Economic Integration, Sectoral Diversification, and Exchange Rate Policy in a Developing Economy

Gabriel Srour
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Abstract

The paper develops a simple three-sector model of a developing country with nominal wage rigidity, in which one sector is thought of as the primary sector and the other two are sectors in which the country can diversify. The paper then analyzes the relationship between the market structure of the nonprimary sectors and equilibrium adjustments to shocks in the primary sector. In particular, the paper examines under what conditions the country should promote one nonprimary sector over another. Among other things, it argues that developing countries should promote those sectors that are more integrated with the outside world.

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### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Introduction</td>
<td>3</td>
</tr>
<tr>
<td>II. The Baseline Model</td>
<td>5</td>
</tr>
<tr>
<td>A. Households</td>
<td>6</td>
</tr>
<tr>
<td>B. Production</td>
<td>8</td>
</tr>
<tr>
<td>C. Price and Wage Setting</td>
<td>9</td>
</tr>
<tr>
<td>D. Long-Run Shares of Trade</td>
<td>10</td>
</tr>
<tr>
<td>III. Equilibrium</td>
<td>11</td>
</tr>
<tr>
<td>A. Flexible Wages</td>
<td>12</td>
</tr>
<tr>
<td>B. Predetermined Wages</td>
<td>12</td>
</tr>
<tr>
<td>IV. All Prices Exogenously Given</td>
<td>14</td>
</tr>
<tr>
<td>V. Prices of Home-Produced Nonprimary Goods Endogenous</td>
<td>16</td>
</tr>
<tr>
<td>VI. Conclusion</td>
<td>20</td>
</tr>
<tr>
<td>Detailed Derivation of the Equilibrium</td>
<td>21</td>
</tr>
<tr>
<td>A. All Prices Exogenously Given</td>
<td>24</td>
</tr>
<tr>
<td>B. Prices of Home-Produced Nonprimary Goods Endogenous</td>
<td>24</td>
</tr>
<tr>
<td>References</td>
<td>30</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

The last twenty years have seen a strong movement towards globalization in the world. Driven by the advent of new information and communication technologies, and by failed experiments in the past, many countries, large and small, are moving away from an earlier protectionist stance towards greater liberalization. Others are seeking yet freer trade through economic and political agreements. In this new environment, questions about exchange rate, trade, and industrial policy have taken new urgency. This paper examines certain effects of economic integration on sectoral diversification and exchange rate policy in developing economies. In contrast, however, to the large body of recent literature, which, in light of the currency crises in the 1990s, examines the role of capital market liberalization, we focus on the effects of integration in the goods markets. This question deserves more attention as an increasing number of countries are taking steps to open their goods markets to trade.

Developing countries have traditionally relied on primary exports, such as agriculture or mining, to generate foreign currency. To name but a few, Colombia has relied on coffee and, more recently, oil. The share of coffee was almost 60 percent of total exports in 1970, and that of oil was about 10 percent. In 1999, the share of coffee dropped to about 10 percent, while that of oil increased to more than 30 percent. In Zambia, metals made up more than 90 percent of exports in 1990, although that share dropped to 60 percent in 2000. The oil and gas sector in Indonesia contributed more than 70 percent of exports at the end of the 1970s. That share subsequently dropped to less than 30 percent in the 1990s. It is therefore not surprising to find that developing countries are highly vulnerable to shocks to their primary sectors.

Indeed, primary exports have been subject, historically, to frequent and important exogenous shocks. The loss of income, and the ensuing balance of payments deficits, due to these shocks were especially destabilizing for developing countries, both in the short and long run. A natural and often-cited policy to enhance a country’s ability to adjust to such shocks is industrial diversification. But while countries like Indonesia have in the past two decades successfully diversified their industry into nontraditional sectors, others have barely reduced their reliance on primary exports. For these countries, diversification remains a difficult challenge, and the question which new industries to promote and invest in is particularly relevant.

Accordingly, we develop a three-sector model with nominal wage rigidity, of the type made popular in the recent New Keynesian open-economy literature. One sector is singled out and is

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2 See for instance Knight (1998) on the impact of financial market liberalization on developing economies; Aghion, Bacchetta, and Banerjee (1999) for analyzing the role of financial liberalization as a source of instability; and Edison et al. (2002) for a survey of the evidence on the links between capital markets liberalization and economic performance.

3 See for instance Cashin et al. (2000) on the persistence of shocks to commodity prices; Cashin et al. (2002) and Cashin and McDermott (2002) on the cyclical and long-run behavior of commodity prices; and Collier and Gunning (2000) and Collier and Dehn (2001) on their impact on developing countries.
thought of as the primary sector of the developing country. We evaluate policies primarily in terms of their ability to offset the effects of shocks to this sector. We allow for two nonprimary sectors in the model, rather than one, to examine under what conditions the developing country should promote one sector over another. The level of economic integration of a particular sector with the rest of the world is measured by the degree of elasticity of substitution between domestic and foreign goods in that sector, and the volume of trade.

First, we show that the efficient exchange rate policy in the context of this model conforms to conventional wisdom. Specifically, following a negative shock to primary goods prices, the local currency must depreciate and, hence, domestic prices must rise, to offset the rigidity in wages. Then, we show that higher economic integration in the form of higher elasticity of substitution between domestic and foreign goods helps to offset the effects of shocks to the primary sector and, therefore, enhances macroeconomic stability and induces less exchange rate volatility. In accordance with this result, we show also that a shift of resources from the less-integrated to the more-integrated nonprimary sector enhances stability in the same fashion.4

The implications of higher economic integration in the form of higher volume of trade are, however, less clear-cut, and depend on the source of higher trade. We show that a higher volume of trade in one nonprimary sector that stems from higher investment in that sector lessens the effects of shocks to the primary sector. A priori, the effects of a shift of resources from one nonprimary sector to the other are unclear, since this raises the volume of trade in one and lowers it in the other. We show, however, that a shift of resources to the sector with higher volume of trade enhances macroeconomic stability.

In contrast, a higher volume of trade that stems from a lower consumption of domestic goods, due, for instance, to access to a greater variety of foreign goods,5 has ambiguous effects on economic stability. But, here too, we show that a shift of resources to the nonprimary sector with larger volume of trade enhances stability.

To sum up, under the three forms of economic integration mentioned above, a shift of resources from the less integrated to the more integrated sector enhances macroeconomic stability. Thus, if the shocks to the primary sectors are substantial, then the home country would be better off investing in a single nonprimary sector. Naturally, if the shocks to the nonprimary sectors are nonnegligible, then further diversification may be called for.

Spurred by the work of Obstfeld and Rogoff (1995, 1996), the past decade has seen widespread application of New Keynesian open-economy models to the study of monetary and exchange

4 Of course, the benefit of such a shift can become marginal if the nonprimary sectors are also subject to important shocks in which case further diversification may be called for.

5 Arguably, this case is more representative of economic integration with the rest of the world than the previous one.
rate policy. These models have been extended to examine the implications of alternative specifications regarding, for instance, the type of rigidities, the price-setting mechanism, and the size of the economy. But the question of sectoral diversification, and its relation with economic integration and exchange rate policy, has not been considered. Indeed, the models employed so far have usually involved symmetric, one-sector (albeit monopolistic-competitive) economies, which are not suited to analyze this question. The studies that come closest to our work are two papers by Tille. Using essentially one-sector representations of the home and foreign countries, Tille (1999) examines the effects of country-wide changes in the elasticity of substitution between domestic and foreign goods on the exchange rate policy. Tille (2002) considers a two-sector model, but his focus is on the relation between industry specialization and exchange rate policy, and the model does not allow for differences in the elasticities of substitution or the volumes of trade between the two sectors.

The rest of the paper is organized as follows. We describe the model in Section II and derive the equilibrium in Section III. In Section IV, we discuss the case where prices of all goods are exogenously given in the world market, and in Section V, we discuss the case where the prices of domestic nonprimary goods are endogenous. We conclude in Section VI.

II. THE BASELINE MODEL

The model is intended to describe the economic setting in many developing countries, if not many small open economies in general. In the model, a developing country, labeled 0, is represented as trading with the rest of the world, labeled 1. The home economy consists of three sectors, $X$, $A$, and $B$. The sector $X$ is singled out, and is referred to as the primary sector. It stands, for example, for an oil or other primary commodities sector, agriculture, textile, or even a tourism sector, whose products are generally low in the production chain and very similar to foreign-made goods. Typically, such a sector makes up a large share of the developing country’s exports, and is subject to frequent and important shocks. Accordingly, in this paper, we will be mostly concerned with the effects of shocks to prices in this sector. We allow for two nonprimary sectors, $A$ and $B$, in the model, rather than one, to examine under what conditions the developing country should promote one sector over another.

