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Missing Link: Volatility and the Debt Intolerance Paradox

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Missing Link: Volatility and the Debt Intolerance Paradox

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Abstract

A striking feature of sovereign lending is that many countries with moderate debt-to-income ratios systematically face higher spreads and more stringent borrowing constraints than others with far higher debt ratios. Earlier research has rationalized the phenomenon in terms of sovereign reputation and countries' distinct credit histories. This paper provides theoretical and empirical evidence to show that differences in underlying macroeconomic volatility are key. While volatility increases the need for international borrowing to help smooth domestic consumption, the ability to borrow is constrained by the higher default risk that volatility engenders.

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I. INTRODUCTION

It is a well documented empirical regularity that developing countries typically face an upward sloping supply schedule for international debt, and may be altogether excluded from international capital markets during bad states of the world (Diaz-Alejandro, 1984; and Sachs, 1989; Eichengreen and Lindert, 1989). In a recent paper, Reinhart, Rogoff, and Savastano (2003) take this evidence one step further. Combining macroeconomic data for the post-1970 period with information about sovereigns' credit histories since the early nineteenth century, they argue that an important subgroup of middle-income countries or "emerging markets" have been *systematically* afflicted by what they call "debt intolerance." That is, even though their external debt-to-GDP ratios are moderate by international standards and substantially lower than those of several high-income countries, these economies are perceived as riskier and unable to tolerate as much debt. Simply put, their sovereign riskiness appears to be out of proportion to the size of the respective debt burdens.

To explain this phenomenon, Reinhart, Rogoff, and Savastano (2003) invoke history. Virtually all these countries have tarnished credit histories, with several of them having defaulted a few times on their public debts. To the extent that those that have defaulted once or more in the past are likely to do so again in the future, the market threshold of what can be considered "safe" borrowing levels for these countries tends to be lower.[2] As a theoretical story, however, this argument raises some important questions. For one, it touches on the contentious issue of what lenders take into account when evaluating sovereign risk.[3] Also, it neither explains what causes serial defaulters to default in the first place, nor how most of today's advanced economies—which have also defaulted several times in their history—managed to graduate out of the debt intolerant "club."

This paper advances a simple but arguably more fundamental explanation for the debt intolerance phenomenon. We contend that the high volatility of macroeconomic aggregates—in

[2] Lindert and Morton (1989) find that countries that defaulted over the 1820–1929 period were, on average, 69 percent more likely to default in the 1930s, and those that incurred in arrears and concessionary reschedulings during 1940–79 were 70 percent more likely to default in the 1980s. The main shortcoming of these estimates, however, is that they are not conditioned by changes in countries' fundamentals. Estimates of credit risk transition probability matrices conditional on a variety of macroeconomic fundamentals are provided in Hu, Kiesel, and Perraudin (2003). Their estimation exercise, however, is limited to the post-1980 period.

[3] For instance, on the one hand, Jorgensen and Sachs (1989) looking at the interwar and early post–World War II comparisons of credit access to sovereigns with distinct repayment records, find that international capital markets have done a fairly poor job in discriminating "bad" from "good" borrowers. Looking at data between 1968 and 1981, Ozler (1993), on the other hand, finds that past repayment record is statistically significant in explaining differences in sovereign spreads across her sample of 26 developing countries.

particular, of domestic output and external terms of trade—is a key factor in the sovereign risk of many developing countries. We argue that this greater volatility is associated with higher default probability, and, as a result, these countries hit borrowing constraints at lower levels of indebtedness. To the extent that such volatility stems from structural and hence slowly evolving factors, the phenomenon can be fairly persistent even if there is scope for these countries to gradually evolve out of this state. In this sense, we view the debt intolerance phenomenon as another—and so far relatively neglected—manifestation of macroeconomic volatility on developing country welfare. The evidence provided in this paper thus bridges a gap between the literature on sovereign debt and that on the adverse effects of macroeconomic volatility on growth and welfare (e.g., Mendoza, 1995, 1997; Ramey and Ramey, 1995; Agénor and Aizenman, 1998; Caballero, 2000; and Acemoglu and others, 2003).

At the core of our proposed explanation to debt intolerance is a simple optimizing model of sovereign borrowing. The model is close in spirit to those advanced in Sachs and Cohen (1985) and Grossman and Van Huyck (1988), which are further discussed and simplified in Obstfeld and Rogoff (1996, Chapter 6). We assume that sovereign borrowing is driven by the desire to smooth consumption in a world of imperfect capital markets, where sovereigns cannot issue income-contingent debt or fully commit to repay, and lenders' enforcement is limited. We introduce frictions associated with costly default and deadweight losses in the model, and study the role of income shocks in determining default probabilities and optimal debt levels.

Two main theoretical results are derived. First, in the model, it is rational for the borrower to default when hit by a sufficiently large negative shock, which justifies the interest rate spreads imposed by risk-neutral lenders who must break even. Accordingly, we show that income volatility raises spreads and lowers the maximum debt threshold beyond which the sovereign borrower is unable to borrow. Second, we model the optimal choice of debt directly, unlike previous studies that have examined default under the simplifying assumption of fixed levels of debt (see Grossman and Van Huyck, 1988, Grossman and Han, 1999, and Alfaro and Kanczuk, 2002). We find that the relationship between volatility and optimal borrowing levels is complex and possibly nonmonotonic. This is not altogether surprising: on the one hand, volatility makes the sovereign more eager to borrow to smooth consumption, but, on the other, it deters borrowing through higher spreads.

We then look at the empirical evidence in light of these theoretical results. We first examine the extent to which volatility explains sovereign risk, over and above countries' repayment histories. Logit estimates of default probabilities in a cross-country panel clearly indicate that it does: output and terms of trade volatility are highly significant in explaining sovereign risk, a result that is robust to the inclusion of the various explanatory variables considered in previous studies. Furthermore, our estimates also show that once volatility variables are included in the regression, the credit history variable used by Reinhart, Rogoff, and Savastano (2003) is no longer statistically significant, thereby suggesting that countries' credit histories are, at least in part, a proxy variable for the effects of volatility on sovereign risk. Finally, we use the same dataset and a regime-switching econometric approach to examine how volatility affects sovereign indebtedness. As noted above, a rise in volatility increases loan demand for consumption smoothing purposes, but also has a supply deterrent effect through higher spreads, which may become binding at times; so we empirically examine the switch between the two regimes. Our empirical results indicate that the supply effect predominates most of the time, so that the net effect of volatility on indebtedness tends to be negative.

The paper is structured as follows. Section II lays out our model to examine the effects of income volatility on default risk and optimal debt. In Section III we empirically relate the model's main propositions to a variety of relevant statistics for a panel spanning 31 years (1970–2001) and 26 emerging markets whose sovereigns have been regular borrowers in international capital markets and for which sufficiently consistent macroeconomic data series are available. Logit regressions for default probabilities and a regime-switching model of debt determination are then estimated to gauge the effects of volatility on those two variables. Section IV concludes with a discussion of the main findings.

II. The Model

As in much of the theoretical literature, sovereign borrowing is assumed to be driven by a desire to smooth consumption in the face of domestic income shocks. This setting can be viewed in terms of either a sovereign that borrows to smooth its own consumption given volatile revenues, or a benevolent government that borrows on behalf of its citizens to smooth their consumption given the variability of national income. We consider a two-period time horizon. The sovereign chooses its level of borrowing in the first period in order to smooth consumption the next period. The specifics of the model are as follows.

A. Domestic Output

The sovereign's autarkic real income in the absence of borrowing is $\overline{Y}+\varepsilon$ in period 2, where $\bar{Y}$ is (the terms of trade adjusted) mean output and $\varepsilon \in [-\varepsilon_m, +\varepsilon_m]$ denotes a random shock with mean zero. We assume that $\overline{Y}-\varepsilon_m > 0$, to rule out negative output. To keep things simple, we do not model the source of these shocks, but one can think of them as emanating from exogenous changes in terms of trade, technology, weather, or any other source outside national control. If the sovereign chooses to borrow, the borrowed funds are invested either in domestic ventures or held as central bank reserves, to augment period-2 consumption. In either case, we assume that they earn the international risk free interest rate *r*. For ease of notation, our formulation uses the gross risk-free interest rate, denoted as R=1+r. Thus, if the sovereign borrows $D > 0$ in the initial period, income in period 2 is:

$$Y_2(D) = \overline{Y} + \varepsilon + RD\,. \tag{1}$$

B. The Debt Contract

Debt is accompanied by a contractual repayment obligation, but the sovereign may choose to default in some states. Debt D in period 1 requires the sovereign to repay $R_L D$ in period 2, where the spread between the contractual rate, R_L, and the international risk-free rate, R, reflects country-specific default risk. In our model, the level of indebtedness affects the likelihood of default, and through that, it affects R_L. We thus derive a functional relation, $R_L = R_L(D)$.

