

# IMF Working Paper

## Incorporating Financial Sector Risk into Monetary Policy Models: Application to Chile

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Monetary and Capital Markets Department

**Incorporating Financial Sector Risk into Monetary Policy Models: Application to Chile****Prepared by Dale Gray,<sup>1</sup> Carlos García, Leonardo Luna, and Jorge E. Restrepo**

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**Abstract****This Working Paper should not be reported as representing the views of the IMF.**

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

This paper builds a model of financial sector vulnerability and integrates it into a macroeconomic framework, typically used for monetary policy analysis. The main question to be answered with the integrated model is whether or not the central bank should include explicitly the financial stability indicator in its monetary policy (interest rate) reaction function. It is found in general, that including distance-to-default (*dtd*) of the banking system in the central bank reaction function reduces both inflation and output volatility. Moreover, the results are robust to different model calibrations: whenever exchange-rate pass-through is higher; financial vulnerability has a larger impact on the exchange rate, as well as on GDP (or the reverse, there is more effect of GDP on bank's equity—i.e., what we call endogeneity), it is more efficient to include *dtd* in the reaction function.

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## I. INTRODUCTION

The integration of the analysis of financial sector vulnerability into macroeconomic models is an area of important and growing interest for policymakers, in both developed and emerging markets. Monetary policy models and financial stability models, by their nature, are very different frameworks. Monetary policy models are widely used by central banks to understand the transmission mechanisms of interest rates to the macroeconomy and inflation. On the other hand, estimating the effect of shocks to vulnerability on the risk of banks in a coherent manner requires both a model of banking sector risk and a tractable methodology for simulating shocks and estimating their effect on various risk measures.

This article analyzes whether monetary policy models should include market-based financial stability indicators (FSIs)<sup>2</sup> or not, and, if so, how. Since the economy and interest rates affect financial sector credit risk, and the financial sector affects the economy, this paper builds a model of financial sector vulnerability and integrates it into a macroeconomic framework, typically used for monetary policy analysis. More specifically, we use the model to answer the question whether the central bank should include explicitly the financial stability indicator in its monetary policy (interest rate) reaction function. The alternative would be to react only indirectly to financial risk by reacting to inflation and GDP gaps, since they already include the effect financial factors have in the economy.<sup>3</sup>

Market-based financial stability indicators (FSIs) summarize both the credit channel and credit risk transmission from distressed borrowers in the economy. Market-based FSIs provide information on the banking sector's financial condition which is related to the quantity of credit extended and the possible or expected effects of this channel on the real economy and GDP (credit expansion and the "financial accelerator").<sup>4</sup> Market-based FSIs also capture the reduced financial soundness of banks when borrowers default in periods of economic distress which leads to lower value of risky debt and thus to lower banking sector assets, higher banking asset volatility. This is a reflection of the economic condition of borrowers and of the real economy. (Note that when the banking sector is in distress, bank assets and bank equity values are lower and volatility of bank assets and bank equity is much higher).

<sup>2</sup> The term FSI used here is an indicator derived from forward-looking market information, including indicators from the CCA model. This should not be confused with the accounting ratio financial stability indicators.

<sup>3</sup> An alternative could also be designed in which the central bank only reacts directly to financial risk whenever financial stability indicator breaches a predetermined threshold.

<sup>4</sup> Bernanke, Gertler, and Gilchrist (1999) introduced financial frictions into a business cycle model. The relation of monetary policy and financial stability is discussed in Walsh (2009) and the literature surveyed therein.

Among the different choices for the market-based FSIs, in this paper we will use distance-to-default (*dtd*) of the banking system, which is an indicator of the riskiness of banks estimated from the contingent claims analysis (CCA) tools developed in finance. The basis of CCA is that the liabilities of a financial institution or firm derive their value from assets which are stochastic. The expected variation (volatility) of assets over a future horizon, relative to the promised payments on liabilities provides a measure of financial distress risk. CCA methodology is frequently used to estimate the probability that an entity (in our case, banks, but also corporations or even governments) will default on its obligations. Forward-looking equity prices plus balance sheet information are used to infer bank asset values and volatility, which are used to calculate probability of default or distress. Due to its explicit focus on risk and probability of default or distress, and its link to market prices of equity, CCA has many advantages. Equity data by nature incorporate the forward-looking expectations of the market in a way that static indicators of bank risk, such as nonperforming loan ratios and provisioning cannot. The high frequency of observations, at least for equity and interest rate data, allow for much faster updating of risk measures than data available only at monthly or quarterly frequencies. The CCA financial risk indicators are calculated for individual banks and then can be aggregated into a system-wide financial stability indicator.

The CCA system-wide FSI is modeled jointly with a practical five equation dynamic, stochastic macroeconomic model used to set monetary policy. The macro model was developed at the Central Bank of Chile at the start of the implementation of fully fledged inflation targeting in 2000 (García, Herrera, and Valdés, 2002), and closely resembles the one proposed by Berg, Karam, and Laxton (2006) as a useful toolkit applicable to the analysis of monetary policy in many small open economies. As they claim, “in the new Keynesian synthesis, there has been a convergence between the useful empirically motivated IS/LM models developed in several policymaking institutions and dynamic stochastic general equilibrium approaches that take expectations seriously and are built on solid microeconomic foundations.”<sup>5</sup>

The specific model used here consists of an equation for the output gap (IS), another for inflation (Phillips curve or aggregate supply), an equation for the exchange rate (interest parity condition), a yield curve relating short and long-run interest rates, and the Central Bank reaction function (Taylor rule). Indeed, the primary tool for macroeconomic management is the interest rate set by the central bank as a reaction to the deviations of inflation from the target and the output gap (Taylor, 1993). It is worth noting that most equations are forward looking in the sense that they include in the right hand side the expected levels of the dependent variables. In addition to the macro equations, a CCA module is included, which interacts with the macro equations affecting each other in several

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<sup>5</sup> Berg, Karam, and Laxton (2006), page 3.

ways. Moreover, the model contains a steady state to which the variables converge, thanks to the reaction of monetary authorities.

Besides de macro model, there is a CCA module, which interacts with the macro equations in several ways. For instance, the output gap includes  $dtd$  as an indicator of financial risk in order to analyze whether it is significant or not. Including an aggregate distance-to-default—credit risk indicator, in the GDP gap equation and testing whether the coefficient is significant or not is a first step to get a better understanding of how the financial sector credit risk affects GDP. It is worth pointing out that the system is perfectly endogenous given that the interest rate and GDP influence the level and volatility of banks equity and at the same time, distance to default affects the country risk premium, GDP and the exchange rate.

The distance-to-default is a good indicator of the financial sector conditions; default risk by its nature is non-linear can affect the future output gap via channels that are different than the effect of interest rates on the output gap (widespread default in the financial sector and associated reduction in risk appetite can significantly affect the output, i.e. affect the tails of the future output gap distribution). For these reasons a case can be made that financial sector default or distress indicators affect the future output gap probability distribution (not just the mean) and should be included in the interest rate reaction function.<sup>6</sup>

Put it in a different way, a shock to the distance to default is not equivalent to the typical demand shock that is included in an aggregate demand equation. This is so, because a shock to distance to default could affect the volatility, and even the skewness and Kurtosis, of the discounted sum of square deviations of inflation and GDP from target and both the central bank and the public are not neutral to these effects. Indeed, low probability large deviation events matter disproportionately to welfare. For these reasons a risk indicator, and risk management in general, has a role to play in monetary policy and central bank behavior in the setting of interest rates.

Finally, in order to assess the inclusion of risk indicators in the monetary authority's reaction function, we construct efficiency frontiers mapping inflation and output volatilities, after the artificial economy is hit with stochastic shocks drawn from a normal distribution. In general, we conclude that it is more efficient to include  $dtd$  in the reaction function because in such a way the central bank is able to reduce the volatility of both inflation and output. Moving the policy interest rate more than what is warranted by the gaps of only inflation and output is efficient because negative shocks to asset prices and liquidity could end up in credit risk crisis with negative systemic consequences over the financial system and production.<sup>7</sup>

<sup>6</sup> The authors would like to thank Jonathan Wright for this observation.

<sup>7</sup> On the other hand, very large  $dtd$  could reflect bubbles in asset prices, which usually have bitter endings.

There is a fast growing literature on monetary policy that includes a financial variable in the monetary policy rule, in addition to the inflation and output gaps. A non exhaustive list includes Christiano et al. (2007), Bauducco et al (2008), Taylor (2008), Curdia and Woodford (2009), Kannan et al (2009), and Gertler and Karadi (2011). They include in the monetary policy reaction function either the interest rate spread or the growth of a credit aggregate.

Section II presents the background of CCA distance-to-default (*dtd*), and discusses the data used in the analysis. Section III lays out the macroeconomic framework, as well as the equations required to simulate distance-to-default, which will be included in the macro setting. Section IV presents the results of the simulations and, finally, Section V concludes and presents possible extensions in this line of research.

