Interest Rate Defenses of Currency Pegs

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Abstract

This paper studies a policy often used to defend a currency peg: raising short-term interest rates. The rationale for this policy is to stem demand for foreign reserves. Yet, this mechanism is absent from most monetary models. This paper develops a general equilibrium model with asset market frictions where this policy can be effective. The friction I emphasize is the same as in Lucas (1990): money is required for asset transactions. When the government raises domestic interest rates, agents want to increase their holdings of domestic currency in order to acquire more domestic-currency-denominated assets. Thus, agents do not run on the reserves of the central bank, and the peg survives. A key implication of the model is that an interest rate defense can always be successful, but at great costs for domestic agents. Hence the reluctance of governments to sustain this policy for long periods of time.

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I. INTRODUCTION

The history of currency pegs is often traumatic. Countries that have fixed exchange rate regimes in the hope of stabilizing their domestic currency are, at times, subject to speculative pressures that result in the depletion of their foreign reserves and the abandonment of the currency peg. This is called a currency crisis. In general, governments try to prevent currency crises by enacting certain policies. This paper is about one of the most commonly used policies to fight currency crises: short-term interest rate increases. On some occasions, this policy is successful and the peg survives the crisis; in other instances, the policy does not prevent the unfolding of the crisis, and the peg is abandoned.

The rationale for the policy is best understood by focusing on the simple mechanism of how currency crises unfold. During a crisis, agents exchange (or borrow) domestic currency at the central bank for foreign currency. With these money balances, agents acquire foreign assets, which they hold until the central bank exhausts its foreign reserves, at which point it is forced to abandon the peg and the domestic currency depreciates. Once this happens, agents need only undo their financial position in domestic currency at a profit. That is, if they borrowed domestic currency to acquire foreign currency, then they need to repay the principal plus interest on that loan. But since the domestic currency has depreciated, agents require less foreign currency to make this payment.

Therefore, the idea behind raising short-term interest rates is to increase the (opportunity) cost of exchanging domestic currency at the central bank to buy foreign assets. This increase in costs decreases the profits from arbitrage, and thus agents stop buying foreign currency from the central bank. In other words, by raising the return on domestic assets the government is trying to increase the demand for domestic currency in the asset market. To the extent that the exchange rate reflects the relative demand of domestic to foreign currency, the increase in the demand of domestic currency will result in the strengthening of the exchange rate. Therefore to capture this mechanism one needs a framework in which the demand for currencies in the asset market is positively affected by the rate of return of the asset denominated in that currency. This paper builds on Helpman and Razin (1985) and Grilli and Roubini (1992) (the latter building on Lucas, 1990) to present a model that delivers the above result.

In the model, asset and goods markets are in separate locations. In both markets, agents need cash in order to make purchases. In this environment, the cash-in-advance constraint in the financial market has important implications for the determination of the exchange rate. In particular, since agents need cash to buy bonds, a higher interest rate on bonds will affect agents’ decisions to trade currencies at the central bank. If agents expect a high return from domestic assets, they will want to hold domestic currency until the time to purchase domestic assets. Therefore, they do not have an incentive to exhaust the central bank’s foreign reserves, and the peg survives. On the other hand, if they expect a devaluation of the domestic currency, they will want to acquire foreign currency at the (lower) exchange rate offered by the central
bank. The standard monetary model without these cash-in-advance constraints cannot deliver this result, the reason being that, in those models, money and bonds are typically substitutes. Therefore, higher interest rates increase the demand for bonds and lower the demand for currency, which, in turn, weakens the exchange rate. Once the cash-in-advance constraint in the asset market is introduced, money and bonds become complements, and hence higher demand for bonds amounts to higher demand for money.

To assess the effectiveness of the interest rate policy in preventing crises, I assume that, initially, the domestic government is fixing the exchange rate. In other words, the central bank stands ready to sell reserves until these are depleted. Once this point is reached, the government cannot guarantee convertibility of the domestic currency and is forced to float it. Call this policy the reserves policy. This paper shows that when the level of reserves is low relative to the expected depreciation rate, there are multiple equilibria. One of these equilibria involves agents running on the reserves of the central bank and a currency crisis occurring. When the level of reserves is high relative to the expected depreciation rate, then the equilibrium is unique, and it involves no crisis. Then I move on to the interest rate policy and ask whether this policy can prevent the crisis. This policy consists of raising interest rates whenever agents want to run on reserves. It will be shown that this policy is indeed effective in avoiding crises, but that it imposes high costs on domestic households via an increase in government’s liabilities. This could explain why many countries are reluctant to raise interest rates to fight currency crises, especially when the required increase in interest rates is large.

With a few notable exceptions, there is a lack of models that deal with interest rate defenses. One of these exceptions is Flood and Jeanne (2003). These authors extend Krugman’s (1979) model of currency crises to study interest rate defenses. In their economy, as well as in Krugman’s, the government uses foreign reserves to finance its ever growing domestic credit. Thus, a main result of these papers is that the timing of the crisis can be fully predicted by the time at which the government runs out of reserves. Flood and Jeanne (2003) evaluate the effects of raising interest rates on the original timing of the crisis (i.e., the time at which the collapse would occur if the government didn’t change rates). They find that increasing rates before the original time of the collapse actually brings the crisis forward, the reason being that higher pre-crisis interest rates increase government’s liabilities. On the other hand, raising rates after the original time of the crisis defers the collapse, the reason being that higher rates after the collapse are associated with higher seigniorage revenues, which allows the government to sustain higher steady-state levels of debt. Lahiri and Végh (2000 and 2003) address the welfare evaluation of interest rate defenses in a general equilibrium framework. In order to be able to conduct an interest rate policy, their models broaden the definition of money by introducing “liquid bonds”. These are assets that provide liquidity services and pay

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2 In this respect, my analysis is reminiscent of Obstfeld’s (1986) seminal contribution. This author showed that currency crises may occur even when there is no current inconsistency in the policies of the government, provided that monetary policy is expected to be looser after the peg is abandoned.
an interest rate determined by the government. In Lahiri and Végh (2000), firms need to borrow in order to pay their wage bill. Hence higher interest rates have a contractionary effect on output. In Lahiri and Végh (2003) higher interest rates translate into higher future inflation taxes. In both settings, the authors find a non-monotonic relationship between welfare and the size of the interest rate increase. Nevertheless, since the government follows a Krugman-type policy (i.e. an ever growing domestic credit), the collapse of the fixed exchange rate is just a matter of time and the interest rate policy is only able to postpone the crisis but not to avert it. The present paper develops a framework in which the interest rate policy eliminates the crisis. In addition, in Lahiri and Végh (2000 and 2003) the increase in interest rates needs to be permanent, whereas in my analysis, as well as in reality, the interest rate increase is short lived. Drazen (2000 and 2003) studies the choice between reserves and interest rate defenses in a context of private information. This author’s contention is that asymmetric information plays a key role in the unraveling of currency crises. In fact, one possible role of interest rate hikes is to provide a signal to agents about the commitment of the central bank to defend the currency. However, when a government believes that a devaluation is likely, it will be unwilling to take a short foreign-currency position (i.e., borrow reserves). Therefore, higher interest rates may indicate a high probability of a crisis. While information factors certainly play a role in currency crises and defenses, I focus my study on the exact mechanism by which interest rate hikes avert a crisis, and thus need a more elaborate model of money demand. For this reason I abstract from information issues in my analysis.

Before closing this section, a word on the real world experience of interest rate defenses is in order. Interest rate hikes have indeed produced mixed results. Some countries have been successful in avoiding crises by raising interest rates. For example, Brazil increased short-term official rates from around 20 percent to over 43 percent in late October 1997 in an attempt to contain speculative pressures on the real. Despite the ongoing turbulence affecting other currencies, the real stayed within its official band. On the other hand, a large number of countries unsuccessfully raised interest rates to sustain the value of their currency. A notable example is Indonesia in mid-August 1997, when overnight rates rose up to close to 100 percent, but, on August 14, the rupiah was allowed to float. Despite this mixed experience with interest rate defenses, we still need a framework for understanding how and when this policy is effective. This is one of the goals of the analysis that follows.

The rest of the paper is organized as follows. Section II presents the economy. Sections III and IV present and discuss the results of the paper. Section V concludes. An appendix provides some of the proofs and derivations of the results.

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3 There is a growing empirical literature that assesses the effectiveness of tightening monetary policy in order to strengthen a currency. See, for instance, Dekle, Hsiao, and Wang (1999), Eichengreen and Rose (2001), International Monetary Fund (1998), Kraay (2003), and Zettelmeyer (2000).
II. THE ECONOMY

First I give an overview of the economy and then I fill in the details.

A. The Outline

Consider an infinitely lived economy in which there are two countries: the domestic country and the foreign country. Each country is populated by a representative household and a government. There is also a central bank in the domestic country. In this economy there are four locations at which trade of different objects can occur (see Figure 1): households can buy and sell goods in the goods market; they can exchange currencies with the domestic central bank; they can buy bonds from the domestic and foreign governments in the asset market; and, finally, they can trade currency with other households in the foreign exchange market (FX). All these transactions need to take place with the cash of the seller. In other words, in all trades agents face cash-in-advance constraints.

There is only one date on which agents face uncertainty in this economy. This is the first period, \( t = 1 \). The uncertainty comes from the existence of a sunspot, \( h \), to be realized at the beginning of date \( t = 2 \), before agents make any decision. The sunspot can take the value...
\( h = r \) with probability \( \pi \), or the value \( h = n \) with probability \( 1 - \pi \). After the sunspot is realized, there is no further uncertainty in the economy.

