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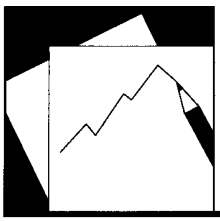

IMF Working Paper

Testing for Cointegration Using the Johansen Methodology when Variables are Near-Integrated

Erik Hjalmarsson and Pär Österholm

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Testing for Cointegration Using the Johansen Methodology when Variables are Near-Integrated

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Abstract

We investigate the properties of Johansen's (1988, 1991) maximum eigenvalue and trace tests for cointegration under the empirically relevant situation of near-integrated variables. Using Monte Carlo techniques, we show that in a system with near-integrated variables, the probability of reaching an erroneous conclusion regarding the cointegrating rank of the system is generally substantially higher than the nominal size. The risk of concluding that completely unrelated series are cointegrated is therefore non-negligible. The spurious rejection rate can be reduced by performing additional tests of restrictions on the cointegrating vector(s), although it is still substantially larger than the nominal size.

JEL Classification Numbers: C12, C15, C32
Keywords: Cointegration, Near-unit-roots, Spurious rejection, Monte Carlo simulations

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I. INTRODUCTION

Cointegration methods have been very popular tools in applied economic work since their introduction about twenty years ago. However, the strict unit-root assumption that these methods typically rely upon is often not easy to justify on economic or theoretical grounds. For instance, variables such as inflation, interest rates, real exchange rates and unemployment rates all appear to be highly persistent, and are frequently modelled as unit root processes. But, there is little *a priori* reason to believe that these variables have an exact unit root, rather than a root close to unity. In fact, these variables often show signs of mean reversion in long enough samples.[1] Since unit-root tests have very limited power to distinguish between a unit-root and a close alternative, the pure unit-root assumption is typically based on convenience rather than on strong theoretical or empirical facts. This has led many economists and econometricians to believe *near-integrated* processes, which explicitly allow for a small (unknown) deviation from the pure unit-root assumption, to be a more appropriate way to describe many economic time series; see, for example, Stock (1991), Cavanagh *et al*., (1995) and Elliott (1998).[2]

Near-integrated and integrated time series have implications for estimation and inference that are similar in many respects. For instance, spurious regressions are a problem when variables are near-integrated as well as integrated, and therefore, it is also relevant to discuss cointegration of near-integrated variables; see Phillips (1988) for an analytical discussion regarding these issues. Unfortunately, inferential procedures designed for data generated by unit-root processes tend not to be robust to deviations from the unit-root assumption. For instance, Elliott (1998) shows that large size distortions can occur when performing inference on the cointegration vector in a system where the individual variables follow near-unit-root processes rather than pure unit-root processes.

The purpose of this paper is to investigate the effect of deviations from the unit-root assumption on the determination of the cointegrating rank of the system using Johansen's (1988, 1991) maximum eigenvalue and trace tests. Unlike inference regarding the cointegrating vectors, this issue has not been investigated much in the literature. The first contribution of the current paper is therefore to document the rejection rates for standard tests of cointegration, using the Johansen framework, in a system where the variables are near-integrated. Through extensive Monte Carlo simulations, we show that the probability of

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[1] For studies relying on cointegration methods, see, for instance, Wallace and Warner (1993), Malley and Moutos (1996), Cardoso (1998), Bremnes *et al*. (2001), Jonsson (2001), Khamis and Leone (2001) and Bagchi *et al*. (2004). Studies arguing the stationarity of these variables include Song and Wu (1997, 1998), Taylor and Sarno (1998), Wu and Chen (2001) and Basher and Westerlund (2006).

[2] Phillips (1988) considers both processes that have roots smaller than unity ("*strongly autoregressive*") and larger than unity ("*mildly explosive*") in his analysis of near-integrated processes. In this paper, however, we only consider the empirically most relevant case of processes with roots less than unity.

reaching an erroneous conclusion regarding the cointegrating rank of the system is generally substantially higher than the nominal size. That is, the nominal size of the test can vastly understate the risks of finding a spurious relationship between unrelated near-integrated variables. In a simple bivariate system, the spurious rejection rate can approach 20 and 40 percent for the maximum eigenvalue and trace tests respectively, using a nominal size of five percent. Even higher rejection rates are found in a trivariate system. The second contribution is to show how a sequence of additional tests on the cointegrating vector(s) can help improve the performance of the tests and reduce the spurious rejection rate. However, even after taking these extra steps, the rejection rate of the test is still considerably larger than the nominal size. This is particularly true for the trivariate system where spurious rejection rates between 15 and 20 percent are documented for nominal five percent tests.

Overall, the performance of the trace test appears worse than that of the maximum eigenvalue test. Both tests, however, have large enough deviations from the nominal size that practitioners should be aware of the problems associated with Johansen's procedures under these circumstances. The proposed sequence of additional tests helps alleviate some of the sensitivity of the Johansen procedures to deviations from the strict unit-root assumption. They do not, however, eliminate the problem.

