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Monetary Policy, Bank Leverage, and Financial Stability

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Research

Monetary Policy, Bank Leverage, and Financial Stability

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Abstract

This paper develops a model to assess how monetary policy rates affect bank risk-taking. In the model, a reduction in the risk-free rate increases lending profitability by reducing funding costs and increasing the surplus the monopolistic bank extracts from borrowers. Under limited liability, this increased profitability affects only upside returns, inducing the bank to take excessive leverage and hence risk. Excessive risk-taking increases as the interest rate decreases. At a broader level, the model illustrates how a benign macroeconomic environment can lead to excessive risk-taking, and thus it highlights a role for macroprudential regulation.

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I. INTRODUCTION

The recent financial crisis in the United States was preceded by a prolonged period of a low monetary policy rate. In discerning the causes of the crisis, one question that commonly arises is whether a low monetary policy rate could create incentives for excessive risk-taking, which can ultimately unfold in a crisis like the one that started in 2007. Shedding light on this question is of great importance because it has implications for the conduct of monetary policy and the design of financial regulation, more so in the current environment of once again low interest rates.

This paper develops a dynamic bank model to investigate this question and gain a better understanding of the incentives to take excessive risk, its impact on financial stability, and possible policy responses to resolve it.

In the model, a decrease in the monetary policy rate, which for simplicity is modeled as an exogenous reduction in the real risk-free rate, raises profitability of lending for two reasons: i) it allows the monopolistic bank to extract a larger surplus from its borrowers, and ii) it decreases the bank's funding costs. However, the presence of limited liability creates a further boost to the returns on lending because losses are bounded.

Contrasted against a constrained social planner benchmark, the model shows that leverage and hence risk-taking are excessive. Moreover, excessive risk-taking gets stronger the lower the monetary policy rate. The more profitable lending becomes, the more attractive it is to take on additional leverage, more so with limited liability. Essentially, the importance of current-period profits relative to the present discounted value of lifetime profits increases to the point that it pays off to take on more risk, despite the fact that in the case of bankruptcy not only current net worth but all future profits are lost. For extremely low levels of interest rates, excessive risk-taking can be substantial.

In the model, whenever the monetary policy rate is lowered, the bank finds it optimal to cut dividends, which reduces leverage, and to increase lending, which pushes leverage in the opposite direction. When the bank does not face costs in adjusting dividends and cannot issue equity, only a sufficiently large reduction in the interest rate generates excessive risk-taking (i.e. the second effect dominates). This result follows from the fact that cutting dividends all the way to zero can be done instantaneously, yet it is not enough to compensate for the increase in leverage generated by the expansion in lending. When banks wish to smooth dividends, leverage unambiguously rises for any reduction in the risk free rate because dividends are adjusted gradually. Moreover, the presence of dividend smoothing amplifies and propagates the increase in bank default risk because the cost in adjusting dividends effectively reduces the opportunity cost of lending.

Finally, I explore how capital requirements imposed on the bank and loan-to-value caps imposed on borrowers affect excessive risk-taking. In this model, capital requirements increase borrowers' leverage and default risk, but on net the risk of bank default is lower because of the extra capital the bank is forced to hold. Loan-to-value caps generate the opposite effect than capital requirements, they make loans safer but the bank riskier. By limiting bank lending, loan-to-value caps make it optimal for the bank to hold less capital and become more fragile. Because capital requirements directly affect bank default risk, they are more effective in reducing excessive bank risk of default at low levels of bank capital, while at high levels both restrictions generate similar outcomes.

At a broader level, the model illustrates how a benign macroeconomic environment (e.g. low risk free rates, asset price booms, increased financial innovation, etc.), which increases lending profitability, could generate excessive risk-taking in the presence of limited liability. Moreover, since the model shows that the degree of excessive risk-taking varies with the macroeconomic environment, it favors the use of macroprudential regulation in the sense that only capital requirements that are contingent with the aggregate state of the economy could eliminate excessive risk-taking.

The predictions of the model are consistent with a number of empirical studies. Jimnez, Ongena, Peydro-Alcalde and Saurina (2007) use a loan-level dataset from the Spanish credit registry and conclude that banks increase lending to risky borrowers when interest rates are low. Ioannidou, Ongena and Peydro (2009) reach similar conclusions with data on Bolivia, while De Nicolo, Dell'Ariccia, Laeven and Valencia (2010) present evidence that corroborates these conclusions with U.S. data. Empirical evidence on the credit channel of monetary policy (Bernanke, Gertler and Gilchrist (1996) and others), also presents suggestive evidence consistent with these studies, since this literature broadly concludes that monetary tightening reduces access to credit for borrowers for whom information problems are more severe, and conversely, monetary easying improves it. If those borrowers for which information problems are more severe are also ex-ante riskier, then one can interpret this evidence as indirect support to the proposition that low monetary policy rates increase risk-taking.

This paper contributes to a scarce literature on the relationship between monetary policy rates and banks' risk-taking and to the growing literature on leverage cycles. On the first topic, Dell'Ariccia, Laeven and Marquez (2010) develop a model to examine the relationship between monetary policy rates and banks' risk-taking, but in a static framework. On the second topic, recent contributions rely on collateral constraints and externalities arising from asset fire sales to generate excessive leverage (e.g. Bianchi (2010), Bianchi and Mendoza (2010), Jeanne and Korinek (2010), and Jeanne and Korinek (2011)). This paper shows that excessive leverage can arise even in absence of fire sale externalities.

The next section presents the depositor-bank and borrower-bank contracts, in which I adopt the familiar Townsend (1979)'s costly state verification. Section III presents a one period version of the model to illustrate the basic channels through which the risk-free rate affects bank profitability. Section IV presents the infinite-horizon version of the model, as well as the construction of the constrained social planner benchmark. Section V discusses the implications of dividend smoothing. Section VI examines the role of regulatory restrictions in the form of capital requirements and loan-to-value caps in reducing bank fragility induced by reductions in the risk-free rate. Section VII concludes.

II. Loan and Deposit Contracts

The focus of the paper is on bank behavior, therefore, the role of depositors and borrowers is kept simple by assuming they live only for one period. Both types are assumed to be risk neutral and negligible in size relative to the bank. Financial frictions are modeled explicitly by imposing asymmetric information between borrowers and the bank, and depositors and the bank. These information problems take the form of costly state verification in both cases, a commonly used device to model financial frictions in the financial accelerator literature (Bernanke, Gertler and Gilchrist (1999), Carlstrom and Fuerst (1997), and others).

A. Bank-Borrower Loan Contract

There is a continuum of identical, risk-neutral borrowers who live for one period only and have access to a common production technology with only capital as an input

$$y_{t+1} = \alpha_{t+1}\mathcal{R}k_t \tag{1}$$

where α_{t+1} is assumed to be i.i.d. mean-one, and continuously distributed over a non-negative support with its cumulative distribution function denoted by F_α, and its corresponding density f_α. $\mathcal{R}$ denotes the exogenously assumed average return on capital. Capital is assumed to be funded with loans from the bank, l_t, and an exogenous endowment normalized for simplicity to 1, $k_t = l_t + 1$. Capital depreciates fully at the end of production, after which entrepreneurs consume any surplus and die. There are no ex-ante information asymmetries between the bank and borrowers, they arise only after the realization of α, which is assumed to be the entrepreneurs' private information, but common to all entrepreneurs. Gale and Hellwig (1985) shows that the optimal financing device in this environment is risky debt.

The information structure used in this contract, α being common to all borrowers and its realization not known to the bank, may come up as unusual since it resembles an aggregate shock and it may be hard to argue that an aggregate shock is not observed by the bank. This route has been chosen for modeling convenience.[2]

Under limited liability, an entrepreneur defaults whenever α falls below the level at which profits are equal to zero, denoted by $\overline{\alpha} = \frac{l_t R_t}{\mathcal{R}(1+l_t)}$. In the event of default, the bank pays monitoring costs (or bankruptcy costs) $1 \geq u > 0$ to observe α, and seizes the project, as in Townsend (1979), Gale and Hellwig (1985), and Williamson (1987). The ex-post return to an entrepreneur can be summarized by

$$\pi(\alpha_{t+1}, l_t, R_t) = \begin{cases} \alpha_{t+1}\mathcal{R}(1+l_t) - l_t R_t & \text{if } \alpha_{t+1} \geq \overline{\alpha}(R_t, l_t) \\ 0 & \text{if } \alpha_{t+1} < \overline{\alpha}(R_t, l_t) \end{cases} \tag{2}$$

[2]One could instead assume α is idiosyncratic and introduce also a source of aggregate risk which is capable of pushing the bank into bankruptcy. This alternative would not change results materially but introduces a complication. The default threshold for the bank, as it will be shown momentarily, would no longer be a simple analytical expression as it is the case with the approach I chose, but instead a numerical object. Since the more natural way to proceed does not add value but only complicates the solution, I chose this simple modeling approach. Still, it is not entirely unrealistic if one interprets the projects as identical homes owned by these borrowers who live in the same neighborhood. When home prices decline, the bank does not know the precise value of the houses, which affects all identically, but the bank can hire an appraiser to learn their values.

where R_t and l_t denote the interest rate and loan amount agreed on the debt contract.

The bank is assumed to be a monopoly and makes "*take-it-or-leave-it*"offers to borrowers including a loan amount and an interest rate R_t that ensures entrepreneurs a return that is at least as good as their opportunity cost, assumed to be equal to the exogenously given risk-free rate. In this environment, this participation constraint always holds with equality, otherwise the bank can increase profits by charging a slightly higher interest rate. This constraint defines implicitly the market-clearing lending rate as a function of the amount of lending and the risk-free rate $R(l_t, \rho_t)$.

Using this function, ex-post bank revenues for the bank are given by

$$g(\alpha_{t+1}, l_t, \rho_t) = \begin{cases} R(l_t, \rho_t)l_t & \text{if } \alpha_{t+1} \geq \overline{\alpha}(R(l_t, \rho_t), l_t) \\ (1-u)\alpha_{t+1}\mathcal{R}(1+l_t) & \text{if } \alpha_{t+1} < \overline{\alpha}(R(l_t, \rho_t), l_t) \end{cases} \tag{3}$$

B. Bank-Depositor Contract

A low realization of α forces the bank to seize entrepreneurs' projects, but if it is sufficiently low, it may also force the bank to liquidate. As in the case of the entrepreneur, the bank defaults if the realization of α falls below the level at which its capital equals zero. Denoting n_{t+1} the banks' capital in period t+1, continuity of α implies that there is a value $\underline{\alpha}$ such that $n_{t+1} = 0 \rightarrow g(\underline{\alpha}, l_t, \rho_t) - i_t d_t = 0$. Where i_t is the deposit interest rate, d_t the amount of deposits, and $g(\underline{\alpha}, l_t, \rho_t)$ the bank revenue function, Equation (3). In line with the costly state verification framework, depositors pay monitoring costs $1 > \omega > u$. The latter implies that depositors are less efficient than the bank in monitoring entrepreneur's projects, justifying in this way the existence of financial intermediation.

