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Current Account and Precautionary Savings for Exporters of Exhaustible Resources

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Current Account and Precautionary Savings for Exporters of Exhaustible Resources [1]

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Abstract

Exporters of exhaustible resources have historically exhibited higher income volatility than other economies, suggesting a heightened role for precautionary savings. This paper uses a parameterized small open economy model to quantify the role of precautionary savings in economies with exhaustible resources, when the only source of uncertainty is the price of the exhaustible resource. Results show that the precautionary motive can generate sizable external sector savings. When aggregated over the sample countries, precautionary savings in 2006 add up to 3.2 percent of GDP. The quantitative importance of the precautionary motive varies considerably across the sample countries and is driven primarily by the weight of exhaustible resource revenues in future income. The parameterized model fares well at capturing current account balances in both cross-section and time-series data.

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I. Introduction

Exporters of exhaustible resources have historically exhibited considerably higher income volatility than other economies. The standard deviation of output growth for oil exporting countries is three times higher than in the typical oil importing country (Figure 1). This observation highlights the importance of the precautionary motive and related accumulation of buffer stock savings for exporters of exhaustible resources. Indeed, exhaustible resource exporters have accumulated sizable external savings over the last decade, contributing significantly towards the global current account imbalances. How much of these savings can be attributed to the precautionary motive?

The purpose of this paper is to estimate the size of precautionary savings for exporters of exhaustible resources. To gauge the link between income volatility and savings, we use a representative agent small open economy model modified to account for an exhaustible resource sector with the price of the exhaustible resource as the only source of uncertainty. To focus on the savings problem, the model abstracts from domestic investment and resource extraction decisions. The representative agent solves a self-insurance problem, whereby accumulation of foreign assets diversifies income away from the volatile exhaustible resources. Precautionary savings are then the change in (external) savings due to this self-insurance motive. The model is parameterized to capture country-specific characteristics of eleven oil and gas exporting economies whose exhaustible resource sector accounts for a significant share of economic activity.

Results show that the precautionary motive can generate sizable external sector savings in exhaustible resource countries. Solving the model from the perspective of year 2006, for the median sample country, precautionary savings amounted to 2.5 percent of output, with large variation in country-specific estimates. When aggregated over those eleven countries, precautionary savings in 2006 add up to 3.2 percent of the total output or 58.3 billion dollars. The quantitative importance of the precautionary motive is driven primarily by the weight of exhaustible resource revenues in expected future income.

The model fares well at capturing the savings/current account behavior in exhaustible resource countries. Under the baseline parameterization, the correlation between current account balances in the model and data for the eleven sample countries is 0.69 for 2006. Moreover, allowing for precautionary savings improves the model's fit over a deterministic baseline. When applied to the case of oil and gas discovery in Norway, the model can account for the balk of the net foreign asset accumulation during the post-1974 period.

In addition to providing estimates for the size of precautionary savings, the paper contributes to the literature by developing a small open economy model with *aggregate* uncertainty that *is not* restricted to a unique stationary equilibrium. As has been repeatedly noted (e.g., Schmitt-Grohé and Uribe (2003) and Ghironi (2007)), the representative agent small open economy model with aggregate uncertainty exhibits unit root dynamics in net foreign assets and therefore does not harbor a well-defined stationary equilibrium. Numerous fixes to this problem have been proposed, all imposing a unique stationary equilibrium. Such a restriction, however, is not satisfactory for studies, such as ours, that look beyond business-cycle dynamics.

The model of this paper avoids the problem by assuming that the only source of aggregate uncertainty is the price of the *exhaustible* resource. Once the resource is exhausted, the model becomes deterministic. As a result of this 'shortcut', the model exhibits a well-defined (non-unique) stationary equilibrium. Similar to the deterministic case, the stationary equilibrium depends on the model's initial conditions as well as transitional dynamics.

This paper deviates from the extensive literature on precautionary savings (e.g., Caballero (1990), Carroll (2001) and references therein) along several dimensions. First, it deals with aggregate, as opposed to idiosyncratic, shocks. Second, the focus is on open economies, with savings taking place through the external sector. Third, while previous literature has concentrated exclusively on advanced economies, the sample in this paper includes both developed and developing countries.

Two previous studies are closely related to our work. Ghosh and Ostry (1997) find that aggregate income uncertainty increases current account balances in a group of advanced economies. Fogli and Perri (2006) show that incentives to accumulate precautionary savings, induced by lower income volatility in the US relative to the rest of the world, can explain a significant share of the US current account deficits since early 1980s. Both studies find that the link between income volatility and external savings is economically significant. For the more volatile and open exporters of exhaustible resources, this channel is likely to have added relevance.

Another strand of related literature deals with the intergenerational allocation of exhaustible resource income. The deterministic version of this paper's modeling framework has been a 'workhorse' model for research and policy advice on this topic.[2] In essence, consumption smoothing considerations are used as a guide for determining the intertemporal allocation of exhaustible resource income. The model can generate the large current account surpluses observed in exhaustible resource countries.

Our paper differs from this literature, since we explicitly model the effects of uncertainty on the external sector dynamics. In the presence of a precautionary motive, the deterministic model overestimates consumption and therefore underestimates the size of savings and the current account balance in the short run, more so the greater the uncertainty attached to the exhaustible resource wealth and the larger the weight of exhaustible resources in economic activity.

The structure of the rest of the paper is as follows: the next section presents the model and its optimal solution; Section III discusses the parameterization procedure, presents results from the baseline model for 2006 and compares a cross-section of optimal current accounts in the model with data; Section IV contains extensive sensitivity analysis of the baseline results; Section V extends the modeling application to a time-series setting and applies it to the case of Norway; finally, Section VI concludes.

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[2] This modeling framework has been advocated in Davis et al (2003) and applied by numerous studies (see, e.g., de Carvalho Filho (2007) for an application to Trinidad and Tobago, Takizawa (2005) for Kuwait and Bailen, Kramarenko (2004) for Iran and Thomas et al.(2008) for a cross-country study).

II. MODELING FRAMEWORK

This section presents a model that captures the savings problem facing producers of exhaustible resources with volatile prices.

A. Small Open Endowment Economy

Consider a small open economy with the following features. Aggregate production technology for the non-exhaustible resource sector is:

$$Y_t = AL_t , \tag{1}$$

where Y_t is output, L_t is labor input and A is a measure of productivity. The model abstracts from domestic investment/savings decisions, motivated by the empirical evidence from the 'resource curse' literature (e.g., Sachs and Warner, 2001), which shows that countries with exhaustible resources historically have exhibited no growth in per capita output. This evidence suggests that domestic investments/savings are not used systematically to diversify income away from the more volatile exhaustible resource sector.[3] In the model, the non-exhaustible resource sector facilitates income diversification only through the exogenous growth in the labor force and the only endogenous channel of diversification is the external sector.

In the model, as in the data, such role is instead assigned to the external sector. From (1) an additional channel for income diversification is provided by the exogenous growth in the labor input – as the size of the labor force increases, so does the output share of the non-exhaustible resource sector.

Labor supply is inelastic and proportional to the size of the labor force. It is the only source of growth in the non-exhaustible resource sector of the economy, growing at a constant rate:[4]

$$L_{t+1} = (1+n)L_t . \tag{2}$$

In the exhaustible resource sector, the extraction technology requires no factor inputs and output follows an exogenously specified sequence:

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[3] In a more general model with endogenous capital, investment would provide an alternative diversification channel only when domestic return on capital exceeds the return on foreign assets. This could be the case if, for example, the economy is credit constrained or experiences a positive productivity shock. Oil revenues can then be used to exploit the temporarily higher domestic returns. However, on the balanced growth path return on domestic capital has to equal the world interest rate, limiting the long-run impact of this diversification channel.

[4] For a discussion of productivity as a source of growth in economies with exhaustible resources and its effects on precautionary savings see section on sensitivity analysis.

$$Z_t \begin{cases} \geq 0 & \text{for } t \leq T \\ = 0 & \text{for } t > T \end{cases}, \tag{3}$$

where Z_t is the quantity extracted. Future extraction quantities, $\{Z_t\}_{t=0}^{T} \geq 0$, are known at time t and such resources are exhausted in period T. The model does not address the question of optimal extraction quantities and instead takes the available projections for future extraction quantities as given. This choice reflects the observation that with few exceptions (e.g. periods of wars), extraction quantities exhibit little variation over time and do not respond systematically to changes in the relative price of the exhaustible resource. For instance, for the countries in our sample, the correlation between oil price and extraction quantities at yearly frequency for 1974-2007 period ranges from -0.2 to 0.1, depending on the assumed time lag.[5]

The aggregate per-period resource constraint of the economy is:

$$C_t + B_{t+1} = (1+r)B_t + e^{p_t} Z_t + Y_t\,. \tag{4}$$

In (4) C_t represents aggregate consumption. The difference between aggregate production and absorption is the trade balance, B_{t+1} - $(1+r)B_t$, where B_t stands for the stock of net foreign assets as of the end of period t-1 and r is the risk-free rate of return. The ownership of the stock of exhaustible resources is assumed to be non-transferable, hence in each period the extracted amount, $e^{p_t} Z_t$, is the only source of revenue. The log of the relative price of the exhaustible resource follows a covariance-stationary AR (1) process:

$$p_{t+1} = \rho p_t + \varepsilon_{t+1}, \tag{5}$$

with p_0 given. In line with empirical findings (e.g., Krautkraemer, 1998, Lin and Wagner, 2007, and references therein), the relative price of the exhaustible resource in the model is assumed to be trendless in the long run.

