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Is the Quantity of Government Debt a Constraint for Monetary Policy?

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Monetary and Financial Systems Department

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Abstract

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This paper derives an interest rate rule for monetary policy in which the interest rate response of the central bank toward an increase in expected inflation falls as debts increase beyond a certain threshold level. A debt-constrained interest rate rule and the threshold level of debt are jointly estimated for Canada during the first decade of its inflation targeting regime of the 1990s. There are three main findings of this paper. First, a high government debt could constrain monetary policy if government spending—rather than taxes—is expected to adjust in future in line with debt service costs. The ‘constraint’ operates through an altered policy transmission mechanism through changes in the IS curve. Second, the effects of the debt-constraint on monetary policy are quite different during booms and recessions. Third, empirical estimates show that Canadian monetary policy might have been constrained by a high government debt-GDP ratio during the 1990s. Policy was less loose than what inflation indicators called for.

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Contents	Page
I. Introduction	3
II. Existing Literature on the Links between Debt and Monetary Policy	4
III. A Possible Motivation for the Empirics.....	5
IV. Joint Estimation of the Threshold Level of Debt and Interest Rate Rule—the Case of Canada.....	12
Empirical model.....	12
Data	15
Estimation results.....	16
V. Summary and Conclusions.....	17
Appendices	
I. Derivation of Optimal Interest Rate Rule Under Discretionary Policy.....	19
References.....	21
Figures	
1. Movements in Some Key Variables in Canada Monthly Data, 1991:11 to 2000:12.....	23
2. J-Statistics for Choosing the Threshold level of Debt.....	24
Tables	
1. GMM Estimates(i) of Parameters of Baseline and Debt-constrained Models, Canada, Monthly Data, 1991.11 to 2000.12	25

I. INTRODUCTION

In response to anecdotal evidence on a potential constraint posed by high government debt levels on monetary policy, this paper extends an existing model to formalize and test for such a constraint. Monetary policy rules of the type introduced in Taylor (1993) typically specify movements in the target short-term nominal interest rate in response to changes in expected inflation and the output gap. Almost all work in this area discuss these rules in the context of industrialized countries, particularly the U.S., Canada, U.K., Sweden, Australia, New Zealand among others; countries where central banks take decisions independent of those of fiscal authorities. The motivation for this paper comes from the fact that monetary authorities have sometimes expressed concern about high levels of government debt. An example of such a concern can be found in the following quote from Bernanke, Mishkin et al (1999), pg.137: “*In the Bank of Canada’s Annual Report, 1993 (released in March 1994), ...[the then Governor]..., called for the reduction of government debt in order to take pressure off interest rates and exchange rates.*” This quote coincided with a rising level of Canadian federal debt to above 50 percent of GDP along with a low CPI inflation rate. However, papers estimating interest rate rules do not account for “pressure” posed by high debt situations.

The goal of this paper is, therefore, to answer the following question: *In an interest rate setting regime, how does the quantity of government debt affect the level of interest rate consistent with achieving the central bank’s monetary policy objectives?* Relatedly, to what extent is the speed of adjustment to the targeted inflation rate affected if the government debt (relative to GDP) is higher than a certain threshold level. This paper derives an augmented interest rate rule that explicitly takes into account the ‘constraint’ posed by a high debt level. The paper modifies an assumption of an existing model (Clarida, Gali and Gertler, 1999) to incorporate ‘pressure’ due to a high government debt in an inflation-targeting monetary policy regime. It uses the experience of Canada during its inflation-targeting regime of the 1990s to estimate a threshold level of debt and test for debt-constraints.

There are three main findings of this paper. First, a high government debt level could constrain monetary policy if government spending—rather than taxes—is expected to adjust in the future in line with debt service costs. The ‘constraint’ operates through the changes in the policy transmission mechanism that change the nature of the IS curve. Second, the effects of the debt-constraint on monetary policy are quite different during booms and recessions. The constraint is larger during a recession. Third, empirical estimates show that Canadian monetary policy was constrained by a high government debt-GDP ratio during the 1990s. Policy was less loose than what inflation indicators called for.

To this end, the existing literature on the links between debt and monetary policy is discussed in the next section. A motivation for the empirical exercise is presented in Section III, which derives a generalized interest-rate rule taking into account debt-constraints. The empirical model with its results for Canada is discussed in Section IV, while Section V summarizes and concludes. All algebraic details are in the Appendix.

II. EXISTING LITERATURE ON THE LINKS BETWEEN DEBT AND MONETARY POLICY

One direct link between debt and monetary policy is seigniorage; a high debt level could lead to monetization of debt that in turn leads to higher inflation. For example, in post war Germany, hyperinflation was used to wipe out the debt. For long-term debt, a surprise-inflation would erode away its real value. Rolling over of short-term debt, on the other hand, would involve higher interest rates (Dornbusch, 1998); but if inflation expectations were perfect, this higher rate would offset the inflationary erosion. For the United States, Dornbusch has shown that a percentage point increase in the interest rate amounts to roughly \$30 billion in extra deficit, or half a percentage point of GDP. There was a long period—mid-1930s to mid-1950s—when bond-yields were kept at or below 2.5 percent to accommodate high debts by pegged, low interest rates. Thus interest rate provides a more useful link between debt and monetary policy, especially in the highly industrial countries where seigniorage accounts for a negligible portion of budgetary finance.

Sargent and Wallace (1981) raised the issue of the monetary policy implications of high government debts, where the real interest rate is above the growth rate of the economy. An effort to reduce inflation by reducing money growth would raise the debt-GDP ratio as bond financing replaced monetary financing. This in turn raises interest payments that raise future budget deficits, leading to higher money growth and inflation in the long run.

It has been pointed out by Goodhart (1999) that the importance of the link between debt and monetary policy has been reduced to second-order level, if at all, in countries like the U.S. and the U.K. However, the question of whether high and rising debt has become an important constraint for policy by making the monetary authorities accept more inflation than they otherwise would (Dornbusch, 1998) or by changing the transmission mechanism of monetary policy, has received little empirical investigation. The European Central Bank (ECB) represents an extreme case where monetary policy is conducted independently of fiscal policies of the participating nations. Yet, high debts (beyond 60 percent of GDP as specified by the Stability and Growth Pact) accumulated by the member nations' governments still clearly represent a risk for the ECB.

In the fiscal theory of price level determination (Leeper, 1989, Sims 1994, Woodford 1998, 2001), if the real primary surplus is determined exogenously, the quantity of debt could affect the price level through a wealth effect upon private consumption. A non-Ricardian tax cut—a tax cut unaccompanied by expectations of future tax increases—makes agents increase their consumption creating excess demand in the market. This excess demand drives up prices until a fall in real wealth—partly comprised of real debt holdings—reduces this excess demand. The maturity structure of government debt would affect monetary conditions too; in particular, the shorter the maturity, larger is the inflation required to reduce the value of the public debt enough to restore equilibrium following an expansionary fiscal shock.