Every period, foreign households receive an endowment of their country-specific good. Domestic households, on the other hand, need to work in order to produce their country-specific good. The production function requires labor only. Households consume goods from both the domestic and the foreign country. Households are coalitions of agents that split at the beginning of each period and reunite at the end of the period. In this coalition, there is a shopper, a seller, and a financial trader (or trader for short) - the next subsection provides a detailed description of the activities of these agents throughout a period.

The domestic central bank is a window where agents trade currency at the official exchange rate of \( \bar{s} \) units of domestic currency per unit of foreign currency. This window is only open when the exchange rate is officially fixed. I assume that, initially, the domestic country is pegging its exchange rate to \( s \). There are a number of stories that can justify such a policy. One of them is that to avoid the temptation to inflate the economy, the government has chosen to commit to a nominal anchor (i.e., the fixed the exchange rate). Once it loses this anchor, that is, when the exchange rate floats, the government starts printing money.\footnote{The conventional wisdom holds that pegs provide more fiscal discipline than flexible regimes. For example, “fixed rates are seen as providing discipline to enable a government to resist the temptation to follow inflationary policies.” Drazen (2000b); or “by acting as a constraint on macroeconomic policies, a fixed exchange rate may enhance the credibility of the central bank’s commitment to maintaining a low and stable rate of money growth.” Agénor and Montiel (1996). Atkeson and Kehoe (2001) discuss the circumstances in which a country will use a fixed exchange rate as a commitment device for not printing money. Giavazzi and Pagano (1988) show how inflation-prone countries can benefit from fixing their exchange rates.}

Both domestic and foreign governments sell one-period discount bonds in the asset market. In order to finance the interest rate burden on these bonds, governments can either print money or levy lump-sum taxes on their citizens. Bonds are sold for cash in the asset market, at a discount price \( q \) for domestic bonds, and \( q^* \) for foreign bonds. Bonds repay in the same currency at the very end of the same period. Thus, cash used to purchase a bond in period \( t \) cannot be used to buy goods until period \( t + 1 \). It is in this sense that bonds have a one-period maturity.

**B. The Details**

This subsection first describes the behavior of the households and the government and provides a definition of equilibrium. In addition, expressions for the prices of goods and bonds are derived, as well as for the exchange rate. Before proceeding, though, a word about notation is in order. Variables in the model are denoted as follows: let \( z_i(h, t) \) denote the value
of a generic variable \( z_i \) for agent \( i \in \{d, f\} \) in period \( t \geq 2 \), when the realization of the sunspot in period \( t = 2 \) was \( h \in \{r, n\} \). Since the value of the sunspot is unknown in the first period, period 1’s variables are denoted as follows: \( z_i(1) \). Subscripts \( d \) and \( f \) denote domestic and foreign agents, respectively.

**Households**

At the beginning of every period, foreign households receive an endowment of their country-specific good, \( y^* \). Domestic households are endowed with one unit of time, which they divide between work (denoted by \( \ell \)) and leisure. Households consume goods from both countries. The household begins the period with holdings of domestic and foreign currency, denoted by \( \{ M_i(h, t - 1), M_i^*(h, t - 1) \} \), respectively, \( i \in \{d, f\} \). At the beginning of each period, the foreign \( \text{seller} \) takes the household’s endowment and travels to the goods market, where he sells it for foreign cash. The domestic \( \text{seller} \) travels also to the goods market, but there he works to produce his country-specific good. The production function is

\[
y(h, t) = \ell(h, t)^\alpha (\alpha > 0).
\]

After producing the good, he sells it for domestic cash. The money obtained from sales of goods cannot be used for current purchases. The \( \text{shopper} \) takes part of the initial money holdings of the household, \( \{ M_i(h, t - 1) - A_i(h, t), M_i^*(h, t - 1) - A_i^*(h, t) \} \), and travels also to the goods market. In that market, the \( \text{shopper} \) will buy domestic and foreign goods on behalf of the household: \( \{ c_i(h, t), c_i^*(h, t) \} \), where \( c_i \) and \( c_i^* \) denote consumption of the domestic and foreign good by household \( i \), respectively. The constraints that the \( \text{shopper} \) faces in the goods market are

\[
P(h, t)c_i(h, t) \leq M_i(h, t - 1) - A_i(h, t),
\]

\[
P^*(h, t)c_i^*(h, t) \leq M_i^*(h, t - 1) - A_i^*(h, t).
\]

Finally, the \( \text{trader} \) of the family takes whatever cash the \( \text{shopper} \) left behind, \( \{ A_i(h, t), A_i^*(h, t) \} \), and heads for the asset market, where he will buy domestic and foreign government bonds. Before entering the asset market, the \( \text{trader} \) has the opportunity of trading currency with the domestic central bank at the official exchange rate of \( \bar{s} \) units of domestic currency per unit of foreign currency. Let \( T_i(h, t) \) denote the units of domestic currency that the \( \text{trader} \) exchanges at the central bank. Thus, the \( \text{trader} \)’s money holdings after trading with the central bank are \( \{ A_i(h, t) - T_i(h, t), A_i^*(h, t) + T_i(h, t)/\bar{s} \} \). To simplify the analysis, assume that agents cannot short-sale currency to the central bank. In other words, if the \( \text{trader} \) wants to obtain foreign currency from the central bank, he must provide domestic currency on the spot, and vice versa for purchases of domestic currency. These no short-sale constraints can be written as

\[
-\bar{s}A_i^*(h, t) \leq T_i(h, t) \leq A_i(h, t).
\]

Also, I abstract from the sequential service constraint in case agents run on the reserves of the central bank and impose the following allocation rule on how the central bank distributes reserves in case agents demand more reserves than the bank has:

\[
T_i(h, t) \leq \bar{s}R(h, t - 1)/2.
\]
This constraint simply states that, in case of a run, agents could receive at most half the reserves that the central bank possesses. In other words, in case both the domestic and the foreign agents run, the central bank splits its reserves equally between the two.\(^5\)

With its new money holdings, the \textit{trader} enters the asset market, where he faces cash-in-advance constraints in purchasing government bonds. Specifically, these constraints are

\begin{align}
q(h, t)B_i(h, t) &\leq A_i(h, t) - T_i(h, t), \\
q^*(h, t)B_i^*(h, t) &\leq A_i^*(h, t) + T_i(h, t)/\bar{s},
\end{align}

where \(\{B_i(h, t), B_i^*(h, t)\}\) are household \(i\)'s holdings of domestic and foreign government bonds.

Before the end of the period, the \textit{seller}, the \textit{shopper}, and the \textit{trader} meet and pool together all the proceeds from the period. Then they pay lump-sum taxes to their respective governments: \(\tau(h, t)\) for domestic agents and \(\tau^*(h, t)\) for foreign agents. Domestic households may also receive a transfer of foreign currency, \(TR(h, t)\), from their government. After that, the foreign exchange market opens. In this market, domestic and foreign households obtain the new money holdings, \(\{M_i(h, t), M_i^*(h, t)\}\), that they carry into the next period. The constraint that domestic households face in this market is

\begin{equation}
P(h, t)g(h, t) - \tau(h, t) + s(h, t)TR(h, t) \\
\geq M_d(h, t) + s(h, t)M_d^*(h, t) - B_d(h, t) - s(h, t)B_d^*(h, t),
\end{equation}

whereas the constraint faced by foreign households is

\begin{equation}s(h, t)[P^*(h, t)g^* - \tau^*(h, t)] + B_f(h, t) + s(h, t)B_f^*(h, t) \geq M_f(h, t) + s(h, t)M_f^*(h, t)
\end{equation}

Initial money holdings are arbitrarily given by \(\{M_{d0}, M_{f0}\}\) and \(\{M_{d0}^*, M_{f0}^*\}\), and they must satisfy \(M_{d0} + M_{f0} = M_0\) and \(M_{d0}^* + M_{f0}^* + R_0 = M^*\), where \(R_0\) is the initial level of reserves that the domestic central bank has.

\(^5\) Strictly speaking the central bank’s allocation rule for reserves is as follows. Let \(\tilde{T}_i\) denote the amount that agent \(i\) effectively obtains from the central bank, and let \(T_i\) be the amount that he demands. Then

- If \(T_i \geq \frac{sR(h, t-1)}{2}, i = 1, 2\) then \(\tilde{T}_i = \frac{sR(h, t-1)}{2}, i = 1, 2\)
- If \(T_i < \frac{sR(h, t-1)}{2}, i = 1, 2\) then \(\tilde{T}_i = T_i, i = 1, 2\)
- If \(\sum T_i > sR(h, t-1), T_i < \frac{sR(h, t-1)}{2}, T_j > \frac{sR(h, t-1)}{2}\) then \(\tilde{T}_i = T_i, \tilde{T}_j = \bar{s}R(h, t-1) - T_i\)

In the equilibria I will derive, I first guess that when agents run, they both would like to demand \(T_i \geq \bar{s}R(h, t-1)/2\), and then I verify that this is the case. Therefore, the relevant restriction is \(T_i \leq \bar{s}R(h, t-1)/2\).
The domestic household’s problem in period 1 is to maximize the objective function

\[
U [c_i(1), c_i^*(1), \ell(1)] + \\
\beta \left\{ (1 - \pi) \sum_{t=2}^{\infty} \beta^{t-2} U [c_i(n, t), c_i^*(n, t), \ell(n, t)] + \pi \sum_{t=2}^{\infty} \beta^{t-2} U [c_i(r, t), c_i^*(r, t), \ell(r, t)] \right\}
\]

subject to constraints (1) to (7). The problem of the foreign household is the same except that the last constraint is replaced by equation (8). Since each realization of the sunspot determines a continuation value for the household, the objective function can be rewritten as

\[
U [c_i(1), c_i^*(1), \ell(1)] + \beta E [V_i(h, M_i(1), M_i^*(1))]
\]

where \(V_i(h, M_i(1), M_i^*(1))\) is the continuation value after the realization of the sunspot, as a function of money holdings at the beginning of period two,

\[
V_i(h, M_i(1), M_i^*(1)) = \max \sum_{t=2}^{\infty} \beta^{t-2} U [c_i(h, t), c_i^*(h, t), \ell(h, t)]
\]

subject to constraints. I assume that

\[
U [c_d(h, t), c_d^*(h, t), \ell(h, t)] = \log c_d(h, t) + \log c_d^*(h, t) + \eta \log [1 - \ell(h, t)]
\]

for domestic households, and

\[
U [c_f(h, t), c_f^*(h, t)] = \log c_f(h, t) + \log c_f^*(h, t)
\]

for foreign households.