The remainder of this paper is organised as follows: Section II gives a brief introduction to Johansen's methodology and Section III presents the Monte Carlo study. In Section IV, we present an empirical illustration of the problems associated with near-integrated variables using U.S. data on CPI inflation and the short nominal interest rate. Section V concludes.

II. TESTING FOR COINTEGRATION USING JOHANSEN'S METHODOLOGY

Johansen's methodology takes its starting point in the vector autoregression (VAR) of order *p* given by

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\mathbf{y}_t$ is an *n*x1 vector of variables that are integrated of order one – commonly denoted I(1) – and $\boldsymbol{\varepsilon}_t$ is an *n*x1 vector of innovations. This VAR can be re-written as

$$\Delta \mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\Pi} \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t \tag{2}$$

where

$$\boldsymbol{\Pi} = \sum_{i=1}^{p} \mathbf{A}_i - \mathbf{I} \text{ and } \boldsymbol{\Gamma}_i = -\sum_{j=i+1}^{p} \mathbf{A}_j \,. \tag{3}$$

If the coefficient matrix $\mathbf{\Pi}$ has reduced rank $r<n$, then there exist nxr matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ each with rank r such that $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$ and $\boldsymbol{\beta}'\mathbf{y}_t$ is stationary. r is the number of cointegrating relationships, the elements of $\boldsymbol{\alpha}$ are known as the adjustment parameters in the vector error correction model and each column of $\boldsymbol{\beta}$ is a cointegrating vector. It can be shown that for a given r, the maximum likelihood estimator of $\boldsymbol{\beta}$ defines the combination of $\mathbf{y}_{t-1}$ that yields the r largest canonical correlations of $\Delta\mathbf{y}_t$ with $\mathbf{y}_{t-1}$ after correcting for lagged differences and deterministic variables when present.[3] Johansen proposes two different likelihood ratio tests of the significance of these canonical correlations and thereby the reduced rank of the $\mathbf{\Pi}$ matrix: the trace test and maximum eigenvalue test, shown in equations (4) and (5) respectively.

$$J_{trace} = -T\sum_{i=r+1}^{n}\ln\left(1-\hat{\lambda}_i\right) \tag{4}$$

$$J_{max} = -T\ln\left(1-\hat{\lambda}_{r+1}\right) \tag{5}$$

Here T is the sample size and $\hat{\lambda}_i$ is the i:th largest canonical correlation. The trace test tests the null hypothesis of r cointegrating vectors against the alternative hypothesis of n cointegrating vectors. The maximum eigenvalue test, on the other hand, tests the null hypothesis of r cointegrating vectors against the alternative hypothesis of $r+1$ cointegrating vectors. Neither of these test statistics follows a chi square distribution in general; asymptotic critical values can be found in Johansen and Juselius (1990) and are also given by most econometric software packages. Since the critical values used for the maximum eigenvalue and trace test statistics are based on a pure unit-root assumption, they will no longer be correct when the variables in the system are near-unit-root processes.[4] Thus, the real question is *how* sensitive Johansen's procedures are to deviations from the pure-unit root assumption.

Although Johansen's methodology is typically used in a setting where all variables in the system are I(1), having stationary variables in the system is theoretically not an issue and Johansen (1995) states that there is little need to pre-test the variables in the system to establish their order of integration. If a single variable is I(0) instead of I(1), this will reveal itself through a cointegrating vector whose space is spanned by the only stationary variable in

[3] For a detailed description of the procedure, see, for example, Johansen (1995).

[4] Based on previous studies – see, for example, Elliott, 1998 – it is no far stretch to conjecture that the Brownian motions in the limiting distribution given in, for instance, Johansen (1988) equation (18) would simply be replaced by the corresponding Ornstein-Uhlenbeck process to which near-unit-root variables converge. As always with near-unit-root variables, the problem is that the local-to-unity parameter is unknown and thus also the percentiles of the limiting distribution.

the model. For instance, if the system in equation (2) describes a model in which $\mathbf{y}_t = \begin{pmatrix} y_{1,t} & y_{2,t} \end{pmatrix}'$ where $y_{1,t}$ is I(1) and $y_{2,t}$ is I(0), one should expect to find that there is one cointegrating vector in the system which is given by $\boldsymbol{\beta} = \begin{pmatrix} 0 & 1 \end{pmatrix}'$. In the case where $\boldsymbol{\Pi}$ has full rank, all *n* variables in the system are stationary.[5]

The fact that stationary variables in a system will introduce restricted cointegrating vectors is something that should be kept in mind in empirical work. That is, it is good econometric practice to always include tests on the cointegrating vectors to establish whether relevant restrictions are rejected or not. If such restrictions are not tested, a non-zero cointegrating rank might mistakenly be taken as evidence in favour of cointegration between variables. This is particularly relevant when there are strong prior opinions regarding which variables "have to" be in the cointegrating relationship. An obvious example is the literature on real exchange rates, where cointegration techniques are very common. After finding support for a cointegrating vector in a system, it is almost always the case that the coefficient on the real exchange rate is normalized to one, thereby forcing it to be part of the cointegrating relationship. However, tests of whether all other coefficients in the cointegrating vector are zero are rarely performed. Even rarer are tests of whether the only cointegrating vector is due to the stationarity of some other variable in the system, despite the fact that the proposed determinants of real exchange rates in many cases can be argued to be stationary.

