Financial Frictions and Sources of Business Cycle

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Abstract

This paper estimates a New Keynesian DSGE model with an explicit financial intermediary sector. Having measures of financial stress, such as the spread between lending and borrowing, enables the model to capture the impact of the financial crisis in a more direct and efficient way. The model fits US post-war macroeconomic data well, and shows that financial shocks play a greater role in explaining the volatility of macroeconomic variables than marginal efficiency of investment (MEI) shocks.

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I. Introduction

The objective of this paper is twofold: first, to find out how well DSGE models which endogenize financial frictions perform in fitting the US post-war macroeconomic data. Second, to study the driving forces of the real business cycle in the US. In the wake of the recent global financial crisis, the ability to understand the impact of financial markets’ health on the real economy has taken center stage. In August 2007, with the onset of the subprime mortgage crisis, money market spreads soared and financial activity became significantly impaired. In particular, term maturities were seriously affected, as banks were unwilling to lend to each other due to concerns over counterparty risk and adverse selection. This disruption reinforced a rapid deterioration in real economic activity through the financial accelerator mechanisms (FA).

The first contribution of the paper is to show that a New Keynesian DSGE model with a Financial Intermediary (FI) sector fits the data well. While there are some recent works to include FIs in these models using different contracts and wedges, little has been done to assess the goodness of fit of such class of models. The two features of this model that improve the fit are the inclusion of (i) the spread between lending and borrowing, as a measure of financial stress and (ii) financial shocks, which are closely related to counter cyclical spreads. These enable my model to capture the impact of a financial crisis in a more direct and efficient way. In an economy without financial frictions, a negative shock to the quality of capital stock disturbs the capital accumulation dynamic, which further deteriorates output modestly. While in the presence of financial frictions, a negative capital quality shock affects the economy through an additional channel, namely, bank balance sheets, by creating significant capital loss in the financial sector.

The model builds on Gertler and Karadi (2011) (hereafter GK (2011)), which extended the benchmark NK model of Smets and Wouters (2007) (hereafter SW (2007)). GK (2011) introduced financial frictions, in the spirit of Bernanke, Gertler, and Gilchrist (1999), hereafter BGG (1999), and Kiyotaki and Moore (1997). The key reason for choosing GK (2011) as my baseline model is that, while it provides a minimal and uniform framework, it also includes FI in a meaningful way that is also nested to SW (2007). I augment the GK (2011) model by adding an extra wedge to account for the efficiency of the investment process. Further, I use a Bayesian estimation technique to estimate the model parameters and evaluate the model’s fit by computing the marginal data density (MDD).

The second contribution of the paper is the analysis of the variance decomposition of the US macroeconomic data. Following the seminal work of Kydland and Prescott (1982), the conventional view about the sources of the business cycle has been centered mainly on technology shocks (TFP). After the experience of the recent financial crisis, the view regarding the driving forces of the business cycle expanded to include investment channels. In line with this idea, Justiniano, Primiceri, and Tambalotti (2010) (hereafter JPT (2010)) argue that a more promising
theory is one that largely attributes fluctuations to investment shocks and, more specifically, to marginal efficiency of investment (MEI) shocks. MEI shocks are exogenous disturbances which affect the efficiency of the process that transforms current investment goods to future productive capital.

![Figure 1: Growth and TED spread (left axis) and credit spread (right axis)](image)

Furthermore, JPT (2010) also argues that the MEI process is highly correlated with credit spread. The external financing premium, or spread, is a good proxy for measuring the counterparty risk in financial markets and can account for most of the fall in real macro variables from 2007 to 2009. As is clear from Figure 1, when the economy experiences a crisis, GDP growth deteriorates, while both the credit spread and the TED spread rise. To corroborate the relationship between investment shocks and the spread, we need a framework that explicitly allows for a financial intermediary sector and can capture the interaction between marginal efficiency of investment and the spread. I extend the GK (2011) model by incorporating MEI shocks in the spirit of JPT (2010).

By performing a variance decomposition analysis, I show that financial shocks are the main driving force behind the volatilities in post-war US macroeconomic data. The shocks in my framework are modeled as exogenous disturbances that affect the quality of capital stock and are highly correlated with the credit spread. My results not only confirm JPT (2010)’s findings, but also go one step further to explicitly decompose financial shocks from MEI shocks. One of the policy implications of this result can be that policies which affect the credit spread are the most effective instruments for recovery during financial crises.

This paper belongs to the body of literature that looks at the supply side of credit. A standard framework for incorporating financial sectors uses the financial accelerator (FA) model, as developed in Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), and BGG (1999). In the supply-side framework, the focus is on credit constraints faced by non-financial borrowers. Recent papers, such as Christiano, Motto, and Rostagno (2013),
Mandelman (2010), Nolan and Thoenissen (2009), Ajello (2010), and Hirakata, Sudo, and Ueda (2011), draw attention to the importance of financial shocks in explaining business cycle fluctuations. Liu, Waggoner, and Zha (2011) emphasizes the key role of shocks on the capital depreciation rate and further argue that these shocks resemble financial shocks.

The paper proceeds as follows. The model is described in section 2. Section 3 details the empirical evaluation of the model. The model fit is presented in section 4. Section 5 shows the result of variance decompositions. The policy implications and impulse responses are discussed in section 6 and section 7 concludes the paper. Data description is included in the appendix.

II. Model

This section describes my model of business cycles for the US economy. It is a standard DSGE model which is based upon GK (2011). The core macro model is kept simple in order to highlight the role of intermediation. Among the models with financial intermediaries, GK (2011) is a good framework for empirical studies for two reasons. Firstly, because the model is nested with CEE (2005) and SW (2007). In an ideal world without financial frictions there is no role for the banking sector, and the model is isomorphic to those standard NK models. Furthermore, both of those models fit the data well and are capable of explaining the sources of business cycle that are proposed in the literature. The second reason for picking the Gertler and Karadi framework is that inclusion of financial intermediaries without making the model too complex has a substantial effect on model dynamics and policy implications.

![Figure 2: Economy with financial intermediary](image_url)

The model has seven sectors: Households (HH), Financial intermediaries (FI), Central bank (CB), Government (Gov), Non-financial goods producer firm (GPF), Capital producer firms (CPF), Monopolistically competitive retail firms (RF). The latter are in the model only to
introduce nominal price rigidities. One important feature of the model is the inclusion of a homogenous financial intermediary sector that contains a moral hazard type of financial accelerator mechanism. Financial intermediaries in this model are simplified and they form a homogenous sector that is meant to capture the entire banking sector, essentially investment banks as well as commercial banks. They transfer funds between households and non-financial firms. The source of financial frictions in the model is the moral hazard incentive constraint that limits the ability of financial intermediaries to obtain funds from households. The second distinguishing feature of the model is allowing for an investment shock that determines the efficiency of newly produced investment goods, the so-called marginal efficiency of investment shock, as in Greenwood, Hercowitz, and Huffman (1988).