Now I present the household’s first order conditions. First note that, before and after the realization of the sunspot, for intratemporal optimality the household must satisfy the conditions that relate domestic and foreign consumption

\[
q(1) \frac{U_{c_i}(1)}{P(1)} = q^*(1) \frac{U_{c_i^*}(1)}{sP^*(1)},
\]

\[
q(h, t) \frac{U_{c_i}[h, t]}{P(h, t)} = q^*(h, t) \frac{U_{c_i^*}[h, t]}{s(h, t)P^*(h, t)},
\]

As for intertemporal substitution, the conditions to be satisfied for the pre-sunspot period are

\[
q(1) \frac{U_{c_i}(1)}{P(1)} = \beta \sum_{h} \pi(h) \frac{U_{c_i}[h, 2]}{P(h, 2)},
\]

\[
q^*(1) \frac{U_{c_i^*}(1)}{P^*(1)} = \beta \sum_{h} \pi(h) \frac{U_{c_i^*}[h, 2]}{P^*(h, 2)},
\]
\[-U_t(1) = \beta \{ \alpha P(1) \ell(1)^{\alpha-1} \} \sum_H \pi(h) \frac{U_{c_t}[h,2]}{P(h,2)}. \]  \hspace{1cm} (13)

The first two equations relate present marginal utility from consumption to discounted expected marginal utility for the second period. The last equation relates marginal utility of leisure to the expected marginal benefit from labor. Similarly, for each continuation period after the realization of the sunspot

\[ q(h,t) \frac{U_{c_t}[h,t]}{P(h,t)} = \beta \frac{U_{c_t}[h,t+1]}{P(h,t+1)}, \hspace{1cm} (14) \]

\[ q^*(h,t) \frac{U_{c_t^*}[h,t]}{P^*(h,t)} = \beta \frac{U_{c_t^*}[h,t+1]}{P^*(h,t+1)}, \hspace{1cm} (15) \]

\[-U_t(h,t) = \beta \{ \alpha P(h,t) \ell(h,t)^{\alpha-1} \} \frac{U_{c_t}[h,t+1]}{P(h,t+1)}. \hspace{1cm} (16) \]

Finally, as refers to trade of currency with the central bank, \( T_i(h,t) \) must satisfy

\[ T_i(h,t) = \min \{ \bar{s}R(h,t-1)/2, A_i(h,t) \} \text{ if } \frac{1}{q(h,t)} < \frac{s(h,t)}{\bar{s}q^*(h,t)}, \hspace{1cm} (17) \]

\[ T_i(h,t) = -\bar{s}A_i^*(h,t) \text{ if } \frac{1}{q(h,t)} > \frac{s(h,t)}{\bar{s}q^*(h,t)}. \hspace{1cm} (18) \]

Equation (17) is the key condition to understand the unraveling of a currency crisis, for it determines when agents run on the reserves of the central bank. With the money holdings that he has available, the trader can follow one of two strategies. On the one hand, he can exchange domestic for foreign currency at the central bank at the official rate \( \bar{s} \), then use the newly acquired foreign currency to buy foreign bonds that repay \( 1/q^*(h,t) \) at the end of the period. Once the devaluation takes place, the returns in domestic currency from following this strategy are \( s(h,t)/\bar{s}q^*(h,t) \). If this amount is larger than \( 1/q(h,t) \) - the returns from holding on to the domestic currency and buying domestic bonds - then agents want to exchange as much currency with the central bank as possible. Given the constraints on the amounts of currency they can exchange at the central bank, \( T_i(h,t) \) will equal the minimum amount between \( \bar{s}R(h,t-1)/2 \) and \( A_i(h,t) \). If the inequality holds in the other direction, that is, if \( 1/q(h,t) > s(h,t)/\bar{s}q^*(h,t) \), then agents want to acquire as much domestic currency as possible.

**The Domestic and Foreign Governments**

This section describes first the behavior of the foreign government and then the behavior of the domestic government under each of the two policies under study.

The foreign government sells domestic bonds in the asset market and taxes foreign agents in order to repay the interest rate costs of its debt. Let \( M^* \) denote the foreign money supply,
which is kept constant forever. For simplicity, assume that every period the foreign
government sells an amount of bonds, \( B^*(h, t) \), equal to a fraction \( x^* \) of the
beginning-of-period foreign money supply: \( B^*(h, t) = x^* M^* \). Therefore, the budget
constraint of the foreign government is given by

\[
M^*(h, t) - M^*(h, t - 1) + \tau^*(h, t) = [1 - q^*(h, t)] B^*(h, t).
\]

Since the foreign money supply is kept constant, lump-sum taxes are the only source of
revenue to finance interest rate payments. Thus, \( \tau^*(h, t) = [1 - q^*(h, t)] x^* M^* \).

The domestic government follows either the reserves policy or the interest rate policy. At the
beginning of period \( t = 1 \), the government irrevocably commits to one of the two policies.
Every period the government must satisfy its budget constraint. When the government is
fixing the exchange rate, it stands ready to trade currency at the official rate \( \bar{s} \). Thus, its budget
constraint is given by

\[
\Delta M(h, t) + \tau(h, t) = \bar{s} \Delta R(h, t) + [1 - q(h, t)] B(h, t) + s(h, t) TR(h, t).
\]

On the other hand, when the government is floating, it does not trade currencies, and the
constraint becomes

\[
\Delta M(h, t) + \tau(h, t) = [1 - q(h, t)] B(h, t) + s(h, t) TR(h, t).
\]

In other words, government resources (money creation, \( \Delta M(h, t) \), plus taxes, \( \tau(h, t) \)) must be
enough to finance government expenditures (change in the value of reserves, \( \bar{s} \Delta R(h, t) \), plus
the interest rate cost of debt, \( [1 - q(h, t)] B(h, t) \), plus transfers to domestic agents,
\( s(h, t) TR(h, t) \)). Let \( T(h, t) \) denote the amount of domestic currency given to the central
bank in exchange for foreign currency. Then, the stock of reserves at the end of the period
equals the beginning-of-period stock minus reserves traded during the period and minus any
transfers of foreign currency, \( TR(h, t) \), that the government makes to domestic agents:

\[
R(h, t) = R(h, t - 1) - T(h, t)/\bar{s} - TR(h, t).
\]

It is assumed that when the government floats, it finances all its expenditures via money
creation. This assumption is meant to capture the experience of many developing economies
after abandoning a fixed exchange rate. It has been argued that these economies used fixed
rate regimes as a way to force their governments to finance public expenditures via taxation.
As soon as the pegs collapsed, these economies restarted using seigniorage as a means of
finance.\(^6\) This paper is not concerned with why this is the case. Rather, it takes as given this

\(^6\) This assumption is not unwarranted. Eichengreen and Rose (2001) present compelling evidence that countries that abandon their peg after a currency crisis experience an increase in the rate of growth of M1, whereas countries that successfully defended the peg kept M1 at a lower and constant growth rate.
behavior of the government and asks whether in this context the two policies under study prevent currency crises.

Next, the reserves policy and the interest rate policy are described. To avoid confusion, variables referring to the interest-rate policy will be denoted with a hat (e.g. \( \hat{\tau}(h, t) \)), and variables referring to the reserves policy are denoted without the hat (e.g. \( \tau(h, t) \)).

The reserves policy consists of selling reserves at the official exchange rate \( \bar{s} \) until these are depleted. Once reserves are depleted, the government abandons the peg. Under the reserves policy, every period the government sells an amount of bonds equal to a constant fraction \( x \) of the beginning-of-period money supply, \( \bar{B}(h, t) = xM(h, t - 1) \). As long as the peg is in place, the government finances the interest rate cost of its debt by levying a lump-sum tax, \( \tau(h, t) \), on domestic consumers. The government makes no transfer of foreign currency to domestic agents; i.e., sets \( T\bar{R}(h, t) = 0 \). If reserves fall to zero, then the government sets taxes to zero and finances its expenditures via seigniorage. Table 1 summarizes the reserves policy. Recall that \( \bar{R}(h, t) \) is determined by equation (21).