The lack of need to *a priori* distinguish between I(1) and I(0) variables is based on the assumption that any variable that is not I(1), or a pure unit-root process, is a stationary I(0) process. This apparent flexibility, therefore, does not make the method robust to near-integrated variables, since they fall into neither of these two classifications. However, the above specification tests of the cointegrating vector suggest a way of making inference more robust in the potential presence of near-unit-root variables. For instance, considering the bivariate case described above, explicitly testing whether $\boldsymbol{\beta} = \begin{pmatrix} 0 & 1 \end{pmatrix}'$ will help to rule out spurious relationships that are not rejected by the initial maximum eigenvalue or trace test.[6] Although we argue that such specification tests should be performed in almost every kind of

[5] This means that the Johansen test can be used as a panel unit root test as suggested by Taylor and Sarno (1998) and Österholm (2004).

[6] One way of viewing tests of such restictions is as unit-root tests within the VAR. Thus, if the first stage rank test is a form of overall panel test of the unit-root assumption in the data, the tests on the cointegrating vector act as supplementary unit-root tests in the cases where either a full set of unit-roots is not found (i.e. $r = 0$) or where stationarity of the entire system (i.e. $r = n$) is not found.

application, they are likely to be extra useful in cases where the variables are likely to have near-unit-roots and the initial test of cointegration rank is biased.[7]

III. MONTE CARLO STUDY

A. Setup

The data generating process (DGP) for the nx1 vector $\mathbf{y}_t$ is given by

$$\mathbf{y}_t = \mathbf{I}\left(1+\frac{c}{T}\right)\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{6}$$

where c is the local-to-unity parameter that, for simplicity, is assumed to be common to all variables, $\mathbf{I}$ is the nxn identity matrix, and $\boldsymbol{\varepsilon}_t$ is an nx1 vector of normally distributed *iid* disturbances such that $E(\boldsymbol{\varepsilon}_t) = \mathbf{0}$ and $E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \mathbf{I}$. We investigate the spurious rejection frequency of the Johansen maximum eigenvalue and trace tests for systems of size $n = (2 \quad 3)$ and set the sample size to $T = (100 \quad 250 \quad 500)$, which covers most empirically relevant cases. For all combinations of n and T, we let c take on values between 0 and -60.[8] The nominal size of all tests is set to five percent.

We estimate the VAR in equation (2). Given the DGP in equation (6), lag length in the VAR is set to the correct value of $p-1=0$. Furthermore, we use the empirically most common specification, which allows for a constant in the cointegrating relationship but no deterministic trend in the data. For notational convenience, the constant term will be suppressed in the following analysis.

Since the variables in the system are completely unrelated, the frequency with which evidence of a cointegrating relationship is found should ideally be equal to the nominal size. However, rejection of the null hypothesis, $H_0 : r = 0$, does not automatically lead to the false conclusion that there is cointegration between the variables in the system. In the *bivariate*

[7] It should be stressed that specification tests on the cointegrating vector are also biased when the variables have near-unit-roots; see Elliott (1998). This may potentially reduce the usefulness of these additional specification tests but does not invalidate them as robustness checks.

[8] This range for c covers most of the plausible values documented in the literature; see, for example, Stock (1991) and Campbell and Yogo (2006).

case, rejecting $H_0 : r = 0$ will not lead to a rejection of the null hypothesis of no cointegration if:

a) $H_0 : r = 1$ is also rejected. For the DGP considered above, this implies that both variables are stationary as the matrix $\mathbf{\Pi}$ has full rank.

b) $H_0 : r = 1$ cannot be rejected but the restriction *i)* $\boldsymbol{\beta}' = \begin{pmatrix} 1 & 0 \end{pmatrix}$ or *ii)* $\boldsymbol{\beta}' = \begin{pmatrix} 0 & 1 \end{pmatrix}$ cannot be rejected either. In either of these cases, we would conclude that there is no cointegration between y_{1t} and y_{2t}. If the restriction in *i)* is judged valid, the conclusion is that y_{1t} is stationary and that it does not have a long-run relationship with y_{2t}. If the restriction in *ii)* is instead judged valid, the conclusion drawn would be symmetric.