The economy is inhabited by a representative household with the following CRRA preferences:

$$\sum_{t=0}^{+\infty} \beta^t L_t U(C_t/L_t)\,, \tag{6}$$

where

[5] From the standpoint of economic theory, this assumption can be restrictive. However, existing economic models fail to capture the observed resource extraction behavior (see, e.g., Sickles and Hartley, 2001, and Lin, 2005).

$$U(C_t/L_t)=\begin{cases}\dfrac{(C_t/L_t)^{1-\sigma}}{1-\sigma} & \text{for } 0<\sigma<1 \text{ or } \sigma>1\\ \log(C_t/L_t) & \text{for} \sigma=1\end{cases}. \quad (7)$$

In (6), β is a subjective discount factor, σ determines the degree of risk aversion and intertemporal elasticity of substitution and L_t is the number of members in the representative household.[6]

B. Optimal Solution

To solve the model, it is instructive to recast all quantities in terms of ratios to the non-exhaustible resource sector output, denoting by small letters variables expressed in terms of effective units of labor, e.g. $y_t = \frac{Y_t}{A\,L_t}$. Then the maximization problem can be formulated as:

$$\max_{\{c_t,b_{t+1}\}} E_0 \sum_{t=0}^{+\infty} \widetilde{\beta}^t U(c_t), \quad (8)$$

subject to:

$$c_t + (1+n)b_{t+1} = (1+r)b_t + e^{p_t} z_t + y_t,$$
$$p_{t+1} = \rho p_t + \varepsilon_{t+1}, \quad (9)$$
$$\lim_{S\to+\infty}\left(\frac{1+n}{1+r}\right)^S b_S = 0,$$

where $\widetilde{\beta} \equiv \beta(1+n)$ and b_0, p_0 are given[7].

It is further assumed that $\beta = 1/(1+r)$, which puts the model solution on a balanced growth path after the exhaustible resources run out. The balanced growth path is characterized by a common constant growth rate, n, for consumption, output and the net foreign asset position, which allows output shares, c_t/y_t and b_t/y_t, to remain constant over time.

Deterministic case, $\varepsilon_t = 0$

In the deterministic case, the expression for optimal consumption is:

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[6] A more general utility form would be $L_t^{\eta} U(C_t / L_t)$, where η is a parameter reflecting valuation of future membership. In (6) we implicitly assign equal weight to all years, which represents a natural benchmark.

[7] The problem in (8)-(9) has a solution if the growth rate of the non-exhaustible resource sector is less than the real interest rate: (1+n)/(1+r)<1.

$$c = y + (r-n)b_0 + \frac{r-n}{1+r}\sum_{t=0}^{+\infty}\left[\frac{1+n}{1+r}\right]^t e^{p_t} z_t\,. \qquad (10)$$

Per-period consumption equals the non-exhaustible resource output, the annuity value from the initial net foreign assets and the annuity value from the discounted steam of future exhaustible resource revenues. For a given exogenous income stream, the consumption-output ratio is set at a constant optimal level and the external balance is used to smooth out any variation in income over time. With the expression for consumption-output ratio in hand, it is straightforward to derive the optimal solution for all the other variables of interest.

Stochastic case, $\varepsilon_t > 0$

In addition to considerations covered by the deterministic case, the risk-averse representative household now faces uncertainty about future exhaustible resource revenues and wants to insure against variation in future income. However, access to insurance markets is cut off by assumption. Most notably, ownership of exhaustible resources is assumed to be nontransferable. Although restrictive, this assumption is justifiable on empirical grounds. The household is also cut off from all asset markets, except the risk-free foreign bond.

With this setup, the household solves a simple self-insurance problem. Holdings of net foreign assets are used to lower the exposure to the uncertain income from exhaustible resources. In the model, *precautionary savings* are defined as the change in net foreign assets due to this self-insurance motive – and for any given period they can be calculated as the difference in net foreign asset positions between the deterministic and stochastic versions of the model.[8]

The exhaustible nature of the resource plays a crucial role in model's solution. It makes the model deterministic from period T onwards and ensures a finite optimal value for expected consumption at the infinite horizon. If instead the resource with the stochastic price was renewable, the optimal solution would feature infinite expected consumption (see Ljungqvist and Sargent (2004)).

To accommodate the precautionary savings motive, the optimal consumption in the stochastic model is upward tilting (until resources are exhausted and uncertainty is resolved) and accompanied by a gradual accumulation of a buffer stock of net foreign assets. In initial periods, consumption is lower than in the deterministic case, but as savings accumulate and the interest income from foreign assets grows, it eventually exceeds consumption in the deterministic case. The stochastic version of the model does not have a closed form solution, but can be solved numerically. Details of the solution method are presented in Appendix A.

[8] This definition of precautionary savings differs from the one used by studies of precautionary savings in models with idiosyncratic shocks, as in e.g. Carroll (2001), where precautionary savings are usually defined as change in the distribution of savings in a stationary steady state.

III. Parameterization and Baseline Model Results

This section applies the model to selected exporters of exhaustible resources. The gist of the exercise is to set model parameters and initial values and then solve for the optimal behavior in the model from the perspective of year 2006. Results reported in this section focus on the quantitative estimates of the size of precautionary savings for parameterized model economies, which require solution of both deterministic and stochastic versions of the model. This section also compares model and actual external sector outcomes.

A. Model Parameterization

The model is applied to eleven exhaustible resource economies – Algeria, Iran, Kazakhstan, Kuwait, Libya, Nigeria, Norway, Qatar, Saudi Arabia, United Arab Emirates and Venezuela. Model economies can differ in terms of their labor growth rate, stock of initial net foreign assets, weight of exhaustible resource revenues in total output and projected future exhaustible resource extraction path.

To parameterize the model, one period is taken as one year in the data. For all sample economies the subjective discount rate is assumed to take a value of $\beta = 1/(1.04)$ and the curvature in CRRA utility is set to $\sigma = 6$. Both values are in the range of what is considered standard in the literature.

The parameterization of the price process for exhaustible resources in (5) is based on our estimates of an AR(1) process for the price of oil with annual data for period 1970-2006. The relevant parameter values are $\rho = 0.9$ and $\sigma_{\varepsilon} = 0.25$. The initial value for the price is the 2006 average price in US dollars per barrel, $p_{2006} = 65$, which is above the 1970-2006 average of $\overline{p} = 42$, (expressed in 2006 prices). All sample countries face the same initial price and the same stochastic process for future prices.

The remaining country-specific model inputs are taken from 2006 data or projections. Growth in labor force, n, is set as each country's average projected growth rate in working age population, based on UN population projections over 2010-2050. The initial net foreign asset stock, b_{2006}, is calculated as the share of net foreign assets over total output in 2006, from the most recent update of the External Wealth of Nations data set (Lane and Milesi-Ferretti (2007)). The initial size of the exhaustible resource sector, $e^{p_{2006}} z_{2006}$, is obtained as the share of the exhaustible resource sector in total output in 2006, based on the national accounts. Projections for the path of extraction of exhaustible resources for the next 30 years are taken from the reports on oil and gas extraction quantity projections for 2010, 2015,...,2030 from the EIA (2007). To project beyond the initial 30-year span of that report, we assume that resource extraction proceeds at a constant level until proven reserves of oil and gas, as reported in BP Statistical Review (2007), are exhausted.

To summarize the procedure, the typical exhaustible resource country is parameterized using standard parameter values and functional forms from the Open Macro literature with one notable exception. As of 2006, in addition to the conventional output, the model economy has access to exhaustible resource revenues which amount to a substantial share of total output. Such revenues, however, will be exhausted at some point in the future. The price of exhaustible resources in 2006 is above the expected long run price and the future path for the price is uncertain.

Country-specific model inputs, summarized in Table 1, exhibit considerable variation across the 11 countries in the sample we analyze. For example, with an initial exhaustible resource output share of 0.25, Norway in 2006 is the least dependent on exhaustible resource revenues. At the other end of the spectrum, Libya's exhaustible resource output share is almost three times larger than its non-exhaustible resource output. In terms of the exhaustible resource lifespan, Norway and Algeria are expected to be the first ones to run out of exhaustible resources, by 2043. On the other end of the spectrum, in Qatar and Iran extraction is projected to continue until 2150. Similarly, there is a large variation in the extraction time profiles, as depicted in Figure 2. By 2030 extraction quantities in Norway are projected to decrease by 50 percent, while in Kazakhstan extraction quantities are projected to triple over the same period.

B. Baseline Model Results

To convey the intuition behind the paper's results, this section first presents a detailed optimal model solution for one of the sample countries – Norway. Next, relevant results for the rest of the sample countries are summarized. Finally, the model outcomes are compared with data.

Norway

The model's solution in the case of Norway is summarized in Figure 3. Panels (a) and (b) show the parameterized process for the price of exhaustible resources and the projected future extraction quantities, both of which were discussed above. Panel (c) shows the resulting revenues from exhaustible resources. *Solid* lines in panels (d), (e) and (f) present the optimal solution to the deterministic version of the model. Note that Y_t on y-axis is *non-exhaustible resource output*, which is equal to total output only after exhaustible resources are depleted.

For the deterministic case, per capita consumption is smoothed perfectly over time, as derived in equation (10). To support this constant level of consumption, the representative household accumulates wealth in periods with exhaustible resource revenues, and consumes interest income from the accumulated assets once exhaustible resource revenues run out. The ratio of foreign assets-to-output increases during the period of asset build-up and stays constant thereafter. To achieve this when labor force is growing over time, Norway must run current account surpluses on its balanced growth path. Since constant per capita consumption can be achieved, curvature in the utility function does not play any role in this deterministic case.

How does the optimal solution change when uncertainty about the exhaustible resource price is added to the model? Combined with the CRRA utility, price uncertainty introduces an additional consideration -- the precautionary savings motive -- to the model. The representative consumer will now save more resources to insure against the proverbial 'rainy day' in the future. To accommodate additional savings, the path of expected consumption initially slopes upwards and then converges to a constant expected level (see panel d). In panels (e) and (f) precautionary savings lead to a higher level of net foreign assets, B_t, and a more positive current account position. The difference in the level of net foreign assets and the current account between the two model solutions, i.e., the size of precautionary savings, is displayed separately in panels (g) and (h).

Quantitatively, in the model economy parameterized to Norway, the precautionary savings motive induces an additional current account surplus of about 0.4 percent of non-exhaustible resource output in the initial year, with surpluses gradually decreasing thereafter. The expected accumulated savings due to the precautionary motive on the balanced growth path amount to slightly more than 4 percent of output, as reported in panel (g).

