Productivity Shocks, Learning, and Open Economy Dynamics

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Abstract

I study the implications of productivity shocks in a model where agents observe the aggregate level of productivity but not its permanent and transitory components separately. The model’s predictions under learning differ substantially from those under full information and are in line with several empirical findings: (i) the response of investment to a permanent shock is sluggish and peaks with delay; (ii) permanent shocks generate positive rather than negative savings on impact; and (iii) saving and investment are highly correlated despite the assumption of capital mobility. Unlike other standard explanations of the Feldstein-Horioka puzzle, learning induces high correlations irrespective of the assumed persistence of shocks.

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I. INTRODUCTION

Ever since Sachs’ (1981) pioneering paper, economists have viewed trade in foreign assets through the current account as a way for an economy to smooth consumption in the face of idiosyncratic income shocks. Embedded in these models is the assumption that agents know the structural model of the economy and can observe in real time the size and persistence of shocks affecting this structure. This is true in particular of productivity shocks, often identified as an important source of current account fluctuations in both small and large countries—see Obstfeld (1986), Mendoza (1991), Baxter and Crucini (1993), and Stockman and Tesar (1995) for examples.

The idea that agents know the structural model of the economy at all times and can assess on impact the exact size and persistence of shocks is being refuted by a growing body of empirical evidence. Using household-level data, Carroll (2003) finds strong support for a model where up-to-date forecasts made by professionals are sluggishly adopted by the rest of the population. Ball and Tchaidze (2002) compare real time estimates of the Non-Accelerating-Inflation Rate of Unemployment (NAIRU) made by academics and professional forecasters with the corresponding estimates obtained with hindsight. In all cases, the real time estimates understated the decline of the NAIRU in the second half of the 1990’s. Related to this finding, Blinder (2000), DeLong (2000), and the Council of Economic Advisors (2000) have argued that the recent combination of high growth and low inflation in the US was due to workers underestimating productivity growth and not adjusting wage aspirations accordingly.

Besides relevant empirical evidence, the idea that agents take time to figure out the underlying state has been used to explain important economic phenomena that cannot be easily accounted for in more standard frameworks. Among recent examples one finds: excess volatility in stock prices and the dividend yield’s predictive power for future returns (Timmerman, 1996); excess smoothness in consumption and the correlation between aggregate consumption and lagged income changes (Pischke, 1995); the output cost of disinflation and lags in monetary policy (Mankiw and Reis, 2001, 2002); the sluggishness in an investment’s response to real interest rate shocks (Moore and Schaller, 2002); and the forward premium and delayed overshooting puzzles (Gourinchas and Tornell, 2001).

This paper abandons the assumption of exact, real time observation of shocks in the context of a standard intertemporal model of the current account with endogenous investment. Uncertainty in the economy comes from idiosyncratic shocks affecting the level of productivity. Firms face convex costs of adjusting capital, and the economy has access to an internationally traded bond that pays a constant, exogenous real interest rate to finance investment or smooth consumption. Fisherian separability applies in the model, so that investment decisions are taken independently of consumption decisions.

The primary innovation of the model is that agents do not directly observe whether any

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2 See Obstfeld and Rogoff (1995) for a survey of this literature.
particular shock to productivity is permanent or transitory, even though they know their relative population variances. Agents form estimates of the permanent and transitory components through Kalman filtering of the level of productivity, and base investment and asset accumulation decisions on these projections. In a sense, this model extends the work of Moore and Schaller (2002) who study investment in a context where agents cannot observe whether shocks to the real interest rate are permanent or transitory. Whereas Moore and Schaller assume a richer structure for the learning process, they do not address consumption and asset accumulation decisions. Given that precise empirical estimates of the signal-to-noise ratio for the Kalman filter are hard to come by, I simulate the model under a large range of possible values.

The introduction of learning in the model generates substantial differences in dynamics that are consistent with some empirical regularities. When shocks can be perfectly observed, investment only reacts to permanent changes in productivity. It is fully financed through a current account deficit, and this foreign debt is repaid through higher expected future income. Under learning, the immediate response of investment is muted since part of the shock is attributed to a temporary component. The response can be as much as a third smaller for plausible values of the signal-to-noise ratio. As agents start revising their expectations of the permanent shock upwards, investment rises, yielding a hump-shaped response. For many plausible values of the signal-to-noise ratio, delay in the peak response of investment is consistent with empirical findings like those of Moore and Schaller (2002) who find a delay of approximately six quarters.

Whereas consumption-smoothing agents would never accumulate foreign assets in the face of a correctly identified permanent shock, learning agents do so by mistakenly taking part of the innovation to be temporary. For about half of the assumed values of the signal-to-noise ratio, the combined responses of saving and investment generate a negative current account response to a permanent shock that is smaller and not larger in absolute terms than that of investment. This is consistent with a central empirical finding in Glick and Rogoff (1995) and Gruber (2002). Learning thus joins sluggishness in consumption adjustment through habit formation as in Gruber (2002) as alternative and mutually compatible explanations for this empirical failure of standard models. Glick and Rogoff (1995) argue that standard models could account for this anomaly if the estimated shocks are stationary but highly persistent and unit root tests fail to reject the null of non stationarity due to lack of power. It should be noted that this explanation is very sensitive to the exact degree of persistence of the shock.

Purely transitory shocks generate protracted dynamics under learning. While investment does not respond to correctly identified one-period shocks, it will go up on impact under learning.

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3 From now on, I will use the word “learning” to denote the evolution of the estimates obtained through the Kalman filter.

4 It is understood in this discussion that the shock is positive. If the shock is negative then all results carry through symmetrically.
This will be followed by a period of disinvestment as agents try to correct for “spurious” additions to the capital stock. The model implies an asymmetry in responses that is worth noting: when the observable level of productivity tracks the permanent component closely, the differences in responses between learning and perfect observation will be small \textit{conditional on a permanent shock}, but will be large \textit{conditional on a transitory shock}.