Notice that for the bank to default, entrepreneurs must have defaulted, given that α is the only source of bankruptcy risk in the model, implying then that $\underline{\alpha} < \overline{\alpha}$. [3]

Therefore, $\underline{\alpha}$ is given by

$$\begin{aligned} g(\underline{\alpha}, l_t, \rho_t) - i_t d_t &= 0 \\ (1-u)\underline{\alpha}\mathcal{R}(1+l_t) - i_t d_t &= 0 \\ \frac{i_t d_t}{(1-u)\mathcal{R}(1+l_t)} &= \underline{\alpha}(i_t, d_t, l_t) \end{aligned} \tag{4}$$

With the bank subject to limited liability, the payoff to a depositor is then given by

[3] At extremely low levels of bank capital the bank may have the incentives to take on an infinite amount of deposits, consume its capital, and default. However, no depositor would lend to the bank when it is imminently bankrupt because there is no interest rate that compensates for the risk of default. Under the chosen calibration, this outcome arises only when capital is negative.

$$h(\alpha_{t+1}, l_t, d_t) = \begin{cases} i_t d_t & \text{if } \alpha_{t+1} \geq \underline{\alpha}(i_t, d_t, l_t) \\ (1-\omega)\alpha_{t+1}\mathcal{R}(1+l_t) & \text{if } \alpha_{t+1} < \underline{\alpha}(i_t, d_t, l_t) \end{cases} \tag{5}$$

which follows from the assumption that deposits are one-period contracts, and that in the event of a shock negative enough to push both, banks and borrowers into bankruptcy, depositors seize the bank and liquidate entrepreneurs' projects.

Risk-neutral depositors supply funds to the bank infinitely elastically at the interest rate that leaves them indifferent between the expected return from a deposit and earning the risk free return. Therefore, the deposit rate that ensures market clearing between deposit supply and demand, denoted by $i(l_t, d_t, \rho_t)$, can be computed implicitly from the following arbitrage condition.[4]

$$\rho_t d_t = i_t d_t (1 - F_\alpha(\underline{\alpha})) + (1-\omega)\mathcal{R}(1+l_t)E\left[\alpha/\alpha < \underline{\alpha}\right] F_\alpha(\underline{\alpha}) \tag{6}$$

where for compactness, from here on, I drop the arguments of $\underline{\alpha}$. As it was also the case for the lending rate, the deposit interest rate needs to compensate for the risk of default. Also, both the lending and deposit rates are a function of the risk free rate because it is the relevant opportunity cost for depositors and borrowers.[5]

Before proceeding, it is important to mention that a well-known criticism of contracts of the type used here is that they are not robust to stochastic monitoring (Mookherjee and Png (1989)), and that they are not ex-post efficient. If lenders could, they would prefer to renegotiate the contract and avoid liquidation. On the latter issue, renegotiation is not possible in this environment because borrowers and depositors live for only one period, and the project has also a one period life. When multi-period contracts are allowed, and liquidation is a choice variable, Krasa and Villamil (2000) show that debt is still optimal, the contract is ex-post efficient, and robust to stochastic monitoring. They also show that the costly state verification model used here can be seen as a reduced form of their more complex setting with a multi-period contracts and costly enforcement.

III. THE BANK'S ONE PERIOD PROBLEM

In illustrating the basic mechanisms of how the risk-free rate affects bank profits, it is useful to begin with a special case of the model, one in which the bank is a static profits maximizer and takes the amount of bank capital as given.

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[4]As in the case of the lending rate, I use a numerical root-finding procedure to solve for the interest rate as a function of its arguments, l_t, d_t, and ρ_t.

[5]Implicit in this assumption is the idea of a minimum scale for the investment project that is larger than the endowment. If this condition is relax and I allow borrowers to invest the endowment, the relevant opportunity cost would be $\mathcal{R}$. While this modification would result in lower profits for the bank, it would not change the results materially.

$$\underset{\{l\}}{Max}\, E\left[g(\underline{\alpha}, l, \rho) - id/\alpha \geq \underline{\alpha}\right] \tag{7}$$

$$\underset{\{l\}}{Max}\, \underbrace{(1 - F_\alpha(\overline{\alpha}))\left[Rl\right] + \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha(1-u)\mathcal{R}(l+1)}_{\textit{Expected Revenues}} - \underbrace{id(1 - F_\alpha(\underline{\alpha}))}_{\textit{Expected Costs}} \tag{8}$$

where for compactness, I have also dropped the arguments of i and R. Assuming the bank holds a given amount of capital equal to q, the balance sheet constraint implies that deposits $d = l - q$. After making this substitution into (8) I solve numerically for the optimal solution for lending as a function of the risk-free rate ρ.[6]

Figure (1) shows the solution as a function of the risk-free rate. A decrease in the risk-free rate causes lending to increase for two reasons. For a given level of lending, a lower risk-free rate lowers the opportunity cost demanded by depositors, Equation (6), and thus lowers expected costs for the bank (last term in Equation (8)). Second, it also implies a lower opportunity cost for borrowers, allowing the monopolistic bank to extract a larger surplus from borrowers, raising expected revenues. The bank then finds it optimal to increase lending because marginal revenues now exceed marginal costs. But the increase in lending causes the risk of default of borrowers to rise, which causes marginal revenues to decrease. However, marginal costs increase by less under limited liability than in absence of it, boosting the bank's incentives to increase lending.[7] The result is then an increase in the bank's expected profits yielding a higher optimal level of lending, which also implies higher leverage and risk of default for borrowers and the bank. Figure (1) shows the solution for lending as a function of ρ, as well as the model's measures of default risk, $F_\alpha(\overline{\alpha})$ for the entrepreneur and $F_\alpha(\underline{\alpha})$ for the bank, both evaluated at the optimal amount of lending. Both rise as the risk-free rate decreases.

IV. THE INFINITE HORIZON CASE

A. Modeling the Bank as a Firm

While the banking literature offers numerous static and short-horizon bank models, it is largely scarce in dynamic, quantitative ones. A few existing models include Valencia (2008), Van Den

[6]The calibration is given in the appendix. A sufficient condition for bank profits to yield an interior solution is that $\overline{\alpha} f_\alpha(\overline{\alpha})/(1 - F_\alpha(\overline{\alpha}))$ is increasing in $\overline{\alpha}$, a condition that is satisfied by the lognormal distribution (see Bernanke et al. (1999)). Intuitively, this tells us that increases in lending generate increases in revenues in the non-default state, but also induces increases in risk. After some point, the increase in risk dominates and expected revenues decrease.

[7]Notice that the marginal expected cost for the bank in absence of limited liability is given by $i'd + i$ whereas with limited liability is given by $i'd + i - F_\alpha(\underline{\alpha})(i'd + i) - idf_\alpha(\underline{\alpha})\frac{\partial \alpha}{\partial l}$, with $F_\alpha(\underline{\alpha})(i'd + i) > 0$ and $idf_\alpha(\underline{\alpha})\frac{\partial \alpha}{\partial l} > 0$.

Heuvel (2002) and Peura and Keppo (2006).[8] The mechanisms illustrated in the static example will remain operational. Furthermore, the loan and deposit contract remain intact, but I introduce a few modifications to the bank's problem. First, bank capital is now endogenous; second, the bank's objective is now to maximize the present discounted value of dividends; and third, the bank cannot issue equity. Gale and Hellwig (1985) showed that in an environment like the one used here, the optimal financing device is risky debt. Thus, the assumption of no equity finance is a simplification to capture the fact that in this environment it is not optimal to issue equity because of the non-observability of α.

The bank starts period t with capital n_t. It observes ρ_t and chooses the amount of deposits, d_t, lending, l_t, and dividends c_t. After making decisions, α_{t+1} is realized, where the $t+1$ subscript is used to highlight the fact that it is unknown when decisions are made. If $\alpha_{t+1} \geq \underline{\alpha}$, the bank collects loans and pays off depositors the agreed amount $i(l_t, d_t, \rho_t)d_t$. It then arrives to the next period with capital n_{t+1}, given by the difference between its total revenues and its liability payments. If $\alpha_{t+1} < \underline{\alpha}$, depositors seize the bank.

The banks' problem is summarized by

$$\underset{\{d_t,c_t,l_t\}}{Max}\, E_t\left[\sum_{t=s}^{t=\infty} {\beta_t}^{s-t} c_t/\alpha_{t+1} \geq \underline{\alpha}\right] \tag{9}$$

$$l_t \leq d_t + \underbrace{n_t - c_t}_{q_t} \tag{10}$$

$$\rho_{t+1} = a_0 + a_1(\rho_t - a_0) + \epsilon_{t+1} \tag{11}$$

$$c_t \geq 0 \tag{12}$$

$$n_{t+1} = \begin{cases} g(\alpha_{t+1}, l_t, \rho_t) - i(l_t, d_t, \rho_t)d_t & = \text{if } \alpha_{t+1} \geq \underline{\alpha} \\ 0 & = \text{if } \alpha_{t+1} < \underline{\alpha} \end{cases} \tag{13}$$

where β_t denotes the discount factor, assumed as $\beta_t = 1/(\rho_t \tau)$ with $\tau > 1$ to rule out self-financing.[9] Equation (9) reflects the objective of maximizing the present discounted value of expected dividends, conditional on the bank not having defaulted. Equation (10) tells us that the bank's liabilities and capital are at least as large as its assets, with q denoting the stock of capital net of dividends. Equation (10) will always hold with equality because the bank has no incentive

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[8]The key difference between the model presented here and Van Den Heuvel (2002) and Peura and Keppo (2006) is that bank capital matters because of the presence of risky debt, while in the cited papers, it matters because of the presence of regulation. The difference with Valencia (2008) is that financial frictions on the bank are here modeled explicitly in the form of risky debt, while in that paper they are exogenously imposed.

[9]In models of this type, the bank would have the incentive to accumulate capital to reduce the likelihood of bankruptcy. If the opportunity cost of holding capital is lower than the risk-free rate, the bank will accumulate capital up to a point where it no longer needs to raise deposits. One could alternatively explicitly model the tax benefits of holding debt to generate positive leverage in equilibrium. The chosen approach is simpler and yields an equivalent outcome. It is a widely used assumption in the precautionary savings literature (see for instance Carroll (2004)).

to raise more deposits than what it needs to finance its chosen amount of lending. This equation can also be interpreted as the balance sheet constraint for the bank. Equation (11) denotes the law of motion for the real risk-free rate, which by assumption follows a mean-reverting process, where ϵ_{t+1} is a mean-zero, i.i.d., random disturbance.[10] Equation (12) is the no-equity-finance restriction. Finally, equation (13) corresponds to the law of motion of bank capital under limited liability.

The Bellman's equation for the problem, after eliminating deposits using equation (10) is given by

$$V(n_t, \rho_t) = \underset{\{c_t, l_t\}}{Max} \{c_t + \beta_t E_t \left[V(n_{t+1}, \rho_{t+1})/\alpha_{t+1} \geq \underline{\alpha} \right]\} \tag{14}$$

$$\rho_{t+1} = a_0 + a_1(\rho_t - a_0) + \epsilon_{t+1} \tag{15}$$

$$c_t \geq 0 \tag{16}$$

$$n_{t+1} = \begin{cases} g(\alpha_{t+1}, l_t, \rho_t) - i(l_t, l_t - q_t, \rho_t)(l_t - q_t) & = \text{if } \alpha_{t+1} \geq \underline{\alpha} \\ 0 & = \text{if } \alpha_{t+1} < \underline{\alpha} \end{cases} \tag{17}$$

If $\alpha_{t+1} \geq \overline{\alpha}$, both the bank and entrepreneurs remain solvent, loans are collected and depositors repaid. If $\underline{\alpha} \leq \alpha_{t+1} < \overline{\alpha}$ only entrepreneurs default. Finally, if $\alpha_{t+1} < \underline{\alpha}$ both default. When the bank defaults, it is assumed its license is withdrawn.[11] Taking into account these possible outcomes, the Bellman's equation, rewritten in terms of capital net of dividends q, is given by

$$V(n_t, \rho_t) = \underset{\{q_t, l_t\}}{Max} \{n_t - q_t + \beta_t E_t \left[(1 - F_\alpha(\overline{\alpha})) \underset{\overline{\alpha}}{V} + \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}} f_\alpha(\alpha) \right] \} \tag{18}$$

subject to (10)-(13).[12] The corresponding first order condition for q by[13]

$$1 = (i_t - (l_t - q_t) i_t^q) \beta_t E_t \left[(1 - F_\alpha(\overline{\alpha})) \underset{\overline{\alpha}}{V^n} + \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V^n}} f_\alpha(\alpha) \right] \tag{19}$$

and the one for lending is given by

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[10]This assumption follows from the intention to capture observed autocorrelation in interest rates, but is not critical for the qualitative nature of results.