Other exporters of exhaustible resources

The optimal size of precautionary savings in other sample model economies is driven by the same considerations – the uncertainty about future income induces the build-up of a buffer stock of savings and leads to an upward tilting consumption path. At the same time, the dynamics of the current account and foreign asset accumulation depends crucially on the exhaustible resource extraction profile and can therefore differ substantially across sample countries. In Norway, where extraction quantities are decreasing over time, consumption smoothing requires larger current account surpluses during the initial periods, while in Kazakhstan, where extraction quantities are projected to triple, consumption smoothing dictates lower current account surpluses or even deficits in the initial years. In summary, although uncertainty increases the size of the current account balances (relative to the deterministic case) in all sample countries, the sign and time profile of aggregate external savings is determined foremost by the resource extraction profile and consumption smoothing considerations.

Optimal 2006 current accounts for sample countries are presented in Table 2. The first column reports the optimal current account balance for year 2006 from the baseline model parameterization. To understand the driving forces behind the reported balances, it is instructive to decompose them into two additive components presented in the second and third columns. The 'consumption smoothing' component of the current account is the optimal current account from the deterministic model, while the 'precautionary savings' component is calculated as the difference between the optimal external savings in stochastic and deterministic versions of the model.

For a given time profile of the exhaustible resource income, values for the 'consumption smoothing' component of the 2006 current account can be derived analytically using equation (10). Any such results, however, are hard to generalize. To see this, consider the following two scenarios that increase total exhaustible resources by the same amount. In the first scenario, all additional resources are added to the 2006 income. The larger the exhaustible resource income in 2006, *ceteris paribus*, the greater the 2006 value of the consumption smoothing component of the current account. This is the case, since out of the additional current period revenues, only its annuity value should be consumed, with the remainder saved (see equation (10)). In the second scenario, all additional resources are added to extend the lifespan of exhaustible resources. In this case, the longer the lifetime of exhaustible resources, c*eteris paribus*, the smaller the consumption smoothing component. At the extreme, if exhaustible resource income is permanent (and constant over time), its entire value is consumed in each period and consumption smoothing component of the current account is zero. In these two cases, the same increase in the exhaustible resource wealth generates opposite effects on optimal external savings in 2006.

Even when the exhaustible resource wealth is kept constant, the slope of the time profile of exhaustible resource revenues matters for the optimal size of the consumption smoothing component. If the future share of exhaustible resource income in output is considerably larger than the current one, consumption smoothing considerations induce a lower current account balance in the current period. This factor explains why, despite sizable exhaustible resource revenues, the optimal 2006 consumption smoothing component of the current account for Kazakhstan and Qatar in the model is negative. Similarly, rapidly decreasing exhaustible resource weights in output contribute to explaining the large initial current account surplus in Norway.

To help evaluating the contributions of the different aspects of the time profile, Figure 4 shows the current and future exhaustible resource weights in output for each of the sample countries. Sizable differences in the 2006 consumption smoothing component of the current account across sample countries is a result of the large cross-country variation in these weights.

The precautionary savings in the model cannot be derived analytically, but are largely also driven by the current and future exhaustible resource shares in output, as reported in Figure 4. In this case, both larger output shares and longer lifespan of exhaustible resources increase the share of household’s total wealth that is exposed to the uncertain exhaustible resource income. Consequently, precautionary savings increase in both factors - the output share and the lifespan of the exhaustible resources. Applying these criteria to Figure 4 explains why among the 11 sample countries, precautionary savings as a share of output are by far the largest in Qatar and smallest in Norway.[9]

[9] To assess the effect of future income uncertainty on precautionary savings more precisely, one should consider the sum of exhaustible resource revenue shares in output, appropriately discounted. This measure needs to be further adjusted for differences between GDP and GNI as well as the increasing uncertainty associated with future price realizations.

Quantitatively, for the baseline calibration, precautionary savings add anywhere between 0.3 to 25.7 percent of total output to the optimal 2006 current account balance. In the extreme case of Qatar, without the precautionary savings motive, the optimal current account would be negative (owing to a back-loaded extraction profile), while with uncertainty added to the model, the optimal current account balance is a surplus of 9 percent of GDP. The median size of precautionary savings in the sample is 2.5 percent of GDP and the mean is 5.9 percent of GDP. When aggregated over the 11 sample countries, savings induced by the precautionary motive in 2006 add up to 58.3 billion dollars, which amounts to 3.2 percent of the sample's GDP.

How do optimal current account balances for 2006 in the model compare with actual outcomes? Figure 5 presents the answer to this question for the eleven sample countries: its x-axis depicts actual 2006 current account balances and its y-axis represents model outcomes, with and without uncertainty. For the full-fledged precautionary savings model, correlation between actual and model outcomes is 0.69, while for the deterministic model the correlation is 0.41. The figure provides a visual confirmation that the precautionary savings motive can significantly increase the optimal external savings.

Overall, results from the baseline model offer two main findings. First, optimal outcomes from the parameterized model prescribe economically significant precautionary savings for exporters of exhaustible resources. Second, optimal model outcomes for the external sector savings are broadly in line with the actual data, as indicated by the positive correlation of 0.69 between the two variables. Adding precautionary savings motive to the deterministic model improves this correlation.

IV. Sensitivity Analysis

The modeling framework of this paper is a simple one. The price of this simplicity comes in the form of several exogenous sequences and an exogenous stochastic process (i.e., labor force, exhaustible resource quantities and prices), all of which can significantly affect model results. Another important parameter with scarce empirical motivation is the curvature in the CRRA utility, which governs the size of precautionary savings. In view of such concerns, our approach in baseline parameterization has been to lean on the available data as much as possible.

This section, in turn, presents extensive sensitivity analysis of the baseline model results. The examination covers four areas: (i) consumer preference parameters, (ii) the source and the size of growth in the non-exhaustible resource sector, (iii) exhaustible resource extraction quantities and (iv) parameters of the exhaustible resource price process. The results of sensitivity tests are summarized in Table 3. The first column in this table numbers the sensitivity test. The second column describes the type of deviation from the baseline that a particular sensitivity test considers. For example, row 2 looks at the case when risk-aversion parameter in higher than in the baseline while all other parameters and initial values are kept unchanged. The remaining columns report the size of consumption smoothing and precautionary savings components of the current account for each sensitivity test. Results are compared with the baseline case, reported in row 1. To conserve space, for each test only mean and median values for sample countries are reported.[10]

A. Preference Parameters

Precautionary savings depend crucially on the value of risk aversion in the utility function, σ. To see this, note that if $\sigma = 0$, utility is linear and the optimal model solution exhibits no precautionary savings. In order to assess the sensitivity of baseline results to this curvature parameter, model is solved for the case of $\sigma = 2$ and $\sigma = 10$. As expected, changes in precautionary savings are substantial. The lower curvature parameter decreases the size of mean and median precautionary savings for sample countries from 5.9 and 2.5 percent of GDP to 1.9 and 1.1 percent of GDP respectively. The higher curvature parameter increases the same statistic to 10.9 and 3.8 percent of GDP respectively (see rows 1, 2 and 3 in Table 3). Since curvature of the utility function does not affect the optimal solution in the deterministic case, the consumption smoothing component of the current account is unaltered and any change in total 2006 current account is entirely due to changes in precautionary savings.

For all considered values of the curvature, the main findings from the baseline model parameterization remain valid. Although with $\sigma = 2$ precautionary savings are reduced, they remain economically significant.

Next, consider the effect of a change in the subjective discount factor, β, reported in rows 4 and 5 of Table 3. Heavier discounting lowers the net present value of exhaustible resource wealth, but at the same time increases the risk-free interest rate that puts the economy on a stable growth path. The net effect for model economies is an increase in the annuity value of exhaustible resource wealth, which increases consumption and lowers savings, as captured by the lower consumption smoothing component of the current account. Higher levels of consumption and less savings imply that a larger share of total income is derived from the uncertain exhaustible resources. In response to this increased uncertainty, the optimal level of precautionary savings raises.

Within the range of considered values of the discount factor, sensitivity tests show relatively minor changes in the consumption smoothing or precautionary savings component of the current account. Furthermore, since the effect on the two components have the opposite sign, the overall impact on the current account balance is muted.

B. Growth in Non-Exhaustible Resource Output

In the model of Section 2, labor is the only source of growth in the non-exhaustible resource sector. The effect of changes in the labor growth rate on the two current account components is

[10] More detailed tables with country-by-country results for each sensitivity test are available from the authors upon request.

similar to that of the subjective discount factor. In the deterministic model, lower labor growth rate increases the per worker return on exhaustible resource wealth, thus raising the level of consumption and lowering the consumption smoothing component of the current account. In addition, it increases the share of future income from uncertain exhaustible resources, which results in more precautionary savings as a share of domestic output.

Change in the sample mean and median levels of the two current account components from 0.5 percentage points higher and lower labor growth rates is reported in rows 6 and 7 of Table 3. Main findings from the baseline parameterization remain valid.

How restrictive is the model's assumption of zero productivity growth in the non-exhaustible resource sector? During 1970-2006, the average growth rate for real output per worker in Norway was close to 2 percent, suggesting that this assumption is indeed restrictive.

To address such concerns, row 8 in Table 3 presents results for the case when time-varying productivity is added to the modeling framework of Section 2. This is done by substituting A_t for A in (1) and assuming that $A_{t+1}=(1+g)A_t$, where $g=0.02$ is the productivity growth rate. With this specification, the interest rate on the stable growth path needs to satisfy $R=(1+g)^{\sigma}/\beta$, which for baseline parameter values implies a 17 percent risk free annual return. Although productivity growth affects external savings though several channels, quantitatively, for this specification, the effect of the substantially higher interest rate dominates. It affects the two components of the current account the same way as an increase in the discount rate – it lowers the consumption smoothing component and increases precautionary savings.