In the *trivariate* case, rejecting $H_0 : r = 0$ will not lead to a rejection of the null hypothesis of no cointegration if:

c) $H_0 : r = 1$ and $H_0 : r = 2$ are also rejected. For the DGP considered above, this implies that all three variables are stationary as the matrix $\mathbf{\Pi}$ has full rank.

d) $H_0 : r = 1$ cannot be rejected but the restriction *iii)* $\boldsymbol{\beta}' = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$, *iv)* $\boldsymbol{\beta}' = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$ or *v)* $\boldsymbol{\beta}' = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$ also cannot be rejected. Similar to the bivariate case *b)*, we would conclude that the only cointegrating vector in the system is due to a stationary variable rather than cointegration between variables.

e) $H_0 : r = 1$ is rejected but $H_0 : r = 2$ is not, at the same time as the restrictions *vi)* $\boldsymbol{\beta}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, *vii)* $\boldsymbol{\beta}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ or *viii)* $\boldsymbol{\beta}' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ cannot be rejected. Just like in *b)* and *d)*, we would conclude that there is no cointegration between variables and that the cointegrating vectors are due to stationary variables.

The interpretation of the restrictions on the cointegrating vector offered above—that variables may be integrated of different orders—is clearly not strictly correct since we know that all variables are near-integrated with the same local-to-unity parameter. However, it is the interpretation that an applied researcher, working within the implicit assumptions of the Johansen framework, would draw. Finally, it should be pointed out that the above testing scheme raises some concerns regarding the properties of the tests under the alternative of

cointegration. In particular, when the matrix $\mathbf{\Pi}$ is found to have full rank—and all n variables in the system accordingly are judged stationary—the ability to actually detect cointegration among stationary near-integrated variables is limited. Although outside the scope of this paper, such issues clearly need to be addressed in a formal extension of the Johansen framework to near-integrated variables.

B. Results

Figures 1 and 2 show the spurious rejection frequencies for the bivariate and trivariate systems respectively. The left columns in both figures show the spurious rejection frequencies when the cointegrating rank of the system alone is taken as evidence of cointegration between variables. This is simply when we conclude that $r = 1$ in the bivariate case and either $r = 1$ or $r = 2$ in the trivariate case. Recall that $r = 0$ or $r = n$ both imply that a correct conclusion has been drawn since the variables in the systems here are completely unrelated. In the right column, on the other hand, the additional tests in *b)*, *d)* or *e)* are also conducted. This means that the correct conclusion of no cointegration between variables can be drawn also for $r = 1$ in the bivariate case and for $r = 1$ or $r = 2$ in the trivariate case and not only for $r = 0$ or $r = n$.

Considering the bivariate system in Figure 1, it is clear from the left column that if one relies exclusively on the estimated rank of the system for inference, there is a large risk of spuriously concluding that completely unrelated variables are cointegrated. When c is small in absolute value, the rejection frequency is close to the nominal size. However, it is evident already for $c = -10$ that the tests are severely over rejecting; in particular, the trace test has very poor properties with a spurious rejection frequency of approximately 18 percent. The problem reaches a peak for a value of $c = -17$, where the maximum eigenvalue and trace tests reach spurious rejection frequencies of approximately 21 and 38 percent respectively, regardless of sample size. As c becomes even larger in absolute value, the rejection frequency falls and approaches zero for $c = -40$. The reason for this is that both the maximum eigenvalue and trace test correctly conclude that $r = 2$; that is, that both variables are stationary. The top row of Figure A1 in the Appendix further illustrates this phenomenon by showing the results for the individual rank tests in the case of $T = 500$. Turning to the right column in Figure 1, it can be seen that if tests of $\boldsymbol{\beta}' = \begin{pmatrix} 1 & 0 \end{pmatrix}$ and $\boldsymbol{\beta}' = \begin{pmatrix} 0 & 1 \end{pmatrix}$ are conducted, after failing to reject that $r = 1$, the spurious rejection frequency falls dramatically for both tests. However, while the problem is alleviated, it is still concluded that there is a cointegrating relationship around ten percent of the time when c is in the neighbourhood of -17; this is the case regardless of the test used.