The interest rate policy consists of setting the bond price of domestic bonds low enough so that the payoff for agents from running on reserves and buying foreign bonds is slightly lower than holding on to the domestic currency and buying domestic bonds. If reserves are depleted, the government abandons the peg and finances its expenditures via seigniorage. Sales of bonds are as follows: for all periods in which there is no run, \( \bar{B}(h, t) = \hat{x}M(h, t - 1) \); whereas for all periods in which a run occurs, the government sells whatever quantity of bonds the market demands at the fixed bond price \( \bar{q}(h, t) \). Let \( \hat{x} \) denote the amount of bonds (as a fraction of the beginning-of-period money supply) sold during the interest rate defense period, then \( \bar{B}(h, t) = \hat{x}M(h, t - 1) \). Table 2 summarizes the interest-rate policy. Transfers under this policy are set to \( \hat{T\bar{R}}(h, t) = -\min\{\hat{T}(h, t)/\bar{s}, 0\} \). In other words, in the event that the government acquires any foreign currency (i.e., \( \hat{T}(h, t) < 0 \)), it gives this amount to domestic agents\(^7\). Recall that \( \hat{R}(h, t) \) is determined by equation (21). Notice that \( x \) indexes the size of government expenditures. A larger \( x \) amounts to larger government bond sales, which need to be financed through taxes or inflation. What varies between a fixed and a floating regime is the fashion in which the government finances its expenditures. Therefore, it may be useful to think of \( x \) as indexing the size of the government.

\(^7\) This can be thought of as a tax rebate.
Table 2: Interest Rate Policy

<table>
<thead>
<tr>
<th>Reserves level</th>
<th>$\Delta \hat{M}(h, t)$</th>
<th>$\hat{\tau}(h, t)$</th>
<th>$\hat{x}(h, t)$</th>
<th>$\hat{T}\hat{R}(h, t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{R}(h, t) &gt; 0$</td>
<td>0</td>
<td>expenditures</td>
<td>$x$ if no run</td>
<td>$\min{\frac{\hat{\tau}(h, t)}{s}, 0}$</td>
</tr>
<tr>
<td>$\hat{R}(h, t) = 0$</td>
<td>expenditures</td>
<td>0</td>
<td>$x$</td>
<td>0</td>
</tr>
</tbody>
</table>

Equilibrium and Prices

**Definition 1** For all $h \in H \equiv \{n, r\}$, given beliefs of agents and given government policy, a competitive equilibrium is a set of quantities $\{c_i(1), c_i^*(1), \ell(1), A_i(1), A_i^*(1), T_i(1), M_i(1), M_i^*(1), B_i(1), B_i^*(1), B(1), B^*, M_0, M^*, R(1), x\}$ and prices $\{P(1), P^*(1), q(1), q^*(1), s\}$ for $t = 1$, and a set of quantities $\{c_i(h, t), c_i^*(h, t), \ell(h, t), A_i(h, t), A_i^*(h, t), T_i(h, t), M_i(h, t), M_i^*(h, t), B_i(h, t), B_i^*(h, t), B^*(h, t), M^*(h, t), R(h, t), s(h, t)\}_{t=2}^{\infty}$ such that (i) given government policy and prices, domestic and foreign households solve their problem, (ii) domestic and foreign governments’ constraints hold for all histories, (iii) all markets clear: (goods) $c_d(1) + c_f(1) = \ell(1)^{\alpha}, c_d(h, t) + c_f(h, t) = \ell(h, t)^{\alpha}, c_d^*(1) + c_f^*(1) = y^*, c_d^*(h, t) + c_f^*(h, t) = y^*$; (monies) $M_d(1) + M_f(1) = M_0, M_d(h, t) + M_f(h, t) = M(h, t), M_d^*(1) + M_f^*(1) + R(1) = M^*, M_d^*(h, t) + M_f^*(h, t) + R(h, t) = M^*$; and (bonds) $B_d(1) + B_f(1) = B(1), B_d(h, t) + B_f(h, t) = B(h, t), B_d^*(1) + B_f^*(1) = B^*, B_d^*(h, t) + B_f^*(h, t) = B^*$, (iv) and beliefs are consistent.

For future reference, it is convenient to derive expressions for goods prices and the exchange rate. Goods prices can be easily obtained by combining cash-in-advance constraints in the goods markets with market clearing for goods and money, along with the cash-in-advance constraints in the asset market and market clearing in those markets:

$$P(h, t) = [M(h, t - 1) - q(h, t)B(h, t) - T(h, t)]/\ell(h, t)^{\alpha}, \quad (22)$$

$$P^*(h, t) = [M^* - R(h, t - 1) - q^*(h, t)B^* + T(h, t)]/s/ y^*. \quad (23)$$

Bond prices, in turn, are obtained from agents’ first order conditions and market clearing. Combine equations (14) and (15) with market clearing for goods. This delivers

$$q(h, t) = \frac{\beta P(h, t)\ell(h, t)^{\alpha}}{P(h, t + 1)\ell(h, t + 1)^{\alpha}} \quad \text{and} \quad q^*(h, t) = \frac{\beta P^*(h, t)y^*}{P^*(h, t + 1)y^*}.$$

Bond prices are equal to the discount factor times the inverse of the change in the amounts of money available in the goods market (which amounts to inflation in this market). In other
words, the bond price reflects the value of time, i.e., the discount factor, corrected by the change in the purchasing power of money, i.e., the inflation rate.

Finally, the exchange rate can be obtained by combining agents’ intratemporal first order conditions (10a) with goods’ market clearing:

\[
s(h, t) = \frac{P(h, t)\ell(h, t)^\alpha q^*(h, t)}{P^*(h, t)\gamma^* q(h, t)}.
\]  

(24)

The exchange rate equals the ratio of the amounts of money used for goods transactions times the ratio of foreign to domestic bond prices. The intuition behind the expression above can be grasped by looking at the first-order conditions along with the budget constraint. Consider the intertemporal first-order condition (14). When \(q(h, t)\) increases, the returns from saving are lower, hence agents want to consume more today. To do this they buy fewer domestic bonds (that is, they take fewer money to the asset market, \(A_i(h, t)\)), which implies that at the end of the period they will have less units of domestic currency (i.e. lower \(B_i(h, t)\)). Since they want to distribute their wealth between both currencies (they like both goods equally), they will demand less foreign currency at the end of the period, so the exchange rate appreciates - \(s(h, t)\) decreases. The opposite effect arises when \(q^*(h, t)\) increases. Similarly, higher \(P(h, t)\) makes today’s consumption more expensive relative to tomorrow’s. Thus agents want to lower \(c_i(h, t)\) and increase \(c_i(h, t + 1)\) by purchasing more bonds today (increasing \(A_i(h, t)\)). At the end of the period they want to split this increase in wealth between the two currencies and so they demand more dollars, which depreciates the exchange rate: \(s(h, t)\) increases.

### III. Analytical Results

In order to gain intuition about the forces at play in the model, the next section considers the simple case in which in period \(t = 1\) households assign probability zero to the event \(h = r\). That is, households do not anticipate the run in the first period and they set \(\pi = 0\). For this particular case it is possible to derive analytical solutions that perfectly illustrate the workings of each one of the two government policies. Appendix IV extends the analysis to the more general case where \(\pi > 0\). Unfortunately, for this case it is only possible to obtain numerical results. Hence, Appendix IV can be seen as a robustness check for the results obtained in Section III.

#### A. Equilibrium Under the Reserves Policy

Suppose first that households behave as follows: during period \(t = 2\), all households run on the reserves of the central bank if the sunspot is \(h = r\), and no household runs if the sunspot is \(h = n\). Given this behavior, one can construct allocations and prices for both realizations \(h \in \{n, r\}\), and then check whether this behavior, along with the quantities and prices
derived, constitute an equilibrium. It will be shown that when the government follows the reserves policy, there are pairs \((x, R_0)\) for which the above behavior, quantities, and prices constitute an equilibrium, and there are other pairs \((x, R_0)\) for which the only equilibrium involves agents never running, regardless of the value of the sunspot.

**Lemma 1** \(q(r, 2)\), satisfies the following two conditions

(i) \(q(r, 2) \leq 0\) as \(R_0 \not\rightarrow M^*[1 - \beta x^*] / (2 - \beta x)\), and

(ii) \(q(r, 2) < \beta\).

**Proof.** See Appendix I ■

The intuition for this lemma is that when a run takes place, the larger \(R_0\) is, the larger the amount of domestic currency that is being exchanged at the central bank, and hence less money is left to buy domestic bonds. Therefore, the price of these bonds needs to fall.\(^8\) Also, as mentioned in the previous section, \(q(r, 2)\) is less than the discount factor because bondholders need to be compensated for inflation.

**Proposition 2** Under the reserves policy, for each \(x\) there is a cutoff value \(R(x)\) such that

(i) \(\forall R_0 \leq R(x)\) there are multiple equilibria: one in which all agents run on the reserves of the central bank, and one in which no agent runs;

(ii) \(\forall R_0 > R(x)\) the equilibrium is unique and involves no agent running on the reserves of the central bank.

**Proof.** See Appendix II ■

The intuition for the proposition above is straightforward. When agents run on the reserves of the central bank there are two opposing forces at play. The first one is that if the run succeeds, the domestic government sets the tax to zero and finances its expenditures via seigniorage. In that case, at the end of the period there will be more domestic currency in the economy and the exchange rate will depreciate. On the other hand, when agents are running on foreign reserves, the central bank is actually dumping foreign currency into the economy. This amounts to an expansion of the foreign money supply, which in turn appreciates the domestic currency. Which one of these forces dominates depends on the relative magnitudes of (i) the

\(^8\) I restrict the maximum level of reserves to be \(M^*[1 - \beta x^*] / (2 - \beta x)\). Levels of reserves above this threshold imply negative bond prices.
depreciation induced by the loosening of the domestic monetary policy (which is indexed by $x$) and (ii) the amount of foreign reserves, $R_0$, sold in the market by the central bank.

