prevent crisis. As a result, the common vulnerability is at its maximum:

$$
\chi(\int_0^N 0 di) = \chi(0) = \chi^{\text{max}}.
$$

The intuition behind the result is straightforward. Under laissez-faire, there is no difference in rescue financing between the first-hit country and the others. So borrower countries have no incentive to avoid being the first-hit country. Therefore, no country will make efforts to prevent crisis. So in equilibrium, global risks will be at its maximum.

Second, consider traditional crisis lending facilities, under which any crisis country receives rescue financing. In this case, the optimization problem of each borrower country is given by

$$
\max_{e^i} w_i = [1 - \tilde{\theta}_1^i - \tilde{\theta}_2^i] \ln(A - r/2) + [\tilde{\theta}_1^i + \tilde{\theta}_2^i] \ln (B - \lambda) + \tilde{\theta}_1^i \ln B - \psi e^i, \tag{42}
$$

which yields \(\frac{\partial w_i}{\partial e^i} = -\psi < 0\) and \(e^{i,T} = 0\), regardless of the other countries’ decisions. Therefore, in Nash equilibrium, we have \(e^{i,T} = 0\) for all the borrower countries, and \(\chi(0) = \chi^{\text{max}}\), the same as under laissez-faire. The reason for the common vulnerability’s being at its maximum here is similar to the laissez-faire case. Under traditional lending facilities, rescue financing is provided to any crisis country regardless of the timing of crises. So countries would have no incentive to strive not to be the first crisis country.

Finally, consider the timing-based lending facility. In this case, rescue financing is provided to any crisis country in the second period, but to no country in the first period. The maximization problem of each borrower country is then given by:

$$
\max_{e^i} w_i = [1 - \tilde{\theta}_1^i - \tilde{\theta}_2^i] \ln(A - r/2) + \tilde{\theta}_1^i (\ln B - \lambda) + \tilde{\theta}_2^i \ln B - \psi e^i. \tag{43}
$$

Differentiating \(w_i\) with respect to \(e^i\) gives

$$
\frac{\partial w_i}{\partial e^i} = \frac{\partial \tilde{\theta}_1^i}{\partial e^i} (\ln B - \lambda) + \frac{\partial \tilde{\theta}_2^i}{\partial e^i} \ln B - \psi = \frac{\partial \tilde{\theta}_1^i}{\partial e^i} (-\lambda) - \psi, \tag{44}
$$
which, given Eq.(38), is positive. So the optimal solution for any borrower country in this case is given by

$$e^{iN} = 1,$$

regardless of the other countries’ efforts for crisis prevention. As a result, all the borrower countries make maximum efforts to prevent crisis in Nash equilibrium. In addition, the common vulnerability is at its minimum: $$\chi = \chi(f_0^N 1di) = \chi(N) = \chi_{\min}.$$ This suggests that in the case of common shocks and vulnerabilities, the timing-based lending facility can raise the countries’ efforts for crisis prevention to its maximum and, by doing so, reduce the risk of a large-scale global crisis. This is in sharp contrast with both laissez-faire and existing crisis lending facilities, under which the countries do not make any crisis prevention effort. It can also be easily shown (in an analogous way to Section 6) that in the case of common shocks, the timing-based facility can be Pareto-superior to both laissez-faire and existing lending facilities.

Note that the timing-based lending facility can resolve a coordination problem that arises under both laissez-faire and existing lending facilities. In the presence of common vulnerabilities, the risk of global crisis could be substantially lowered only when all the countries put crisis prevention efforts at the same time. From the standpoint of an individual country, however, its crisis prevention efforts can little affect the chance of its having a crisis, but only delay the timing of crisis. Under both laissez-faire and existing facilities, however, borrower countries do not have any incentive to make crisis prevention efforts, because all the crisis countries are treated the same regardless of the timing of crises. In contrast, the timing-based lending facility, by differentiating the first crisis country from the others, promotes competition among borrower countries not to become the first country. The competition could then induce all the countries to undertake their voluntary crisis prevention efforts, which helps resolve the coordination problem.