A. Households

The economy is populated with a continuum of identical households. Households’ utility is a function of consumption with habit formation and labor. Each household saves by lending funds to competitive financial intermediaries or, potentially, to the government. Within each household there are two types of agents. At any point in time 1 − f fraction of households are "workers" and the remaining f fraction of them are "bankers"; moreover, they can also switch between occupations with a historically independent probability. This set up allows for having a similar coefficient of intertemporal time preference between bankers and workers, which results in perfect consumption smoothing. Workers are monopolistic competitors in the labor market, where they get a wage for supplying their labor. They, further, return their wages to the households, while bankers manage a financial intermediary and also transfer earnings back to households. With i.i.d probability 1 − θ, a banker exits next period, so at each period, a total measure of (1 − θ)f bankers randomly become workers and the same fraction of workers should transit to being bankers to keep the measure of each type constant. Upon exit, bankers transfer retained earnings to the households and new bankers receive some ”startup” funds from the family (household). In order to keep the measure in each occupation constant, the model allows the same fraction of agents to transit from being workers to being bankers. There is a finite horizon for bankers to prevent the limiting case of funding the investment opportunities entirely from bankers’ capital.

In my representative agent setup, households consume \(C_t\), buy one period real bond \(B_t\) which pays the gross real rate \(R_t\) from banks or the government, supply labor \(L_t\) in exchange for wage \(W_t\), pay lump-sum tax \(T_t\) to the government, and receive the net transfer \(\Pi_t\) which is the funds transferred from existing bakers minus the funds transferred to new bankers. The household utility maximization problem is the following:

\[
\max_{C_t, L_t} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i b_{t+i} \left[ \log(C_{t+i} - hC_{t+i-1}) - \frac{X}{1 - \phi} L_{t+i}^{1+\phi} \right] \tag{1}
\]

s.t. \(C_t + B_{t+1} = W_t L_t + R_t B_t + \Pi_t - T_t\) \tag{2}
where $\chi$ is the coefficient of leisure, $h$ is the habit persistence parameter, $\phi$ is the inverse Frisch parameter, $b_t$ is intertemporal preference shock, which affects both the marginal utility of consumption and the marginal disutility of labor. It follows an AR(1) process, as below, with the $\rho_b$ persistence and $\sigma_b$ variance:

$$\log b_t = \rho_b \log b_{t-1} + \sigma_b \epsilon_{b,t},$$

where $\epsilon_{b,t} \sim i.i.d.N(0,1)$. Since technological progress is non-stationary, the log utility ensures the existence of a balanced growth path. Note that I do not assume an exogenous process for the coefficient of leisure, $\chi$. $\beta$ is the time discount factor. In another exercise, which is not reported here, I found that variance decomposition analysis shows that this shock is not an important shock in explaining the volatility of macroeconomic data. The first order condition of the household optimization problem implies:

$$\rho_t W_t = \chi_t L_t^\phi$$

$$\beta E_t \Lambda_{t,t+1} R_{t+1} = 1,$$

where marginal utility of consumption is $\rho_t = b_t(C_t - hC_{t-1})^{-1} - \beta h E_t b_{t+1}(C_{t+1} - hC_t)^{-1}$ and $\Lambda_{t,t+1} \equiv \frac{\rho_{t+1}}{\rho_t}$. Equation 5 is known as the Euler equation.

**B. Financial Intermediaries**

Bankers transfer funds from savers (households) to investors (good producer firms). Financial intermediaries are indexed by $j$. At the beginning of the period, each bank raises deposits $B_{jt}$ from households in the retail financial market at the risk-free rate $R_{t+1}$, which is paid at period $t+1$. Then they purchase financial claims $S_t$ from goods producer firms with relative price $Q_t$. Banker’s equity capital, or net worth, at the end of period $t$ is $N_t$, which is the difference between the assets and the liabilities. Hence, the balance sheet of the intermediary $j$ is given by:

$$\begin{array}{c|c}
\text{Assets} & \text{Liabilities} \\
Q_t S_t & B_t \\
& N_t \\
\end{array}$$

$$N_{j,t} = Q_t S_{j,t} - B_{j,t}.$$

At period $t + 1$ claims on non-financial firms pay out the stochastic return $R_{k,t+1}$. Note that, as I will show later, both $R_{k,t+1}$ and $R_{t+1}$ are endogenous variables. Bankers’ net worth dynamic evolves as net earnings on assets minus the interest payments on liabilities:
\( N_{jt+1} = R_{kt+1}Q_{jt}S_{jt,t} - R_{t+1}B_{jt} \)  
\( N_{jt+1} = R_{t+1}N_{jt,t} + (R_{kt+1} - R_{t+1})Q_{jt}S_{jt,t} \).  

(7)  

(8)

where \( (R_{kt+1} - R_{t+1}) \) represent the interest rate spread or the external financing premium. Net worth grows at rate \( R \) and any expansion comes from excess return on financial assets \( (R_{kt+1} - R_{t+1})Q_{jt}S_{jt,t} \). It follows that the spread is related to financial frictions in the model. The necessary condition for the bankers to stay in business is that the discounted expected return on non-financial claims should be greater than or equal to the discounted cost of borrowing from households; this implies that the discounted spread should be nonnegative:

\[ \beta^i \Lambda_{t,t+1+i}(R_{kt+1+i} - R_{t+1+i}) \geq 0, \forall i \geq 0, \]

where at time \( t \), \( \beta^i \Lambda_{t,t+1+i} \) is the stochastic discount factor for \( i \) period ahead return. The banker accumulates net worth before exit and her value function at time \( t \), \( V_{jt,t} \), is to maximize her net present value of her expected equity capital. The expected terminal wealth follows by:

\[
\max \{ V_{jt} \} \equiv (1 - \theta)E_{t}\{ \sum_{i=0}^{\infty} \theta^i \beta^i \Lambda_{t,t+1+i}(N_{jt+1+i}) \} \\
\equiv (1 - \theta)E_{t}\{ \sum_{i=0}^{\infty} \theta^i \beta^i \Lambda_{t,t+1+i}(R_{t+1+i}N_{jt+i} + (R_{kt+1+i} - R_{t+1+i})Q_{t+i}S_{jt+i}) \}.
\]

I can rewrite the value of the bank at the end of period \( t - 1 \), as a Bellman equation:

\[
V_{t-1}(S_{jt-1}, B_{jt-1}) = E_{t-1}\{ \Lambda_{t-1,t}N_{jt,t} \} + \max_{S_{jt,t}} V_{t}(S_{jt,t}, B_{jt,t}).
\]

(10)

To solve the decision problem, one can show that the value function is linear:

\[
V_{jt} = \nu_{t}Q_{jt}S_{jt,t} + \eta_{t}N_{jt,t},
\]

(11)

where \( \nu_{t} \) and \( \eta_{t} \) are time varying parameters. Note that \( \nu_{t} \) is the marginal value of assets at the end of period \( t \) and \( \eta_{t} \) is the marginal cost of deposits, as follows:
\[ \nu_t = E_t\{(1 - \theta)(R_{kt+1} - R_{t+1}) + \beta \Lambda_{t,t+1}\theta x_{t,t+1}\nu_{t+1}\} \]  
(12)

\[ \eta_t = E_t\{(1 - \theta)R_{t+1} + \beta \Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}\} \]  
(13)

\[ x_{t,t+1} \equiv \frac{Q_{t+S_j,t}}{Q_{t+S_j,t}} \]  
(14)

\[ z_{t,t+1} \equiv \frac{N_{j,t}}{N_{j,t}}. \]  
(15)