Because it is hard empirically to justify such a high interest rate, an alternative specification is also considered. Assume that instead of the utility specified in (6), the household maximizes consumption per effective unit of labor, $U(C_t/Y_t)$. Under this specification, the interest rate on the stable growth path satisfies $R=(1+g)/\beta$, which with baseline parameters amounts to 6 percent.

Results for this sensitivity test are presented in row 9. In contrast to the previous case, the consumption smoothing component of the current account now increases relative to the baseline, because interest rate increase is smaller and the added growth in the non-exhaustible resource sector makes exhaustible resource revenues relatively more temporary. Consequently, more of the revenues are saved. The added growth also decreases economy's dependence on uncertain exhaustible resources, and therefore lowers precautionary savings.

Both 'productivity growth' scenarios show that, although levels of precautionary savings can be significantly altered, the main finding from the baseline model – economically non-negligible levels of precautionary savings – survives introduction of productivity growth into the model.

Furthermore, Norway has outperformed the rest of the sample in terms of labor productivity. For the sample as a whole, in line with the findings of the literature on the 'resource curse' (see e.g. Sachs and Warner (2001)), the average labor productivity growth rate over the 1970-2006 period is close to zero and, thus, supports the baseline parameterization.

C. Path and Lifespan of Exhaustible Resource Extraction

The exhaustible resource extraction path for the model is obtained by combining estimates of proven oil and gas reserves and projected future extraction quantities until 2030. However, these underlying estimates and projections are *highly* uncertain, because, among other factors, (i) much of the input data are self-reported and unverified by an independent source; (ii) estimates such as 'proven reserves' by definition cover only a fraction of the country's exhaustible resource potential; and (iii) future extraction technologies and costs are uncertain. Furthermore, available estimates might be biased. Some experts have argued that OPEC countries over report the level of proven reserves (see, e.g., Zittel and Schindler (2007)). At the same time, the limited coverage of concepts such as 'proven reserves' suggests that underreporting is equally likely.

Sensitivity analysis of model's results with respect to exhaustible resource quantities covers three different scenarios. In the first scenario, the total reserves of the exhaustible resource are changed by varying the weight of exhaustible resource income in total output in each period, keeping the lifespan of resources fixed. In the second scenario, the change in total exhaustible resource reserves is achieved by varying the lifespan of reserves, keeping the exhaustible resource output weights constant. In both cases we consider a 10 percent increase/decrease in total exhaustible resource reserves. The third scenario considers the case of constant extraction profiles, which is a common assumption in the literature (see e.g. Thomas et al. (2008)). In this case it is assumed that the path of exhaustible resource extraction does not vary over the initial period until 2030. Instead, it remains constant at the 2006 level until proven reserves are exhausted.

Results for the first two scenarios are reported in rows 10-13 of Table 3. As already conveyed in discussion of the results in the baseline model, an increase in per-period quantities of exhaustible resources should increase precautionary savings as well as the consumption smoothing component of the current account. In contrast, an increase in the lifespan of exhaustible resources should increase precautionary savings, but decrease the consumption smoothing component. Overall, a 10 percent change in total exhaustible resource reserves leads to relatively minor deviations from the baseline results. Notice that a change in reserves has a smaller effect when introduced by varying the lifespan of exhaustible resource, because intertemporal discounting substantially lowers the effect of additional revenues from period T onwards on the net present value of exhaustible resource wealth.

Since results for constant extraction profiles vary considerably across countries, they are reported separately country-by-country in Table 4. When a constant extraction profile is assumed instead of an increasing one, the consumption smoothing component of the current account increases and precautionary savings decrease. This effect drives the ‘constant extraction quantities’ results for all sample countries except Norway, where extraction in the baseline is declining over time and the opposite result holds. Countries with the steepest extraction profiles, i.e. Qatar and Kazakhstan, exhibit a very sizable decrease in precautionary savings and increase in consumption smoothing component of the current account. In case of a flat extraction profile, the current account is by construction restricted to non-negative values. For countries where baseline extraction profiles are close to flat, e.g., Iran and Venezuela, deviations from the baseline specification have only a minor effect on the optimal external position.

D. Process for Exhaustible Resource Prices

The parameterized process for the exhaustible resource price is another crucial input in the baseline model. Taking as given the appropriateness of the AR(1) process for our exercise, rows 14-17 in Table 3 report sensitivity results with respect to two key inputs – persistence of the AR(1) process and its expected mean value.

A more persistent price process increases the size of precautionary savings. When combined with an above average exhaustible resource price, it also makes the convergence of the expected price back to its mean value more gradual, increasing the 2006 consumption smoothing component of the current account. As a result, the total current account balance also increases. Quantitative effects from altering the persistence of the price process can be substantial. For example, increase in the persistence from $\rho=0.90$ to $\rho=0.95$ more than triples the size of precautionary savings for the median country in the sample, while decreasing it to $\rho=0.85$ cuts precautionary savings by a half.

Higher expected mean value of the price process decreases the 2006 consumption smoothing component of the current account, but increases precautionary savings. The former decreases, because higher expected average price implicitly lowers the size of the initial positive 2006 price shock, or could even reverse its sign. With a smaller positive shock it is optimal to consume more of the current period’s exhaustible resource income. In contrast, optimal precautionary savings increase, since higher average price raises the weight of exhaustible resources in total output and thus increases uncertainty about the total future wealth.

This result offers an interesting insight into the optimal behavior for exhaustible resource exporters. Recent persistent increase in the price of exhaustible resource has motivated many exporting countries (correctly or incorrectly) to increase the expected long-run price of the resource, which in turn justifies higher current consumption levels. However, it should be borne in mind that higher consumption levels embed a discount to account for the implicitly assumed increase in the weight on the economy of the uncertain exhaustible resource revenues in the long run.

Quantitatively, an increase in the mean of the exhaustible resource price by two-standard-deviations increases precautionary savings on average by 30 percent. For all sample counties consumption smoothing considerations dominate precautionary savings, and the aggregate current account balance falls.

Finally, we should note that for all sensitivity tests the correlation between the current account in the model and data remains above 0.5 (above 0.64, if the case of $\sigma = 2$ is excluded). Furthermore, in all cases the correlation on the stochastic model was higher than in the deterministic one.

V. Extension to Time-Series of Optimal outcomes

Each year can potentially bring a new exhaustible resource price shock and new information about future exhaustible resource quantities. In response, the optimizing household should re-solve the infinite horizon optimization problem, taking into account the latest information. Since the baseline exercise in Section III considers only one such shock, an obvious question follows: how does the size of precautionary savings in 2006 compare to other years?

To answer the question, this section extends the baseline exercise to cover multiple historical price shocks. In particular, it compares the size of precautionary savings for 2006 with estimates for each year between 1974 and 2006, using the parameterization for Norway (chosen because of better data coverage). In addition, this extension allows comparison of historical current account data with optimal model outcomes, presented at the end of this section.

A. Setup and Parameterization

The application is implemented in the following steps. First, using the estimated exhaustible resource price process, we interpret historical prices as representing positive or negative price shocks. Figure 6 shows the actual exhaustible resource price for each year between 1974 and 2006. When actual prices are above (below) expected prices in the parameterized AR(1) process, they represent a positive (negative) shock to the exhaustible resource price.

Second, Figure 7 shows yearly updates for the sum of future exhaustible resource quantities, proxied by changes in proven oil and gas reserves. After accounting for current period extraction, changes in this measure represent new information about future exhaustible resource quantities. Both prices and quantities of exhaustible resources show sizable changes over time.

Next, given the exhaustible resource price, we set the extraction quantity in the first period so that the model matches the share of mining to non-mining GDP in data. Then, using annual data for proven reserves from BP Statistical Review (2007), we calculate the lifetime of reserves, assuming a constant extraction profile, i.e. continued extraction at the current (i.e., year ‘t’) levels going forward. This procedure is repeated for each year of interest. Note that here we deviate from the time-varying exhaustible resource quantity profile that was assumed for 2006-2030 in the baseline exercise. This is done, since data on future extraction profiles from the perspective of each of the years between 1975 and 2006 are not available. The remaining model parameter values are the same as in the exercise of the previous section and the initial net foreign asset position for Norway is taken from data for 1974 (Lane and Milesi-Ferretti (2007)).

This exercise amounts to sequentially solving the infinite horizon model for each year of interest. Optimizations for subsequent years are connected though the optimal choice of net foreign assets, b_{t+1}. In particular, the optimal choice of b_{t+1} from the perspective of year t is taken as the initial value, b_t, for the problem that is solved from the perspective of a subsequent year. Both deterministic and stochastic versions of the model are solved.

B. Results

Figure 8 reports the results of this exercise. For each year, it presents the optimal consumption-savings decision given: (i) the exhaustible resource price shock, (ii) the exhaustible resource quantity shock and (iii) the initial net foreign asset position. Notice that all variables represent actual realizations, rather than expected values, since only the optimal decisions for the 'current' year are included in the solution reported in Figure 8, i.e., for year 't' only optimal response of c_t and b_{t+1} are reported in the figure.

In contract to Figure 2, precautionary savings are not 'front-loaded' anymore. Instead, they correlate positively with the level of proven reserves of oil and gas (see Figure 7). As proven reserves tripled over the 1980-2000 period, the difference in current account surpluses between the deterministic and stochastic versions of the model increase from less than 0.5 percent of non-exhaustible resource output in the early 80s to more than 1 percent of output by 2000 (see panel d in Figure 8). As a result, over the 1975-2006 period accumulated precautionary savings amount to 17 percent of output (panel c) and 10 percent of total external savings.

Figure 9 compares model outcomes with historical current account and net foreign asset data for Norway. The model's fit with data is qualitatively encouraging, despite the failure of the baseline parameterization to capture quantitatively the current account deficits in the late 70s and the size of surpluses during the post-2000 period.

VI. Conclusions

This paper introduces precautionary savings motive into the framework of a deterministic small open economy model to study optimal consumption-savings choices in economies with exhaustible resources. It also contributes to the literature by introducing aggregate uncertainty into the model without imposing a unique stationary equilibrium.