To model this, we follow Sachs and Cohen (1985) and assume that lenders have access to an enforcement technology that allows them to capture a fraction η of a defaulter's aggregate period-2 income, $Y_2(D)$. We assume that η lies between 0 and 1. It is rational in this two-period context for the sovereign borrower to repay whenever the repayment obligation is less than what the lender can capture in case of default. Given proportional capture, it will be rational to repay for high realizations of output (i.e., high realizations of ε). For low realizations, borrowers will choose to default and lenders will confiscate a fraction of the output. Thus repayments are state-contingent:

$$P(\varepsilon, R_{L,} D) = Min[R_L D, \eta Y_2(D)]. \tag{2}$$

With this structure, there exists a critical value e such that the borrower will repay if and only if the random shock $\varepsilon \geq e$. Assuming that $-\varepsilon_m < e < \varepsilon_m$, we have

$$P(\varepsilon, R_L, D) = \begin{cases} R_L D & for \quad e \leq \varepsilon \leq \varepsilon_m \\ \eta[\bar{Y} + \varepsilon + RD] & for \quad -\varepsilon_m \leq \varepsilon < e \end{cases}$$

where
$$e(R_L, D) \equiv \frac{[R_L - \eta R]D}{\eta} - \bar{Y} \tag{3}$$

C. Lenders' Supply Schedule and Interest Rate Spreads

Given positive capture rates, default is partial. The size of the default is given by the difference between the contractual repayment obligation and actual repayments:

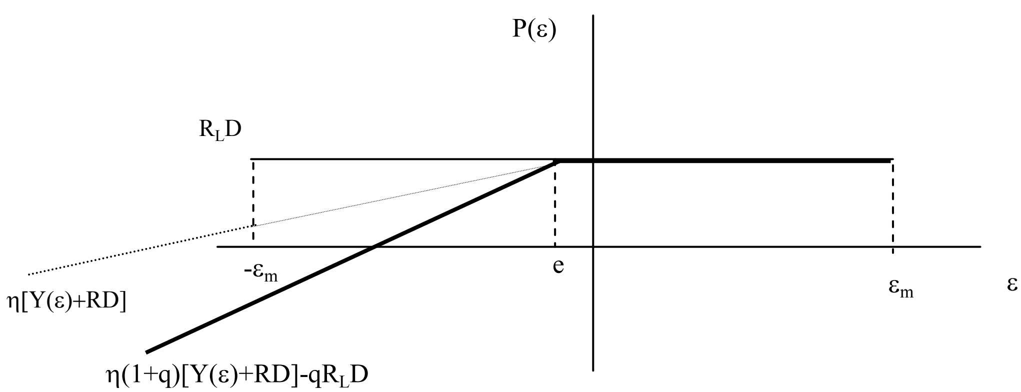
$$S(\varepsilon, D) = R_L D - P(\varepsilon, R_L, D) \tag{4}$$

In addition to the above direct costs (and possibly administrative, legal and political costs), default may also involve a negative externality—say, if default in one country increases the risk of default by other borrowers through contagion effects. In this model we assume that the magnitude of such spillover costs is proportional to size of the default. In particular, we assume that a default of size *S* imposes a cost (1+*q*)*S* on the lender, where the parameter $q \geq 0$ is a measure of the spillover costs. In the presence of such costs, the net return to lenders is given by contractual repayments less total default costs:

$$P^*(\varepsilon, R_L, D) = R_L D - (1+q)S(\varepsilon, D). \tag{5}$$

The existence of a wedge between repayments and the return to lenders is shown in Figure 1. Note that in our structure, the wedge is increasing and linear in the size of the default.[4] This structure implies that the default, when it occurs, is renegotiation proof. In contrast, models that imply a discrete jump in the costs associated with default are open to the possibility that lender and borrower renegotiate to avoid these fixed costs.

Figure 1. The Return to Lenders



In keeping with the standard assumptions in the literature, we assume that the capital market is competitive and that lenders are risk neutral. This implies that for a given level of borrowing *D*, lenders must choose R_L to break even:

$$\int_{-\varepsilon_m}^{\varepsilon_m} P^*(\varepsilon, R_L, D)\pi(\varepsilon)d\varepsilon = RD, \tag{6}$$

where $\pi(\varepsilon)$ is the density function of random shocks:

$$\int_{-\varepsilon_m}^{\varepsilon_m} \pi(\varepsilon)d\varepsilon = 1 \ . \tag{7}$$

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[4] Unlike the structure here, some models assume that default costs display a discrete jump at the point of default. In such cases default may not be renegotiation proof, due to the possibility that the lender and borrower renegotiate to avoid these discrete costs. With our structure, "small" defaults will have only small costs.

Combining (5) and (6), the break-even condition can be written as:

$$(R_L - R)D = \int_{-\varepsilon_m}^{e(R_L,D)} \eta(1+q)[e(R_L,D)-\varepsilon]\pi(\varepsilon)d\varepsilon \qquad (8)$$

Equation (8) defines a functional relationship between the level of borrowing *D* and the contractual interest rate R_L that allows the lender to break even. This functional relationship, which we denote as $R_L(D)$, allows us to measure the markup over the risk-free rate *R* necessitated by default risk. Since this relationship is central to our story about the role of volatility in accounting for the debt intolerance phenomenon, Proposition 1 explores its properties.

Proposition 1

(a) $R_L(D)$ *is well-defined for levels of debt in some bounded interval* $[0, D_{\max})$, *where* $D_{\max}$ *depends, inter alia, on the probability distribution of shocks,* $\pi(\varepsilon)$.

(b) $R_L(D)=R$ *for* $D \in [0, \frac{\eta}{1-\eta}\frac{\overline{Y}-\varepsilon_m}{R}]$. For higher values of *D,* $R_L(D)$ *exceeds R and is strictly increasing in D.*

(c) $R_L(D)$ *is increasing in the variance of shocks.*

A formal proof is in the Appendix, but the intuition can be outlined here. The spread between the contractual interest rate R_L and the risk-free interest rate *R* reflects default risk. When debt is low relative to mean income (i.e, when $D/\overline{Y}$ is small), the threat of capture precludes default, so that competitive interest rates will equal the risk-free rate. At higher *D*, default becomes more likely. An increase in the level of debt clearly increases default risk since contractual repayment obligation rises at the rate R_L while the cost of default (in terms of loss through capture) rises at the strictly lower rate, ηR. In order for the lender to break-even, the interest-rate spread $R_L(D)-R$ must rise with *D*. Finally, R_L is increasing in the variance of shocks: the return to lenders is concave in ε, so that an increase in variance needs to be compensated by higher R_L in order that lenders break-even.

While these results hold quite generally, it is useful to specialize the analysis to the case in which the distribution of output shocks is uniform:[5]

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[5] We use the uniform distribution for presentational purposes. Other standard distributions, such as the normal distribution, make the analysis less tractable but lead to similar conclusions.

$$\pi(\varepsilon)=\left\{\begin{array}{cc}\dfrac{1}{2\varepsilon_m} & \text{if } -\varepsilon_m\le\varepsilon\le\varepsilon_m\\ 0 & \text{otherwise}\end{array}\right\}. \tag{9}$$

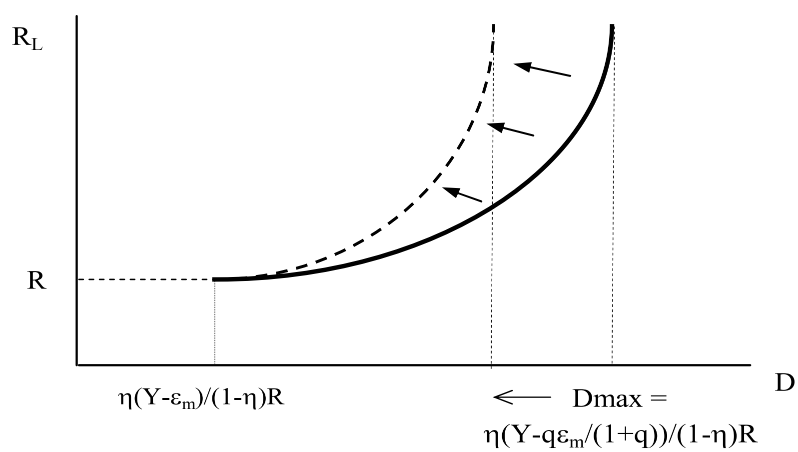
For this case, the Appendix shows that the spread function takes the following form:

$$R_L(D)=\left(\begin{array}{cc} R & \text{for } D\le\dfrac{\eta(\overline{Y}-\varepsilon_m)}{(1-\eta)R}\\ R+\left[\sqrt{\dfrac{\eta\varepsilon_m}{D(1+q)}}-\sqrt{\eta\dfrac{\overline{Y}}{D}-R(1-\eta)-\dfrac{\eta q\varepsilon_m}{D(1+q)}}\right]^2 & \text{for } \dfrac{\eta(\overline{Y}-\varepsilon_m)}{(1-\eta)R}<D<D_{\max}\end{array}\right) \tag{10}$$

where $D_{\max}=\dfrac{\eta}{(1-\eta)R}\left[\overline{Y}-\dfrac{q\varepsilon_m}{(1+q)}\right]$.

Figure 2 displays $R_L(D)$ as a function of levels of debt, given uniformly-distributed shocks. At low levels of debt, potential income losses from capture are large enough to preclude default, so that R_L equals R. Debt above $D_{\max}$ is not sustainable because the required R_L rises without bound. Note that this credit ceiling, $D_{\max}$, is inversely related to volatility. For the uniform distribution, volatility depends directly on ε_m; the larger is ε_m, the lower is the credit ceiling. This is depicted in the diagram as an inward shift of loan supply schedule. Clearly, the first point is not new: previous models of sovereign debt have postulated the existence of a vertically sloped loan supply schedule for sufficiently large values of D (see, e.g., Sachs, 1984; Sachs and Cohen, 1985). What is new here is how we relate such a threshold to the underlying volatility of shocks.

Figure 2. $R_L(D)$ as a function of D



D. The Sovereign Borrower's Optimal Choice of Debt

The sovereign borrower has a concave utility function and cares only about the expected utility of period-2 consumption:

$$E[U(C_2(\varepsilon, D))] \tag{11}$$

where $U'(C) > 0$ and $U''(C) < 0$. Consumption in period 2 is given by aggregate output net of repayments:

$$C_2(\varepsilon, D) = \overline{Y} + \varepsilon + RD - P(\varepsilon, R_L, D) \tag{12}$$

Consumption in the default and the non-default states is given by:

$$C_2(\varepsilon, D) = \begin{Bmatrix} C_L = (Y + \varepsilon + RD)(1-\eta) & \text{if } \varepsilon < \text{e} \\ C_H = Y + \varepsilon + (R - R_L)D & \text{if } \varepsilon > \text{e} \end{Bmatrix} \tag{13}$$

The borrower's problem is to choose D to maximize expected utility,

$$Max_D \int_{-\varepsilon_m}^{e(R_L,D)} U(C_L)\pi(\varepsilon)d\varepsilon + \int_{e(R_L,D)}^{\varepsilon_m} U(C_H)\pi(\varepsilon)d\varepsilon \tag{14}$$

subject to the condition that contractual interest rates satisfy the break-even condition

$$R_L = R_L(D). \tag{15}$$

For any *D*, let $\phi(D)$ denote the ex-ante probability of default. Clearly, ϕ varies with *D* directly and through R_L, as the latter affects the repayment obligation. We have the following proposition.