In the economy without financial frictions, the banker wants to expand her assets by borrowing from households infinitely. In order to limit bankers’ borrowing ability, financial frictions in the form of the moral hazard problem is introduced. In each period the banker can choose to divert a fraction \( \lambda \) of her total assets \( Q_tS_{j,t} \) back to her family. If a bank diverts assets for its personal benefit, it defaults on its debt. The creditors can reclaim the remaining fraction \( 1 - \lambda \) of funds. Since the creditors are aware of this moral hazard incentive, they will restrict the amount they lend to the bank. Let \( V_t(S_{j,t}, B_{j,t}) \) be the maximized value of \( V_t \), given the balance sheet configuration \( (S_{j,t}, B_{j,t}) \) at the end of period \( t \). Next, the moral hazard incentive condition ensure that the bank does not divert funds:

\[ V_t(S_{j,t}, B_{j,t}) \geq \lambda Q_tS_{j,t}. \]  
(16)

The value of staying in business should be greater than diverting some fund and defaulting on the rest. When the constraint binds, assets will be a levered ratio \( \phi_t \) of net worth:

\[ Q_tS_{j,t} = \frac{\eta_t}{\lambda - \nu_t} N_{j,t} \]  
(17)

\[ Q_tS_{j,t} = \phi_t N_{j,t}, \]  
(18)

where \( \phi_t = \frac{\eta_t}{\lambda - \nu_t} \) is the leverage ratio of financial intermediary.

**Net worth Dynamic:**

The net worth of existing bankers evolves as:

\[ N_t = \theta[(R_{k,t} - R_t)\phi_t + R_t]N_{t-1}, \]

Note that the net worth of the existing banker does not include any wealth shock, as, in another exercise which is not reported here, I found that variance decomposition analysis shows that this shock plays virtually no role in explaining the business cycle. At each period, a fraction of \( 1 - \theta \) bankers become workers and transfer their accumulated net worth \( Q_tS_t \) to households. A
fraction \( \frac{\omega}{1-\theta} \) of this transfer will be given to new bankers who enter at period \( t + 1 \), \( 0 < \omega < 1 \) as their start-up net worth, \( N_{nt} \):

\[
N_{nt} = \frac{\omega}{1 - \theta}(1 - \theta)Q_tS_{t-1} = \omega Q_tS_{t-1}.
\] (19)

Aggregate net worth at any period \( t \) is the sum of the net worth of existing bankers \( (N_{et}) \) and new bankers \( (N_{nt}) \):

\[
N_t = N_{et} + N_{nt}
\] (20)

\[
N_t = \theta[(R_{k,t} - R_t)\phi_t + R_t]N_{t-1} + \omega Q_tS_{t-1}.
\] (21)

Financial intermediaries also issue loans to non-financial goods producer sectors by purchasing their financial claims at unit price of capital. The loans are further used in production of intermediary goods. In the following section I describe the role of the intermediate goods producer firm in detail.

C. Intermediate Good Producer Firm

Non-financial goods producer firms are competitive. They have Cobb-Douglas constant returns-to-scale technology that uses labor and capital to produce intermediate good \( Y_t \). At the end of any period \( t \), they issue stocks \( (S_t) \) equal to the number of units of capital they need. They, further, sell the stocks at the price of unit of capital \( Q_t \), in order to raise the required capital, which is going to be used in the next period production. Hence, by the arbitrage condition, the following equality should hold:

\[
Q_tK_{t+1} = Q_tS_t.
\]

I will show in the next section that the shadow price \( Q_t \) is drawn endogenously from profit maximization of the capital producer firm. Production takes place at time \( t \). Capital is not mobile, but labor is perfectly mobile across firms and this allows expressing aggregate output \( Y_t \) as a function of effective aggregate capital \( K_t \) and aggregate labor hours \( L_t \) as below. Note that \( U_t \) is the time-varying utilization rate of the capital and \( \xi_t K_t \) is the effective quantity of capital:

\[
Y_t = A_t(U_t\xi_tK_t)^\alpha L_t^{1-\alpha}
\] (22)

\[
\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t}
\] (23)

\[
\log \xi_t = (1 - \rho_\xi) \log \xi_{ss} + \rho \log \xi_{t-1} + \sigma_\xi \epsilon_{\xi,t}.
\] (24)
\(\alpha\) is the share of capital in the production function. AR(1) \(A_t\) is aggregate productivity which follows a Markov process, with \(\epsilon_{A,t} \sim i.i.d.N(0,1)\). The model is stationary so \(A_t\) does not contain stochastic growth. The shock \(\xi_t\) is meant to capture the exogenous disturbances to the quality of productive capital; it also resembles distortions to the capital depreciation rate. The later captures the microeconomic frictions that effects intertemporal capital accumulation choice. \(\xi_t\) follows AR(1) stochastic exogenous process with \(\epsilon_{\xi,t} \sim i.i.d.N(0,1)\). Following BGG (1999), retailers purchase the intermediate good from non-financial good producer firms at the wholesale price \(P_{mt}\) and transform it into a composite final good, whose price index is \(P_t\). With this notation, \(P_t/P_{mt}\) denotes the markup of final over intermediate goods. Production takes place at \(t+1\) and intermediate goods are sold to retail firms at the price \(P_{m,t+1}\). Firm’s profit is given by:

\[
\Upsilon_{t+1} = E_t\beta A_{t+1}[P_{m,t+1}Y_{t+1} + (Q_{t+1} - \delta(U_{t+1}+1))\xi_{t+1}K_{t+1} - R_{k,t+1}Q_tK_{t+1} - W_{t+1}L_{t+1}] . \tag{25}
\]

The model assumes that the replacement price of depreciated capital is unity and, given that \(Q_t\) is the price of capital stock, \((Q_{t+1} - \delta(U_{t+1}))\xi_{t+1}K_{t+1}\) gives the value of the leftover capital. The depreciation rate varies with utilization and is assumed to follow the functional form:

\[
\delta(U_t) = \delta_{ss} + \frac{b}{1+\zeta}U_t^{1+\zeta} , \tag{26}
\]

where \(\delta_{ss}\) is determined by the steady state and \(\zeta\) is the elasticity of utility cost. Optimal choice of wage \(W_t\), utilization rate \(U_t\), and rental rate \(R_{k,t}\) are given by the first order conditions of firm’s state-by-state profit maximization problem, as below:

\[
\frac{\partial \Upsilon_t}{\partial L_t} = 0 : P_{m,t}(1-\alpha)\frac{Y_t}{L_t} = W_t \tag{27}
\]

\[
\frac{\partial \Upsilon_t}{\partial U_t} = 0 : P_{m,t}\alpha\frac{Y_t}{U_t} = \delta'(U_t)\xi_tK_t \tag{28}
\]

\[
\frac{\partial \Upsilon_{t+1}}{\partial K_{t+1}} = 0 : R_{k,t+1} = \frac{P_{m,t+1}\alpha Y_{t+1}K_{t+1} + (Q_{t+1} - \delta(U_{t+1}))\xi_{t+1}}{Q_t} \tag{29}.
\]

The last equation relates the return on capital, \(R_{k,t}\), to the financial shock, \(\xi_t\). This implies when there is a negative exogenous disturbances to the quality of productive capital, rental cost increases and this further deteriorates the net worth of bankers. This is the key to understanding
the dynamic of spread; later, in section 6, I show how the correlation between spread and capital quality shock plays a major role in explaining volatility in US macroeconomic variables. In the next section I will describe the role of the capital producer firm and the dynamic of capital accumulation. Presence of a representative capital producer sector allows for a meaningful way of decomposing out financial shock from marginal efficiency of investment shock, through the dynamic of capital. I will also show how the shadow price is related to the MEI shock.

