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Inflation Targeting and Exchange Rate Management in Less Developed Countries*
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Abstract

We analyze coordination of monetary and exchange rate policy in a two-sector model of a small open economy featuring imperfect substitution between domestic and foreign financial assets. Our central finding is that management of the exchange rate greatly enhances the efficacy of inflation targeting. In a flexible exchange rate system, inflation targeting incurs a high risk of indeterminacy where macroeconomic fluctuations can be driven by self-fulfilling expectations. Moreover, small inflation shocks may escalate into much larger increases in inflation ex post. Both problems disappear when the central bank leans heavily against the wind in a managed float.

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1 Introduction

There is a great deal of confusion about how to manage the exchange rate in less developed countries (LDCs) that practice inflation targeting (IT). Conventional wisdom holds that policy should respond to the exchange rate one step removed, only after fluctuations in the rate affect inflation or real output (Taylor, 2001; Clarida et al., 2001). Including the exchange rate directly in the interest rate rule serves no useful purpose and risks creating confusion in the public’s mind about whether control of inflation is truly the central bank’s main priority.

Incomplete pass through and concerns about balance sheet effects on firms and banks that carry dollar-denominated liabilities qualify this conclusion, but only a little. Overall, theory offers scant support for targeting the exchange rate. The dominant view is that the central bank should stay out of the foreign exchange market. Some of the literature even asserts that a flexible, market-determined exchange rate is a precondition for successful adoption of IT.

Policy makers evidently disagree. Most emerging market and many low- and middle-income countries subscribe to some form of IT. Few, if any, however, allow the exchange rate to float freely. Despite the strictures of theory, interventions in the foreign exchange market are frequent and large (BIS, 2005; International Monetary Fund, 2011; Berganza and Broto, 2012). The universal preference is to carefully manage the path of the exchange rate (Mishra et al., 2010).

The embarrassing contradiction between what theory prescribes and what central banks do has motivated the development of new frameworks for the analysis of IT in developing countries. Benes et al. (2013) and Ostry et al. (2012) argue that theory’s obsession with whether the exchange rate should appear in the interest rate rule is misplaced. In practice, sterilized purchases/sales of foreign exchange are the main instrument for managing the exchange rate. But with two instruments for two targets, there is no reason to fear that a crawling peg or a tightly managed float will undermine credibility of IT. It should be easy to explain to the public that the interest rate will adjust to steer inflation to its target level while net sales of foreign exchange reserves limit undesirable fluctuations in the exchange rate.\footnote{The qualifications are emphasized in Calvo (2000), Mishkin (2000a, 2000b), Ghironi and Rebucci (2002), Elekdag and Tchakarov (2004), and Moron and Winkelreid (2005). Cespedes et al. (2004), Laxton and Pesenti (2003), Cavoli and Rajan (2006), and Batini et al. (2007) conclude, however, that the weight on the exchange rate target should be small or zero.}

\footnote{See, for example, Masson et al. (1997).}{\footnote{Adherents of the conventional wisdom remain sceptical. Blejer and Leone (2000) contend, for example, that the co-existence of multiple anchors leads sooner or later to policy conflicts, confusion, and a loss of credibility. Mishkin}}
Two instruments for two targets explains why management of the exchange rate to maintain export competitiveness, stabilize food prices, or protect firm and bank balance sheets does not conflict with pursuit of IT. It does not, however, fully reconcile theory and practice. In many countries, sterilized reserve sales seem to be an integral part of the IT framework as opposed to a separate policy instrument deployed to achieve a separate objective (Chang, 2007). This and the steady accumulation of episodes in which central banks could not regain control of inflation until they intervened to stem rapid depreciation of the currency has started to erode confidence in the conventional wisdom, albeit tentatively and with qualifications. After surveying the empirical record, Truman (2003), concludes that the monetary authorities in LDCs have to be on guard against paths of the exchange rate incompatible with IT. Mishkin (2008, p.11) concurs, but also warns that it is dangerous to for monetary policy to “focus too much on limiting exchange rate movements.” In a similar vein, although the IMF officially favors maximum exchange rate flexibility, its own country reports often acknowledge that effective IT entails periodic substantial intervention in the foreign exchange market.5 LDC policy makers probably find these tentative, mixed assessments more confusing than helpful, but at present nothing better is on offer. The fundamental problem, as Chang (2007) and O’Connell (2008) rightly observe, is that the existing literature lacks a coherent theoretical framework for thinking about the potential complementarity of exchange rate and inflation targeting.

This paper squarely confronts the issue of whether coordination of monetary and exchange rate policy is good or bad for IT in developing countries. We conduct the analysis in a two-sector model of a small open economy featuring imperfect substitution between domestic and foreign financial assets. Our central finding is that policy makers have been right all along: tight management of the exchange rate through sterilized foreign exchange purchases/sales greatly enhances the viability of forward-looking IT—the type of IT most countries profess to practice. The case for what we call exchange-rate-anchored IT rests on a set of strongly positive, complementary results for targeting efficiency and uniqueness of the equilibrium path.

(2008) expresses the same concern.

4See Kumhof et al. (2007), Chang (2007) and Al-Eyd et al. (2012) for accounts of such episodes in Latin America and the Central Asian republics. It is interesting to note in this connection that when Colombia adopted IT in 1999 the central bank explicitly reserved the right to intervene in the foreign exchange market to prevent excessive volatility of the exchange rate.

5Thus the 2007 country report for Peru dutifully acknowledges the view in the existing literature that “IT with flexible exchange rate is the best option for monetary policy,” but then observes that “the central bank’s stronger emphasis on exchange rate stability, relative to non-dollarized inflation-targeting countries, has complemented that on price stability.” The 2009 country report goes even further, agreeing with Peruvian policy makers that the conditions are not yet right for using the exchange rate as a macroeconomic buffer and hence that Peru should maintain a large reserve buffer stock and intervene frequently to limit exchange rate fluctuations.
**Determinacy in Managed vs. Floating Exchange Rate Regimes**

Our first selection criterion is that monetary and exchange rate policies should guarantee determinacy—a unique equilibrium. In this regard, we subscribe to the policy assessment of policy rules proposed by Woodford (2003). Rules that lead to indeterminacy—multiple equilibria—can induce fluctuations in the economy that are driven by self-fulfilling expectations, not by fundamentals, and that can be associated with a large degree of volatility of inflation, output and other macro variables. As such, these rules can induce macroeconomic instability.\(^6\)

We find that when the central bank follows the conventional wisdom and lets the exchange rate float, IT incurs a high risk of indeterminacy. As usual, the Taylor coefficient in the interest rate rule is subject to an upper bound.\(^7\) The upper bound is much smaller, however, than in the canonical closed-economy model of IT. Moreover, the lower bound on the Taylor coefficient is often substantially above unity. Squeezed from above and below, the determinacy region is generally small and, in a minority of important cases, non-existent.

Open economy interactions and the structural characteristics of LDC economies underlie these strong results. The smallish upper bound on the Taylor coefficient reflects the presence of a large flex-price tradables sector in small open LDCs (e.g., Zanna, 2003, De Fiore and Liu, 2005). To see why the lower bound may be far above unity, consider the state of the economy, ex ante, when the central bank raises the nominal interest rate one percentage point to combat arbitrary expectations that the rate of currency depreciation and nontradables inflation will increase one percentage point. At first glance, the increase in the interest rate neutralizes both the incentive for capital flight and the impact of higher inflation on aggregate demand. But this is quite wrong. Higher expected inflation spurs substitution away from domestic currency toward foreign currency.\(^8\) In a flexible exchange rate system, the increase in demand for foreign currency is satisfied by a spot depreciation of the currency and an upward jump in the real exchange rate. The spot depreciation, in turn, creates inflationary pressures in the sticky-price nontradables sector by inducing substitution toward nontraded goods in

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\(^6\) “[Indeterminacy includes] equilibria in which endogenous variables such as inflation and output respond to random events that are completely unrelated to economic ‘fundamentals’ (i.e., to the exogenous disturbances that affect the structural relations that determine inflation and output) and also equilibria in which ‘fundamental’ disturbances cause fluctuations in equilibrium inflation and output that are arbitrarily large relative to the degree to which the structural relations are perturbed. Thus in such a case, macroeconomic instability can occur owing purely to self-fulfilling expectations. This is plainly undesirable, if one’s objective is to stabilize inflation and/or output.” Woodford (2003).

\(^7\) The upper bound is characteristic of rules that target future inflation (Bernanke and Woodford, 1997; Bullard and Mitra, 2002; Levin et al., 2003).

\(^8\) Demand for foreign currency increases under the plausible assumption that domestic and foreign currency are Edgeworth substitutes in the provision of liquidity services.
consumption, by reducing labor supply, and by increasing tradables sector employment and output. Easy substitution between domestic and foreign bonds alleviates some of the pressure on the exchange rate, but also sharply reduces the upper bound on the Taylor coefficient. Consequently, for widely varying degrees of currency and bond substitution, sunspot equilibria abound at ordinary values of the Taylor coefficient (e.g., 1.5). To prevent indeterminacy, the increase in the real interest rate must be large enough to counteract all the inflationary pressures created by incipient depreciation of the real exchange rate, but not so large as to exceed the nearby upper bound. Which is easier said than done. Since policy makers do not know their exact coordinates in the parameter space, finding the right value of the Taylor coefficient amounts to guessing right. IT in a flexible exchange rate system is like throwing darts in the dark; it is important to be lucky.

The solution to the problem is to fortify IT with some mechanism that limits feedback effects between sticky-price nontradables inflation and depreciation of the currency. Targeting the real exchange rate in the interest rate rule is not the answer: sunspot equilibria remain a significant threat even when the coefficient on the real exchange rate is large. The right approach is to enlist the aid of a second instrument. When the central bank pegs the real exchange rate through sterilized purchases/sales of foreign exchange, the Taylor principle holds: active policy is necessary and sufficient for a unique equilibrium provided the coefficient on the inflation target does not exceed a large upper bound. Pegging the rate of crawl of the currency at the target inflation rate and leaning against the wind in a managed float also work well.

**Targeting Efficiency**

Obviously the proposed IT regime should not allow small inflation shocks to escalate into much larger increases in inflation ex post. Invoking this second criterion further strengthens the case for exchange-rate-anchored IT. *Attempting* to target inflation does not consistently translate into actual successful targeting of inflation when the central bank intervenes only to stabilize the path of the real exchange rate or lets the nominal exchange rate fluctuate too much in a pure or lightly managed

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9 Spot depreciation decreases labor supply by reducing the real consumption wage. On the other hand, the increase in the real interest rate tends to increase labor supply by reducing aggregate consumption. Hence the overall impact on labor supply is ambiguous.

10 In this regard, our paper is also related to the literature on purchasing-power-parity rules (PPP), whereby the government adjusts the nominal exchange rate in response to real exchange rate deviations from an intermediate target. Starting with the seminal work by Dorbusch (1982), there was an important literature that analyzed how, abstracting from inflation targeting, PPP rules affect fundamental-driven business cycles of an economy (see Montiel and Ostry, 1991, and Calvo et al., 1995). More recently the literature on PPP rules has focused on how these rules can actually induce multiple equilibria and, therefore, destabilize the economy by inducing fluctuations that are driven by people’s beliefs rather than fundamentals (see Uribe, 2002, and Zanna, 2009).
float. Impulse responses for these IT regimes reveal large, erratic fluctuations in the rate of currency depreciation and frequent wide misses of the inflation target. In some runs, the quarterly rates of currency depreciation and inflation jump from 1% to 4-6% following a shock that directly increases the inflation rate only a half percentage point.