All goods can be traded. Home-produced and foreign-produced primary goods are assumed to be perfect substitutes for each other, whereas nonprimary goods are differentiated according to the country of origin. The model is static in that it admits only one period.

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6 See Lane (2001) and Bowman and Doyle (2003) for surveys of the literature on new open-economy macroeconomics; Obstfeld (2001) for a broad review of international macroeconomics; and Clarida, Gali, and Gertler (2001).

7 See Betts and Devereux (1996), Devereux and Engel (1998, 2000), and Gali and Monacelli (2002).

8 The model can easily be extended to include nontraded goods as well.
A. Households

Households in the home economy are identical and their total number is assumed constant and normalized to 1. The periodic utility of a household is

\[
\frac{1}{1-\sigma} C^{1-\sigma} - \frac{1}{1+\phi} N^{1+\phi} + \chi \ln \left( \frac{M}{P} \right)
\]

(1)

where \( N \) is the number of hours worked, \( \frac{M}{P} \) are real money balances held, and \( C \) is consumption of a composite of goods from the various sectors and countries:

\[
C = \left( \frac{C_X}{\gamma_X} \right)^{\gamma_X} \left( \frac{C_A}{\gamma_A} \right)^{\gamma_A} \left( \frac{C_B}{\gamma_B} \right)^{\gamma_B} , \quad \gamma_X + \gamma_A + \gamma_B = 1,
\]

(2)

\[
C_A = \left[ \gamma_{0A}^a C_{0A}^{a_A} + \gamma_{1A}^a C_{1A}^{a_A} \right]^{\eta_A}, \quad \gamma_{0A} + \gamma_{1A} = 1,
\]

(3)

\[
C_B = \left[ \gamma_{0B}^b C_{0B}^{a_B} + \gamma_{1B}^b C_{1B}^{a_B} \right]^{\eta_B}, \quad \gamma_{0B} + \gamma_{1B} = 1,
\]

(4)

\( C_X \) denotes consumption of primary goods, and \( C_i \), \( (l=0,1, \ i=A,B) \), denotes consumption of goods produced in country \( l \) of type \( i \). The elasticity of substitution, \( \eta (\nu) \), between home-produced and foreign-produced A-goods (B-goods) is assumed to be greater than or equal to the elasticity of substitution between goods across different sectors, which, given the Cobb-Douglas specification, equals 1.\(^9\)

Households choose consumption goods and money balances after shocks are realized, subject to the budget constraint

\[ 9 \text{ The limit case of an elasticity of substitution between home-produced and foreign-produced goods equal to 1 is identified with the Cobb-Douglas functional form.} \]
\[ P_X C_X + \sum_{i} P_i C_i + M = R + \Pi + T + M_{-1}, \]  

where \( \Pi \) denotes dividends, \( M_{-1} \) is the initial stock of money balances held, \( T \) is lump-sum transfers (equal to the increase in money balances \( M - M_{-1} \)), \( P_X \) is the price of primary goods, \( P_i \) (\( l = 0,1, i = A, B \)), is the price of goods produced in country \( l \) of type \( i \), and \( R \) is the household’s income from labor, all measures denominated in local currency.

The price index of \( i \)-goods, \( P_i \), and the aggregate price index, \( P \), are defined as follows\(^{10}\)

\[ P_A = \left[ \gamma_{0A} P_A^{i-A} + \gamma_{1A} P_A^{i-A} \right]^{1-\eta}, \]  

\[ P_B = \left[ \gamma_{0B} P_B^{i-B} + \gamma_{1B} P_B^{i-B} \right]^{1-\nu}, \]  

and

\[ P = P_X P_A P_B. \]  

Standard optimization implies that a household’s holding of money balances and expenditures on the various types of goods are:

\[ P_i C_i = \gamma_i P C, \quad (i = X, A, B), \]  

\[ P_l C_l = \gamma_{lA} \left( \frac{P_l}{P_A} \right)^{1-\eta} P_A C_A, \quad (l = 0,1) \]  

\[ P_l C_l = \gamma_{lB} \left( \frac{P_l}{P_B} \right)^{1-\nu} P_B C_B, \quad (l = 0,1) \]

\(^{10}\) The price index of a composite good can be defined as the minimum expenditure needed to buy one unit of the composite good.
and

\[
\frac{M}{P} = \chi C^\sigma. \tag{12}
\]

**B. Production**

All domestic firms are identical and have decreasing returns-to-scale technologies. Production requires a continuum of differentiated types of labor, \( j \), uniformly distributed over the unit interval (\( j \in [0,1] \)). Each household is associated with one type of labor, and the number of households of each type is constant and equal to 1.

Specifically,

\[
Y_i = \frac{1}{1-\alpha} A_i l_i^{1-\alpha}, \quad 0 \leq \alpha < 1, \quad \tag{13}
\]

\[
L_i = \left( \int_0^1 l_i(j) \frac{1}{j^\lambda} dj \right)^{\frac{1}{\lambda - 1}}, \quad \lambda \geq 1, \quad \tag{14}
\]

where \( Y_i \) denotes output by a firm in sector \( i \), \( l_i(j) \) is labor input of type \( j \), and \( A_i \) is a sector-wide technological shock equal to 1 in steady state.

Firms take prices and wages as given and choose their volume of output after shocks are realized. Profit maximization therefore implies that demand by a firm in sector \( i \) for labor is

\[
L_x = \left[ \frac{A_x P_x}{W_x} \right]^{\frac{1}{\alpha}}, \quad L_i = \left[ \frac{A_i P_{ii}}{W_i} \right]^{\frac{1}{\alpha}} \quad (i = A, B) \quad \text{and} \quad l_i(j) = \left[ \frac{W_i}{W_i(j)} \right]^\lambda L_i, \quad \tag{15}
\]

where \( W_i(j) \) is the wage for labor of type \( j \) in sector \( i \), and \( W_i \) is the wage-index for labor employed in sector \( i \).\(^{11}\)

\(^{11}\) \( W_i \) can be interpreted as the minimum cost of a unit of composite labor.
\[ W_i = \left( \int_0^1 W_i(j)^{1-\lambda} \, dj \right)^{\frac{1}{1-\lambda}}. \]  

(16)

Naturally, the higher the effective markup over wages, the larger the output and the larger the demand for labor.

### C. Price and Wage Setting

We assume throughout that the law of one price holds for all goods. Furthermore, on the basis that the domestic market is small, the prices of primary goods and all foreign goods are assumed to be determined in the world market and exogenously given. We will consider, however, both the case where the price of home-produced nonprimary goods is exogenously given and the case where it is endogenously determined.

Labor is assumed to be perfectly mobile across firms and sectors. Households of the same type share equally all their resources and jointly fix their wages at the beginning of the period, before shocks are realized, and without discrimination as to the firm or sector of employment. Accordingly, households of the same type earn the same wage and share labor equally.

Maximization of expected utility implies

\[ W(j) = \frac{\lambda}{\lambda - 1} E\left[ l(j)^{1+\phi} \right], \]

(17)

where \( E \) is the expectation operator, \( l(j) \) denotes the total demand for labor per household of type \( j \), and \( C(j) \) is consumption per household of type \( j \).

Under the model specification, wages are rigid in the sense that they cannot readjust after shocks are realized. We view, however, the outcome under flexible wages (that is, the outcome that obtain if wages could adjust after shocks are realized) as the benchmark that policymakers seek to approximate.

Optimization under flexible wages leads to an identical formula for wages as the one above without the expectation operator, e.g.,

\[ \frac{W(j)}{P} = \frac{\lambda}{\lambda - 1} C(j)^{\phi} l(j)^{\phi}. \]

(18)
D. Long-Run Shares of Trade

Consistent with the fact that wages are equal across sectors, and firms have identical technologies and take prices and wages as given, prices of all goods are assumed to be equal in steady state. It follows that all domestic firms produce the same output, $\bar{Y}$, in steady state, and

$$\bar{C}_i = \gamma_i \bar{C}, \quad (i = 0,1), \quad (i = A, B), \quad \text{(19)}$$

$$\bar{C}_i = \gamma_i \bar{C}, \quad (i = X, A, B), \quad \text{(20)}$$

$$\bar{C} = n\bar{Y}, \quad \text{(21)}$$

where bar superscripts denote steady-state values, $n_i$ is the number of firms in sector $i$, and $n = n_x + n_A + n_B$ is the total number of firms in the home country.