The above discussion, together with Section 6, suggests that the timing-based lending facility can be Pareto-superior to both laissez-faire and existing facilities regardless of whether crises are caused by contagion or common shocks. Therefore, even under uncertainty over whether crises are due to crisis contagion or common shocks, the timing-based facility can bring about a better outcome than laissez-faire and existing lending facilities.  

To illustrate this, for example, consider the case where crises can occur as a result of crisis contagion with probability $$\beta \in (0, 1)$$ and common shocks with probability $$1 - \beta$$. In addition, define the world’s welfare to be a weighted sum of the utility in the cases of crisis contagion and common shocks with each having a weight of $$\beta$$ and $$1 - \beta$$. In this case, we can easily show that the new lending facility is Pareto-superior to both laissez-faire and existing facilities. The result is intuitively obvious given that the timing-based facility induces higher
B. Reverse Timing Order and Racing for Delays

The basic model assumes that autonomous crises occur earlier than imported crises, given that in reality an imported crisis is less likely to precede the occurrence of a crisis from which it is infected. However, we cannot totally rule out the possibility that an autonomous crisis takes place after an imported crisis, particularly when countries try not to become the first crisis country. A variant of the basic model can be easily applied to the case of potential reversions of the timing order between autonomous and imported crises.

In this variant, assume that it may take time for a country that had a bad productivity shock to have a full-blown crisis. As a result, a bad autonomous shock may generate a crisis (and so observable by anybody) in the second rather than the first period. More specifically, if a country has a bad autonomous productivity shock, it may have a crisis in the second period with probability \( \pi_A^i > 0 \) or in the first period with probability \( 1 - \pi_A^i \). In addition, if it has a bad contagion shock, the country may have a crisis in the first period with probability \( \pi_C^i > 0 \) or in the second period with probability \( 1 - \pi_C^i \). So \( \pi_A^i \) and \( \pi_C^i \) represent the chances of the reversion of the timing order of crises (note that the basic model assumes \( \pi_A^i = \pi_C^i = 0 \)). In the following, we examine both cases where the probabilities of timing reversions are exogenously given and where they are endogenously determined.

First, consider the case where the probabilities of reversions are exogenously given by \( \pi_A^i = \pi_A \) and \( \pi_C^i = \pi_C \). In this case, the probability of a country’s having a crisis in the first period is

\[
\theta_1^i = (1 - \pi_A^i)\theta_A^i + \pi_C^i(1 - \theta_A^i)\theta_C^i,
\]  

and the probability of having a crisis in the second period is

\[
\theta_2^i = \pi_A^i\theta_A^i + (1 - \pi_C^i)(1 - \theta_A^i)\theta_C^i.
\]

In this case, the crisis probability of each country \( (\theta^i) \) is the same as in the basic model (that is, \( \theta^i = \theta_1^i + \theta_2^i = \theta_A^i + (1 - \theta_A^i)\theta_C^i \)). Under both laissez-faire and existing lending facilities, the optimization problem of a borrower country in this variant model then becomes the same as that of the basic model. As a result, the optimal level of crisis prevention efforts is determined to be \( e^{iL} = 1 \) under laissez-faire, and \( e^{iT} = 0 \) under existing lending facilities.

Under the timing-based lending facility, the optimization problem in this variant is slightly altered. The government of each borrower country solves the following optimization problem [P4]’ instead of [P4]:

\[
[P4]' \max_{e_i} w_i = \left[ 1 - \theta_A(e^i) - (1 - \theta_A(e^i))\theta_C^i \right] \ln(A - r/2)
\]

welfare in both cases of crisis contagion and common shocks.
\[ +((1 - \pi_A)\theta_A(e^i) + \pi_C(1 - \theta_A(e^i))\theta_C^i)(\ln B - \lambda) + (\pi_A\theta_A(e^i) + (1 - \pi_C)(1 - \theta_A(e^i))\theta_C^i)\ln B - \psi e^i. \] (48)

We can easily show that the derivative of \(w_i\) with respect to \(e^i\) in Eq.(48) is positive as long as \(\pi_A\) is lower than a certain threshold level, \(\pi^{th}\langle 1\). As a result, the optimal solution under the timing-based facility in this variant of the model is the same as that of the basic model: \(e^{i,N} = 1\).