[11]It translates into imposing the value function at zero equity to be zero.

[12]For compactness I introduce the following notation: $\underset{\overline{\alpha}}{V} = V(n_{t+1}, \rho_{t+1}|_{\alpha_{t+1} \geq \overline{\alpha}})$ and $\overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}} = V(n_{t+1}, \rho_{t+1}|_{\overline{\alpha} \geq \alpha_{t+1} \geq \underline{\alpha}})$, and where a superscript n will denote its derivative with respect to bank capital.

[13]Derivations are shown in the appendix.

$$
\begin{aligned}
0 = & \overbrace{E_t\left(\left[R_t + l_t R_t^l - i_t - (l_t - q_t) i_t^l\right](1 - F_\alpha(\overline{\alpha})) V^n_{\overline{\alpha}}\right)}^{\substack{\textit{Expected Marginal Profits} \\ \textit{if borrowers repay}}} \\
& \underbrace{+ E_t\left(\int_{\underline{\alpha}}^{\overline{\alpha}} \left[\alpha(1-u) - i_t - (l_t - q_t) i_t^l\right] \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}}{}^n f_\alpha(\alpha)\right)}_{\substack{\textit{Expected Marginal Profits} \\ \textit{if borrowers default}}}
\end{aligned}
\tag{20}
$$

where I have also dropped the arguments of i_t, R_t, and their derivatives, denoted by i^l, i^q, and R^l. Equation (19) equates the marginal value of dividends with the marginal value of bank capital. The marginal value of capital is composed of two terms, $(i_t - (l_t - q_t) i_t^q)$, which corresponds to the marginal cost of raising deposits that could be saved if the bank instead increased capital. This term represents the current-period component of the marginal value of bank capital. The second component represents the present marginal value of future dividends, since earnings retained today will generate profits in future periods.

Optimal capital, in this model, is driven by the balance of two forces: when bank capital is high its marginal value is lower than the discount factor, implying that it pays off to distribute dividends and decrease capital. On the other hand, at low levels of bank capital, the high risk of bankruptcy makes capital very valuable, more so than dividends, making it attractive to increase capital.

Equation (20) tells us that the optimal amount of lending is such that marginal profits equal zero. The terms in the squared brackets denote the marginal profits under default and no-default of borrowers, weighted by the marginal value of bank capital in each state. The terms inside each corresponding set of brackets reflect the current period marginal profits, when borrowers default and when they do not.

1. A Constrained Social Planner Benchmark

It is useful to develop a benchmark to illustrate the increase in risk-taking implied by limited liability. This benchmark can be thought of a constrained social planner problem in which the best he can do is to internalize the losses depositors face when the bank defaults, but everything else remains the same. This benchmark model is given by

$$
V(n_t, \rho_t) = \underset{\{q_t, l_t\}}{Max} \{ n_t - q_t + \beta_t E_t \left[(1 - F_\alpha(\overline{\alpha})) V_{\overline{\alpha}} + \int_0^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{0}{V}} f_\alpha(\alpha)\right] \}
\tag{21}
$$

with first order conditions identical to (19) and (20), except for the limits of integration in the default state, which before ranged from $\underline{\alpha}$ to $\overline{\alpha}$ and now from 0 to $\overline{\alpha}$, reflecting the fact that the planner cares about the default region.

Figure 2 shows the marginal value of bank capital for the baseline and benchmark models. The appendix elaborates on the calibration and the numerical solution method. The figure illustrates how leverage is determined. Essentially, I construct a version of equation (19) expressed in terms of leverage.[14] The horizontal line corresponds to the discount factor, with the risk-free rate equal to its unconditional mean. The marginal value of bank capital is higher in the benchmark model, more so at low levels of bank capital or when the risk of default is high. Limited liability makes holding capital less valuable because the bank does not internalize the losses it would impose on depositors if it defaulted. Consequently, optimal leverage is higher (or the capital-to-assets ratio lower) in the baseline model. Limited liability makes it attractive then to take on more leverage whenever profitability of lending increases. In the particular case of this paper, this profitability arises from decreases in the monetary policy rate. However, it is easy to see that there are other sources of increased profitability that could be exploited in this model that could generate similar effects (i.e. asset price booms, increases in financial development, increases in productivity).

B. Optimal Decision Rules

The model is solved using backwards induction starting at some hypothetical last period of life at which the bank distributes all its capital in dividends. Using this assumption as an initial condition, I iterate backwards applying the method of endogenous gridpoints (Carroll (2006)) until the optimal decision rules for lending and dividends satisfy a given convergence criteria. Figure 3 shows the converged decision rules as a function of the state variables of the problem. The corresponding shapes are intuitive, both are increasing in bank capital. Lending increases as the risk-free rate goes up because it becomes more profitable to lend, while dividends decrease when the risk-free rate goes down because the discount factor also goes down. Notice also the kink in the dividends function, which corresponds to the points where the no-equity-finance constraint binds. The arrows in the figures denote the location of the steady state. The steady state values are shown in Table 1, on which I elaborate later.

I obtain similar decision rules for the benchmark model and compute the difference in the banks' probability of default, $F_\alpha(\underline{\alpha})$, between the baseline and benchmark models for each point in the state space, evaluated at each corresponding set of optimal decision rules. Figure 4 shows the outcome. As expected from the analysis in the previous section, default risk is overall higher in the baseline model, but varies in a non-monotonic way with the risk-free rate. Recall that a reduction in the risk-free rate increases lending profitability and also reduces the attractiveness of distributing dividends (see Figure 3). Cutting dividends reduces leverage, while increases in lending raise leverage. The non-monotonicity of the response of excessive risk-taking to the risk-free rate arises from the fact that for some region in the state space, the increase in leverage generated by the increase in lending trumps the reduction in leverage generated by the decrease in dividends. Concretely, for any level of bank capital, there is a large enough reduction in the risk-free rate such that leverage increases because the first effect is stronger. For smaller

[14] In constructing this graph, I evaluate equation (19) using the lending decision rule (see section B for details) at different values of beginning-of-the period bank capital, n, and compute the corresponding leverage at each of these points as the ratio of bank capital to loans, with dividends equal to zero. This procedure gives me a mapping from leverage to the marginal value of bank capital, depicted in the figure.

reductions in the risk-free rate, the second effect dominates. This becomes clear for values of bank capital around 1 in Figure 4.

C. Response to Interest Rate Shocks

Using the optimal decision rules derived earlier, I now simulate the model to study its dynamic properties. The experiment involves a one time reduction in the risk-free rate, in period 4, starting from the steady state. Given these initial conditions, the decision rules determine the optimal dividend and lending decisions. Together with the corresponding transition equations for the risk-free rate and bank capital, I can compute the expected value of the state variables for the next period, integrating across possible realizations of α and ρ. These values become the state variables for the following period which feed into the optimal decision rules to get the amounts of dividends. The process is repeated for 20 quarters. The outcome is shown in Figure 5. The panel shows the responses to a one-standard deviation (approximately 0.7 percent) and two-standard deviation reduction in the risk-free rate.

Lending increases sharply in both cases, but the role of the no-equity-finance constraint in shaping the sign of the relationship, highlighted in the previous section, becomes evident now. The bank cuts dividends in both cases. When the shock is small, this reduction in dividends allows the bank to increase lending without increasing leverage, in fact, leverage goes down and decreases even further when the profits of new loans materialize. There is a gradual adjustment towards equilibrium as the interest rate goes back up and the bank adjusts lending and dividends accordingly. Therefore, while the riskiness of loans goes up, the riskiness of the bank does not. When the shock is large, the increase in lending is larger, and while dividends are cut all the way down to zero, the increased in optimal lending is large enough that leverage and risk of default rise. But the risk of default rises more in the baseline than in the benchmark case, resulting in excessive risk-taking. During this period financial fragility increases since it would take a smaller negative shock–smaller than in steady state–to push the bank and borrowers into bankruptcy.

V. THE MODEL WITH DIVIDEND SMOOTHING

I now modify the model to allow for equity financing and dividend smoothing. It is important to examine these features because in reality, banks can issue equity, although it is costly, and dividends tend to be sticky. Several alternatives exist to incorporate these modifications. For instance, one could assume that recapitalization is costly and arrives with a delay, as in Peura and Keppo (2006), or one could simply impose dividends adjustment costs as in Jermann and Quadrini (2009). These options ultimately imply that adjusting bank capital is costly. An even simpler way to incorporate this feature is to assume risk-averse shareholders, which can be justified by assuming that shareholders do not hold diversified portfolios and a significant fraction of their wealth is invested in bank shares. The immediate implication of such an assumption is dividend smoothing. The gains from introducing these modifications are that now the model implications for the behavior of dividends is closer to the data, but as pointed out in Gale and Hellwig (1985), risk aversion complicates the shape of the optimal contract, therefore, the optimality of the contract becomes less straightforward. I assume that these equity injections come from existing shareholders who also manage the bank.

The objective of these risk-averse shareholders is now to maximize the present discounted value of utility derived from dividends. Equation (9) now becomes

$$\underset{\{d_t, c_t, l_t\}}{Max}\, E_t \left[\sum_{t=s}^{t=\infty} {\beta_t}^{s-t} u(c_s) / \alpha_{s+1} \geq \underline{\alpha} \right] \tag{22}$$

where $u(\cdot)$ denotes the utility function, which satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. The problem is subject to (10), (11), and (13), but I no longer impose constraint (12). The Envelope Theorem implies the following Euler equations for the problem.[15]

$$u^c(n_t - q_t) = (i_t - (l_t - q_t) i_t^q) \beta_t E_t \left[u^c(\underset{\overline{\alpha}}{c_{t+1}})(1 - F_\alpha(\overline{\alpha})) + \int_{\underline{\alpha}}^{\overline{\alpha}} u^c(\underset{\underline{\alpha}}{\overset{\overline{\alpha}}{c_{t+1}}}) \right] f_\alpha(\alpha) \tag{23}$$

$$\begin{aligned} 0 = {} & E_t \left[(1 - F_\alpha(\overline{\alpha})) u^c(\underset{\overline{\alpha}}{c_{t+1}}) \left(R_t + l_t R_t^l - i_t - (l_t - q_t) i_t^l \right) \right] f_\alpha(\alpha) \\ & + E_t \left[\int_{\underline{\alpha}}^{\overline{\alpha}} u^c(\underset{\underline{\alpha}}{\overset{\overline{\alpha}}{c_{t+1}}}) \left(\alpha(1-u) - i_t - (l_t - q_t) i_t^l \right) \right] f_\alpha(\alpha) \end{aligned} \tag{24}$$

The left-hand side of equation (23) corresponds to the marginal utility of dividends. As before, the bank chooses how much dividends to distribute as to equate this term with the marginal value of capital, the right-hand side of equation (23). The same intuition as in the model with risk neutrality applies. The marginal value of capital includes the current-period cost of raising deposits and the discounted expected marginal utility from future dividends. In the case of lending, equation (24) is identical to equation (20), with the marginal utility replacing the marginal value of bank capital because the Envelope Theorem implies that $u^c(c_t) = V^n(n_t, \rho_t)$.