The literature closest to the subject of this paper is the analysis of the distinction between 'active' and 'passive' monetary and fiscal policies depending on the responsiveness to government debt shocks (Leeper, 1991). Monetary policy is active and fiscal policy passive if future direct lump-sum taxes back debt shocks entirely. Fiscal policy is active and

monetary policy is passive if fluctuations in real debt generate current or future money creation—thus deficit shocks increase inflation now or in the future. Prices, under the second set of policies, depend on the aggregate supply of government liabilities and nominal interest rates depend on the ratio of money to debt. Leeper shifts the focus away from average towards marginal sources of fiscal revenues—even when, on average, government debt is backed entirely by direct taxes, innovations to debt may be systematically monetized.

This paper looks at how a forward-looking interest rate rule can be generalized to incorporate debt as a constraint on the monetary authorities' ability to use the short-term interest rate as an instrument for monetary policy. The paper assumes that when government debt reaches a certain level, the fiscal authorities are expected to reduce government spending in the future rather than assume direct taxes to back debt shocks. There is no seigniorage revenues to depend on at any time. An econometric model based on these implications is formulated to estimate the threshold level of debt beyond which it acts as a constraint for policy. The example of Canada in the early 1990s has been used to demonstrate the application of the econometric exercise to estimate a threshold level of debt during its inflation-targeting regime.

III. A POSSIBLE MOTIVATION FOR THE EMPIRICS

The model is an extension of an optimization-based closed economy framework by Clarida, Gali and Gertler (1999) (henceforth, CGG) who had derived optimal interest rate rules under discretion and commitment. Their two-step procedure of deriving the optimal interest rate rule under discretionary policy is adopted here. First, a baseline rule is derived by minimizing the central bank's loss function subject to the expectation IS and Aggregate Supply functions. Next, the rule is re-derived for the case when a debt-constraint operates—the IS equation is modified to incorporate debt in the analysis. For this purpose, the assumption regarding the path of expected changes in government spending is altered.

Monetary policy is active and fiscal policy is passive if direct lump sum (net) taxes back debt shocks entirely (Leeper, 1991). The monetary authority is not constrained by current budgetary conditions; it is free to choose a decision rule that depends on past, present or future expected variables influencing its target variables. The forward-looking version of the Taylor rule proposed by CGG (1999) had assumed this policy-coordination scheme. In their model, fiscal behavior supported the prevailing monetary policy by raising net taxes enough to prevent explosive debt paths. However, a reason for concern by the central bank about a very high debt level at certain times could be the unresponsiveness of net taxes to debt beyond a certain debt level. Given that there is no role for seigniorage revenues, government spending is expected to be curtailed in proportion with debt service costs. This is when the monetary transmission mechanism or the speed of adjustment is affected by changing the interest-elasticity of the output gap—the slope of the IS curve changes. In terms of a forward-looking interest rate rule, the responsiveness of the interest rate to expected inflation is different under a debt-constraint. The interest rate rule derived in this paper intends to capture such debt-constraints.

A standard optimization problem for central banks operating under inflation targeting is laid out in Box 1 using the model in CGG. Here the target for inflation is taken as zero; the

analysis will not change qualitatively if, as in countries like Canada and the U.K., the target is different from zero or is stated as a range rather than as a point. It should be noted that in some of the inflation-targeting countries, like Canada and New Zealand, the inflation target is jointly determined and announced by both the government and the central bank (Bernanke, Laubach, et al, 1999); however, it is often the central bank that is responsible for meeting the target. An independent central bank will not target any fiscal policy variable, as the latter is the sole concern of the fiscal authorities.

The autoregressive disturbance term, ε_t , of the IS equation is crucial to this analysis and is a function of expected changes in government consumption relative to expected changes in potential output. Government consumption is assumed to evolve exogenously under an active monetary policy regime.

Box 1. The optimal interest rate rule under discretionary monetary policy (Clarida, Gali, Gertler, 1999)

The objective function of central banks is characterized by a quadratic loss function increasing in, both present and future expected, inflation and output variability. The central bank minimizes this loss function over the target variables, inflation and output gap, subject to the state of the economy. Under discretionary policy, the problem to be solved is the same each period; the monetary authorities make no attempt to manipulate private sector beliefs. This objective function can be written as follows:

$$(1) \quad \max -\frac{1}{2}[\alpha x_t^2 + \pi_t^2]$$

This function penalizes for deviations of (log of) output, y , from its trend, z , and for deviations of inflation, π , from its target level. Here the target for inflation is taken as zero for simplification. The variable $x \equiv y - z$, represents the output gap. The parameter α represents the relative weight put on output in the objective function—the weight on inflation is normalized to 1.

The state of the economy is summarized by the following set of equations:

$$(2) \quad x_t = -\varphi[i_t - E_t \pi_{t+1}] + E_t x_{t+1} + \varepsilon_t$$

$$(3) \quad \pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t$$

$$\varepsilon_t = \mu \varepsilon_{t-1} + \hat{\varepsilon}_t$$

$$u_t = \rho u_{t-1} + \hat{u}_t$$

$$\hat{\varepsilon}_t \sim i.i.d.(0, \sigma_\varepsilon^2), \hat{u}_t \sim i.i.d.(0, \sigma_u^2), |\mu| < 1, |\rho| < 1$$

Equation (2) is an expectational IS curve relating the present output gap, x_t , to the expected real interest rate and future expected output gap. As with traditional Keynesian analysis, there is a negative relationship between the real interest rate and the output gap. Consumption smoothing objectives also lead individuals to respond to an increase in future expected output by raising current consumption and, hence, the current output gap. This equation can be derived from a consumption Euler equation resulting from a utility maximization exercise of a representative agent under standard assumptions as in CGG.

Equation (3) is a Phillips curve assuming Calvo (1983) pricing—each of the monopolistically competitive firms chooses its nominal price to maximize profits subject to constraints on the frequency of future price adjustments; equation (3) is derived by aggregating individual firm pricing decisions. Current inflation depends upon both current and future expected output and inflation—inflation is a result of increases in marginal costs partly due to variations in excess demand and partly arising from “cost push” factors, u_t ; this disturbance is also an autoregressive error term. Both \hat{u}_t and $\hat{\varepsilon}_t$ are i.i.d. disturbances with zero-mean and constant variances.

CGG uses a two-step procedure to solve this optimization problem. In the first step, objective function (1) is maximized at time t with respect to π_t and x_t subject to the Phillips Curve (PC) equation (3). In the second step, the first order conditions for π_t and x_t are put back into the IS equation (2) to get the optimal interest rate rule. The resulting (target) interest rate rule for the baseline model is as follows:

$$(4) \quad i_t = \beta_\pi E_t \pi_{t+1} + \frac{1}{\varphi} \varepsilon_t$$

where, $\beta_\pi = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha} > 1$

The monetary policy rule derived in Box 1 incorporates concern for both output and inflation variability and responds to both demand and supply side shocks. It is important to

note that $\beta_\pi > 1$ —an increase in expected inflation calls for short term nominal interest rates to be moved up more than proportionately so that an increase in real rates influences real output and ultimately inflation.