Learning generates correlations between estimates of permanent and transitory components of productivity which in turn generate positive saving-investment correlations despite capital mobility. Importantly, the correlation value is found to be robust to the assumed value of the signal-to-noise ratio and is close to estimates for small open economies reported by Baxter and Crucini (1993). The fact that positive saving-investment correlations arise despite Fisherian separability provides an alternative interpretation of the Feldstein-Horioka puzzle that does not rest on low international capital mobility. The literature has advanced several plausible explanations for the Feldstein-Horioka puzzle which do not require limits on capital movements. The results in this paper should be compared most directly with explanations based on the propagation of productivity shocks, such as Obstfeld (1986), Mendoza (1991), Baxter and Crucini (1993), and Stockman and Tesar (1995) among others.\footnote{Other possible explanations of the puzzle include: governments’ targeting of the current account (Summers, 1988), the presence of a non-tradables sector with different capital/labor intensities (Engel and Kletzer, 1989) and iceberg costs to trade (Obstfeld and Rogoff, 2001).} In those models, however, the assumption of persistent yet stationary (or trend stationary) productivity is essential; permanent shocks do not generate positive saving responses, and sufficiently transitory ones do not generate investment responses.\footnote{The issue of the plausibility of trend stationarity in productivity is discussed later in the paper.} Here, the assumption of highly persistent productivity is absolutely non essential since it is the learning process that generates the correlations. Indeed, productivity shocks are assumed to be both permanent and fully transitory, and I show that under perfect observation of shocks, the model would imply zero saving-investment correlations.

The paper is organized as follows. Section II reviews the model and derives expectations of the permanent and transitory components of productivity under both learning and perfect observation of shocks. The limitations of the simplifying assumptions in the model are discussed. Section III compares simulated saving, investment and current account dynamics under learning and perfect observation. It discusses saving and investment correlations and Feldstein-Horioka-style regressions implied by the model. Section IV concludes.

II. THE MODEL

A. Consumption and Saving

Assume the economy is populated by a representative agent who maximizes the present value of future utility streams, where utility is time-separable. The agent can save through the
acquisition of an internationally traded foreign bond that pays real rate of interest \( r \). If one assumes that uncovered real interest parity holds and that the country is small in capital markets, then investment is chosen to maximize the present discounted value of the country’s future wealth irrespective of the consumption profile (see Ghosh, 1995).

Mathematically, the agent will:

\[
\begin{align*}
\text{Max} & \quad V_t = \sum_{s=t}^{\infty} E_t \left( \frac{1}{1+r} \right)^{s-t} U(C_s) \\
\text{s.t.} & \quad B_{t+1} = (1+r)B_t + Y_t - I_t - C_t \\
& \quad 0 = \lim_{s \to \infty} \left( \frac{1}{1+r} \right)^{s-t} E_t B_{s+1}
\end{align*}
\]

taking net output \( Y - I \) as given. \( B_t \) denotes the net stock of foreign assets at time \( t \). The transversality condition rules out explosive accumulation of debt or the running of a Ponzi scheme. Assume that utility is quadratic. In this case the current account or net increase in foreign assets \( B_{t+1} - B_t \), is given by:

\[
CA_t = -\sum_{s=t+1}^{\infty} E_t \left( \frac{1}{1+r} \right)^{s-t} \Delta(Y_s - I_s)
\]

The current account is just the present discounted value of expected future declines in net income. Agents with consumption-smoothing desires will accumulate foreign assets whenever net current income is above net permanent income. This is the open economy version of Campbell (1987) “saving for a rainy day equation.”

B. Investment

Output in this model is given by:

\[
Y_t = A_t K_t^\alpha \left[ 1 - \frac{g}{2} \left( \frac{I_t}{K_t} \right) \right]
\]

s.t. \( I_t = K_{t+1} - K_t \) and \( 0 < \alpha < 1 \)

where \( A_t, K_t, \) and \( I_t \) are productivity, capital, and investment at time \( t \). This is a standard production function with convex costs of adjusting capital. I assume no depreciation and inelastic supply of labor to keep the model as simple as possible. Firms will choose the path of investment so as to maximize the present discounted value of future profits:

\[
\begin{align*}
\text{Max} & \quad \Pi_t = \sum_{s=t}^{\infty} E_t \left( \frac{1}{1+r} \right)^{s-t} \left[ Y_s - I_s \right] \\
\text{s.t.} & \quad Y_s = A_s K_s^\alpha \left[ 1 - \frac{g}{2} \left( \frac{I_s}{K_s} \right) \right]
\end{align*}
\]
Taking a first order linear approximation around the steady state yields:

\[ I_t = \lambda_1 I_{t-1} + \eta \sum_{s=t}^{\infty} \lambda_2^{s-t+1} (E_s A_{s+1} - E_{t-1} A_s) \]  

(5)

with \( \lambda_1, \lambda_2 < 1 \) and \( \eta > 0 \)

(See the Technical Appendix for details).

Investment depends on lagged investment as the firm spreads adjustment of the capital stock through many periods in order to minimize costs of adjustment. In addition, expected increases in productivity raise the target level of capital.

C. Productivity

The Productivity Process

To close the model, I assume that productivity is a random walk with noise. More precisely:

\[ A_t = P_t + u_t \]

\[ P_t = P_{t-1} + \epsilon_t \]

The shocks \( \epsilon_t \) and \( u_t \) are assumed to be iid with zero mean and variance \( \sigma_\epsilon^2 \) and \( \sigma_u^2 \) respectively, serially and mutually uncorrelated at all leads and lags. Estimating the exact degree of persistence of productivity shocks is empirically controversial. Following Prescott (1986), the literature has typically identified productivity shocks through the Solow residual. Some authors have questioned whether the Solow residual is a measure of true productivity changes. For instance, Rotemberg and Summers (1990) have found that labor hoarding can help explain why total factor productivity is procyclical. In a similar vein, Braun and Evans (1998) show that the Solow residual tends to rise substantially in the fourth quarter of every year and the magnitude of the change (16% on average) makes the productivity story implausible. Indeed, the authors claim that the seasonal changes in the residual can be best explained by a combination of demand shifts, labor hoarding, and increasing returns.