Results show that allowing for uncertainty in the price of exhaustible resources significantly increases the optimal level of saving in the model economies, more so for countries where exhaustible resources dominate economic activity. With the baseline parameterization the median size of precautionary savings in the sample is 2.5 percent of GDP and the mean is 5.9 percent of GDP. When aggregated over all sample countries, precautionary savings in 2006 add up to 58.3 billion dollars, which amounts to 3.2 percent of the sample's GDP.

The model fares well at capturing savings/current account behavior in exhaustible resource countries. The correlation between the optimal 2006 current account in the model and data for 11 sample countries is 0.69. Model also does reasonably well at capturing historical current account time series data for Norway. Extensive sensitivity analysis shows that the main finding from the baseline model – economically non-negligible levels of precautionary savings – is not driven by the particular parameter values used in the baseline parameterization.

There are several important issues that are left for future research. First, the model cannot easily incorporate the effect of productivity growth on the optimal consumption-savings outcomes. While several ways to add productivity growth to the model were examined, none of them is fully satisfactory.

Second, the 'small open economy' assumption is employed throughout the paper. When taken separately, each of the exhaustible resource economies is likely too small to affect outcomes in the rest of the world. However, as a group, exhaustible resource countries can generate sizable savings, which could affect the world interest rate.

Next, we have abstracted from investment requirements for the extraction of exhaustible resources. For economies that are in the process of expanding their exhaustible resource output, e.g., Kazakhstan, this assumption might be problematic, since expansion of the resource extraction capacity requires large upfront investments. In such instances, current account deficits might be driven not only by the consumption smoothing and precautionary savings motives, but also by the need for investment.

Finally, we do not model the uncertainty surrounding future extraction quantities. Data suggests that for various reasons there is a considerable amount of such uncertainty at the yearly time horizon. Similar concerns apply to the estimates of the total stock of available exhaustible resources. Since extraction quantities do not appear to respond systematically to prices, it is then likely that exhaustible resource countries face more uncertainty about future revenues than our model allows for.

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APPENDIX A: SOLUTION METHOD (CASE OF *n* = 0)

The problem we want to solve is:

$$
\begin{aligned}
& Max_{\{C_t\}_{t_0}^{\infty}} E_{t_0}\left[\sum_{t=t_0}^{\infty} \beta^t U\left(C_t\right)\right] \\
& s.t. \quad B_{t_0} = B_0 \\
& \qquad B_{t+1} = B_t(1+r) + Y_t - C_t \\
& \qquad Y_t = Y^{no} + Z_t P_t \\
& \qquad \{P_t\} \text{ is a random sequence}
\end{aligned}
$$

This problem does not have a steady-state solution, so one needs a shortcut to solve it. The shortcut we use is to shut down the randomness of the model beyond the depletion date.

Then we can solve it recursively. In the first period after depletion, the consumption-savings decision going forward is:

$$
\begin{aligned}
& V\left(B_T\right) = Max_{\{C_t\}_{T}^{\infty}} E_{t_0}\left[\sum_{t=T}^{\infty} \beta^t U\left(C_t\right)\right] \\
& s.t. \quad B_{t+1} = B_t(1+r) + 1 - C_t \\
& \qquad Y_t = Y^{no}
\end{aligned}
$$

This problem has a closed-form solution (assuming for now zero growth in the non-oil sector and a CRRA utility function):

$$
V\left(B_T\right) = \frac{\left(Y^N + rB_T\right)^{1-\sigma}}{(1-\beta)(1-\sigma)}
$$

For each of N values for B_T (it can be a large positive or negative number), we can calculate the value of the program going forward.

Then we can solve it recursively back to the initial period. For T-1, output in the exhaustible resource sector is still positive, but the uncertainty with regards to oil prices is revealed before the consumption decisions, so the problem to be solved is:

$$
\begin{aligned}
& V_{T-1}\left(B_{T-1}, P_{T-1}\right) = Max_{\{C_{T-1}\}}\left[U\left(C_{T-1}\right) + \beta V_T(B_T)\right] \\
& s.t. \quad B_T = B_{T-1}(1+r) + 1 + Z_{T-1}P_{T-1} - C_{T-1}
\end{aligned}
$$

This problem can be solved numerically over a grid on B_{T-1}, P_{T-1} with NT nodes.

For all periods up to *T-2*, the program has to incorporate the uncertainty over prices going forward:

$$
\begin{aligned}
&V_t\left(B_t, P_t\right) = Max_{\{C_t\}}\left[U\left(C_t\right) + \beta E_t V_{t+1}(B_{t+1}, P_{t+1})\right] \\
&s.t. \quad B_{t+1} = B_t(1+r) + 1 + Z_t P_t - C_t
\end{aligned}
.
$$

Table 1: Country-Specific Model Parameters and Initial Values

	Labor force growth rate, %	Initial NFA position, % of GDP	Initial ER revenues, % of GDP	Proven ER reserves, bn brls oil equivalent	Lifetime of ER
Algeria	0.7	64	47	41	2043
Iran	0.5	37	27	314	2150
Kazakhstan	0.1	-38	32	59	2055
Kuwait	1.0	226	58	113	2083
Libya	0.9	280	73	50	2075
Norway	0.2	72	25	27	2043
Nigeria	2.1	33	38	69	2045
Qatar	0.7	122	62	175	2150
Saudi Arabia	1.5	102	54	309	2064
United Arab Emirates	1.1	161	37	136	2087
Venezuela	0.9	28	35	107	2096

Data sources: IMF World Economic Outlook, IMF International Financial Statistics, British Petroleum (2007) Energy Information Administrations (2007), United Nations (2006), Lane and Milesi-Ferretti (2007).
Note: Parameters and initial values not reported in the table are the same for all countries.

Table 2: Optimal 2006 Current Accounts in the Baseline Model

	CA in 2006, % of GDP	of which:	
		Consumption smoothing	Precautionary savings
	(1)	(2)	(3)
Algeria	20.7	17.2	3.5
Iran	8.8	7.7	1.1
Kazakhstan	-4.6	-10.8	6.1
Kuwait	25.4	19.8	5.6
Libya	46.1	30.7	15.4
Norway	15.0	14.7	0.3
Nigeria	22.7	21.6	1.1
Qatar	10.5	-15.2	25.7
Saudi Arabia	32.0	29.5	2.5
United Arab Emirates	13.1	10.7	2.4
Venezuela	15.4	14.0	1.5
Sample mean	18.6	12.7	5.9
Sample median	15.4	14.7	2.5

Source: Authors' calculations

Table 3: Summary of Sensitivity Analysis for Current Account Components

Row number	Deviation from baseline	Consumption smoothing, % of GDP		Precautionary savings, % of GDP	
		Mean	Median	Mean	Median
		(1)	(2)	(3)	(4)
1	Baseline	12.7	14.7	5.9	2.5
2	$\sigma' = \sigma + 4$	12.7	14.7	10.9	3.8
3	$\sigma' = \sigma - 4$	12.7	14.7	1.9	1.1
4	$\beta' = \beta + 0.005$	15.0	15.6	5.2	2.0
5	$\beta' = \beta - 0.005$	10.9	13.9	6.8	3.0
6	$n_i' = n_i + 0.005\ \forall i$	18.5	17.6	3.9	1.6
7	$n_i' = n_i - 0.005\ \forall i$	6.9	11.8	8.2	3.6
8	$A_{t+1} / A_t = 0.02\ \forall t$ (Case 1)	10.3	10.0	8.9	3.6
9	$A_{t+1} / A_t = 0.02\ \forall t$ (Case 2)	24.5	23.0	2.5	1.2
10	$z_t' = 0.9 z_t\ \forall t$	11.5	13.2	4.8	2.1
11	$z_t' = 1.1 z_t\ \forall t$	13.9	16.1	7.2	2.9
12	$T_i' = 0.9 T_i\ \forall i$	13.4	14.8	5.7	2.4
13	$T_i' = 1.1 T_i\ \forall i$	12.2	14.6	6.0	2.5
14	$\rho' = \rho - 0.05$	11.6	13.5	3.9	1.2
15	$\rho' = \rho + 0.05$	13.4	17.4	13.7	7.9
16	$\overline{p}' = \overline{p} - 10$	18.8	17.9	3.2	1.4
17	$\overline{p}' = \overline{p} + 10$	7.0	12.2	8.1	3.1

Source: Authors' calculations

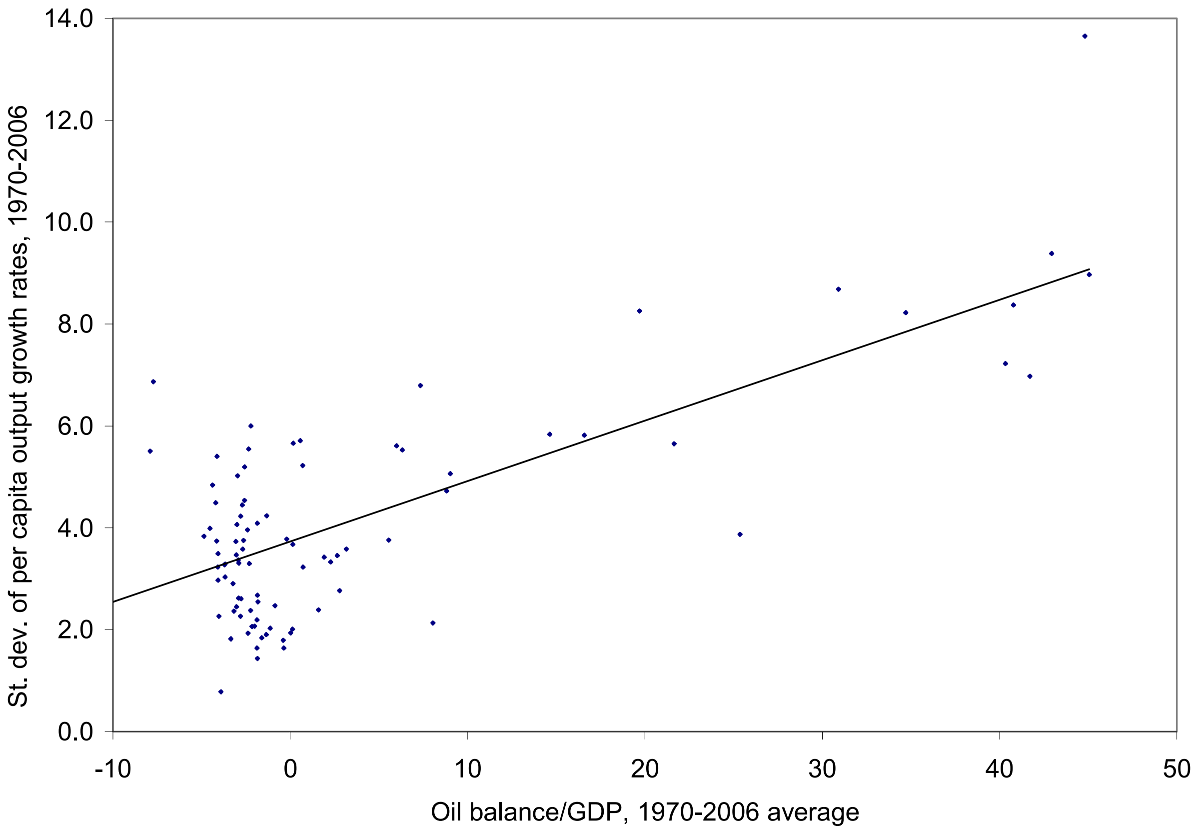
Note: In the column reporting the type of deviation from baseline 'primed' parameters represent the deviation and subscript *i* indexes sample countries.