<u>Proposition 2</u>

An interior maximum to the sovereign borrower's optimization problem, if it exists, is given by D^* such that:

$$\phi(D^*) = \frac{1}{1+q} \frac{\int_{-\varepsilon_m}^{e(R_L(D^*),D^*)} U'(C_L)\ \pi(\varepsilon)d\varepsilon}{\int_{-\varepsilon_m}^{e(R_L(D^*),D^*)} U'(C_L)\ \pi(\varepsilon)d\varepsilon + \int_{e(R_L(D^*),D^*)}^{+\varepsilon_m} U'(C_H)\ \pi(\varepsilon)d\varepsilon} \tag{16}$$

Once again, the proof is in the Appendix but the intuition can be outlined here. In the model, debt is costly: in outcomes where default does not occur, the cost of carrying debt is $D(R_L - R)$. At the same time, debt is valuable because it enables partial insurance against adverse shocks.

Default on positive levels of debt yields results in consumption (net of capture) of $(1-\eta)[\overline{Y}+RD+\varepsilon]$, while consumption in the absence of debt, would have been $\overline{Y}+\varepsilon$. If so, debt provides partial insurance whenever $(1-\eta)RD>\eta[\overline{Y}+\varepsilon]$.[6]

The condition for the maximum in equation (16) highlights the role of the deadweight loss parameter *q* in determining the nature of the optimum. If *q*=0, the problem does not admit an interior maximum with a strictly concave utility function. To see why note that with *q*=0, the condition for an interior maximum reduces to:

$$\phi=\frac{\int_{-\varepsilon_m}^{e}U'(C_L)\pi(\varepsilon)d\varepsilon}{\int_{-\varepsilon_m}^{e}U'(C_L)\pi(\varepsilon)d\varepsilon+\int_{e}^{\varepsilon_m}U'(C_H)\pi(\varepsilon)d\varepsilon}. \qquad (17)$$

For a strictly concave utility function, the expression on the right is strictly larger than ϕ, thus ruling out an interior solution.[7]

E. Volatility and Optimal Debt

How does the optimal level of debt and likelihood of default vary with volatility of shocks? For simplicity, we examine this for the special case with uniformly-distributed shocks. We have the following proposition.

Proposition 3 *Consider the case with uniformly-distributed shocks*

(*a*) *At the optimum,* $e(R_L(D^*),D^*)$, *the threshold below which the borrower defaults, is increasing in the volatility of shocks.*

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[6] Note that if the chosen *D* is so low that that this condition is violated at all ε, it must be violated at $\varepsilon=-\varepsilon_m$ in particular. We know from Proposition 1(b) that $R_L=R$ for that range: while debt provides no insurance in this case, it has no opportunity cost, given that borrowed resources can be invested domestically at rate *R*. Second-order conditions suggest that it would be rational then for the borrower to choose higher levels of debt. Indeed, this chain of argument implies that at optimally chosen levels of debt, the probability of default is strictly positive, and consequently, that $R_L>R$.

[7] To see why, note that if *U* were linear, the expression on the right hand side would be exactly ϕ, so that the borrower would be indifferent to the choice of *D*. With strictly concave utility, *U'* is a decreasing function, so that the right hand side would be strictly greater than ϕ.

(*b*) *The effect of volatility on the level of optimal debt is ambiguous and will depend on the borrower's degree of risk aversion.*

The first part of the proposition states that at the optimum, the threshold value $e(R_L(D^*), D^*)$ is increasing in volatility. This has a key implication for the ex-ante probability of default. Consider cases where the distribution of shocks is symmetric around its mean value of zero. It is reasonable to assume that the ex-ante probability of default is less than one half (see Rose and Spiegel (2002) for a similar assumption, and Eaton and Gersovitz (1995) for a model that justifies such an assumption). If so, $e(R_L(D^*), D^*)$ must be negative. In that case, as volatility rises, an increase in $e(R_L(D^*), D^*)$ would increase the ex-ante probability of default.[8]

The intuition behind the second result is not altogether surprising. Greater volatility increases the risk-premium for any given level of debt (in effect, as suggested by Proposition 1(c), it shifts the R_L-schedule upward); this makes debt costlier, reducing the incentive to borrow. At the same time, greater volatility in consumption increases the incentive to borrow to smooth consumption. Given these opposing tendencies, the overall effect could well be quantitatively small and ambiguous in direction.

The above claims are best illustrated by means of a numerical example. We choose the following parameter values: η=0.3, q=0.3, R=1.05.[9] We also set $\overline{Y}=100$ so that D becomes the debt ratio in percent. We consider different utility specifications for the borrower, within the standard constant elasticity of substitution (CES) class:

$$U(C)=\frac{c^{1-\theta}}{1-\theta}$$

For this class of utility functions, the intertemporal elasticity of substitution is given by $1/\theta$. We choose values ranging from θ=1 (which corresponds to the logarithmic utility function) to θ=2 (the implied inter-temporal elastiticity of 0.5 is closer to values assumed in the real business cycle literature). We also consider alternative values for volatility of shocks, as given by the range of values $[-\varepsilon_m, \varepsilon_m]$ for the uniformly-distributed shocks. The

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[8] While we state this result for the case of uniformly-distributed shocks, it holds more generally.

[9] The chosen value for η is consistent with an average share of trade flows in GDP of 30 percent, and also with what defaulters ended up transferring to creditors during the debt crisis of the 1980s (about 3 percent of their annual GDP's times 10 years), according to Cohen's (1992) estimates. The value of 1.05 for R corresponds to a risk free interest rate of 5 percent which is close to the average yield by the relevant benchmark instruments. Finally, q=0.3 is conservative guess, but one which alternative calibrations indicate that would not to alter the thrust of the results.

effect of alternative assumptions about the inter-temporal elasticity and volatility are reported in Table 1.

Table 1. Optimal Debt as a Function of Volatility, and Associated Default Probabilities for η=0.3, R=1.05 and $\overline{Y}=100$, q=0.5

Volatility	Optimal D (and associated default probability in parentheses)			
	θ=1	θ=1.3	θ=1.5	θ=2
$\varepsilon \in$ [-30, 30]	corner solution	29.26 (3%)	30.82 (10%)	33.17 (23%)
$\varepsilon \in$ [-40, 40]	26.52 (6%)	29.68 (18%)	30.94 (24%)	33.03 (36%)
$\varepsilon \in$ [-50, 50]	26.90 (19%)	29.71 (29%)	30.82 (34%)	32.44 (44%)
$\varepsilon \in$ [-60, 60]	27.05 (27%)	29.43 (37%)	30.38 (42%)	31.62 (49%)

Reading down the columns, we find that the effect of volatility on optimal choice of debt is small and, in some cases, nonmonotonic. On the other hand, the ex-ante probability of default is unambiguously increasing in volatility. Lastly for given levels of volatility (reading across any row from the left to the right), we find that optimal borrowing rises with the borrower's degree of risk aversion.

It is useful to contrast the role of volatility in this model with that in Eaton and Gersovitz's (1981) classic model of sovereign borrowing. In their infinite horizon model, default results in permanent exclusion from capital markets. To the extent that greater volatility increases the penalty of exclusion for borrowers, higher levels of volatility increase the incentive to repay and can support higher levels of debt. In our two-period model, we consider the impact of volatility not only on the desire to borrow for consumption smoothing, but also on the terms on which they borrow (i.e., the spread). If so, greater volatility may well lead to lower levels of borrowing, as the above simulations illustrate. Clearly, a multi-period extension of our model would moderate this conclusion to some extent: the penalty for default (in terms of inability to smooth consumption) would be larger in a multi-period context; this would lower the default threshold and the risk premium required to break even. The precise outcome may depend on the specifics of the model—in particular on the time-horizon of the borrower, the intertemporal elasticity of substitution, and the severity of the market exclusion penalty. But ultimately the proof the pudding is in the eating. And as the econometric results presented next overwhelmingly indicate, volatility appears to be positively associated with higher default risk and lower borrowing on average.

III. EMPIRICS

In light of the above model, testing the proposed explanation for debt intolerance requires empirically establishing two results. First, default risk should rise with income volatility holding other factors constant. In general, default risk can be measured by the interest rate spread on sovereign debt, or by the observed frequency of sovereign “credit events” such as defaults and rescheduling of repayment. Given the lack of long series on sovereign spreads, we choose, as the dependent variable in our regressions, the actual incidence of credit events over the period from 1970 to 2001.[10]

The second testable implication of the theory is that, while countries with greater volatility desire more debt than less volatile ones, they face more stringent borrowing conditions. Within the confines of the two-period set-up, this is because lending to more volatile countries is riskier: they face higher spreads, which dampens borrowing. This effect may be self-reinforcing in a richer multi-period context. Suppose greater volatility leads to greater frequency of default and raises the spreads. With higher spreads, the value of access to capital markets goes down, making default less costly. If so, the terms of access to capital markets may remain poor for highly volatile countries.

Table 2 reports some relevant descriptive statistics for a set of 26 developing countries that are mostly middle-income economies and which have been regular customers in private international capital markets.[11] Because a substantial share of these countries’ external borrowing has been undertaken by the respective national governments whose debt servicing problems have typically been the main trigger of sovereign defaults, the reported debt statistics exclude the domestic private sector external obligations. As in the remainder of the discussion, the focus is then on public sector external debt.[12]

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[10] The main source of discontinuity in emerging markets spread data series is due to the transition from syndicated loan as the main borrowing instrument in the 1970s and 1980s to bond financing in the 1990s. Moreover, existing bond spread data for much of the 1990s suffers from a coverage bias, since only the countries that defaulted in the 1980s and converted their debt into Brady bonds are represented. Unavailability of sufficiently long country spread series partly explains why other researchers also used actual information on credit events in probit or logit specifications in their empirical analyses of sovereign risk (e.g., Feder and Just, 1977; Detragiache and Spilimbergo, 2001; and Reinhart, 2002).