Figure 6 shows the optimal decision rules as a function of the state variables. A noticeable difference between this figure and Figure 3 is the absence of the kink in the dividends policy function because the no-equity-financing constraint has been removed.

I also construct a benchmark case in which the bank internalizes the losses imposed on its creditors when it defaults.[16] Figure 7 depicts the difference in risk of default between the model

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[15] I have dropped the arguments of the dividends function for compactness and denote $\underset{\overline{\alpha}}{c_{t+1}} = c(n_{t+1}, \rho_{t+1}|_{\alpha_{t+1} \geq \overline{\alpha}})$ and $\underset{\underline{\alpha}}{\overset{\overline{\alpha}}{c_{t+1}}} = c(n_{t+1}, \rho_{t+1}|_{\overline{\alpha} \geq \alpha_{t+1} \geq \underline{\alpha}})$.

[16] As before, the crucial change lies in the lower bound of the integrant when computing expected profits for the bank across realizations of α, which in the benchmark it covers the entire default region, instead of only the region where the bank remains solvent.

with dividend smoothing and its corresponding constrained social planner benchmark. At first glance, the figure is similar to Figure 4, the sensitivity of excess-risk to reductions in the risk-free rate is much higher at low levels of bank capital. However, the relationship is now monotonic because of the absence of the no-equity-financing constraint.

Table 1 shows the steady state values of the endogenous variables for different calibrations. A couple of properties of the model are worth highlighting. First, the model generates a precautionary motive, when the standard deviation of borrowers' project value increases, the bank's capital-to-assets ratio increases. Notice how raising σ_α from 0.06 to 0.10 raises the target capital-to-assets ratio from 17.62 percent to 18.03 percent in the benchmark case when $\tau = 1.025$ and shareholders are risk neutral. This precautionary motive gets stronger in the model with dividend smoothing as one would expect because now shareholders dislike dividend volatility.[17] However, this precautionary motive arises in the baseline model only when the discount factor is low, or shareholders are more patient. The logic follows from the fact that in this model, a change in borrowers' project risk, σ_α, has two effects: first, it increases upside returns without modifying the downside because of limited liability, and second, it increases the volatility of future profits. The first effect reduces the incentive to hold more capital, while the second one creates the incentive to hold more capital to self-insure against this volatility. The more the bank discounts the future, the lesser the importance of the second effect because the bank has less desire to stay in business for a prolonged period of time. In that case, the bank's optimal leverage and risk of default are higher, increasing the importance of limited liability. When the bank has a low discount factor, the bank is more willing to accumulate capital, resulting in a leverage level farther away from the bankruptcy point and thus reducing the relevance of limited liability. In the table, one can see how when $\tau = 1.005$ the mechanism just described emerges in all cases. A second result is that the change in σ_α also affects the size of the bank. Notice how in all cases, the equilibrium amount of lending is substantially reduced when σ_α increases. This is because the importance of financial frictions depend on risk and liquidation costs. By increasing risk, financial frictions become more quantitatively important, making it more expensive to borrow, and thus reducing credit intermediation.

A. Response to Interest Rate Shocks

As in section C, I examine the dynamic responses of the model with dividend smoothing following a reduction in the risk-free rate, contrasting them with its corresponding benchmark. The results are shown in Figure 8. There is no qualitative difference between the responses to a small or large interest rate reduction. In both cases, the bank cuts dividends, but more so in the benchmark model because the marginal value of bank capital is higher. The Capital-to-assets ratio declines because of the sharp increase in lending, resulting in risk of default rising more in the baseline than benchmark model. Notice also that the increase in risk of default not only is larger than in the previous case, under risk-neutrality, but also persists for a longer period of time. This follows from the desire of shareholders to smooth dividends. A sharp increase in dividends, as it would happen under risk neutrality, would decrease the marginal utility of dividends, compressing

[17] A key driver of this result is the decreasing nature of the marginal value of bank capital. Valencia (2008) elaborates in more detail about this behavior and Valencia (2010) presents empirical evidence that supports it.

the right-hand side of Equation (24), for a given amount of lending. This induces the bank to lend more instead of distributing dividends.

VI. POLICY EXPERIMENTS

There are several ways in which policy intervention could help correct the distortion introduced by limited liability. Here, I am going to examine the role of two specific alternatives. One is to require the bank to hold more capital–through capital requirements–thus reducing the likelihood the bank goes bankrupt, and the other is to limit how much risk banks can take on their assets, for instance, through loan-to-value caps. The purpose is to contrast them and examine which one could be more effective in reducing excessive risk-taking.

Capital requirements are introduced as follows: I assume that when the capital-to-assets ratio, measured as q/l, decreases below an exogenously given regulatory minimum, λ, shareholders are forced to inject capital to cover the shortfall.[18] However, this happens before uncertainty is realized, which implies that the bank can still go bankrupt. In keeping this exercise simple I also assume that this recapitalization cannot take place once the bank has gone bankrupt. This assumption serves the purpose of ruling out the ability of the bank to always replenish capital, in which case depositors would never face a loss. The problem is identical to the one in section V, with the addition of the following constraint:

$$c_t = \begin{cases} c_t^* & \textit{if } q_t \geq \lambda l_t \\ q_t - \lambda l_t & \textit{if } q_t < \lambda l_t \end{cases} \tag{25}$$

where c_t^* denotes the unconstrained solution for dividends, whereas $q_t - \lambda l_t$ reflects the capital shortfall the bank needs to inject to meet capital requirements.[19]

Recall that in this model I am assuming that borrowers' equity is fixed, which implies that lending is equivalent to borrowers' leverage from which one can compute the corresponding loan-to-value ratio. Assuming that the regulatory authority does not want the loan-to-value ratio of borrowers to exceed Θ, the loan-to-value constraint becomes[20]

$$l_t \leq \tfrac{\Theta}{1-\Theta} \tag{26}$$

[18] One can introduce this restriction in several ways. The key feature is that violating this constraint is costly. The specifics of how this cost is modeled may have somewhat different quantitative but no qualitative results.

[19] Imposing the restriction on capital net of dividends, q, implies that the regulator cares about what capital the bank has left after the distribution of dividends.

[20] Recall that the value of borrowers' assets is given by $l_t + 1$ where equity equals 1. The loan-to-value ratio is given by $\frac{l_t}{l_t+1}$, which cannot exceed Θ.

Constraints (25) and (26) are imposed one at a time. Optimal decision rules for the bank are obtained as before, which for brevity are omitted here but are of similar shape as those in Figure 6. In solving the model with capital requirements, I choose $\lambda = 10\%$ and $\lambda = 15.38\%$, where the latter corresponds to the steady state capital-to-assets ratio that arises in the benchmark model (see Table 1). I also choose $\Theta = 85\%$ and $\Theta = 86.43\%$. The latter is the equilibrium loan-to-value that arises in the benchmark model. Table 2 shows the steady state values for the models discussed in this section, where for convenience I have also reproduced the steady state values for the model without these constraints.

Notice how the steady state capital-to-assets ratio is higher when there are capital requirements. Moreover, the steady state level exceeds the regulatory requirement. The bank holds a buffer of capital to reduce the likelihood of hitting the constraint. The optimal capital-to-assets ratio declines, relative to the baseline, when loan-to-value caps are in place because the constraint on lending makes it less attractive for the bank to accumulate capital since it will not be able to expand lending as much as it wished when the risk-free rate goes down. Notice also how the introduction of capital requirements causes the riskiness of loans to increase (denoted in the table as borrowers' default probability), but the bank's probability of default goes down because of the extra capital the bank holds. In contrast, the introduction of loan-to-value caps decreases the riskiness of loans, but increases the riskiness of the bank, since in that case the bank holds less capital.

Figure (9) shows the familiar plot for excessive risk of default for the model with capital requirements. As expected, the higher the capital requirements, the lower the excess risk of bank default. Capital requirements set at $\lambda = 15.38\%$ can substantially reduce excessive bank risk-taking, but cannot eliminate it entirely because for very low levels of the interest rate, the profitability of lending increases sufficiently as to make it optimal for the bank to take on more risk, despite the presence of capital requirements. Since the latter do not factor in the stronger incentives the bank has to take on excessive risk when the risk-free rate is low. At a broader level, this implication of the model can be interpreted as a justification for regulation that is contingent on the aggregate state of the economy, such as countercyclical macro prudential regulation. Interestingly, this implication of the model hints at a potential conflict between a regulator who cares only about financial stability and the monetary authority who may choose to lower interest rates to stimulate economic activity, which in part takes place through the supply of credit.

Figure (10) shows the outcome for the model with loan-to-value caps. However, for visual convenience I only include the solution for $\Theta = 85\%$. When interest rates are low, the bank wants to expand lending but it is limited by the constraint, especially at low levels of capital. Therefore, risk-taking is reduced relative to the baseline model. In fact, there is a region in the state space where risk of default in the baseline model is below the one under the benchmark. This happens because the loan-to-value cap is binding, limiting the riskiness of loans and generating too little risk-taking in this region. At high levels of the risk-free rate, the constraint on lending may no longer bind, which generates higher leverage and risk of default because optimal capital is lower than in the benchmark model (recall that the bank has less incentives to accumulate capital in the presence of the loan-to-value caps because it knows it will be constrained in how much it can lend if interest rates are low).

Finally, Figure 11 shows the dynamic responses to a reduction in the risk-free rate. In the figure I have included the results from the benchmark model (blue solid line), the model with capital requirements (dotted brown line), and the model with loan-to-value caps (red dashed line). In conducting these simulations I set $\Theta = 86.43\%$ and $\lambda = 15.38\%$. Both correspond respectively, to the steady state loan-to-value ratio and capital-to-assets ratio that arise in the benchmark model.

In the model with loan-to-value caps, lending increases by less than in any of the other two models. Dividends increase because of the additional profits generated by increased lending, which cannot be used to expand lending further because of the loan-to-value constraint. However, the bank does not adjust dividends instantaneously because it wishes to smooth dividends. Therefore, bank default risk increases slightly on impact in both scenarios (small and large shock) following the increase in lending and leverage, but it decreases in subsequent periods even below the pre-shock levels because of excess capital.

For the model with capital requirements, the qualitative nature of the responses vary depending on whether the regulatory constraint becomes binding or not. Dividends become negative in the case of the large shock because of the forced recapitalization implied by the constraint and the capital-to-assets ratio increases. As a result, the risk of default is lower than the one arising in the benchmark model for a few quarters, but as the bank uses up the extra capital, leverage and the risk of default increase. When the shock is small, there is no equity injection, thus the risk of default increases but less than in the benchmark model.