Table 4: Optimal Current Account with Constant Extraction Quantities

	Consumption smoothing, % of GDP		Precautionary savings, % of GDP	
	baseline	$z_t=z_{2006}, \forall t$	baseline	$z_t=z_{2006}, \forall t$
	(1)	(2)	(4)	(5)
Algeria	17.2	21.7	3.5	2.4
Iran	7.7	9.2	1.1	0.9
Kazakhstan	-10.8	8.5	6.1	1.7
Kuwait	19.8	28.1	5.6	3.5
Libya	30.7	34.9	15.4	12.4
Norway	14.7	13.3	0.3	0.5
Nigeria	21.6	25.9	1.1	0.5
Qatar	-15.2	24.4	25.7	5.2
Saudi Arabia	29.5	31.3	2.5	2.0
United Arab Emirates	10.7	19.0	2.4	1.2
Venezuela	14.0	15.4	1.5	1.3
Sample mean	12.7	21.1	5.9	2.9
Sample median	14.7	21.7	2.5	1.7

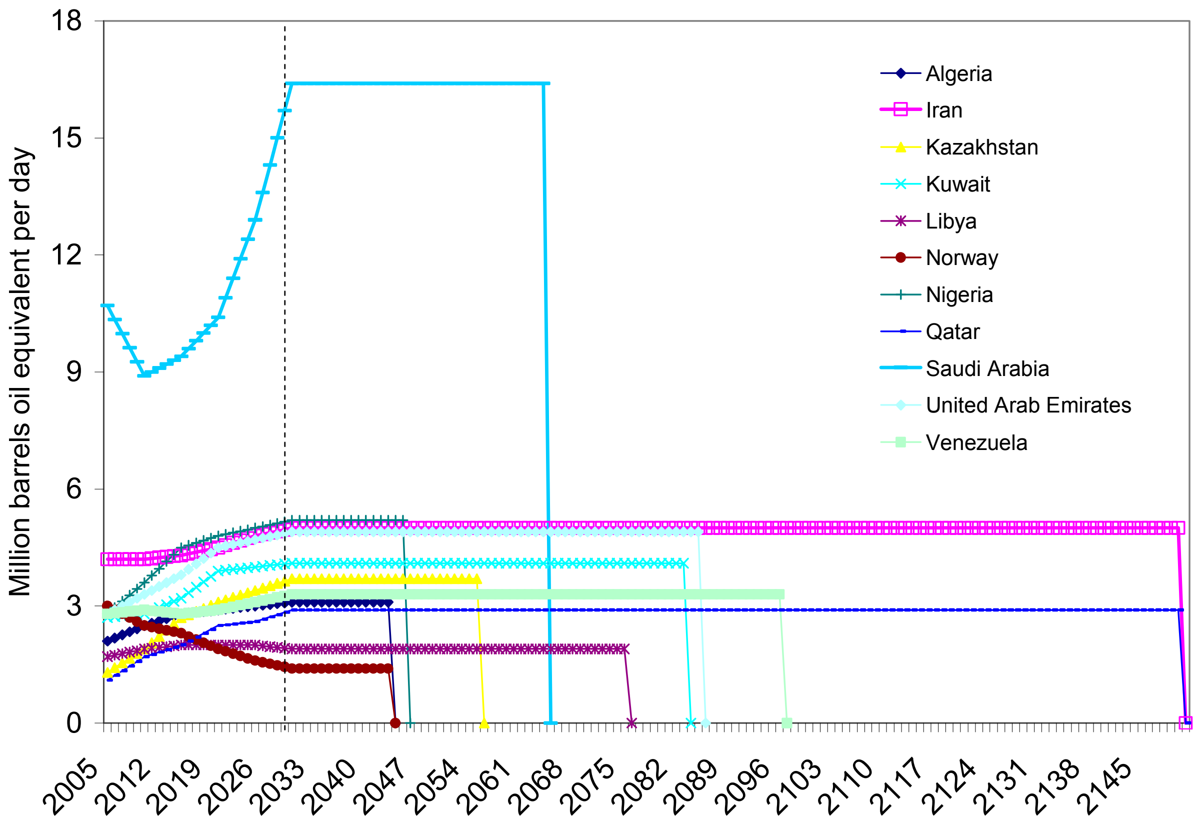
Source: Authors' calculations

Figure 1: Output Volatility and Oil Balance



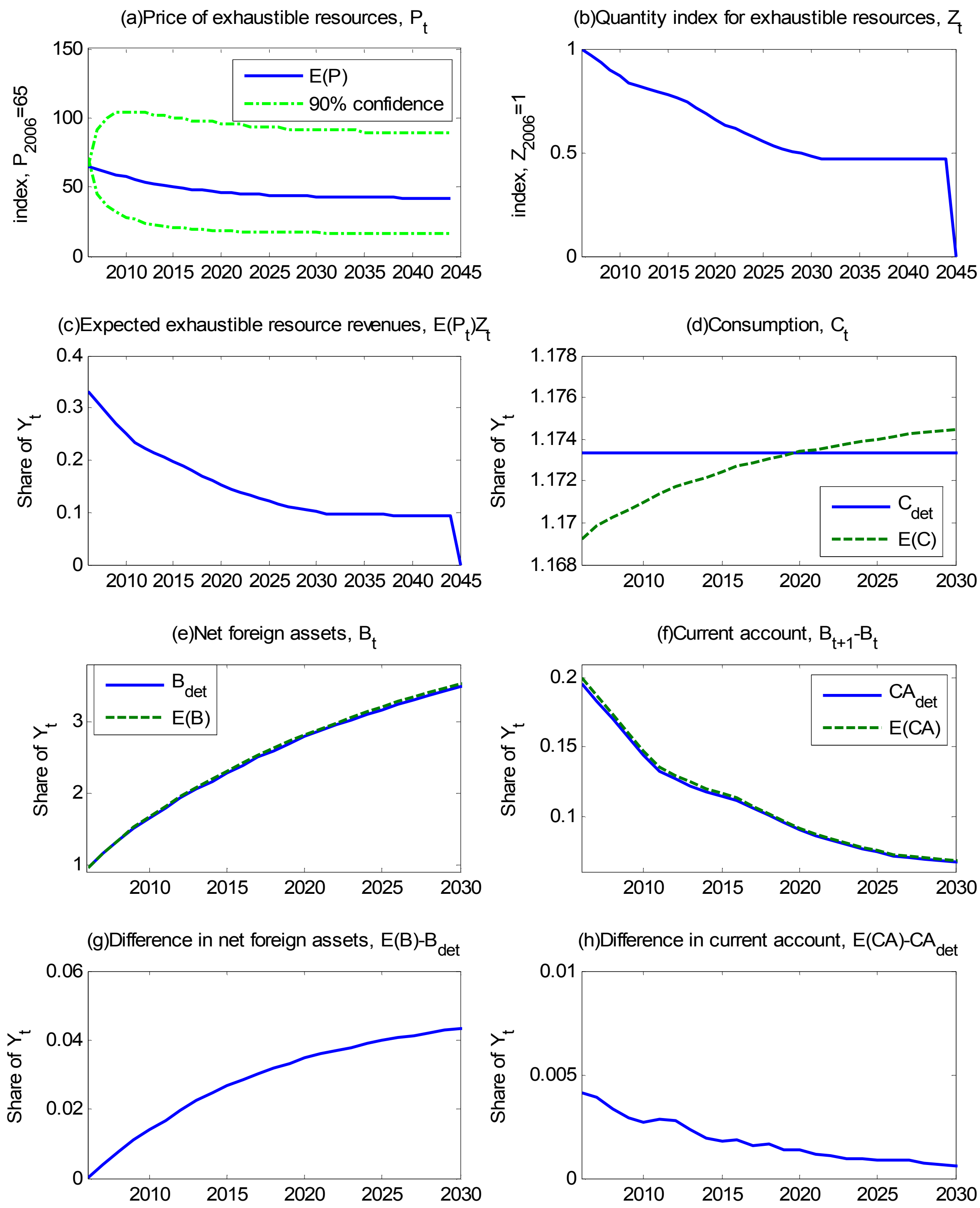
Source: IMF World Economic Outlook

Figure 2: Projected Production of Liquids (oil and gas)