This problem disappears, however, if the central bank pegs the rate of currency depreciation or leans heavily against the wind in a tightly managed float. Active monetary policy then suppresses inflationary shocks in the expected manner. In LDCs, successful IT requires two nominal anchors secured by two instruments.

The rest of the paper is organized into six sections. Section 2 lays out the model and calibrates it with parameter values appropriate for a LDC. Following this, Sections 3 and 4 present a mix of numerical and analytical results for forward-looking IT, first in a flexible exchange rate system and then in alternative schemes that target the exchange rate. Section 5 discusses the results from sensitivity tests and Section 6 compares impulse responses for different IT regimes. The final section expands on the policy implications of the results, emphasizing the pros and cons of exchange-rate-anchored IT.

2 The Benchmark Model

The economy produces a homogeneous traded good and a mix of differentiated nontraded goods. World prices are fixed at unity, so domestic prices of traded goods equal the nominal exchange rate. Firms in the tradables sector operate on their supply curves, viewing all prices as parametric. The nontradables sector is New Keynesian: prices are sticky à la Calvo (1983) and output is determined by aggregate demand. To simplify the analysis, labor is assumed to be immobile across sectors, but the implications of such assumption are also discussed. Time is discrete but our results also hold in continuous time.\footnote{Discrete time allows us to investigate the role of the timing of monetary policy rules—e.g. forward-looking rules vs. contemporaneous rules.}

In specifying the financial side of the model, we aim for flexibility and generality. The type and degree of integration into world capital markets varies greatly across developing and emerging market economies. In some countries, substitution between domestic and foreign currency prevails; in others, bond substitution dominates. Mixed cases of currency and bond substitution are also quite common (e.g., Turkey, Costa Rica, the Philippines). Accordingly, we assume the private sector divides its financial wealth between domestic bonds, domestic currency, foreign currency, and foreign
bonds/loans. Foreign currency provides liquidity services and purchases of foreign bonds/loans incur
transaction costs. Hence uncovered interest parity does not tie down the yield on domestic bonds.
The central bank controls the path of the interest rate.

Variable names are familiar or at least mnemonic. One notational convention needs to be kept in
mind, however. Throughout the traded good serves as the numeraire; thus, unless otherwise noted,
prices and variables are deflated by the nominal exchange rate.

2.1 Households

Preferences of agents qua consumers are given by

\[ C = C_x^{1-\gamma} C_n^\gamma, \]  

where \( C_x \) is consumption of traded goods and

\[ C_n = \left[ \int_0^1 c_j^{(\epsilon-1)/\epsilon} dj \right]^{\epsilon/(\epsilon-1)} \]

is a CES aggregate of consumption of nontraded varieties \( c_j \).

The representative agent solves a two-stage optimization problem. In the first stage \( C_x, C_n, \) and \( c_j \) are chosen to minimize the cost of purchasing the composite consumer good \( C \). This yields a set of demand functions and associated price indices:

\[ e_{j,t} = \left( \frac{p_{j,t}}{P_{n,t}} \right)^{-\epsilon} C_{n,t}, \]  

\[ C_{n,t} = \gamma \left( \frac{P_{n,t}}{P_t} \right)^{-1} C_t, \]  

\[ P_{n,t} = \left[ \int_0^1 p_{j,t}^{1-\epsilon} dj \right]^{1/(1-\epsilon)}, \]  

\[ P_t = AP_{n,t}^\gamma, \]  

and

\[ \tilde{P}_t = e_t P_t, \]

where \( A \) is a constant and \( e, P_n, \) and \( \tilde{P} \) denote the nominal exchange rate, the relative price of the
nontraded good, and the exact consumer price index.

After choosing the best mix of traded goods and nontraded varieties, the agent solves the problem

$$\max \sum_{t=0}^{\infty} \beta^t \left[ \frac{C^t_{1-1/\tau}}{1-1/\tau} + G \left( \frac{m_t}{P_t}, \frac{F_t}{P_t} \right) - h_x \frac{L_{sx,t}^{1+\psi}}{1+\psi} - h_n \frac{L_{sn,t}^{1+\psi}}{1+\psi} \right], \quad (7)$$

subject to

$$b_t + b_t^* + F_t + m_t + P_tC_{si,t} = w_{x,t}L_{sx,t} + w_{n,t}L_{sn,t} + T_t + \frac{R_{t-1}}{\pi_{e,t}}b_{t-1} + R_{t-1}^*b_{t-1}^* + F_{t-1} + \frac{m_{t-1}}{\pi_{e,t}} - \frac{\Lambda}{2} (b_t^* - \tilde{b}^*)^2, \quad (8)$$

where

$$G \left( \frac{m_t}{P_t}, \frac{F_t}{P_t} \right) = k_o \frac{\left[ b_t^{1/\sigma} (m_t/P_t)^{\sigma-1}/\sigma + (1 - k_2)^{1/\sigma} (F_t/P_t)^{\sigma-1}/\sigma \right]^{\sigma/(\sigma-1)}}{1-1/\tau}.$$ 

$m$ and $F$ denote domestic and foreign currency; $\tau$ is the intertemporal elasticity of substitution; $\beta$ is the discount factor; $L_{si}$ is labor supply in sector $i$; $b$ is the stock of domestic bonds and $R$ the gross nominal interest rate; $\sigma$ is the elasticity of substitution between domestic and foreign currency in the provision of liquidity services; $b^*$, $R^*$, and $\pi_{e,t} = e_t/e_{t-1}$ are the stock of foreign bonds, the gross nominal interest rate in dollars, and the gross rate of currency depreciation; $w$ is the wage; $T$ is profits less lump-sum taxes; and $\psi$ is the Frisch elasticity of labor supply. Liquidity services are a CES aggregate of real holdings of domestic and foreign currency. In the budget constraint (8), the term $\Lambda (b_t^* - \tilde{b}^*)^2/2$ captures portfolio adjustment costs associated with purchases/sales of foreign bonds.

On an optimal path, consumption is governed by the Euler equation

$$C_t^{-1/\tau} = \frac{\beta R_t}{\pi_{t+1}} C_{t+1}^{-1/\tau}, \quad (9)$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, while the marginal rate of substitution between work and consumption equals the real wage in each sector

$$h_i \frac{L_{si,1/\psi}}{C_t^{-1/\tau}} = \frac{w_{i,t}}{P_t}, \quad \text{for} \quad i = n, x. \quad (10)$$
Moreover, the marginal rate of substitution between consumption and (foreign and domestic) currency equals income foregone from not holding domestic bonds implying that

\[
G_F \left( \frac{m_t}{P_t} \cdot \frac{F_t}{P_t} \right) = \beta C_{t+1}^{-1/\tau} \frac{P_t}{P_{t+1}} \left( \frac{R_t}{\pi_{e,t+1}} - 1 \right),
\]

(11)

and

\[
G_m \left( \frac{m_t}{P_t} \cdot \frac{F_t}{P_t} \right) = \beta C_{t+1}^{-1/\tau} \frac{R_t - 1}{\pi_{t+1}},
\]

(12)

where \( G_F \equiv \partial G/\partial (F/P) \), and \( G_m \equiv \partial G/\partial (m/P) \) and the degree of substitutability between domestic and foreign currency depends on the value of \( \sigma \) in the liquidity services function, with \( \sigma > \tau \) required for the two currencies to be Edgeworth substitutes. Regarding bonds, the return on domestic bonds equals the return on foreign bonds net of portfolio adjustment costs

\[
R_t = \frac{R_t^* \pi_{e,t+1}}{1 + \Lambda (b_t^* - b^*)},
\]

(13)

where the parameter \( \Lambda \) determines the ease of substitution between domestic and foreign bonds. For some emerging market economies, a small \( \Lambda \) may be appropriate. Elastic capital flows then keep the domestic interest rate \( (R_t) \) close to the foreign interest rate \( (R_t^* \pi_{e,t+1}) \). In low-income countries, where \( \Lambda \) is comparatively large, the spread \( (R_t^* \pi_{e,t+1}/R_t - 1) \) fluctuates much more.

### 2.2 Firms

Firms hire labor and fixed, sector-specific factors in competitive markets. Cobb-Douglas production functions convert the inputs into output:\textsuperscript{12}

\[
Q_{x,t} = a_x L_{x,t}^{\theta_x},
\]

(14)

and

\[
Q_{n,t} = \int_0^1 a_n L_{n,t}^{\theta_n},
\]

(15)

with sectoral labor demands

\[
L_{x,t} = \frac{\theta_x}{w_{x,t}} Q_{x,t},
\]

(16)

\textsuperscript{12}The fixed factors are subsumed in \( a_x \) and \( a_n \). Firms in the nontradables sector operate identical linearly homogeneous production functions and face the same prices for labor and the fixed factor. This allows aggregation over firm output and use of a sector-level production function.
Following Calvo (1983), prices in the nontradable sector change when firms receive an informative signal about the state of the market. Each period a fraction \(1 - \omega\) of firms receive the requisite signal. The average price quote lasts \((1 - \omega)^{-1}\) periods and the probability of receiving a new signal is independent of signals received by other firms and the duration of the current price quote. Upon receiving a signal, the firm chooses its price so as to maximize the present discounted value of real profits. If a firm does not receive a signal, it increases its price in percentage terms by the steady-state inflation rate. This leads to the usual New Keynesian Phillips Curve.\(^{13}\) Let

\[ H_t = \frac{w_{n,t}}{\theta_n} \left( \frac{Q_{n,t}}{a_n} \right)^{(1-\theta_n)/\theta_n} \]  

(18)

denote marginal cost. Then well-known analytical derivations yield

\[ \hat{\pi}_{n,t} = \beta \hat{\pi}_{n,t+1} + \kappa (\hat{H}_t - \hat{P}_{n,t}), \]  

(19)

where a circumflex indicates a percentage deviation from the steady state and \(\kappa \equiv (1 - \beta \omega)(1 - \omega)/\omega\).

### 2.3 Monetary and Fiscal Policy

Until Section 4, the central bank targets headline inflation via the forward-looking rule

\[ R_t = R^* \left( \frac{\pi_{t+1}}{\pi^*} \right)^{\alpha}, \]  

(20)

where \(\pi^*\) is the target inflation rate and \(R^* = \pi^*/\beta\) is the steady-state interest rate. When \(\alpha > 1\), monetary policy abides by the Taylor principle.

Fiscal policy is rudimentary. Following standard practice in the IT literature, we assume that lump-sum taxes adjust in the background to satisfy the government budget constraint.\(^{14}\)

\(^{13}\)See Yun (1996).

\(^{14}\)Since the path of transfers does not affect the equilibrium path of the economy, it is not necessary to expend an equation writing down the government budget constraint.
2.4 Market-Clearing Conditions

Wages adjust to clear the two labor markets

\[ L_{i,t} = L_{si,t}, \quad (21) \]

while firms produce to demand in the nontradables sector

\[ Q_{n,t} = \int_0^1 c_j dj = \left[ \int_0^1 \left( \frac{P_{j,t}}{P_{n,t}} \right)^{-e} dj \right] C_{n,t}. \quad (22) \]

Summing the private and public sector budget constraints produces the accounting identity that the current account surplus and net foreign asset accumulation equal the difference between national spending and national income:

\[ X_t - X_{t-1} = (R_{t-1}^* - 1)b_{t-1}^* + Q_{x,t} + P_{n,t}Q_{n,t} - P_tC_t, \quad (23) \]

where \( X \equiv F + b^* + S \) is net foreign assets and \( S \) stands for the stock of foreign exchange reserves. Equation (23) applies to all exchange rate regimes. In a pure float, reserves equal zero; in a fixed rate system or a managed float, they vary to enforce the desired path of the exchange rate.