Even accepting that Solow residuals are the best way to capture productivity changes, there is no consensus on the persistence of the residual’s process. Glick and Rogoff (1995) cannot reject the null of a unit root, while Backus, Kehoe, and Kydland (1992) among others find

7 The implications of linearizing investment are discussed below.

8 I abstract from a deterministic trend in productivity which would not alter the conclusions of the model. Non-trivially, the assumption of a random walk with noise does not rule out the possibility of negative productivity. I keep this specification to be consistent with Glick and Rogoff (1995) and Gruber (2002).
productivity shocks to be highly persistent but not permanent using similar but not identical data. The assumption of stationary but highly persistent shocks can generate high saving-investment correlations (see Obstfeld (1986), Mendoza (1991) and Baxter and Crucini (1993)). The choice of permanent and fully transitory shocks in equation 6 will make clear that the predictions of the model such as positive saving-investment correlations arise not because of particular persistence assumptions but solely through the addition of learning.

These assumptions for productivity imply that:

\[
I_t = \lambda_t I_{t-1} + \eta \beta \left[ E_t P_t - E_{t-1} P_{t-1} \right] \text{ with } \beta = \frac{\lambda_2}{1 - \lambda_2} 
\]  

(7)

\[
CA_t = \gamma (A_t - E_t P_t) - I_t \text{ with } \gamma > 0
\]

(See the Technical appendix for details).

Since by definition \(CA_t = S_t - I_t\) where \(S_t\) is domestic investment, \(S_t = \gamma (A_t - E_t P_t)\).

**Perfect Observation of Productivity Shocks**

In my framework, the analogous assumption to full observation of shocks would be that agents observe both \(P_t\) and \(u_t\) separately in each period. In this case, \(E_t P_t - E_{t-1} P_{t-1} = \epsilon_t\) and \(A_t - E_t P_t = A_t - P_t = u_t\).

Thus, from equation 7:

\[
I_t = \lambda_t I_{t-1} + \eta \beta \epsilon_t 
\]  

(8)

\[
S_t = \gamma u_t 
\]

\[
CA_t = S_t - I_t
\]

Positive permanent shocks raise the target level of capital and induce investment in the economy. Moreover, investment is fully financed through external borrowing. Since borrowing can be repaid through higher (expected) future output, it is not optimal for agents to reduce consumption in order to finance investment. Under perfect observation then, the response of the current account to a (positive) permanent shock cannot be smaller in absolute terms than that of investment.9

9 As Obstfeld and Rogoff (1996) note, the negative effect of permanent shocks on savings is a second order effect, and is thus lost in our linearization. Glick and Rogoff (1995) find a negative effect of permanent shocks on savings in their linearized model. However, the authors claim that \(\alpha_j\), the effect of investment on output in the steady state, is negative. Under their assumption of no depreciation this parameter should be zero, not negative. One can show that if \(\alpha_j = 0\) their model would also imply a zero effect of \(\epsilon_t\) on \(S_t\). In any event, a negative effect on saving would make the current account response to a permanent shock even larger in absolute terms.
Investment does not respond to transitory shocks since one-period shocks vanish before new capital can be made operational. Saving does rise with temporary shocks as agents will save part of the temporary output increase to smooth consumption in the future. In this model, the amount of saving will make (expected) consumption constant in the future. Note that the separation of consumption and investment decisions together with the assumption of uncorrelated permanent and transitory shocks imply zero saving-investment correlation under perfect observation of shocks.

**Learning**

Suppose now that agents can only observe the level of aggregate productivity \( A_t \). Even though they do not observe the permanent and transitory components separately, they are assumed to know the population variances \( \sigma_e^2 \) and \( \sigma_u^2 \). Agents need to form projections of the permanent component of productivity on which to base investment and saving decisions. From equation 7, the innovation to investment is proportional to \( E_t P_t - E_{t-1} P_{t-1} \). For a given initial condition \( P_0 \), \( E_t P_t - E_{t-1} P_{t-1} \) can be written as \( E_t e_t + (E_t e_{t-1} - E_{t-1} e_{t-1} + \ldots + E_t e_1 - E_{t-1} e_1) \).

Given that agents no longer observe the individual shocks separately and that the information set available to agents at time \( t \) is richer than that of \( t-1 \) by \( A_t \), \( E_t e_j \) need no longer equal \( E_{t-1} e_j \) for any \( 1 \leq j \leq t-1 \). This means that investment under learning will not only respond to what agents believe to be the current permanent innovation, but also to updates of past beliefs. If agents perceive that they underestimated permanent shocks in the past, they will raise investment today; the opposite arises if they believe they overestimated the shocks.