However, this rule, along with other forms of Taylor rules, does not reflect any concern posed by a high level of government debt; yet central bankers have expressed concern over high debt levels and how the latter could constrain their ability to use the nominal interest rate as an instrument to fight inflation. Recognizing that the interest rate response to higher expected inflation could be an approximation to a more general function, this rule is generalized to incorporate concerns over debt.

As a result of the derivation from a consumption Euler equation, the government consumption term, G_t , is embedded in the error term of the IS equation as follows:

$$(5) \quad \varepsilon_t \equiv E_t \{ \Delta z_{t+1} - \Delta g_{t+1} \}$$

where,

$$g_t \equiv - \log \left(1 - \frac{G_t}{Y_t} \right) .$$

This particular form of the IS-shock is a result of the log-linearization of the consumption Euler equation, and the subsequent substitution of the market clearing condition, $Y_t = C_t + G_t$, where Y_t is the current output level, and C_t is current consumption.² The shock to the IS curve arises from the relative shocks to the trend output and to government consumption.

Under an active monetary policy regime, direct lump sum taxes (net of transfers) support debt shocks entirely. An alternative scenario is suggested in this paper. It is one with a very high debt-to-GDP ratio. When debt-GDP ratio is above a certain threshold level, the expected change in government consumption is assumed to depend on the existing level and growth in debt and its servicing costs. This is because taxes (net of transfers) are no longer expected to back debt shocks entirely. *Instead, government purchases will be expected to fall in the future with increases in real debt service costs if the debt level is high and growing.* The government budget constraint needs to be taken into account to form expectations about future growth in government consumption.

The government budget constraint can be written as a ratio to output in the following form:

$$\Delta b_t = (r_t - \hat{y})b_{t-1} + \frac{G_t}{Y_t} - \left(\frac{tx - tr}{Y_t} \right)$$

² See footnote 11 in CGG(1999).

Here b_t is the debt-to-GDP ratio, r_t is real interest rate, \hat{y} is growth rate of output, approximated by $y_t - y_{t-1}$, $\frac{G_t}{Y_t}$ is government purchases to GDP ratio and net tax (tax (tx) – transfer (tr)) expressed as ratio to GDP is given by $\frac{tx - tr}{Y_t}$.

The identity relates the growth of government debt—all of which is assumed to be marketable securities issued domestically by the government—to its interest payment obligations on past debt and its primary deficit. The latter comprises of the excess of government purchases and transfer payments beyond total tax collections. Seigniorage as a source of revenue for the government has not been included as these revenues are very small and insignificant for the major industrial countries.

When debt is high and growing, net taxes evolve as an exogenous process such that its change is unpredictable. Then, government spending is assumed to evolve as follows:

$$(6) \quad \left(\frac{G_{t+1}}{Y_{t+1}} \right) = \phi_0 - \phi_1 [(r_{t+1} - \hat{y}_{t+1}) b_t] + \eta_{t+1}, \quad \eta_t \sim i.i.d(0, \sigma_\eta^2)$$

$$\phi_1 > 0 \text{ if } b_t > b^*$$

$$\phi_1 = 0 \text{ if } b_t \leq b^*$$

Equation (6) captures the fact that fiscal authorities expect spending to decrease with higher debt service costs if debt-GDP ratio is above a threshold level of b^* . With very high debt, net tax revenue is no longer expected to be ‘sufficient’ to completely offset debt-shocks. The onus is instead on government spending to fall. It is assumed that the monetary authorities expect government spending to fall in proportion to expected debt-service payments.³ But, if the future interest rate is expected to be lower, then expected government spending need not fall as much. At this time fiscal policy puts pressure on monetary policy indirectly by changing the constraint that the central bank operates under. The threshold represents the point at which future government spending is expected to fall, but the extent to which it does so depends upon the future interest rate. However, when debt is below the threshold level, then monetary policy is free from fiscal concerns and there is no attention paid to the level of debt.

Substituting for $\frac{G_t}{Y_t}$ in (5) from (6), algebraic manipulations are made, as is shown in the Appendix, to derive a new IS equation with debt-constraints:

³ There is some empirical evidence (see for example, Ardagna et al, 2004) on higher debt leading to higher long-term interest rates, but this ‘price’ effect is not modeled here. Also note that this is in contrast to Leeper (1991) which assumes that government spending is constant.

$$(7) \quad x_t = -(\varphi + \phi_1 b_{t-1})[i_t - E_t \pi_{t+1}] + E_t x_{t+1} + E_t [\phi_1 b_t \{i_{t+1} - \pi_{t+2}\}] - \phi_1 E_t \{\Delta x_{t+1} b_t - \Delta x_t b_{t-1}\} + \varepsilon_t'$$

In this new IS equation, real interest elasticity parameter, φ , interacts with the past period debt-GDP ratio, b_{t-1} , to *reinforce* the effect of a high interest rate on the output gap, x_t . This reinforced effect takes into account the fact that a higher interest rate necessitates a higher fiscal adjustment—government spending needs to go down even more—thus having a larger effect on the output gap. The second term on the right hand side, $E_t x_{t+1}$, is a term that lends the IS curve its expectational nature and was also present in the previous IS equation (2). There are two new terms—one related to future expected debt servicing costs and the other related to interactions between debt and growth in the output gap. These extra terms complicate the nature of the IS relationship, bringing in future expected real interest rates and debt servicing costs. The error term, $\varepsilon_t' \equiv (1 - \phi_1 b_t) E_t (\Delta z_{t+1}) + \phi_1 b_{t-1} E_t \Delta z_t - E_t (\Delta b_{t+1} - \Delta b_t)$, is an autoregressive (AR) process and is assumed, as a simplification, to have the same AR coefficient as ε_t in (2).

Re-solving the problem with an unchanged objective function, equation (1), an unchanged PC, equation (3), and the newly derived IS, equation (7), the new interest rate rule takes the following form:

$$(8) \quad i_t = \beta_\pi' E_t \pi_{t+1} + \frac{\phi_1 b_t}{\varphi + \phi_1 b_{t-1}} r_{t+1}^e + \frac{\lambda}{\alpha} \left(\frac{\phi_1 b_{t-1}}{\varphi + \phi_1 b_{t-1}} \right) \pi_{t-1} + \frac{\varepsilon_t'}{\varphi + \phi_1 b_{t-1}}$$

where,

$$\beta_\pi' \equiv \left\{ \beta_\pi + \left(\frac{\varphi}{\varphi + \phi_1 b_{t-1}} - 1 \right) \beta_\pi + \phi_1 \frac{\varphi}{\varphi + \phi_1 b_{t-1}} \left(\frac{(\alpha \rho - \lambda) b_{t-1} - \lambda (1 - \rho) b_t}{\alpha \rho \varphi} \right) \right\}$$

$$\beta_\pi = 1 + \frac{(1 - \rho)\lambda}{\rho \varphi \alpha} > 1$$

$$r_{t+1}^e \equiv E_t (i_{t+1} - \pi_{t+2})$$

There are several things to notice about this interest rate rule. First, if $b \leq b^*$, that is $\phi_1 = 0$, we get back the earlier benchmark rule of CGG given by (4). Second, apart from expected inflation, policy also depends upon lagged inflation rate. This indicates that the central bank looks at past inflation beyond its role in forecasting future inflation, and could be a sign of a more gradual adjustment towards the target.⁴ Third, there is a new variable that the central