Lastly, the definition of \( P_n \) provides the following law of motion

\[ P_{n,t} = P_{n,t-1} \frac{\pi_{n,t}}{\pi_{e,t}}. \quad (24) \]

2.5 Discussion of the Model

We pause to comment on the implications of mobile vs. sector-specific labor and to supply evidence of the ongoing relevance of currency substitution in IT countries. The sensitivity of the results to the form of the Taylor rule, the Cobb-Douglas specifications of utility and production functions, and other aspects of the model will be discussed in Section 5.

2.5.1 Immobile vs. Mobile Labor

Most multi-sector IT models treat labor as sector-specific (e.g., Carlstrom et al. 2006; Airaudo and Zanna, 2012). But this preference is arbitrary. The extent to which labor reallocates across sectors depends on the time horizon, the magnitude and duration of the shocks hitting the economy, and
geography (i.e., physical proximity of the tradables and nontradables sectors). Doubtless the truth lies somewhere between the polar extremes of zero and perfect labor mobility.

In our model, labor mobility matters. When the central bank raises the real interest rate, labor demand and the wage usually fall more in the sticky-price nontradables sector than in the tradables sector. (In fact, the labor demand curve shifts to the right in the tradables sector when the currency depreciates.) If labor is intersectorally mobile, the wage differential disappears ex post. Since smaller wage cuts in the nontradables sector limit the reduction in inflation, monetary policy is less effective the greater the degree of labor mobility.

We have analyzed the alternative benchmark model that allows for full intersectoral labor mobility. Many of the results are similar to those in the current model, but some differ significantly. We report these results in subsequent sections, which are available upon request.

\subsection*{2.5.2 Is Currency Substitution Relevant in IT Countries?}

In a word, yes. Inflation has dropped to a single digit in most LDCs. Nevertheless, dollarization ratios are high in much of Latin America, Sub-Saharan Africa, East Asia (e.g., Cambodia, Vietnam, the Philippines), Eastern Europe, and the former Soviet republics. The high-dollarization group includes the majority of emerging market economies that practice IT.

Dollarization does not automatically imply currency substitution. The common finding in empirical studies that span forty years and diverse countries, however, is that the elasticity of currency substitution ($\sigma$) is very high. Estimates for Latin America, based on data from the seventies and eighties, range from 1.5 to 7.15 More recent estimates for sample periods with low inflation return similar values. The average estimate of $\sigma$ for the IT countries in Table 1 is 2.91. Clearly, currency substitution is not a relic of the high-inflation past.

We do not find any of this particularly surprising. Network externalities provide a straightforward and convincing explanation for the persistence of high dollarization ratios in countries with low inflation. It is also plausible that the growth of global economic integration, the advent of electronic banking, and learning-by-experience make it easier to engage in currency substitution today compared to earlier periods (Sharma et al., 2005). Related to this point, currency substitution does not require that foreign currency circulate alongside domestic currency as a competing medium of exchange. In many countries, cross-border deposits and other liquid foreign assets are large and easily converted.

\footnote{15 See Buffie et al. (2008) for a list of references.}
into domestic currency; as such, they qualify as near-monies readily substitutable with \( M_0/M_1 \). Substitution may occur, moreover, across liquidity services other than transactions. Informed observers often assert, for example, that domestic and foreign currency jointly satisfy a precautionary demand for liquid funds (Selcuk, 2005; Menon, 2008, Samreth, 2011). The near-money characterization of cross-border deposits and/or the precautionary motive can explain the twin stylized facts that (i) even in countries with no visible dollarization, domestic money demand decreases with the rate of currency depreciation, controlling for interest rates and inflation and (ii) demand for foreign assets increases with inflation, controlling for interest rates and the rate of currency depreciation, even when dollars are not used directly as a medium of exchange. Our model applies as long as these effects operate through some channel or another.

2.6 Calibration

Table 2 lists the parameter values used to calibrate the model in the base case together with the range of values examined in sensitivity tests. The time period is one quarter and the constants \( h_i \) in equation (10) are chosen so that \( w_x = w_n \) at the steady state. Standard values are assigned to the elasticity of labor supply and the elasticity of substitution between differentiated varieties. Other variables take values appropriate for a LDC. With respect to the parameters that govern capital flows:

- The value for the elasticity of currency substitution in the base case, two, is at the low end of empirical estimates. This conservative choice biases the results in favor of determinacy.

- Median dollarization ratios in 2012 were 30–35% in Latin America and Sub-Saharan Africa and 45% in Eastern Europe and the Central Asian republics (Imam et al., 2014). These numbers, however, do not take into account cross-border deposits or holdings of foreign currency outside

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16 See, for example, the evidence for Nigeria presented in Yinusa and Akinlo (2008) and the empirical estimates of substitutability between domestic money and dollar deposits held in the United States in Sharma et al. (2005).

17 See the first-order conditions in (11) and (12). Absent currency substitution, domestic money demand should not depend on the rate of currency depreciation and the demand for foreign currency should not depend on the inflation rate.

18 The complete list of references is very long. For a start, see Rodriguez and Turner (2003), Chaisrisawatsuk et al. (2004), Bjornland (2005), Yu (2006, 2007), Lebre de Freitas and Veiga (2006), Akhtaruzzaman (2007), and Imam et al. (2014).

19 A discount rate of .983 is slightly higher than the rate assumed by Colombia’s central bank (Hamann et al., 2006) and consistent with data on the real interest rate in emerging market economies (Elekdag et al., 2005; Garcia-Cicco et al., 2010) and in Sub-Saharan Africa (Fedelina and Kundino, 2003); the target inflation rate is equals the rate in Turkey, Brazil, and Colombia; and the consumption share of nontradables equals its value in Peru (Castillo et al., 2011).
the domestic banking system, which are known to be large (Feige, 2002; Feige et al., 2002). Our best educated guess is that the true dollarization ratio is around 50%. The accuracy of this guess is not important. As will become apparent shortly, the results for determinacy are insensitive to wide variations in the dollarization ratio, i.e., \( F/(m + F) \).

- The parameter \( \phi \) determines the degree of substitutability between domestic and foreign bonds. Write the first-order condition (13) as \( \Lambda \frac{[\hat{b}_t^d - \hat{b}_t^*]/Y]}{IRD} = \frac{1}{\phi} \), where \( Y \) is annual GDP at the initial steady state and \( IRD \equiv R_t^* \frac{\pi_{t+1}}{\hat{R}_t} - 1 \) is the interest rate differential. The term \( \phi \equiv \frac{[\hat{b}_t^d - \hat{b}_t^*]/Y]}{IRD} \) is the ratio of capital flows, measured as a percentage of annual GDP, to the \( IRD \). Since \( \Lambda \) is unobservable, we calibrate to \( \phi \). Then \( \Lambda \) can be backed out from \( \Lambda = \frac{1}{\phi \cdot Y} \). The value in the base case, unity, implies that a one percentage point increase in the domestic interest rate induces capital inflows equal to one percent of annual GDP. This intermediate degree of capital mobility is consistent with the estimate of \( \Lambda \) for emerging market economies in Chang et al. (2013), with the estimated response of capital flows to the interest rate differential in De Gregorio et al. (2000), and with the finding that the offset coefficient for capital flows is far below unity (.15 – .60) in LDCs and transition economies (Ljubaj et al., 2010).21,22

We should also say a few words about our decision to fix the length of the average sticky-price quote—a key parameter—at six months (\( \omega = .50 \)) in the base case. It is more common to assume that the average price quote lasts one year (\( \omega = .75 \)). Estimates for developed countries provide some support for this choice. But there is also growing evidence that \( \omega \) is on the order of .5 or less in LDCs (Table 3).23 Respect for this emerging stylized fact conditions many of our conclusions. Most notably, we do not judge IT to be successful unless it produces satisfactory results for \( \omega = .25 - .50 \) as well as the “developed-country values” \( \omega = .67 - .75 \).

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20 In Turkey, for example, estimates suggest that holdings of foreign currency are roughly equal to M1 (Selcuk, 2005).
21 Since a one percentage point increase in the quarterly interest rate equals a four point increase in the annualized rate, \( \phi = 1 \) implies a value of .25 for the coefficient relating capital flows to the interest rate differential. Three of the four estimates in De Gregorio et al. lie between .19 and .25 (Table 4, first panel); the fourth equals .10. The estimated value of \( \Lambda \) in Chang et al. is .22, while \( \phi = 1 \) is equivalent to \( \Lambda = .25 \) in the calibration of our model. (Chang et al. assume that portfolio adjustment costs depend on the share of domestic bonds in total bond holdings, but variation in this ratio probably mirrors debt flows induced by variations in the spread between the domestic interest rate and the external borrowing rate.)
22 The offset coefficient reflects both currency and bond substitution. If capital flows stemming from currency substitution were removed, the offset coefficient would be lower.
23 It is now standard practice to set the prior mean for \( \omega \) at .5 in Bayesian estimates of DSGE models for LDCs.
3 Doing It By the Book: IT in a Pure Float

In a pure float the transition path is governed by a core system of six first-order difference equations in $X$, $C$, $P_n$, $\pi_n$, $b^*$, and $\pi_e$ (see Appendix A). Since the system has three state variables ($X$, $b^*$, and $P_n$), saddle-point stability requires three eigenvalues inside the unit circle.

Table 4 and Figures 1a-1h show the lower and upper bounds on the Taylor coefficient ($\alpha$) in a wide variety of cases. The size and location of the determinacy region varies considerably across the different runs, but well-defined patterns are also evident. Seven results stand out:

- **Weakly active monetary policy ensures determinacy when currency substitution does not operate.** Trade in bonds is the only source of capital flows in Figure 1a. The upper bound ranges from 1.34 to 1.97 depending on the share of nontraded goods in aggregate consumption ($\gamma$) and the elasticity of capital flows ($\phi$), but the lower bound always equals unity. Thus weakly active monetary policy ($\alpha < 1.20$) guarantees determinacy. Unfortunately, this result is limited to the case where all capital flows derive from substitution between domestic and foreign bonds. The rest of the bullet points emphasize that the lower bound is large—and that determinacy may be unattainable—when moderately or highly elastic currency substitution confounds the monetary transmission mechanism.

- **The lower bound increases strongly with the elasticity of substitution between domestic and foreign currency ($\sigma$).** This holds true throughout Figure 1b and Table 4. The effect is especially damaging, however, when the average price quote is short-lived, the consumption weight of tradables is large, and the elasticity of labor supply is low. The next result elaborates on the meaning of “especially damaging.”

- **In numerous potentially relevant cases, the determinacy region is completely empty.** The entry Null means that the equilibrium is indeterminate for passive and active policy alike. This occurs with dismaying frequency in runs where $\psi < .25$, $\omega = .25 - .33$, and $\gamma < .40$.

- **Both the lower and upper bound on $\alpha$ are sensitive to the degree of demand-side openness ($1 - \gamma$), the degree of price stickiness ($\omega$), the intertemporal elasticity of substitution ($\tau$), and the Frisch elasticity of labor supply ($\psi$).** Figures 1c-1f highlight these results. In the base case, where the average price quote lasts six months and the weight of traded goods in the CPI is 60%, the

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24 Alternative values for $\psi$, $\tau$, $\omega$, etc. do not affect the lower bound.
equilibrium is unique for $\alpha = 1.84 - 2.07$. The determinacy region expands to $1.74 - 2.46$ when the average price quote persists for one year ($\omega = .75$) and to $1.53 - 2.40$ when the consumption weight of tradables decreases to 40% ($\gamma = .60$). It contracts sharply for $\tau = .50 - .80$, and disappears altogether when either the Frisch elasticity of labor supply is below .22 or the average sticky price quote lasts less than five months.