Figure 1. Spurious Rejection Frequency for Bivariate System

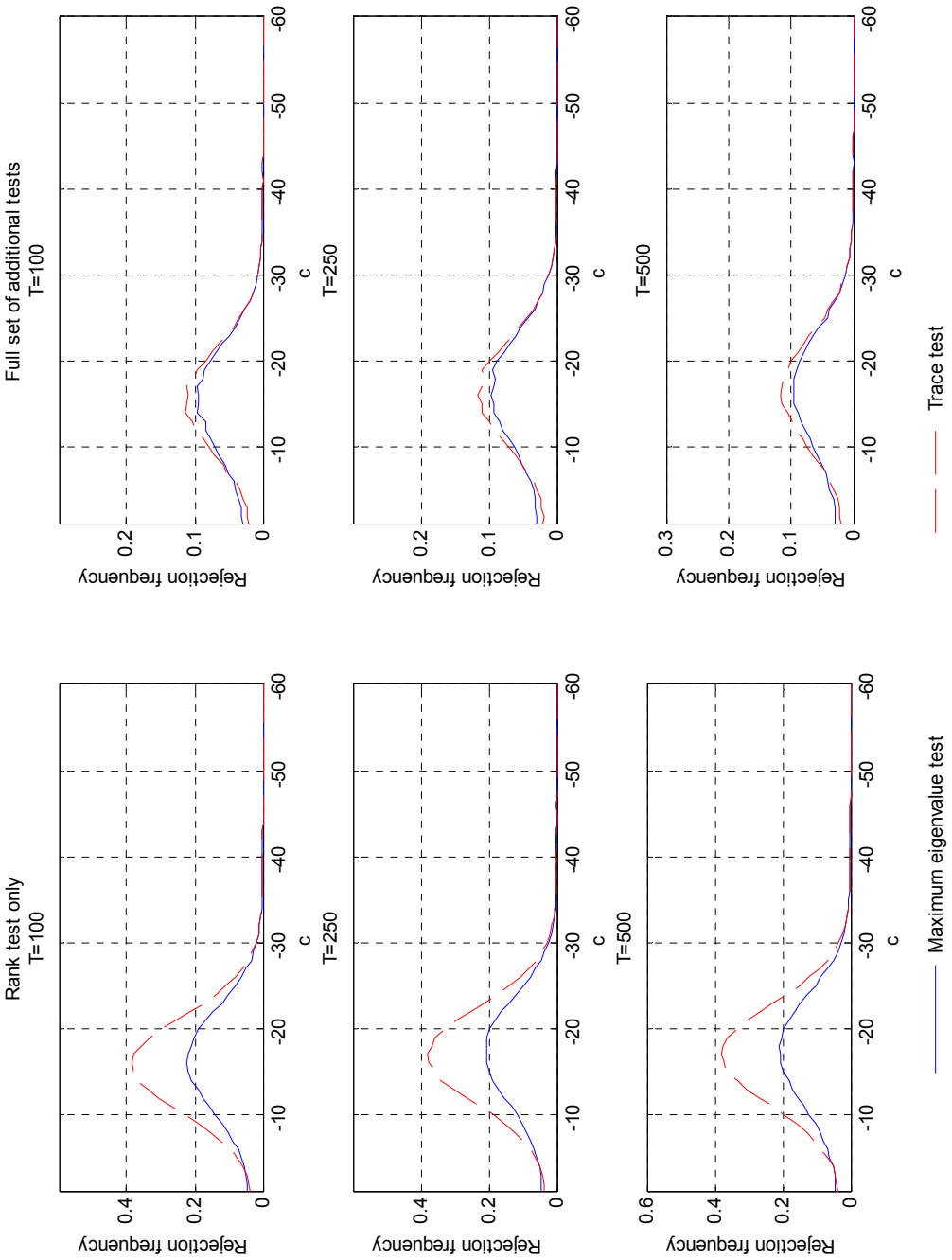
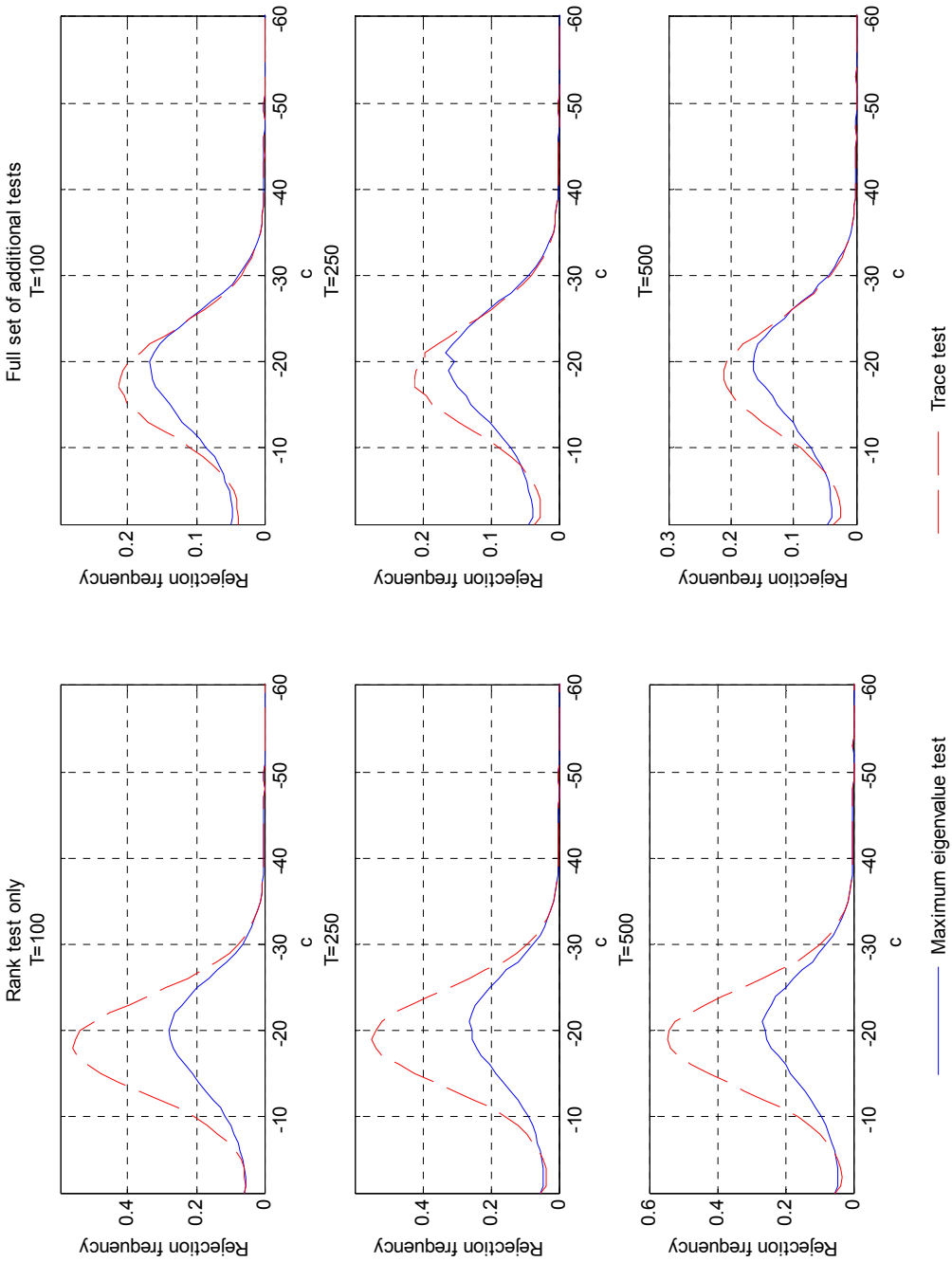