D. Capital Producer Firm

Capital producer firms are competitive. At the end of each period $t$, they buy the depreciated capital from intermediate good producer firms at price $\bar{Q}_t$. They repair depreciated capital, and sell both the refurbished and the new capital at price $Q_t$. Let $I_t$ denote the investment expenditures; then capital accumulation dynamic is given by:

$$K_{t+1} = (1 - \delta(U_t))\xi_t K_t + \gamma_t \Phi\left(\frac{I_t}{K_t}\right)K_t.$$ (30)

The second term shows the adjustment cost of capital and $\gamma_t$ represents the MEI shock which follows an AR(1) stochastic process. MEI shock represents exogenous disturbance to the process by which investment goods are transformed into installed capital to be used in production. JPT (2010) argues that the MEI shocks represent disturbances to the process by which investment goods are turned into capital ready for production. The classical Keynesian theory relates MEI to the responsiveness of investment to changes in the interest rate spread. A fall in interest rate spread should decrease the cost of investment relative to the potential yield and, as a result planned capital investment projects on the margin may become worthwhile. Therefore, the efficiency of the investment process is closely linked to the health of the financial system. In the real world, because of the presence of friction in financial market, the investment process is not entirely efficient and it encounters randomness, uncertainty, and waste of physical resources while adjusting the investment rate. Introducing the MEI shock is a way to capture such inefficiencies:

$$\log \gamma_t = \rho_i \log \gamma_{t-1} + \sigma_\gamma \epsilon_{\gamma,t},$$ (31)

where $\epsilon_{\gamma,t} \sim i.i.d. N(0, 1)$. JPT (2010) shows that the business cycles are driven primarily by MEI shocks, as opposed to the shocks that affect the transformation of consumption into investment goods (IST shocks). In the next section, I will present evidence that in the presence of the financial frictions, MEI shocks play only a minor role in explaining the business cycle fluctuations; this is while the capital quality shocks can account for most of the cyclical variations in the aggregate macroeconomic variables.
Following BGG (1999), I assume that there are increasing marginal adjustment costs in the production of capital, which I capture by assuming that investment expenditures of $I_t$ yield a gross output of new capital goods $\Phi\left(\frac{I_t}{K_t}\right)K_t$, where $\Phi$ is an increasing and concave function where $\Phi(0) = 0$. I include adjustment costs to permit a variable price of capital. In equilibrium, given the adjustment cost function, the price of a unit of capital in terms of the numeraire good, $Q_t$, is given by:

$$Q_t = (\Phi'\left(\frac{I_t}{K_t}\right))^{-1}. \quad (32)$$

I assume the following conventional form for $\Phi(.)$:

$$\Phi\left(\frac{I_t}{K_t}\right) = [1 - S\left(\frac{I_t}{I_{t-1}}\right)] \frac{I_t}{K_t}, \quad (33)$$

where the function $S$ captures the presence of adjustment costs in investment. As in CEE (2005), firms face quadratic adjustment costs on construction of new capital. In particular, the costs are quadratic in deviations of the growth in investment from a value of unity:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\eta_i}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2. \quad (34)$$

Note that $S'(.) > 0, S''(.) > 0, S(1) = S'(1) = 0$. $\eta_i$ is the inverse elasticity of net investment to the price of capital. The law of capital motion is then given by:

$$K_{t+1} = \xi_t(1 - \delta(U_t))K_t + \gamma_t[1 - S\left(\frac{I_t}{I_{t-1}}\right)]I_t. \quad (35)$$

### D.1. Tobin’s Q

Capital producer firm’s profit per unit of capital at period $t$ is given by:

$$\Psi_t = Q_t\gamma_t\Phi\left(\frac{I_t}{K_t}\right) - \frac{I_t}{K_t} - (\bar{Q}_t - Q_t). \quad (36)$$

Firm maximizes its total discounted profit:

$$\max_{I_t} \left\{ E_t \sum_{i=t}^{\infty} \beta^i \Lambda_{t,i}\{Q_i\gamma_i\Phi\left(\frac{I_i}{K_i}\right)K_i - I_i - (\bar{Q}_i - Q_i)K_i\} \right\}. \quad (37)$$

---

$^1$As in Kiyotaki and Moore (1997), therefore the idea is to have asset price variability which contributes to volatility in entrepreneurial net worth.
I rewrite the profit equation by substituting adjustment cost from above, as follows:

\[
\max_{I_t} \{ E_t \sum_{i=1}^{\infty} \beta^i \Lambda_{t,i} \{ Q_i \gamma_i [(1 - S(I_t/I_{t-1}))I_t] - I_i - (\bar{Q}_i - Q_i)K_i \} \}. \tag{37}
\]

The profit maximization problem of capital producer firms implies that the shadow price of capital is given by:

\[
Q_t = \arg \max_{I_t} \{ E_t \sum_{i=1}^{\infty} \beta^i \Lambda_{t,i} \{ Q_i \gamma_i [(1 - S(I_t/I_{t-1}))I_t] - I_i - (\bar{Q}_i - Q_i)K_i \} \} - Q_t \gamma_t [1 - S(I_t/I_{t-1})] - S'(I_t/I_{t-1})(I_t/I_{t-1})] + \beta E_t \{ Q_{t+1} \Lambda_{t,t+1} \gamma_{t+1} S'(I_{t+1}/I_t)(I_{t+1}/I_t)^2 \} = 1. \tag{38}
\]

E. Retail Firms

The retail sector in this model is a standard RBC sector that generates nominal rigidity, following CEE (2005). At every point in time t, perfectly competitive firms purchase the intermediate goods \( Y_{mt} \) at price \( P_{mt} \) and re-package them and set the price on a staggered basis, as in Calvo (1983). The derivations of formulas in this section are standard, therefore without going into the details of equations, I just mention the essence of the sticky price mechanism. The wholesale retail output is:

\[
Y_t = Y_{mt} D_t, \tag{40}
\]

where \( D_t \) is price dispersion and is given by:

\[
D_t = \varpi D_{t-1} \pi_t^{-\gamma_P \epsilon_P} \pi_t^{\epsilon_P} + (1 - \varpi) \left( 1 - \varpi \right)^{\gamma_P (1 - \varpi)} \pi_t^{\epsilon_P (1 - \varpi) - \gamma_P \epsilon_P}. \tag{41}
\]