- **Determinacy requires aggressive, highly active policy when currency substitution is moderately or highly elastic.** The most popular value for the Taylor coefficient in the literature, 1.5, delivers determinacy in 50% of the runs for $\sigma = .5 - 1$. But this result is special to the case of low currency substitution. The success rate plunges to a woeful 6.7% when $\sigma = 1.5 - 3$.

- **Asset substitution may expand or contract the determinacy region depending on the elasticity of currency substitution.** Easier substitution between domestic and foreign bonds decreases both the upper and the lower bound on the Taylor coefficient. In Figure 1g, where the effects are roughly proportionate, the size of the determinacy region changes little as $\phi$ rises from 1 to 5. But when currency substitution is highly elastic, the upper bound decreases much faster than the lower bound. The gap between the two bounds in Figure 1h contracts from $1.97 - 2.35$ at $\phi = .50$ to $1.94 - 2.13$ at $\phi = 2$ to a single point at $\phi = 3.05$. High capital mobility across all financial assets (i.e., currency and bonds) may be incompatible with forward-looking IT in a flexible exchange rate system.\(^{25}\)

- **The upper and lower bounds are highly insensitive to the degree of dollarization and to the net foreign debt of the country.** The results for $F = .16$ and $b^* = 1$ in Table 4 are virtually identical to the results in the base case (where $F = .32$ and $b^* = -1$). What counts are the elasticities of currency and asset substitution, not the degree of dollarization or the size of the net foreign debt.

There is a straightforward, familiar logic to some of these results. We already know from models of IT in closed-economy set-ups—e.g., Airaudo and Zanna (2012)—and open-economy frameworks—e.g., De Fiore and Liu (2005) and Zanna (2003)—that the upper bound on the Taylor coefficient is inversely related to (i) the overall degree of price stickiness in the economy and (ii) the elasticity of aggregate demand with respect to the interest rate. Figures 1c and 1d are corollaries of the first

\(^{25}\)A second region of determinacy emerges in the “neighborhood” of perfect capital mobility. In Figure 1h with an extended horizontal axis, determinate equilibria reappear around $\phi = 8$. The determinacy region expands slowly from $\alpha = 1.69 - 1.73$ at $\phi = 8.1$ to $\alpha = 1.14 - 1.43$ at $\phi = 50$.  

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principle: since the tradables sector is a flexprice sector in small open LDCs, the upper bound is decreasing in both the length of the average sticky-price quote \((1 - \omega)^{-1}\) and the weight of traded goods in the CPI \((1 - \gamma)\). Figures 1g and 1h are corollaries of the second principle: tight money is more effective in reducing aggregate demand when capital inflows cause the currency to appreciate; hence easy substitution between domestic and foreign bonds requires a tighter upper bound to prevent overly active monetary policy and oscillatory sunspot equilibria.\(^{26}\)

The explanation for large lower bounds is more involved. To fix ideas, suppose the Taylor coefficient is 1.5 and the public suddenly expects nontradables inflation to increase. When all goes well, the increase in the real interest rate reduces aggregate consumption expenditure while attracting capital inflows that cause the nominal exchange rate to appreciate to the point where \(\pi_{e,t+1}\) plus the marginal nonpecuniary benefits of foreign currency match the increase in \(R_{t}\).\(^{27}\) This is the textbook story of exchange rate shooting (Dornbusch, 1976). Both appreciation of the nominal exchange rate and lower aggregate spending reduce demand for nontraded goods, so \(\pi_{n}\) moves in the opposite direction from private expectations. The interest rate and exchange rate channels work together to prevent sunspot equilibria.

But the textbook outcome is not guaranteed. The complicating factor is that expectations of higher inflation may trigger capital outflows and depreciation of the real exchange rate even though the central bank raises the real interest rate. When \(\sigma > \tau\), domestic and foreign currency are Edgeworth substitutes. Decreases in the real money supply then increase the marginal utility of foreign currency, creating an incentive for capital flight. This effect is quantitatively large for \(\sigma = 1 - 3\). If the Taylor coefficient is too small (i.e., too close to unity), it dominates the incentive to substitute away from foreign currency and foreign bonds toward domestic bonds. In such cases, incipient capital outflows force a spot depreciation of the real exchange rate at time \(t\). It is a small step from this to the existence of sunspot equilibria. Active policy does not ensure determinacy unless the increase in the real interest rate is large enough that the deflationary pull of lower aggregate demand outweighs the inflationary pressure associated with real depreciation.\(^{28}\)

\(^{26}\)Oscillatory sunspot equilibria, in which inflation cycles above and below the target level, emerge when monetary policy is too effective: above the upper bound, expectations that higher inflation will trigger an overly large increase in the interest rate that leads to lower inflation, which triggers an excessively large decrease in the interest rate that leads to another period of high inflation, etc., prove self-fulfilling. Hence the upper bound is smaller when easy substitution between domestic and foreign bonds increases the elasticity of aggregate demand with respect to the interest rate.

\(^{27}\)In the case of domestic vs. foreign bonds/loans, marginal portfolio adjustment costs enter as a wedge between \(R_{t}\) and \(R_{t}^{*}\pi_{e,t+1}\).

\(^{28}\)More precisely, the deflationary pull of lower aggregate demand has to offset to a sufficient degree the inflationary pressure associated with real depreciation. As shown in Section 6, the equilibrium path may be unique when nontradables
Intuitive explanations for the other results now follow naturally. Higher values for \( \sigma \) increase the lower bound on \( \alpha \) by increasing the elasticity of capital flows with respect to inflation. Demand-side openness raises the lower bound by increasing the direct impact of real depreciation on current and future inflation; less sticky prices do the same indirectly as real depreciation induces larger increases in nontradables inflation when price quotes are short-lived.

One puzzle remains, namely: why is the impact of asset substitution (i.e., bond substitution) so sensitive to the elasticity of currency substitution? In Figure 1g, the lower bound decreases monotonically with \( \phi \). This is the expected outcome. Since substitution from foreign toward domestic bonds offsets substitution from domestic toward foreign currency, higher values of \( \phi \) reduce the likelihood that expectations of higher inflation will precipitate net capital outflows and a sudden spike in the nominal exchange rate. It follows immediately that smaller increases in the interest rate suffice to prevent sunspot equilibria. This logic seems to break down, however, at high values for the elasticity of currency substitution. In Figure 1h, the relationship between the lower bound and \( \phi \) is weakly non-monotonic (hard to see because of the scale of the figure).

What changes going from Figure 1g to Figure 1h is that an increase in the real interest rate no longer increases the return on domestic bonds relative to foreign bonds. To illustrate, suppose that \( \hat{\pi}_n = .01 \) and \( \hat{\pi}_e = .018 \), ex ante, when \( \gamma = .50 \) and \( \sigma = 2 \). For \( \alpha = 1.5 \), the real interest rate then increases seven-tenths of a percentage point, while the nominal interest rate rises three-tenths of a point more than the rate of currency depreciation. The monetary transmission mechanism works as advertised; most notably, the positive spread between the return on domestic vs. foreign bonds attracts capital inflows that reduce the lower bound by limiting depreciation of the currency. But when \( \sigma = 3 \) (Figure 1h), stronger substitution from domestic toward foreign currency causes the nominal exchange rate to rise more, say 5%. In this case, the real interest rate increases more (1.5 percentage points), but the nominal interest rate rises less than the return on foreign bonds (\( \tilde{R}_t = .045 \) vs. \( \hat{\pi}_e = .05 \)). Capital outflows stemming from bond substitution now reinforce outflows stemming from currency substitution. As \( \phi \) increases, domestic and foreign bonds become closer substitutes and the tug-of-war between the inflationary pull of greater currency depreciation and deflationary pull of a larger increase in the real interest rate tips, at some point, in favor of a net inflationary effect. This is what happens in Figure 1h: after decreasing from 1.98 at \( \phi = .1 \) to 1.94 at \( \phi = 2.3 \), the lower bound rises to 2.0 at \( \phi = 3.05 \). inflation rises and the real exchange rate depreciates at time t because nontradables inflation does not rise enough to validate original expectations.
3.1 Policy Issues

Four serious challenges confront LDCs that wish to practice IT in a floating exchange rate system. First, it is difficult to locate the region of determinate equilibria. Policy makers in LDCs operate with fuzzy, imprecise information about the likely values of \( \tau, \omega, \sigma \) and other key parameters. The intertemporal elasticity of substitution probably lies between .2 and .75, but estimates of \( \sigma \) range from 1 to 6, and the “emerging stylized fact” that \( \omega \leq .50 \) needs to be confirmed for additional countries. Given the paucity of information, the central bank cannot be confident that its choice of \( \alpha \) is compatible with determinacy. Nor can it circumvent the information problem by choosing a value for the Taylor coefficient that exceeds the largest lower bound in Table 4. This strategy is quite risky because the largest lower bound exceeds many of the upper bounds in the potentially relevant parameter space. It would be unfortunate, for example, to set \( \alpha = 2.10 \) when the country’s true coordinates are \( \omega = .50, \tau = .5, \) and \( \sigma = 1 \) (where unique equilibria obtain only for \( \alpha = 1.30 - 1.58 \)).

The second issue also relates to the problem of parameter uncertainty. Suppose, optimistically, the central bank knows that a Taylor coefficient of two will ensure determinacy, given the range of parameter uncertainty. The determinacy problem is solved, but a new dilemma arises: since aggressive interest rate rules amplify policy errors, IT itself may become a source of macroeconomic instability (Orphanides, 2001; Benhabib and Eusepi, 2005).

The third issue concerns the socio-political acceptability of high values of the Taylor coefficient. Internal and external shocks are larger and more frequent in LDCs than in developed countries. In practice, therefore, highly activist policy may prove infeasible. A Taylor coefficient of 1.3 is probably acceptable when dealing with an 8–12 percentage point shock to annual inflation (roughly what LDCs faced in the world food and oil price increases of 2007 - 2009). But if the coefficient has to exceed two, IT is in trouble: policy makers cannot be expected to combat inflation shocks with brutal, intolerably large increases in the real interest rate (Fraga et al., 2003).

Finally, even if we assume away the daunting informational requirements for finding the determinacy region (when it exists) and ignore the technical and socio-political constraints on active monetary policy, IT in a flexible exchange rate system suffers from a fourth drawback: sometimes benign neglect of the exchange rate helps dampen inflationary shocks; all too often, however, it allows the initial shock to escalate through depreciation of the real exchange rate into a much larger shock, resulting in a wide miss of the inflation target. This problem will be analyzed in more detail when we compare impulse responses for alternative types of IT in Section 6.
4 Coordinating Monetary and Exchange Rate Policy

It is transparent from the preceding analysis that the key to reconciling active policy with determinacy is to find some way to limit nominal and/or real depreciation of the currency. Accordingly, in this section we investigate the efficacy of alternative methods of targeting the exchange rate.

4.1 Targeting the Exchange Rate the Wrong Way

There is a right way and a wrong way to target the exchange rate. The wrong way is to encumber the Taylor rule with a target for the nominal \((e)\) or real exchange rate \((1/P_n)\). The usual criticism of such rules is that they undermine credibility of the inflation target. To which we can add the objection that they are largely ineffective. The rule

\[
\hat{R}_t = \alpha \hat{\pi}_{t+1} - v \hat{P}_{n,t}
\]

leads to the results on display in Figures 2a and 2b. Positive values for \(v\) raise the upper bound and reduce the lower bound. But the increase in the size of the determinacy region is small even when \(v\) is large, and the problem cases highlighted earlier in Figure 1 are still there—indeterminacy is virtually unavoidable when \(\omega\) and \(\psi\) are low.\(^{29}\) In any event, there is a much better way to target the exchange rate—a way that fully solves the indeterminacy problem, that proves more helpful in neutralizing inflation shocks, and that does not risk creating confusion about which target takes priority in the central bank’s interest rate rule.