Figure 2. Spurious Rejection Frequency for Trivariate System



The results for the trivariate system shown in Figure 2 are qualitatively very similar to those from the bivariate system, but the problem of spurious rejection is quantitatively worse for the larger system. There is a large interval of values for *c*, for which both the maximum eigenvalue and trace tests have very high spurious rejection frequencies, regardless of whether we look solely at the rank (left column) or conduct the additional tests after determining the rank (right column). For a *c* of approximately -18 to -20, the rejection frequency is at its highest. Even if the additional tests on the cointegrating vectors are conducted, the maximum eigenvalue and trace tests have unacceptably high rejection rates: 16 and 21 percent respectively regardless of sample size. Finally, for $c = -40$ or smaller, the spurious rejection frequency is virtually zero as both tests always conclude that the rank of $\mathbf{\Pi}$ is equal to three. This is again further illustrated in the bottom row of Figure A1 in the Appendix.

Summing up, neither the maximum eigenvalue nor the trace test is reliable in terms of assessing whether variables are cointegrated when the data do not have exact unit roots. For reasonable values of *c*, the spurious rejection frequency can be several times higher than the nominal size.

IV. AN EMPIRICAL ILLUSTRATION

We next turn to an empirical application where it can be argued that the DGP underlying the series is potentially near-integrated. Given the high persistence of nominal interest rates and inflation in many countries, a popular approach to test the Fisher hypothesis in more recent years has been to employ cointegration techniques; see, for example, MacDonald and Murphy (1989), Wallace and Warner (1993), Crowder and Hoffman (1996) and Junttila (2001). This makes sense to some extent as it has been pointed out that the Fisher hypothesis is better interpreted as a long-run equilibrium condition (Summers, 1983). However, much research has questioned the implicit or explicit assumption in these papers that inflation and the nominal interest rate are I(1); see, for example, Wu and Zhang (1996), Culver and Papell (1997), Lee and Wu (2001), Wu and Chen (2001) and Basher and Westerlund (2006). The existence of exact unit-roots in either inflation or nominal interest rates is thus far from certain, and it is interesting to revisit the question of cointegration between them in the light of the above Monte Carlo study.

We use monthly data on US nominal interest rate, denoted i_t, and CPI inflation, π_t, from January 1974 to October 2006. Data were provided by the Board of Governors of the Federal Reserve System and are shown in Figure 3 below.

Table 1 shows the results from the Augmented Dickey-Fuller (Said and Dickey, 1984) unit root test, where lag length has been established using the Akaike (1974) information criterion. As can be seen, the null hypothesis of a unit root cannot be rejected for either variable. In addition, Table 1 shows the 95% confidence intervals for the local-to-unity

parameter c and the corresponding autoregressive root $\rho=1+c/T$, for each of the variables. These are obtained by inverting the ADF test statistic as described in Stock (1991). The range of possible values for c clearly covers the values for which the largest spurious rejection rates were recorded in the Monte Carlo study.