Overall, this exercise suggest that when λ and Θ are set equal to the steady state levels that arise under the benchmark, both can reduce excessive risk-taking. However, because capital requirements force the bank to hold more capital and thus have a direct impact on bank risk of default, they are more effective in reducing excessive risk-taking without being too distortionary. However, since I have examined only regulatory restrictions that are not contingent on the aggregate state of the economy, excessive risk-taking cannot be entirely eliminated. Therefore, the model favors the use of macroprudential regulation that is contingent on the aggregate economy, in this case, the risk-free rate. In earlier sections I noted that one could exploit other sources of cyclical variation in lending profitability, such as asset prices or the return on capital, financial innovation through variations in liquidation costs, etc. In the presence of limited liability, these other sources of changes in lending profitability could yield similar results than those presented here.

VII. Conclusions

Recent empirical evidence suggests that banks increase risk-taking when monetary policy rates are low. This paper develops a dynamic bank model to understand what may lead banks to adopt this behavior and the distortions that may cause risk-taking to be excessive. In the model, a decrease in the risk free rate increases the profitability of lending for two reasons: it reduces the funding costs of the bank and increases the surplus the monopolistic bank can extract from its borrowers. Because of limited liability, the bank increases the riskiness of its loans and the riskiness of the bank itself above what is suggested by a constrained social planner benchmark in which the bank internalizes the losses it imposes on depositors when it defaults. Consequently, risk-taking is excessive. Simulations of the model suggest that a reduction in the risk-free rate exacerbates excessive risk-taking because the lower the interest rate, the more attractive it is to lend more and increase leverage, increasing the importance of limited liability.

While the above result is unambiguous if banks face costs in adjusting dividends, in absence of these costs, this behavior depends on the size of the reduction in interest rates and the initial capital position of the bank. I consider how capital requirements and loan-to-value caps reduce excessive risk-taking. In this model, capital requirements perform better than imposing loan-to-value caps in reducing excessive risk-taking because they directly affect the banks' probability of default, despite the fact that capital requirements make loans riskier. But because the incentives to take excessive risk intensify as the risk-free rate decreases, regulatory restrictions that do not take into account the state of the economy (in this case the risk-free rate) cannot eliminate excessive risk-taking entirely. These results also highlights how in the presence of limited liability, excessive risk-taking can arise from benign macroeconomic conditions.

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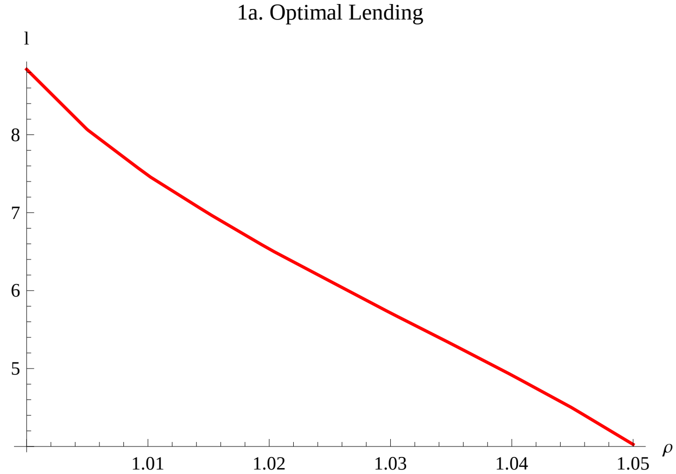
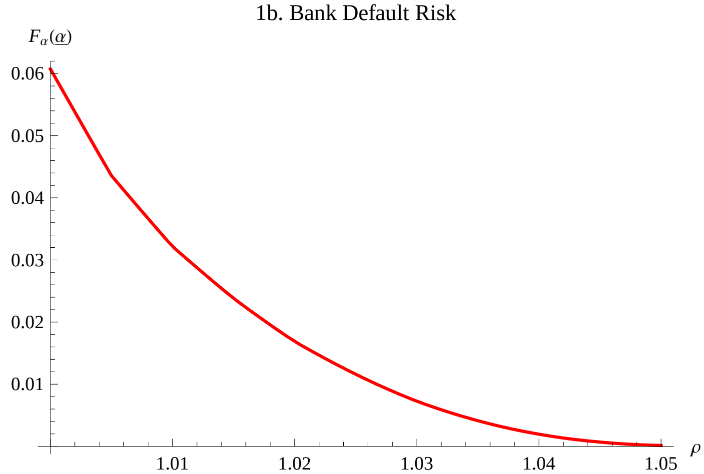
Figure 1. Optimal Lending and Default Risk: The One Period Case

1a. Optimal Lending



1b. Bank Default Risk



1c. Borrower Default Risk

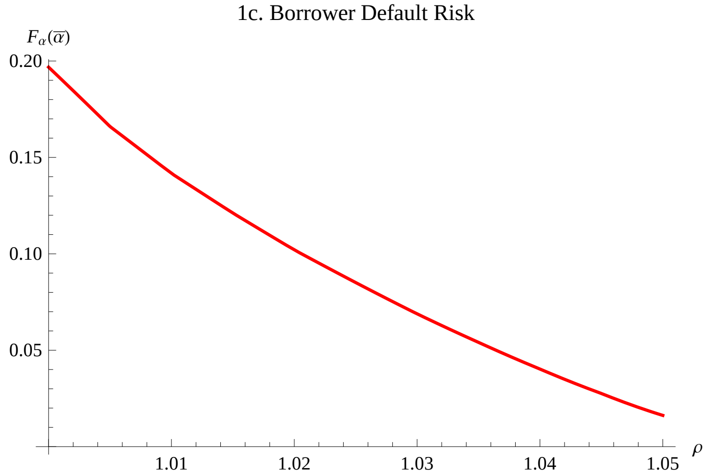


Figure 2. Marginal Value of Bank Capital

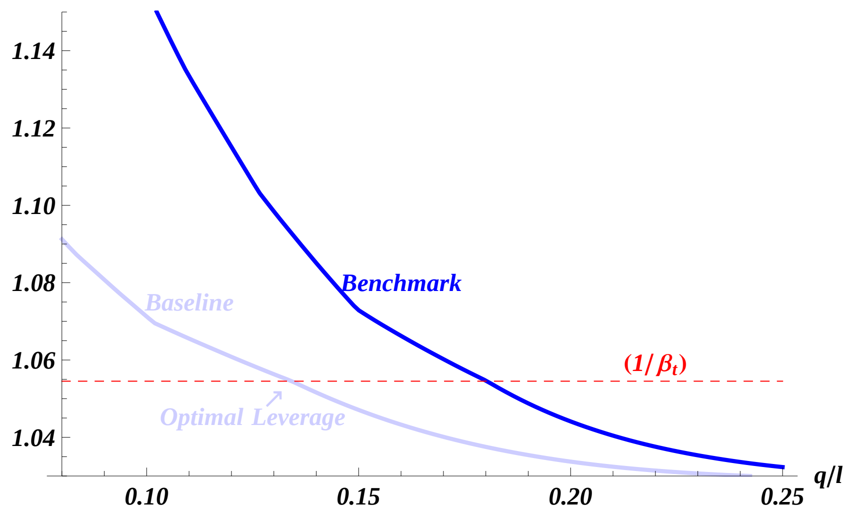


Figure 3. Lending and Dividends Optimal Decision Rules

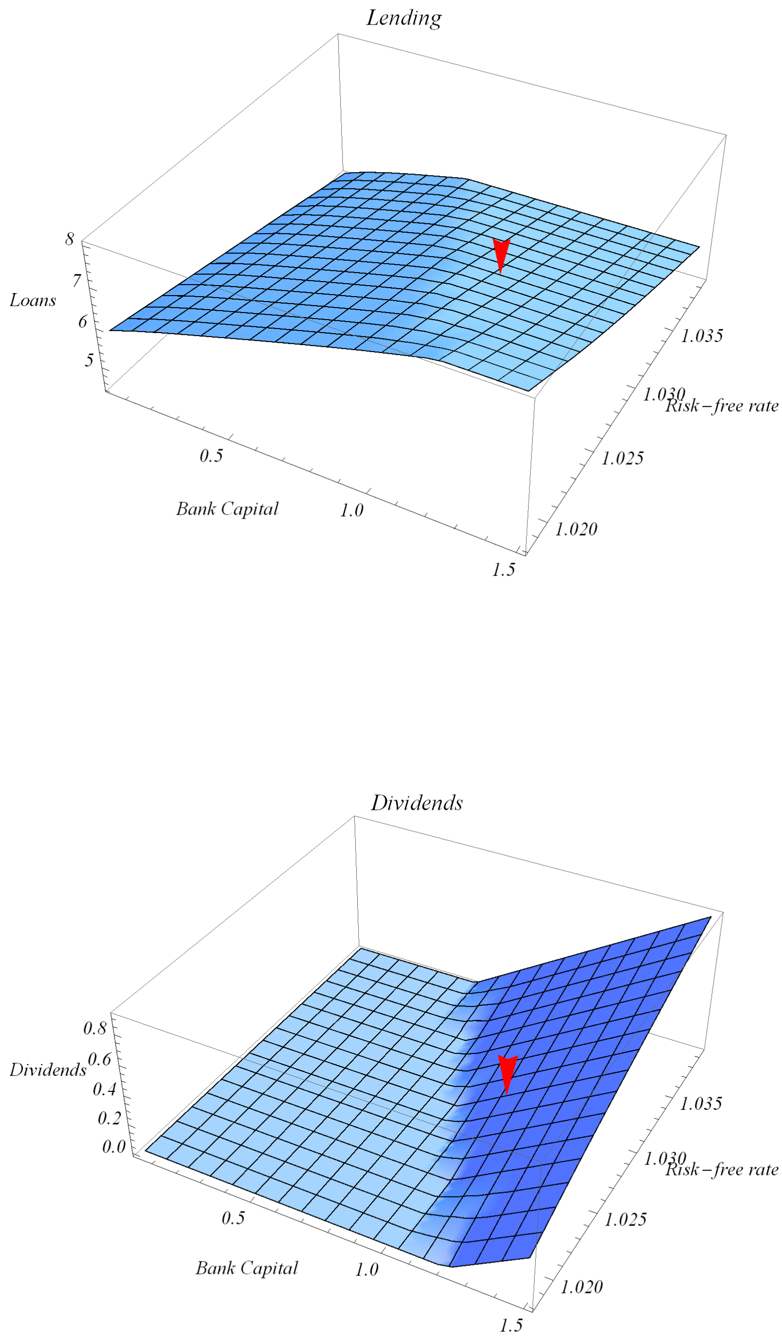
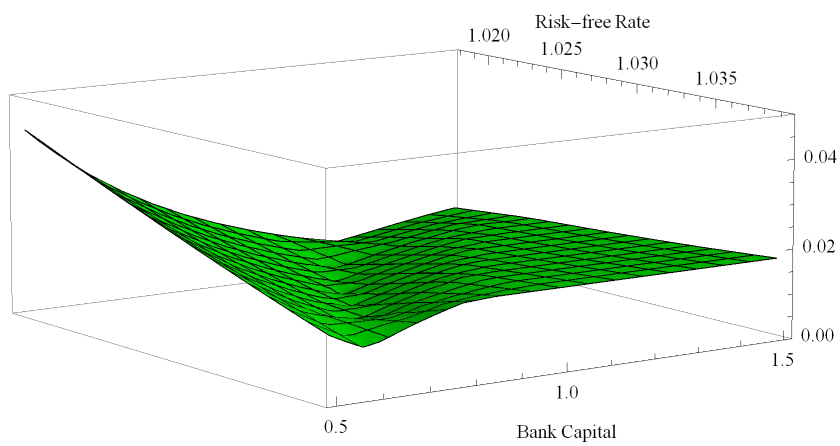
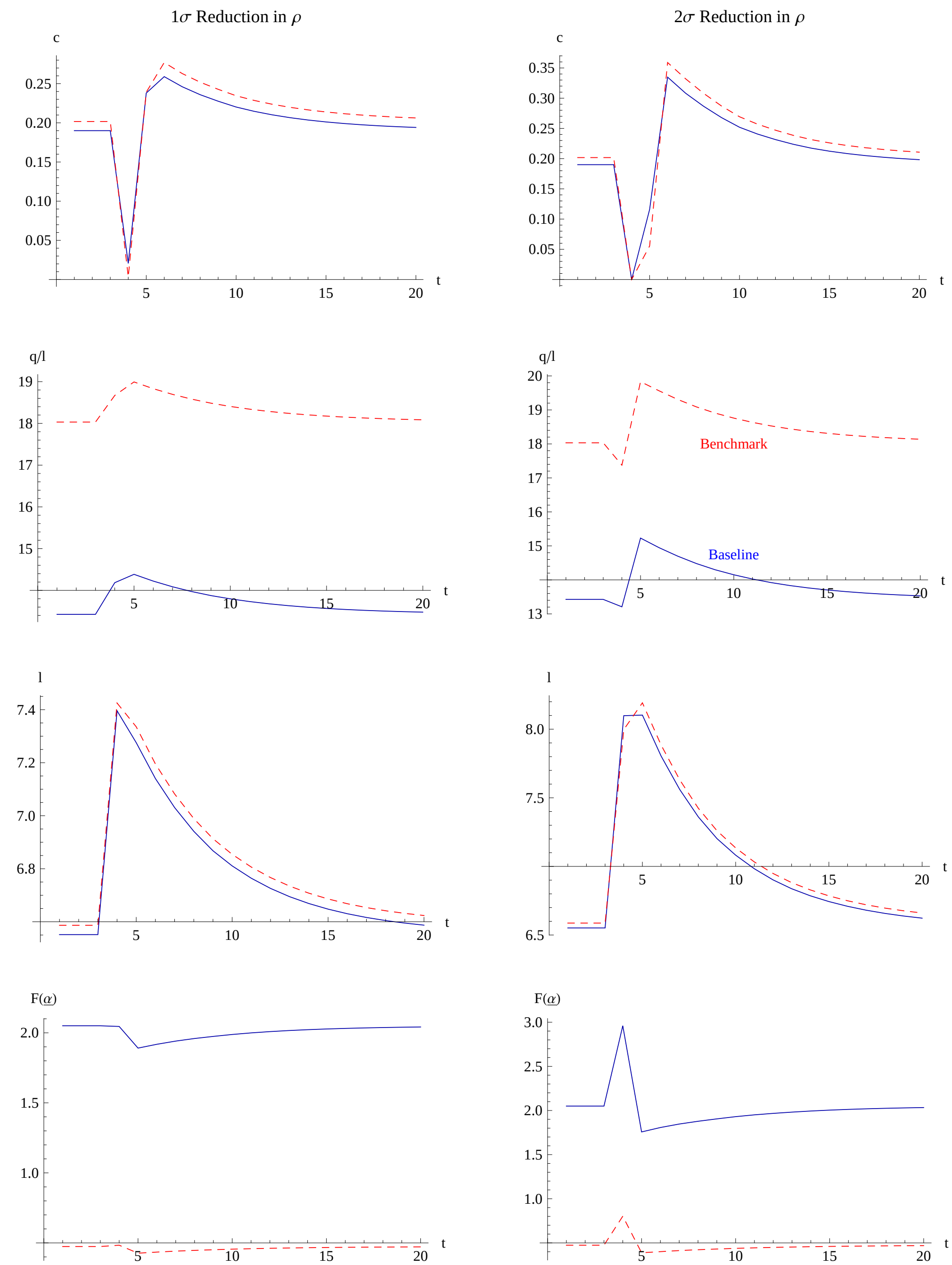