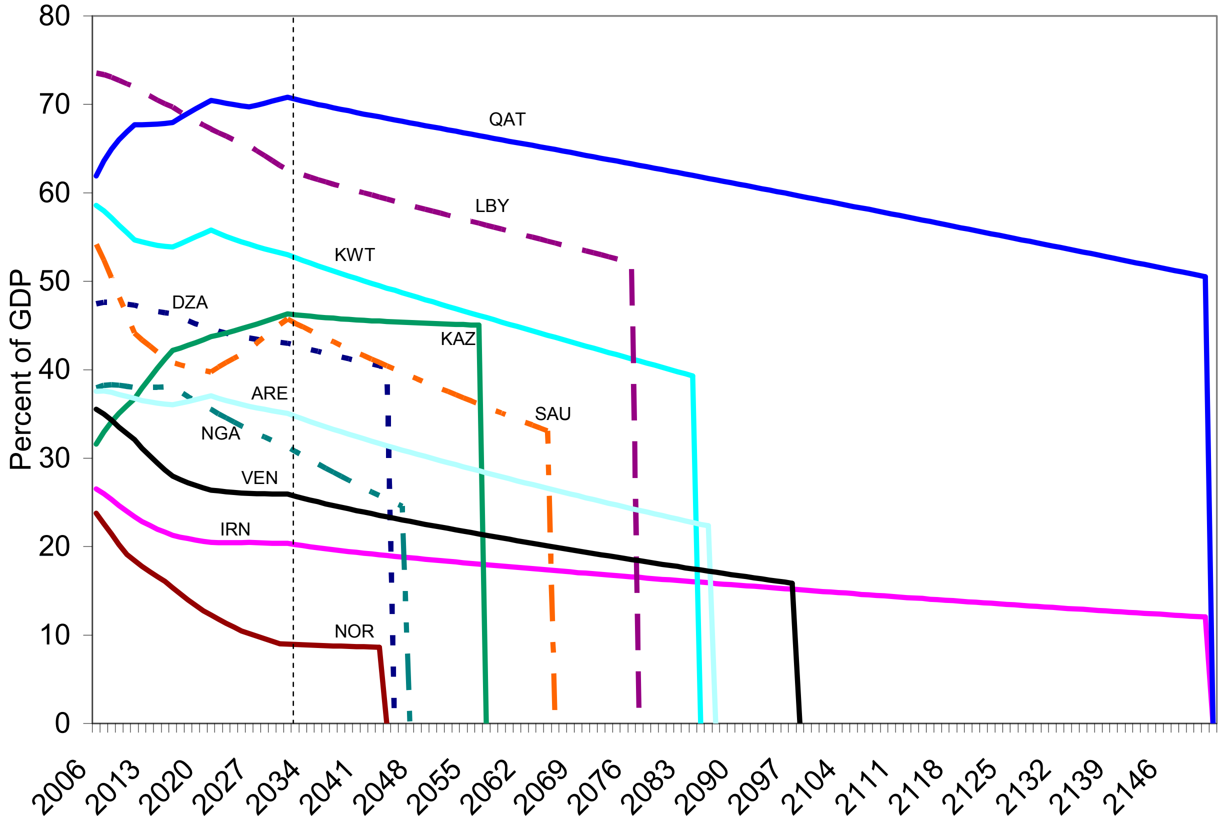
Source: Data for 2005, 2010, 2015,…,2030 from Energy Information Administration (2007). Quantities for other years between 2005-2030 extrapolated using a linear trend. After 2030 production quantities kept constant until proven reserves, as reported in column 4 of Table 1, are exhausted.

Figure 3: Optimal Model Solution for Norway, t_0=2006



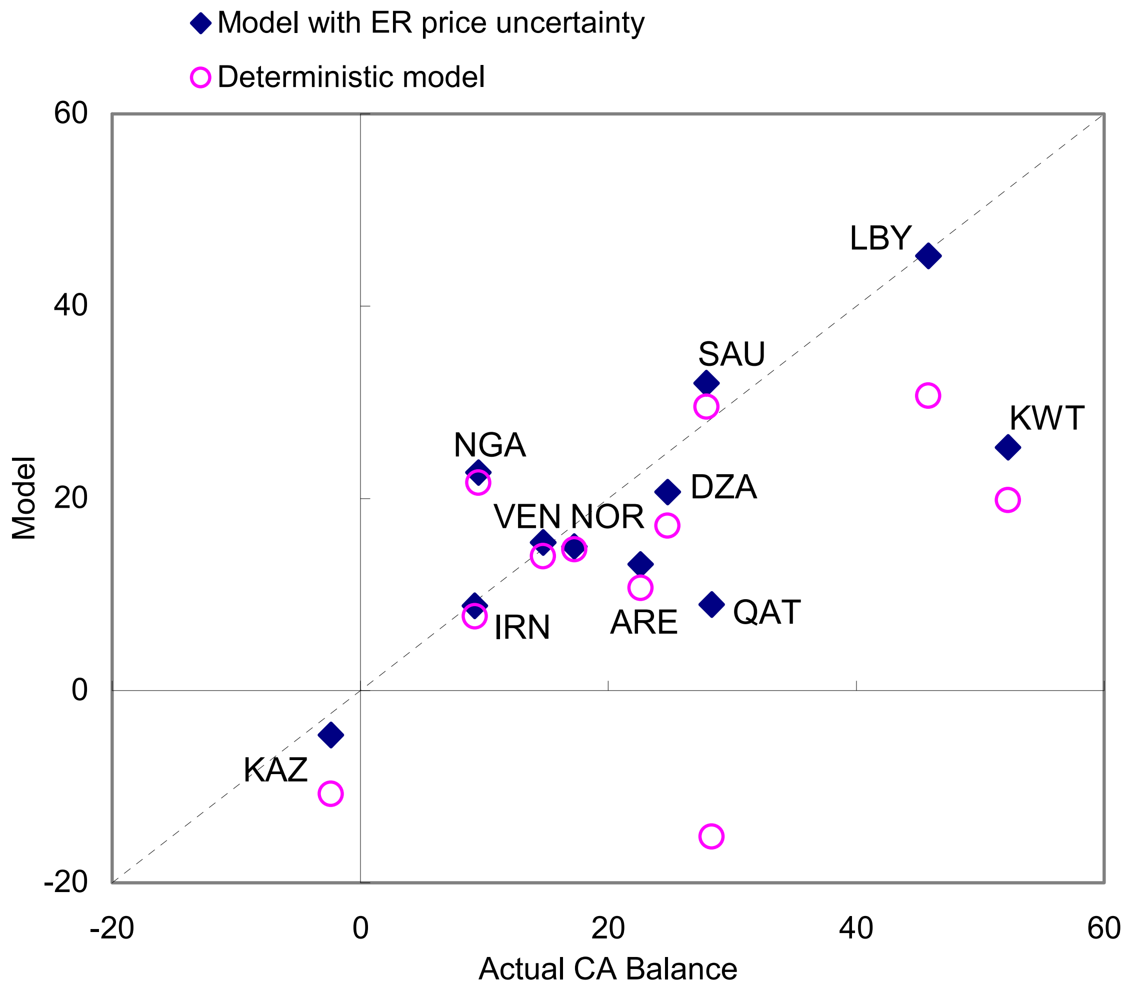
Source: Author's calculations

Figure 4: 'Output at Risk': Expected Share of Exhaustible Resource Revenues in GDP



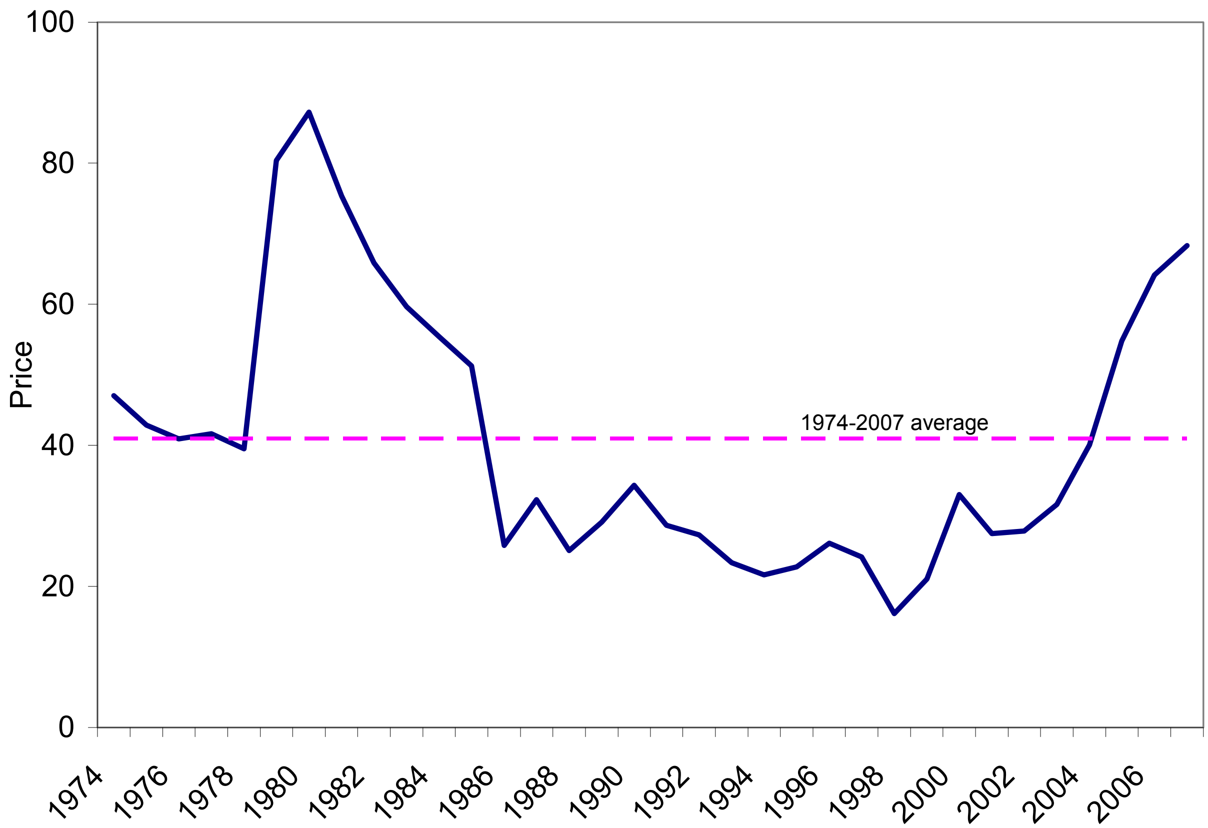
Source: Authors' calculations

Figure 5. Actual and Model-Based 2006 Current Account Balances (in percent of GDP)