Figure 3. Data

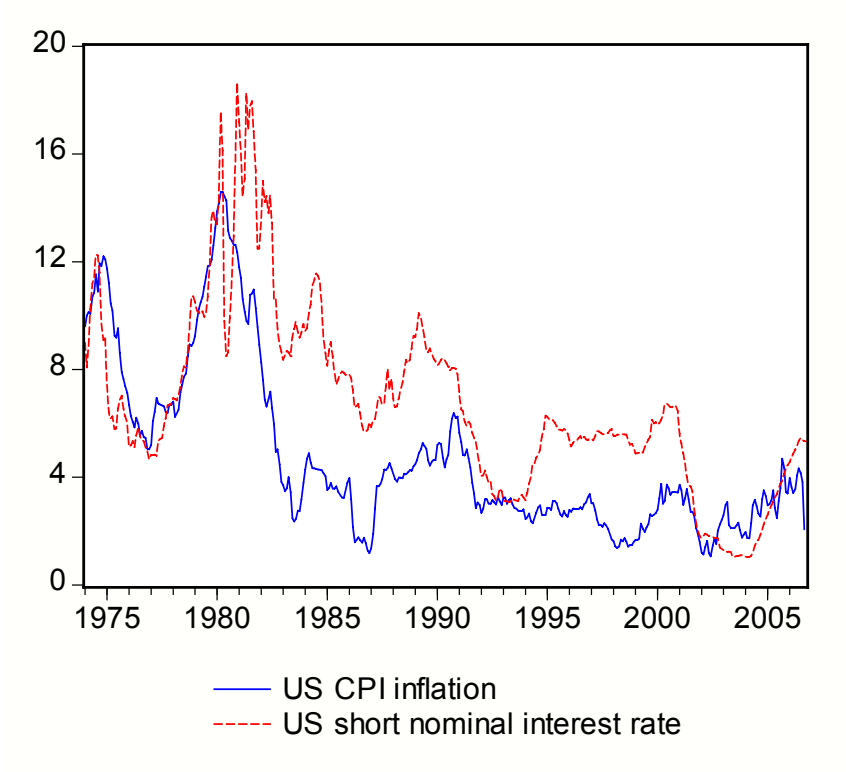


Table 1. Results from Augmented Dickey-Fuller Test

	τ_{ADF}	95% confidence interval for c	95% confidence interval for ρ
π_t	-2.271 (0.182)	[-18.019, 2.776]	[0.954, 1.007]
i_t	-2.722 (0.071)	[-24.144, 1.680]	[0.939, 1.004]

Note: p-value in parentheses ().

Next, we turn to the issue of determining the cointegrating rank of the system, which is done by estimating equation (2) with $\mathbf{y}_t = (\pi_t \quad i_t)'$. Lag length is set to $p-1=10$ based on the Akaike information criterion and the constant is restricted to allow for an intercept in the cointegrating relationship but no deterministic trend in the data. Table 2 shows the results from the cointegration tests. Both tests reject the null of zero cointegrating vectors. The

hypothesis that there is one cointegrating vector cannot be rejected on the other hand; that is, based on the cointegration test, there is no support for both variables in the system being stationary. Based solely on the evidence in Table 2, we would conclude that there exists a cointegrating relationship.

Table 2. Results from Cointegration Test

Null hypothesis	J_{trace}	J_{max}
$r = 0$	22.045	16.402
	(0.028)	(0.042)
$r = 1$	5.642	5.642
	(0.220)	(0.220)

Note: p-value in parentheses ().

Table 3. Results from Hypothesis Tests on the Cointegrating Vector

Restriction	Test statistic
$\boldsymbol{\beta}' = (0 \quad 1)$	6.911
	(0.009)
$\boldsymbol{\beta}' = (1 \quad 0)$	0.391
	(0.532)

Note: p-value in parentheses ().

Typically, finding that the rank of $\boldsymbol{\Pi}$ is one in the system above is taken as evidence for cointegration between the nominal interest rate and inflation. Following good econometric practice, we should, however, also test whether the cointegrating vector satisfies either the restriction $\boldsymbol{\beta}' = (0 \quad 1)$ or $\boldsymbol{\beta}' = (1 \quad 0)$. As shown in the Monte Carlo study above, these additional tests also substantially reduce the risk of spuriously concluding that near-integrated variables are cointegrated. The results are given in Table 3 and, as can be seen, $\boldsymbol{\beta}' = (0 \quad 1)$ is rejected whereas the restriction $\boldsymbol{\beta}' = (1 \quad 0)$ is not. Our conclusion is hence that the above finding of a cointegrating vector does not lend support for cointegration between the nominal interest rate and inflation. Instead, based on conducted tests, the empirical evidence points to the nominal interest rate and inflation being integrated of different orders. In such a case, no long-run equilibrium relationship can exist between the two.[9]

[9] Stationary inflation but integrated nominal interest rate is consistent with a unit root in the real interest rate. Support for a unit root in the real interest rate can be found in, for example, Rose (1988).