The (long-run) share of net exports of primary goods in total output is therefore

$$\alpha_X \equiv \frac{n_x \bar{Y}_x - \bar{C}_x}{n\bar{Y}} = \frac{n_x}{n} - \gamma_X; \quad \text{(22)}$$

the share of exports of nonprimary goods is

$$\alpha_{0i} \equiv \frac{n_i \bar{Y}_i - \bar{C}_{0i}}{n\bar{Y}} = \frac{n_i}{n} - \gamma_{0i}; \quad (i = A, B), \quad \text{(23)}$$

while the share of imports of nonprimary goods is

$$\alpha_{1i} = \frac{\bar{C}_{1i}}{n\bar{Y}} = \gamma_{1i}, \quad (i = A, B). \quad \text{(24)}$$

Since there is only one period, the share of total imports of nonprimary goods, $\alpha_{1A}$, must balance the share of total exports

$$\alpha_{1A} + \alpha_{1B} + \alpha_{2A} + \alpha_{2B} = \alpha_X + \alpha_{0A} + \alpha_{0B}. \quad \text{(25)}$$
III. Equilibrium

We focus on symmetric equilibriums, whereby all households and all firms within a sector have identical outcomes. Under these circumstances, households of all types earn the same wage, $W$, supply the same amount of labor, $L$, and consume the same baskets of goods, $C$. In equilibrium, total supply of labor must equal total demand, and total expenditures must equal total income. In other words,

$$L = n_X L_X + n_A L_A + n_B L_B,$$  \hspace{1cm} (26)

and

$$PC = n_X P_X Y_X + n_A P_A Y_A + n_B P_B Y_B.$$  \hspace{1cm} (27)

Substituting the expressions for labor demand, $L_i$, and output, $Y_i$, by a firm in sector $i$, it follows,

$$L = \Omega \left[ \frac{P}{W} \right]^{\frac{1}{\alpha}},$$  \hspace{1cm} (28)

$$C = \frac{1}{1-\alpha} \Omega \left[ \frac{P}{W} \right]^{\frac{1-\alpha}{\alpha}},$$  \hspace{1cm} (29)

and

$$\frac{M}{P} = \frac{\chi}{(1-\alpha)^{\sigma}} \Omega^\sigma \left[ \frac{P}{W} \right]^{\frac{\sigma(1-\alpha)}{\alpha}},$$  \hspace{1cm} (30)

where

$$\Omega = n_X \left[ \frac{A_X P_X}{P} \right]^{\frac{1}{\alpha}} + n_A \left[ \frac{A_A P_{0A}}{P} \right]^{\frac{1}{\alpha}} + n_B \left[ \frac{A_B P_{0B}}{P} \right]^{\frac{1}{\alpha}}.$$  \hspace{1cm} (31)

The system of equations above is “closed” by adding the equation for wages and specifying a rule for the money supply.
A. Flexible Wages

Under flexible wages, $\frac{W}{P} = \frac{\lambda}{\lambda - 1} C^\sigma L^\phi$. Substituting the expressions for $C$ and $L$ found above, it follows

$$
\left(\frac{W}{P}\right)^\Theta = \left(\frac{1}{1-\alpha}\right)^{a_\sigma} \left(\frac{\lambda}{\lambda - 1}\right)^{a} \Omega^{a(\phi + \sigma)}, \quad (32)
$$

$$
C^\Theta = \left(\frac{1}{1-\alpha}\right)^{a_\phi} \left(\frac{\lambda - 1}{\lambda}\right)^{1-a} \Omega^{a(1+\phi)}, \quad (33)
$$

$$
L^\Theta = (1-\alpha)^{a_\sigma} \left(\frac{\lambda - 1}{\lambda}\right) \Omega^{a(1-\sigma)}, \quad (34)
$$

where,

$$
\Theta = \alpha + \phi + \sigma(1-\alpha). \quad (35)
$$

Of course, under flexible wages, money is neutral and has no real effect on the economy.

B. Predetermined Wages

Under predetermined wages,

$$
W = \frac{\lambda}{\lambda - 1} E\left[\frac{L^{1+\phi}}{PC^\sigma}\right]. \quad (36)
$$

Now, monetary policy can be used to reproduce the level of real wages that obtains under flexible wages. For this it suffices that the money supply be set so that

$$
P^{-\Theta} = \left(\frac{1}{1-\alpha}\right)^{a_\sigma} \left(\frac{\lambda}{\lambda - 1}\right)^{a} \Omega^{a(\phi + \sigma)} W^{-\Theta}, \quad (37)
$$

hence
\[ M^\Theta = X^\Theta \left( \frac{1}{1-\alpha} \right)^{\frac{\sigma}{1-\alpha}} \left( \frac{\lambda-1}{\lambda} \right)^{\alpha+\sigma(1-\alpha)} \Omega^{\alpha\phi(\sigma-1)} W^\Theta. \]  

(38)

To the extent that, under both flexible and predetermined wages, the rest of the aggregate variables depend in the same fashion on real wages, this monetary policy rule would achieve with predetermined wages the same equilibrium outcome that obtains under flexible wages, and is therefore considered efficient. Unless stated otherwise, this monetary policy is assumed to be implemented from now on.

Accordingly, under both flexible and predetermined wages, the behavior of the macro variables is governed by the behavior of \( \Omega \) alone. To a first-order approximation, in terms of log-deviations from steady state, we have

\[ \alpha \hat{\Omega} = \frac{n_X}{n} (\hat{p}_X' - \hat{p}'_X) + \frac{n_A}{n} (\hat{p}_0A' - \hat{p}'_0) + \frac{n_B}{n} (\hat{p}_0B' - \hat{p}'_0) + \frac{n_X \hat{\alpha}_X + n_A \hat{\alpha}_A + n_B \hat{\alpha}_B}{n}, \]

(39)

where \( \hat{\Omega} \) amounts to the effect of price changes on income from foreign trade. Thus, not surprisingly, the composition of trade has a bearing on the desired monetary policy and efficient equilibrium adjustments to shocks only through the effects on income from foreign trade.

The desired monetary policy rule described above can be expressed equivalently as an exchange rate policy rule:

\[ \hat{s} = \hat{p}'_X - \hat{p}' = \frac{\gamma_X + (\phi + \sigma)}{\Theta} \alpha_X \hat{p}_X' + \frac{\gamma_A \gamma_{0A} + (\phi + \sigma)}{\Theta} \alpha_{0A} \hat{p}_0A' + \frac{\gamma_B \gamma_{0B} + (\phi + \sigma)}{\Theta} \alpha_{0B} \hat{p}_0B' + \frac{\gamma_A \gamma_{1A} - (\phi + \sigma)}{\Theta} \alpha_{1A} \hat{p}_1A' + \frac{\gamma_B \gamma_{1B} - (\phi + \sigma)}{\Theta} \alpha_{1B} \hat{p}_1B' + \frac{(\phi + \sigma) n_X \hat{\alpha}_X + n_A \hat{\alpha}_A + n_B \hat{\alpha}_B}{n}. \]

(40)

where \( s \) is the nominal exchange rate (expressed as the price of a unit of domestic currency in foreign currency).

To complete the derivation of the equilibrium, we need to determine the relative prices of goods. We consider first the case where prices of all goods (denominated in foreign currency) are determined exogenously in the world market.
IV. All Prices Exogenously Given

Note first that if all prices are exogenously given, then the effect of a domestic technological innovation on \( \Omega \), and, hence, on the monetary policy response and economic equilibrium, is independent from the composition of trade in the home country. A positive technological shock\(^{12} \) leads to an increase in income, consumption, real wages, and real balances. To achieve the higher real wages, local prices must fall, which is achieved by not allowing the money stock to rise proportionately to output. Employment may increase if the substitution effect of the shock exceeds the income effect, i.e., if \( \sigma < 1 \).

Consider now shocks to world prices. Assuming that such shocks are sector-wide and prices within each sector move in tandem,\(^{13} \Omega \), and hence, all macro variables, are least volatile when there is balanced trade within each sector, i.e., \( \alpha_x = 0 \) and \( \alpha_{oi} = \alpha_{il} \quad (i = A, B) \). Under these circumstances, any change in income from exports is exactly offset by an equal change in the cost of imports.