Note that the \( \varpi \) is the Calvo probability, which is the probability of keeping the price fixed. \( \gamma_P \) is the measure of price indexation and \( \epsilon_P \) is the elasticity of substitution. The recursive formulation of the optimal price choice is given by:
\[ Y_t = Y_t P_{mt} + E_t \left[ \beta \omega \Lambda_{t+1} \frac{\pi_t^{-(\gamma_P \epsilon_P)}}{\pi_{t+1}^{-(\gamma_P \epsilon_P)}} F_{t+1} \right] \]  

(42)

\[ Z_t = Y_t + E_t \left[ \beta \omega \Lambda_{t+1} \frac{\pi_t^{\gamma_P(1-\epsilon_P)}}{\pi_{t+1}^{(1-\epsilon_P)}} Z_{t+1} \right] \]  

(43)

\[ \pi_t^* = \frac{1}{\tau_X \epsilon_P - 1} \frac{\epsilon_P}{Z_t} \pi_t. \]  

(44)

Therefore, it will follow that the inflation dynamic has the below standard form:

\[ \pi_t^{1-\epsilon_P} = \omega \pi_{t-1}^{\gamma_P(1-\epsilon_P)} + (1 - \omega) \pi_t^*^{(1-\epsilon_P)}. \]  

(45)

\section*{F. Central Bank}

The central bank has a conventional monetary policy instrument by which it sets the nominal interest rate following a feedback rule of the form:

\[ \left( \frac{i_t}{\bar{i}} \right) = \left( \frac{i_{t-1}}{\bar{i}} \right)^{\rho_r} \left( \frac{\pi_t}{\pi^*} \right)^{\kappa_{\pi}} \left( \frac{Y_t}{Y^*} \right)^{\kappa_y} \left( 1 - \rho_r \right) e^{\sigma_i \epsilon_{i,t}}, \]  

(46)

where \( \rho_r \) sets the degree of interest rate smoothing and \( i \) is the steady state of the gross nominal interest rate. The interest rate responds to the deviations of inflation from its steady state and the output gap; \( \kappa_\pi \) and \( \kappa_y \) are the corresponding control parameters. Finally, the monetary policy rule is also disturbed by a monetary policy shock \( \epsilon_{i,t} \sim i.i.d.N(0,1) \). The nominal and risk-free interest rates satisfy the Fisher equation:

\[ i_t = R_{t+1} E_t \pi_{t+1}, \]  

(47)

where \( R_t \) is real interest rate (RIR).
G. Government Budget Constraint

Government finances its exogenous expenditure $G_t$ by collecting the lump-sum tax $T_t$ (or subsidies) that also appear in households’ budget constraint. Consequently, the government’s budget constraint is given by:

$$G_t = T_t,$$  \hspace{1cm} (48)

where government spending is subject to a stochastic shock and follows as:

$$G_t = \frac{1}{1 + g_t}Y_t,$$  \hspace{1cm} (49)

$$\log g_t = (1 - \rho_g) \log g_{ss} + \rho_g \log g_{t-1} + \sigma_g \epsilon_{g,t},$$  \hspace{1cm} (50)

$$\epsilon_{g,t} \sim i.i.d. \mathcal{N}(0, 1),$$  \hspace{1cm} (51)

where $g_{ss}$ is the steady state value of government spending.

H. Resource Constraint

Output is divided between consumption, investment, adjustment cost of investment, and government consumption $G_t$; hence, the aggregate resource constraint is given by:

$$Y_t = C_t + I_t + S(I_{t-1}I_t) + G_t.$$  \hspace{1cm} (52)

Equations (1) to (52) determine twelve endogenous variables. The stochastic behavior of the system of linear rational expectations equations is driven by six exogenous disturbances: intertemporal preference shifter ($b_t$), total factor productivity ($A_t$), marginal efficiency of investment ($\gamma_t$), capital quality ($\xi_t$), government spending ($g_t$), and monetary policy ($\epsilon_i$) shocks. In the next section I will explain the empirical evaluations of the model.

III. Empirical Evaluation

The model presented in the previous section is estimated with Bayesian estimation techniques. I use four key macroeconomic quarterly US time series as observable variables: the log difference
of real GDP per capita, the log difference of real investment per capita, the log difference of the GDP deflator, and the federal funds rate. The model is expressed in log deviation from steady state for simulation purposes. Real per-capita GDP is obtained by dividing the real GDP by the civilian non-institutional population (16 years and over). Real per-capita investment is the nominal private investment, divided by the civilian non-institutional population and the GDP deflator. The inflation rate is the log difference of the GDP deflator. All series are seasonally adjusted. The interest rate is the Federal Funds Rate. GDP and investment are expressed in 100 times log. The inflation rate and interest rate are expressed on a quarterly basis. GDP, investment, and the deflator are obtained from the US Department of Commerce, Bureau of Economic Analysis. Civilian non-institutional population (16 years and over) is obtained from US Department of Labor, Bureau of Labor Statistics. Federal Funds Rate is obtained from Federal Reserve Economic Data - FRED - St. Louis Fed. The corresponding measurement equation is:

\[
\begin{bmatrix}
\Delta \log(GDP_t) \\
\Delta \log(INV_t) \\
\Delta \log(P_t) \\
FedFund_t
\end{bmatrix} = 
\begin{bmatrix}
Y(t) - Y(t - 1) \\
I(t) - I(t - 1) \\
\pi(t) \\
i(t)
\end{bmatrix},
\]

where \(\Delta \log\) stands for the log difference. I use quarterly data from 1962Q1 to 2004Q4 and estimate the posterior modes by maximizing the log posterior function, which combines the prior information on the parameters with the likelihood of the data. In what follows I explain prior assumptions and details of my Bayesian estimation steps.

A. Prior Distributions

The standard errors of the innovations are assumed to follow an inverse-gamma distribution with a mean of 0.10 and a standard deviation of 2.0, which corresponds to a rather loose prior. The covariance matrix of the innovations is diagonal. The persistence of AR(1) processes is beta distributed with a mean of 0.5 and a standard deviation of 0.2. Priors for structural parameters are set following the standard measures in the literature (SW (2007), JPT (2010)). Table 1 summarizes the prior and posterior distributions of the parameters in the model. Inv. Gamma represents Inverse Gamma distribution.