Figure 2 illustrates the above point from a slightly different angle. This figure depicts the marginal payoffs for an agent of following each of two strategies. The dashed line represents the payoff from running when everybody else is running, $s(r, 2)/\bar{s} q^*(r, 2)$. In other words, these are the payoffs from exchanging one unit of domestic currency at the official rate and investing the proceeds in foreign assets. The solid line represents the payoffs from not running when everybody else is running, $1/q(r, 2)$, that is, investing a unit of domestic currency in domestic bonds. As the picture shows, there exists a cutoff value below which running is a strategy that yields higher payoffs than not running. The opposite is true to the right of the cutoff point. The cutoff value $R(x)$ is higher the higher $x$ is.

An immediate policy implication of these results is that countries with large reserves relative to the size of the expected depreciation will not be subject to currency crises. Since agents

---

9 These payoffs have been obtained for the arbitrary set of parameters: $\beta = 0.95$, $\eta = 2$, $\alpha = 0.7$, $y^* = 1$, $M^* = 1$, $x^* = 0.1$, and $\bar{s} = 1$. Other parameter values deliver similar results.

10 This payoff is decreasing in $R_0$ because larger sales of reserves appreciate the domestic currency, i.e. lower $s(r, 2)$.

11 This payoff is decreasing in $R_0$ by virtue of lemma 1.
know that the country has large reserves to be deployed in case there is a run, they realize that the payoff from running is lower than the payoff from not joining the run.\textsuperscript{12} Thus, agents will refrain from running.

\textbf{B. Equilibrium Under the Interest Rate Policy}

The previous section showed how, for given $x$, a sufficiently low level of reserves gives rise to multiple equilibria, one of which entails a currency crisis. Suppose now that a country is in the crisis zone, that is, $(x, R_0)$ are such that $R_0 \leq R(x)$. Can a policy of raising interest rates when $h = r$ prevent the crisis? This section shows that the answer to this question is yes but that such a policy entails high costs for domestic households via an increase in the domestic government’s liabilities.

To understand how the interest rate policy works, recall that when $h = r$, agents want to run on reserves because they expect a payoff of $s(r, 2)/\bar{s}q^*(r, 2)$ from running. Therefore, the central bank has to offer a return on its bonds that is at least as good as the payoff agents get from joining the run. In other words, the domestic bond price under the interest rate policy, $\hat{q}(r, 2)$, needs to be such that $1/\hat{q}(r, 2) = s(r, 2)/\bar{s}q^*(r, 2)$.\textsuperscript{13} In order to implement such a policy, the central bank needs to sell as many bonds as the market demands at this price. In order to pay the interest rate on these bond sales, the government will levy a lump-sum tax $\hat{\tau}(r, 2)$ on domestic agents. We have the following proposition.

\textbf{Proposition 3} Let $(x, R_0)$ be such that $R_0 \leq R(x)$. Under the interest rate policy, in equilibrium, agents do not to run on the reserves of the central bank, and thus the peg survives.

\textbf{Proof.} The proof and intuition for this result are straightforward. Suppose that $h = r$, and all households believe that a run is going to occur. Consider then the problem of an individual household that needs to decide whether to run on reserves or not. When everybody is running, the household’s payoffs from joining the run are $s(r, 2)/\bar{s}q^*(r, 2)$. On the other hand, the payoff from not running and buying a domestic bond is $1/\hat{q}(r, 2)$, which is (an $\varepsilon > 0$) higher than the payoff from the run. Since all agents think that way, none of them will want to run, and therefore the crisis does not occur. QED. \hfill \blacksquare

Appendix III provides details for the derivation of the equilibrium variables under the interest rate policy. It should be mentioned, though, that the equilibrium has the following feature.

\textsuperscript{12} A good case in point is China which, in the midst of the East Asian crisis, boasted that its level of foreign reserves was over US$ 140 billions.  
\textsuperscript{13} Strictly speaking, $1/\hat{q}(r, 2)$ needs to be an $\varepsilon > 0$ higher than $s(r, 2)/\bar{s}q^*(r, 2)$ for agents not to be indifferent between running and not running.
When the government lowers its bond price, agents exchange foreign currency for domestic currency at the central bank in order to buy a larger quantity of domestic bonds. As a result, there is less foreign currency available to buy foreign bonds, and the foreign bond price decreases so that the returns from domestic and foreign assets are equal. In other words, under the interest rate policy, foreign interest rates track domestic interest rates. Section IV presents a version of the model for which this feature is not present.

To illustrate the magnitude of the interest rate defense consider Figure 3 and Table 3. Figure 3 shows bond prices during the crisis period for the same parameters used above for the reserves and the interest rate policies, as a function of the size of bond sales, \( x \). Recall that \( x \) can be interpreted as indexing the size of government expenditures. When the reserves policy fails and the government monetizes its deficit, higher \( x \) results in higher rates of money growth and higher inflation rates. Since agents want to be compensated for future inflation, the larger \( x \) is, the lower the bond price, \( q(r, 2) \), will be. Similarly, larger \( x \) implies larger depreciations of the exchange rate.\(^{14}\) Therefore, under the interest rate policy, a larger \( x \) results in a lower bond price, \( \hat{q}(r, 2) \) during the defense period. Table 3 illustrates the same point but in terms of interest rates (i.e. \([1 - q(r, 2)] / q(r, 2)\)). An interesting fact about this table is that low deficits require less aggressive interest rate defenses. When the size of the deficit is large and, thus,

\(^{14}\)Recall that \( s(r, t) = \gamma(r)s(r, t - 1) \), where \( \gamma(r) \) is the domestic growth factor.
Table 3: Bond Prices

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\gamma$</th>
<th>$\frac{1-q(r,2)}{q(r,2)}%$</th>
<th>$\frac{1-q(r,2)}{q(r,2)}%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.55</td>
<td>5.84</td>
<td>5.90</td>
</tr>
<tr>
<td>0.3</td>
<td>2.08</td>
<td>7.45</td>
<td>8.32</td>
</tr>
<tr>
<td>0.5</td>
<td>4.58</td>
<td>10.08</td>
<td>14.44</td>
</tr>
<tr>
<td>0.7</td>
<td>8.97</td>
<td>14.71</td>
<td>33.47</td>
</tr>
<tr>
<td>0.9</td>
<td>16.79</td>
<td>22.94</td>
<td>127.18</td>
</tr>
</tbody>
</table>

the rate of money growth in case of a crisis would be high, the interest rate defense needs to be very aggressive (e.g., an interest rate of 127 percent for $x = 0.9$).

C. Welfare Comparison Across Policies

Suppose that a country is in the crisis zone defined above (i.e. $(x, R_0): R_0 \leq R(x)$). Should this country adopt an interest rate policy in order to avoid a currency crisis? To answer this question one needs to assess the costs and benefits of raising interest rates. The benefits of raising rates can be thought of as the costs that would be incurred if the government followed the reserves policy instead and that are now avoided. Namely, when the peg collapses and the government monetizes its deficit, there is inflation in the economy. Inflation acts as a tax on the proceeds from labor and thus discourages domestic production. As a result, domestic welfare will be lower. On the other hand, by raising interest rates, the government increases its public deficit for that period. In order to finance this increase in liabilities, it is necessary to levy a (large) lump-sum tax on domestic households, which lowers their welfare. In other words, since government bonds are held by domestic and foreign agents, but taxes are paid by domestic agents, the interest rate policy amounts to a transfer of resources from domestic to foreign agents.\footnote{Notice that if domestic agents were the only ones to purchase domestic bonds, then Ricardian equivalence would hold and the redistributive effects from this policy would be nil.} One would like to know which of these effects is stronger. Figure 4 plots the continuation utility for each of the two policies. That is,

$$U_{d}^{pol} = U_{d}^{pol}(r, 2) + \beta U_{d}^{pol}(r, 3)/(1 - \beta)$$

where $pol \in \{reserves \text{ policy, interest rate policy}\}$. As shown, utility is higher under the reserves policy than under the interest rate policy.

To develop an intuition for this result, it is useful to look at consumption levels during the crisis period. Figure 5 shows domestic agent’s consumption of the domestic good during a crisis period for the reserves and interest rates policies. The first period consumption level is also depicted for comparison purposes. Notice that consumption levels are depicted as functions of bond sales, $x$. Consumption during the first period is higher than consumption during the second (crisis) period, regardless of the policy the government follows. To see why,
Figure 4: Utility Levels for Domestic Agents

Figure 5: Consumption of Domestic Goods by Domestic Agents
Table 4: Change in Domestic Consumption of Domestic Good

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\frac{c_d(t, 2) - c_d(t, 1)}{c_d(t, 1)} %$</th>
<th>$\frac{c_d(t, 2) - c_d(t, 1)}{c_d(t, 1)} %$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-0.23</td>
<td>-0.25</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.84</td>
<td>-1.22</td>
</tr>
<tr>
<td>0.5</td>
<td>-1.76</td>
<td>-3.61</td>
</tr>
<tr>
<td>0.7</td>
<td>-2.93</td>
<td>-10.60</td>
</tr>
<tr>
<td>0.9</td>
<td>-1.24</td>
<td>-38.63</td>
</tr>
</tbody>
</table>

suppose first that the government is following the reserves policy.\footnote{First-period consumption is decreasing in $x$ because the size of the tax during the first period is an increasing function of $x$.}  In the second (crisis) period, the government starts printing money. This results in higher inflation, which affects negatively the work effort of domestic households. Since now labor is lower, agents have a lower income and are not able to consume as much. Suppose now that the government is following the interest rate policy. When the crisis hits, the government will levy a large tax on domestic agents. This lowers dramatically their income and, hence, their consumption. The effect of the tax is not transitory because, in this economy, money holdings allow agents to carry wealth from one period to the next: if an agent starts a period with large money holdings, he can invest a large fraction of these in bonds that repay at the end of the period. During the crisis period, the large tax leaves domestic agents with low end-of-period money holdings, that is, low wealth, and that is why the effect of this policy is permanent.