V. CONCLUSION

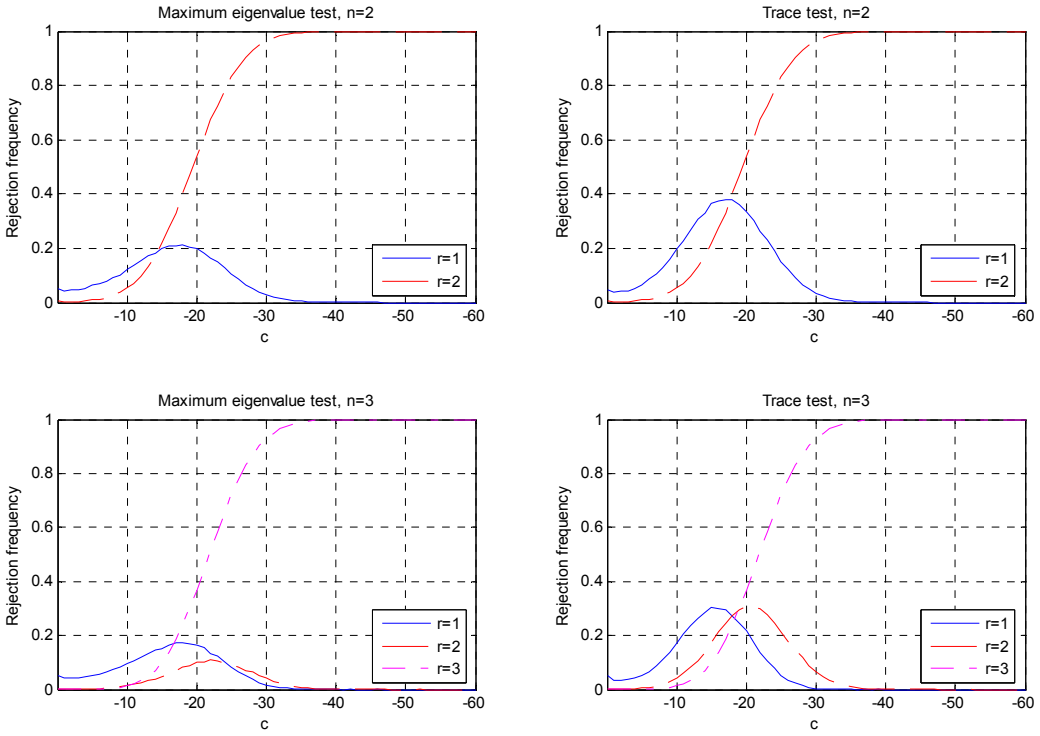
This paper has investigated the properties of Johansen's maximum eigenvalue and trace tests for cointegration under the empirically relevant situation of near-integrated variables. Overall, the results show that there is a substantial probability, much larger than the nominal size of the test, of falsely concluding that completely unrelated series are cointegrated. We find that a systematic check of additional tests on the cointegrating vector(s)—based on Johansen's claim that there is little need to pre-test variables for unit roots—helps reduce the spurious rejection frequency. However, the spurious rejection frequency remains large and appears to increase with the number of variables in the system, even after applying such specification tests.

The results are obtained in a Monte Carlo simulation under perfect circumstances. That is, the data are normally distributed and the lag-length in the VAR in levels is known and equal to one. In practice, we do not have the benefit of being given the correct model—neither in terms of the variables in the system nor the lag length—and the problems shown in this paper are likely to be exacerbated.

The findings in this paper further illustrate the sensitivity of cointegration methods to deviations from the pure unit-root assumption, as originally noted by Elliott (1998) in regards to inference on the cointegrating vectors. Since unit-root tests cannot easily distinguish between a unit root and close alternatives, this raises a precautionary note to the interpretation of results from cointegration studies. In particular, it raises questions regarding the conclusions drawn in previous studies that have relied on cointegrating methods despite having found evidence of stationarity of the included variables; see, for example, Crowder and Hoffman (1996) and Granville and Mallick (2004). One way of making the Johansen procedure more robust to near-unit-roots may be through a Bonferroni type bounds procedure as proposed by Cavanagh *et al*. (1995) for inference on the cointegrating vector and by Hjalmarsson and Österholm (2007) for residual-based tests of cointegration.

APPENDIX

Figure A1. Frequency with which the Rank Tests Conclude that the Cointegrating Rank is r



Note: Sample size is T=500. Nominal size is 5%.

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