## **II. RISK MEASURES FROM CONTINGENT CLAIMS ANALYSIS**

### **A. Background**

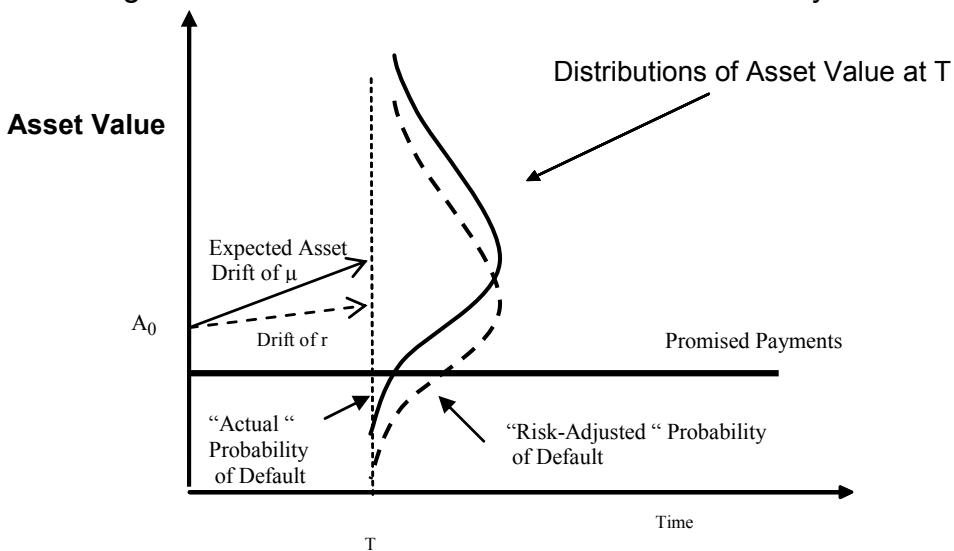
This section introduces the contingent claims approach (CCA), which uses forward looking information to build risk indicators for the banking system, and have important implications for monetary policy, as will be clear in the third section. This approach provides a methodology to combine balance sheet information with widely used finance and risk management tools to construct marked-to-market balance sheets that better reflect underlying risk. The risk adjusted balance sheets use option pricing tools to value the liabilities which are modeled as claims on stochastic assets. It can be used to derive a set of risk indicators, including distance-to-default that can serve as barometers of risk for firms, financial sector vulnerability, and sovereign risk.

A contingent claim is any financial asset whose future payoff depends on the value of another asset. The prototypical contingent claim is an option—the right to buy or sell the underlying asset at a specified exercise price by a certain expiration date. A call is an option to buy; a put is an option to sell and the value of each is contingent on the price of the underlying asset to be bought or sold. Contingent claims analysis is a generalization of the option pricing theory pioneered by Black-Scholes (1973) and Merton (1973). Since 1973, option pricing methodology has been applied to a wide variety of contingent claims. In this paper we focus on its application to the analysis of credit risk and guarantees against the risk of default, and their links to macroeconomic and financial developments.

The contingent claims approach is based on three principles: (i) the values of liabilities are derived from assets; (ii) liabilities have different priority (i.e., senior and junior claims); and, (iii) assets follow a stochastic process. The liabilities consist of senior claims (such as senior debt), subordinated claims (such as subordinated debt) and the junior claims (equity or the most junior claim). For a bank, as the value of its total assets decline, the debt that it owes to other institutions becomes riskier, and its value declines, while credit spreads on its risky debt rise.

Balance sheet risk is the key to understanding credit risk and the probability of crisis. Default happens when assets cannot service debt payments, that is, when assets fall below a distress barrier comprising the total value of the firm's liabilities. Uncertain changes in future asset value, relative to promised payments on debt, is the driver of default risk. Figure 1 illustrates the key relationships. The uncertainty in asset value is represented by a probability distribution at time horizon T. At the end of the period the value of assets may be above the promised payments indicating that debt service can be made, or below the promised payments leading to default. The area below the distribution in Figure 1 is the "actual" probability of default. The asset-return probability distribution used to value contingent claims is not the "actual" one but the "risk-adjusted" or "risk-neutral" probability distribution, which substitutes the risk-free interest rate for the actual expected return in the distribution. This risk-neutral distribution is the dashed line in Figure 1 with expected rate of return  $r$ , the risk-free rate. Thus, the "risk-adjusted" probability of default calculated using the "risk-neutral" distribution is larger than the actual probability of default for all assets which have an actual expected return ( $\mu$ ) greater than the risk-free rate  $r$  (that is, a positive risk premium).<sup>8</sup>

Figure 1. Distribution of Asset Value and Probability of Default



Source: Adapted from Gray and Malone (2008).

The calculation of the actual probability of default is outside the CCA/Merton Model but such a probability can be calculated by combining the CCA/Merton model with an equilibrium model of underlying asset expected returns to produce estimates that are consistent for expected returns on all derivatives, conditional on the expected return on the

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<sup>8</sup> See Merton (1992, pp. 334–343; 448–450).

asset. One does not have to know expected returns to use the CCA/Merton models for the purpose of value or risk calculations, but for calibration into actual probabilities such data are necessary. The value of assets at time  $t$  is  $A(t)$ . The asset return process is

$dA/A = \mu_A dt + \sigma_A \varepsilon \sqrt{t}$ , where  $\mu_A$  is the drift rate or asset return,  $\sigma_A$  is equal to the standard deviation of the asset return, and  $\varepsilon$  is normally distributed, with zero mean and unit variance.

Default occurs when assets fall to or below the promised payments,  $B$ . Therefore, it is the price in which the option is exercised. The probability of default is the probability that

$A_t \leq B$  which is:  $\text{Prob}(A_t \leq B) = \text{Prob}\left(A_0 \exp\left[\left(\mu_A - \sigma_A^2/2\right)t + \sigma_A \varepsilon \sqrt{t}\right] \leq B\right) = \text{Prob}\left(\varepsilon \leq -d_{2,\mu}\right)$

Since  $\varepsilon \sim N(0,1)$ , the “actual” probability of default is  $N(-d_{2,\mu})$ ,

where  $d_{2,\mu} = \frac{\ln(A_0/B) + (\mu_A - \sigma_A^2/2)t}{\sigma_A \sqrt{t}}$  is distance-to-default with a drift of  $\mu_A$  and  $N(\cdot)$  is the cumulative standard normal distribution.

The probability distribution at time  $T$  is shown in Figure 1 above (dashed line) with drift of the risk-free interest rate,  $r$ . The risk adjusted probability of default is  $N(-d_2)$ ,

where  $d_2 = \frac{\ln(A_0/B) + (r - \sigma_A^2/2)t}{\sigma_A \sqrt{t}}$ . This is distance-to-default with a drift of  $r$ , the risk-free rate.

## Calculating Implied Assets and Implied Asset Volatility

The value of assets is unobservable, but it can be implied using CCA.

In the Merton Model for firms, banks and non-bank financials with traded equity use equity,  $E$ , and equity volatility,  $\sigma_E$ , and the distress barrier in the following two equations to solve for the two unknowns  $A$ , asset value, and  $\sigma_A$ , asset volatility (see Crouhy, Mark and Galai, 2000). The first equation is the equation for equity,  $E$ , valued using the Black-Scholes-Merton formula for pricing call options:

$$E = AN(d_1) - B \exp(-r \cdot t)N(d_2)$$

The second equation relates the volatility of equity and value of equity to the implied asset volatility and asset value (Merton 1973, 1974).

$$E\sigma_E = A\sigma_A N(d_1),$$

where  $d_2$  was already defined and  $d_1 = d_2 + \sigma_A \sqrt{t}$ . Since there are two equations and two unknowns (asset value,  $A$ , and asset volatility,  $\sigma_A$ ) an iteration procedure is used to find the

values of the unknowns. Therefore,  $d_1$  and  $d_2$  can be calculated because they depend on A and  $\sigma_A$ .

Financial fragility is intimately related to probability of default. Shocks to prices or liquidity frequently end up being converted into credit risk crisis, as the income flows of banks' debtors weaken and they run into difficulties servicing their loans to banks. Default is hard to handle in traditional macro models in part due to assumptions which usually exclude such possibility. In addition, flow-of-funds and accounting balance sheets cannot provide measures of risk exposures which are forward-looking estimates of losses. CCA, on the other hand, is a framework that explicitly includes and estimates the probability of default.

Since there is a nonzero chance of default, the value of debt is risky and therefore less than the value of risk free debt:

$$\text{Risky debt} + \text{Guarantee against default} \equiv \text{Risk-free debt}$$

The value of "risky" debt can therefore be modeled as the default-free value of the debt less the expected loss:

$$\text{Risky debt} \equiv \text{Risk-free debt} - \text{Guarantee against default}$$

Given that this guarantee is an asset of uncertain value the debt can be thought of and modeled as a contingent claim.