Such a state of affairs is, however, unrealistic because of inherited endowments in capital and resources.\(^{14} \) Accordingly, we assume from now on that the home country is a net exporter of primary goods, i.e., \( \alpha_x \geq 0 \), and we examine the effects of a change in the world price of primary goods, other (foreign-currency-denominated) prices kept constant.

Then, \( \tilde{\alpha} \Omega = \alpha_x \hat{p}_x \).\(^{15} \) A drop in the foreign price of primary goods, \( \hat{p}_x \), entails a drop in income from exports of primary goods, and hence, a drop in real wages and aggregate consumption. The lower cost of labor induces output and employment in the nonprimary sectors to increase, and thus to partly offset the lower income from exports of primary goods.

The effect on employment and output in the primary sector, however, depends on the model’s parameters: the larger the effect of the shock on income (i.e., the larger the share of exports of primary goods, \( \alpha_x \)), the smaller the capacity in the nonprimary sectors to absorb new labor (i.e., the steeper the decrease in returns to scale, \( \alpha \)), the larger the income effect relative to the substitution effect as incorporated in the household’s utility function (i.e., the smaller the parameter \( \phi \) and the larger the parameter \( \sigma \)), then, the smaller the drop (if any) in employment.

---

\(^{12} \) Except for the relative weights of the sectors that they affect, the different types of technological innovations have identical implications.

\(^{13} \) And fluctuations across sectors are uncorrelated.

\(^{14} \) By equipping all firms with the same decreasing returns to scale technology, we have essentially reduced technological differences between sectors, and hence comparative advantage between countries, to differences in endowments of capital, i.e., number of firms, in each sector.

\(^{15} \) We neglect to mention technological shocks.
and output in the primary sector. If $\sigma \leq 1$, then the substitution effect of a drop in wages on the household's utility dominates the income effect, and output and employment in the primary sector unambiguously decrease as wages decline less than the relative price of primary goods. If $\sigma > 1$, then output and employment in the primary sector can actually increase (see appendix for a formal proof). In any case, as can be seen from the expression of $L$ in terms of $\Omega$, aggregate labor always decreases if $\sigma < 1$ following a drop in primary goods prices, and increases if $\sigma > 1$.16

With regard to the exchange rate, we have

$$\hat{s} = \left( \gamma_x + \frac{(\phi + \sigma)}{\Theta} \alpha_x \right) \hat{p}_x'$$

(41)

Not surprisingly, this policy (which achieves with predetermined wages the same equilibrium outcome that obtains under flexible wages) calls for a depreciation of the local currency following a drop in the relative price of primary goods. This is intended to raise the price level above its steady state and achieve the downward adjustment in real wages that would obtain if wages were flexible.

The higher the volatility of prices in the primary sector, the larger the relative size of the primary sector (and, hence the larger the share of exports, $\alpha_x$, the share of domestic consumption, $\gamma_x$, being kept equal), the smaller the capacity in the nonprimary sectors to absorb new labor, i.e., the steeper the decrease in returns to scale, $\alpha$, the greater the effect of a price change on income, and the larger the ensuing change in real wages if wages were flexible. Hence, the greater the required adjustment in the exchange rate, and the greater the need for a flexible exchange rate.

However, a higher share of exports that stems from lower consumption of local primary goods,17 would call for a more flexible exchange rate policy if, and only if, $\frac{\phi + \sigma}{\Theta} > 1$, i.e., if $\sigma > 1$.18 In that case, as shown earlier, households are willing to accept low enough real wages to actually increase labor following the negative shock to income.

16 While the case $\sigma > 1$ seems more plausible in developing countries, the implication that aggregate labor increases following a negative shock is counterintuitive. This result depends, however, on the assumption of full labor mobility, and does not necessarily hold otherwise.

17 Which may obtain, for instance, as a result of export subsidies or taxes on domestic consumption.

18 To see this note that $\gamma_x + \frac{\phi + \sigma}{\Theta} \alpha_x = n_x + \left( \frac{\phi + \sigma}{\Theta} - 1 \right) \alpha_x$, and $\frac{\phi + \sigma}{\Theta} - 1 = \frac{\alpha (\sigma - 1)}{\Theta}$.
Note that the composition and volume of trade within the nonprimary sectors do not affect the conclusions above. Thus, to the extent that shocks to the nonprimary sectors can be neglected (and prices are exogenous), the market structure of the nonprimary sectors has no bearing on economic stability or monetary policy.

V. PRICES OF HOME-PRODUCED NONPRIMARY GOODS ENDOGENOUS

Assume now that prices of primary goods and foreign–produced goods are still determined in the world market and exogenously given, but prices of home-produced nonprimary goods adjust to equate supply and demand. The rationale is that whereas it is too small to influence prices of primary goods and foreign-produced goods, the developing country may account for a significant share of the market (demand or supply) for home-produced goods, and can, therefore, influence their prices. As a consequence, prices of domestic nonprimary goods will also reflect the costs of domestic inputs, i.e., wages.

Specifically, in analogy with domestic demand, we suppose that foreign demand for home-produced nonprimary goods is as follows,

$$ C_{0A}^f = \alpha_{0A} n \bar{Y} \left( \frac{P_{\text{level}}^f}{P_A^f} \right)^{\mu_A} \left( \frac{P_{0A}}{P_A} \right)^{-\eta}, \quad (42) $$

and

$$ C_{0B}^f = \alpha_{0B} n \bar{Y} \left( \frac{P_{\text{level}}^f}{P_B^f} \right)^{\mu_B} \left( \frac{P_{0B}}{P_B} \right)^{-\nu}, \quad (43) $$

where $C_{0i}^f$, ($i = A, B$), denotes foreign consumption of home-produced $i$-goods, $\alpha_{0i}$ and $\alpha_{0i} n \bar{Y}$ are, respectively, the share and volume of exports of $i$-goods in steady state, $P_{\text{level}}^f$ is the foreign aggregate price-level (not to confuse with the domestic aggregate price-level denominated in foreign currency, $P_f$), and $\mu_i$ is the elasticity of substitution between $i$-goods and other goods in the foreign country. Whereas the elasticity of substitution between $i$-goods and other goods equals 1 in the domestic country, $\mu_i$ can be greater than 1 to reflect, for instance, that the foreign country may have access to other goods not available in the home country. The elasticity of substitution between home-produced and foreign-produced $i$-goods is, however, the same, e.g., $\eta$ or $\nu$. $P_{\text{level}}^f$ is assumed to be exogenously given, and unaffected by changes in
the price of primary goods.\textsuperscript{19} This, combined with the fact that \( \mu_i \) can be greater than 1, entails that the price elasticity of foreign demand for home-produced nonprimary goods is larger than that of domestic demand. Again, all prices are assumed equal in steady state.

In equilibrium, prices of home-produced nonprimary goods adjust to equate demand with supply:

\[
C_{0i} + C_{f0i} = n_i Y_i, \quad (i = A, B),
\]

hence

\[
\gamma_{0A} \left( \frac{P_{0A}}{P_A} \right)^{-\eta} C_A + \alpha_{0A} n Y \left( \frac{P_{f\text{level}}}{P_A} \right)^{\mu_A} \left( \frac{P_{0A}}{P_A} \right)^{-\eta} = n_A Y_A,
\]

which, after substituting the expressions for \( C_A \) and \( Y_A \), becomes

\[
\gamma_{0A} \gamma_A \left( \frac{P}{P_A} \right) C + \alpha_{0A} n Y \left( \frac{P_{f\text{level}}}{P_A} \right)^{\mu_A} \left( \frac{P_{0A}}{P_A} \right)^{-\eta} = n_A Y_A.
\]

Similarly, for \( B \)-goods, we get

\[
\gamma_{0B} \gamma_B \left( \frac{P}{P_B} \right) C + \alpha_{0B} n Y \left( \frac{P_{f\text{level}}}{P_B} \right)^{\mu_B} \left( \frac{P_{0B}}{P_B} \right)^{-\eta} = n_B Y_B.
\]

Keeping prices of foreign-produced goods constant, the system of equations above can be solved for \( \hat{P}_{0A} \) and \( \hat{P}_{0B} \) and, hence, for \( \hat{\Omega} \) and \( \hat{s} \). Specifically, one finds\textsuperscript{20}

\[
\Lambda \hat{p}_{0A} = \Pi \hat{p}_{Xf}.
\]

\textsuperscript{19} Since prices of foreign goods have been assumed to be exogenously given, while prices of domestic goods are endogenous, one cannot expect the foreign and home countries to have the same market structure. One possible specification is that, even though the foreign country is large, its consumption of home-produced \( i \)-goods is relatively small because of stronger preference for foreign goods or because of greater access to a variety of other goods. Under these conditions, the foreign price-level, \( P_{f\text{level}} \), is not identical to the home price-level denominated in foreign currency, \( P_{f} \), and can be assumed to be exogenous.