⁴ This can be compared to Leeper (1991)'s observation that a passive authority's decision rule depends on current and past variables and can be viewed as backward looking.

bank's policy responds to, independent of past or future inflation rates—the future expected short-term real interest rate, r^e_{t+1} . A rise in the expected short-term real interest rate, *ceteris paribus*, calls for a rise in the target nominal interest rate, in order to take account of exogenous factors other than expected inflation and the output gap, which would lead to an increase in the actual nominal rate⁵. Fourth, the coefficients of the three variables on the right hand side of equation (8) vary with the (present or past) debt level.

The first term on the right hand side of (8) is the response of short-term nominal interest rate to expected future inflation—a higher expected inflation calls for a hike in the nominal rate as a ‘lean against the wind’ policy (see Appendix). However, this response might not be greater than β_π . In order to analyze how the policy maker's response to expected inflation alters in the case of debt above the threshold, assume that debts are high and near the maximum level, b .⁶ Imposing this condition on (8), the latter can be re-written as:

$$(9) \quad i_t = \left\{ \beta_\pi + \left(\frac{\varphi}{\varphi + \phi_1 b} - 1 \right) \beta_\pi + \frac{\phi_1 \phi b}{\varphi + \phi_1 b} \left(\frac{\rho(\alpha + \lambda) - 2\lambda}{\alpha \rho \varphi} \right) \right\} E_t \pi_{t+1} + \frac{\phi_1 b}{\varphi + \phi_1 b} r^e_{t+1} + \frac{\lambda}{\alpha} \left(\frac{\phi_1 b}{\varphi + \phi_1 b} \right) \pi_{t-1} + \frac{\varepsilon'_t}{\varphi + \phi_1 b}$$

The interest rate response to $E_t \pi_{t+1}$ is given by

$$(10) \quad \beta'_\pi \equiv \beta_\pi + \left(\frac{\varphi}{\varphi + \phi_1 b} - 1 \right) \beta_\pi + \frac{\phi_1 \varphi}{\varphi + \phi_1 b} \left(\frac{(\rho(\alpha + \lambda) - 2\lambda)b}{\alpha \rho \varphi} \right)$$

Whether β'_π is larger than β_π depends upon the relative magnitudes and signs of the second and the third terms of (10).

If $b \leq b^*$, there is no attention paid to high debt levels by monetary authorities and the original baseline rule given by (4) applies. However, if $b > b^*$, then $\left(\frac{\varphi}{\varphi + \phi_1 b} - 1 \right) < 0$, given $\varphi > 0$. This implies that the second term of (10) is negative. The sign of the third term of (10) is ambiguous unless we make assumptions about the relative magnitudes of α and λ , given $\rho < 1$. If $\alpha \leq \lambda$ —weight given to output variations is ‘low’ in the central bank's objective function (1)—then the third term is negative. If $\alpha > \lambda$, then the ambiguity remains. In other words, if the policy maker is tough (low α) towards inflation—as during a nearly pure inflation-targeting regime—then $\beta'_\pi < \beta_\pi$.

Furthermore, β'_π decreases if b increases and $\alpha \leq \lambda$. To see this, we can partially differentiate (10) with respect to b :

⁵ This could present a case for a time-varying natural rate component in the interest-rate rule. The nominal rate is adjusted to close the ‘gap’ between the actual real rate and the natural rate as in Woodford (2003).

⁶ A maximum level of debt is assumed to exist.

$$\begin{aligned}
\frac{\partial}{\partial b} \beta'_\pi &\equiv \frac{\partial}{\partial b} \left\{ \beta_\pi + \left(\frac{\varphi}{\varphi + \phi_1 b} - 1 \right) \beta_\pi + \frac{\phi_1 \varphi}{\varphi + \phi_1 b} \left(\frac{(\rho(\alpha + \lambda) - 2\lambda)b}{\alpha \rho \varphi} \right) \right\} \\
&= -\frac{\phi_1 \varphi \beta_\pi}{(\varphi + \phi_1 b)^2} + \frac{\phi_1}{\alpha \rho (\varphi + \phi_1 b)^2} [\varphi \{ \rho(\alpha + \lambda) - 2\lambda \}] \\
&< 0, \text{ if } \alpha \leq \lambda.
\end{aligned}$$

The conclusion is ambiguous if $\alpha > \lambda$. Summarizing the central bank's response to expected inflation: in an inflation-targeting regime (low α), the policy responds less to expected inflation if debts are high and this response falls if debts increase over time. This does not necessarily imply that policy is accommodating; rather, the same expected inflation would need a more gradual response due to the extra constraints from the IS curve. This could translate into the central bankers feeling 'pressure' when debts are growing—the coefficient of, and hence the ability to fight, inflation being constrained by high and growing debts.

The sensitivity of the current short-term interest rate to expected future real interest rates, as is given by the second term on the right hand side of (9), is positive. Independent of whether there is a boom or a recession in the economy, if debts are high, the interest rate is moved up if future expected real interest rate, r^e_{t+1} , goes up. High debts can pose a constraint on the monetary authorities if there are recessionary pressures—with falling output gap and the inflation rate—calling for a fall in the interest rate. Indeed, this was the case in Canada during 1993-94. On the other hand, if the economy is in a boom—although the coefficient for expected inflation is low when debts are high—and if r^e_{t+1} is higher, it 'helps' the monetary authorities to increase interest rates to some extent.

The third term of equation (9) is the interest rate response to the past period inflation rate—it is a positive response when debts are high. This makes the central bank pay attention to the lagged inflation rate beyond its ability to forecast future inflation, making the adjustment process more gradual.

In what follows, an empirical model is set up to estimate both the threshold level of debt and a debt-constrained policy rule (using the estimated threshold level) for the first decade of inflation targeting regime in Canada during the 1990s.