4.2 Targeting the Exchange Rate the Right Way

The right way to target the exchange rate is through sterilized sales/purchases of foreign exchange. We examine two variants of this two-instruments-for-two-targets strategy.

4.2.1 Pegging the Real Exchange Rate

In a pure float the Taylor coefficient often has to exceed a large lower bound to ensure a unique rational expectations equilibrium. When the coefficient is above unity but below the lower bound, IT runs into trouble because the increase in the real interest rate fails to prevent capital flight and depreciation of the real exchange rate. Since real depreciation increases real marginal costs in the
sticky-price nontradables sector, expectations of higher inflation prove to be self-validating. There is no macroeconomic anchor. The feedback effects running from expectations of higher inflation to capital flight to depreciation of the real exchange rate back to inflation destabilize the economy.

Using foreign exchange sales to peg the real exchange rate solves the instability problem by creating a firewall that insulates real marginal cost in the nontradables sector from capital flows and depreciation of the currency. Since $\hat{R}_t > \hat{\pi}_{t+1} = \hat{\pi}_{n,t+1} = \hat{\pi}_{e,t+1}$, the interest rate always increases more than the rate of currency depreciation. This does not eliminate the possibility of capital outflows because lower holdings of domestic currency increase the non-pecuniary yield of foreign currency when the two currencies are Edgeworth substitutes (i.e., $\sigma > \tau$). If the non-pecuniary yield rises more than $R_t/\pi_{e,t+1}$, the demand for foreign currency increases, putting pressure on the exchange rate. But then the firewall is activated: sterilized sales of foreign exchange reserves buttress the real exchange rate, severing the link that connects depreciation of the currency to marginal costs and nontradables inflation.

The upshot of all this is that the real exchange rate peg allows IT to function in much the same way as in the closed economy. When $P_n$ and $\pi_e$ drop out of the dynamic system, the Euler equation for aggregate consumption and the New Keynesian Phillips Curve decouple from the difference equations for $X$ and $b^*$. In Appendix B we derive

$$\begin{bmatrix} \hat{C}_{t+1} \\ \hat{\pi}_{n,t+1} \end{bmatrix} = \begin{bmatrix} 1 - \tau(\alpha - 1)f_4 & \tau(\alpha - 1)/\beta \\ -f_4 & 1/\beta \end{bmatrix} \begin{bmatrix} \hat{C}_t \\ \hat{\pi}_{n,t} \end{bmatrix},$$

(26)

where

$$f_4 \equiv \kappa \frac{\tau[1 + \psi(1 - \theta_n)] + \theta_n \psi}{\beta \theta_n \psi \tau} > 0.$$  

Stability of the full system requires that the subsystem in (26) possess two eigenvalues outside the unit circle.\(^{30}\) This yields (see Appendix B)

**Proposition 1** Suppose the central bank pegs the real exchange rate. The necessary and sufficient condition for a unique equilibrium is then

$$1 < \alpha < 1 + \frac{2(1 + \beta)\theta_n \psi}{\kappa \{\tau[1 + \psi(1 - \theta_n)] + \theta_n \psi\}}.$$  

\(^{30}\)It is evident from inspection of (23) that the full system contains a unit root. This does not cause any problems. The path for net foreign assets does not “drift.” Stability of the $C$-$\pi_n$ subsystem ensures that $X$ converges to a stationary level. (See the impulse responses in Section 6.)
Remark 1 The upper bound is independent of the consumption share of traded goods \((1 - \gamma)\), the degree of dollarization \((F)\), the net foreign debt \((b^*)\), factor cost shares in tradables production \((\theta_x)\), and the elasticities of currency substitution and bond substitution \((\sigma \text{ and } \phi)\).

The upper bound in Proposition 1 is exactly equal to the upper bound in the standard one-sector, sticky-price, closed-economy model. The correct comparison, however, is with the closed-economy model that incorporates both sticky- and flex-price sectors (e.g. the model in Airaudo and Zanna, 2012). This solution can be retrieved by by setting \(F\), \(b^*\), and \(\phi\) equal to zero. The salient point that emerges from the comparison is that IT works better in the open economy if the central bank rejects the conventional wisdom and pegs the real exchange rate (Table 5). The upper bound ranges from 1.22 to 2.20 in the closed economy. In the open economy, by contrast, the upper bound is comfortably large, except, perhaps, in the case where prices adjust quickly in the nontradables sector \((\omega = .25 - .33)\).

Unfortunately, theory disputes the notion that policy makers can rescue the Taylor principle by enforcing a hard peg of the real exchange rate. A purist would object that it is improper to mix an instrument rule (the interest rate rule) with a targeting rule (peg the real exchange rate). Put differently, if the interest rate adjusts only to deviations of inflation from its target value, is it not dubious to assume that foreign exchange sales/purchases adjust instantaneously to prevent any changes in the real exchange rate? In practice, real exchange rate targeting also takes the form of an instrument rule. Highly efficient targeting will deliver results close to the nice results in Table 5. But how much efficiency does this presume? How closely does the instrument rule have to approximate the ideal—the target rule?

This objection does not apply to the next rule we analyze: pegging the rate of currency depreciation. There are no practical or conceptual problems involving in buying or selling foreign exchange as needed to enforce a pre-announced path for the nominal exchange rate. LDCs have done this for decades.

4.2.2 Pegging the Rate of Crawl \((\pi_e)\)

Benign neglect of the exchange rate allows depreciation of the currency, capital flight, increases in the real exchange rate, and nontradables inflation to form a self-validating loop. Pegging the real exchange rate short-circuits the unstable loop, reconciling determinacy with reasonable values for the Taylor coefficient and socially acceptable increases in the real interest rate. So does pegging the rate of crawl at the target inflation rate. With \(\pi_e\) constant, the real exchange rate always decreases \((P_n \uparrow)\) following a positive inflation shock. Since the exchange rate channel reinforces the interest
rate channel, the equilibrium is saddle point stable under active policy. Surprisingly, passive policy also works. Although aggregate consumption rises when passive policy allows expectations of higher inflation to reduce the real interest rate, the resulting inflationary pressures are effectively neutralized by appreciation of the real exchange rate. This precludes sunspot equilibria. In Appendix B we prove

**Proposition 2** When the central bank pegs the rate of crawl of the nominal exchange rate at the target inflation rate,

$$\alpha < 1 + \frac{(1 - \gamma)(1 + \psi)}{\gamma\{\tau[1 + \psi(1 - \theta_n)] + \theta_n\psi\}}$$  \hspace{1cm} (27)

is necessary and sufficient for a unique equilibrium.

**Remark 2** The upper bound is independent of the degree of price stickiness ($\omega$), the degree of dollarization ($F$), the net foreign debt ($b^*$), and the elasticities of currency substitution and bond substitution ($\sigma$ and $\phi$).

Central banks usually prefer to combat inflation shocks with an increase in the real interest rate. Pegging the rate of crawl is competitive with pegging the real exchange rate therefore only if the upper bound on $\alpha$ in Proposition 2 is sufficiently large. This requirement is satisfied. Outside of a few runs where $\gamma = .7 - .8$, the upper bounds in Table 6 are higher than any realistic value for $\alpha$ under aggressive, activist monetary policy.\(^{31}\)

4.2.3 Managed Floats

Pegging the nominal or real exchange rate solves the indeterminacy problem through overkill. Consider, for example, the base case. For a hard peg of the nominal exchange rate, the equilibrium path is determinate as long as the Taylor coefficient is less than 4.3. This is inefficient. Since the upper bound is much larger than necessary, intermediate exchange rate regimes exist that offer a superior tradeoff—more exchange rate flexibility together with a determinacy region that, while smaller, does not contrain the exercise of active monetary policy. What is not clear is where these intermediate regimes lie on the continuum of exchange rate flexibility. Does the best tradeoff of determinacy space for exchange rate flexibility generally occur closer to the fixed or the flexible end of the continuum?

\(^{31}\)It should also be noted here that the upper bounds are much larger when labor is intersectorally mobile (see the online appendix).
Turning to the business at hand, we pair the Taylor rule with reserve sales that lean against the wind when the rate of currency depreciation deviates from its target level \( \pi^* \), viz:

\[
\frac{S_t - S_{t-1}}{S_{t-1}} = -\chi (\pi_{\epsilon,t} - \pi^*), \quad \text{with} \quad \chi > 0.
\] (28)

Going from a number for \( \chi \) to the right adjective for the managed float requires information on the ratio of reserves sales in the float to sales required to enforce a hard peg. For the shocks analyzed in Section 6, the ratios associated with \( \chi = .25, 1, \) and 5 are \(.13 -.24, .37 -.54, \) and \(.70 - .84 \). Assuming these ratios are representative, we call the managed float lite when \( \chi \leq .50 \), medium when \( \chi = 1 - 1.50 \), and heavy when \( \chi \geq 5 \).

Figures 3a-3d contain a mix of positive and negative results. The good news is that small values of \( \chi \), on the order of \(.25 \), reduce the lower bound to unity in every case covered by Table 4. Consequently, determinate equilibria always exist for some \( \alpha > 1 \). Moreover, the determinacy region is fairly large in many of the worst-case scenarios where indeterminacy is unavoidable under a pure float. Figure 3b is a case in point. For \( \omega = .33 \), determinate equilibria cease to exist in a flexible exchange rate system when \( \sigma \) exceeds 1.16. But if the central bank operates a light managed float with \( \chi = .25 \), determinacy obtains for \( \alpha = 1.92 \) even when \( \sigma \) equals three.

The bad news is that the upper bound is not very responsive to \( \chi \). This creates problems when the elasticity of currency substitution is relatively low. In Figure 3c the upper bound is only \( 1.31 - 1.38 \) for \( \chi = 5 - 10 \). The corresponding range in Figure 3d is \( 1.50 - 1.60 \). By contrast, the upper bounds for a hard peg of the nominal exchange rate are 2.9 and 4.7 (see Table 6). Leaning against the wind always increases the size of the determinacy region, but the impact varies considerably across the relevant parameter space. A light lean is often highly effective. There are also cases, however, in which a heavily managed float proves a poor substitute for a hard peg.

5 Sensitivity Analysis

For the most part, our results are robust to structural changes in the model. We concentrate therefore on how the form of the Taylor rule affects the prospects for determinacy.\(^{32}\)

\(^{32}\)Further sensitivity analysis results are available upon request.
5.1 Alternative Structural Specifications

To simplify the model, we abstracted from imported inputs and assumed sector-specific labor, Cobb-Douglas utility and production functions, purely forward-looking Calvo-pricing, and full asset market participation. None of these assumptions is critical. The determinacy results change little in variants of the model that allow for limited asset market participation, inertial inflation, intersectorally mobile labor, and imported intermediate inputs. They are also insensitive to the elasticity of substitution between inputs in the sectoral production functions and to the elasticity of substitution between traded and nontraded consumer goods when CES functions replace Cobb-Douglas functions in the specifications of preferences and technology. The robustness of the results extends to combinations of structural changes, not just to changes introduced singly.\footnote{Typically the alternative structural model shows slightly higher upper and lower bounds than the benchmark model.}

There is an important exception to the rule. Recall that the equilibrium is always indeterminate when highly elastic currency substitution ($\sigma = 3$) combines with easy substitution between domestic and foreign bonds—$\phi > 3.05$ in Figure 1h. This result takes a much more threatening form when labor is intersectorally mobile labor: determinate equilibria disappear once $\phi$ exceeds 1.57.