To derive the expressions for investment, saving and the current account under learning, it is necessary to compute \( E_t P_t \). The problem of signal extraction for a random walk with noise has been extensively studied in the literature. Let \( P_{t|t} \) be the period \( t \) Kalman filter estimate of \( P_t \). It can be shown that:

- If the disturbances \( e \) and \( u \) are Normally distributed, then \( P_t \) is Normally distributed and \( P_{t|t} \) is its conditional mean.
- \( P_{t|t} \) satisfies the recursion:
  \[
  P_{t|t} = k_t A_t + (1-k_t) P_{t-1|t-1} \quad \text{for some } k_t, 0 < k_t < 1
  \]
  (9)
- For a given initial condition \( P_0 \) and a given distribution of \( P_0 \), \( P_{t|t} \) is such that:
  \[
  \lim_{t \to \infty} P_{t|t} = k_A A_t + (1-k) P_{t-1|t-1}
  \]

10 Anderson and Moore (1979) and Harvey (1989) provide thorough discussions and proofs for the particular case of the random walk with noise.
\[
where \( k = \frac{q + \sqrt{q^2 + 4q}}{2 + q + \sqrt{q^2 + 4q}} \) and \( q = \frac{\sigma_t^2}{\sigma_u^2} \).
\]

That is, when agents have been living long enough learning converges to a single, unique rule. Note that this rule is identical to the adaptive rule proposed by Muth (1960).

- Finally, if the Normality assumption of the errors is dropped, then \( P_{t|t} \) is still the best of all linear estimators of \( P_t \) in the sense of minimizing the mean squared error.\(^{11}\)

Assume that agents have been living long enough to have converged to a stable rule. The coefficient \( k \) increases monotonically with the signal-to-noise ratio \( q \). When the signal-to-noise ratio is high, the level of productivity will track the permanent component closely and agents will find it optimal to respond strongly to changes in the level of productivity when forecasting the permanent component. Inversely, a low signal-to-noise ratio means that the level of productivity can deviate far from the permanent component. In this case, agents will slowly integrate changes in the level of productivity into their projections of the permanent component.

The expressions for investment, saving and the current account under learning can now be derived. From equation 7 and the discussion of the Kalman filter:

\[
I_t = \lambda_t I_{t-1} + \eta \beta k \left[ A_t - P_{t-1|t-1} \right] \\
S_t = \gamma (1 - k) \left[ A_t - P_{t-1|t-1} \right] \\
CA_t = S_t - I_t
\]

Note that the perceived innovations driving investment and saving are perfectly correlated. Intuitively, a high level of productivity at time \( t \) may come from three sources: a permanent shock at time \( t \), a transitory shock at time \( t \), or shocks before time \( t \) that were more permanent than previously thought. Rational learning agents will assign non-zero probabilities to all three possibilities.

I close this section by discussing some of the simplifying assumptions made with respect to learning. In particular, linearization of investment and quadratic utility make the model easy to solve as investment and saving become linear in the perceived permanent and temporary innovations. The downside is that they impose certainty equivalence: the response of investment and saving to the perceived permanent and temporary innovations do not depend

\(^{11}\) The assumption that \( e \) and \( u \) are not correlated is also non-essential. In this case, \( k \) will also depend on this correlation. We assumed no correlation between shocks to make saving-investment correlations zero under perfect observation.
on the variability of these estimates.\textsuperscript{12} In other words, learning agents behave in the model as if $E_P$ was the true estimate of $P$, even though they know that this projection will change over time. The justification for this strong assumption is to keep the model tractable. This is the same justification given by other authors to impose certainty equivalence in their own learning models (see Timmerman (1997), Swanson (2002), Moore and Schaller (2002)).

III. IMPLICATIONS OF THE MODEL

A. Model Parameterization

A key coefficient determining the dynamics of saving, investment, and the current account is the coefficient $k$. $K$ depends uniquely on the signal-to-noise ratio $q = \frac{\sigma^2}{\sigma_v}$. There is no consensus on the empirical value of $q$. Estimates of productivity using the Solow residual typically assume one source of shocks, and the question then is to estimate its degree of persistence.

Shapiro (1987) constructs an alternative productivity measure based on movements in the prices of factors of production. He argues that the R-squared of a regression of the Solow residual on this estimate provides an estimate of the share of productivity’s variance that is due to supply rather than demand shocks. The mapping from supply and demand shocks to permanent and transitory shocks need not be automatic however. His estimated R-squared for aggregate US manufacturing is 0.75, with some intra-industry variation. Using GDP rather than productivity data, Campbell and Mankiw (1987a, 1987b, 1989) build on the work of Nelson and Plosser (1982) and conclude: “our estimates suggest that shocks to GNP are largely permanent.”\textsuperscript{13} Kuttner (1994), using a latent variable approach to estimate potential output, also reached a similar conclusion for the prevalence of permanent shocks. In order to account for the uncertainty in the value of the signal-to-noise ratio, results will be presented for a wide range of signal-to-noise ratios: $q \subset [0.5, 3.5]$ in increments of 0.1.

Figure 1 displays the coefficient $k$ as a function of the assumed values of the signal-to-noise ratio. A perfect averaging between observable productivity and past expectations occurs when $q = 0.5$. The coefficient $k$ rises gradually and peaks at around 0.8 for the largest signal-to-noise ratio.

\textsuperscript{12} As Swanson (2000) points out, certainty equivalence in the projection does not imply certainty equivalence in the observable variable $A$. As can be seen in equations 11, the response to the observable variable will depend on the signal-to-noise ratio through $k$.

\textsuperscript{13} See page 858 of Campbell and Mankiw (1989) for the quote.
The remaining parameter values are borrowed from Glick and Rogoff (1995) who estimate a very similar model. They are: $\lambda_i = 0.9$, $n\beta = 0.35$, $\gamma = \frac{1}{1+r}$. Since Glick and Rogoff estimate the model with annual data, I set $r = 3\%$.

### B. Simulated Dynamics

In this section I compare the dynamic response of the variables in the system under perfect observation and under learning. In both cases, the economy is assumed to start at the steady-state, with a stable learning rule.\(^{14}\) Investment is set at zero at the start of the simulation, called period 0.\(^{15}\) In period 1, the economy is shocked with a unit permanent shock (Figure 2) or a unit temporary shock (Figure 3). For comparison purposes, the graphs show the results under three values of $k$: 0.5, 0.65, and 0.8 corresponding to the lower bound, the middle

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\(^{14}\) Otherwise, investment, saving and current account dynamics would be affected by the evolution of the unstable learning rule. This would be uninteresting in itself and would distract from the pure effects of imperfect observation and learning under a stable rule.