IV. JOINT ESTIMATION OF THE THRESHOLD LEVEL OF DEBT AND INTEREST RATE RULE—THE CASE OF CANADA

Empirical model

For the purpose of estimating and testing for an interest rate rule given by equation (8), a threshold regression model is employed to simultaneously choose the threshold debt level and the estimates of the debt-constrained rule. An iterative dummy variable approach is used

in setting up the model and estimating it by generalized method of moments (GMM). The econometric model is written as:

$$(11) \quad i_t^T = \alpha + \beta_\pi E_t(\pi_{t+n} | \Omega_t) + \theta_1(DUM)E_t(\pi_{t+n} | \Omega_t) + \gamma\pi_{t-1} + \theta_2(DUM)\pi_{t-1} \\ + \sigma r_{t+1}^e + \theta_3(DUM)r_{t+1}^e + \mu z_t + \varepsilon_t$$

The monetary policy instrument is the target short-term interest rate, i_t^T , which responds to expected future inflation n -period ahead, $E_t\pi_{t+n}$, future expected real interest rate (exogenously given), r_{t+1}^e , and to other variables such as exchange rates and foreign interest rates contained in the vector z . Expectations on inflation are formed given the time- t information set, Ω_t . The policy maker takes lagged inflation rate into account only when debts are above the threshold; lagged inflation has, therefore, been included in (11). Concern about high debts is incorporated using the dummy variable DUM defined as follows:

$$DUM = 1, \text{ when debt-GDP ratio} > \text{estimated threshold level } (b^*) \\ = 0, \text{ otherwise.}$$

As has been explained in the previous section, the coefficients of (11) are expected to have the following range of values:

$$\beta_\pi > 1, \gamma = 0, \sigma = 0, \mu > 0, \theta_1 < 0, \theta_2 > 0, \theta_3 > 0.$$

In order to eliminate the unobserved expectation term, $E_t\pi_{t+n}$, the realized n -period ahead inflation rate, π_{t+n} , is added and subtracted from the right hand side of (11) and the interest rate rule is re-written in terms of the realized variables and a new error term, v_t . Here v_t is a linear function of the forecast error of inflation (adjusted for the dummy variable) and the exogenous disturbance, ε_t :

$$(12) \quad i_t^T = \alpha + \beta_\pi\pi_{t+n} + \theta_1(DUM)\pi_{t+n} + \gamma\pi_{t-1} + \theta_2(DUM)\pi_{t-1} \\ + \sigma r_{t+1}^e + \theta_3(DUM)r_{t+1}^e + \mu z_t + v_t$$

Assuming that the actual interest rate adjusts to the target rate with a lag, we can write the actual interest rate as a weighted average of the past two⁷ periods' rates and the target rate set in this period:

$$i_t = \psi_1 i_{t-1} + \psi_2 i_{t-2} + (1 - \psi_1 - \psi_2) i_t^T \quad |\psi_1 + \psi_2| < 1$$

⁷ The number of lags is decided empirically.

Substituting for i_t^T from above into (12), the actual short term interest rate is:

$$(13) \quad i_t = \tilde{\alpha} + \tilde{\beta}_\pi \pi_{t+n} + \tilde{\theta}_1(DUM)\pi_{t+n} + \tilde{\gamma}\pi_{t-1} + \tilde{\theta}_2(DUM)\pi_{t-1} \\ + \tilde{\sigma}r_{t+1}^e + \tilde{\theta}_3(DUM)r_{t+1}^e + \tilde{\mu}z_t + \psi_1 i_{t-1} + \psi_2 i_{t-2} + v_t$$

The parameters with a (\sim) represent the actual parameters in (12), adjusted for the interest rate-smoothing parameters, ψ_1 and ψ_2 . For example, $\tilde{\beta}_\pi = (1 - \psi_1 - \psi_2)\beta_\pi$ and so on.

If $w_t \in \Omega_t$ is the vector of variables within the bank's information set, that are orthogonal to v_t , we have a set of orthogonality conditions that can be exploited to estimate model (13) by GMM. These are $E[v_t | w_t] = 0$, which implies:

$$(14) \quad E[i_t - \tilde{\alpha} - \tilde{\beta}_\pi \pi_{t+n} - \tilde{\theta}_1(DUM)\pi_{t+n} - \tilde{\gamma}\pi_{t-1} - \tilde{\theta}_2(DUM)\pi_{t-1} \\ - \tilde{\sigma}r_{t+1}^e - \tilde{\theta}_3(DUM)r_{t+1}^e - \tilde{\mu}z_t - \psi_1 i_{t-1} - \psi_2 i_{t-2} | w_t] = 0$$

The parameter vector $[\alpha, \beta, \theta_1, \gamma, \theta_2, \sigma, \theta_3, \mu, \psi_1, \psi_2]$ and the threshold level of debt are then jointly estimated with the instruments being elements of the vector w_t . The set of instruments is chosen such that they are potentially useful in forecasting future inflation. We are assuming that the central bank sets interest rates according to (11) taking into account all relevant information available at that time.

For the right hand side variables of (11), lagged values of the interest rate, output gap, core inflation rate, the US Federal Funds Rate, and a long term indexed bond rate have been used as instruments. These are discussed in more detail with the results. Since there are more instruments than parameters to estimate, Hansen's J -statistic has been used to test for over-identifying restrictions in the system. The p-value (the minimum level of significance at which the null hypothesis of over-identifying restrictions being valid, is rejected) is reported with the estimation results.

In order to choose the threshold level of debt, we search over a grid of values for debt-GDP ratio ranging from 46 percent to 52 percent (almost the maximum during this period)⁸. The step size is 0.1 percentage point. The threshold level of debt is chosen so as to minimize the objective function of the GMM—that is the one with the lowest J -statistic.

⁸ The range ensures convergence of the iterations for GMM estimation of the parameters and the threshold level of debt-GDP ratio.

Data

Data for Canada has been used to test if a high debt level constrained monetary policy during its inflation targeting regime in the 1990s. The choice of the country was given by the motivation stated in the introduction—the central bank had expressed concern over fiscal issues, especially debt. A second reason for the choice was the ready availability of high-frequency data. Canada adopted inflation targeting in 1991, and this fact guides the choice of the sample period—the first decade of its inflation targeting regime.

Monthly data for Canada is taken from the Bank of Canada, DRI Economics Database, and International Financial Statistics (IMF), and ranges from 1991:11 to 2000:12. The Overnight Rate is the short-term interest rate, i , the Bank of Canada uses as an instrument for monetary policy. The Consumer Price Index (CPI) is used to calculate the yearly rate of inflation, π ; the core inflation rate, used only to predict overall future inflation, is included in the set of instruments. The output gap, x , is calculated by linearly de-trending the log of the Index of Industrial Production (IIP)⁹.

The monthly series for Total Interest-bearing Marketable Debt issued by the government of Canada (Federal) is divided by GDP to derive the debt-GDP ratio.¹⁰ The debt-GDP ratio, although not used directly in actual estimation, guides the definition of the dummy variable, DUM.

The series for expected future short-term real interest rate is proxied by the interest rate on long-term indexed bonds. Unavailability of data on a short-term indexed bond prevented the use of a term structure relationship to derive the future expected short-term real interest rate. A series for expected inflation was constructed by taking the difference in rates for a long-term nominal bond and the long term indexed bond—this series would have been exogenous. However, it failed to provide convincing results for the stance of monetary policy. This could also provide a guide to the horizon, n , that the policy maker considers while taking a decision—near-to-medium term (a quarter to about a year ahead) as opposed to long term (ten to thirty years). The horizon for expected inflation is chosen as 12, suggesting that the bank looks around a year ahead while setting interest rates. Reflecting the openness of the Canadian economy, the annual percentage change in the exchange rate (C\$/USD) has been used as z .