Figure 4. Excessive Risk of Bank Default



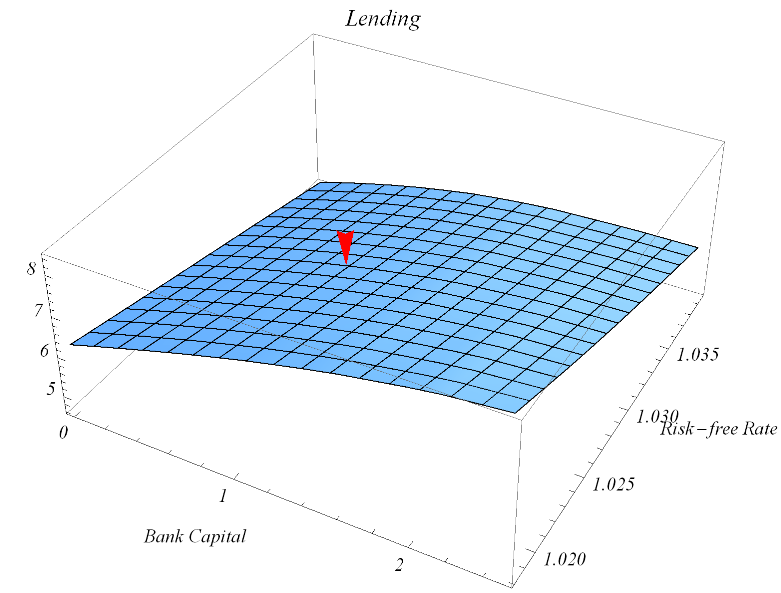
Note: Bank risk of default evaluated at the optimal decision rules in the baseline model minus the one corresponding to the benchmark model, in which the bank internalizes the losses imposed on depositors when it goes bankrupt.

Figure 5. Responses to an Interest Rate Shock



Note: State variables are initialized at the corresponding steady state values, with the shock assumed to hit in period 4 for one time only. The expectations of future values of bank capital is computed integrating over possible values of α.

Figure 6. Optimal Decision Rules with Dividend Smoothing



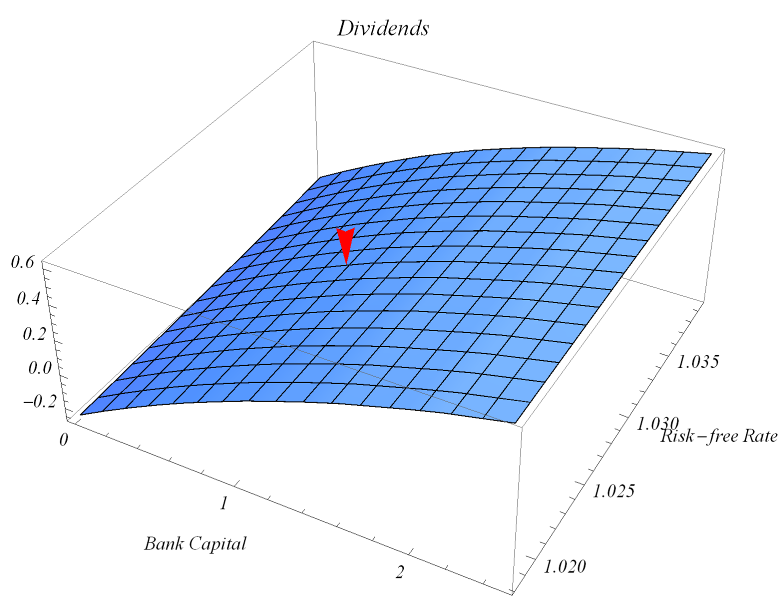
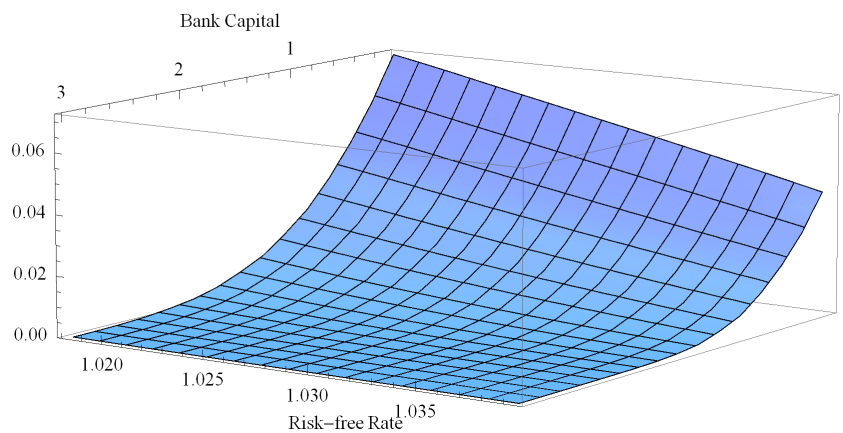
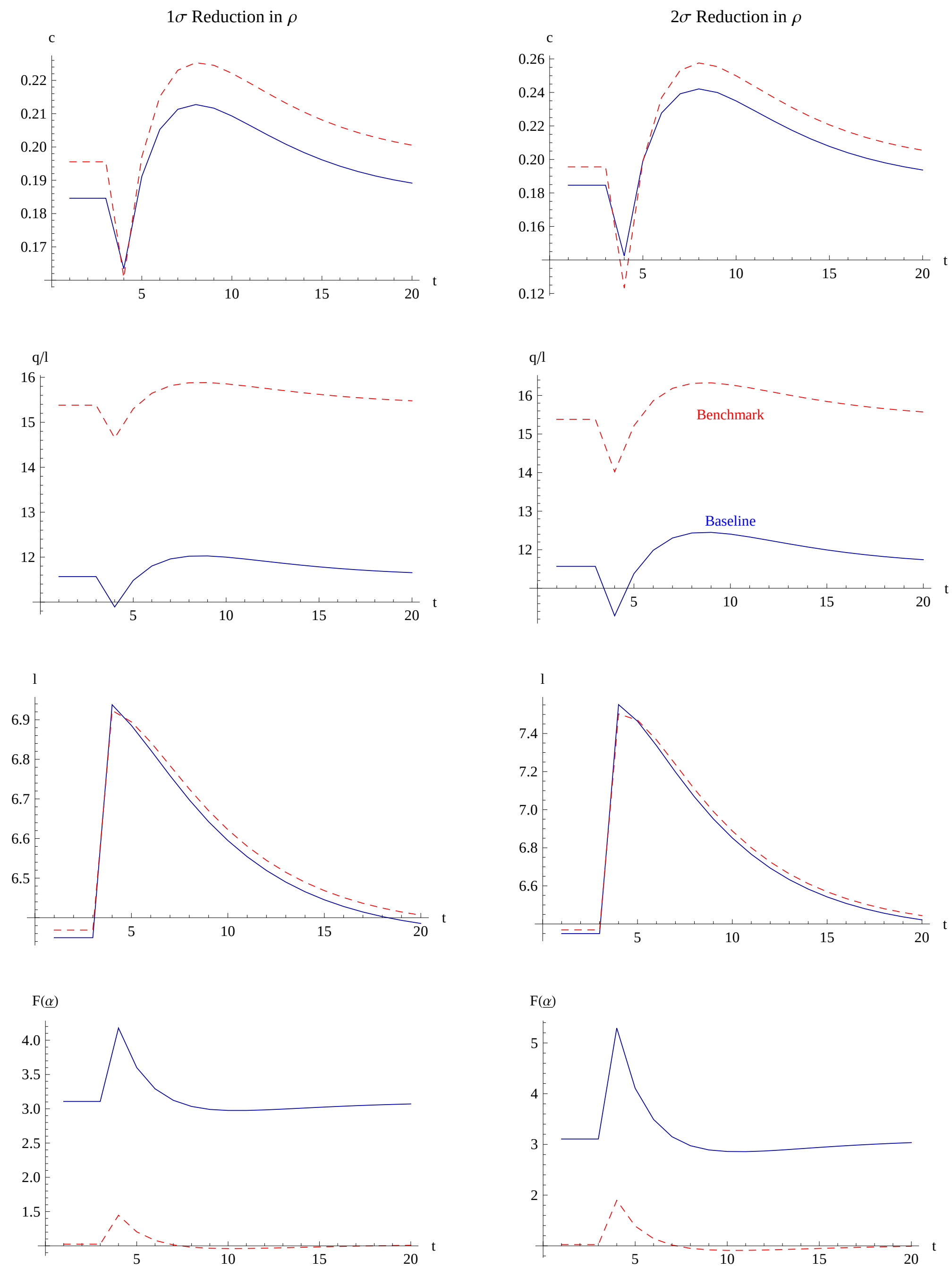