[11] We exclude low-income countries because they rely much more on concessional and official multilateral debt, for which our model is less relevant. A similar cutoff has been adopted by RSS.

[12] Moreover, private sector debt statistics are not very reliable for emerging markets. This is because they rely on accurate balance-of-payment recording of private sector transactions or alternatively rely on firm level survey data, rarely available for the entire thirty-year period.

The first noteworthy feature of the data is the recurrence of credit events (defined as defaults or rescheduling) in some countries and the complete absence of such events in others. Studies that consider longer periods confirm the impression of serial correlation in default (see Lindert and Morton (1989) and Reinhart, Rogoff and Savastano (2003), for instance). Table 2 also suggests that this pattern is not necessarily correlated with per capita income (measured in thousands of U.S. dollars, Ypc_us). Some serial defaulters (e.g., Argentina) are several times richer than countries that never defaulted (e.g. India).

Table 2. Selected Macroeconomic and Debt Statistics, 1970-2001

	In-sample Credit Events		Ypc_us	D/Y	D/X	$\sigma_{\Delta yr}$	$\sigma_{\Delta TOT}$
	Defaults	Reschedulings	US$	%	%	%	%
Argentina	2	0	7.7	23.5	252.1	5.1	13.6
Brazil	1	0	3.5	22.5	252.6	4.3	11.8
Chile	2	1	4.6	31.8	133.5	6.1	12.6
Colombia	0	0	1.9	23.5	152.5	2.4	14.7
Costa Rica	1	0	4.0	52.5	153.4	3.6	8.7
Ecuador	2	0	1.1	57.5	200.4	5.6	21.8
Mexico	1	0	5.8	27.7	173.6	3.7	18.9
Panama	1	0	3.4	62.3	92.9	5.4	11.2
Peru	1	2	2.0	37.5	247.5	5.5	11.8
Uruguay	1	0	6.0	34.0	169.2	4.1	11.5
Venezuela	1	0	5.0	36.0	132.8	4.4	31.8
India	0	0	0.5	17.6	247.1	3.0	6.6
Pakistan	1	0	0.4	52.7	427.5	2.6	13.5
Malaysia	0	0	3.9	24.2	39.1	6.6	7.6
Indonesia	0	1	0.7	35.7	143.4	4.1	15.7
Philipines	1	0	1.0	33.1	120.9	3.6	8.3
Thailand	0	0	2.0	14.6	49.2	4.4	9.5
Singapore	0	0	23.0	2.1	0.4	4.0	4.4
Korea	0	0	9.8	6.6	23.1	3.8	6.6
Turkey	1	0	3.0	22.4	231.3	4.4	6.2
South Africa	1	0	2.9	2.0	7.5	2.3	6.5
Egypt	1	0	1.5	43.9	215.1	4.1	12.8
Bulgaria	1	0	1.5	52.8	133.1	5.5	18.3
Russia	2	0	1.8	48.0	132.5	6.7	13.9
Hungary	0	0	4.8	44.6	117.8	4.5	11.6
Poland	1	0	4.1	36.5	162.6	6.3	3.7
Mean	0.85	0.15	4.1	32.5	154.3	4.5	12.1

Sources: Credit events from Lindert and Morton (1989), Beim and Calomiris (2001), and IMF desks. Other data are from the IMF *International Financial Statistics*, *World Economic Outlook*, and World Bank databases, and the authors' own calculations.

The second interesting feature of the data is that an average emerging market debt to GDP ratio (*D/Y*) of 33 percent is not only unremarkable, but also a lot lower than those commonly observed for most OECD countries. Within our sample, the vast majority of serial defaulters have low to moderate debt ratios. Year-by-year inspection of the data (not shown here for constraints of space) indicates that most of them the debt to GDP ratios on the eve of their defaults was under 40 percent. Table 2 also indicates that once debt is scaled by exports (*D/X*) rather than by GDP, the correlation between default events and debt burdens tighten considerably, even though some important outliers remain. As shown below, regression results corroborate this *prima-facie* association, lending support to the widespread use of the debt to export ratio as a risk indicator in countries' credit rating assessments.

But arguably the most striking association highlighted in Table 2 is that between those credit events and macroeconomic volatility. All the serial defaulters in our sample (Argentina, Chile, Ecuador, Peru, and Russia) had unconditional output volatility ($\sigma_{\Delta y_r}$) well above the sample average. Since the latter is, in turn, about twice as high as the average output volatility of OECD economies during the same period, it follows that serial defaulters are indeed highly volatile economies. In contrast, non-defaulting countries such as Colombia, India, Korea, Thailand, Singapore, and Hungary all have below average output volatility.[13] Of course, such association does not imply causality, since default events may themselves be a source of output volatility. Yet, using external terms-of-trade (TOT) as an indicator of both the exogenously determined component and the purchasing power of national output,[14] it also appears that countries with more volatile TOT appear to default more often. This can be seen from the fact that nearly all Latin American emerging markets (in the first twelve rows of the Table) had both eventful credit histories and relatively high TOT volatility, whereas all default-free Asian economies (positioned in the middle of the Table) had much lower TOT volatility. TOT volatility has also been relatively high amongst all Eastern European defaulters with the exception of Poland.

To understand these relationships more rigorously, and to test their robustness to the inclusion of other variables featuring in previous studies, we estimate a discrete choice model of

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[13] South Africa would also fit this story if not for the external political sanctions that triggered the 1985 default. Malaysia is the biggest outlier in the default-volatility association but its outlier behavior is crucially dependent on the inclusion of the Asian crisis period in the sample. In fact, when the Asian crises years of 1997-99 are taken out of the sample, the association between output volatility and default frequency is further reinforced for Asian economies.

[14] Use of external terms of trade as a yardstick to the exogenous component of domestic income volatility is consistent with evidence from the developing country business cycle literature which estimates that TOT accounts for nearly one half of overall income volatility in developing countries (Mendoza, 1995). Kose and Reizman (2001) report similar estimates disaggregating between export and import prices.

the default probability. As suggested by the theoretical discussion of Section II, the default probability can be generally written as the following function:

$$\phi = f(D, R, \overline{Y}, \eta, q, \sigma_\varepsilon) \tag{18}$$

where $\partial\phi/\partial D > 0, \partial\phi/\partial R > 0, \partial\phi/\partial\overline{Y}, \partial\phi/\partial\eta < 0$, and $\partial\phi/\partial\sigma_\varepsilon > 0$.

In deciding whether to model the discrete choice between the non-event "0" (non-default in our case) and the realization of the event "1" (default), empirical researchers have been divided between the use of a logit or a probit specification. In most cases the differences are not significant (see Greene, 2000, pp. 815 for a discussion and references). One approach is to choose on the basis of standard maximum likelihood criteria given same set of left-hand side variables. On this basis, we chose a logit specification because it fits the data slightly better.

Table 3 reports the results for a variety of alternative specifications. To mitigate potential endogeneity biases, all ratios and level variables enter the regressions lagged one period, and the respective *z*-statistics are corrected for country-specific heteroscedasticity using the standard White procedure. The list of explanatory variables includes the following. We take the U.S. 10-year bond rate, deflated by current US CPI, as a proxy for the risk free interest rate, and denote it as r^*. We include export to GDP as an explanatory variable in some regressions: this may be viewed as a proxy for the capture rate η, which the existing literature typically associates with trade disruption (Bulow and Rogoff, 1988; Rose, 2002).[15] The volatility variable σ_{ygap} refers to the volatility of potential real output, measured as its ten-year rolling standard deviation.[16]

Column (1) of Table 3 reports the results of a specification that includes the risk free interest rate r^*, the volatility of output, and the ratios of debt to potential output and export to GDP. Estimated coefficients on both the risk-free rate r^* and the output volatility variable σ_{ygap} take on the expected sign and are highly significant statistically. The coefficients on *D/Yp* and *X/GDP* have the right sign, but these are estimated with much less precision. But since they have a similar order of magnitude and opposite signs, this suggests that they can be combined in one single indicator – the ratio of debt to exports. Column (2) reports the results with the debt to export variable, which is clearly statistically significant at 5 percent. As before, r* and

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[15] We also experimented with the ratio of exports plus imports to GDP, but the export to GDP ratio was the openness indicator closest to statistical significance.

[16] Potential real GDP was derived from an HP-filter with the smoothing parameter λ set to 7 as suggested in Pesaran and Pesaran (1997, p. 47) for annual data. The use of ten-year averages allows for slowly evolving changes in the underlying distribution of shocks over time for any given country. Rolling averages for volatility measures have also been used in studies on the impact of terms of trade instability on economic growth (e.g., Mendoza, 1997; Blattman, Hwang, and Williamson, 2003).

σ_{ygap} remain important determinants of default risk and the regression passes the Wald test for joint significance with flying colors. Moreover, while a pseudo-R^2 of 0.23 may appear low, it is in fact marginally higher than in other empirical studies applying logit/probit models to sovereign risk analysis (cf., Detragiache and Spilimbergo, 2001; and Reinhart, 2002).