Six parameters are fixed in the estimation procedure. The steady state of the depreciation rate is fixed at 0.02 and the exogenous spending-GDP ratio is set at 0.2. Both of these parameters would be difficult to estimate unless the investment and exogenous spending ratios would be directly used in the measurement equation. Discount factor (\(\beta\)) is set to 0.99, inverse Frisch elasticity of labor is set to 0.275, banker survival rate is set to 0.971 following GK (2011), and proportional transfer to entering bankers (\(\omega\)) is set to \(2 \times 10^5\).
**Bayesian Estimation.** The Metropolis-Hastings algorithm is used to obtain a complete picture of the posterior distribution and to evaluate the marginal likelihood of the model. I simulate the model for 20,000 Metropolis-Hastings iterations. The model is estimated over the full sample period. Figure 3 shows the multivariate convergence statistic of the MCMC simulation. The red and blue lines represent specific within- and between-chain measures. The interval statistic, presented in the top panel, is constructed around the parameter mean. The M2 statistic, presented in the middle panel, is a measure of the variance. M3, depicted in the bottom panel, is based on third moments. Simulation converges when the red and blue lines get close and settle down. As is clear in the graph, convergence only occurs after 10,000 draws.\(^2\)

![Figure 3: Brooks and Gelman’s convergence diagnostics](image)

Figure 4 shows the normalized history of financial shocks along with the normalized default risk spread, which I define as the spread between BAA-rated corporate bond yields and AAA-rated corporate bond yields. The shaded areas are the NBER recession bars. Smoothed shocks are a reconstruction of the values of unobserved shocks over the sample, using all the information contained in the sample of observation. They are computed via the Kalman smoother. One

---

\(^2\)I use a csminwel optimization algorithm that is known to have good properties for exploring the likelihood surface, instead of fmincon (Matlab built in optimization procedure, which is Newton based). By adopting a Bayesian approach and computing Markov chain Monte Carlo (MCMC) optimization steps, I sampled the posterior distribution for a very large set of points. Sampling posterior topology is an optimization procedure in and of itself. After sampling a large set of points, I didn’t find a point whose log-posterior value was superior to the one found during the first optimization round (csminwel).
can observe that historical recessions coincide with the rise in spread and financial shocks. The correlation between the financial shock and spread is small but positive. My conjecture is that if I extend the estimation period to cover the great recession, the impact of financial shocks will be more visible.

B. Posterior Estimates

Table 1 gives the mean, mode, and the 5th and 95th percentiles of the posterior distribution of the parameters obtained by the Metropolis-Hastings algorithm. A number of observations are worth making regarding the estimated processes for the exogenous shock variables. Overall, the estimation results are consistent with JPT (2010) and SW (2007)’s findings; furthermore, data appears to be very informative on the stochastic processes for the exogenous disturbances. The government spending, productivity, and capital quality processes are estimated to be the most persistent; the estimated mean of the AR(1) coefficients are 0.98, 0.96, and 0.89, respectively. Also, the estimated means of the standard error of the shock to the government spending, productivity, and capital quality processes are 0.65, 0.68, and 0.37, respectively. The high persistence of these processes implies that they are important shocks in explaining the forecast error variance of the real variables; Table 4 shows the variance decomposition of the shocks to the main variables. Turning to the estimates of the main behavioral parameters, it turns out that the mean of the posterior distributions are typically relatively close to the mean of the prior assumptions. The model suggests a significantly large habit parameter (0.95). Finally, turning to the monetary policy reaction function parameters, the mean of the long-run reaction coefficient to inflation is estimated to be relatively high, with an estimated mean of 2.19, compared to the one in JPT (2010). The degree of interest rate smoothing as the coefficient on the lagged interest rate, $\rho_i$, is estimated to be 0.39, which is smaller than SW (2007). Policy appears to react strongly to the output gap (0.20). In the next section, I will investigate the
Table 1: Prior Distributions and Posterior Parameter Estimates in the Model

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Distribution</th>
<th>Mean</th>
<th>S.D.V</th>
<th>Mean</th>
<th>Mode</th>
<th>5%</th>
<th>95%</th>
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<tr>
<td>$\sigma_a$</td>
<td>Std Tech.</td>
<td>Inv. Gamma</td>
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<td>Inv. Gamma</td>
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<td>0.50</td>
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<td>0.50</td>
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<td>0.82</td>
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**Prior**

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**Fixed**

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<th>Mean</th>
<th>Mode</th>
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fit of the model using the Bayesian model comparison method.

IV. Model Fit

How well does the model fit the data, given my posterior estimates? In this section, I try to answer this question. Bayesian econometricians use marginal data density (MDD) to assess how well the model fits data. I address the above question by first comparing the Laplace approximation of the log data density of my estimation to the one by SW (2007). This comparison can be insightful, as these two models are roughly nested\(^{[3]}\). Moreover, the SW (2007) model is widely used in policy environments and is proven to have a good data fit; therefore, it performs as a good benchmark to evaluate the fit of my model. When comparing two models, \(M_1, M_2\) given the history of data, \(Y^T\), posterior probability of model \(i\) can be computed as follows:

\[
\Pi_{i,T} = \frac{\Pi_{i,0} P(Y^T|M_i)}{\Pi_{i,0} P(Y^T|M_i) + \Pi_{j,0} P(Y^T|M_j)},
\]

(53)

where \(P(Y^T|M_i)\) shows the marginal data density and is computed as below:

\[
P(Y^T|M_i) = \int P(\theta^i|M_i) L(\theta^i|Y^T, M_i) d\theta^i.
\]

(54)

The Laplace approximation of log data density of my model is estimated to be \(-629.58\). I repeat the same exercise for the SW (2007) model and estimate it using the same observables: the log difference of real GDP, real investment, the log difference of the GDP deflator, and the Federal Funds Rate. The Laplace approximation of log data density of the SW (2007) model is estimated to be \(-676.02\).

Table 2 summarizes these results. The smaller absolute value of MDD for the model with FF implies that the model fits the data well. The marginal data density of my model is close to that of SW but not identical. The reason is that the two models have a different number of parameters, and marginal data density penalizes the more complex model. More precisely, the SW model has 36 free parameters, while the GK model has 27 parameters—one-third fewer. However, it is still the case that the model with financial frictions is simpler and yet demonstrates a reasonable fit. The MDD is an out-of-sample measure of model fit and is somewhat better for the GK model. Table 2 shows the Laplace approximations of the log data density for the two models, estimated over a time horizon similar to the original SW

\(^{[3]}\)By turning off the financial frictions in my model and adding wage rigidities, I would get the SW (2007).
Table 2: Marginal Data Density Test

<table>
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<tr>
<th>Model comparison</th>
<th>my model</th>
<th>SW (2007)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laplace approx. of Log data density</td>
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<td>-676.02</td>
</tr>
<tr>
<td>Number of shocks</td>
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<td>Number of observables</td>
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<td>Number of Estimated Parameters</td>
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</tbody>
</table>

(2007) sample (1962Q1-2004Q4). I also estimate the model for a period that encompasses the latest financial crisis (1962Q1-2010Q4)\(4\) and the results regarding relative predictive power remain unchanged. This confirms the reasonable fit of the model with financial frictions. More generally, the sample period that I use includes several periods of financial stress or crisis. Having a measure of financial stress, such as the spread between lending and borrowing, enables my model to capture the impact of a financial crisis in a more direct and efficient way. The SW model captures these effects through the second round impact on real and nominal variables. The other feature of the model with financial friction that leads to this result is the presence of financial shocks, which are closely related to the countercyclical spread. In an economy without financial frictions, an exogenous drop in quality of capital stock harms the capital accumulation process and increases the rate of depreciation; this will eventually lower output modestly. However, in the presence of financial frictions, a negative capital quality shock affects the economy through an additional channel, namely the bank balance sheet, by creating a significant capital loss in the financial sector. Section 6 provides more in-depth microeconomic interpretations of such financial shocks.