This suggests that even when autonomous crises are allowed to occur later than imported crises, the main results of the basic model would not change as long as the chances of timing reversions are not too high.

Second, consider the case where each country can affect the probabilities of timing reversions. More specifically assume that

\[
\begin{align*}
\pi_A^i &= f(s^i/\mu_s) + \pi_A, \\
\pi_C^i &= -f(s^i/\mu_s) + \pi_C, \quad f' > 0, \quad f(1) = 0,
\end{align*}
\] (49)

where \(s^i ([0,s^{max}])\) represents a country’s efforts to have a crisis in the second rather than the first period, and \(\mu_s\) is the average of \(s^i\) (i.e., \(\mu_s = \int_0^N s^i di/N\)). The positive first derivative \(f' > 0\) tells us that the greater efforts a country makes (compared to those of the other countries), the more likely the country is to have a crisis in the second period. In addition, \(f(1) = 0\) indicates that if all the countries make the same amount of efforts, the chances of the timing reversion do not change. For simplicity, assume that \(s^i\) does not entail any costs. Assume also that each country seeks to maximize the utility of the representative consumer by choosing \(e^i\) and \(s^i\).

Under both laissez-faire and existing lending facilities, borrower countries in this variant of the model do not have any incentive to delay the crisis. Therefore, the optimal solution for \(s^i\) could be zero for all the borrower countries.

Under the timing-based facility, each government would try hard to raise its own \(\pi_A^i\) by choosing the maximum \(s^i\). Note that such an effort would substantially reduce the chance of having a crisis in the first period only if the other countries did not raise \(s^i\). However, every country has an incentive to raise \(\pi_A^i\) by raising \(s^i\) to its maximum. As a result, in equilibrium, all the borrower countries put their maximum efforts: \(s^i = s^{max}\). Given \(f(1) = 0\), it also follows that no country would succeed in raising \(\pi_A^i\).

The above discussion suggests that the optimization problem of this variant of the model becomes the same as that of the basic model. As a result, the introduction of a potential racing for crisis delays would not alter the main results of the paper. \(^{27}\)

\(^{27}\) One may imagine a hypothetical case where under the timing-based lending facility,
C. Crisis-Prevention Efforts Without Bound

In the basic model, we focus on the case where the timing-based lending facility and laissez-faire induce the same amount of crisis prevention efforts. A critical assumption needed to make such a case was that crisis prevention efforts, \( e^i \), cannot be greater than one. However, we can modify the basic model to allow crisis prevention efforts not to be bounded from the above. In this more general case, the equilibrium level of crisis prevention efforts could differ between the timing-based lending facility and laissez-faire.

Assume that crisis prevention efforts could take any nonnegative value, that is, \( e^i \geq 0 \), and suppose that
\[
\frac{\partial^2 \theta^i_A}{\partial (e^i)^2} > 0. \tag{50}
\]
We can then have an interior solution for the optimal crisis prevention efforts under both laissez-faire and the timing-based lending facility. Under laissez-faire, we can derive the optimal (and equilibrium) crisis prevention efforts by equating \( \frac{\partial w^i}{\partial e^i} \) in Eq.(17) to zero, that is,
\[
\frac{\partial w^i}{\partial e^i} = \frac{\partial \theta^i_A}{\partial e^i} [(1 - \gamma abn_1)(- \ln (A - r/2) + \ln B - \lambda)] - \psi = 0. \tag{51}
\]
Under the timing-based lending facility, the optimal crisis prevention effort is derived from
\[
\frac{\partial w^i}{\partial e^i} = \frac{\partial \theta^i_A}{\partial e^i} [(1 - \gamma a(b - 1)n_1)( - \ln (A - r/2) + \ln B - \lambda)] - \psi = 0, \tag{52}
\]
which equates \( \frac{\partial w^i}{\partial e^i} \) in Eq.(29) to be zero.