Table 3. Logit Estimates of Default Probabilities with Output Gap Volatility
(marginal effects with robust z-statistics in parentheses)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
r*	0.43	0.62	0.61	0.75	0.57	0.39	0.17	0.19
	(3.72)**	(3.90)**	(4.20)**	(4.39)**	(3.69)**	(4.37)**	(4.06)**	(3.98)**
σ_{10}**_y_{gap}**	0.37	0.56	0.49		0.58	0.35	0.13	0.14
	(3.80)**	(4.13)**	(2.91)**		(4.38)**	(4.29)**	(2.87)**	(2.73)**
D/X		0.003	0.003	0.004	0.003	0.03	0.0004	
		(2.30)*	(2.25)*	(2.15)*	(2.30)*	(2.50)*	(0.74)	
D/GDPp	0.02							
	(1.58)							
X/GDP	-0.03							
	(-1.69)							
Def_freq			0.02	0.02				
			(1.07)	(1.91)				
REER_gap						0.08	0.01	0.03
						(3.98)**	(2.28)*	(2.16)*
Fxnet/M					-0.01			
					(-1.76)			
DS_X							0.01	0.01
							(6.47)**	(6.92)**
pseudo-R^2	0.26	0.23	0.24	0.19	0.24	0.31	0.49	0.49
Wald χ^2	38.4	27.3	37.7	32.4	29.3	56.2	78.3	73.1
No. of Obs.	588	588	588	588	588	588	588	588

Columns (3) and (4) report the results of experimenting with the credit history variable used in the Reinhart, Rogoff, and Savastano paper—the proportion of years the country was in default since 1820. Column (3) indicates that this variable is not statistically significant at any conventional level. Interestingly, however, once the volatility variable σ_{ygap} is dropped from the regressions as shown in column (4), the credit history variable becomes significant at the 5 percent borderline. This suggests that the credit history indicator is a catch-all variable proxying the more fundamental effects of underlying macroeconomic volatility on sovereign risk. In other words, this result suggests that countries that defaulted more in the past are more likely to default more in the future to the extent that the underlying sources of output volatility in these economies go on unabated.

Also, importantly, our results indicate that the significance of the volatility variable is robust to the inclusion of a wide array of explanatory variables featuring in the sovereign debt literature. The ratio of net foreign exchange reserves to imports (*Fxnet/M*) may capture liquidity factors and, as such, is widely used in empirical analyses of country risk (Edwards, 1984; Eichengreen and Portes, 1986; Cantor and Packer, 1996; Hu, Kiesel, and Perraudin, 2003). As shown in column (5), however, this variable falls short of statistical significance at 5 percent. Its failure in improving the model's fit is clearly corroborated by the virtually unchanged pseudo-R^2 of the regression which includes it relative to the one that does not (see column (3)). On the other hand, an indicator of real exchange rate misalignment (the REER gap), which also features prominently in empirical studies of currency and debt crises (see, for example, Frankel and Rose, 1996), does much better.[17] This is not surprising since this variable captures debt-denomination effects on sovereign risk which, while abstracted from the simple model of Section II, are deemed to be important (see Eichengreen and Haussman, 1999).

The additional variable found to be most significant is the ratio of debt service to exports, with the inclusion of this variable substantially improving the fit of the regressions as shown in the last two columns of Table 3. This again is not surprising since, in a world where debt maturity varies widely across countries and over time, the debt service to export ratio is arguably a more effective proxy for next period's debt servicing costs featuring in the theoretical model. And partly because of its obvious colinearity with the *D/X* variable, the inclusion of the debt service/export variable clearly dwarfs the former. Column (8) thus reports estimates where the *D/X* variable is dropped and the *DS/X* variable enters as the only debt burden indicator. Finally, we have tested these best fit models of columns (7) and (8) to the addition of several variables that appear in other studies, including per capita income, real GDP growth, and regional dummies. None of them proved to be statistically significant at 5 or 10 percent.

A further robustness test to the hypothesis that domestic volatility raises default risk consists of checking whether this holds for alternative volatility measures. In particular, one potential criticism of the results of Table 3 is that output gap volatility is partly endogenous: that is, it may be a by-product of high default risk perceptions and possibly a lingering outcome of the country's previous repayment history. Another potential concern is that the output volatility measure of Table 3 does not distinguish between expected and unexpected shocks to GDP. Even though this distinction does not play a role in the theoretical set-up of Section II, it may be important in practice; so it needs to be considered.

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[17] As others have done, we measure misalignment by deviations of the IMF's real effective exchange rate index from a univariate trend which, in our case, is again derived from an HP-filter with the smoothing parameter λ set to 7.

Table 4. Logit Estimates of Default Probabilities with Alternative Volatility Measures
(marginal effects with robust z-statistics in parentheses)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
r*	0.46	0.49	0.46	0.45	0.20	0.23	0.22	0.19
	(4.51)**	(4.79)**	(4.73)**	(4.53)**	(3.74)**	(3.72)**	(3.97)**	(3.82)**
REER_gap	0.10	0.10	0.09	0.10	0.03	0.03	0.04	0.03
	(4.17)**	(4.10)**	(4.85)**	(4.25)**	(2.07)*	(1.97)*	(2.47)*	(2.13)*
D/X	0.00	0.00	0.00	0.00				
	(1.86)	(1.73)	(1.77)	(1.86)				
DS_X					0.02	0.02	0.02	0.02
					(6.31)**	(6.38)**	(6.48)**	(6.38)**
σ_{10}**_tot**	0.03			0.03	0.02			0.01
	(4.01)**			(3.73)**	(4.16)**			(3.84)**
σ_5**_tot**		0.05				0.02		
		(3.62)**				(3.36)**		
$\sigma_{10}\_\varepsilon_{wtot}$			0.09				0.04	
			(2.28)*				(2.44)*	
$\sigma_{10}\_\varepsilon_{xtot}$				0.06				0.02
				(1.27)				(1.79)
No. of Obs.	588	588	588	588	588	588	588	588
Wald χ^2	40.99	40.33	35.52	41.61	58.96	57.63	69.14	63.73
Pseudo R^2	0.30	0.30	0.28	0.30	0.49	0.48	0.47	0.49

Estimation results in Table 4 address both types of concerns. As before, all explanatory variables are lagged one period except for the TOT indicator (which, as discussed earlier, can be taken as exogenous), and the respective *z*-statistics are corrected for country-specific heteroscedasticity. Using TOT volatility as an instrument for domestic output volatility, the estimates show that our previous results hold: TOT volatility is not only statistically significant, but also the overall fit of the regressions do not change much. This is so, irrespective of whether one uses the debt-stock-to-export ratio (D/X) as the indicator of debt burden (and hence of the gains of defaulting) or, alternatively, the debt service to export ratio (DS/X). This result also holds whether one uses 5-year or 10-year rolling standard deviations of TOT. The only noticeable difference with regard to results in Table 3 is that the D/X variable is only significant at 10 percent.

Table 4 also shows (columns (3) and (7)) estimates with 10-year rolling standard deviations of the residuals of a country-specific real GDP growth forecasting equation (σ_{10_ε}), aimed at capturing unanticipated shocks to output. Following Ramey and Ramey (1995), such growth forecasting equation includes 2 lags of real GDP levels, a linear time trend, and a segmented trend broken 1974.[18] This shock volatility indicator has the expected positive sign and is also significant at 5 percent, even though the classic generated regressor bias problem tends, if anything, to detract from its statistical significance. Finally, we consider a small variant of the former measure by including 2 lags of TOT in the growth forecasting equation. This makes the residual ($\sigma_{10\_\varepsilon_{xtot}}$) less correlated with the TOT volatility indicator and likely to be capturing more of unexpected shocks associated with other variables, such as fiscal and monetary policies. The results reported in columns (4) and (8) indicate that this measure is not significant at 5 percent, what may be simply due to the generator bias problem noted above. In any event, TOT volatility remains highly statistically significant in both cases.

Having shown that default probability is positively and significantly related to output and terms of trade volatility controlling for other factors, we now turn to the evidence pertaining the impact of volatility on indebtedness levels. Having established in section 2 that, while the net effect of volatility on indebtedness is ambiguous on purely theoretical grounds but that sensible model calibrations suggest the direction of the effect to be mostly negative, the remainder of this section tests this hypothesis.

As discussed in Eaton and Gersovitz (1981), econometric estimation of the effect of income volatility on debt levels is not trivial. This is not only because of the potential presence of a debt supply credit ceiling under which standard OLS estimates tend be inconsistent due to the truncated nature of the distribution, but also because such a credit ceiling is shifting according to the various parameters of the model (See Maddala, 1983 for a comprehensive discussion and further references). One way of modelling this problem, which has been advanced in Maddala and Nelson (1974) and used by Eaton and Gersovitz (1981) among others, is to assume that debt at any given point in time is determined within either of the two regimes: one in which the demand schedule intersects a horizontal or mildly sloping supply schedule so that debt is essentially demand determined; or one in which the supply constraint becomes binding as debt approaches $d_{\max}$.

Since there is likely to be a switch between these two regimes in practice and given that $d_{\max}$ is unobserved, the proposed estimation technique that allows for this possibility amounts to estimating the following system:

[18] As discussed in their paper, this measure is consistent with the hypothesis of a unit root as well as with the alternative of a trend-stationary or a segmented-trend stationary real GDP.

$$
\begin{aligned}
d_{i_t}^* &= g(\sigma_{y_{it}}, \eta_{it}, q_{it}) \\
d_{\max i_t} &= h(\sigma_{y_{it}}, \eta_{it}, q_{it}, z_{0_{it}}) \qquad (.19) \\
d_{it} &= \min(d_{\max i_t}, d_{it}^*)
\end{aligned}
$$

where d_t^* is a point in the demand schedule before d approaches the maximum debt threshold regime. The main estimation challenges in this case are that: i) d_{it}^* and $d_{\max i_t}$ are unobserved; ii) there must be a meaningful way to distinguish the supply constrained regime from its alternative, the "unconstrained" market equilibrium regime. Conditional upon the latter requirement, a maximum likelihood method for this type of model has been advanced by Maddala and Nelson (1974). In what follows, we thus estimate (19) by full maximum likelihood using OLS estimates as the starting values for the non-linear optimization. Regarding identification, it we discriminate between the two regimes by introducing in the dmax equation a dummy variable $z_{0_{it}}$ which equals one for periods in which the country is in default (when indebtedness is known to be supply constrained) and zero otherwise.