Table 4 presents the change in consumption in the crisis period as a percentage of consumption in the first period. As mentioned, the fall in consumption is always larger when the government follows the interest rate policy than when it follows the reserves policy.

The above ranking of utility levels has important policy implications. Since the interest rate policy always delivers lower utility, this suggests a reason for the reluctance of countries to pursue interest rate defenses.

IV. DISCUSSION AND CONCLUDING REMARKS

A. Discussion

As mentioned above, an unsatisfactory aspect of the model is that under the interest rate policy, the foreign bond price decreases with the domestic bond price. This is due to the fact that, in the model, when the sunspot is realized, financial traders are free to exchange with the
central bank as much currency as they posses. As a result, interest rate parity holds because the foreign bond price is allowed to move. In the real world, however, the foreign bond price (i.e., the bond price of the country to whose currency the domestic currency is pegged) is not affected by changes in the domestic bond price. Thus one is left with the question of whether interest rate parity holds in the data or, on the contrary, whether there are unexploited arbitrage opportunities. Fortunately, there is a growing body of literature that assesses whether covered and uncovered interest rate parity holds for different countries and different data frequencies (see for instance Chinn and Dooley 1997; Flood and Rose 2002; and Kumhof 2001). An interesting finding of this literature is that, for emerging economies, interest rate parity does not hold during times of crises but does during tranquil times. One reason why this may be so, as pointed out by several authors, is that in most developing economies money markets and foreign exchange markets are segmented and, therefore, arbitrage opportunities cannot be fully taken advantage of. Hence the observed deviations from interest rate parity. To assess the effects of the lack of integration between money and foreign exchange markets on interest rate parity, I consider an extension of the model in which there is some degree of segmentation in the assets market. In particular, I limit the amounts of money that financial traders can trade with the central bank before proceeding to the bonds market. Thus, assume that only a fraction $\lambda$ of the cash balances that the trader has available initially can be exchanged at the central bank. In other words, constraint (3) becomes

$$-\lambda \bar{s}_t A^{*}_t(t) \leq T_i(h, t) \leq \lambda A^{*}_t(t).$$  \hspace{1cm} (25)

The effect of this constraint will be as follows. Suppose that the government follows an aggressive interest rate policy (i.e. low domestic bond prices). Then, for returns on foreign bonds lower than the return on domestic assets, traders would like to exchange foreign currency for domestic currency: that is, they would like to set $T_i(h, t)$ as negative as possible. The more negative $T_i(h, t)$ is, the less liquidity there would be in the foreign bond market and, hence, the lower $q^*(h, t)$ would be in equilibrium. Nevertheless, at one point the left-hand side of the constraint would be binding (the smaller $\lambda$ is, the tighter the constraint becomes). At this point there is no further effect of domestic bond prices on foreign bond prices and uncovered interest rate parity does not hold. Figure 6 illustrates this point (for $\lambda = 5$ percent): notice that for high values of $x$ (i.e. aggressive interest rate policy) there is a wedge between domestic and foreign bond prices.

B. Concluding Remarks

This paper has been devoted to developing a simple general equilibrium framework to study currency crises and one of the policies used to fight them: short-term interest rate hikes. The

\footnote{For this version of the model it is necessary that the sunspot is realized once the family has split. Otherwise, the segmentation constraint would not have any effect, for agents would adjust their currency holdings as desired before splitting}
Figure 6: Bond Prices under Interest-Rate Policy ($\lambda$=5 percent)

The rationale for the policy is to increase demand for the domestic currency in the asset market. In other words, if demand of a currency is (positively) linked to the returns of the assets denominated in that currency, then increases in domestic rates of return should translate into increased demand for domestic currency in the asset market. This mechanism is introduced into the model through a cash-in-advance constraint in the asset market. As the analysis shows, this friction is crucial for rendering interest-rate policies effective in defending a currency peg.

In order to be explicit about the major forces at play, I considered a two-country economy. In such a model, demand for currency and financial assets is endogenously determined. Therefore, domestic and foreign interest rates are determined in equilibrium. This, in turn, delivers the somewhat counterfactual implication that the foreign interest rate tracks the domestic one whenever the interest rate defense is enacted.\(^\text{18}\) I believe that this feature of the model is useful in that it draws attention to an area where more research is needed. Indeed, some researchers have suggested that interest rate parity fails during times of distress. These deviations may certainly accrue from imperfections in the money and foreign exchange markets. Identifying and incorporating these frictions into economic models constitute, in my opinion, one of the next steps in the research agenda.

\(^{18}\) With no further frictions, capital is free to move until rates of return are equal, all else constant.
Finally, the analysis also sheds some light on the costs incurred by raising interest rates to defend a currency. As the model shows, the returns from higher interest rates are gathered by both domestic and foreign agents. On the other hand, the costs of this policy translate into tax increases (via the government’s budget constraint\(^{19}\)), which are borne by domestic agents alone. Therefore, interest rate defenses are costly because they amount to a transfer of resources from domestic agents to foreigners.

\(^{19}\)Note that in the analysis I have not allowed the government to default on its debt.
Proof of Lemma 1: The equation determining \( q(r, 2) \) can be expressed as follows

\[
q(r, 2) = \beta \frac{\left( \frac{M_0 - sR_0}{M_0} - q(r, 2) \right) x}{M_0 (1 + x) - sR_0} \left[ 1 - q(r)x \right] - \frac{1 - q(r)x}{z_3} q(r, 2)
\]

where \( M_0 = s [M^* [1 - \beta x^*] - R_0] / [1 - \beta x] \). I will prove the lemma in three steps:

(i) \( \frac{\partial q(r, 2)}{\partial R_0} < 0 \)

The expression that determines \( q(r, 2) \) defines the following function

\[
F[q(r, 2), R_0] \equiv q(r, 2) - \beta \frac{M_0 [1 - q(r, 2)x] - sR_0}{M_0 (1 + x - q(r, 2)x) - sR_0} [1 - q(r, 3)x] = 0
\]

by the implicit function theorem

\[
\frac{\partial q(r, 2)}{\partial R_0} = -\frac{F_{R_0}}{F_{q(r, 2)}}
\]

After some algebraic manipulations it can be shown that \( F_{R_0}, F_{q(r, 2)} > 0 \) and, therefore, \( \partial q(r, 2)/(\partial R_0 < 0 \).

(ii) \( q(r, 2) \searrow 0 \) as \( R_0 \nearrow M^* [1 - \beta x^*] / (2 - \beta x) \)

Simply note that as \( R_0 \nearrow M^* [1 - \beta x^*] / (2 - \beta x) \), \( z_1 \searrow 0 \) and \( z_2 \searrow [1 - q(r)x] \). Since \( q(r, 2) = \left\{ \frac{[\beta + z_2] \pm \sqrt{(\beta + z_2)^2 + 4\beta z_1 z_3}}{2z_3}, q(r, 2) \searrow 0, \right. \)

(iii) \( q(r, 2) < \beta \).

This immediately follows from the facts that \( \partial q(r, 2)/\partial R_0 < 0 \) and \( q(r, 2) = q(r, 3) = \beta/\gamma < \beta \) for \( R_0 = 0 \). Then for any \( R_0 > 0 \) it must be the case that \( q(r, 2) < \beta \).
PROOF OF PROPOSITION 2

Since the sunspot appears only in the second period, and after that there is no more uncertainty, one can truncate each history of the model in three parts: (i) the first period before uncertainty is resolved, (ii) the second period when the sunspot is revealed and agents decide whether to run on reserves or not, and (iii) the steady state reached after each realization of the sunspot. In the following sections, however, it will be convenient to solve the model backwards: first the continuation steady-states, then the period of the sunspot, and finally the first period.

Suppose first that households behave as follows: during period $t = 2$, all households run on the reserves of the central bank if the sunspot is $h = r$, and no household runs if the sunspot is $h = n$. Given this behavior, one can construct allocations and prices for both realizations $h \in \{n, r\}$, and then check whether this behavior along with the quantities and prices derived constitute an equilibrium. It will be shown that when the government follows the reserves policy, there are pairs $(x, R_0)$ for which the above behavior, quantities and prices constitute an equilibrium, and there are other pairs $(x, R_0)$ for which the only equilibrium involves agents never running, regardless of the value of the sunspot.