Figure 7. Excessive Risk of Bank Default in Model with Dividend Smoothing



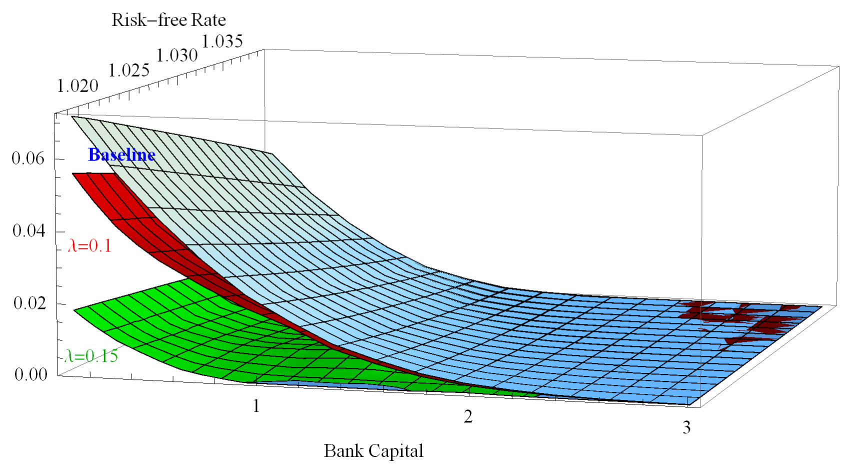
Note: Bank risk of default evaluated at the optimal decision rules in the baseline model minus the one corresponding to the benchmark model, in which the bank internalizes the losses imposed on depositors when it goes bankrupt.

Figure 8. Responses to an Interest Rate Shock in Model with Dividend Smoothing



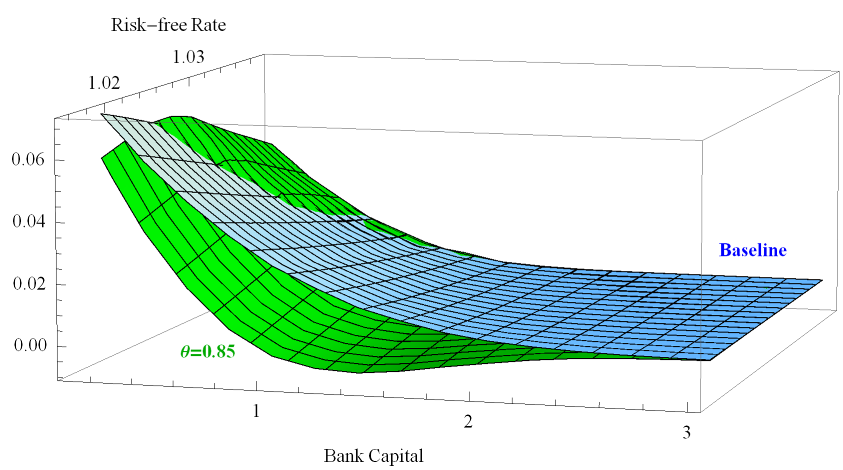
Note: State variables are initialized at the corresponding steady state values, with the shock assumed to hit in period 4 for one time only. The expectations of future values of bank capital is computed integrating over possible values of α.

Figure 9. Excessive Risk of Bank Default and Capital Requirements



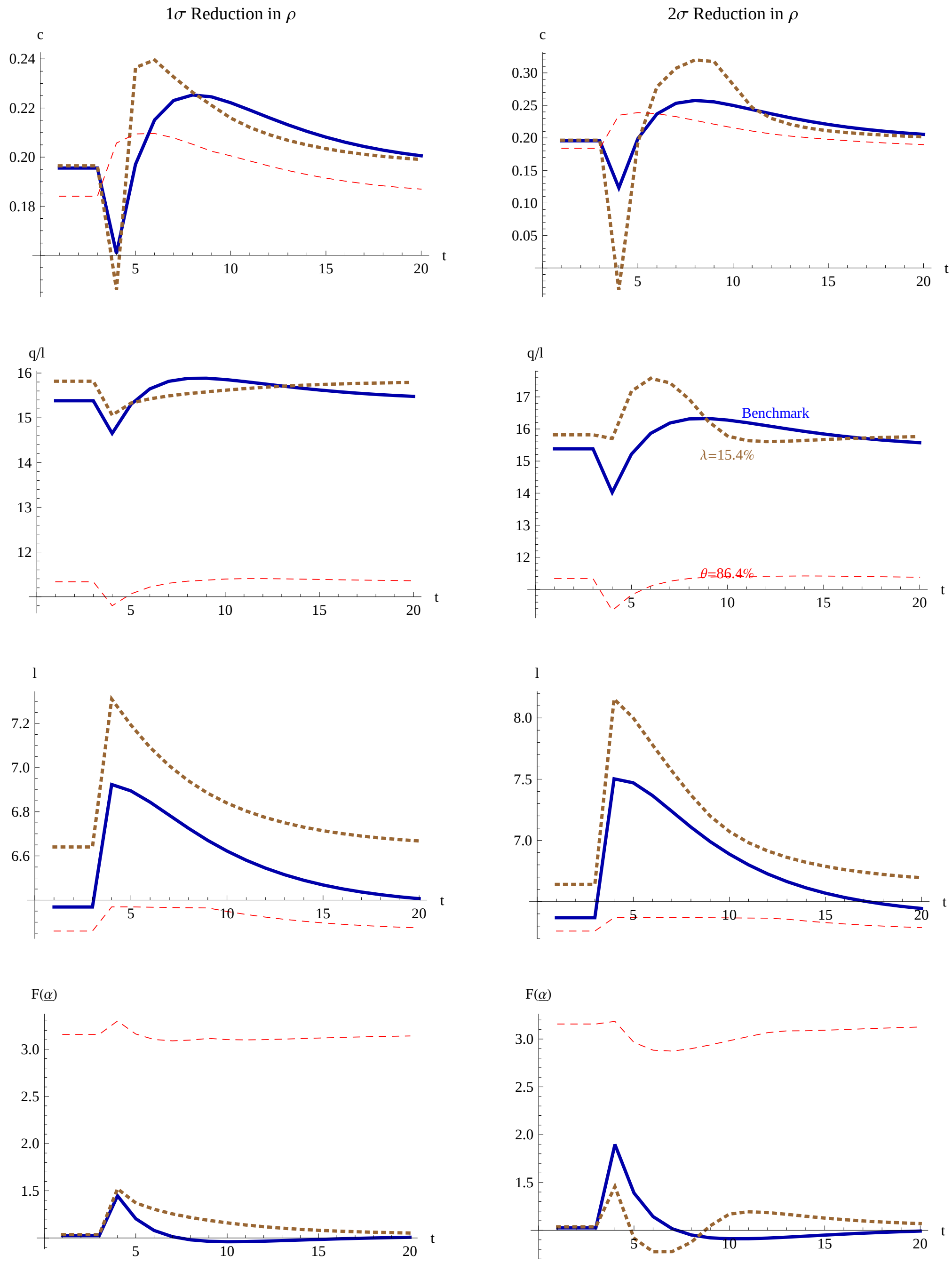
Note: Bank risk of default evaluated at the optimal decision rules in the baseline model minus the one corresponding to the benchmark model, in which the bank internalizes the losses imposed on depositors when it goes bankrupt (blue). The difference is plotted also for baseline model with capital requirements at 10 percent (red) and 15.4 percent (green).

Figure 10. Excessive Risk of Bank Default and Loan-to-value Caps



Note: Bank risk of default evaluated at the optimal decision rules in the baseline model minus the one corresponding to the benchmark model, in which the bank internalizes the losses imposed on depositors when it goes bankrupt (blue). The difference is plotted also for baseline model with loan-to-value caps at 85 percent (green).

Figure 11. Responses to an Interest Rate Shock with Regulatory Restrictions



Note: State variables are initialized at the corresponding steady state values, with the shock assumed to hit in period 4 for one time only. The expectations of future values of bank capital is computed integrating over possible values of α. Blue solid line corresponds to Benchmark model, dashed red line to model with loan-to-value caps at 86.4 percent and brown dotted line to model with capital requirements at 15.4 percent.

Table 1. Steady State Values

Variable		Model							
		Risk Neutral Shareholders				Dividend Smoothing			
		τ=1.025		τ=1.005		τ=1.025		τ=1.005	
		σ_α=0.06	σ_α=0.10	σ_α=0.06	σ_α=0.10	σ_α=0.06	σ_α=0.10	σ_α=0.06	σ_α=0.10
Bank Capital	Baseline	2.39	**1.07**	3.84	2.02	1.90	**0.92**	3.10	1.73
	Benchmark	2.70	1.39	4.11	2.31	2.12	1.18	3.17	1.97
Dividends	Baseline	0.31	**0.19**	0.33	0.22	0.30	**0.18**	0.32	0.21
	Benchmark	0.32	0.20	0.34	0.22	0.31	0.20	0.32	0.22
Loans	Baseline	13.48	**6.55**	14.66	7.18	12.58	**6.35**	13.94	7.05
	Benchmark	13.54	6.59	14.66	7.19	12.62	6.37	13.89	7.05
Bank Capital net of Dividends	Baseline	2.09	**0.88**	3.51	1.80	1.60	**0.73**	2.78	1.53
	Benchmark	2.39	1.19	3.78	2.08	1.82	0.98	2.85	1.75
Capital-to-assets ratio	Baseline	15.47 %	**13.43 %**	23.95%	25.12%	12.71 %	**11.57 %**	19.92 %	21.66 %
	Benchmark	17.62 %	18.03 %	25.76%	28.99%	14.38 %	15.38 %	20.52 %	24.95 %
Spread	Baseline	3.80 %	**3.94 %**	3.91 %	4.19 %	3.69 %	**3.83 %**	3.85 %	4.15 %
	Benchmark	3.82 %	4.06 %	3.91 %	4.19 %	3.73 %	3.99 %	3.85 %	4.16%
Bank Probability of Default	Baseline	0.35 %	**2.05 %**	6x10^{-4}%	0.03 %	1.24 %	**3.11 %**	0.02 %	0.15 %
	Benchmark	0.09 %	0.47 %	8x10^{-5}%	4x10^{-3} %	0.51 %	1.02 %	0.01 %	0.03 %
Borrowers' probability of Default	Baseline	14.62 %	**10.03 %**	17.21%	12.44 %	12.63 %	**9.26 %**	15.63 %	11.91 %
	Benchmark	14.76 %	10.16 %	17.20%	12.45 %	12.71 %	9.33 %	15.52 %	11.93 %

Numbers in bold correspond to baseline calibration in each corresponding version of the model. Benchmark refers to a version of the model in which the bank internalizes the losses imposed on depositors when it goes bankrupt.