The results are reported in Table 5. In light of the theoretical model of Section II, we start with a baseline specification which expresses the debt to GDP ratio as function of underlying income volatility (as before, proxied by the 10-year rolling standard deviation of the output gap) and trade openness.[19] Clearly, such a highly parsimonious model should not be expected to fully capture the complexity of sovereign indebtedness decisions. Yet, as it turns out, its prediction regarding the effects of volatility on borrowing are not overturned by richer specifications. Consistent with our theoretical model, higher income volatility shifts downwards the maximum debt threshold (*dmax*), with column (1) estimates indicating that a 1 percentage point change in the underlying real GDP volatility leading to a 12 percent decline in the *dmax*, all else constant—this semi-elasticity estimate being basically unchanged across specifications. Likewise consistent with the model, greater trade openness (a proxy for default costs, as already discussed) tends to increase *dmax*, while the coefficient on the default period dummy Zo also takes on the expected positive sign. Regarding the unconstrained regime *d**, the baseline specification estimates are no less sensible. Consistent with the consumption smoothing motive for borrowing, volatility affects debt positively; and while the respective coefficient is imprecisely estimated (as witnessed by the z-statistic of 0.66), we shall see below that it will become highly statistically significant in more comprehensive specifications. The openness indicator takes on a negative sign and is highly significant, supporting the view that higher default costs in a volatile environment with non-trivial default probabilities tend to discourage borrowing.

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[19] As others have done (e.g., Eaton and Gersovitz, 1981), we express those ratio variables in natural logs. Using the export to GDP ratio instead of the ratio of export plus imports to GDP does not alter the thrust of the results. In the absence of other information, we assume q to be constant throughout the estimation.

This baseline specification is then augmented in column (2) by the (one-period lagged) GDP growth rate. Considering the effects of economic growth on optimal debt is important, *inter alia*, for the reasons laid out in Eaton and Gersovitz (1981): on the one hand, a higher growth rate of domestic income tends to encourage borrowing for Fisherian reasons (i.e., some of the future income is desired now), but on the other hand, higher growth may reduce lender's capture power, for instance, by lowering the cost of a future credit embargo. Our estimates indicate that while the effect of growth in the supply constrained regime is consistent with the Eaton-Gersovitz mechanism, its effect on optimal debt in the unconstrained regime is opposite to that postulated by the Eaton-Gersovitz demand for borrowing, i.e., higher GDP growth tends to discourage rather than encourage borrowing. This result, however, is not implausible and can in fact be easily rationalized. [20] More relevant to the core of our hypothesis is the fact that the signs and statistical significance of the estimated coefficients on both the unconstrained and constrained regimes are consistent with this paper's proposed explanation for debt intolerance—the main difference with the baseline specification being the coefficient on the volatility variable in the unconstrained regime which now appears to be highly significant statistically. In addition, this highly positive coefficient suggests that the volatility-induced effect on debt demand is strong before the supply constraint kicks in with a vengeance. On average, inspection of the fitted values for this regression indicates that 30 percent of the fitted valued lie on the d* regime, with 70 percent lying on the constrained regime, thereby indicating that the supply constraint is binding most of the time for these countries. This finding is clearly consistent with the view of the debt intolerance being a systematic rather than episodic phenomenon.

The remainder of Table 5 basically reinstates the robustness of the above results. Adding countries' U.S. dollar per-capita income as an explanatory variable (see column 3), and using TOT instead of real GDP variance only has an impact on the magnitude of the effect rather than on its direction or statistical significance. Finally, estimates reported in column (4) add a variable that the political economy literature have deemed as an important determinant of fiscal performance and hence of debt accumulation—namely, country's degree political stability (Alesina and Drazen, 1991; Cukierman, Edwards, and Tabellini, 1992).[21]In tandem with the findings of this literature which postulate that politically less stable countries tend to run more persistent fiscal deficits and hence demand more debt, we find that greater political *stability* tends to lower debt. At the same time, the estimates also show that the inclusion of this

[20] It is possible, for instance, that this opposite sign reflects the short-comings of proxying future growth potential on the basis of lagged growth (although Eaton and Gersovitz use the same lagged indicator) and also some possible multicollinearity between volatility and the growth rate indicator for the reasons highlighted in Ramey and Ramey (1995), that is, the existence of a statistically significant association between volatility and growth. Indeed, the sharp change in the coefficient of the volatility variable once growth is included in the unconstrained regime suggests that multicollinearity plays a role. Finally, it may also be conjectured that higher growth tends to improve the sovereign budget hence mitigating borrowing needs.

[21] The political stability variable used is the Handon House indicator which ranges from 0 (maximum political instability) to 1 (fully stable democracy).

additional variable does not change the thrust of the previous results—with volatility and openness remaining significant determinants of sovereign indebtedness. Finally, as in previous specifications, the model's fitted values classify that the majority of observations (77 percent) as belonging to the supply constrained regime, thus clearly indicating that volatility depresses rather than encourages borrowing most of the time.

Table 5. Determinants of Sovereign Debt: Regime Switching Model Estimates
(Dependent Variable: External Debt to GDP, z-statistics in parenthesis)

	(1)		(2)		(3)		(4)		(5)	
	d*	dmax	d*	dmax	d*	dmax	d*	dmax	d*	dmax
σ10_Yg	7.23	-12.01	30.65	-11.18	57.49	-11.25	82.43	-10.80	---	---
	(0.66)	(-8.85)	(2.49)	(-8.60)	(4.17)	(-8.63)	(3.91)	(-8.54)	---	---
(X+M)/GDP	-1.22	0.84	-1.22	0.75	-0.41	0.74	-0.78	0.74	-0.71	0.84
	(-6.20)	(26.82)	(-6.41)	(24.93)	(-2.01)	(23.44)	(-3.23)	(24.19)	(-3.45)	(28.51)
Zo	---	0.60	---	0.52	---	0.52	---	0.5	---	0.49
	---	(7.96)	---	(7.68)	---	(7.43)	---	(7.50)	---	(6.15)
Growth			-17.24	-3.21	-17.19	-3.21	-12.59	-2.92	-8.72	-3.31
			(-6.48)	(-7.24)	(-6.89)	(-7.24)	(-4.91)	(-6.71)	(-3.46)	(-6.60)
Yus$_pc					-1.07	---	-0.87	---	-0.58	---
					(-7.25)	---	(-6.50)	---	(-5.37)	---
Pol. Stab.							-2.63	---	-1.28	---
							(-3.18)	---	(-2.58)	---
σ10_TOT									5.08	-0.81
									(3.49)	(-5.54)
ML	-871.3		-818.3		-792.6		-781.4		-806.3	
No. of Obs.	710		710		710		710		710	

IV. Conclusions

The fact that most sovereign defaults have taken place among countries with low to moderate debt to income ratios is puzzling. This is especially so when one notes that many other sovereigns have far higher debt ratios and continue to borrow at much lower spreads. While reputation and cross-country differences in credit histories have been invoked as reasons, such explanations raise a number of thorny questions as discussed above.

This paper has argued that cross-country differences in underlying macroeconomic volatility is at least part of the answer and a key missing link that reconciles the standard theory of sovereign borrowing with the empirical evidence on the "debt intolerance" phenomenon. At the root of our argument is not something new: it is well documented that many emerging markets are more volatile than both their advanced counterparts and other developing country peers, and that this volatility comes from diverse sources, such as commodity specialization and institutions that are conducive to destabilizing economic policies (c.f., Gavin, Hausmann, Perotti, and Talvi, 1996; Talvi and Végh, 2002; and Acemoglu, Johnson, Robinson, Thaicharoen, 2003). Given that, this paper has shown that this subgroup of countries tends to carry a higher default risk and face a lower credit ceiling, even when one controls for a host of other variables. In addition, our econometric estimates indicate that the such supply constraints are binding most of the time (over two-thirds of the sample observations), thereby suggesting that market intolerance to higher indebtedness among this group of countries is more of a systematic than an episodic phenomenon. This finding corroborates that of Reinhart, Rogoff, and Savastano (2003) using a different methodology and a slightly different country coverage.

This paper's emphasis on the role of volatility in sovereign risk obviously does not rule out other factors previously identified in the literature. One such factor is currency-denomination balance sheet mismatches and the associated role of exchange rate misalignment in debt crises. While the purpose of isolating the role of income volatility on sovereign borrowing in a tractable way has led us to abstract from the balance sheet channel in our theoretical analysis, such effects have been controlled for in our regressions; and as seen above, the respective results corroborate the importance of this variable, as previous researchers have also found. Similarly, by focusing on the effects of underlying or "structural" macroeconomic volatility on debt servicing, we are not necessarily rejecting an autonomous role for sovereign reputation. What our results do suggest is that macroeconomic volatility is a fundamental factor that, among other things, can easily manifest itself in unsound credit histories and hence help shape reputation.

Some implications follow directly from these results. First, contrary to the classic Eaton and Gersovitz (1981) mechanism—which suggests that volatility might lower the incentive to default—we find that volatility does not raise countries' credit ceilings, but quite the opposite. Our findings also help qualify one key inference drawn by Reinhart, Rogoff, and Savastano (2003). Since in their view sovereigns' reputation built over decades or centuries is a crucial determinant of debt intolerance, overcoming the latter would require many of today's emerging economies to dramatically lower their debt ratios to the point where their default risk is sufficiently low (their estimated threshold being as low as 15 percent in some cases), so that debt becomes "sustainable"; this would then make possible a gradual build-up of reputation which would eventually enhance these countries' borrowing capacity. Aside from the point that their own empirical analysis suggests that gradual deleveraging is hard to accomplish and that reputation building is a painfully slow process, our model cautions that such debt reducing strategies may be suboptimal if they preclude feasible consumption smoothing and do not ultimately address the sources of domestic income volatility.

This takes us to a paradoxical aspect of the debt intolerance phenomenon highlighted by this paper's results. On the one hand, more volatile countries need international borrowing the most for smoothing consumption purposes; but on the other hand, these are precisely the countries which will face the most stringent constraints on their borrowing capacity because of the default risk that volatility itself engenders. So, by reducing volatility, a country can improve its maximum indebtedness threshold but at the same time reduce its desire for debt. Which effect will prevail is of course an empirical issue which in practice will partly depend on other motives driving international borrowing besides consumption smoothing. Provided that these other motives are sufficiently weighty, reducing macroeconomic volatility should translate in more rather than less emerging market borrowing. In addition, because the sovereign spread is a well-known benchmark to the setting of interest rates facing the domestic private sector, by reducing the former, lower macroeconomic volatility should also be instrumental in helping reduce the latter and thus positively affect economic growth. This channel linking volatility and sovereign spreads is thus one other plausible explanation for the inverse relationship between output and terms-of-trade volatility and economic growth extensively documented elsewhere (Ramey and Ramey, 1995; Mendoza, 1997; Agénor and Aizermann, 1998; and Blattman, Hwang, and Williamson, 2003).