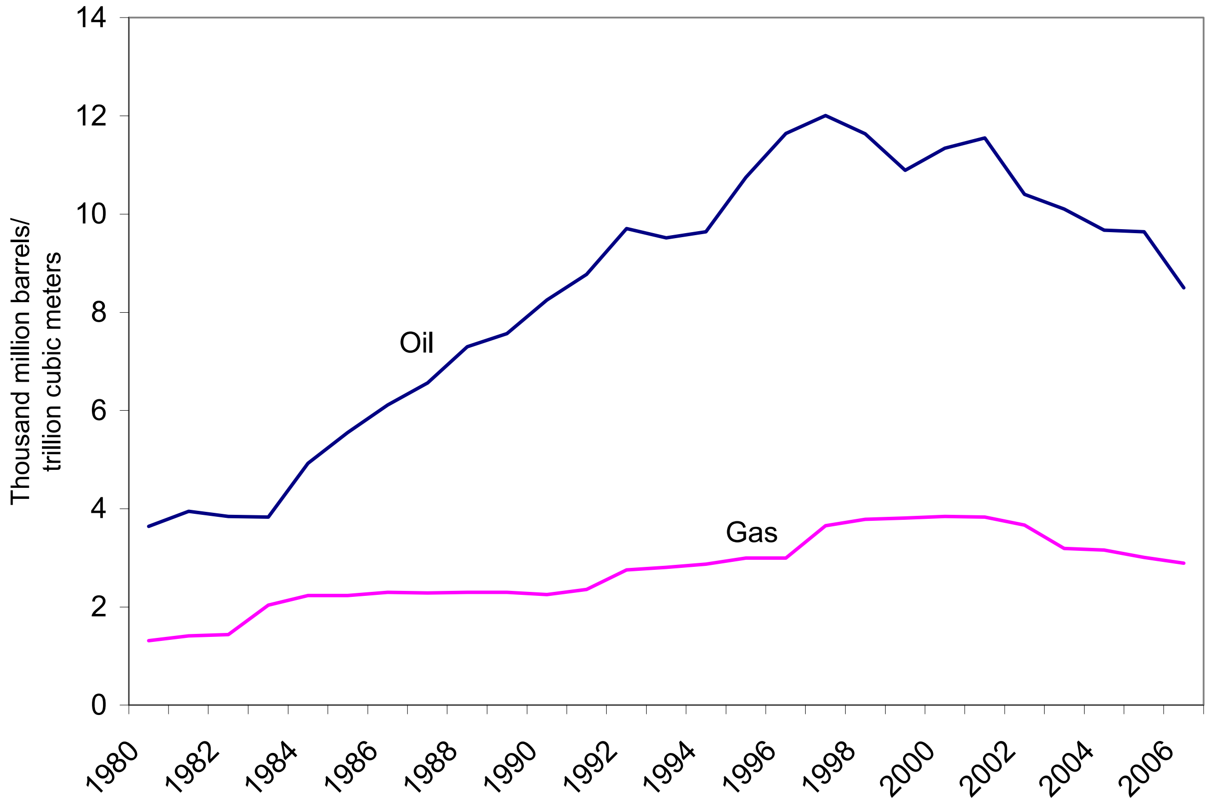
Source: IMF World Economic Outlook and authors' calculations

Figure 6: Price of Exhaustible Resources, in 2006 Dollars



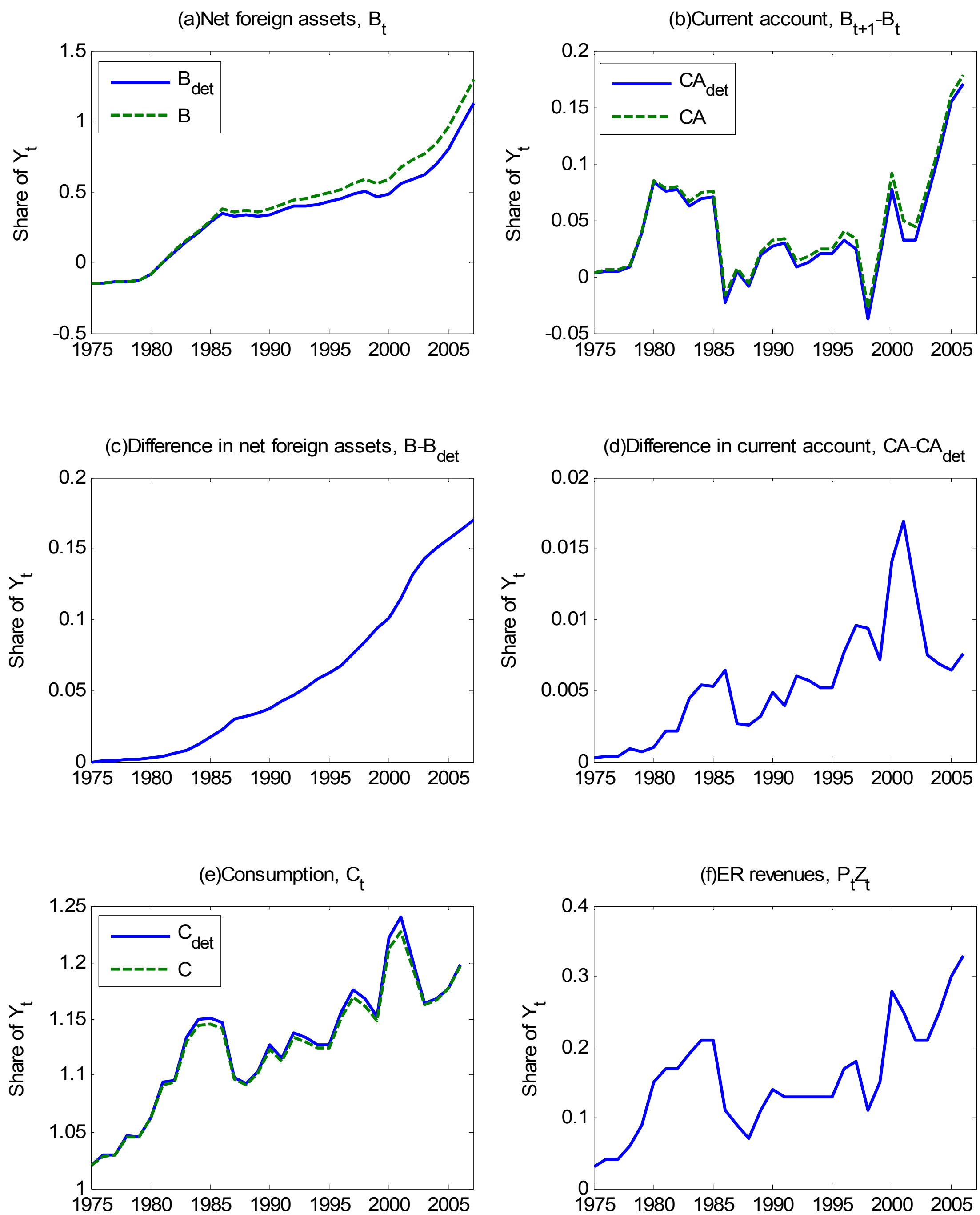
Sources: British Petroleum (2007) and authors' calculations

Figure 7: Proven Reserves of Oil and Gas for Norway



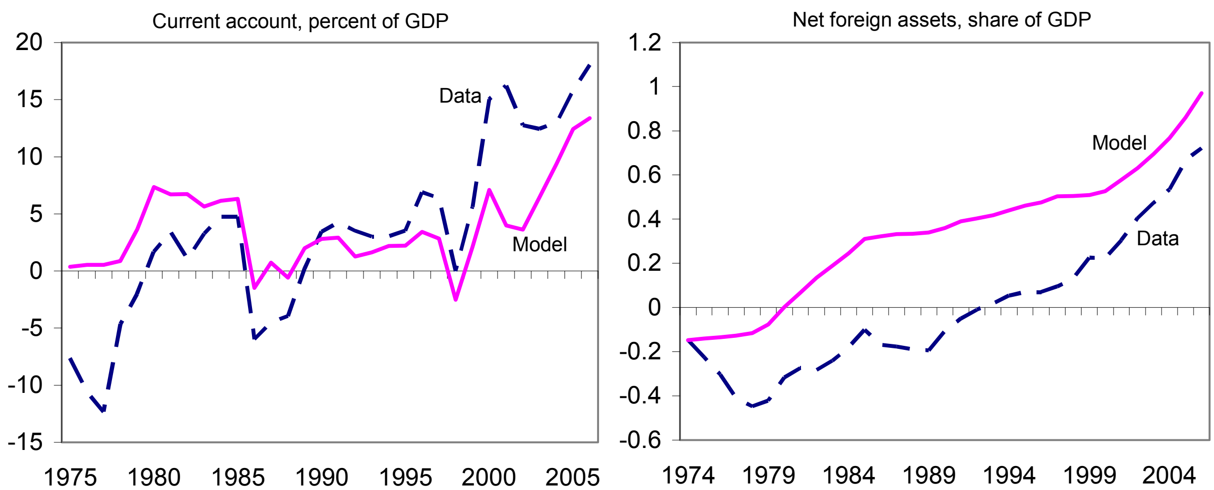
Source: British Petroleum (2007)

Figure 8: Time-Series of Optimal Outcomes for Norway, t_0={1975,1976,…,2006}



Source: Authors' calculations

Figure 9: Comparison of Norway's CA and NFA in the Model and Data



Source: Authors' calculations