This identity holds both conceptually and in terms of value. If the debt is collateralized by a specific asset, then the guarantee against default can be modeled as a put option on the asset with an exercise price equal to the face value of the debt. The debt holder is offering an implicit guarantee as it is obligated to absorb the losses if there is default. However, often a third party is the guarantor, as is the case when the government guarantees the deposit liabilities of banks or the pension-benefit promises of firms.<sup>9</sup>

Using the Black-Scholes-Merton equation for pricing contingent claims (shown above), the value of risky debt is a function of the default free value of debt (i.e. distress barrier) at time 0, asset level at time 0, volatility of the asset, the time horizon until the expiration date of the claim, and the risk-free interest rate. Since 1973, the Merton Model methodology has been applied to a wide variety of corporations and financial institutions, as well as sovereigns.<sup>10</sup> It has been applied to analyze systemic risk and government contingent liabilities (see Gray,

<sup>9</sup> The CCA framework is an extension of Merton's models of risky debt (1974) and deposit insurance (1977).

<sup>10</sup> See Gray, Merton, Bodie (2007).

2009, and Gray and Jobst 2010). Appendix I provides some suggestions on how to extend the CCA model.

Banks do not frequently default, and regulators are likely to be less interested in the probability of such an event than they are in the possibility that bank assets will fall below a level at which the authorities might be expected to intervene<sup>11</sup>. One useful threshold is a minimum capital threshold. This barrier would be the default barrier plus say 8 percent of assets. The CCA model can be used in this analysis. This model would give “distance-to-minimum capital” as well as “distance-to-default.”

## **B. Calculating Risk Indicators for Individual Banks or Financial Institutions**

Domestic equity markets provide pricing and volatility information for the calculation of implied assets and implied asset volatility in corporate, bank and nonbank financial institutions. The simplest method solves two equations for two unknowns, asset value and asset volatility. Details are shown in Merton (1974) and Crouhy et al. (2000). Levonian (1991) used explicit option prices on bank equity to measure equity volatility and calibrate Merton Models for banks. Moody’s-KMV has successfully applied its version of the CCA model to measure the implied assets values and volatilities and to calculate expected default frequencies (EDFs) for over 35,000 firms and financial institutions in 55 countries around the world KMV (1999 and 2001).

For unlisted corporates and banks, the relationship between the accounting information and the risk indicators, of companies with traded equity, can be used as a guide to map accounting information of companies without traded equity to default probabilities and risk indicators for institutions that do not have traded equity (An example is Moody’s RiskCalc for corporate sectors in many countries and for banks in the U.S.).

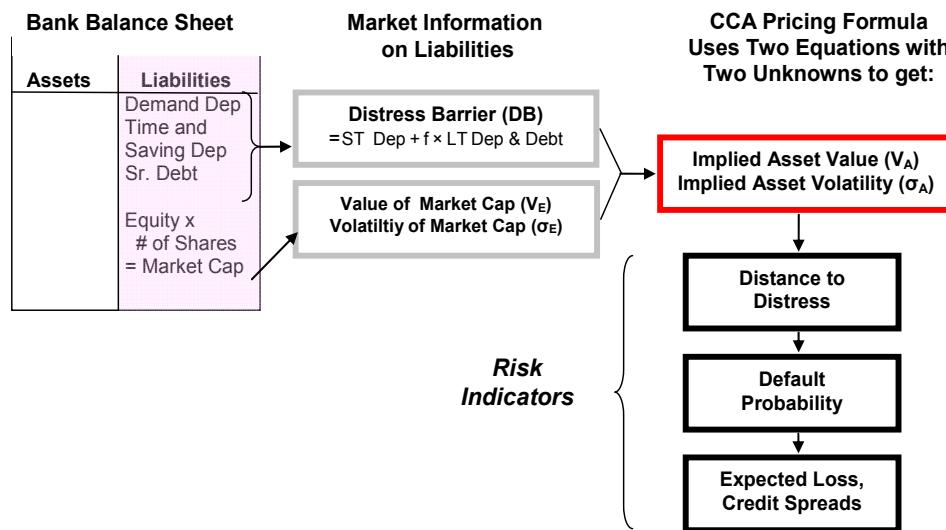
The CCA model for banks and financial institutions uses a time series of the daily market capitalization, the volatility of the market capitalization, and the distress barrier (derived from book values of deposits and debt) to estimate a time series of the implied market value of bank assets and asset volatility. Several useful risk indicators can be calculated for each bank or institution: (i) distance-to-default; (ii) the risk adjusted and actual probabilities of default; (iii) the expected losses (implicit put option) to depositors and debt holders; (iv) potential size of financial guarantees of the public sector; and, (v) sensitivity of risk indicators to changes in underlying bank assets, asset volatility or other factors. The steps used to calculate the implied assets and asset volatility of the individual bank or financial institution, and the risk indicators, is shown in Figure 2.

<sup>11</sup> This has not been the case for many banks in the last sub-prime crisis.

### C. A Distance-to-Default Indicator for Chile

The strategy to compute a risk indicator based on the CCA model described in the last sections II.A and II.B was applied for Chile. The indicator was computed by treating the portfolio of banks in the system as one “large bank.” Since not all banks have shares quoted in the stock market, a sample of the largest banks was used, including approximately 50 percent of total bank’s assets, 65 percent of the total amount of bonds issued by the banking system, and more than 80 percent of market capitalization of the banking industry.<sup>12</sup> Then, the market capitalization, the volatility of the market capitalization, and the default free value of debt (derived from book values of deposits and debt) were used to simultaneously estimate a time series of the implied market value of bank assets and asset volatility (Gray, Echeverría, and Luna, 2007).

Figure 2. Calibrating Bank CCA Balance Sheets and Risk Indicators



Source: Adapted from Gray and Jones (2006).

It is important to point out that although we include 80 percent of bank’s equity, there could be a bias given that small banks, which could be riskier, are excluded. Nevertheless, Luna and Gómez (2008) after comparing an aggregated risk indicator with an aggregation of individual indicators conclude that their behavior regarding levels and volatility are very similar. In addition, the authors state that contagion through interbank lending would be very limited due to the small share of total assets it represents. Moreover, the introduction of a real

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<sup>12</sup> See table 1 in Gray, Echeverría, and Luna (2007).

time interbank settlement system substantially reduced settlement risks in Chile.<sup>13</sup> Yet, using ad hoc methods of aggregating data from different banks can lead to mismeasurement of systemic risk, by averaging heterogeneous agents and, implicitly, assuming that the measures of different banks' risks are not correlated. Consequently, the methodology should be used as a complement to the regular stress tests for banks' and adequate surveillance analysis of the risks for banks' financial stability.

In the current setting we are implicitly assuming that interest rates are non stochastic. Therefore, we have only one stochastic process, bank's assets. Total debt includes monthly information supplied by the Superintendency of Banks (SBIF) on short-term debt plus a portion of long-run debt.<sup>14</sup>

On the equity side, daily numbers of shares and their prices for the selected banks were obtained from the Santiago Stock Exchange. However, calculating implicit equity volatility from call options on banks shares is not possible because such derivatives do not exist in Chile. Therefore, a direct measure of stock volatility was obtained with a simple model of conditional heteroskedasticity, with a one year horizon.<sup>15</sup> Recent work on this issue has shown that at least for the S&P500 the volatility obtained with a similar model is highly correlated with the VIX, which is computed based on the implicit volatility from options on the stocks included in this index (Alfaro and Siva, 2008).

In theory, shares' prices should equal the present discounted value of the flow of dividends. In practice, it is also true that these prices could also change due to many factors different to movements in fundamentals, namely, abundant liquidity, market overreactions to good news, herd behavior, or a different risk assessment than that of the authorities.

Despite all the caveats above mentioned, risk indicators derived from market prices and balance sheet information, such as the distance-to-default have proved to be good predictors of financial stress, risk ratings, and several credit risk indicators.<sup>16</sup> Several studies show that

<sup>13</sup> For simplicity, we did not consider explicitly the volatility of foreign debt. In Chile, bank's foreign debt in the analyzed period represented only 7 percent of total debt.

<sup>14</sup> A linear transformation of the balance sheet data is performed in order to get daily data.

<sup>15</sup> Echeverría, Gómez, and Luna (2008) include a detailed analysis of measuring distance-to-default, in which they consider alternative strategies to obtain direct volatility.

<sup>16</sup> Tudela and Young (2003) find that the distance-to-default measure anticipates changes in the risk ratings of banks in Europe. Moody's KMV CreditEdge model uses this framework to estimate daily default probabilities for 35,000 corporations and financial institutions worldwide and their research shows very good predictive power of the CCA based indicators.

the model is robust, since it correctly reflects and anticipates the behavior of other measures of bank's financial fragility, such as risk ratings and various indicators of portfolio quality.<sup>17</sup>

The information on equity and debt is used to compute the implicit value of assets and its volatility with the Black-Scholes-Merton system described in sections III.A and III.B, in order to solve the system of nonlinear equations for asset and asset volatility (Gray, Merton and Bodie, 2006). However, the value of assets and their volatility require the calculation of  $d_1$  and  $d_2$ , the latter being an exact measure of distance-to-default ( $dtd$ ). Therefore, in practice this system is complemented by two additional equations, one for  $d_1$  and another for  $d_2$ , and solved simultaneously to obtain  $A_0, \sigma_A, d_1, d_2$ , and also  $N(-d_2)$ , which corresponds to probability of default.