Finally, our theoretical and empirical analyses both suggest an alternative channel through which countries' borrowing capacity can be increased without lowering volatility and depressing sovereign loan demand. This channel is the lenders' "capture technology," as simply represented by parameters η and q in our model. While the effectiveness of this mechanism is obviously constrained by the limits imposed by national sovereignty, it is plain that if an economy is open enough so that default entails potentially significant trade and other output losses (a higher η), and debt recovery plus spillover default losses are not overly high (i.e., q is sufficiently low), then lenders will be more assured that default is less likely. This will shift downward the loan supply schedule, thereby raising the sovereign's credit ceiling. The empirical significance of this mechanism is overwhelmingly supported by our econometric results which indicate that higher openness reduces default probability and raises the maximum debt threshold. Hence, provided that it does not generate some volatility of its own, greater trade openness naturally emerges as instrumental in mitigating the impact of higher domestic volatility on default risk.

References

Acemoglu, D., S. Johnson, J. Robinson, and Y. Thaicharoen, 2003, "Institutional Causes, Macroeconomic Symptoms: Volatility, Crises and Growth," *Journal of Monetary Economics,* Vol. 50, pp. 49-123.

Agénor, Pierre-Richard, and Joshua Aizenman, 1998, "Contagion and Volatility with Imperfect Capital Markets," *Staff Papers*, International Monetary Fund, Vol. 45, pp. 207–35.

Alesina, Alberto, and Drazen, Allan, 1991, "Why Are Stabilizations Delayed?" *American Economic Review*, Vol. 81, pp. 1170–88.

Alfaro, Laura, and Fabio Kanuczuk, 2002, "Sovereign Debt with Adverse Selection: A Quantitative Approach," Harvard Business School Working Paper 02-082 (Cambridge, Massachusetts: Harvard Business School).

Beim, David O., and Charles W. Calomiris, 2001, *Emerging Financial Markets* (New York: McGraw-Hill/Irwin).

Blattman, Chris, Jason Hwang, and Jeffrey Williamson, 2003, "Terms of Trade and Economic Performance in the Periphery 1870-1940," NBER Working Paper 9940 (Cambridge, Massachusetts: National Bureau of Economic Research).

Bulow, Jeremy, and Kenneth Rogoff, 1989, "Sovereign Debt: Is to Forgive or Forget?" *American Economic Review*, Vol. 79, pp. 43–50.

Caballero, Ricardo, 2000, "Macroeconomic Volatility in Latin America: A View and Three Case Studies*, Economia,* Vol. 1, pp. 31–108.

Cantor, R. and F. Parker, 1996, "Determinants and Inputs of Sovereign Credit Ratings," Federal Reserve Board of New York, *Economic Policy Review*, Vol. 2, pp. 37–53.

Cohen, Daniel, 1992, "The Debt Crisis: A Post-Mortem," *NBER Macroeconomics Annual*, (Cambridge, Massachusetts: National Bureau of Economic Research) pp. 65–101.

Cukierman, Alex, Sebastian Edwards, and Guido Tabellini, 1992, "Seignorage and Political Instability," *American Economic Review*, Vol. 82, pp. 537–55.

Detragiache, Enrica, and Antonio Spilimbergo, 2001. "Crises and Liquidity: Evidence and Interpretation," IMF Working Paper 01/02 (Washington: International Monetary Fund).

Díaz-Alejandro, Carlos F., 1984, "Latin American Debt: I Don't Think We Are in Kansas Anymore," *Brookings Papers on Economic Activity*, Vol. 2, pp. 335–403.

Eaton, Jonathan, and Mark Gersovitz, 1981, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economics and Statistics*, Vol. 48, pp. 284–309.

———, 1995, "Some Curious Properties of a Familiar Model of Debt and Default," *Economics Letters*, Vol. 63, pp. 367–71.

Edwards, Sebastian, 1984, "LDC Foreign Borrowing and Default Risk: An Empirical Investigation, 1976-80," *American Economic Review*, Vol. 74, No.4, pp. 726-34.

Eichengreen, Barry, and Richard Portes, 1986, "Debt and Default in the 1930s: Causes and Consequences," *European Economic Review*, Vol. 30, pp. 599–640.

———, and Peter Lindert (eds.), 1989, *The International Debt Crisis in Historical Perspective*, (Cambridge, Massachusetts: MIT press).

———, and Ricardo Haussman, 1999, "Exchange Rate Regimes and Financial Fragility," NBER Working Paper 7418 (Cambridge, Massachusetts: National Bureau of Economic Research).

Feder, Gershon, and Richard E. Just, 1977, "A Study of Debt Servicing Capacity Applying Logit Analysis," *Journal of Development Economics,* Vol. 4, pp. 25–38.

Frankel, Jeffrey and Andrew Rose, 1996, "Currency crashes in emerging markets: An empirical treatment," *Journal of International Economics,* Vol. 41, pp. 351–66.

Gavin, Michael, Ricardo Hausmann, Roberto Perotti, and Ernesto Talvi, 1996, "Managing Fiscal Policy in Latin America and the Caribbean: Volatility, Procyclicality, and Limited Creditworthiness," Working Paper 326 (Washington: Office of the Chief Economist, Inter-American Development Bank).

Greene, William H., 2000, *Econometric Analysis* (New Jersey: Prentice Hall).

Grossman, Herschel I., and John B.Van Huyck, 1988, "Sovereign Debt as a Contigent Claim: Excusable Default, Repudiation, and Reputation," *American Economic Review*, Vol. 78, pp. 1088–97.

———, and Taejoon Hahn, 1999, "Sovereign Debt and Consumption Smoothing," *Journal of Monetary Economics*, Vol. 44, pp. 149–58.

Hu, Yen-Ting, Rudiger Kiesel and William Perraudin, 2003, "The Estimation of Transition Matrices for Sovereign Credit Ratings," *Journal of Banking and Finance*, Forthcoming.

Jorgensen, Erika, and Jeffrey Sachs, 1989, "Default and Renegotiation of Latin American Foreign Bonds in the Interwar Period," in Eichengreen and Lindert (eds.), op.cit, pp.48–85.

Kletzer, Kenneth, 1997, "Volatility, External Debt, and Fiscal Risk: Simulations of the Impact of Shocks on Fiscal Adjustment for Thirteen Latin American Countries," Inter-American Development Bank Working Paper 358 (Washington: Inter-American Development Bank).

Kose, M. Ayhan, and R. Reizman, 2001, "Trade Shocks and Macroeconomic Fluctuations in Africa," *Journal of Development Economics*, Vol. 65, pp. 55–80.

Lindert, Peter H., and Peter J. Morton, 1989, "How Sovereign Debt Has Worked," in *Developing Country Debt and Economic Performance*, ed. by Jeffrey Sachs, (Chicago: National Bureau of Economic Research).

Maddala, G.S., 1986, *Limited Dependent and Qualitative Variables in Econometrics*, (Cambridge, Massachusetts: Cambridge University Press).

———, and F. Nelson, 1974, "Maximum Likelihood Methods for Markets in Disequilibrium," *Econometrica*, Vol. 42, pp. 1013–30.

Mendoza, Enrique, 1995 "Terms of Trade, the Real Exchange Rate, and Economic Fluctuations", *International Economic Review*, Vol. 36, pp. 101–37.

———,1997, "Terms of Trade Uncertainty and Economic Growth: Are Risk Indicators Significant in Growth Regressions," *Journal of Development Economics*. Vol. 54, pp. 323–56.

Obstfeld, Maurice, and Kenneth Rogoff, 1996, *Foundations of International Macroeconomics*, (Cambridge, Massachusetts: MIT Press).

Ozler, Sule, 1993, "Have Commercial Banks Ignored History," *American Economic Review*, Vol. 83, pp. 608–20.

Pesaran, M. H., and B. Pesaran, 1997, *Microfit 4.0*, (Oxford: Oxford University Press).

Sachs, Jeffrey, 1984, "Theoretical Issues in International Borrowing," Princeton Studies in International Finance, No. 54.

———, 1989, *Developing Country Debt and Economic Performance*, (Chicago: University of Chicago Press).

———, and Daniel Cohen, 1985, "LDC Borrowing with Default Risk," *Kredit und Kapital*, pp. 211–35.

Ramey, Garey and Valery Ramey, 1995, "Cross-Country Evidence on the Links between Volatility and Growth," *American Economic Review,* Vol. 85, pp. 1138–51.

Reinhart, Carmen M., 2002, "Credit Ratings, Default, and Financial Crises: Evidence from Emerging Markets**,**" *World Bank Economic Review*, Vol. 16, pp. 151–70.

———, Kenneth Rogoff, and Miguel Savastano, 2003, "Debt Intolerance", NBER Working Paper 9908 (Cambridge, Massachusetts: National Bureau of Economic Research).

Rose, Andrew K., 2002, "One Reason Countries Pay Their Debts: Renegotiation and International Trade," CEPR Discussion Paper 3157 (London, United Kingdom: Centre for Economic Policy Research).

——— and Mark M. Speigel, 2002, "A Gravity Model of Sovereign Lending: Trade, Default and Credit," NBER Working Paper 9285 (Cambridge, Massachusetts: National Bureau of Economic Research).

Talvi, Ernesto and Carlos Vègh, 2002, "Tax Base Variability and Procyclical Fiscal Policy," NBER Working Paper 7499 (Cambridge, Massachusetts: National Bureau of Economic Research).