5.2 Targeting Core Inflation

Switching the target from CPI to core inflation goes some distance toward solving the indeterminacy problem (Table 7), provided the intertemporal elasticity of substitution is not too low. The upper bound on the Taylor coefficient is no longer a worry in the base case or in runs where $\omega$ and $\tau$ exceed .5. Note also that the lower bound drops to unity for $\tau = .75 - 1$ and that the equilibrium is always unique when the average sticky-price quote lasts one year ($\omega = .75$). Targeting core inflation solves the indeterminacy problem for parameter values macroeconomists deem appropriate for developed countries ($\tau = .50 - 1$, $\omega \geq .67$).

These results are better but far from satisfactory. The lower bounds cluster between 1.5 and 2.8 when $\omega \leq .50$ and $\sigma \geq 1$, so indeterminacy is still a threat and the socio-political acceptability of highly active monetary policy is still an issue in some parts of the relevant parameter space. Moreover, the caveat that the intertemporal elasticity of substitution not be too low is hugely important. Numerous empirical estimates suggest that $\tau$ might be on the order of .2-.3 in LDCs. But for $\tau = .2$, the lower bound soars from 1.65 at $\sigma = .50$ to 4 – 7.1 at $\sigma = 1 – 1.5$. The extreme sensitivity of the results to $\tau$ opens up a big divide between developed and less developed countries. In LDCs, targeting core
inflation is fraught with risk: it may help, but it may also render the indeterminacy problem insoluble.

5.3 Flexible Inflation Targeting

Under flexible IT, the central bank targets both real output and the inflation rate, viz.:

\[ \hat{R}_t = \alpha \hat{\pi}_{t+1} + \xi \hat{Y}_t, \quad \xi > 0, \quad (29) \]

where \( Y_t \equiv Q_{x,t} + P_{n,o}Q_{n,t} \). Empirical estimates of interest rate rules generally place \( \xi \) between .1 and .5 in LDCs.

In the closed economy the prospects for determinacy improve significantly when the central bank opts for flexible IT over strict IT (e.g., Galí et al., 2004 and Huang and Meng, 2007). This result does not survive—even qualitatively—in our open economy model. The determinacy region expands a small amount in some cases, with the lower bound decreasing slightly more than the upper bound (Figure 4b). In other cases, however, the two bounds move toward each other at every point: in Figure 4c, the determinacy region for flexible IT lies completely inside the determinacy region for strict IT.

5.4 Contemporaneous Inflation Targeting

Carlstrom et al. (2006) proved that contemporaneous IT guarantees determinacy in the closed economy and conjectured that the result would generalize to the open economy. We can confirm that it does. Exchange rate policy and the choice of price index do not matter if the central bank targets current inflation: even in a flexible exchange rate system, Core- and CPI-IT are compatible with a unique equilibrium under active policy (\( \alpha > 1 \)).

This is an exceptionally strong, clean result. Its realism is open to dispute, however. Contemporaneous interest rate rules presume that policy makers have access to precise information on the current values of inflation and other variables. Bullard and Mitra (2002) worry that the requisite information is unavailable in many developed countries, let alone LDCs. McCallum (1999) asserts more strongly that contemporaneous rules are “non-operational.” Consistent with Bullard-Mitra-McCallum view, most central banks practice forward-looking IT; few, if any, target contemporaneous inflation.

We do not need to take sides in this debate. Although contemporaneous IT guarantees determinacy in a flexible exchange rate system, it does not challenge our thesis that tight management of the exchange rate is essential for effective IT in LDCs. As shown in the next section, all flexible exchange
rate regimes suffer from low targeting efficiency. Absent intervention to limit exchange rate volatility, aggressive manipulation of the interest rate fails to prevent small shocks from driving inflation far above its target level. The central bank appears inept when, in truth, it is simply under-equipped.

6 Comparing Impulse Responses

Intervention of the central bank in the foreign market was a very useful complement to interest rate policy. It was perceived that the increases of the interest rate alone were not sufficient to correct the depreciation of the [Colombian] peso or to curb inflation expectations. In other words, the exclusive use of interest rates would have required much larger movements than those observed, probably introducing inefficient output volatility. (Vargas, 2005. Our emphasis.)

The results for determinacy favor contemporaneous IT with a flexible exchange rate, forward-looking IT regimes that peg either the nominal or the real exchange rate, and a managed float that leans against the wind. Macroeconomists who believe strongly that $\omega > .67$ will also endorse core IT with a flexible exchange rate. The determinacy results eliminate some contenders; on their own, however, they do not settle the issue of which IT regime works best.

To build an open-and-shut case for tight management of the exchange rate, we need to demonstrate in scenarios where concerns about determinacy are not decisive that exchange-rate-anchored IT (i) does a better job than flexible exchange rate IT of minimizing macroeconomic volatility and (ii) does not entail unrealistically large reserve sales. The ensuing analysis focuses therefore on a subset of cases where $\omega = .75$ and $\sigma = 3$. Since the dynamics for inflation are sensitive to the degree of labor mobility (see the discussion in Section 2.5.1), we present one run from the alternative benchmark model that postulates perfect labor mobility. This is an effort to satisfy one of Bernanke and Woodford’s (2005) more demanding criterion: a reliable IT regime should deliver satisfactory results not only for alternative values of deep parameters, but also for alternative model specifications.

We analyze the impact of a 1.25% AR1 cost-push shock in the nontradables sector (equivalent to a .5% inflation shock), with persistence parameter $\rho = .90$, in three scenarios where $\tau = .20$, $\omega = .75$, $\sigma = 3$, and $\alpha = 1.5$. Scenario A assumes $\psi_i = .5$. For Scenario B, we set $\psi_x = .01$ and $\psi_n = 3$ to mimic

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34 The quote is lifted from Chang (2007).
35 The value for $\sigma$ equals the average of the estimates in Table 1, while $\omega = .75$ ensures determinacy in the runs for core IT.
an economy with limited wage flexibility in the sticky-price nontradables sector and inelastic supply in a natural-resource-based export sector. In Scenario C, labor is intersectorally mobile and \( \psi = 0.20 \). Although the interest rate rule targets only the inflation rate (to enhance the central bank’s reputation as an inflation fighter), we adopt a broad definition of macroeconomic volatility. The impulse responses track the paths of inflation, currency depreciation, sectoral output \((Q_x, Q_n)\), the real exchange rate \((RER = 1/P_n)\), and the real wage. In runs where the central bank operates a managed float (MF-IT) or pegs either the real exchange rate (RERA-IT) or the rate of crawl of the nominal exchange rate (NERA-IT), we also report reserve sales/purchases measured as a percentage of initial annual GDP.

Jumping straight to the punch line, NERA-IT is the clear winner.\(^{36}\) Figure 5 shows the outcome against the runner-up, contemporaneous IT with a flexible exchange rate.\(^{37}\) In all three scenarios—and many others not presented here—NERA-IT dampens the inflation shock. Nontradables output and the real exchange rate fluctuate more than in the competing flexible exchange rate regimes. Counterbalancing this, the real wage decreases much less. The peg does require permanent reserve sales, but the losses are tolerable in Scenarios A and C and manageable, if large, in Scenario B.\(^{38,39}\)

NERA-IT and the flexible exchange rate IT regimes fare poorly. Under contemporaneous IT, quarterly inflation increases \(0.8 - 3.5\) percentage points despite exceptional downward flexibility of the real wage. RERA-IT accepts slightly higher inflation \((\pi \text{ increases } 1.2 - 4 \text{ points})\) and slightly larger real wage cuts as the price of stabilizing the real exchange rate. But core IT is not even remotely competitive with contemporaneous IT, let alone NERA-IT: propelled by a large spot depreciation of the currency at \(t = 0\), core and CPI inflation jump from \(1\%\) to \(2.8 - 5.5\%\) and \(3.2 - 6.4\%\), respectively. Ironically, NERA-IT and contemporaneous CPI-IT target core inflation more effectively than core IT.

These results are disturbing, for they suggest that the solutions presented earlier understate the severity of the indeterminacy problem. The model implicitly assumes the annual inflation rate can jump from \(4\%\) to \(15 - 25\%\) without undermining credibility of IT. In reality, credibility might disappear very quickly, especially in countries struggling to overcome a recent history of lax monetary policy

\(^{36}\) It is beyond the scope of the current paper to carry out a formal welfare analysis. There is little doubt, however, that most policy makers would strongly prefer the solution offered by NERA-IT to the solutions offered by RERA-IT and flexible exchange rate IT.

\(^{37}\) See the online appendix for head-to-head comparisons of NERA-IT with RERA-IT and Core-IT.

\(^{38}\) Reserves are initially \(16.7\%\) of annual GDP, the figure in Berganza and Broto’s (2012) sample of emerging markets engaged in IT. The long-run decrease in reserves is \(2.3-3\%\) of GDP in Scenarios A and C and \(5.2\%\) of GDP in Scenario B.

\(^{39}\) This is consistent with the finding in Berganza and Broto (2012) that reserve sales are highly effective in reducing exchange rate volatility in emerging markets that practice IT. See the discussion in Chang (2007) of intervention episodes in Chile, Peru, and Colombia for additional supporting evidence. In some cases, however, successful intervention did require large reserve sales (e.g., \(20\%\) of the total stock in Brazil 2001).
and high inflation. And if this happens, macroeconomic chaos is sure to follow. Loss of the IT
anchor would trigger greater capital flight, more rapid depreciation of the currency, and escalating
and probably unstable increases in inflation.

Safeguarding against this threat—against inflation surges that destroy credibility—requires a stronger
response of the interest rate to inflation in policy regimes that allow the exchange rate to float. The
pessimistic results in Sections 3-5 are thus overly optimistic: in all likelihood, the true values for the
lower bound on the Taylor coefficient are much larger and the number of Null entries much greater than
stated. The next section shows that light- and medium-weight managed floats join contemporaneous
and core IT on the list of major casualties.

6.1 NERA-IT vs. Managed Float

Although the determinacy region for a lightly managed float is smaller than the determinacy
region for NERA-IT, it is large enough in the majority of the parameter space to accommodate highly
active monetary policy. If Managed Float Lite (MFL) also matched NERA-IT’s success in neutralizing
inflation shocks, then our model would agree in spirit with conventional wisdom. Allowing the exchange
rate to float freely is a bad idea, and a hard peg would be advisable in some cases. (Recall the tight
upper bounds in Figures 4c and 4d). But most LDCs could practice effective IT in conjunction with
a managed float close to the flexible exchange rate end of the continuum. MFL-IT would dominate
NERA-IT on the grounds that, ceteris paribus, more exchange rate flexibility is better.

Figure 6 collects the results for the head-to-head comparison of NERA-IT and a managed float
where \( \chi \) equals unity (first row) or five (second row). In the runs for \( \chi = 1 \), cumulative reserve sales
amount to 37 – 54% of reserve sales under NERA-IT; for \( \chi = 5 \), sales are 70 – 84% as large.

Sadly, the case for MFL-IT falls apart in Figure 6. When \( \chi = 1 \), the quarterly inflation rate jumps
from 1% to 3.3 – 3.9%. And the right label for \( \chi = 1 \) is MFM (Managed Float Medium), not MFL. If
we drop \( \chi \) to .25 as called for by MFL, the inflation rate soars to 4.6 – 5.8%, undermining credibility
and casting doubt on whether determinate, stable equilibria really exist.