\textsuperscript{20} We ignore from now on technological shocks.
\[ \Lambda \hat{p}_0 = \Gamma \hat{p}_X \]  

(49)

\[ \alpha \hat{\omega} = K \hat{p}_X \]  

(50)

and

\[ \hat{s} = H \hat{p}_X \]  

(51)

where \( \Lambda, \Gamma, \Pi, H, \) and \( K \) are coefficients that depend on the model’s parameters, and the behavior of which can be examined numerically.\(^{21}\)

The equilibrium adjustment to a negative shock to primary goods prices (and the role of the parameters \( \alpha, \phi, \) and \( \sigma \)) are similar to those when all prices are exogenously given. In particular, the domestic currency must depreciate to allow real wages to fall and thus ease the shock to output and employment. The extra spin in this case is that the larger domestic output in the nonprimary goods sector induced by the depreciation can be liquidated only if the relative price of home-produced to foreign-produced nonprimary goods, \( \frac{P_{0i}}{P_i} \), falls. This particular effect entails that the market structure in the nonprimary sector has a bearing on economic adjustments to shocks and stability.

Leaving the primary sector, that is, \( n_X, \gamma_X, \) and \( \alpha_X \), unchanged, one can show:

1. The higher the elasticities of substitutions \( \eta, \nu, \mu_A, \) and \( \mu_B \), the lower the coefficients \( K \) and \( H \). The reason is that the higher elasticities of substitution entail a smaller drop in the relative price of domestic nonprimary goods and, hence, a smaller loss of income from exports, following a negative shock to the price of primary goods. Thus, higher economic integration, in the form of higher elasticities of substitution, \( \eta, \nu, \mu_A, \) and \( \mu_B \), enhances stability of the aggregate variables as well as that of the exchange rate.

2. All else being equal between the two nonprimary sectors (except the elasticities of substitution, \( \eta, \nu, \mu_A, \) and \( \mu_B \)), a reallocation of resources from the nonprimary sector with smaller elasticities to the one with larger elasticities enhances the stability of the aggregate variables. Numerically, one shows that at the point where the two sectors are symmetric in all but the elasticities of substitution, \( \eta, \nu \) (resp. \( \mu_A, \) and \( \mu_B \)), \( \frac{\partial K}{\partial n_A} \) (keeping the total number of firms \( n \) fixed) has the opposite sign of \( (\eta - \nu) \) (resp. \( \mu_A - \mu_B \)).

\(^{21}\) The numerical estimations are available from the author upon request.
This proposition takes a stronger form in the extreme case where there is no local demand for domestic nonprimary goods (i.e., $\gamma_{0A} = \gamma_{0B} = 0$). Then, $K$ decreases continuously as more resources are reallocated from the nonprimary sector with smaller elasticity of substitution to the one with larger elasticity of substitution. In general, however, $K$ traces a bell shape as $n_A$ increases from 0 to $n$. The reason is provided in the following result.

3. All else being equal between the two nonprimary sectors (except their sizes and shares of exports), the aggregate variables are more stable when resources are concentrated in one sector. Put differently, higher investment in the sector with higher volume of trade enhances macroeconomic stability. The reason is that, in both cases, demand for domestic products is relatively more price elastic. Hence, income is less affected by negative shocks. Numerically, $K$ is largest when $n_A = n_B$, and $K$ decreases as one or the other nonprimary sector grows larger relative to the other.

The reason is that the nonprimary sector with more capital (i.e., more firms) devotes a larger share of its output to exports. Because exports are more price elastic than local consumption, prices change relatively less in that sector and, hence, cause a relatively smaller change in income from trade following a shock. (In the extreme case where there is no local demand for home-produced nonprimary goods, i.e., $\gamma_{0A} = \gamma_{0B} = 0$, stability is unaffected by the relative size of the sectors.)

Whether greater concentration of resources in one sector calls for a more or less flexible exchange rate depends on the model’s parameters. Typically, a higher concentration of resources in one sector calls for a more flexible exchange rate policy if $\frac{\phi + \sigma}{\Theta} > 1$, i.e., if $\sigma > 1$.

4. All else being equal, a higher volume of trade in a nonprimary sector that stems from a lower share of domestic consumption of home- relative to foreign-produced goods in that sector requires smaller adjustments in the exchange rate following shocks. The effect on aggregate variables is, however, ambiguous ($K$ traces a bell curve as the volume of trade increases).

The reason is that, since foreign demand for domestic goods is more elastic than local demand, the larger the share of exports relative to local consumption, the smaller the decrease in the relative price of domestic nonprimary goods, and, hence, the smaller the drop in real wages. The effect on income from trade is, however, ambiguous, since the volume of exports if higher.

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22 This claim is shown numerically for certain values of the model’s parameters.

23 This can be due, for instance, to industry specialization and/or economic integration in the form of greater access to other foreign goods.
Nonetheless, a shift of resources from the sector with smaller volume of trade, say B, to the one with larger volume of trade, say A, lowers volatility. (Numerically, one shows that at the point where the two sectors are symmetric in all but the volume of trade, \( \frac{\partial K}{\partial n_A} < 0 \) if sector A has larger volume of trade.) The reason is that the shift of resources to sector A causes domestic prices in that sector to drop less following a negative shock to primary goods. This will have a first-order benefit, because sector A already devotes a larger volume of its output to exports than sector B.

VI. CONCLUSION

The paper examines certain effects of economic integration on sectoral diversification and exchange rate policy in developing economies. It develops a three-sector model with nominal wage rigidity, whereby one sector is thought of as the primary sector of the developing country, and the other two are sectors in which the country can diversify. The paper then examines the relationship between the market structure of the nonprimary sectors and macroeconomic adjustments to shocks in the primary sector. In particular, it examines under what conditions the developing country should promote one nonprimary sector over another.

It is shown first that the efficient exchange rate policy in the context of this model conforms to conventional wisdom: the local currency must depreciate following a negative shock to primary goods prices to offset the rigidity in wages and limit the increase in unemployment. Then, it is shown that higher economic integration, whether in the form of higher elasticity of substitution between domestic and foreign goods, or in the form of higher volume of trade that stems from higher investment in export-oriented industries, lessens the effects of shocks to the primary sector and enhances macroeconomic stability. The reason is that, in both cases, economic integration makes demand for domestic products relatively more price elastic. Hence, income is less affected by negative shocks.

However, higher economic integration in the form of a higher volume of trade that stems from a drop in the domestic consumption of home goods relative to foreign goods has ambiguous effects on macroeconomic stability. The higher volume of trade implies a more price elastic demand for home goods and, hence, promotes stability, but it also implies greater effects on income from trade following shocks, which impairs stability. Nonetheless, in all three forms of economic integration, a shift of resources from the less-integrated to the more-integrated nonprimary sector is shown to enhance macroeconomic stability.