\(^{15}\) This is with no loss of generality, since what we are interested is to compare deviations from the steady-state level generated by the shock.
point, and the upper bound in the assumed range. To aid in ocular inference, I keep the same scale for each variable across all informational assumptions.

Figure 2. Dynamic Responses to a Unit Permanent Shock
Permanent Shocks
Starting with Figure 2, the top panel graphs the evolution of the Kalman filter estimate of the permanent component. A high signal-to-noise ratio means agents react more strongly to changes in the level of productivity when forecasting permanent components. That is why their initial guess is higher for high values of $k$, and their rate of convergence is faster.

Investment under perfect observation reacts positively to the shock, and then decays as firms spread the cost of adjustment through many periods. Note the differences under learning: the initial response of investment is more muted since agents take part of the shock to be temporary. The differences can be substantial: for the medium value of $k = 0.65$ the impact response is a third smaller. As agents start revising their expectations of the permanent component upwards, investment rises and peaks with some delay.\textsuperscript{16} The peak occurs after one period (corresponding to a year here) for $k = 0.8$, two periods for $k = 0.65$, and three periods for $k = 0.5$. That is, the first two cases—and all values of $k$ in between—are in line

\textsuperscript{16} Since firms correct for past over or under investment, accumulated investment over an infinite period after the shock should be identical under all informational assumptions. Indeed, accumulated investment over 100 periods after the shock equals 3.4999 in all four cases considered.
with Moore and Schaller (2002) who find a mean lag of 6.3 quarters in the response of investment to real shocks using quarterly data for the period 1960:2 to 1997:4. For the extreme case of $k = 0.5$, the delay is twice that found by Moore and Schaller.

The differences in saving dynamics are even more important. Whereas correctly identified permanent shocks induce no saving in this model, learning agents save that part of the shock which they take to be temporary. Positive savings are greater and more persistent the lower the signal-to-noise ratio since beliefs are slower to converge. Notice that in the model agents do not correct for past mistakes in saving as they do in investment. They only respond to what they think is the current temporary innovation. Past over or under-saving is seen as “water under the bridge.” This is perhaps too simplistic, but is a direct result of the highly tractable utility function.

Positive saving responses counteract the effect of investment on current account deficits. Positive saving responses to permanent shocks are an important finding in Glick and Rogoff (1995) who estimate a very similar model using G-7 data. Gruber (2002) also finds positive saving responses in a different specification that allows for habits in consumption. The finding is puzzling for standard models, since the predicted response of saving to a permanent shock is never positive in these models. If there is no instantaneous adjustment of capital, a permanent shock to productivity pushes perceived permanent income above current income and saving falls.\(^\text{17}\) Even if current income could jump immediately to the new level of permanent income, saving would be zero.

\(^{17}\) In linearized models the effect on saving is zero as can be seen in Figure 2.
To examine the implications of the model for relative current account / investment responses in more detail, Figure 4 plots the impact responses of the two variables as a function of the signal-to-noise ratio. For all values of $q$ the estimated response of investment is below Glick and Rogoff’s point estimate of 0.35. The difference is not large especially for high values of $q$, even though the estimated response is still outside their (tightly estimated) 95% confidence interval equal to $[0.29 – 0.41]$. For the bottom half of the range of signal-to-noise ratios the model has the counterfactual implication of a positive current account response. For the upper half, the model generates a negative current account response. Throughout this

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18 This is not surprising since my calibration—borrowed from Glick and Rogoff—is such that the point estimate of 0.35 is perfectly matched under perfect information. Learning lowers the response of investment to a permanent shock given that part of the shock is thought to be transitory.

19 The bottom half is close to the values implied by the estimates of Carroll and Samwick (1997), using household income data, in which transitory shocks are likely more pronounced than in aggregate data.
range, the difference between the response of investment and the absolute value of the current account is stable and close to Glick and Rogoff’s point estimate of 0.18.

Gruber (2002) provides an alternative explanation for Glick and Rogoff’s finding based on habit formation in consumption. Under habit formation, a permanent shock will raise income by more than consumption since habit-forming agents smooth consumption levels but also consumption growth. The exact difference in current account and investment responses is a function of the habit formation parameter. Gruber shows that Glick and Rogoff’s estimated difference is consistent with reasonable values of the parameter. Learning and habit formation thus provide alternative and mutually compatible explanations for this finding.20

Transitory Shocks
Figure 3 plots the dynamic responses to a one-period shock. Whereas such shocks have a very short lived effect under perfect observation, they can generate protracted dynamics under learning. First, investment reacts positively to the shock as firms believe it to be partly permanent. When they start revising their expectations of the permanent component downwards, investment dips below zero. Firms are trying to undo their initial spurious accumulation of capital. The immediate saving’s response is more muted under learning. In the following period, agents observe a decline in productivity and assign part of that decline to the belief that there has been a negative transitory shock in that period. This pushes saving below zero. Such rapid swings in saving are not present in the data. Once again, the responses of saving highlight the naive assumption that past over or under-saving are viewed as “water under the bridge.”

Finally, note the asymmetry implied by the model. When the signal-to-noise ratio is high, responses to permanent shocks will be close to those under perfect observation. The opposite is true when transitory shocks hit the economy. If the signal-to-noise ratio is high, agents will commit small mistakes in the likely event of a permanent shock, but large mistakes in the unlikely event of a transitory shock. When the signal-to-noise ratio is close to one, agents will make average-size mistakes in the equally likely event of a permanent or transitory shock.