Figure 1 shows movements in some of the key variables used in this analysis. The shaded region in the debt-GDP ratio graph gives the level of debt at which the central bank Governor had expressed concerns about the debt level. For the short sample—1991 to 2000—monthly

⁹ Parameter estimates and the threshold level of debt were very similar when we de-trended output by means of an exponentially smoothed trend.

¹⁰ Since data on GDP is only available quarterly, the quarterly series has been extrapolated into a monthly one, to use it as a denominator for the debt-GDP ratio.

data for inflation rate, output gap and expected real interest rate are all stationary. Estimation results are discussed in the next sub-section.

Estimation results

The iterative procedure simultaneously estimates the parameters of (13) and the threshold level of debt doing a grid search over 46 percent through 52 percent of the debt-GDP ratio, with a step size of 0.1 percentage point. The series of J-statistics for the GMM estimations using the grid search is given in Figure 2. The J-statistic-minimizing threshold level of debt is estimated to be 50.2 percent of GDP. Debt was above this level from June 1993 through May 1997. The standard error bands around a Hodrick-Prescott ‘trend’ is also shown in the diagram, although the bands do not show the confidence intervals for choosing the J-statistic minimizing threshold level of debt.

The parameter estimates from GMM estimation is given in Table 1. The first column provides parameter estimates for the baseline model such as those used by CGG (1998). Besides expected inflation (with a horizon of 12 months), the rate of change of the Canadian nominal exchange rate¹¹ is included as one of the indicator variables for monetary policy. The instruments used for both the baseline and the debt-constrained models are the first through sixth, ninth and twelfth lags of the following variables: overnight interest rate, the core inflation rate, interest rate on the long-term indexed bond, output gap, and the Fed Funds Rate.¹² A test for over-identifying restrictions revealed a p-value for the *J*-statistic to be 0.96 and 0.99 for the baseline and the debt-constrained models respectively. We therefore cannot reject the null hypothesis of the instruments used being valid.

The adjustment of the nominal interest rate is considerably smoothed, given that the estimate of the smoothing parameter ($\psi_1 + \psi_2$) is 0.93. The estimate for β_π is 0.76—considerably less than 1. Absent other explanations, this might suggest that the Bank seems to have been accommodating inflation instead of targeting it. However, one explanation for this low coefficient on expected inflation could be the presence of a break in the parameter due to high debt levels. We therefore use the threshold-GMM estimates to test for the presence of a debt-constraint. These results are presented in the second column of Table 1.

There seems to be a significant presence of a debt-constraint during the years in which debt was above the threshold level of 50.2 percent of GDP. At other times, the Bank of Canada was indeed following a ‘lean against the wind’ strategy of fighting inflation—the estimate of β_π is 1.69, substantially above 1. Pressure due to high debt-levels is quantified by the significantly negative estimate of θ_1 —for every percentage point increase in expected inflation, the debt-constrained interest rate response is 0.99 (θ_1) percentage point less than

¹¹ The level of the nominal exchange rate is non-stationary.

¹² For the baseline model, we have used the same set of instruments as those for the debt-constrained one, except for lags of the indexed bond rate for the latter.

that during normal times. The overall effect of a one percent increase in inflation is 0.70 ($=\beta_{\pi} - \theta_1$) percentage point increase in the overnight rate for the debt-constrained model during high debt levels. This is almost the same as the corresponding 0.76 percentage point found for the baseline model.¹³

There is no evidence of the Bank using the past inflation rate in its policy formulation, besides its role in predicting future inflation, with or without debt constraints. The interest rate on long-term indexed bond, proxying for the future expected short-term real rate, has a positive effect on the overnight rate, as was expected in Section 3. During high debt levels, the overnight rate is revised upward, independent of concerns for inflation. For instance, if there is a simultaneous 1 percentage point increase in the 12-month ahead inflation expectation and a 50-basis point increase in the rate on indexed bond, the additional effect of high debts on the overnight rate is $(\theta_1 + \theta_3 / 2)$ 0.63 percentage point below normal. There is no separate response of the overnight rate to changes in the U.S. Federal Funds Rate and the output gap, besides their role in predicting inflation.

V. SUMMARY AND CONCLUSIONS

This paper derives an interest rate rule for monetary policy in which the interest rate response of the central bank toward an increase in expected inflation falls as debts increase beyond a certain threshold level. A debt-constrained interest rate rule and the threshold level of debt are jointly estimated for Canada during the first decade of its inflation targeting regime of the 1990s.

There are three main findings of this paper. First, a high government debt could constrain monetary policy if government spending is expected to adjust in future in line with debt service costs. The ‘constraint’ operates through an altered policy transmission mechanism through changes in the nature of the IS curve. The policy responds less to expected inflation if debts are high and this response falls if debts increase over time. This does not necessarily imply that policy is accommodating; rather, the same expected inflation would need a more gradual response due to the extra constraints from the IS curve. This could translate into the central bankers feeling ‘pressure’ when debts are growing—the coefficient of, and hence the ability to fight, inflation being constrained by high and growing debts. Second, the ‘constraining’ effects of high debt on monetary policy are quite different during booms and recessions. The effects depend upon the level of the equilibrium real interest rate, proxied in this paper by expected future short term rate. Third, empirical estimates show that Canadian monetary policy might have been constrained by a high government debt-GDP ratio during the 1990s. Policy was less loose than what inflation indicators called for.

¹³ Empirical work done by the Canadian Department of Finance—estimating a monetary policy reaction function for Canada—shows a close fit between actual and predicted short-term interest rates since the beginning of 1995. The close fit coincides with the adoption of the 1995 Budget that earnestly promoted fiscal consolidation.

The threshold level of debt-GDP ratio has been estimated to be 50.2 percent, slightly higher than 48 percent, the level at which the Bank of Canada governor had warned about pressures of high debts on monetary policy. For the case of Canada during the nineties, it has been shown that the interest rate response to expected inflation has been significantly constrained by debt. When debt is above the threshold level, for every one percentage point increase in expected inflation, the interest rate response is 0.99 percentage point below ‘normal’ (a scenario without debt constraints). In addition, the interest rate is adjusted upward for an increase in future expected real interest rate. During the high debt period, the inflation rate and the output gap were low, calling for loosening of monetary policy. However, debt-constraints appeared to prevent the central bank from loosening enough.

There are policy implications for the findings of this paper. Even though major industrialized countries have independent central banks, we have established a channel through which fiscal decisions—leading to higher debts—could “constrain” monetary authorities in their policy decisions by changing the nature of the IS curve. This analysis serves as a caution for many developing and emerging economies considering moving towards an inflation-targeting framework, without adequately lowering the level of public debt. The findings also serve as a reminder that debt-constraints can alter the monetary policy transmission mechanism in industrialized countries as well, and even the United States could find itself operating under a debt-constraint if its public debt crosses a certain threshold level.