An illustrative *approximation* to  $dtd$  could be computed by defining it as the difference between the implicit market value of assets (A) and the distress barrier (DB), divided by one standard deviation of the value of assets:  $dtd \approx (A - DB)/A\sigma_A$ .<sup>18</sup> Put it in words, this indicator corresponds to the number of standard deviations there are from the current level of assets to the distress barrier, given the level of equity and its volatility, the distress barrier, the interest rate and the period analyzed. The larger is this indicator, the safer is the banking system. It is also possible to compute the probability of default with this formula under the assumption that  $dtd$  is normally distributed.

Figure 3 shows the time pattern of  $dtd$  for the Chilean banking system estimated with the Black-Scholes-Merton approach from 1997 to 2006, along with a three-month moving average.<sup>19</sup> It is clear in the figure that the period of higher risk for the banking system was during the severe shock the Chilean economy experienced during the Asia crises which reduced exports and slowed growth in 1998 and 1999. Since then, the Chilean banking system has gradually reduced its risk, though this trend appears to have leveled off in late 2005.<sup>20</sup> Other periods where markets assessed suddenly higher risk for the Chilean banks are easily discerned, for example, the decline in world stock markets following the collapse of the internet bubble in 2000 and the period preceding the Brazilian presidential elections in the third quarter of 2002.

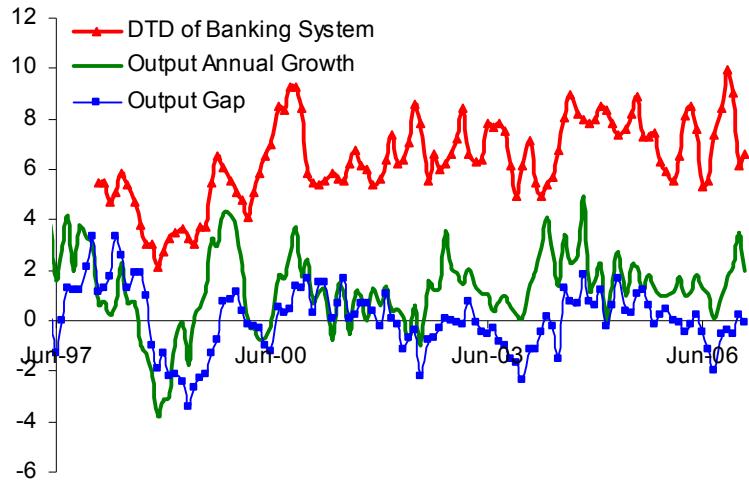
<sup>17</sup> See Chan-Lau (2006), Chan-Lau and Gravelle (2005); also Chan-Lau et al. (2004).

<sup>18</sup> See KMV (2001).

<sup>19</sup> As already said, The CCA risk indicators shown in Figure 3 are taken from Gray, Echeverría, and Luna (2006), who used daily market capitalization for the banks obtained by the Central Bank of Chile from the *Bolsa de Santiago*. Bank debt was obtained from the Central Bank of Chile's database. Financial practitioners use various methods for estimating the volatility of daily asset returns. Two frequently used methods model daily volatility either as a GARCH(1,1) or as a moving average process. The GARCH(1,1) methodology for all banks in the sample was used in this case, but the results of the moving-average model are similar.

<sup>20</sup> As we see below, this leveling-off has occurred at a very low level of risk.

Figure 3. Distance-to-Default for the Banking System



Source: Authors' own calculations.

In Figure 3, it is also clear that there is a relation between distance-to-default of the banking system with both GDP annual growth and the output gap. The regressions with output and output gap as the dependent variable with  $dtd$ , as one of the independent variables, are shown in Appendix II.  $Dtd$  has a significant impact on both output and output gap. Other systemic risk indicators that could be used are described in detail in Gray, Merton, and Bodie (2007 and 2008). And also in Goodhart et al. (2006a and 2006b), Gray and Walsh (2008), Gray and Malone (2008), Haldane et al. (2007), and Segoviano (2006a, 2006b).

### III. LINKING MACROFINANCE INDICATORS TO A SIMPLE DYNAMIC, STOCHASTIC MACROECONOMIC POLICY MODEL

In this section, we will lay out an integrated, “macrofinance policy model” in which risk indicators for the financial system as a whole are incorporated directly into a macroeconomic policy model. Our focus here will be on a modular exposition of the parts of the model and the equations that comprise these parts, as well as giving intuition for how they are linked together and can be used for the analysis of a wide range of policies.

Examples of forward-looking indicators of systemic risk derived from the contingent claim analysis (CCA) model are distance-to-default ( $dtd$ ), expected loss (i.e., implicit put option), or the default probability weighted by the assets of individual financial institutions. The macro model used here incorporates the CCA risk indicator  $dtd$ , whose derivation is described below.<sup>21</sup>

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<sup>21</sup> A related issue is whether an indicator of market risk appetite such as the VIX should be included in monetary policy models along with the risk indicator. This could help estimate the impact of the credit risk

(continued...)

The first module of our model consists of equations for the most important macro variables. There is an equation for the output gap, an equation for inflation, an equation the real exchange rate, a yield curve and a Taylor rule for setting the domestic policy rate, which is a short term interest rate, set by the central bank. The second module is used to model distance to default.

Distance-to-default for the banking system is included in the GDP gap equation, the parity condition, and the policy rate reaction function. The model parameters are estimated using historical data, including the distance-to-default indicator. Even though the equations have empirical support as shown in Appendix II, this is mostly a theoretical exercise in which some of the parameters of the model are modified (calibrated) in order to assess how the results of the simulations change with them. The approach can be used to examine the tradeoffs between GDP and inflation, with and without the inclusion of distance-to-default for the banking system in the monetary authorities reaction function.<sup>22</sup>

### **Module 1: Output, Inflation, Exchange Rate, and a Taylor rule**

The five equation dynamic stochastic macroeconomic model used to set monetary policy was already briefly described. This version of the model was built in the Central Bank of Chile at the start of the implementation of fully fledged inflation targeting in 2000, and closely resembles the one by Berg, Karam, and Laxton (2006). An application of it to the design of monetary policy in Chile, using efficiency frontiers, is found in García, Herrera and Valdés (2002). It is one example of a class of models that can be used for policy analysis in small open economies and that, as stated above, are empirically motivated (IS/LM type) but at the same time share many features of the dynamic stochastic, micro founded, general equilibrium models used by central banks.<sup>23</sup>

#### Equation for output gap

$$\begin{aligned} ygap_t = & \beta_1 ygap_{t+1} + \beta_2 ygap_{t-1} + \beta_3 ygap_{t-2} + \beta_4 ygap_{t-3} + \beta_5 (r_{t-1}) \\ & + \beta_6 (rl_{t-2}) + \beta_7 (q_{t-4}) + \beta_8 (dtd_t) + \varepsilon_t^y \end{aligned} \quad (1)$$

indicator on the GDP gap, adjusted for changes in risk appetite. Also, risk indicators for a group of institutions could include the correlation, or dependence, structure observed between the institutions.

<sup>22</sup> There are several other interesting routes to take in linking risk analytics more closely with macroeconomic models. These include incorporating default risk and a risk premium into the Mundell-Fleming model to separate out the effects of changes in interest rates due to changes in the market for liquidity, and changes in interest rates due to changes in the risk premium on debt (see Gray and Malone, 2009).

<sup>23</sup> Berg, Karam, and Laxton (2006), page 3.

Where  $ygap$  corresponds to the output gap, i.e., the log deviation of GDP with respect to its trend,  $r$  is the short-run real interest rate,  $rl$  the long-run real interest rate,  $q$  is the real exchange rate, and  $dtd$  is distance-to-default, which is also modeled here. As was explained in detail above,  $dtd$  is a financial risk indicator that could reflect, in general, the financial conditions that the economy faces. Finally,  $\varepsilon_t^y$  is a shock to GDP. All variables are expressed as log deviations from steady state.

Phillips curve

$$\begin{aligned}\Delta\pi_t = & \alpha_1[(\pi_{t+1}^e + \pi_t)/2 - \pi_{t-1}] + \alpha_2[(\pi_{t-2} + \pi_{t-3} + \pi_{t-4})/3 - \pi_{t-1}] \\ & + \alpha_3[(q_{t-1} - q_{t-4})/3 - \pi_{t-1}] + \alpha_4[(ygap_{t-1} + ygap_{t-2})/2] + \varepsilon_t^\pi\end{aligned}\quad (2)$$

Where  $\pi_t$  stands for inflation,  $\pi_{t+1}^e$  inflation expectations next period,  $q_t$  is the real exchange rate, and  $\varepsilon_t^\pi$  is a cost-push shock.