---

20 Glick and Rogoff argue that their puzzling finding can be explained if productivity shocks are stationary but highly persistent and unit root tests fail to reject the null of non-stationarity. The problem with this explanation is that the implied responses of investment and the current account are highly sensitive to the degree of persistence of the shock. In the authors’ own calculations, the response of investment and the current account to a unit permanent shock are 0.35 and -0.97 respectively. If the autoregressive root of the shock falls to 0.95 (still implying a half life of 13.5 years), these responses become 0.3 and 0.05.
C. Saving-Investment Correlations

Ever since Feldstein and Horioka (1980), economists have tried to understand why domestic saving and investment move so closely together in a world of supposedly open capital markets. As Obstfeld and Rogoff (2001) argue, the correlation between saving and investment may have fallen a bit in the last two decades, but it remains a temporarily robust and puzzling fact despite the many explanations advanced in the literature. In Figures 2 and 3 the responses of saving and investment under learning are always of the same sign. Indeed, the saving-investment correlations over the twenty periods following the shock can be easily computed:

<table>
<thead>
<tr>
<th>Permanent Shock</th>
<th>Temporary Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 0.8$</td>
<td>0.45</td>
</tr>
<tr>
<td>$k = 0.65$</td>
<td>0.41</td>
</tr>
<tr>
<td>$k = 0.5$</td>
<td>0.37</td>
</tr>
</tbody>
</table>

But these correlations cannot be taken as the saving-investment correlations implied by the model. First, shocks were assumed to be of unit size in both cases. A proper comparison would scale the size of the shocks to the assumed signal-to-noise ratio. Moreover, the assumption that the economy received a shock at time $t$ and then received no shocks for twenty periods is only useful for illustrative purposes. To get implied saving-investment correlations the model has to be simulated with properly scaled shocks every period. I simulate the model as follows: the economy is assumed to have started from zero initial conditions and to live for a hundred periods. In each period permanent and transitory shocks hit the economy, where the shocks are assumed to come from zero mean, Normal distributions with relative variances given by the signal-to-noise ratio. Beliefs evolve according to equation 10 and saving and investment according to equation 11. I compute the value of saving and investment in period 100. Saving-investment correlations and regressions of investment on saving are then obtained by using the computed values across five thousand repetitions of the experiment. The assumption of a hundred periods is arbitrary but not important per se: it simply guarantees that results will be independent of the assumed initial conditions.

21 See Tesar (1991) for an early review of this literature. Since then, the international real business cycle literature (Mendoza (1991), Baxter and Crucini (1993), Stockman and Tesar (1995) among others) has attempted to explain time series correlations in saving and investment through the propagation of trend-stationary productivity shocks. This literature will be further discussed in the paper.

22 Our discussion of the Glick and Rogoff finding is not affected by these considerations since that finding involves impact responses to a unit permanent shock.
Figure 5a. Saving-Investment Correlations

Figure 5b. Saving-Investment Correlations with Different Initial Conditions
Figure 5a plots simulated saving-investment correlations for the assumed range of signal-to-noise ratios. Notwithstanding simulation error, correlations are constant and close to 0.45. As was previously discussed, the perceived innovations in the permanent and transitory component are perfectly correlated in this model regardless of the value of $q$. What drives the correlation between saving and investment down to 0.45 are costs of adjusting capital. Saving depends on past shocks through the Kalman filter. Investment on the other side depends on past shocks both through the Kalman filter and through investment inertia. These two effects need not be perfectly correlated. For robustness’ sake, Figure 5b replicates Figure 5a except that all initial conditions were set equal to one, a hundred times the assumed standard deviation of transitory shocks. The two are virtually indistinguishable: after some time, even large differences in initial conditions cease to matter.

To see how implied correlations compare with empirical values, consider the following international saving-investment correlations from Baxter and Crucini (1993):

<table>
<thead>
<tr>
<th>Country</th>
<th>S-I correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.86</td>
</tr>
<tr>
<td>Japan</td>
<td>0.80</td>
</tr>
<tr>
<td>Germany</td>
<td>0.68</td>
</tr>
<tr>
<td>France</td>
<td>0.31</td>
</tr>
<tr>
<td>Italy</td>
<td>0.39</td>
</tr>
<tr>
<td>Canada</td>
<td>0.61</td>
</tr>
<tr>
<td>Australia</td>
<td>0.54</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Except for the two largest economies where the correlation is highest, my estimate is quite close to the average value in the other six which equals 0.53. Discriminating between the US and Japan—and perhaps Germany too—and the rest is important since my model is one of a small open economy that takes interest rates as given, and induced changes in interest rates are one plausible reason why large countries have larger correlations (see Baxter and Crucini (1993)). Also, the fact that the value of the correlation is robust to the assumed value of $q$ is an attractive feature of the model.

As was previously mentioned, other papers in the literature have managed to generate saving-investment correlations in a world of capital mobility through the propagation of productivity shocks. But in a world of consumption-smoothing agents, the assumption of stationary (or trend stationary) but highly persistent productivity is essential. Indeed, Obstfeld (1986), Mendoza (1991), Baxter and Crucini (1993) and Stockman and Tesar (1995) among others.

23 The only exception is if all shocks are of global nature, but this imposes strong restrictions on the correlation of shocks among countries or on the speed of international diffusion of idiosyncratic shocks.
assume such productivity processes. Given that measures of productivity based on the Solow residual are highly imprecise, estimates of the persistence of the productivity process are at best contentious. In this model, the exact root of the productivity process is absolutely non-essential since correlations are generated by the learning process. Indeed, I have assumed both fully permanent and fully transitory shocks and shown that under perfect observation of shocks saving-investment correlations are zero.