From a comparison between Eqs. (51) and (52), we have
\[
e^i_L < e^i_N. \tag{53}
\]
This suggests that the timing-based lending facility can be more effective even in preventing moral hazard (or inducing crisis prevention efforts) than laissez-faire. The countries may precipitate rather than postpone crisis: if a country was already hit by crisis, some other countries with high chances of crisis would try to strategically default to use the window of opportunity to receive rescue financing. In reality, however, such a case is unlikely to happen. Given huge welfare cost of crisis (even when being bailed out), the countries would not try to precipitate crisis unless the chance of having a crisis in the near future is extremely high. Moreover, when no country has been yet hit by crisis, each country would exert much crisis prevention efforts so as not to become the first crisis country. As a result, no country would have a chance of crisis high enough to induce them to precipitate crisis after another country is hit by crisis.
reason is as follows. Recall that the utility in the event of crisis in the second period is given by $\ln B$ for the case of the timing-based facility, and $\ln B - \lambda$ for laissez-faire. So a country has higher utility in the event of crisis in the second period under the timing-based lending facility than under laissez-faire. The crisis-time utility gain from the timing-based facility is magnified if the chance of crisis in the second period ($\theta_2^i$) increases, while the chance is raised by crisis prevention efforts (recall that $\theta_2^i = (1 - \theta_A^i)\theta_C^i$). Therefore, under the timing-based facility, crisis prevention efforts would raise the welfare of a borrower country more than under laissez-faire, which induces the country to exert more crisis prevention efforts. 28

VIII. Conclusion

This paper has proposed a new lending facility that differentiates rescue financing provided to different countries based on the timing of crises. This new facility basically would take significant account of the timing of crises: it would refuse loans to the country that started a crisis but unconditionally provide rescue financing to countries where crises occurred within a short period after the initial crisis. Given that the key to developing a desirable rescue-lending facility lies in how to reduce moral hazard and crisis contagion at the same time, the paper has introduced a simple model and used it to explore how laissez-faire, existing crisis-lending facilities, and the proposed timing-based facility each affect the problems of moral hazard and crisis contagion. It suggests that laissez-faire may be effective in abating the moral hazard problem but ineffective in containing crisis contagion. Existing international crisis-lending facilities may be effective in resolving the contagion effect but ineffective in abating the moral hazard problem. In contrast, the newly proposed lending facility could effectively resolve both moral-hazard and contagion problems at the same time, particularly under information asymmetry. Hence, the timing-based lending facility merits attention as an alternative that would overcome the shortcomings of existing lending facilities. Using a variant of the model, this study has shown that even where crises are caused by common shocks instead of crisis contagion, the timing-based lending facility would be more effective in reducing risks of global crisis than the laissez-faire approach or use of existing facilities.

Note that the logic of the timing-based lending facility proposed here is general and could be easily applied to the crisis-managing authorities or lender of last resort in the domestic context, by replacing “countries” by “firms” and “crisis” by “default.” The domestic lender of last resort may use this type of lending facility, based on the timing order of crises, to effectively mitigate both the problems of moral hazard and default contagion.

28 Corsetti, Guimarães, and Roubini (2003) suggest that contingent liquidity assistance provided by the international lender of last resort can promote countries to undertake efficiency-enhancing reform efforts.
Proof of (36)

To formally prove this, express $\Delta W_{N-T}$ as a function of $\theta_{A}^{\text{min}}$, that is, $\Delta W_{N-T} = \Gamma(\theta_{A}^{\text{min}})$. With $\theta_{A}^{\text{min}} < 1/2$, we then have

$$\frac{\partial \Gamma(\theta_{A}^{\text{min}})}{\partial \theta_{A}^{\text{min}}} = N[-[1 + \gamma a(b - 1)N(1 - 2\theta_{A}^{\text{min}})](\ln(A - r/2) - \ln B) - \lambda - \frac{1}{1 - \theta_{A}^{\text{min}}}] < 0.$$  \hspace{1cm} (54)