The results for \( \chi = 5 \) are considerably better, and perhaps acceptable. But \( \chi = 5 \) is a heavily
managed float: on a line where zero corresponds to a pure float and unity to a fixed exchange rate,
\( \chi = 5 \) maps to a point close to .75. The bottom line is clear: LDCs can tolerate some, but not much
flexibility; successful, effective IT requires tight management of the exchange rate, if not a hard peg.
6.2 Fear of Appreciation

Inflation surges fueled by capital outflows and rapid depreciation of the currency occur in many scenarios. They are not, however, the only challenge facing flexible exchange rate IT. The polar opposite problem of unexpected, large capital inflows and rapid appreciation is also quite common. In Figure 7, excessive appreciation of the nominal exchange rate causes large, disruptive fluctuations in the real exchange rate and sectoral output. NERA-IT eliminates much of the unwanted volatility. The real exchange rate appreciates only 2% on impact, and the reductions in nontradables output and the real wage are a small fraction of the reductions in the floating rate regimes.40

6.3 External Shocks

Besides domestic shocks, we have examined oil price and world market interest rate shocks. (It is straightforward to add imported inputs to the model in Section 2.) Since these shocks hit the exchange rate directly, the comparison between NERA-IT and flexible exchange rate IT is even more favorable to NERA-IT than the comparison for domestic shocks. The comparison is especially striking for an interest rate shock. Under flexible exchange rate core IT, a 50 basis point rise in the external borrowing rate increases inflation by $1 - 1.8$ percentage points for $\gamma = .4 - .6$ in Scenarios A-C.41 Flexible exchange rate contemporaneous IT also struggles, allowing inflation to jump $6 - 2.7$ points. But for NERA-IT the number is exactly zero ($\pi_t = \pi^*, \forall t$). The interest rate shock affects output, the real exchange rate, and nontradables inflation solely through its impact on the rate of depreciation.42 NERA-IT neutralizes the shock simply by fixing the rate of depreciation at the target inflation rate.

6.4 Reconciling Theory and Practice

The results for targeting efficiency are decisive. They provide a complete explanation of why LDCs engaged in IT hold large stocks of foreign exchange reserves and intervene frequently in the foreign exchange market.  

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40 The increase in tradables output in the flexible exchange rate regime is an artifact of the large increase in labor supply and the large decrease in the real wage associated with the large decrease in consumption. Tradables sector output would contract sharply if the nominal wage were rigid downward.

41 The results pertain to an AR1 shock with persistence parameter $p = .90$ in an economy where private foreign debt is initially 50% of GDP.

42 This result reflects the fact that reserves do not earn interest. In general equilibrium, the higher cost of debt service is financed entirely by a reduction in foreign exchange reserves. (Reserves decrease permanently by 2.46% of annual GDP.) Since reserves do not pay interest, there is no income effect associated with adjustment to the shock. Hence $b^*$ and $S$ are the only variables that change on the transition path. (Formally, $P_e$, $\pi_n$, and $C$ form a 3x3 subsystem independent of $R^*$. ) Adjustment is not costless, however. National wealth declines and the smaller stock of reserves leaves the economy more vulnerable to future external shocks.
exchange market. Intervention is essential to counteract both domestic and external shocks, to prevent excessive depreciation in some cases and excessive appreciation in others. This “fear of everything”—fear of floating in any direction—is now ubiquitous among LDC central banks. The dataset in Vavra (2014) classifies countries as IT Pure (IT with floating exchange rate) or IT with FXI (IT with frequent foreign exchange intervention). In 2006, all eight developed countries and eight of seventeen EMEs (emerging market economies) were classified as IT Pure. Since then, EMEs have deserted IT Pure en masse. At the end of 2013, six of eight developed countries were still classified as IT Pure, but twenty-four of twenty-five EMEs practiced IT with FXI.

7 Concluding Remarks

In this paper we have analyzed the role of exchange rate policy in LDCs that practice forward-looking IT. Our results dispute the dominant view in the literature. Conventional wisdom counsels central banks to let the exchange rate float. This is dangerous advice, for benign neglect of the exchange rate does not invariably translate into a benign path for the exchange rate. In LDCs, where foreign currency still competes with domestic currency as a supplier of liquidity services, expectations of higher inflation can easily prove self-fulfilling once capital flight triggers rapid depreciation of the currency. Consequently, sunspot equilibria proliferate in much of the potentially relevant parameter space. The indeterminacy problem is especially acute in regimes that target future headline inflation. In CPI-IT, the set of Taylor coefficients compatible with a unique equilibrium is circumscribed by a lower bound substantially larger than unity and an upper bound much smaller than in the closed economy. The gap separating the two bounds is small in many cases and nonexistent in some. Moreover, in the region of unique equilibria, the location of the upper and lower bounds is case sensitive. Because of this and uncertainty surrounding the values of numerous key parameters, it is easy to for policy makers to choose wrong—to choose a sensible value for the Taylor coefficient that misses the determinacy region.

Robust compatibility with the Taylor principle is one of two criteria we employed to rank IT regimes. The other criterion assumes determinacy and asks whether the policy regime contains inflationary shocks without causing excessive instability in the other important macroeconomic variables. In a flexible exchange rate system, CPI- and Core-IT fail this second test, badly. Impulse responses warn that a small inflation shock may combust, via capital flight and a precipitate spike in the exchange rate, into an increase in the inflation rate seven times larger. In the model, this does not create any additional, troublesome complications. The computer calmly steers the economy onto the path
that leads back to the original equilibrium. In practice, a sudden huge surge in inflation is likely to destroy confidence in the competence of the central bank and/or its commitment to the inflation target. Policy makers are overly familiar with this problem. We suspect it is the main reason the conventional wisdom has such a small following among LDCs.

Exchange-rate-anchored IT produces much better results. Pegging either the real exchange rate or the rate of nominal depreciation affords policy makers maximum flexibility in the conduct of active monetary policy: the lower bound on the Taylor coefficient drops to unity, while the upper bound jumps to a level that is, at the very least, comfortably large. The two rules do not score equally well, however, on our other criterion. RERA-IT still allows capital flight and endogenous depreciation of the currency to strongly magnify inflation shocks. Pegging the rate of crawl or leaning heavily against the wind in a managed float (MFH-IT) solves this problem: ripple effects disappear and inflation shocks are successfully contained. Exchange-rate-anchored IT works. NERA-IT always passes both tests. MFH-IT also scores well; it ranks slightly below NERA-IT only because of a few marginal grades on the determinacy test.

We close by returning to an observation made at the start of the paper. The stylized fact that virtually all LDCs now support IT with tight management of the exchange rate is a serious problem for existing theory. The objection of the existing literature to exchange-rate-anchored IT recalls the joke that “It works in practice, but does it work in theory?” Conversely, the defense offered for floating exchange rate IT is dangerously close to “It doesn’t work in practice, but it does work in theory” (ignoring the analysis here). Our results may prove controversial, but at least they reconcile theory with current practice. Confronted with the stylized fact that LDCs overwhelmingly reject the conventional wisdom to float in favor of tight management of the exchange rate, we answer: “What is the puzzle? Exchange-rate-anchored IT works both in practice and in theory, while flexible exchange rate IT is problematic in theory and has performed poorly in practice.” None of this denies that exchange-rate-anchored IT has shortcomings. The risks involved in tightly managing the path of the exchange rate are well known, and the opportunity cost of amassing a large stockpile of reserves to support IT is not trivial. But LDC central banks have learned from experience that in this business you get what you pay for; effective IT cannot be done on the cheap.

43 Developing countries have often faced difficulties in sustaining fixed exchange rate regimes. They have been forced to abandon such regimes in response to a gradual but sustained loss of international reserves that typically culminates in a full-blown speculative attack. (see Calvo and Vegh, 1999, Burnside et al. 2001, and Corsetti and Mackowiak, 2006, among others).
Several extensions of our analysis are worth pursuing. These may include extending our framework to include investment and other relevant features for LDCs, such as rule of thumb consumers. These extensions may play a role in the determinacy analysis and may actually reinforce our results. Moreover, one can try to provide empirical support to the macroeconomic mechanisms described in this paper. For instance, one can try to estimate our model using data for a LDC and test “indirectly” for the presence of indeterminacy. This is a challenging task and deserves a stand-alone paper, since it involves not restricting the parameter space to ensure determinacy, as is frequently done in the estimations of dynamic stochastic general equilibrium models. Recent empirical contributions, however, suggest that it is possible to estimate these models even in the presence of indeterminacy problems.\textsuperscript{44} We leave this task for future research.

\textsuperscript{44}See Farmer et al. (2015).
Table 1: Estimates of the Elasticity of Currency Substitution

<table>
<thead>
<tr>
<th>Country (Study)</th>
<th>Estimate (Range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico (Garcia-Cicco, 2010)*</td>
<td>3.9</td>
</tr>
<tr>
<td>Peru (Felices and Tuesta, 2010)*</td>
<td>4.1</td>
</tr>
<tr>
<td>Georgia (Loiseau-Aslanidi, 2012)*</td>
<td>2.95 (1.02-5.30)</td>
</tr>
<tr>
<td>Vietnam (Shinichi and Binh, 2005)*</td>
<td>3.0 (1.02-5.30)</td>
</tr>
<tr>
<td>Russia (Friedman and Verbetsky, 2001)</td>
<td>2.90 (1.40-5.08)</td>
</tr>
<tr>
<td>Cambodia (Samreth, 2010)</td>
<td>2.06, 3.89</td>
</tr>
<tr>
<td>Czech Republic*</td>
<td>1.72</td>
</tr>
<tr>
<td>Israel*</td>
<td>1.78</td>
</tr>
<tr>
<td>Poland*</td>
<td>5.0</td>
</tr>
<tr>
<td>Slovak Republic* (Selcuk, 2003)</td>
<td>1.28</td>
</tr>
<tr>
<td>Turkey*</td>
<td>1.37</td>
</tr>
<tr>
<td>Kyrgyz Republic</td>
<td>3.59 (3.03-4.17)</td>
</tr>
<tr>
<td>Tajikistan</td>
<td>3.23 (3.13-3.33)</td>
</tr>
<tr>
<td>Kazakhstan</td>
<td>5.90 (3.33-7.69)</td>
</tr>
<tr>
<td>Indonesia*</td>
<td>1.92</td>
</tr>
<tr>
<td>Korea*</td>
<td>2.42</td>
</tr>
<tr>
<td>Singapore</td>
<td>3.66</td>
</tr>
<tr>
<td>Thailand*</td>
<td>1.59</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>2.96</strong></td>
</tr>
<tr>
<td><strong>Average for IT Countries</strong></td>
<td><strong>2.91</strong></td>
</tr>
</tbody>
</table>

1 Unless otherwise noted, the estimates refer to GMM estimates of the elasticity of currency substitution in a CES liquidity services function. A star refers to countries that practice inflation targeting.

2 Computed from the semi-elasticity of foreign currency deposits with respect to its own return. The estimate of 3.0 assumes the intertemporal elasticity of substitution equals .35; the range lets the intertemporal elasticity vary from .2 to 1.

3 Averages of the quarterly estimates for 1996 of the Morishima elasticity of substitution between foreign currency and domestic currency.
Table 2: Calibration of the Model (time period = 1 quarter).