Finally, there are a few ways that this analysis can be left for future work. First, this paper modifies an assumption of a particular model, by CGG, without proposing a fully-specified model or checking whether the Blanchard-Kahn condition (Blanchard and Kahn, 1980) for a unique saddle-path equilibrium holds. Second, the estimate of the threshold level of debt is based on minimizing the J -statistics of the GMM regressions, but proper confidence intervals could not be computed. Computing such confidence intervals would greatly enhance the quality of the analysis. Third, historical episodes from other countries (like the United Kingdom) with inflation targeting in the backdrop of high government debt could serve to strengthen the empirical results.

DERIVATION OF OPTIMAL INTEREST RATE RULE UNDER DISCRETIONARY POLICY

Baseline Rule (CGG (1999))—refer to Box 1 in the text

Step 1: Choose x_t and π_t to maximize objective (1), given inflation equation (3)

$$\text{FOC: } x_t = -\frac{\lambda}{\alpha}\pi_t \quad (A1)$$

This is a ‘lean against the wind’ policy. Fighting against inflation calls for a lowering of the output gap. The extent to which this policy is going to be successful depends on the parameters λ and α . A higher λ indicates a greater response of inflation to output gap variations as is seen in the Phillips curve and the extent to which the policy maker will be helped in her task; a higher α indicates a higher relative weight put on output variability in the objective function and so this is going to ‘hurt’ the policy maker in her fight against inflation.

Step 2: Combining (A1) with Phillips Curve and assuming Rational Expectations and using the IS equation (2) we solve for i_t to get the baseline interest rate rule:

$$i_t = \beta_\pi E_t \pi_{t+1} + \frac{1}{\varphi} \varepsilon_t \quad (A2)$$

where,
$$\beta_\pi = 1 + \frac{(1-\rho)\lambda}{\rho\varphi\alpha} > 1$$

Debt-constrained Rule

The original IS equation (2) was derived from an Euler equation from optimization framework, after substituting $Y_t - G_t \equiv C_t$, log-linearizing and writing $y_t \equiv x_t + z_t$, where z_t is trend output and x_t is the output gap or the cyclical part of output and y_t is $\log(Y_t)$. The resulting IS equation (replicating (2)) was the following:

$$x_t = -\varphi[i_t - E_t \pi_{t+1}] + E_t x_{t+1} + \varepsilon_t \quad (2)$$

The error term $\varepsilon_t \equiv E_t \{ \Delta z_{t+1} - \Delta g_{t+1} \}$

$$g_t \equiv -\log\left(1 - \frac{G_t}{Y_t}\right) \quad (A3)$$

where G_t is Government consumption.

Using the government spending equation, (6a) to substitute for $\frac{G_t}{Y_t}$ in (A3), we have

$$\Delta g_{t+1} = g_{t+1} - g_t$$

$$= -[\log(1 - \phi_0 + \phi_1(i_{t+1} - E_{t+1}\pi_{t+2} - y_{t+1} + y_t)b_t) - \log(1 - \phi_0 + \phi_1(i_t - E_t\pi_{t+1} - y_t + y_{t-1})b_{t-1})]$$

$$= -\log\left(\frac{1-\phi_0 + \phi_1(i_{t+1} - E_{t+1}\pi_{t+2} - y_{t+1} + y_t)b_t}{1-\phi_0 + \phi_1(i_t - E_t\pi_{t+1} - y_t + y_{t-1})b_{t-1}}\right)$$

Using approximation $\frac{1+\delta}{1+\psi} \approx 1 + \delta - \psi$, when δ and ψ are small,

$$\approx -\log[1 - \phi_0 + \phi_1(i_{t+1} - E_{t+1}\pi_{t+2} - \Delta y_{t+1})b_t + \phi_0 - \phi_1(i_t - E_t\pi_{t+1} - \Delta y_t)b_{t-1}]$$

Further using approximation $\log(1+a) \approx a$, for small a , and rearranging:

$$= -\phi_1\{i_{t+1} - E_{t+1}\pi_{t+2} - \Delta y_{t+1}\}b_t - \phi_1\{i_t - E_t\pi_{t+1} - \Delta y_t\}b_{t-1} \quad (A4)$$

Replacing (A4) in (A3):

$$\varepsilon_t \equiv E_t\{\Delta z_{t+1} - \Delta g_{t+1}\}$$

$$= E_t[\Delta z_{t+1}] + E_t[\phi_1\{i_{t+1} - E_{t+1}\pi_{t+2} - \Delta y_{t+1}\}b_t] - E_t[\phi_1\{i_t - E_t\pi_{t+1} - \Delta y_t\}b_{t-1}] \quad (A5)$$

Substituting for ε_t from (A5) in (2), and writing $y_t \equiv x_t + z_t$, we come upon the new IS curve given by (7), replicated here for convenience:

$$x_t = -(\varphi + \phi_1 b_{t-1})[i_t - E_t\pi_{t+1}] + E_t x_{t+1} + E_t[\phi_1 b_t \{i_{t+1} - \pi_{t+2}\}] - \phi_1 E_t \{\Delta x_{t+1} b_t - \Delta x_t b_{t-1}\} + \varepsilon_t' \quad (7)$$

$$\text{Here } \varepsilon_t' \equiv (1 - \phi_1 b_t)E_t(\Delta z_{t+1}) + \phi_1 b_{t-1}E_t \Delta z_t$$

In deriving the new interest rate rule, we use the same first order conditions as (A1), since the objective function and the Phillips Curve have not changed; substituting these conditions in the new IS equation (7), we get the new interest rate rule as in (8) which is also replicated here:

$$i_t = \left[\frac{(\lambda + \varphi\alpha\rho - \lambda\rho) + \phi_1(\alpha\rho b_{t-1} + \lambda\rho b_t - \lambda b_t - \lambda b_{t-1})}{\alpha\rho(\varphi + \phi_1 b_{t-1})} \right] E_t \pi_{t+1} + \left(\frac{b_t}{\varphi + b_{t-1}} \right) E_t (i_{t+1} - \pi_{t+2}) + \frac{\lambda}{\alpha} \left(\frac{\phi_1 b_{t-1}}{\varphi + \phi_1 b_{t-1}} \right) \pi_{t-1} + \frac{\varepsilon_t'}{\varphi + \phi_1 b_{t-1}}$$

or,

$$i_t = \left\{ \beta_\pi + \left(\frac{\varphi}{\varphi + \phi_1 b_{t-1}} - 1 \right) \beta_\pi + \frac{\phi_1 \varphi}{\varphi + \phi_1 b_{t-1}} \left(\frac{(\alpha\rho - \lambda)b_{t-1} - \lambda(1 - \rho)b_t}{\alpha\rho\varphi} \right) \right\} E_t \pi_{t+1} + \frac{\phi_1 b_t}{\varphi + \phi_1 b_{t-1}} r_{t+1}^e + \frac{\lambda}{\alpha} \left(\frac{\phi_1 b_{t-1}}{\varphi + \phi_1 b_{t-1}} \right) \pi_{t-1} + \frac{\varepsilon_t'}{\varphi + \phi_1 b_{t-1}}$$

$$\text{where, } \beta_\pi = 1 + \frac{(1 - \rho)\lambda}{\rho\varphi\alpha} > 1$$

$$r_{t+1}^e \equiv E_t(i_{t+1} - \pi_{t+2})$$

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FIGURE 1: MOVEMENTS IN SOME KEY VARIABLES IN CANADA MONTHLY DATA, 1991:11 TO 2000:12.