Table 3: Theoretical Moments (standard deviation and 3 lags autocorrelation)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Auto Corr 1 Model</th>
<th>Data</th>
<th>Auto Corr 2 Model</th>
<th>Data</th>
<th>Auto Corr 3 Model</th>
<th>Data</th>
<th>STD Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Growth</td>
<td>0.35</td>
<td>0.23</td>
<td>0.19</td>
<td>0.21</td>
<td>0.09</td>
<td>0.08</td>
<td>0.98</td>
<td>0.86</td>
</tr>
<tr>
<td>Investment Growth</td>
<td>0.59</td>
<td>0.53</td>
<td>0.32</td>
<td>0.37</td>
<td>0.15</td>
<td>0.20</td>
<td>2.47</td>
<td>2.26</td>
</tr>
<tr>
<td>Inflation Rate</td>
<td>0.84</td>
<td>0.87</td>
<td>0.67</td>
<td>0.82</td>
<td>0.55</td>
<td>0.79</td>
<td>0.66</td>
<td>0.62</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.95</td>
<td>0.94</td>
<td>0.87</td>
<td>0.86</td>
<td>0.77</td>
<td>0.79</td>
<td>1.06</td>
<td>0.83</td>
</tr>
</tbody>
</table>

\(4\) my model’s MDD is -709.80 and the SW model’s MDD is -778.21.
Next, I compare a set of statistics implied by the model to those measured in the data. In particular, I cross compare theoretical moments of estimated variables with the observable variables used for the estimation. Table 3 reports the standard deviation and three lags autocorrelation coefficients of the observable variables included in the estimation, namely output growth, investment growth, inflation rate, and interest rate. One can observe from Table 3 that the estimation matches the moments in the data relatively well.

V. Drivers of Business Cycle

Investment shocks have proven quite successful in explaining long-run growth and have been adopted as a standard feature of many DSGE models. Following Papanikolao (2011), investment shocks change the resource cost of producing new capital goods and, thus, through the Euler equation, have an effect on consumption and output. In the New Keynesian DSGE model of JPT (2010), without financial frictions, analysis of variance decomposition shows that business cycles are driven primarily by shocks that affect the transformation of investment goods into installed capital (MEI shocks), rather than the transformation of consumption into investment goods (IST shocks). In my model, the presence of financial frictions provides a suitable framework for emphasizing, first, the role of interest rate spread and, second, the wedge between MEI shocks and financial shocks in explaining the US business cycle. The variance decompositions of variables in the model are computed. Table 4 reports the contribution of the four most important shocks in the model to the variance of macroeconomic variables at business cycle frequencies. The first result to stress is that MEI processes play virtually no role in business cycles, after introducing financial shocks into the model.

The second result that emerges from Table 4 is that capital quality shocks are the key drivers of business cycle fluctuations. They are responsible for 49, 75, 36, 62, and 60 percent of the variance of output, consumption, investment, spread, and welfare, respectively. The remarkable feature of the model that leads to these results is the close relationship between financial shocks and countercyclical credit spread. A financial shock in this model is the shock which affects the quality of productive capital stock, and it is highly correlated with the spread. Hence, policies which affect the credit spread are the most effective instruments in aiding recovery during financial distress. Both results are in favor of JPT (2010), as the common feature of both models is the importance of the shocks which are correlated with the credit spread. In addition, I go one step further and decompose the financial shock from the MEI.

Presence of capital quality shock is the novel feature of the GK (2011) model. As it is argued there, in an economy without financial frictions, a negative shock to the quality of capital stock

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5 The results are qualitatively robust over different estimation sample sizes: 1962Q1-1985Q4, 1986Q1-2004Q4, and 1962Q1-2010Q4.
Table 4: Variance Decompositions (%)

<table>
<thead>
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<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>35.7101</td>
<td><strong>48.6919</strong></td>
<td>14.7556</td>
<td>0.8422</td>
<td>0.0002</td>
<td>0</td>
</tr>
<tr>
<td>Consumption</td>
<td>17.2169</td>
<td><strong>75.8795</strong></td>
<td>6.7251</td>
<td>0.1783</td>
<td>0</td>
<td>0.0002</td>
</tr>
<tr>
<td>Investment</td>
<td>58.5572</td>
<td><strong>36.4861</strong></td>
<td>2.0442</td>
<td>2.9112</td>
<td>0.0011</td>
<td>0.0001</td>
</tr>
<tr>
<td>Capital</td>
<td>9.922</td>
<td><strong>89.4107</strong></td>
<td>0.3658</td>
<td>0.3015</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Gov’t Spending</td>
<td>4.2054</td>
<td><strong>5.7342</strong></td>
<td>89.9612</td>
<td>0.0992</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Asset Prices</td>
<td>34.0244</td>
<td><strong>53.3743</strong></td>
<td>0.7714</td>
<td>11.8145</td>
<td>0.0153</td>
<td>0</td>
</tr>
<tr>
<td>Welfare</td>
<td>11.8152</td>
<td><strong>59.8949</strong></td>
<td>28.15</td>
<td>0.1398</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Inflation</td>
<td>6.9589</td>
<td><strong>50.946</strong></td>
<td>0.9901</td>
<td>41.1024</td>
<td>0.0025</td>
<td>0</td>
</tr>
</tbody>
</table>

6Households spend part of their income on installing capital goods for future production. Following the seminal paper by Kydland and Prescott (1982), it is assumed that installing capital goods takes time; the so-called time-to-build period.

7For example, suppose Blackberry mobile phone technology becomes obsolete as touch screen smart phone technology is introduced (market capitalization of BB dropped from 80 billion USD to 4 billion USD in October 2013). There could be three channels through which this shock is transmitted. First, capital stock for pro-

6This is while in the presence of financial frictions, negative capital quality shock effects the economy through an additional channel, namely, the bank balance sheet channel, by creating significant capital loss in the financial sector. Prior to the negative capital quality shock, financial intermediaries were highly leveraged and, when the shock hits the economy, intermediaries suffer severely from deterioration of their net asset position, which in turn induces a tightening of credit and a significant downturn in the economy.

7Therefore, one can summarize three channels through which financial shock transmit into
First, capital stock, de facto, depreciates much more rapidly than originally expected. Second, output of intermediate producers suffers. Third, the financial balance sheets suffer as stock valuations fall.

This process of capital destruction is also similar to the ones argued by JPT (2010), Gertler and Kiyotaki (2010), Ajello (2011) and Liu, Waggoner, and Zha (2011). The common feature of all is the impact of a financial shock on the credit spread. To corroborate the above interpretation, it is explicitly shown below that a financial shock is closely related to spreads or the external financing premium. The key point is that the quality factor, $\xi$, plays a role in determining the rate at which non-financial goods producer firms borrow capital from financial intermediaries.