Exchange rate equation (interest parity condition):

$$q_t = \delta_1 q_{t+1} + \delta_2 q_{t-1} + (r - rf) + \delta_3(dtd_{t-1}) + \varepsilon_t^q \quad (3)$$

The real exchange rate depends on lags and leads of itself, the domestic policy rate, the foreign policy rate, the risk indicator which embeds both the sovereign spread for domestic debt, and the sovereign spread for foreign debt and a shock. According to uncovered interest rate parity, the expected change in the spot exchange rate should be related to the differential between the domestic and foreign interest rates, plus some risk premium.

The long-run interest rate (yield curve) equation describes the relationship between long run ( $rl_t$ ) and short run ( $r_t$ ) interest rates:

$$(rl_t) = \xi_1(rl_{t+1}^e) + \xi_2(rl_{t-1}) + (1 - \xi_1 - \xi_2)(r_t) + \varepsilon_t^{rl} \quad (4)$$

Reaction Function (Taylor rule)

$$r_t = \rho(r_{t-1}) + (1 - \rho)\left\{\rho rl^{eq} + \theta[\gamma(\pi_{t+1} + \pi_t + \pi_{t-1})/3 + (1 - \gamma)(ygap_{t-1})] + \zeta(dtd_t)\right\} + \varepsilon_t^r \quad (5)$$

The monetary policy interest rate depends on its own lag, the expected inflation gap, output gap, distance-to-default and a policy shock. While including a measure of financial stability in the Taylor rule for setting interest rates may be able to improve efficiency (welfare), in particular if financial stability affects output, accurate regulation and supervision of financial institutions could be a better way of targeting financial stability.

## Module 2: Distance-to-Default Model for the Banking System

This module completes the whole system to be simulated simultaneously. The value of assets A is derived from the Black & Scholes (B&S) model,

$$A = [E + B * \exp(-r \cdot t)N(d_2)] / N(d_1) \quad (6)$$

where  $E$  is the value of the Equity (or the same, the value of the call option),  $B$  is the value of the default barrier, where ' $r$ ' is the risk free interest rate and ' $t$ ' is time -fixed in the model to one year. Finally  $N(\cdot)$  is the normal cumulative distribution function and  $d_1$  and  $d_2$  were derived from the B&S model as described in section II<sup>24</sup>:

$$d_1 = d_2 + \sigma_A \sqrt{t} , \text{ and} \quad (7)$$

$$d_2 = \frac{\ln(A_0 / B) + (r - \sigma_A^2 / 2)t}{\sigma_A \sqrt{t}} \quad (8)$$

Note that  $d_2$  is equal, precisely, to distance-to-default ( $dtd = d_2$ ).

It is apparent from equation 8 that assets volatility,  $\sigma_A$  and assets value,  $A$  are crucial for finding  $dtd$ . Thus, the system of non linear equations requires an equation for  $\sigma_A$  to have a solution:

$$\sigma_A = (\sigma_E * E) / [A * N(d_1)] \quad (9)$$

where,  $\sigma_E$  stands for volatility of equity.<sup>25</sup>

It is important to point out that bank's equity ( $E$ ) and its volatility ( $\sigma_E$ ) were initially set constant. However, the results obtained with the model simulations were counterintuitive regarding distance-to-default. Indeed, after a cost push shock hit the economy inflation went up as expected, GDP fell, and, as a reaction to the inflationary pressures, the interest rate increased. While this negative economic scenario was taking place, distance-to-default was growing signaling a sounder economic situation of the banking industry and the businesses in general, which is not a sensible outcome. It goes without saying that the efficiency frontiers obtained were not satisfactory either. By the same token, after a positive shock to GDP, which was accompanied by an interest rate hike, distance-to-default fell as if the economy were more vulnerable. This is so because in the model higher interest rates have a negative effect on the level of assets, even if the economy is in better shape.

<sup>24</sup> Dynare has an explicit function built in for the cumulative normal distribution function.

<sup>25</sup> A thorough explanation is found in Gray and Malone (2008).

In consequence, a new strategy of modeling both  $E$  and its volatility,  $\sigma_E$ , was adopted. As the reader may recall, distance-to-default affects the macro variables in several ways: through affecting GDP, the real exchange rate, and the interest rate in equations (1), (3) and (5) of the macro model, respectively. In the following equations, GDP affects banks' capital,  $E$ , as well as its volatility,  $\sigma_E$ , also affecting, through this channel, distance-to-default, and making the whole system of equations completely endogenous. Moreover, another channel of endogeneity is the effect that the interest rate has on assets,  $A$ , as well as on the volatility of equity,  $\sigma_E$ :<sup>26</sup>

$$E = \rho_1 E(-1) + 0.01 * ygap_t \quad (10)$$

$$\sigma_E = 0.1 + 3 * (r_t) - (ygap_{t+1} + ygap_t + ygap_{t-4}) / 3 \quad (11)$$

The parameters of the macro model (see Table 1) were estimated for carrying out monetary policy analysis, with few exceptions which were calibrated in the yield curve  $\xi$ , in the reaction function  $\theta$ ,  $\gamma$ ,  $\zeta$  and the ones related to  $dtd$  in the interest parity condition and the Phillips curve, which were used in the sensitivity analysis of the next section.

**Table 1. Parameters of the Macro Model**

$\beta_1 = 0.1$	$\alpha_3 = 0.05$
$\beta_2 = -0.1$	$\alpha_4 = 0.15$
$\beta_3 = -0.6$	$\delta_1 = 0.3$
$\beta_4 = -0.4$	$\delta_2 = 0.6$
$\beta_5 = -0.5$	$\delta_3 = -0.04$
$\beta_6 = -0.5$	$\xi_1 = 0.5$
$\beta_7 = 0.02$	$\xi_2 = 0.45$
$\beta_8 = 0.2$	$\rho = 0.8$
$\alpha_1 = 0.3$	$\rho_1 = 0.9$
$\alpha_2 = 0.5$	$\theta = 1.3$
$\alpha_3 = 0.05$	$\zeta = 0.5—1.0—1.5$
$\gamma = 0.2—0.3...—1.2$	

Source: Authors' estimates.

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<sup>26</sup> It is worth mentioning that the spread put is another measure of risk that could be used alternatively. It is described in Gray, Merton and Bodie (2008) and Gray and Malone (2008) as a function of the value of the Put option, the default barrier, the risk free rate and time:  $spread\_put = -1/t * \log(1 - PUT/BB * \exp(-r*t)) - 0.00925382$  Even though the *spread put* is a useful concept, it was not used in the simulations performed with the model here.

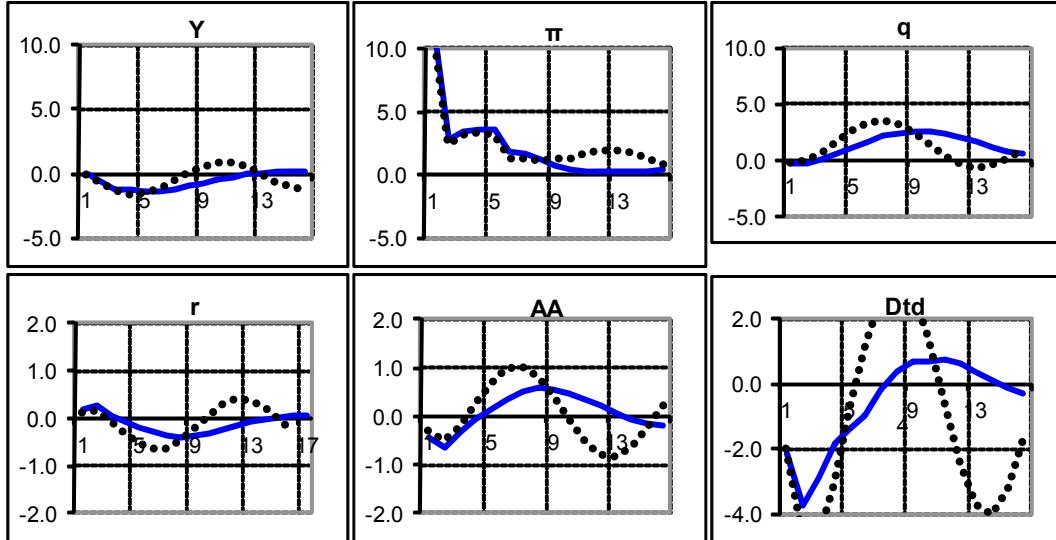
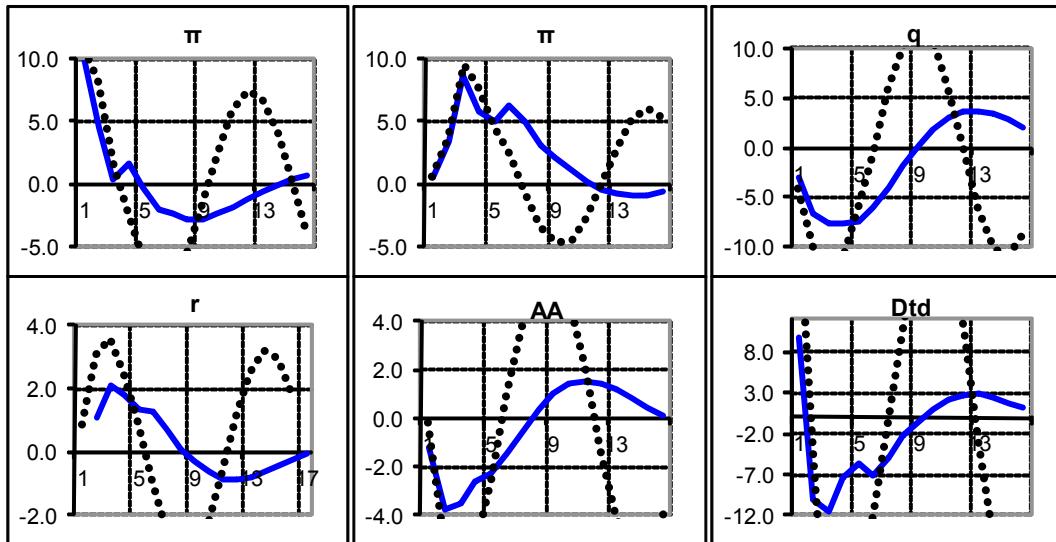
#### IV. STOCHASTIC SIMULATIONS AND POLICY ANALYSIS