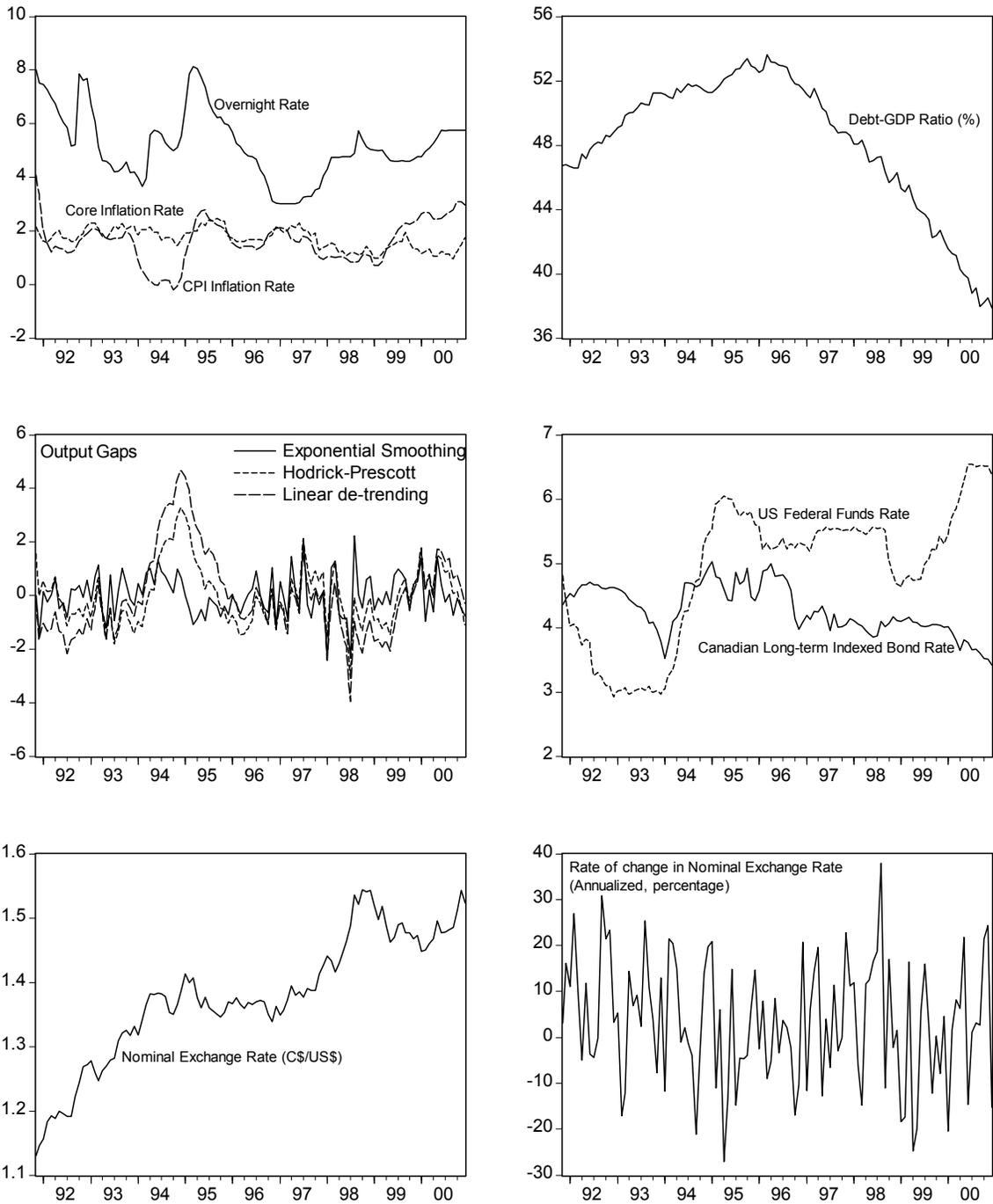
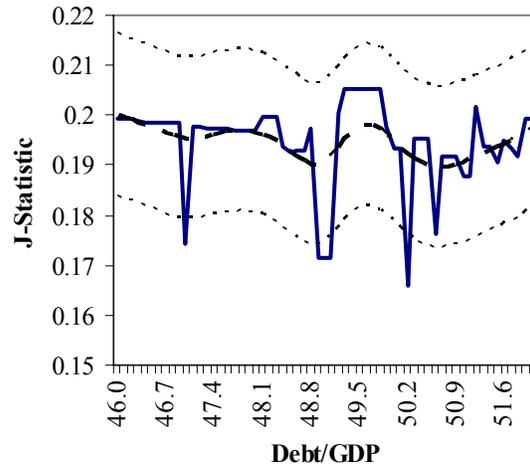


FIGURE 2: J-STATISTICS FOR CHOOSING THE THRESHOLD LEVEL OF DEBT



Note: The light broken lines are ± 2 *S.E. bands around a Hodrick-Prescott trend (dark broken line). The reason for the apparent persistence of the J-statistic between debt/gdp ratios from 48.9 through 49.1 is the non-existence of actual debt/gdp ratios of 48.9 and 49.0. The effective dummy variable is, therefore, at a debt/gdp of 49.1.

TABLE 1: GMM ESTIMATES⁽ⁱ⁾ OF PARAMETERS OF BASELINE AND DEBT-CONSTRAINED MODELS, CANADA, MONTHLY DATA, 1991:11 TO 2000:12

Parameter Name	Baseline Estimates	Debt-constrained Estimates
α	2.81 (0.75)	5.44 (3.42)
β_{π}	0.76 (0.21)	1.69 (0.28)
θ_1	--	-0.99 (0.45)
γ	--	-0.40 (0.33)
θ_2	--	0.23 (0.43)
σ	--	-1.13 (0.78)
θ_3	--	0.72 (0.28)
μ	0.07 (0.02)	0.05 (0.02)
ψ_1	1.24 (0.04)	0.96 (0.02)
ψ_2	-0.31 (0.04)	-0.06 (0.02)
<i>J</i> – statistic	0.18	0.17
<i>p</i> -value for <i>J</i> -statistic	0.96	0.99

Note: (i) Statistically significant (at 5% level) estimates are in **bold**. The standard errors of the estimates, given in parenthesis, are adjusted from the ones derived from estimation of (9'') by the delta-method.