Figure 5: Normalized impulse response functions to MEI and capital quality shocks

In the New Keynesian DSGE model of JPT (2010) without financial frictions, 37% of output producing Blackberry devices, de facto, depreciates much more rapidly than originally expected. Second, output of intermediate producers, say those who produce Blackberry parts or software, suffers. Third, the financial balance sheets of banks, which lend to such firms such as Blackberry, suffer as stock valuations fall. One possible microeconomic interpretation is that a large number of goods are produced using good-specific capital. In each period, as a fraction of goods becomes obsolete, the capital used for producing those goods becomes worthless. In aggregate, a capital quality shock reflects the economic obsolescence of capital, which in turn leads to deterioration of the balance sheets of financial intermediaries.

\[
\begin{align*}
\text{spread}_{t+1} & = (R_{k,t+1} - R_{t+1}) \\
R_{k,t+1} & = P_{mt+1}a \frac{Y_{t+1}}{K_{t+1}} + (Q_{t+1} - \delta(U_{t+1}))\xi_{t+1} \\
R_{t+1} & = (\beta\Lambda_{t,t+1})^{-1} \\
\text{spread}_{t+1} & = \text{spread}_t + b_{t+1}\xi_{t+1} \\
\text{spread}_t & \propto \xi_t
\end{align*}
\]
variation and 79% of investment volatility are explained by the marginal efficiency of investment shocks, while the TFP shock is the second most significant. In an economy with information asymmetry, where the spread between borrowing and lending is non-zero, costly state verification puts in place a mechanism to monitor the risk in banks’ activities. A deterioration of quality of productive capital adversely affects output and demand for productive capital, lowers good producer firm’s stock valuation and their collateral value, and, ultimately, financial intermediaries’ asset valuation in their balance sheets drops. As is shown in the above figure, a capital quality shock implies a procyclical price of capital. On the other hand, a marginal efficiency of investment shock implies that the value of net worth (or the stock market equity) is countercyclical. This is the main reason that, in the presence of the financial accelerator block, the data favors the capital quality shock over the marginal efficiency of investment shock.

VI. Impulse Response Functions

Figure 1.5 shows the impulse response functions of the economy to a technology shock and a monetary policy shock. In each case, the direction of the shock is set to produce a recession. Impulse response functions are computed as the expected future path of the endogenous variables conditional on a shock in period 1 of one standard deviation to the exogenous variables, evaluated at the estimated parameters’ modes. The direction of impulse responses is sensible, just as conventional theory would suggest, but the magnitudes of the responses are amplified as compared to those of the standard New Keynesian models without financial frictions. Figure 1.5 shows the responses of 8 key variables: output, hours of work, investment, capital, inflation, spread, shadow price of capital stock, and interest rate.

The technology shock is a negative one standard deviation innovation in TFP, with a quarterly autoregressive factor of 0.96. As compared to SW (2007), the magnitude of responses upon impact and at the extrema (peak effect) are amplified. The amplification effect comes from the bank balance sheet channel, which produces a positive feedback between endogenous variables. After the shock hits the economy, investment declines sharply, and this reduces the shadow price of capital stock. Therefore, the asset side of intermediaries’ balance sheets deteriorates, which further limits the lending. Since the supply of funds for non-financial firms goes down, the spread goes up and their cost of funding increases. As a result, demand for capital declines, and this further reduces output, investment and the shadow price. The inflation and the interest rate go up but come back to their steady states relatively quickly. Household consumption falls, and they have to work more (hours of work increases). Output and investment go down in a humpshape manner.

The monetary shock is an unanticipated one standard deviation increase in the short-term nominal interest rate. The degree of interest rate smoothing by the central bank in this model is less than in the SW (2007) model and, as a result, the peak effects in impulse responses occur at earlier horizons than SW (2007) impulse responses; moreover, financial frictions lead
Figure 6: Impulse responses to technology (blue) and monetary policy (red) shocks to amplification in responses due to the strongly countercyclical premium. In line with SW (2007)’s findings the peak effect of a policy shock on inflation occurs before its peak effect on output.

Figure 1.6 exhibits the impulse response functions to a capital quality shock (financial shock). A negative one standard deviation financial shock can create a recession of plausible dynamic and magnitude with two remarkable features: firstly, output, investment, consumption, interest rate, and capital stock decline severely. Secondly, the recovery of the economy is slow, due to the prolonged deleveraging. These key results come from the close relationship between financial shocks and the countercyclical premium. Following the shock, the model delivers a large drop in asset price, driving it 1.1% below its steady state level on impact. This acts as a shock to the intermediaries’ balance sheets and their asset quality declines substantially. The spread soars as a consequence and increases the cost of funding. Since banks are highly leveraged, this produces an enhanced decline in the capital stock, which recovers very slowly. Output initially falls about 0.5 percent and decreases further to about 0.97 percent, relative to zero, within five quarters. Investment drops to 1.4 percent below zero upon impact and follows a similar humpshape response as output. Hours of work initially drop to 0.16 percent below steady state but quickly ramp up and increases to a maximum of 1 percent after 20 quarters; it goes down slowly but the steady state value remains positive. One of the shortcomings of this model is the lack of wage rigidities, which further affects the response of hours of work.
Figure 7: Impulse responses to capital quality shock

Therefore, to get a more accurate response of hours, one would need to include appropriate labor market imperfections in the model. Recovery of the economy is slow and in line with evidence from the recent financial crisis. During this period, the intermediary sector is deleveraging by building up equity relative to assets. It is widely known that banking crises preceded by credit bubbles are typically followed by prolonged deleveraging. Data shows that the average deleveraging period is 5 to 6 years and typically starts 2 years after the onset of a financial crisis. It is clear from impulse responses that output recovery occurs only after 6 quarters and it takes 5 years for it to return to the pre-crisis steady state.

VII. Conclusion

I developed a New Keynesian DSGE model which features endogenous market friction, based on the model of Gertler and Karadi (2011), with financial shocks and marginal efficiency of investment shocks. The model parameters are estimated using a Bayesian technique. The fit of the model is further assessed by comparing the marginal data density (MDD) of the model with the frictionless economy described in Smets and Wouters (2007). I found that a Bayesian model comparison would pick the model with financial frictions over the baseline frictionless

9McKinsey “Episodes of Deleveraging.”
This paper further studies the sources of business cycle fluctuations in the spirit of Justiniano, Primiceri, and Tambalotti (2010). The presence of financial frictions in my model provides a framework which emphasizes the role of the credit spread and the wedge between marginal efficiency of investment shocks and financial shocks in explaining the US business cycle. The analysis of variance decomposition shows that with the presence of financial intermediaries explicitly in the model, financial shocks are the predominant source of variability in key macroeconomic variables at business cycle frequencies. A key conclusion of this paper is that policies that reduce the costs associated with financial transactions would be most effective in facilitating economic recovery. Moreover, when analyzing the impact of economic policies on business cycle developments, it would be essential to use macroeconomic models that explicitly incorporate a financial sector.
References


A Appendix - Data

output = LN( GDPC96 / LNSindex ) * 100
investment = LN( ( FPI / GDPDEF ) / LNSindex ) * 100
inflation = LN( GDPDEF / GDPDEF(-1) ) * 100
interest rate = Federal Funds Rate / 4

GDPC96 : Real Gross Domestic Product - Billions of Chained 1996 Dollars, Seasonally Adjusted Annual Rate. Source: US Department of Commerce, Bureau of Economic Analysis


FPI : Fixed Private Investment - Billions of Dollars, Seasonally Adjusted Annual Rate. Source: US Department of Commerce, Bureau of Economic Analysis