In sum, if the shocks to the primary sector dominate other shocks, then the home country may be better off investing in a single nonprimary sector. If the nonprimary sectors are also subject to important shocks, then further diversification may be called for.
DETAILED DERIVATION OF THE EQUILIBRIUM

Recall, in equilibrium,

\[ L = \Omega \left[ \frac{P}{W} \right]^\alpha, \quad (1) \]

\[ C = \frac{1}{1-\alpha} \Omega \left[ \frac{P}{W} \right]^{1-\alpha}, \quad (2) \]

\[ L_x = \left[ \frac{A_x P_x}{P} \right] \left[ \frac{P}{W} \right]^\alpha, \quad (3) \]

\[ L_i = \left[ \frac{A_i P_i}{P} \right] \left[ \frac{P}{W} \right]^\alpha, \quad (i = A, B), \]

and

\[ \frac{M}{P} = \frac{\nabla}{(1-\alpha)^\alpha} \Omega^\sigma \left[ \frac{P}{W} \right]^{\sigma(1-\alpha)}, \quad (4) \]

where

\[ \Omega = n_x \left[ \frac{A_x P_x}{P} \right]^\alpha + n_A \left[ \frac{A_i P_i}{P} \right]^\alpha + n_B \left[ \frac{A_B P_B}{P} \right]^\alpha, \quad (5) \]

or in log-deviations from steady state,
\[ \alpha \hat{\Omega} = \frac{n_x}{n} (\hat{p}_X - \hat{p}_f) + \frac{n_A}{n} (\hat{p}_{0A} - \hat{p}_f) + \frac{n_B}{n} (\hat{p}_{0B} - \hat{p}_f) + \frac{n_x \hat{a}_x + n_A \hat{a}_A + n_B \hat{a}_B}{n} \]

\[ = \alpha_X \hat{p}_X + \left( \alpha_{0A} \hat{p}_{0A} - \alpha_{1A} \hat{p}_{1A} \right) + \left( \alpha_{0B} \hat{p}_{0B} - \alpha_{1B} \hat{p}_{1B} \right) + \frac{n_x \hat{a}_x + n_A \hat{a}_A + n_B \hat{a}_B}{n} \]  

(6)

Under flexible wages

\[ \left( \frac{W}{P} \right)^\Theta = \left( \frac{1}{1-\alpha} \right)^{\alpha} \left( \frac{\lambda}{\lambda-1} \right)^{\alpha} \Omega^{\alpha(\phi+\sigma)} , \]  

(7)

\[ C^\Theta = \left( \frac{1}{1-\alpha} \right)^{\alpha+\phi} \left( \frac{\lambda-1}{\lambda} \right)^{1-\alpha} \Omega^{\alpha(1+\phi)} , \]  

(8)

\[ L^\Theta = (1-\alpha)^{\sigma} \left( \frac{\lambda-1}{\lambda} \right)^{\alpha+\sigma} \Omega^{\sigma(1-\sigma)} , \]  

(9)

\[ \bar{Y}^\Theta = \left( \frac{1}{1-\alpha} \right)^{\alpha+\phi} \left( \frac{\lambda-1}{\lambda} \right)^{\sigma} \Omega^{\sigma(1-\sigma)} \]  

(10)

\[ L_X = \left[ \frac{A_X P_X P}{W} \right]^{\frac{1}{\alpha}} , \quad L_i = \left[ \frac{A_i P_{0i} P}{W} \right]^{\frac{1}{\alpha}} \quad (i = A, B) \]  

(11)

where

\[ \Theta = \alpha + \phi + \sigma(1-\alpha) . \]  

(12)

Thus, to reproduce with predetermined wages the level of real wages that obtains under flexible wages, it suffices to set the money supply so that
\[ P^{-\Theta} = \left( \frac{1}{1-\alpha} \right)^{\alpha} \left( \frac{\lambda}{\lambda-1} \right)^{\alpha} \Omega^{\alpha(\phi+\sigma)} W^{-\Theta}. \] (13)

Substituting this expression in that of real money balances provided above, it follows

\[ M^{\Theta} = \chi^{\Theta} \left( \frac{1}{1-\alpha} \right)^{\alpha} \left( \frac{\lambda}{\lambda-1} \right)^{\alpha+\sigma(1-\alpha)} \Omega^{\alpha(\phi-1)} W^{\Theta}. \] (14)

Alternatively, using the representations of \( P, P^f, \) and \( \Omega, \) in log-deviations, e.g.,

\[ \hat{p} = -\frac{(\phi+\sigma)}{\Theta} \alpha \hat{\Omega}, \] (15)

and

\[ \hat{p}^f = \gamma_X \hat{P}_X^f + \gamma_{0,A} \hat{P}_A^f + \gamma_A \hat{P}_A^f + \gamma_{0,B} \hat{P}_B^f + \gamma_B \hat{P}_B^f, \] (16)

the monetary policy rule above can be expressed in the form,

\[ \hat{s} = \hat{p}^f - \hat{p} = \left( \gamma_X + \frac{(\phi+\sigma)}{\Theta} \alpha_X \right) \hat{P}_X^f + \left( \gamma_{0,A} + \frac{(\phi+\sigma)}{\Theta} \alpha_{0,A} \right) \hat{P}_0^A + \left( \gamma_A \gamma_{0,B} + \frac{(\phi+\sigma)}{\Theta} \alpha_{0,B} \right) \hat{P}_0^B + \left( \gamma_B \gamma_{1,B} - \frac{(\phi+\sigma)}{\Theta} \alpha_{1,B} \right) \hat{P}_1^B \] (17)

where \( s \) is the nominal exchange rate (expressed as the price of a unit of domestic currency in foreign currency).
A. All Prices Exogenously Given

Under flexible wages, the markup of prices over wages in the primary goods sector, in log-deviation from steady state (with prices of nonprimary goods kept constant) is

\[ \hat{p}_X - \hat{w} = \hat{p}_X^f - \hat{p}^f + \hat{p} - \hat{w} = \left(1 - \gamma_X - \frac{\phi + \sigma}{\Theta} \left(\frac{n_X}{n} - \gamma_X\right)\right) \hat{p}_X^f. \] (18)

If \( \sigma \leq 1\), then \( \frac{\phi + \sigma}{\Theta} \leq 1\) and \( \hat{p}_X - \hat{w} \) has the same sign as \( \hat{p}_X^f \) (since \( 1 - \gamma_X \geq \frac{n_X}{n} - \gamma_X \)). In other words, a drop in the price of primary goods induces a smaller drop in wages. It follows that, in this case, output and labor in the primary goods sector, as well as aggregate labor, unambiguously decrease with a decrease in the relative price of primary goods. If \( \sigma > 1\), however, then \( \frac{\phi + \sigma}{\Theta} > 1\) and the sign of \( \hat{p}_X - \hat{w} \) is ambiguous. Simple derivatives show that the larger is \( \alpha \), the smaller is \( \phi \), and the larger is \( \sigma \), the larger is \( \frac{\phi + \sigma}{\Theta} \), and, hence, the smaller is \( 1 - \gamma_X - \frac{\phi + \sigma}{\Theta} \left(\frac{n_X}{n} - \gamma_X\right) \), the larger the drop in wages, and the more likely are \( \hat{p}_X - \hat{w} \) and \( \hat{p}_X^f \) of opposite signs.

Likewise, the larger the share of exports of primary goods, \( \alpha_X = \frac{n_X}{n} - \gamma_X \), the smaller is \( 1 - \gamma_X - \frac{\phi + \sigma}{\Theta} \left(\frac{n_X}{n} - \gamma_X\right) \), the smaller the drop (if any) in labor and output in the primary goods sector, and the more likely is labor and output in this sector to actually increase following a decline in the relative price of primary goods.

B. Prices of Home-Produced Nonprimary Goods Endogenous

Equality between supply and demand of home-produced nonprimary goods imply

\[ \gamma_{0, \lambda} \frac{P}{P_A} C + \alpha_{0, \lambda} n \bar{Y} \left(\frac{P^{\text{level}}}{P_A}\right)^{\mu_A} = n \frac{1}{1 - \alpha} A^\mu \left(\frac{P_{0, \lambda}}{P_A}\right)^{\frac{1}{\alpha}} \left[\frac{P_{0, \lambda}}{W}\right]^{1 - \alpha}, \] (19)

hence
\[ \gamma_{0,A} P^C + \alpha_{0,A} n \bar{Y} \left( \frac{P_{level}}{P_A} \right)^{\mu_A} = n_A \left( 1 - \alpha \right) A_A \left( \frac{P_{0,A}}{P_A} \right)^{1-\alpha} \left( \frac{P_A}{P} \right)^{\alpha} \left[ \frac{P}{W} \right]^{1-\alpha}. \]  

(20)

Substituting the expressions for \( C \) and \( \frac{P}{W} \), it follows

\[ \gamma_{0,A} P^A \left( \frac{1}{1-\alpha} \right)^{\frac{1}{\alpha}} (\frac{\lambda-1}{\lambda})^{\frac{1-\alpha}{\Theta}} \alpha(\frac{1+\phi}{\Theta}) + \alpha_{0,A} n \bar{Y} \left( \frac{P_{level}}{P_A} \right)^{\mu_A} \]

\[ = n_A \left( \frac{1}{1-\alpha} \right)^{\frac{1}{\alpha}} A_A \left( \frac{P_{0,A}}{P_A} \right)^{1-\alpha+\alpha\gamma} \left( \frac{P_A}{P} \right)^{\alpha} \left( \frac{\lambda}{\lambda-1} \right)^{\frac{1-\alpha}{\Theta}} \alpha(\frac{1+\phi}{\Theta})^{1-1}. \]