D. Saving-Investment Regressions

The original Feldstein-Horioka result was based on cross-sectional regressions rather than time-series correlations. More importantly, their result not only pointed to high correlations but also to movements in saving and investment of similar size. The latter is a more stringent condition, and what prompted the authors to argue against high capital mobility in the world. To see how the model performs in the context of a Feldstein-Horioka style regression, Figure 6 shows the coefficient on a regression of simulated investment on simulated saving together with the estimated 95% confidence band. Here, the value of the coefficient is not independent of the signal-to-noise ratio. A greater ratio will raise the size of investment responses to shocks relative to saving for two reasons. First, investment ultimately responds to permanent shocks, and their relative variance is growing. Second, a higher signal-to-noise ratio lowers the response of saving to shocks since the share attributed to transitory components on impact decreases.

Figure 6. Feldstein-Horioka Coefficient

Results in Figure 6 overshoot empirical estimates. Tesar (1991) re-estimates Feldstein-Horioka style regressions using data on OECD countries spanning the period 1960–94. Estimated coefficients are robust to the sub-period chosen and are on average close to 0.85–
Obstfeld and Rogoff (2001) extend the sample period to include recent observations and find that the coefficient has fallen over time to around 0.6. Coefficient values \([0.6, 0.9]\) are compatible with low values of the signal-to-noise ratio, approximately between 1 and 1.75. Unfortunately, this range of \(q_s\) also generates counterfactual positive current account responses to permanent shocks. The range above two overshoots the actual values in the data, by generating investment responses that are too high relative to saving.

Ultimately, the performance of the model has to be judged relative to other models driven by exogenous shocks. This is not an easy task since most of these papers report saving-investment correlations but not regression estimates. Mendoza (1991) reports a simulated standard deviation of investment almost twice that of saving under his benchmark parameterization, which also suggests a coefficient above 1. Also, the value of the coefficient implied by these models may vary greatly with the assumed root of the productivity process.

IV. CONCLUDING REMARKS

Productivity shocks have been identified as an important source of saving, investment, and current account fluctuations. This paper departs from the usual assumption made in open economy models that agents observe the size and persistence of shocks affecting productivity in real time. Instead, I assume that agents only observe the level of productivity and gradually learn to disentangle its permanent and transitory components.

The introduction of learning induces substantial differences in the dynamics of saving, investment, and the current account. Under reasonable parameterizations, the model can explain three empirical regularities: why the response of investment to productivity shocks is sluggish and peaks with delay, why increases in investment following permanent shocks are not fully financed through current account deficits, and why saving and investment may be highly correlated despite high capital mobility. Contrary to other models that explain the saving-investment correlations through the propagation of productivity shocks, positive correlations in this model do not hinge on the assumption of stationary (or trend stationary) but persistent productivity.

Several restrictive assumptions were made to keep the model tractable and the differences between perfect observation and learning transparent. Besides this being a one-country, one-good model, linearization of the investment function and the assumption of quadratic utility impose certainty equivalence on the perceived innovations of productivity. That is, investment and saving react under learning as if their estimates were the true values of the process, even though agents know that these estimates are likely to change through time. The effect of estimates’ variability on the response of saving and investment is likely to be important and is sidestepped in this paper. Studying whether the dynamics obtained are robust in a non certainty equivalent model is a topic for future research.
TECHNICAL APPENDIX

A. Derivation of Linearized Investment

The firm solves the following maximization problem:

\[
\max_{\{I_s\}_s} \Pi_t = \sum_{s=t}^{\infty} E_t \left( \frac{1}{1+r} \right)^{s-t} \left[ Y_s - I_s \right]
\]

(12)

s.t. \( Y_s = A_s K_s^\alpha \left( 1 - \frac{g}{2} \frac{I_s^2}{K_s} \right) \)

and \( I_s = K_s - K_{s-1} \)

Let \( l_t \) be the Lagrange multiplier in period \( t \). The first order conditions (FOCs) of this maximization problem are:

\[
\frac{\partial \Pi}{\partial K_{t+1}} = E_t \frac{1}{1+r} \left\{ \alpha A_{t+1} K_{t+1}^{\alpha-1} + A_{t+1} K_{t+1}^\alpha \frac{g}{2} \left( \frac{I_{t+1}^2}{K_{t+1}} \right)^2 \right\} - l_t = 0
\]

(13)

\[
\frac{\partial \Pi}{\partial I_t} = -1 - A_t K_t^\alpha g \frac{I_t}{K_t} + l_t = 0
\]

Using the expression for \( l_t \) and using the definition of investment yields a unique FOC:

\[
F = \frac{1}{1+r} E_t \left\{ \alpha A_{t+1} K_{t+1}^{\alpha-1} + A_{t+1} K_{t+1}^\alpha \frac{g}{2} \left( \frac{K_{t+2}^2 - K_{t+1}^2}{K_{t+1}} \right)^2 + 1 \right\}
\]

(14)

\[
-\alpha A_{t+1} K_{t+1}^{\alpha-1} g \left( \frac{K_{t+2}^2 - K_{t+1}^2}{K_{t+1}} \right) + A_{t+1} K_{t+1}^\alpha g \frac{K_{t+2} - K_{t+1}}{K_{t+1}} = 0
\]

\[
-1 - A_t K_t^\alpha g \frac{K_{t+1} - K_t}{K_t} = 0
\]

To linearize the FOC, compute the partial derivatives of \( F \) with respect to all of its arguments, which evaluated at the steady state simplify to:

\[
\frac{\partial F}{\partial K_t} = gr - gr + g \frac{AK^{\alpha-1}}{\alpha} = \frac{gr}{\alpha}
\]

(14)

\[
\frac{\partial F}{\partial K_{t+1}} = \frac{1}{1+r} \left( (\alpha-1) r \frac{1}{K} \frac{gr}{\alpha} - \frac{gr}{\alpha} \right)
\]
\[
\frac{\partial F}{\partial K_{t+2}} = \frac{1}{1+r} \alpha \frac{gr}{K} \\
\frac{\partial F}{\partial A_t} = -\frac{\alpha^2}{K} g K - K = 0 \\
\frac{\partial F}{\partial A_{t+1}} = \frac{1}{1+r} \alpha^2 K^{-a-1}
\]