**Continuation for $h = r$ for $t \geq 3$:** After a run in period 2, agents know that the government is financing its expenditures via money creation. Therefore, domestic nominal variables will grow at the rate of domestic money growth. In order to solve for a steady state it is convenient to render all variables of the model stationary. As in Lucas and Stockey (1987), normalize nominal variables by their respective beginning-of-period money supplies, $M(r, t - 1)$ and $M^*(r, t - 1)$. This transformation delivers a new set of constraints for agents. In particular, cash-in-advance constraints in the goods markets are now: $p(r, t)c_i(r, t) \leq m_i(r, t - 1) - a_i(r, t)$ and $p_i(r, t)c_i^*(r, t) \leq m_i^*(r, t - 1) - a_i^*(r, t)$. Cash-in-advance in the bonds markets are $a_i(r, t) \geq q(r, t)b_i(r, t)$ and $a_i^*(r, t) \geq q^*(r, t)b_i^*(r, t)$. And finally, the budget constraint for domestic agents is $p(r, t)\ell(r, t)^a + b_d(r, t) + e(r, t)b_d^*(r, t) \geq \gamma(r, t)m_d(r, t) + e(r, t)m_d^*(r, t)$ and $e(r, t)[p(r, t)y^* - t^*(r, t)] + b_f(r, t) + e(r, t)b_d^*(r, t) \geq \gamma(r, t)m_f(r, t) + e(r, t)m_f^*(r, t)$ for foreign agents, where $e(r, t) = s(r, t)M^*(r, t - 1)/M(r, t - 1)$. Solving for the first order conditions of the household and manipulating as before delivers expressions for domestic and foreign steady-state bond prices: $q(r) = \beta/\gamma(r)$ and $q^*(r) = \beta$, where $\gamma(r)$ is the domestic money growth factor in the steady state, and it is given by the government’s budget constraint: $\gamma(r, t) = \gamma(r) \equiv M(r, t)/M(r, t - 1) = 1 + [1 - q(r)]x$. In the post-crisis steady state, the domestic bond price equals the discount factor adjusted by the rate of money growth. As one would expect, larger rates of money growth imply lower bond prices since bondholders need to be compensated for inflation. Combining $\gamma(r)$ and $q(r)$ above delivers a quadratic equation in $q(r)$ which determines the equilibrium domestic bond price.\textsuperscript{20}

\textsuperscript{20}The equation for $q(r)$ is

$$q(r) = \left\{ (1 + x) \pm \sqrt{(1 + x)^2 - 4x\beta} \right\} / 2x.$$
Normalized goods prices are found as before \( p(r) = \frac{1 - q(r) x}{\ell(r)^\alpha} \) and \( p^*(r) = \frac{1 - q^*(r) x^*}{y^*} \), and the exchange rate is given by

\[
s(r, t) = e(r) \frac{M(r, t - 1)}{M^*} \text{ where } e(r) = \left( \frac{1 - q(r) x}{1 - q^*(r) x^*} \right) \frac{q^*(r)}{q(r)}
\]

In other words, \( s(r, t) = \gamma(r, t) s(r, t - 1) \); higher rates of money growth imply higher rates of depreciation. Let the continuation utility for \( h = r \), from period 3 on, be denoted by

\[
\tilde{V}(r, M_k(r, 2), M^*_k(r, 2)) = U[c_i(r), c^*_i(r)] / (1 - \beta).
\]

**Period** \((h, t) = (r, 2)\): Now consider the household’s problem in period 2 when \( h = r \) is realized. The proposed behavior is that agents run on reserves. So,

\[
T(r, 2) = T_d(r, 2) + T_f(r, 2) = R_0.\]

When reserves are exhausted, the domestic government sets taxes to zero and reverts to seigniorage to finance its expenditures. Therefore, from the government’s budget constraint, end-of-period domestic money supply is

\[
M(r, 2) = M_0 + [1 - q(r, 2)] x M_0 - \ddot{s} R_0. \]

Bond prices are

\[
q(r, 2) = \beta \frac{P(r, 2) \ell(r, 2)^\alpha}{P(r, 3) \ell(r, 3)^\alpha} \quad \text{and} \quad q^*(r, 2) = \beta \frac{P^*(r, 2) y^*}{P^*(r, 3) y^*}.
\]

One can substitute the expressions for goods prices (equations (22) and (23)) into the equations above to obtain

\[
q(r, 2) = \beta \frac{M_0 - \ddot{s} R_0}{x M_0} - q(r, 2)
\]

\[
M_0(1 + x) - \ddot{s} R_0 \quad [1 - q(r) x] - \left[ 1 - q(r) x \right] q(r, 2)
\]

\[
q(r, 2) = \frac{z_2 + \beta \pm \sqrt{(z_2 + \beta)^2 - 4 \beta z_1 z_3}}{2 z_3}.
\]

The larger of the solutions for this equation is larger than one, and thus cannot be an equilibrium bond price. The other solution is between zero and one. Also,

\[
q^*(r, 2) = \beta \frac{M^* - R_0 - q^*(r, 2) x^* M^* + R_0}{M^* [1 - q^*(r) x^*]} = \beta \frac{[1 - q^*(r, 2) x^*]}{[1 - q^*(r) x^*]} = \beta
\]

The higher root is always larger than one and hence cannot be an equilibrium bond price, whereas the smallest root is between zero and one.
Continuation for \( h = n, t \geq 2 \): when \( h = n \) agents do not run in the second period. Hence, the government maintains the peg and taxes domestic consumers to finance its expenditures. To solve for bond prices proceed as before. Since there is no money growth in either country goods prices are constant, \( P(n, t) = P(n, t + 1) \) and \( P^*(n, t) = P^*(n, t + 1) \). Therefore, bond prices equal the discount factor, \( q(n, t) = q^*(n, t) = \beta \), and \( T_i(n, t) = 0 \) satisfies the households’ optimality conditions. Goods prices are found as before
\[ P(n, t) = M_0 [1 - q(n, t)x^*] / y^* \quad \text{and} \quad P^*(n, t) = \{ M^* [1 - q^*(n)x^*] - R_0 \} / y^* , \]
and the exchange rate is given by
\[ s(n, t) = \frac{P(n, t) \ell(n, t) \alpha}{P^*(n, t) y^*} q^*(n, t) . \]
Notice that \( s(n, t) = \bar{s} \). Finally, let the continuation utility for \( h = r \), from period 3 on, be denoted by \( \bar{V}(n, M_i(n, 2), M_i^*(n, 2)) = U[c_i(n), c_i^*(n)] / (1 - \beta) \).

First period, \( t = 1 \): Since agents are not anticipating the possibility of a run, they set \( \pi(h = r) = 0 \). The intertemporal first-order conditions (11) and (12) reduce to
\[ q(1)U_{c_i}(1) / P(1) = \beta U_{c_i}[n, 2] / P(n, 2) \quad \text{and} \quad q^*(1)U_{c_i^*}(1) / P^*(1) = \beta U_{c_i^*}[n, 2] / P^*(n, 2) \]
for \( i = d, f \). Combining these equations with market clearing for goods delivers expressions for bond prices
\[ q(1) = \beta \frac{P(1) \ell(1) \alpha}{P(n, 2) \ell(n, 2) \alpha} \quad \text{and} \quad q^*(1) = \beta \frac{P^*(1)y^*}{P^*(n, 2)y^*} . \]
Recall now that foreign goods prices are given by
\[ P^*(1) = \{ M^* [1 - q^*(1)x^*] - R_0 + T(1) / \bar{s} \} / y^* , \quad \text{and} \quad P^*(n, 2) = \{ M^* [1 - q^*(n, 2)x^*] - R(1) \} / y^* . \]
But also recall that \( q^*(n, 2) = \beta \) and \( R(1) = R_0 - T(1) / \bar{s} - TR_0 \). Therefore, \( q^*(1) = \beta \). Now it remains to determine \( q(1) \) and \( T(1) \). From the expression for \( q(1) \) above,
\[ q(1) = \beta \frac{M_0 [1 - q(1)x] - T(1)}{M_0 [1 - \beta x]} , \]
but also notice that \( T(1) = T_d(1) + T_f(1) \), where \( T_i(1) \) is determined by agent \( i \)'s first-order conditions. In particular, \( T_i(1) = \min \{ A_i(1), \bar{s} R_0 / 2 \} \) if \( 1/q(1) < 1/q^*(1) \), \( T_i(1) = -\bar{s} A_i^* \) if \( 1/q > 1/q^* \), and \( T_i(1) \in [-\bar{s} A_i^*(1), \min \{ A_i(1), \bar{s} R_0 / 2 \}] \) if \( q(1) = q^*(1) \). Therefore, \( q(1) = \beta \) and \( T_i(1) = 0 \) for \( i = d, f \) satisfies household's first-order conditions. That is, bond prices for the first period are \( q(1) = q^*(1) = \beta \). When agents do not expect any inflation, the price of a one-period discount bond paying one unit of currency in the next period should be equal to the discount factor. All that remains is to solve for the domestic money supply. When the government is fixing the exchange rate, the domestic money supply is endogenously determined. In particular, from the expression for the exchange rate
\[ \bar{s} = P(1) \ell(1) \alpha \left( \frac{q^*(1)}{q(1)} \right) = \frac{M_0 [1 - \beta x]}{M^* [1 - \beta x^*] - R_0} , \]
which implies \( M_0 = \bar{s} [M^* [1 - \beta x^*] - R_0] / [1 - \beta x] \).
So far I have constructed quantities and prices assuming that households run when \( h = r \) and that they do not run when \( h = n \). Therefore, it remains to verify that given the prices and quantities obtained, the proposed behavior is indeed optimal. In other words, is it optimal for an individual household to run on reserves when everybody else is running? From the household’s first order condition for \( T_1(r, 2) \), joining the run is optimal as long as the condition \( 1/q(r, 2) < s(r, 2)/\bar{s}q^*(r, 2) \) is satisfied. Recall that

\[
s(r, 2) = \frac{P(r, 2)\ell(r, 2)^{\alpha} q^*(r, 2)}{P^*(r, 2)y^* q(r, 2)}
\]

and that \( P(r, 2)y = M_0 [1 - q(r, 2)x] - \bar{s}R_0, \) \( P^*(r, 2)y^* = M^* [1 - q^*(r, 2)x^*] - R_0 + \frac{\bar{s}R_0}{\bar{s}}, \) and \( q^*(r, 2) = \beta \). Making substitutions, the condition \( 1/q(r, 2) \leq s(r, 2)/\bar{s}q^*(r, 2) \) can be rewritten as

\[
\bar{s} < \frac{M_0 [1 - q(r, 2)x] - \bar{s}R_0}{M^* (1 - \beta x^*)},
\]

and, substituting further for \( M_0 \), rewrite as

\[
\underbrace{M^* (1 - \beta x)}_{\text{lhs}} (1 - \beta x^*) < \underbrace{[M^* (1 - \beta x^*) - R_0] [1 - q(r, 2)x] - (1 - \beta x) R_0}_{\text{rhs}}.
\]