(21)

Taking log-deviations from the steady state, and recalling \( n \bar{Y} = \left( \frac{1}{1-\alpha} \right)^{\frac{1}{\alpha}} (\frac{\lambda-1}{\lambda})^{\frac{1-\alpha}{\Theta}} n \Theta \),

\[ \gamma_{0,A} \gamma_{A} + \alpha_{0,A} \left[ \hat{p} - \hat{p}_A + \frac{\alpha(1+\phi)}{\Theta} \right] + \alpha_{0,A} \mu_A \left[ \hat{P}_{level} - \hat{p}_A^{f} \right] \]

\[ = \frac{1}{\alpha} \hat{a}_A + \frac{1-\alpha + \alpha \eta}{\alpha} \left( \hat{p}_{0,A} - \hat{p}_A \right) + \left( \frac{\alpha(1+\phi)}{\Theta} - 1 \right) \hat{\Omega} \]

(22)

hence

\[ \left( \frac{1}{\alpha} - \frac{\alpha_{0,A}}{n_A} \right) \left[ \hat{p} - \hat{p}_A \right] + \frac{\alpha_{0,A}}{n_A} \mu_A \left[ \hat{P}_{level} - \hat{p}_A^{f} \right] = \]

\[ \frac{1}{\alpha} \hat{a}_A + \frac{1-\alpha + \alpha \eta}{\alpha} \left( \hat{p}_{0,A} - \hat{p}_A \right) + \left( \frac{\alpha(1+\phi)}{\Theta} - 1 \right) \frac{\alpha_{0,A}}{n_A} \hat{\Omega} \]

(23)

Substituting the expression for \( \hat{\Omega} \), expanding the various price indexes into its constituents, and using the fact that the foreign price-level, \( \hat{p}_{level}^{f} \), is insignificantly affected by the shocks under consideration, we have
\[
\begin{align*}
\left[ \frac{1}{\alpha} \alpha_{0A} \right] \gamma_{0A} + \left( \frac{1}{\alpha} \frac{1 + \phi}{\Theta} \alpha_{0A} \right) \alpha_{0A} - \frac{1}{\alpha} \left( (\eta - 1)(1 - \gamma_{0A}) + \frac{\alpha_{0A}}{n_A} (\mu - 1) \gamma_{0A} \right) \hat{p}_{0A} \\
+ \left[ \frac{1}{\alpha} \alpha_{0B} \right] \gamma_{0B} + \left( \frac{1}{\alpha} \frac{1 + \phi}{\Theta} \alpha_{0B} \right) \alpha_{0B} \right] \hat{p}_{0B} = \\
- \left[ \frac{1}{\alpha} \alpha_{0A} \right] \gamma_{X} + \left( \frac{1}{\alpha} \frac{1 + \phi}{\Theta} \alpha_{0A} \right) \alpha_{X} \right] \hat{p}_{X} + \left( \frac{1 + \phi}{\Theta} \alpha_{0A} \right) \frac{n}{\alpha} n_A \hat{\alpha}_x + n_B \hat{\alpha}_A + n_B \hat{\alpha}_B + \frac{1}{\alpha}
\end{align*}
\]

and the corresponding equation for B-goods is

\[
\begin{align*}
\left[ \frac{1}{\alpha} \alpha_{0B} \right] \gamma_{0A} + \left( \frac{1}{\alpha} \frac{1 + \phi}{\Theta} \alpha_{0B} \right) \alpha_{0A} \right] \hat{p}_{0A} + \\
\left[ \frac{1}{\alpha} \alpha_{0B} \right] \gamma_{0B} + \left( \frac{1}{\alpha} \frac{1 + \phi}{\Theta} \alpha_{0B} \right) \alpha_{0B} - \frac{1}{\alpha} \left( (\eta - 1)(1 - \gamma_{0B}) + \frac{\alpha_{0B}}{n_B} (\mu - 1) \gamma_{0B} \right) \hat{p}_{0B} \\
- \left[ \frac{1}{\alpha} \alpha_{0B} \right] \gamma_{X} + \left( \frac{1}{\alpha} \frac{1 + \phi}{\Theta} \alpha_{0B} \right) \alpha_{X} \right] \hat{p}_{X} + \left( \frac{1 + \phi}{\Theta} \alpha_{0B} \right) \frac{n}{\alpha} n_B \hat{\alpha}_x + n_B \hat{\alpha}_A + n_B \hat{\alpha}_B + \frac{1}{\alpha}
\end{align*}
\]

The system of two equations above can be solved to provide expressions for \( p_{0A}^f, p_{0B}^f \) as functions of \( p_X^f \) and the technological shocks:

\[
\Lambda p_{0A}^f = \Pi p_X^f + N \tag{26}
\]

\[
\Lambda p_{0B}^f = \Gamma p_X^f + T \tag{27}
\]

where
\[ \Lambda = \Lambda(n_A, n_B) = \]
\[ \left[ \frac{1}{\alpha} - \frac{\alpha_{0,d,n}}{n_A} \right] \gamma_{A} \gamma'_{0,A} + \frac{1}{\alpha} \left( 1 + \frac{(1 + \phi) \alpha_{0,d,n}}{\Theta n_A} \right) \alpha_{0,A} \left[ \frac{1}{\alpha} - \frac{\alpha_{0,b,n}}{n_B} \right] \gamma_{B} \gamma'_{0,B} + \frac{1}{\alpha} \left( 1 + \frac{(1 + \phi) \alpha_{0,b,n}}{\Theta n_B} \right) \alpha_{0,B} \]
\[ - \left[ \frac{1}{\alpha} - \frac{\alpha_{0,d,n}}{n_A} \right] \gamma_{A} \gamma'_{0,B} + \frac{1}{\alpha} \left( 1 + \frac{(1 + \phi) \alpha_{0,d,n}}{\Theta n_A} \right) \alpha_{0,A} \left[ \frac{1}{\alpha} - \frac{\alpha_{0,b,n}}{n_B} \right] \gamma_{B} \gamma'_{0,A} + \frac{1}{\alpha} \left( 1 + \frac{(1 + \phi) \alpha_{0,b,n}}{\Theta n_B} \right) \alpha_{0,B} \]
\[ - \left[ \frac{1}{\alpha} - \frac{\alpha_{0,b,n}}{n_B} \right] \gamma_{B} \gamma'_{0,A} + \frac{1}{\alpha} \left( 1 + \frac{(1 + \phi) \alpha_{0,b,n}}{\Theta n_B} \right) \alpha_{0,B} \left[ \frac{1}{\alpha} - \frac{\alpha_{0,d,n}}{n_A} \right] \gamma_{A} \gamma'_{0,B} + \frac{1}{\alpha} \left( 1 + \frac{(1 + \phi) \alpha_{0,d,n}}{\Theta n_A} \right) \alpha_{0,A} \]
\[ + \frac{1}{\alpha} \left( \eta - 1 \right) \left( 1 - \gamma_{0,A} \right) + \frac{\alpha_{0,d,n}}{n_A} (\mu - 1) \gamma_{0,A} \right]\]
\[ \left[ \frac{1}{\alpha} \left( 1 - \frac{(1 + \phi)}{\Theta} \right) \left( \frac{\alpha_{0,d,n}}{n_A} - \frac{\alpha_{0,b,n}}{n_B} \right) \left( \gamma_{A} \gamma'_{0,B} - \gamma'_{0,A} \alpha_{0,B} \right) \right] \]  

(28)

\[ \Pi = \left[ \frac{1}{\alpha} \left( 1 + \phi \right) - 1 \right] \left( \frac{\alpha_{0,d,n}}{n_A} - \frac{\alpha_{0,b,n}}{n_B} \right) \left( \gamma_{B} \gamma'_{0,B} \alpha_{X} - \alpha_{0,b} \gamma'_{0,B} \right) \hat{p}_{X}^{f} \]
\[ + \frac{1}{\alpha} \left( \eta - 1 \right) \left( 1 - \gamma_{0,B} \right) + \frac{\alpha_{0,b,n}}{n_B} (\mu - 1) \gamma_{0,B} \right]\]
\[ \left[ \frac{1}{\alpha} \left( 1 - \frac{(1 + \phi)}{\Theta} \right) - 1 \right] \left( \frac{\alpha_{0,d,n}}{n_A} - \frac{\alpha_{0,b,n}}{n_B} \right) \left( \gamma_{B} \gamma'_{0,A} \alpha_{X} - \alpha_{0,b} \gamma'_{0,A} \right) \hat{p}_{X}^{f} \]  