In order to understand how the model works, we first obtained impulse responses with and without including  $dtd$  in the reaction function (Figure 4). Next, we assess different alternatives of monetary policy as well as calibrations of the model by building efficiency frontiers with the volatilities of GDP and inflation (García, Herrera and Valdés, 2002; Laxton and Pesenti, 2003).

Specifically, we measure the responses of GDP, inflation, the exchange rate, the monetary policy interest rate,  $r$ , the CCA-derived risk indicator,  $dtd$ , and assets following a shock of 100 basis points to GDP and inflation.

After a shock to inflation (cost-push shock) hits the economy, output falls taking the output gap ( $ygap$ ) to negative levels. On the contrary, the interest rate tends to initially increase, which jointly with the output gap reduction increase financial vulnerability and take distance-to-default down significantly (Figure 4). The reduction in distance-to-default is so large that an otherwise increasing interest rate ends up falling while the exchange rate increases. This is so because the exchange rate is not only affected by the interest rate but also by  $dtd$  through the risk premium.

Figure 4. Responses to a Shock to Inflation and to a shock to Output Shock

**Responses to a Shock to Inflation or Cost Push Shock****Responses to a Shock to GDP (y)**

Source: Authors' calculations.

In the case of a positive shock to the output gap, GDP and inflation increase, and so do interest rates, while the exchange rate falls in agreement with economic intuition. The system takes around four years to return to equilibrium after the shock (Figure 4).<sup>27</sup> It is clear in the

<sup>27</sup> A not reported negative shock to distance-to-default causes an initial small drop in  $ygap$ . However, due to the fact that  $dtd$  is included in the policy reaction function, the original shock is followed by a reduction in the MPR. Moreover, arbitrage through the uncovered interest parity as well as the respective hike of the risk premium result in a large real depreciation. Thus, the interest rate and the exchange rate fuel a GDP expansion.

figure that responses to both shocks are more volatile whenever  $dtd$  is not included in the reaction function.

In general, the model works as expected according to standard economic intuition. There is strong interaction among macro variables, and  $dtd$  has a large impact on the monetary policy rate, the real exchange rate, and even the output gap.

The efficiency frontiers are built combining the volatility of inflation and GDP that results after the economy is hit repeatedly by shocks drawn from a normal distribution. Using Dynare, the artificial economy was simulated during 200 periods, repeatedly, and average standard deviations of the variables were computed between periods 100 and 120 across the repetitions. The purpose of the exercise is to compare frontiers that were obtained with a combination of ten weights, in the policy rule, for both the inflation and the output gap objectives, respectively, using three different weights on distance to default.<sup>28</sup> Additional frontiers are obtained using a similar procedure, but changing one the parameters of the model. *Whenever a frontier is closer to the origin the volatility tradeoff is smaller, and it is possible to say that the policy choice is better for the central bank and the society as a whole.*

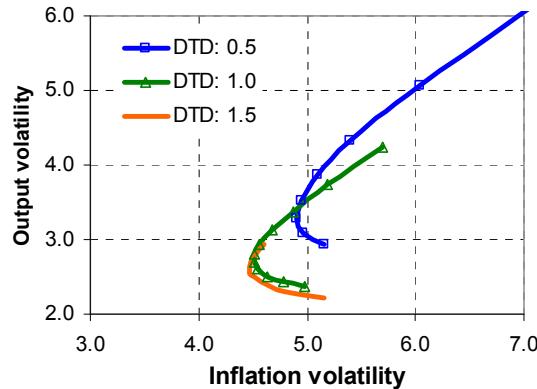
Each of the figures below includes three frontiers. All were obtained with a traditional Taylor Rule in which besides inflation and *GDP* gaps ( $\theta=0.5$ ,  $\rho =0.6$  and  $\gamma =0.6$ ),  $dtd$  is included. The first line results from a rule in which  $dtd$  has a small weight (with a7 coefficient  $\zeta =0.5$ ) i.e., authorities react only weakly to the risk indicator (blue line). The other lines in the figure correspond to alternative reaction functions for monetary policy that have a larger weight of  $dtd$ , with coefficients  $\zeta$  equal to 1 and 1.5, respectively (green and orange lines). In summary, besides reacting to inflation and *GDP* gaps the monetary authority also reacts to distance-to-default, in such a way that when  $dtd$  is large, the monetary authority increases the interest rate but when the banking system is close to default the central bank reduces the interest rate more than what is warranted by the gaps of inflation and output alone. This is so because negative shocks to asset prices and liquidity could end up in credit risk crisis with systemic consequences on lending and production. On the other side, a very large  $dtd$  could be the result of asset bubbles, which are usually associated with financial turmoil when they burst.

- **Reaction size to  $dtd$  in the policy rule**

The size of the reaction to  $dtd$  in the Taylor rule has a very significant effect on the results. Indeed, the larger the coefficient associated to  $dtd$  in the authorities reaction function, the closer to the origin is the frontier obtained with the simulations (lowest line in Figure 5). Therefore, the stabilization of  $dtd$  by the central bank contributes to stabilizing volatilities of

both GDP and inflation, which fall more with a larger coefficient on  $dtd$  but with diminishing marginal gains. Indeed, it is clear from the figure that increasing the coefficient from 0.5 to 1 generates a large reduction in volatility of GDP and inflation while using a coefficient of 1.5 improves the trade off only marginally.

Figure 5. Efficiency Frontiers  
Base model reaction to  $dtd: 0.5, 1.0, 1.5$

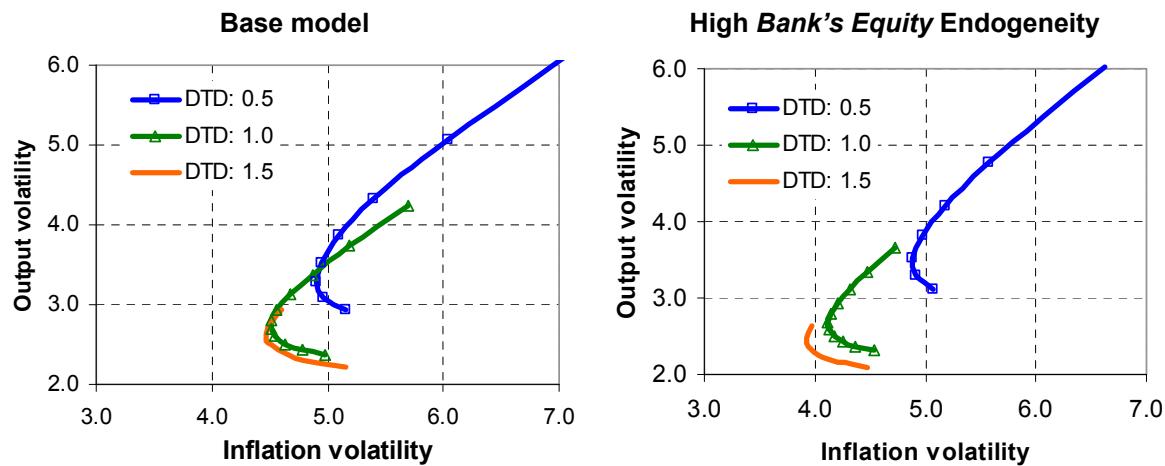


Source: Authors' calculations.