Table 2. Steady State Values With and Without Regulatory Restrictions

Variable	Model				
	Baseline	With Capital Requirements		With Loan-to-Value Caps	
		$\lambda = 10\%$	$\lambda = 15.38\%$	$\Theta = 86.43\%$	$\Theta = 85\%$
Bank Capital	0.92	0.98	1.25	0.89	0.75
Dividends	0.19	0.19	0.20	0.18	0.18
Loans	6.35	6.42	6.64	6.26	5.67
Bank Capital net of Dividends	0.73	0.80	1.05	0.71	0.57
Capital-to-assets ratio	11.57 %	12.41 %	15.82 %	11.33 %	10.03 %
Loan-Deposit Spread	3.83%	3.88 %	4.02 %	3.82 %	3.75 %
Bank Probability of Default	3.11 %	2.55 %	1.04 %	3.16 %	3.18 %
Borrowers' probability of Default	9.26 %	9.53 %	10.37 %	8.93 %	6.75 %

$\lambda = 15.38\%$ and $\Theta = 86.43\%$ correspond to the steady state values of capital-to-assets ratio and loan-to-value ratio respectively, in the benchmark model.

Appendix I. Calibration

The objective of the calibration is to get leverage and intermediation spreads in the banking sector in steady state similar to those seen in the data. Since in this model, the economy is always being hit by shocks, the steady state is computed as the value of the state variables where the model converges, using the expected value of future capital and interest rates.

The bankruptcy costs parameters, $u = 0.1$ and $\omega = 0.13$, are taken from the literature and are within the range of values used in the literature (see for instance Bernanke et al. (1999)). The parameters of equation (11) are estimated from quarterly data on treasury bond yields, expressed in real terms using the GDP Deflator, from 1985 to 2010. The estimation yields the following results: $a0 = 1.0288$, $a1 = 0.82$, and ϵ is $N(0, \sigma_\epsilon)$ with $\sigma_\epsilon = 0.0067$.

The average spread between yields on 6-month financial CD's and conventional mortgage rates between 1985-2011 in the U.S. is 3%. By choosing $\sigma_\alpha = 0.1$ and $\mathcal{R} = 1.06$ I obtain a steady state spread between lending and deposit rates of 3.8% in the model with dividend smoothing and 3.9% in the one without it. $\tau = 1.025$ generates a steady state bank capital-to-asset ratio of about 11.6% in the model with dividend smoothing and 13.4% in the one without it. The average capital-to-assets ratio for U.S. commercial banks was 10.7% as of end-2009.

For the model with dividend smoothing, I chose a utility function of the CARA type, $u(x) = -\frac{1}{\gamma}e^{\gamma x}$, which allows for negative dividends (i.e. equity financing). The coefficient of risk aversion, γ, is chosen to approximate the ratio of volatility of dividends to volatility of changes in bank capital to the median value for US commercial banks over the period 1978Q1-2010Q2 which turns out to be 0.5.

APPENDIX II. NUMERICAL SOLUTION

The model and all its variants presented in this paper are solved using backwards induction and the method of endogenous gridpoints developed by Carroll (2006). The solution is implemented as follows:

1. Guess an initial marginal value function $V_T^n(n, \rho)$, which under the assumption that the bank distributes all its capital in dividends at some hypothetical last period of life results in being equal to 1 for all values of bank capital and the risk-free rate.
2. Construct a discrete approximation to the normally distributed interest rate shocks using a Gaussian quadrature with 7 points, and construct a vector of possible values of the risk-free rate, covering the range $a_0 \pm 3\sigma_\epsilon$, collected in $\vec{\rho}$.
3. Construct a vector $\vec{q}$ of values of bank capital net of dividends q. For each $q \,\epsilon\, \vec{q}$ and for each $\rho \,\epsilon\, \vec{\rho}$, I use a numerical root-finding routine to find the optimal amount of lending l_{T-1} that solves equation 20, using the initial guess for the marginal value function.
4. Using the solution for lending on hand, I find the optimal amount of capital q^* that solves equation 19 for each $\rho \,\epsilon\, \vec{\rho}$. For each $q \,\epsilon\, \vec{q}$, if $q > q^*$, then the optimal solution for dividends is $c^*_{T-1} = q - q^*_{T-1}$, and the beginning-of-period bank capital is given by $n_{T-1} = q + c^*_{T-1}$.
5. The solutions yield two sets of triples $\{\rho_{T-1}, n_{T-1}, c^*_{T-1}\}$ and $\{\rho_{T-1}, n_{T-1}, l^*_{T-1}\}$. Using piecewise linear interpolation I construct continuous functions $l_{T-1}(n, \rho)$ and $c_{T-1}(n, \rho)$.
6. Evaluating (19) with these optimal solutions I update the marginal value function and repeat the above procedure to obtain a new pair of policy functions $l_{T-2}(n, \rho)$ and $c_{T-2}(n, \rho)$.
7. If Max $[\|l_{T-2}(n, \rho) - l_{T-1}(n, \rho)\|, \|c_{T-2}(n, \rho) - c_{T-1}(n, \rho)\|] < 0.00001$ for all n and ρ, stop, if not, repeat the above sequence until this convergence condition is satisfied.

In the case of the version with risk aversion, the above procedure is modified as follows:

1. Begin with a guess for the dividends policy function $c_T(n, \rho) = n$, which corresponds to the same assumption as before of distributing all available capital in dividends at the end of the bank's life.
2. Modify step 4 above as follows, for each $q \,\epsilon\, \vec{q}$ and the corresponding solution for lending obtained from step 3 above, I obtain the optimal amount of dividends using $c^*_{T-1} = u^{c^{-1}}(\Delta)$, where $u^{c^{-1}}(\cdot)$ corresponds to the inverse of the marginal utility function and Δ corresponds to the right-hand side of equation (23).
3. Use the optimal amount of dividends on hand to obtain the beginning-of-period bank capital $n_{T-1} = q - c^*_{T-1}$. The remaining steps are identical to the those for the baseline model.

When capital requirements or loan-to-value caps are introduced, the only modification to the above procedure is to set the optimal solution to either the unconstrained solution or to the one that satisfies the corresponding constraint.

Appendix III. Mathematical Derivations

The first order condition for dividends is given by

$$
\begin{aligned}
0 &= -1 + \beta E_t \left[(1 - F(\overline{\alpha})) V^n_{\overline{\alpha}} \tfrac{dn_{t+1}}{dq} + \tfrac{dn_{t+1}}{dq} \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}}{}^n f_\alpha(\alpha) \right] \\
1 &= \beta E_t \left[V^n_{\overline{\alpha}} (1 - F(\overline{\alpha}))(i_t - (l_t - q_t) i^q_t) + (i_t - (l_t - q_t) i^q_t) \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}}{}^n f_\alpha(\alpha) \right] \\
1 &= \beta (i_t - (l_t - q_t) i^q_t) E_t \left[V^n_{\overline{\alpha}} (1 - F(\overline{\alpha})) + \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}}{}^n f_\alpha(\alpha) \right]
\end{aligned}
$$

The first order condition for loans is given by

$$
\begin{aligned}
0 &= (1 - F(\overline{\alpha})) E_t V^n_{\overline{\alpha}} \left[R_t + l_t R^l_t - i_t - (l_t - q_t) i^q_t \right] - \cancel{E_t f(\overline{\alpha}) \frac{d\overline{\alpha}}{dl} V_{\overline{\alpha}}} \\
&\cancel{+ E_t f(\overline{\alpha}) \frac{d\overline{\alpha}}{dl} V(n_{t+1}, \rho_{t+1}|_{\alpha = \overline{\alpha}})} - \underbrace{E_t f(\underline{\alpha}) \frac{d\underline{\alpha}}{dl} V(n_{t+1}, \rho_{t+1}|_{\alpha = \underline{\alpha}})}_{=0} \\
&+ E_t \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}}{}^n \left[\alpha (1 - u) R_t - i_t - (l_t - q_t) i^l_t \right] f_\alpha(\alpha) \\
0 &= (1 - F(\overline{\alpha})) E_t V^n_{\overline{\alpha}} \left[R_t + l_t R^l_t - i_t - (l_t - q_t) i^l_t \right] \\
&+ E_t \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}}{}^n \left[\alpha (1 - u) - i_t - (l_t - q_t) i^l_t \right] f_\alpha(\alpha)
\end{aligned}
$$

For the model with dividend smoothing, the first order conditions are given by

$$
\begin{aligned}
0 &= -u^c(n_t - q_t) + \beta E_t \left[(1 - F(\overline{\alpha})) V^n_{\overline{\alpha}} \tfrac{dn_{t+1}}{dq} + \tfrac{dn_{t+1}}{dq} \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}}{}^n f_\alpha(\alpha) \right] \\
u^c(n_t - q_t) &= \beta E_t \left[V^n_{\overline{\alpha}} (1 - F(\overline{\alpha}))(i_t - (l_t - q_t) i^q_t) + (i_t - (l_t - q_t) i^q_t) \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}}{}^n f_\alpha(\alpha) \right] \\
u^c(n_t - q_t) &= \beta (i_t - (l_t - q_t) i^q_t) E_t \left[V^n_{\overline{\alpha}} (1 - F(\overline{\alpha})) + \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}}{}^n f_\alpha(\alpha) \right]
\end{aligned}
$$

The Envelope Theorem implies

$$
\begin{aligned}
V^n &= \beta(i_t - (l_t - q_t)i_t^q)E_t\left[\underset{\overline{\alpha}}{V}^n(1 - F(\overline{\alpha})) + \int_{\underline{\alpha}}^{\overline{\alpha}} \overset{\overline{\alpha}}{\underset{\underline{\alpha}}{V}}{}^n f_\alpha(\alpha)\right] \\
V^n &= u^c(n_t - q_t)
\end{aligned}
$$

which after rolling it forward one period, we obtain the corresponding Euler equation for dividends

$$
u^c(n_t - q_t) = \beta(i_t - (l_t - q_t)i_t^q)E_t\left[u^c(c_{t+1})(1 - F(\overline{\alpha})) + \int_{\underline{\alpha}}^{\overline{\alpha}} u^c(c_{t+1})f_\alpha(\alpha)\right]
$$

and using the same Envelope theorem logic, the Euler equation for loans is identical to the risk-neutral case shown above.

$$
\begin{aligned}
0 &= (1 - F(\overline{\alpha}))E_t u^c(c_{t+1})\left[R_t + l_t R_t^l - i_t - (l_t - q_t)i_t^l\right] \\
&+ E_t \int_{\underline{\alpha}}^{\overline{\alpha}} u^c(c_{t+1})\left[\alpha(1 - u) - i_t - (l_t - q_t)i_t^l\right] f_\alpha(\alpha)
\end{aligned}
$$