Now notice that for \( R_0 = 0, lhs < rhs \) if and only if \( q(r, 2) < \beta \) which it is the case by lemma 1. Also by lemma 1, as \( R_0 \gtrsim M^* [1 - \beta x^*]^{2 - \beta x}, q(r, 2) \lesssim 0, \) and thus, \( rhs \lesssim 0 \). Therefore, as \( R_0 \lesssim 0, lhs < rhs, \) whereas \( R_0 \gtrsim M^* [1 - \beta x^*]^{2 - \beta x}, \) implies \( lhs > rhs \). If it can be shown that \( rhs \) decreases monotonically, then the cut-off value \( R(x) \) is unique. To prove that indeed \( rhs \) decreases monotonically I proceed by contradiction. That is, suppose that there are \( R_2 < R_1 \) such that \( rhs (R_2) < rhs (R_1) \). Then

\[
[M^* (1 - \beta x^*) - R_2] [1 - q(r, 2; R_2)x] - [M^* (1 - \beta x^*) - R_1] [1 - q(r, 2; R_1)x] < (1 - \beta x) [R_2 - R_1] < 0
\]

Since \( \partial q(r, 2; R_0)/\partial R_0 < 0, [1 - q(r, 2; R_2)x] < [1 - q(r, 2; R_1)x] \). Thus

\[
0 < - [R_2 - R_1] [1 - q(r, 2; R_1)x] < [M^* (1 - \beta x^*) - R_2] [1 - q(r, 2; R_2)x] - [M^* (1 - \beta x^*) - R_1] [1 - q(r, 2; R_1)x] < (1 - \beta x) [R_2 - R_1] < 0
\]

Clearly a contradiction. QED.
EQUILIBRIUM UNDER THE INTEREST RATE POLICY

Continuation $h \in \{n, r\}$, $t \geq 3$: Under the interest rate policy there is no run in period $t = 2$. Hence the government does not switch to inflationary finance. Since there is no money growth $\hat{q}(r, t) = \hat{q}^*(r, t) = \beta$, $\forall t \geq 3$ (with these bond prices $\hat{T}_i(r, t) = 0$ satisfies household’s first order conditions). The exchange rate is given by $\hat{s}(r, t) = \frac{P(r, t)\ell(r, t)^\alpha}{P^*(r, t)y^*} \frac{\hat{q}^*(r, t)}{\hat{q}(r, t)}$. Substituting the expressions for prices delivers $\hat{s}(r, t) = \bar{s}$.

The period $h = r$, $t = 2$: Consider now the period of the defense. The exchange rate is given by $\hat{s}(r, 2) = \frac{P(r, 2)\ell(r, 2)^\alpha}{P^*(r, 2)y^*} \frac{\hat{q}^*(r, 2)}{\hat{q}(r, 2)}$, and bond prices are given by

$$\hat{q}(r, t) = \beta \frac{P(r, 2)\ell(r, 2)^\alpha}{P(r, 3)\ell(r, 3)^\alpha} \quad \text{and} \quad \hat{q}^*(r, t) = \beta \frac{P^*(r, 2)y^*}{P^*(r, 3)y^*}.$$  

Therefore, $\hat{s}(r, 2) = \frac{P(r, 3)\ell(r, 3)^\alpha}{P^*(r, 3)y^*} \frac{\hat{q}(r, 2)}{\hat{q}^*(r, 2)} = \hat{s}(r, 3) = \bar{s}$. In equilibrium the arbitrage condition in period $t = 2$, $\frac{1}{\hat{s}(r, 2)\hat{q}^*(r, 2)}$, must hold with equality. For if it holds with the inequality $<$, then agents want to run, and if it holds with $>$ then agents want to set $\hat{T}_i(r, 2) = -\bar{s} \hat{A}_i^*(r, 2)$ which in turn implies $\hat{q}^*(r, 2) = 0$. Thus, $\hat{q}(r, 2) = \hat{q}^*(r, 2)$ in equilibrium. In other words, when the domestic government lowers the domestic bond price, agents will exchange foreign currency for domestic currency until the foreign bond price decreases enough. Finally from the equations for $\hat{q}(r, 2)$ and $\hat{q}^*(r, 2)$ above, it is possible to get expressions for the amount of domestic currency traded at the central bank, $\hat{T}(r, 2)$, and the amount of domestic bonds sold by the government, $\hat{x}(r, 2)$.

$$\hat{T}(r, 2) = \bar{s} \left[ M^* - R_0 \right] \left[ \frac{\hat{q}(r, 2)}{\beta} - 1 \right].$$

Note that since $\hat{q}(r, 2) < \beta$, $\hat{T}(r, 2) < 0$. That is, the period of the defense there is an inflow of foreign currency to the central bank. Recall that under the interest rate defense the government sets transfers to be equal to $\hat{T}(r, 2) = -\min\{\hat{T}(r, 2)/\bar{s}, 0\}$. Thus, at the end of the period, agents receive the inflow of foreign currency as a transfer from the government. Finally, solving for $\hat{x}(r, 2)$ yields

$$\hat{x}(r, 2) = \frac{1}{\left\{ M^* \left[ 1 - \beta x^* \right] - R_0 \right\} \hat{q}(r, 2)} \begin{cases} M^* \left[ 2 - \beta x^* - \beta x - (1 - \beta x) (2 - \beta x^*) \frac{\hat{q}(r, 2)}{\beta} \right] \\ -R_0 \left[ 2 - 2 (1 - \beta x) \frac{\hat{q}(r, 2)}{\beta} - \beta x \right] \end{cases}.$$  

First period, $t = 1$: The first period under the interest rate policy is simply the same as the first period under the reserves policy. This is so because agents assign $\pi = 0$ to the event $h = r$. Therefore, the problem they solve in the first period is the same regardless of the policy in place.
NUMERICAL RESULTS

In this appendix I return to the more generic case in which \( \pi(h = r) = \pi > 0 \). Unfortunately, for this case it is not possible to derive analytical solutions for the model. Instead, I solve several examples with the computer. The forces at play are the same as in the previous section, with the only exception being that now bond prices in the first period need to reflect the possibility that in the second period a crisis may occur. In other words, since the value of money at the beginning of period 2 depends on whether there is a crisis, first-period bond prices need to capture this uncertainty. The model can solved as above, but note that equations (11) and (12) can be written as

\[
q \frac{U_{c_1}}{P} = \beta \sum_H \pi(h) \frac{U_{c_1}[h, 2]}{P(h, 2)} = \beta \left[ \pi \frac{U_{c_1}[r, 2]}{P(r, 2)} + (1 - \pi) \frac{U_{c_1}[n, 2]}{P(n, 2)} \right]
\]  

(26)

and

\[
q^* \frac{U_{c^*_1}}{P^*} = \beta \sum_H \pi(h) \frac{U_{c^*_1}[h, 2]}{P^*(h, 2)} = \beta \left[ \pi \frac{U_{c^*_1}[r, 2]}{P^*(r, 2)} + (1 - \pi) \frac{U_{c^*_1}[n, 2]}{P^*(n, 2)} \right].
\]

(27)

In the rest of this section I will report results for a range of values of \((x, R_0)\) for which solutions are interior. This range is \( \mathcal{R} \equiv \{(x, R_0) : x \in [0.2, 0.55], R_0 \in [0, 0.12]\} \). For this range of values of \((x, R_0)\), similar results as the ones obtained in the previous section can be obtained. In particular, results 1 and 2 below mirror propositions 1 and 2. Result 3 states the same ranking of utilities as in the previous section.

Result 1: For any pair \((x, R_0) \in \mathcal{R}\), when the government follows the reserves policy, there is an equilibrium in which agents run on reserves when they observe \( h = r \).

Result 2: For any pair \((x, R_0) \in \mathcal{R}\), when the government follows the interest rate policy, the equilibrium is unique and it involves agents not running on reserves.

Result 3: The interest rate policy delivers lower levels of welfare than the reserves policy.

Figure 7 shows welfare levels under the two different policies for parameter values \( \beta = 0.95, \eta = 2, \alpha = 0.7, \pi = 0.7, y^* = 1, M^* = 1, x^* = 0.1 \). Different parameters deliver the same results and ranking of utilities.
Figure 7: Utility for $\pi=0.7$
References