Proof of Proposition 1

(a) $R_L(D)$ is well-defined only for $D \in [0, D_{\max})$. To see why note, first, that the lowest possible value for R_L is *R*. This follows from the fact that if $R_L < R$, repayments will be less that *RD* for every realization, ruling out the possibility that the lender breaks even for any positive level of debt *D*. Given this, the gain from defaulting $R_L D - \eta(\overline{Y} + \varepsilon + RD)$ is increasing in *D*. If so, then for some *D* high enough, the borrower will default with probability one. This implies that, given $\overline{Y}$ and *R*, there exists some *D*, call it $D_{\max}$, such that the lender cannot break-even for debt levels beyond this.

(b) The borrower will never default if, for the lowest possible realization of $\varepsilon = -\varepsilon_m$, $\eta(\overline{Y} - \varepsilon_m + RD) \le RD$. If so, setting $R_L = R$ will allow the lender to break even. Solving the last inequality for *D,* it follows that once *D* lies in the range $[0, \frac{\eta}{1-\eta}\frac{\overline{Y}-\varepsilon_m}{R}]$, default is never rational. This establishes the first part . For the second part, write the break even condition as:

$$RD = \int_{-\varepsilon_m}^{e(R_L,D)} [\eta(1+q)(\overline{Y} + \varepsilon + RD) - qR_L D]\pi(\varepsilon)d\varepsilon + \int_{e(R_L,D)}^{+\varepsilon_m} R_L D\pi(\varepsilon)d\varepsilon .$$

Note that the probability of default, for given R_L and *D,* is

$$\phi(R_L, D) = \int_{-\varepsilon_m}^{e(R_L,D)} \pi(\varepsilon)d\varepsilon .$$

Implicit differentiation of the break-even condition and simple manipulation yields

$$\frac{dR_L}{dD} = \frac{1}{D}\left(-R_L + R + R(1-\eta)\frac{(1+q)\phi}{1-(1+q)\phi}\right) \tag{20}$$

This derivative is positive as long as the expression in parenthesis on the right is positive. To see this note, once again from the break-even condition, that as $\overline{Y} + \varepsilon > 0$ for all ε, we have:

$$RD > \int_{-\varepsilon_m}^{e(R_L,D)} [\eta(1+q)RD - qR_L D]\pi(\varepsilon)d\varepsilon + \int_{e(R_L,D)}^{+\varepsilon_m} R_L D\pi(\varepsilon)d\varepsilon ,$$

which implies that:

$$RD > \phi[\eta(1+q)RD - qR_L D] + (1-\phi)R_L D .$$

This in turn implies that $R > \phi\eta(1+q)R + [1-\phi(1+q)]R_L$, which is sufficient to establish that the term in parenthesis is positive.

(c) The expected repayment $P^*(\varepsilon, R_L, D)$ on the left hand side of (6) is concave in shocks. By Jensen's inequality, an increase in the volatility of shocks lowers the expected value of repayments for a given R_L. To restore the break-even requirement, R_L must rise.

Deriving $R_L(D)$ for the uniform distribution.

For the uniform distribution (i.e., $\pi(\varepsilon)=1/2\varepsilon_m$), where the following pairs of roots obtain:

$$\left\{R_L=R+\left[\sqrt{\frac{\eta\varepsilon_m}{D(1+q)}}+\sqrt{\eta\frac{\bar{Y}}{D}-R(1-\eta)-\frac{\eta q\varepsilon_m}{D(1+q)}}\right]^2,\ e=RD\left(\frac{1-\eta}{\eta}\right)-\bar{Y}+\varepsilon_m\left[\sqrt{\frac{1}{(1+q)}}+\sqrt{\frac{\bar{Y}}{\varepsilon_m}-\frac{RD(1-\eta)}{\eta\varepsilon_m}-\frac{q}{1+q}}\right]^2\right\} \quad (21)$$

$$\left\{R_L=R+\left[\sqrt{\frac{\eta\varepsilon_m}{D(1+q)}}-\sqrt{\eta\frac{\bar{Y}}{D}-R(1-\eta)-\frac{\eta q\varepsilon_m}{D(1+q)}}\right],\ \mathrm{e}=RD\left(\frac{1-\eta}{\eta}\right)-\bar{Y}+\varepsilon_m\left[\sqrt{\frac{1}{(1+q)}}-\sqrt{\frac{\bar{Y}}{\varepsilon_m}-\frac{RD(1-\eta)}{\eta\varepsilon_m}-\frac{q}{1+q}}\right]^2\right\} \quad (22)$$

The restrictions that $R_L(D)$ be real and no smaller than 1 and that $-\varepsilon_m < e < +\varepsilon_m$ limit the range of *D* for which an economically meaningful solution can be extracted from (21) and (22). The requirement that the solution be a real number implies that:

$$D \le \frac{\eta}{R(1-\eta)}\left[\bar{Y}-\frac{q\varepsilon_m}{(1+q)}\right]. \quad (23)$$

In addition, our requirement that $e < +\varepsilon_m$ rules out (21) as a possible solution, whereas the requirement that $-\varepsilon_m < e$ in (22) yields:

$$D > \frac{\eta}{(1-\eta)}\frac{(\bar{Y}-\varepsilon_m)}{R}, \quad (24)$$

which implies that, given $\bar{Y}$ and R, the greater the lenders' capture power η, the higher has to be the variance of shocks to generate a solution for the optimal debt problem in this setting.

Proof of Proposition 2

Let $R_L(D)$ be a functional relationship that defines the break-even constraint. Given $R_L(D)$, the borrower's optimization problem:

$$Max_D \int_{-\varepsilon_m}^{\mathrm{e}(R_L(D),D)} U(C_L)\pi(\varepsilon)d\varepsilon + \int_{e(R_L(D),D)}^{+\varepsilon_m} U(C_H)\pi(\varepsilon)d\varepsilon \quad (25)$$

where C_L and C_H stand for consumption with and without partial default, respectively.

$$\begin{aligned} C_L &= (1-\eta)(\bar{Y}+\varepsilon+RD) \\ C_H &= \bar{Y}+\varepsilon+[R - R_L^*(D)]D \end{aligned} \quad (26)$$

Taking the derivative with respect to D yields the first order condition for an interior maximum (provided it exists): $V_D = 0$, where

$$V_D = \int_{-\varepsilon_m}^{e(D,R_L^*(D))} U'[(Y+\varepsilon+RD)(1-\eta)]\ (1-\eta)R\ \pi(\varepsilon)d\varepsilon$$
$$+\int_{e(D,R_L^*(D))}^{\varepsilon_m} U'[(Y+\varepsilon+(R-R_L^*(D))D](R-R_L^*(D)-\frac{\partial R_L^*}{\partial D}D)\ \pi(\varepsilon)d\varepsilon \tag{27}$$

Noting that the break-even constraint implies

$$\frac{\partial R_L}{\partial D} = -\frac{1}{D}\left[R_L - R(1+(1-\eta)\frac{(1+q)\phi}{1-\phi(1+q)})\right]. \tag{28}$$

Using this relation in the first-order condition, the first-order condition reduces to

$$V_D = \int_{-\varepsilon_m}^{e(D)} U'[C_L(D,\varepsilon)]\ \pi(\varepsilon)d\varepsilon - \frac{(1+q)\phi}{1-\phi(1+q)}\int_{e(D)}^{\varepsilon_m} U'[C_H(D,\varepsilon)]\ \pi(\varepsilon)d\varepsilon = 0 \tag{29}$$

Rearranging (29) yields:

$$\phi = \frac{1}{1+q}\frac{\int_{-\varepsilon_m}^{e} U'(C_L)\pi(\varepsilon)d\varepsilon}{\int_{-\varepsilon_m}^{e} U'(C_L)\pi(\varepsilon)d\varepsilon + \int_{e}^{\varepsilon_m} U'(C_H)\pi(\varepsilon)d\varepsilon} \tag{30}$$

Proof of Proposition 3

(a) Define $e^{**} = e(R_L(D^*), D^*)$. We can write

$$\frac{de^{**}}{d\varepsilon_m} = \frac{de}{dD}\frac{dD}{d\varepsilon_m}|_{R_L \text{ fixed}} + \frac{de}{dR_L}\frac{dR_L}{d\varepsilon_m}|_{D \text{ fixed}}$$

Each expression on the right is positive.

(b) The sign of $\frac{\partial D^*}{\partial \varepsilon_m}$ is the same as the sign of the cross-partial $V_{D\varepsilon_m}$. For uniformly distributed shocks, we have

$$V_D = \int_{-\varepsilon_m}^{e(D)} U'[C_L(D,\varepsilon)]\ \frac{1}{2\varepsilon_m}d\varepsilon - \frac{(1+q)\phi}{1-\phi(1+q)}\int_{e(D)}^{\varepsilon_m} U'[C_H(D,\varepsilon)]\ \frac{1}{2\varepsilon_m}d\varepsilon = 0$$

so that:

$$V_{D\varepsilon_m} = \frac{-1}{2\varepsilon_m^2}\left(\int_{-\varepsilon_m}^{e(D)} U'[C_L(D,\varepsilon)]\ \frac{1}{2\varepsilon_m} d\varepsilon - \frac{(1+q)\phi}{1-\phi(1+q)} \int_{e(D)}^{\varepsilon_m} U'[C_H(D,\varepsilon)]\ d\varepsilon \right)$$
$$+\frac{1}{2\varepsilon_m}\left(U'[C_L(D,-\varepsilon_m)] + U'[C_L(D,e)]\frac{de}{d\varepsilon_m} \right)$$
$$+\frac{(1+q)\phi}{1-\phi(1+q)}\frac{1}{2\varepsilon_m}\left(-U'[C_H(D,\varepsilon_m)] + U'[C_H(D,e)]\frac{de}{d\varepsilon_m} \right)$$
$$+\frac{(1+q)\phi}{1-\phi(1+q)}\frac{1}{2\varepsilon_m}\left(\int_{e(D)}^{\varepsilon_m} U''[C_H(D,\varepsilon)]\ d\varepsilon] \right)\frac{dR_L}{d\varepsilon_m}$$

Note that the first two terms disappear due to the first order condition for an interior maximum. Collecting the remaining terms and simplifying, we note that the last two terms are negative, while the rest are positive. The net effect could go either way.