- **Endogenous effect on bank's equity ( $E$ ) and its volatility ( $\sigma_e$ )**

This experiment consists of increasing substantially the effect of GDP on both bank's equity and equity's volatility. This is implemented by augmenting the coefficient of  $ygap$  (from (0.01 to 0.1) in equation (10) and (1 to 1.5) in equation (11). If the feedback from GDP to Bank's equity and  $dtd$  (endogeneity) is stronger, the gains by reacting strongly to  $dtd$  are even larger than in the base model (Figure 6). In fact, a comparison of both panels in the figure shows that the volatility reduction of both variables, included in the frontier, is much larger here than in the base model.

Figure 6. Efficiency Frontier and Endogeneity of Bank's Equity

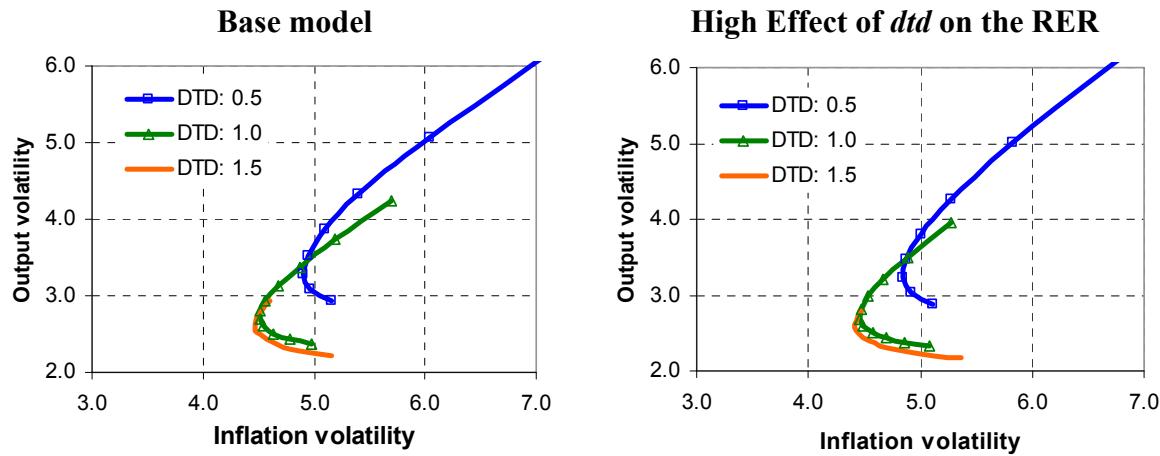


Source: Authors' calculations.

- **Effect of  $dtd$  on real exchange rate**

In this experiment the effect (coefficient) of  $dtd$  in the (risk premium) exchange rate equation (3) was increased (from 0.04 to 0.5). (Figure 7, right panel). Again, the lowest line, which represents the frontier obtained with a larger weight on  $dtd$  in the reaction function, includes points that are closer to the origin than any point in the middle or the green lines. Thus, this policy should be preferred by the central bank. The gains in terms of volatility are very similar in both panels of Figure 7. The right panel of the figure only shows small differences with respect to the baseline model. The shape of the frontiers obtained in this experiment indicates that putting more weight on  $dtd$ , generates here a larger reduction in inflation volatility.

Figure 7. Efficiency Frontier and Interest Parity Condition

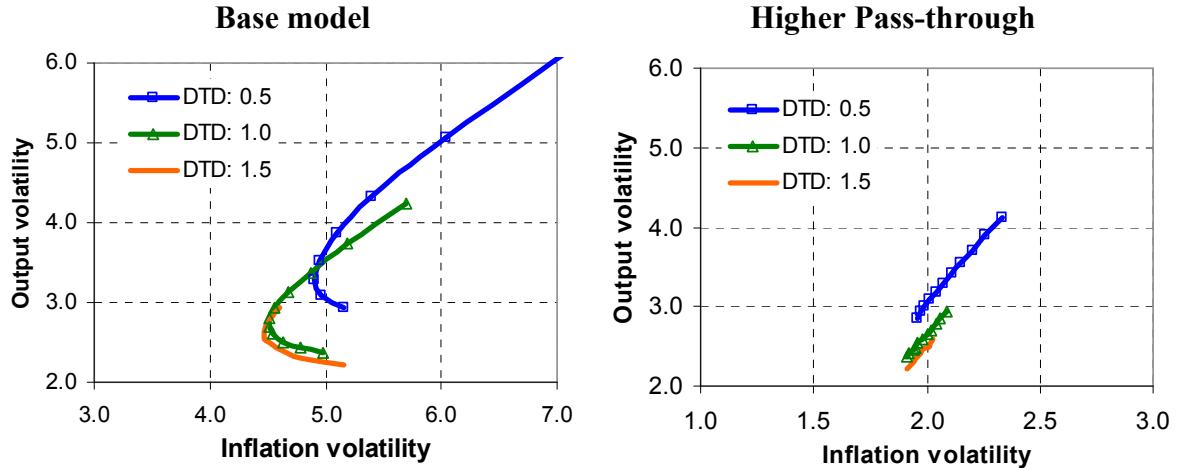


Source: Authors' calculations.

- **Higher pass-through**

Were the pass-through from exchange rate to inflation larger (0.7 instead of 0.05), the central bank policy would be more efficient by reacting to  $dtd$ . Indeed, by reacting to  $dtd$  the central bank is able to reduce volatility of output but not of inflation (Figure 8). As shown in the figure, the frontiers move downwards whenever the coefficient associated to  $dtd$ , in the monetary policy rule, increases. In fact, a high level of pass-through is an important issue in very open economies. If prices are very flexible and quickly reflect any movement of the exchange rate, a risk premium hike could affect output negatively while increasing inflation at the same time. Therefore, the authority will face a trade-off between stabilizing inflation and output.

Figure 8. Efficiency Frontier and Pass-Through



Source: Authors' calculations.

In summary, the simulations of the macro model show that it is more efficient for the central bank to put a larger weight on  $dtd$  in the reaction function given that inflation and output volatility decrease. Whenever pass-through from exchange rate to prices is very high, including  $dtd$  in the reaction function will reduce output volatility without increasing the variability of inflation. In addition, whether financial vulnerability or  $dtd$  has a larger impact on the exchange rate, or GDP has a larger effect on bank's equity and, through it, on  $dtd$  – more endogeneity, it is more efficient to include  $dtd$  in the reaction function because in that way the central bank is able to reduce the volatility of both inflation and output.

## V. CONCLUSIONS

This main objective of this article was the integration of the analysis of financial sector vulnerability into macroeconomic models, which is an area of important and growing interest for policymakers, in both developed and emerging markets. This paper uses contingent claims analysis (CCA) tools, developed in finance, to construct financial stability indicators in a standard monetary policy model. The financial sector risk affects the economy while the economy (GDP) and interest rates affect financial sector credit risk.

The new framework is simple, and a practical way to include financial sector risk in monetary policy analysis. Indeed, the model has the main variables analyzed by policymakers, but is small enough to understand easily how it works. Even though, it is an artificial economy used to be stochastically simulated, the empirical evidence supports the model. In addition, impulse responses behave in accordance with economic intuition.

The main question to be answered with the integrated model was whether or not the central bank should explicitly include the financial stability indicator in the interest rate reaction function. The alternative is to react only indirectly to financial risk by reacting to inflation and GDP gaps, since they already include the effect financial factors have in the economy. In order to reach the objective, efficiency frontiers were built with the volatility of inflation and output obtained from stochastic simulations. It is found in general, that including *dtd* in the reaction function reduces both inflation and output volatility. Moving the policy interest rate more than what is consistent with the gaps of inflation and output is efficient because negative shocks to asset prices and liquidity could end up in credit risk crisis with negative systemic consequences over the financial system and GDP.

A set of exercises were also performed in which some of the parameters of the model were calibrated to reflect and assess actual differences among economies regarding exchange rate pass-through, the relation between financial risk and exchange rate through the parity condition (risk premium), and the endogeneity of the financial indicator, namely, the degree in which the macro variables, GDP and interest rates, affect distance-to-default through bank's assets, bank's equity, and equity volatility. Whenever the pass-through from the exchange rate to inflation is higher, when the impact of financial vulnerability (*dtd*) on the exchange rate is larger; as well as whenever the effect of GDP on bank's equity–endogeneity—is stronger, it is more efficient to include *dtd* in the reaction function, with a large coefficient.

Finally, this is the first analysis of this subject and there are numerous refinements and extensions that could be introduced in the future. A nonexhaustive list includes: (i) exploring the use of other financial sector risk indicators; (ii) combinations of financial scenarios (strong, normal, fragility); (iii) changes in the dynamics of the macro model; (iv) adoption of a more micro-founded general equilibrium macro model; (v) using it to analyze the effects of counter

cyclical capital requirements, and other macroprudential policies, and (vi) how the monetary policy model and framework presented here might include risk-adjusted GDP measures.<sup>29</sup> These extensions are left for future research.

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<sup>29</sup> See Gray, Jobst and Malone 2010.