Given (35), we also have

$$\lim_{\theta_{A}^{\text{min}} \to 0} \Gamma(\theta_{A}^{\text{min}}) = N[\theta_{A}^{\text{max}}[1 + (1 - \theta_{A}^{\text{max}})(a - 1)\gamma N(b - 1)](\ln(A - r/2) - \ln B) \hspace{1cm} (55)$$

$$-\psi] > 0.$$

Finally, $\lim_{\theta_{A}^{\text{min}} \to \theta_{A}^{\text{max}}} \Gamma(\theta_{A}^{\text{min}})$ is given by

$$\lim_{\theta_{A}^{\text{min}} \to \theta_{A}^{\text{max}}} \Gamma(\theta_{A}^{\text{min}}) = N[-(\theta_{A}^{\text{max}})(1 - \theta_{A}^{\text{max}})(\gamma N(b - 1))(\ln(A - r/2) - \ln B) \hspace{1cm} (56)$$

$$-\theta_{A}^{\text{max}} \lambda - \psi + \ln(1 - \theta_{A}^{\text{max}})] < 0.$$

Therefore, there exists a threshold level, $\theta_{A}^{th} (\leq \theta_{A}^{\text{max}})$, which satisfies $\Gamma(\theta_{A}^{th}) = 0$. It then follows that for any $\theta_{A}^{\text{min}} < \theta_{A}^{th}$, we have $\Gamma(\theta_{A}^{\text{min}}) > 0$, that is $W_{N} > W_{T}$. 

Financing of Reserves for Bailouts

The basic model assumes that the international crisis lender already raised the reserves for rescue financing so that countries now do not have to make contributions for that. This appendix considers a different case, where no reserves were already raised and therefore the international crisis lender has to raise it from the countries at the beginning of the initial period. Let \( R \) denote the amount of the reserves needed, and \( \omega \) be the fraction of the initial capital endowment that each country has to contribute.

Under laissez-faire, where no rescue financing is provided to the countries in crisis, the amount of the reserves needed for rescue financing (\( R \)) is zero, and so is \( \omega \). Therefore, this variant of the model has the same optimization problem and solution (that is, \( e_i^L = 1 \)) as the basic model.

Under existing lending facilities, the international crisis lender provides rescue financing regardless of the timing of crises. The rescue financing to a crisis country amounts to \( \rho = rk_i^f = rk(1 - \omega) = (r/2)(1 - \omega) \). If a borrower country is not hit by crisis, its representative consumer consumes \( (A - r/2)(1 - \omega) \). If the country is hit by crisis, the representative consumer consumes \( B(1 - \omega) \), while the country does not incur the cost due to nonrepayment, \( \lambda \). Using these, the maximization problem of a borrower country is given by

\[
[P2]^\prime \quad \max_{e_i} w_i = [1 - \theta_A(e_i^f) - (1 - \theta_A(e_i^f))\gamma(a - 1)(b - 1)n_1] \ln((A - r/2)(1 - \omega)) \\
+ [\theta_A(e_i^f) + (1 - \theta_A(e_i^f))\gamma(a - 1)(b - 1)n_1] \ln(B(1 - \omega)) - \psi e_i^f 
\]

which, under Condition (i), yields \( e_i^T = 0 \), as in the basic model.

Under the timing-based lending facility, rescue financing is provided to any crisis country in the second period, but to no country in the first period. As a result, the optimization problem of each borrower country is given by:

\[
[P4]^\prime \quad \max_{e_i} w_i = [1 - \theta_A(e_i^f) - (1 - \theta_A(e_i^f))\gamma a(b - 1)n_1] \ln((A - r/2)(1 - \omega)) \\
+ \theta_A(e_i^f)(\ln(B(1 - \omega)) - \lambda) \\
+ (1 - \theta_A(e_i^f))(\gamma a(b - 1)n_1) \ln(B(1 - \omega)) - \psi e_i^f, \tag{58}
\]

which, under Condition (i), yields \( e_i^N = 1 \).