The linearized FOC can then be written as:

\[
E_t \left[ K_t^* + \left( \frac{1}{1+r} \left( \alpha - 1 \right) r - \frac{\alpha}{gK} - 1 \right) - 1 \right] K_{t+1}^* + \frac{1}{1+r} K_{t+2}^* 
= \frac{\alpha}{g(1+r)} - \frac{1}{g(1+r)r} \alpha^2 K^{-a-1} E_{t+1} A_{t+1}
\]

(15)

The left-hand side of equation 15 can be expressed as a polynomial in the lag operator, yielding:

\[
\left[ 1 + \left( \frac{\alpha (\alpha - 1)r}{gK} - 2 - r \right) L + (1 + r) L^2 \right] K_{t+2}^* = \frac{\alpha}{g} - \frac{1}{gr} \alpha^2 K^{-a-1} E_{t+1} A_{t+1}
\]

or

\[
(1 - \lambda_1 L)(1 - \lambda_0 L) K_{t+2}^* = \frac{\alpha}{g} - \frac{1}{gr} \alpha^2 K^{-a-1} E_{t+1} A_{t+1}
\]

(16)

where \( \lambda_1 \) and \( \lambda_0 \) are the roots of the polynomial, and are such that \( 0 < \lambda_1 < 1 \) and \( \lambda_0 > 1 \).

Solving equation 17 for \( K_t^* \) yields:

\[
K_t^* = \lambda_1 K_{t+1}^* + \frac{\alpha}{g} \left( 1 - \lambda_0 L \right)^{-1} + \frac{1}{gr} \alpha^2 K^{-a-1} \sum_{s=t}^{\infty} \left( \frac{1}{\lambda_0} \right)^{s-t+1} E_{t-s} A_s
\]

(18)

Since \( I_t = K_t - K_{t-1} \), equation 18 implies

\[
I_t = \lambda_1 I_{t+1} + \frac{\alpha}{gr} \alpha^2 K^{-a-1} \sum_{s=t}^{\infty} \left( \frac{1}{\lambda_0} \right)^{s-t+1} \left[ E_{t-s} A_{s+1} - E_{t-s} A_s \right]
\]

(19)

Equation 19 is equivalent to equation 5 in the text, with \( \eta = \frac{1}{gr} \alpha^2 K^{-a-1} \) and \( \lambda_2 = \frac{1}{\lambda_0} \).

Since productivity is a random walk with noise and errors iid and zero mean, it has to be true that \( E_{t-A_{t+1} - A_s} = E_{t-A_{s+1} - A_t} \forall s \geq t \). Equation 19 then implies:

\[
I_t = \lambda_1 I_{t+1} + \eta \beta \left[ E_{t-A_{t+1}} - E_{t-s} A_s \right] = \lambda_1 I_{t+1} + \eta \beta \left[ E_{t-P_s} - E_{t-s} P_{s+1} \right]
\]

(20)

which is the first part of equation 7 in the text.
B. Derivation of the Linearized Current Account

On the consumer side, if utility is quadratic then it is know that:

$$\text{CA}_s = -\sum_{s=t+1}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} E_t [\Delta Y_s - \Delta I_s]$$  \hspace{1cm} (21)

To linearize output, note that:

$$Y_t = A_t K_t^\alpha \left[ 1 - \frac{g}{2} \left( \frac{I_t^2}{K_t} \right) \right]$$  \hspace{1cm} (22)

Let * denote deviations from the steady-state. Taking a first order linear approximation to output yields:

$$Y_t^* = \left[ -\frac{\alpha AK^\alpha}{K} g \left( \frac{T}{K} \right) \right] I_t^* + \left[ K^\alpha - \frac{\alpha AK^\alpha}{K} g \left( \frac{T}{K} \right) \right] A_t^* + \left[ \alpha \frac{AK^{\alpha-1}}{K} \right. + \left. \frac{\alpha AK^\alpha}{K} \right] \frac{g}{2} \left( \frac{T^2}{K} \right) K_t^*$$

In the steady state $I_t^* = K^\alpha = 0$ since I assume no depreciation, and $\alpha AK^{\alpha-1} = r$. This means that linearized output can be written as:

$$Y_t^* = rK_t^* + K^\alpha A_t^*$$  \hspace{1cm} (23)

or $\Delta Y_t = rI_{t-1} + K^\alpha \Delta I_t$

Finally, from equation 20 $E_t I_s = \lambda_{s-t} I_t \ \forall s \geq t+1$ since $E_t (A_s - A_{s-1}) = 0 \ \forall s \geq t+2$.

Combining expectations of future investment with the expression for linearized output into equation 21 yields:

$$\text{CA}_t = \frac{1}{1+r} \left[ \lambda_t I_t - I_t + rI_t \right] + \left( \frac{1}{1+r} \right)^2 \left[ \lambda_t^2 I_t - \lambda_t I_t + r\lambda_t I_t \right] + \left( \frac{1}{1+r} \right)^3 \left[ \lambda_t^3 I_t - \lambda_t^2 I_t + r\lambda_t^2 I_t \right] + \ldots = \frac{K^\alpha}{1+r} \left[ E_t A_{t+1} - A_t \right]$$  \hspace{1cm} (24)

which simplifies to:

$$\text{CA}_t = I_t \left[ \frac{1+r-\lambda_t}{1+r} \frac{1+r}{1+r-\lambda_t} \right] - \frac{K^\alpha}{1+r} \left[ E_t A_{t+1} - A_t \right]$$

$$= I_t - \gamma [E_t P_t - A_t], \ \text{where} \ \gamma = \frac{K^\alpha}{1+r}$$  \hspace{1cm} (27)

This completes the proof.
REFERENCES