## APPENDIX I. EXTENSIONS OF THE MERTON MODEL

Numerous extensions of the original Merton Model have been developed by relaxing some of its assumptions. Restrictions of the model include the assumptions that: (i) default can occur only at the maturity date of the debt; (ii) there is a fixed default barrier; (iii) there is a constant risk-free rate; and, (iv) asset volatility is constant. Cossin and Pirotte (2001) provide a good summary of extensions of the Merton Model. Black and Cox (1976) extended the Merton Model to relax the assumptions (i) and (ii) above by introducing a “first passage time” model where default can occur prior to the maturity of the debt if the asset falls below a specified barrier function for the first time.

Although the strict theoretical condition in the Merton Model for default is that the value of assets is less than the required payments due on the debt, in the real world, default typically occurs at much higher asset values, either because of a material breach of a debt covenant or because assets cannot be sold to meet the payments (“inadequate liquidity”) or because the sovereign decides to default and induce a debt renegotiation rather than sell assets. To capture these real-world conditions for default in the model, we specify a market value of total assets at which default occurs. We call this level of assets that trigger default the “distress barrier.” This barrier can be viewed as the present value of the promised payments discounted at the risk-free rate. The approach used in the KMV model sets the barrier level equal to the sum of the book value of short-term debt, promised interest payments for the next 12 months, and half of long-term debt (see Crouhy, et. al. (2000) and KMV (1999, 2001)).

In the 1990s the KMV model was based on the VK model (Vasicek and Kealhofer) which has multiple layers of liabilities and several confidential features. MKMV’s EDF (expected default frequency) credit measure is calculated using an iterative procedure to solve for the asset volatility. This distance-to-default was then mapped to actual default probabilities using a database of detailed real world default probabilities for many firms. The MKMV distance-to-default and the CEDF (cumulative expected default probabilities) are calculated as follows:

$$DD_{KMV} = f\left( \frac{\ln(A_0 / B_t) + (\mu_A - \sigma_A^2 / 2)t}{\sigma_A \sqrt{t}} \right)$$

$$CEDF_t = f(DD_{KMV}(t))$$

Note that this definition of  $DD_{KMV}$  includes the real drift of the asset,  $\mu_A$ , whereas the distance-to-default from the Merton approach has  $r$  for the asset drift. Since MKMV estimates the actual default probabilities, the risk neutral default probabilities are calculated

from the correlation of the implied asset with the market, the market Sharpe Ratio, and time horizon.

The Merton Model has been extended to include stochastic interest rates as well. Shimko, Tejima, and Van Deventer (1993) include a Vasicek interest rate term structure model which relaxes assumption (iii) above allowing the risk free interest rate to change and including the correlation of asset return with the interest rate. There are two stochastic factors, the asset and the interest rate and this model is frequently called the STV model. Longstaff and Schwartz (1995) take the Black and Cox (1976) model and add in stochastic interest rates, similar to the way STV includes interest rates.

The CreditGrades model (2002) includes a diffusion of a firm's assets and a first passage time default with a stochastic default barrier. The model was modified to incorporate equity derivatives (Stamicar and Finger 2005). Recent research has studied the relationship between the volatility skew implied by equity options and CDS spreads (Hull et al., 2004). They establish a relationship between implied volatility of two equity options, leverage and asset volatility. This approach is, in fact, another way of implementing Merton's Model to get spreads and risk-neutral default probabilities directly from the implied volatility of equity options. A similar approach using several equity options is discussed in Zou (2003).

Financial support for liquidity and potential credit risk from the authorities is likely to be provided before the "default" barrier is reached. A minimum capital barrier, or simply a capital barrier, can be defined in addition to the default barrier. For instance, the default barrier plus 8 percent of market value of assets could be used as the "minimum 8 percent capital" barrier. The area between the minimum capital barrier and the default barrier represents the probability of falling below minimum capital but not as far as default. The value of this area is calculated as the implicit put option below the minimum capital barrier minus the implicit default put option. We will call the value of the area as the "capital barrier put option" or "capital barrier expected loss." This is particularly relevant to the central bank as it is a measure loss directly related to liquidity support/financial support which would be needed to get the bank asset level above the minimum capital level.

Finally, contingent claims models can be used to assess systemic risk in portfolios of financial institutions, including the correlation or dependence structure (IMF Global Financial Stability Report, 2009, April 1).

## APPENDIX II. REGRESSION RESULTS OF OUTPUT AND OUTPUT GAP ON DISTANCE-TO-DEFAULT OF THE BANKING SYSTEM

The first regression is on GDP growth:

$$\Delta y_t = c + \alpha_1 r_{t-1} + \alpha_2 \Delta dtd_{t-1} + \alpha_3 \Delta e_{t-1} + \alpha_4 \Delta y_{t-1} + \varepsilon_t$$

Dependent Variable: DLOG(YS,0,3)  
 Sample (adjusted): 1998M05 2007M02  
 Included observations: 106 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.011	0.002	4.830	0.000
R(-1)	-0.001	0.000	-3.723	0.000
<b>DLOG(TCR(-1),0,3)</b>	<b>0.046</b>	<b>0.019</b>	<b>2.438</b>	<b>0.017</b>
DLOG(DTDS(-1),0,3)	0.012	0.003	3.551	0.001
DLOG(YS(-1),0,3)	0.463	0.074	6.283	0.000
R-squared	0.574	Mean dependent var		0.009
Adjusted R-squared	0.557	S.D. dependent var		0.013
S.E. of regression	0.008	Akaike info criterion		-6.677
Sum squared resid	0.007	Schwarz criterion		-6.552
Log likelihood	358.890	F-statistic		34.036
Durbin-Watson stat	1.912	Prob(F-statistic)		0.000

The second is a regression on the output gap:

$$gap_t = c + \alpha_1 \Delta dtd_{t-1} + \alpha_2 \Delta e_{t-1} + \alpha_4 gap_{t-1} + \varepsilon_t$$

Dependent Variable: YGAP  
 Sample (adjusted): 1998M02 2007M02  
 Included observations: 109 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.736	0.470	-3.691	0.000
DLOG(TCR(-3),0,3)	4.134	1.639	2.522	0.013
<b>LOG(DTDS(-1))</b>	<b>0.934</b>	<b>0.256</b>	<b>3.653</b>	<b>0.000</b>
YGAP(-1)	0.513	0.082	6.275	0.000
YGAP(-3)	0.225	0.072	3.113	0.002
R-squared	0.661	Mean dependent var		-0.035
Adjusted R-squared	0.648	S.D. dependent var		1.201
S.E. of regression	0.712	Akaike info criterion		2.204
Sum squared resid	52.766	Schwarz criterion		2.328
Log likelihood	-115.126	F-statistic		50.695
Durbin-Watson stat	1.842	Prob(F-statistic)		0.000

These regressions show that changes in *dtd* are significant in explaining both GDP quarterly growth (equation #1) and the output gap (equation #2) with the expected (positive) sign.

### APPENDIX III. EXTENSIONS AND FURTHER APPLICATIONS

The central bank may expand its set of policy instruments to better accommodate its multiple objectives. Additional tools that can be used to target financial stability include the reserve requirements for banks and other measures of capital adequacy, such as Value-at-Risk based measures advocated in Basel II. A rule can be specified for targeting, such a measure of capital adequacy,  $C$ , as follows:

Capital adequacy rule for the banking sector

$$C_t = \phi_1 C_{t-1} + (1 - \phi_1)[\eta_2 ygap_t + \eta_3 fsigap_t] + \varepsilon_{10,t}$$

The closer the parameter  $\phi_1$  is to one, the more continuity is built into the capital adequacy requirement. The *fsigap* is the financial stability indicator. As in the case of interest rates, some continuity is important, because significant changes in capital adequacy requirements, or interest rates, in a short amount of time can also potentially contribute to instability as banks move en masse to comply with new requirements. The second term in the above rule, which is multiplied by the coefficient  $1 - \phi_1$ , allows the central bank to use capital adequacy requirements, or other variables that affect the risk profile of the banking sector, to respond to deviations of inflation, output, and financial stability from their targets.<sup>30</sup>

Lower capital adequacy requirements, by stimulating lending, may be able to contribute to higher investment that stimulates output when output is below target. Likewise, more stringent capital adequacy requirements can help increase the financial stability indicator when it is below target, by lowering the probability of banking sector instability or widespread defaults.

Finally, the sovereign and the central bank will choose the coefficients of their decisions rules to maximize their objective functions.

In a similar vein, a paper by P. N'Diaye (2009) explores how prudential regulations can support monetary policy in reducing output fluctuations while maintaining financial stability using a new framework that blends a standard model for monetary policy analysis with a contingent claims model of financial sector vulnerabilities. The results suggest that binding countercyclical prudential regulations can help reduce output fluctuations and lessen the risk of financial instability.<sup>31</sup>

<sup>30</sup> See Gray and Malone, 2008, for details.

<sup>31</sup> See Papa N'Diaye, Countercyclical Macroprudential Policies in a Supporting Role to Monetary Policy, 2009.

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