The above discussions suggest that the assumption that the international crisis lender needs to raise the reserves for rescue financing does not change the equilibrium level of crisis prevention efforts of the basic model.
This variant of the model slightly changes the magnitude of the world’s welfare under existing lending facilities and the timing-based facility. Under existing lending facilities, total reserves needed for rescue financing (denoted by $R_T$) and an individual country’s contribution (denoted by $\omega^T$) are given by

\[
R_T = \theta^A_N (1 + (1 - \theta^A_N)\gamma(a - 1)(b - 1)N)(r/2) (1 - \omega),
\]

\[
\omega^T = \frac{(1 + (1 - \theta^A_N)\gamma(a - 1)(b - 1)N)(r/2)}{1 + (1 - \theta^A_N)\gamma(a - 1)(b - 1)N).}
\]

Using these, the world’s welfare under existing rescue financing facilities (denoted by $W_T$) is

\[
W_T = N[(1 - \theta^A_N)\gamma(a - 1)(b - 1)N\theta^A_N \ln(A - r/2) + (\theta^A_N + (1 - \theta^A_N)\gamma(a - 1)(b - 1)N\theta^A_N) \ln B + \ln(1 + \omega^T)] + N[\ln((r/2)(1 - \omega^T))].
\]

In the case of the new lending facility, total reserves needed for rescue financing and an individual country’s contribution are given by

\[
R_N = (1 - \theta^A_N)\gamma(a - 1)(b - 1)N\theta^A_N N^2 (r/2)(1 - \omega)
\]

\[
\omega^N = \frac{(1 - \theta^A_N)\gamma(a - 1)(b - 1)N\theta^A_N N (r/2)}{1 + (1 - \theta^A_N)\gamma(a - 1)(b - 1)N (r/2)}.
\]

which yields the world’s welfare to be

\[
W^N = N[(1 - \theta^A_N)\gamma(a - 1)(b - 1)N\theta^A_N \ln(A - r/2) + \theta^A_N \ln B - \lambda)
\]

\[
+ (1 - \theta^A_N)(\gamma(a - 1)(b - 1)N\theta^A_N) \ln B + \ln(1 - \omega^N) - \psi]
\]

\[
+ N[\ln((r/2) + \ln(1 - \omega^N) + \ln(1 - \theta^A_N)).
\]

Suppose that the welfare gain from providing rescue financing is larger than the welfare costs due to having reserves, more specifically

\[
\gamma a N\theta^A_N (1 - \theta^A_N)(\ln(A - r/2) - \ln B) > 2 \ln(1 - \omega^N).
\]

Given (35) and (65), it then follows that there exists a threshold level of $\theta^A_N$ below which we have

\[
\Delta W^N - \Delta W^{N^T} > 0
\]

which suggests that the main results of the paper are not affected by assuming that the international crisis lender needs to raise reserves for bailouts.

Furthermore, we may introduce another variant of the model to examine the case where the rescue financing provided to crisis countries ($\rho$) is just a pure lending that needs to be
repaid later (see Krueger, 2002; Jeanne and Zettelmeyer, 2001; Joshi and Zettelmeyer, 2003). In this variant, we introduce an additional period, \( t = 3 \). Assume that each country has additional income in \( t = 3 \), denoted by \( V^i \), which is different from output at the end of the second period, \( y^i (= A^i k^i) \). Suppose that countries can repay \( \omega \) to the international crisis lender in \( t = 3 \) out of \( V^i \). We can then show, in an analogous way to the above, that this variation of the model does not alter the main results of this study.
REFERENCES


Krueger, Anne, 2002, “Preventing and Resolving Financial Crises: The Role of Sovereign Debt Restructuring,” a speech at Latin American Meeting of the Econometric Society, IMF.