(29)
\[
\Gamma = \left[ \frac{1}{\alpha} \left( \frac{1 + \phi}{\Theta} - 1 \right) \left( \frac{\alpha_{0B} n}{n_B} - \frac{\alpha_{0A} n}{n_A} \right) \right] \left( \gamma_d \gamma_{0A} \alpha_X - \alpha_{0A} \gamma_{0X} \right) \hat{p}_X^f
+ \left[ \frac{1}{\alpha} + \left( \eta - 1 \right) \left( 1 - \gamma_{0A} \right) + \frac{\alpha_{0A} n}{n_A} \left( \mu_A - 1 \right) \gamma_{0A} \right] \left[ \left( \frac{1}{\alpha} - \frac{\alpha_{0B} n}{n_B} \right) \gamma_X + \left( \frac{1}{\alpha} - \frac{\left( 1 + \phi \right) \alpha_{0B} n}{n_B} \right) \alpha_X \right] \hat{p}_X^f
\]

\[
N = \left[ \left( \frac{1}{\alpha} - \frac{\alpha_{0B} n}{n_B} \right) \gamma_{B} \gamma_{0B} + \frac{\alpha_{0A} n}{n_A} \left( 1 + \frac{\left( 1 + \phi \right) \alpha_{0B} n}{n_B} \right) \alpha_{0B} \right]
+ \left[ \frac{1}{\alpha} - \left( \frac{1}{\alpha} - \frac{\alpha_{0B} n}{n_B} \right) \gamma_{B} \gamma_{0B} + \frac{\alpha_{0A} n}{n_A} \left( 1 + \frac{\left( 1 + \phi \right) \alpha_{0B} n}{n_B} \right) \alpha_{0B} \right]
+ \left[ \frac{1}{\alpha} - \left( 1 - \gamma_{0B} \right) + \frac{\alpha_{0B} n}{n_B} \left( \mu_B - 1 \right) \gamma_{0B} \right]
\]

\[
T = \left[ \frac{1}{\alpha} - \left( \eta - 1 \right) \left( 1 - \gamma_{0A} \right) + \frac{\alpha_{0A} n}{n_A} \left( \mu_A - 1 \right) \gamma_{0A} \right]
+ \left[ \frac{1}{\alpha} - \left( \frac{1}{\alpha} - \frac{\alpha_{0B} n}{n_B} \right) \gamma_{B} \gamma_{0B} + \frac{\alpha_{0A} n}{n_A} \left( 1 + \frac{\left( 1 + \phi \right) \alpha_{0B} n}{n_B} \right) \alpha_{0B} \right]
- \left[ \frac{1}{\alpha} - \left( 1 - \gamma_{0B} \right) + \frac{\alpha_{0B} n}{n_B} \left( \mu_B - 1 \right) \gamma_{0B} \right]
\]

Substituting the expressions above into \( \Omega \), it follows
\[
a\hat{\Omega} = K \hat{p}_X^f + Q, \quad \text{and} \quad \hat{s} = H \hat{p}_X^f + R,
\]

where

\[
\begin{align*}
K &= K \left( n_A, n_B \right) = \alpha_X + \frac{\alpha_{0A}}{\alpha} \left( \frac{1 + \phi}{\Theta} - 1 \right) \left( \frac{\alpha_{0A} n}{n_A} - \frac{\alpha_{0B} n}{n_B} \right) \left( \alpha_{0A} \gamma_{B} \gamma_{0B} - \alpha_{0A} \gamma_{0A} \right) \\
&+ \frac{\alpha_{0B}}{\lambda} \left[ \frac{1}{\alpha} + \left( \eta - 1 \right) \left( 1 - \gamma_{0A} \right) + \frac{\alpha_{0A} n}{n_A} \left( \mu_A - 1 \right) \gamma_{0A} \right] \left( \frac{1}{\alpha} - \frac{\alpha_{0B} n}{n_B} \right) \gamma_X + \left( \frac{1}{\alpha} - \frac{\left( 1 + \phi \right) \alpha_{0B} n}{n_B} \right) \alpha_X \\
&+ \frac{\alpha_{0A}}{\lambda} \left[ \frac{1}{\alpha} + \left( \eta - 1 \right) \left( 1 - \gamma_{0B} \right) + \frac{\alpha_{0B} n}{n_B} \left( \mu_B - 1 \right) \gamma_{0B} \right] \left( \frac{1}{\alpha} - \frac{\alpha_{0A} n}{n_A} \right) \gamma_X + \left( \frac{1}{\alpha} - \frac{\left( 1 + \phi \right) \alpha_{0B} n}{n_B} \right) \alpha_X
\end{align*}
\]
\[ Q = \frac{\alpha_{0.4} N}{\Lambda} + \frac{\alpha_{0.8}}{\Lambda} T + \frac{n_A \hat{a}_X + n_A \hat{a}_A + n_B \hat{a}_B}{n} \]

\[ = \frac{1}{\Lambda} \left[ \left( \frac{1}{\alpha} - \frac{\alpha_{0.8} n}{n_B} \right) (\alpha_{0.4} \gamma_{0.4} - \alpha_{0.8} \gamma_{0.8}) - \frac{1}{\alpha} \alpha_{0.4} \right] \left[ \left( \frac{1 + \phi}{\Theta} \right) \frac{\alpha_{0.4} n}{n_A} - \frac{1}{\alpha} \right] \frac{n_A \hat{a}_X + n_A \hat{a}_A + n_B \hat{a}_B}{n} \]

\[ - \frac{1}{\Lambda} \left[ \left( \frac{1}{\alpha} - \frac{\alpha_{0.4} n}{n_A} \right) (\alpha_{0.4} \gamma_{0.4} - \alpha_{0.8} \gamma_{0.8}) + \frac{1}{\alpha} \alpha_{0.8} \right] \left[ \left( \frac{1 + \phi}{\Theta} \right) \frac{\alpha_{0.8} n}{n_B} - \frac{1}{\alpha} \right] \frac{n_A \hat{a}_X + n_A \hat{a}_A + n_B \hat{a}_B}{n} \]

\[ + \frac{\alpha_{0.4}}{\Lambda} \left[ - (\nu - 1)(1 - \gamma_{0.8}) + \frac{\alpha_{0.8} n}{n_B} (\mu_B - 1) \gamma_{0.8} \right] \left[ \left( \frac{1 + \phi}{\Theta} \right) \frac{\alpha_{0.4} n}{n_A} - \frac{1}{\alpha} \right] \frac{n_A \hat{a}_X + n_A \hat{a}_A + n_B \hat{a}_B}{n} \]

\[ + \frac{\alpha_{0.8}}{\Lambda} \left[ - (\eta - 1)(1 - \gamma_{0.4}) + \frac{\alpha_{0.4} n}{n_A} (\mu_A - 1) \gamma_{0.4} \right] \left[ \left( \frac{1 + \phi}{\Theta} \right) \frac{\alpha_{0.8} n}{n_B} - \frac{1}{\alpha} \right] \frac{n_A \hat{a}_X + n_A \hat{a}_A + n_B \hat{a}_B}{n} \]

\[ + \frac{n_A \hat{a}_X + n_A \hat{a}_A + n_B \hat{a}_B}{n} \]

\[ H = \gamma_X + \gamma_{A +} \Pi + \gamma_{B +} \Gamma + \frac{(\phi + \sigma)}{\Theta} K \]

\[ R = \left( \gamma_{A +} \frac{\phi + \sigma - \alpha_{0.4}}{\Theta} \right) N + \left( \gamma_{B +} + \frac{\phi + \sigma}{\Theta} \alpha_{0.8} \right) T + \frac{(\phi + \sigma)}{\Theta} \frac{n_A \hat{a}_X + n_A \hat{a}_A + n_B \hat{a}_B}{n} \]

The behavior of \( K \) and \( H \) in relation to the model’s parameters, such as the elasticities of substitution and the volume of trade, is examined numerically, although, in some special cases, one can provide analytical arguments.\(^{24}\)

\(^{24}\) For example, one can show analytically that at the point where the two nonprimary sectors are perfectly symmetric except for the elasticities of substitutions, \( \eta \) and \( \nu \), \( \frac{\partial K}{\partial n_A} \) has the opposite sign of \( (\eta - \nu) \).
REFERENCES