<table>
<thead>
<tr>
<th>Variable/Parameter</th>
<th>Base Case</th>
<th>Alternative Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of domestic currency to quarterly GDP</td>
<td>.32</td>
<td></td>
</tr>
<tr>
<td>Ratio of foreign currency to quarterly GDP</td>
<td>.32</td>
<td>.16</td>
</tr>
<tr>
<td>Elasticity of substitution between differentiated varieties (ε)</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Discount factor (β)</td>
<td>.983</td>
<td>.99</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution (τ)</td>
<td>.35</td>
<td>.2 - 1</td>
</tr>
<tr>
<td>Consumption share of nontraded goods (γ)</td>
<td>.4</td>
<td>.3 - .8</td>
</tr>
<tr>
<td>Frisch elasticity of labor supply (ψ)</td>
<td>.5</td>
<td>.2, 1.5</td>
</tr>
<tr>
<td>Elasticity of substitution between domestic and foreign currency (σ)</td>
<td>.5-3</td>
<td></td>
</tr>
<tr>
<td>Ratio of foreign bonds/loans to quarterly GDP</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>Elasticity of foreign bond flows (ϕ)</td>
<td>1</td>
<td>.1 - 5</td>
</tr>
<tr>
<td>Average duration of a price quote [1/(1 – ω)]</td>
<td>.5</td>
<td>.25 - .75</td>
</tr>
<tr>
<td>Cost shares of labor (θ_{nl}, θ_{xl})*</td>
<td>.50</td>
<td>.67</td>
</tr>
<tr>
<td>Quarterly inflation rate</td>
<td>1%</td>
<td></td>
</tr>
</tbody>
</table>
Table 3: Estimates of the Calvo Sticky-price Parameter ($\omega$) in LDCs.

<table>
<thead>
<tr>
<th>Country (Study)</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil (Barros and Matos, 2008)</td>
<td>.40</td>
</tr>
<tr>
<td>Brazil (Gouvea, 2007)</td>
<td>.27</td>
</tr>
<tr>
<td>Brazil (Furlani et al., 2010)</td>
<td>.27</td>
</tr>
<tr>
<td>Brazil (de Castro et al., 2011)</td>
<td>.74</td>
</tr>
<tr>
<td>Brazil (Minella and Souza-Sobrinho, 2013)</td>
<td>.50</td>
</tr>
<tr>
<td>Chile (Del Negro and Schorfheide, 2005)</td>
<td>.40-.46</td>
</tr>
<tr>
<td>Chile (Cespedes et al., 2005)</td>
<td>.55-.67</td>
</tr>
<tr>
<td>Chile (Tovar, 2006a)</td>
<td>.44</td>
</tr>
<tr>
<td>Chile (Median and Soto, 2007)</td>
<td>.74</td>
</tr>
<tr>
<td>Chile (Catao and Pagan, 2009)</td>
<td>.48</td>
</tr>
<tr>
<td>Chile (Caputo et al., 2007)</td>
<td>.42-.53</td>
</tr>
<tr>
<td>Korea (Elekdag et al., 2005)</td>
<td>.48-.53</td>
</tr>
<tr>
<td>Korea (An and Kang, 2011)</td>
<td>.71, .81</td>
</tr>
<tr>
<td>Korea (Tovar, 2006b)</td>
<td>.41</td>
</tr>
<tr>
<td>Mexico (Garcia-Cicco, 2010)</td>
<td>.16</td>
</tr>
<tr>
<td>Mexico (Tovar, 2006a)</td>
<td>.37</td>
</tr>
<tr>
<td>Colombia (Tovar, 2006a)</td>
<td>.43</td>
</tr>
<tr>
<td>Peru (Castillo et al., 2006)</td>
<td>.34-.52</td>
</tr>
<tr>
<td>China (Zheng and Guo, 2013)</td>
<td>.41</td>
</tr>
<tr>
<td>Pakistan (Choudhary et al., 2011)</td>
<td>.25</td>
</tr>
<tr>
<td>South Africa (Steinbach et al., 2009)</td>
<td>.49</td>
</tr>
<tr>
<td>South Africa (Alapanda et al., 2010)</td>
<td>.46</td>
</tr>
<tr>
<td>South Africa (Gupta and Steinbach, 2013)</td>
<td>.63</td>
</tr>
<tr>
<td>Turkey (Cebi, 2012)</td>
<td>.59</td>
</tr>
<tr>
<td>Israel (Binyamini, 2007)</td>
<td>.47</td>
</tr>
</tbody>
</table>
Table 4: Values of $\alpha$ Compatible with Determinacy in a Flexible Exchange Rate System ($\phi = 1$).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\sigma = .5$</th>
<th>$\sigma = 1$</th>
<th>$\sigma = 1.5$</th>
<th>$\sigma = 2$</th>
<th>$\sigma = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>1.11-1.45</td>
<td>1.45-1.73</td>
<td>1.69-1.92</td>
<td>1.84-2.07</td>
<td>1.96-2.26</td>
</tr>
<tr>
<td>$\tau = .20$</td>
<td>1.29-1.64</td>
<td>1.65-1.95</td>
<td>1.87-2.15</td>
<td>1.99-2.30</td>
<td>2.08-2.49</td>
</tr>
<tr>
<td>$\tau = .50$</td>
<td>1-1.33</td>
<td>1.30-1.58</td>
<td>1.54-1.76</td>
<td>1.71-1.89</td>
<td>1.85-2.09</td>
</tr>
<tr>
<td>$\tau = .75$</td>
<td>1-1.20</td>
<td>1.21-1.41</td>
<td>1.42-1.57</td>
<td>1.59-1.69</td>
<td>1.77-1.87</td>
</tr>
<tr>
<td>$\tau = 1$</td>
<td>1-1.12</td>
<td>1-1.30</td>
<td>1.17-1.44</td>
<td>1.31-1.55</td>
<td>1.50-1.71</td>
</tr>
<tr>
<td>$\omega = .25$</td>
<td>1.13-1.21</td>
<td>Null</td>
<td>Null</td>
<td>Null</td>
<td>Null</td>
</tr>
<tr>
<td>$\omega = .33$</td>
<td>1.12-1.28</td>
<td>1.46-1.50</td>
<td>Null</td>
<td>Null</td>
<td>Null</td>
</tr>
<tr>
<td>$\omega = .67^1$</td>
<td>1-1.65</td>
<td>1.42-1.99</td>
<td>1.66-2.23</td>
<td>1.80-2.42</td>
<td>1.92-2.65</td>
</tr>
<tr>
<td>$\omega = .75$</td>
<td>1-1.73</td>
<td>1-1.09, 1.35-2.10</td>
<td>1-1.07, 1.59-2.36</td>
<td>1-1.07, 1.74-2.46</td>
<td>1-1.07, 1.86-2.66</td>
</tr>
<tr>
<td>$\gamma = .30$</td>
<td>1.12-1.34</td>
<td>1.54-1.61</td>
<td>Null</td>
<td>Null</td>
<td>Null</td>
</tr>
<tr>
<td>$\gamma = .60$</td>
<td>1.08-1.78</td>
<td>1.33-2.07</td>
<td>1.46-2.26</td>
<td>1.53-2.40</td>
<td>1.59-2.58</td>
</tr>
<tr>
<td>$\psi = .20$</td>
<td>1.12-1.33</td>
<td>1.47-1.58</td>
<td>1.73-1.76</td>
<td>Null</td>
<td>Null</td>
</tr>
<tr>
<td>$\psi = 1.5^1$</td>
<td>1-1.55</td>
<td>1.41-1.86</td>
<td>1.63-2.07</td>
<td>1.77-2.23</td>
<td>1.86-2.45</td>
</tr>
<tr>
<td>$\beta = .99$</td>
<td>1.11-1.45</td>
<td>1.38-1.69</td>
<td>1.51-1.84</td>
<td>1.57-1.93</td>
<td>1.62-2.05</td>
</tr>
<tr>
<td>$\theta_{nl} = \theta_{xl} = .67$</td>
<td>1.10-1.50</td>
<td>1.44-1.80</td>
<td>1.67-2.00</td>
<td>1.81-2.15</td>
<td>1.92-2.36</td>
</tr>
<tr>
<td>$\psi = .20$ and $\omega = .33$</td>
<td>1.13-1.20</td>
<td>Null</td>
<td>Null</td>
<td>Null</td>
<td>Null</td>
</tr>
<tr>
<td>$b^* = 1$</td>
<td>1.11-1.45</td>
<td>1.46-1.73</td>
<td>1.70-1.93</td>
<td>1.85-2.07</td>
<td>1.96-2.27</td>
</tr>
<tr>
<td>$F = .16$</td>
<td>1.09-1.44</td>
<td>1.43-1.70</td>
<td>1.68-1.88</td>
<td>1.85-2.02</td>
<td>1.96-2.22</td>
</tr>
</tbody>
</table>

$^1$ Determinacy region includes $\alpha = 1-1.03$ for $\sigma = 1-2$ and $\alpha = 1-1.02$ for $\sigma = 3$. 
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Closed Economy 1</th>
<th>Closed Economy 2</th>
<th>Open Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>3.83</td>
<td>1.64</td>
<td>3.83</td>
</tr>
<tr>
<td>$\tau = .20$</td>
<td>4.90</td>
<td>1.88</td>
<td>4.90</td>
</tr>
<tr>
<td>$\tau = .50$</td>
<td>3.23</td>
<td>1.51</td>
<td>3.23</td>
</tr>
<tr>
<td>$\tau = .75$</td>
<td>2.64</td>
<td>1.37</td>
<td>2.64</td>
</tr>
<tr>
<td>$\tau = 1$</td>
<td>2.30</td>
<td>1.30</td>
<td>2.30</td>
</tr>
<tr>
<td>$\omega = .25$</td>
<td>1.64</td>
<td>1.22</td>
<td>1.64</td>
</tr>
<tr>
<td>$\omega = .33$</td>
<td>2.05</td>
<td>1.33</td>
<td>2.05</td>
</tr>
<tr>
<td>$\omega = .67$</td>
<td>9.58</td>
<td>1.64</td>
<td>9.58</td>
</tr>
<tr>
<td>$\omega = .75$</td>
<td>17.46</td>
<td>2.20</td>
<td>17.46</td>
</tr>
<tr>
<td>$\psi = .20$</td>
<td>2.61</td>
<td>1.47</td>
<td>2.61</td>
</tr>
<tr>
<td>$\psi = 1.5$</td>
<td>5.29</td>
<td>1.72</td>
<td>5.29</td>
</tr>
</tbody>
</table>

1 Closed Economy 1 is the standard closed-economy model with a single sticky-price sector. Closed Economy 2 is the two-sector closed-economy model with a sticky-price sector and a flex-price sector.

2 The upper bound is independent of the consumption share of traded goods, the degree of dollarization, factor cost shares in tradables production, and the elasticities of currency and bond substitution.
Table 6: Upper bound on the Taylor Coefficient when the Central Bank Fixes the Rate of Crawl of the Exchange Rate at the Target Inflation Rate.\(^1\)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>(\gamma = .30)</th>
<th>(\gamma = .40)</th>
<th>(\gamma = .50)</th>
<th>(\gamma = .60)</th>
<th>(\gamma = .70)</th>
<th>(\gamma = .80)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>6.1</td>
<td>4.3</td>
<td>3.2</td>
<td>2.5</td>
<td>1.9</td>
<td>1.5</td>
</tr>
<tr>
<td>(\tau = .20)</td>
<td>8.0</td>
<td>5.5</td>
<td>4.0</td>
<td>3.0</td>
<td>2.3</td>
<td>1.8</td>
</tr>
<tr>
<td>(\tau = .50)</td>
<td>5.0</td>
<td>3.6</td>
<td>2.7</td>
<td>2.1</td>
<td>1.7</td>
<td>1.4</td>
</tr>
<tr>
<td>(\tau = .75)</td>
<td>3.9</td>
<td>2.9</td>
<td>2.3</td>
<td>1.8</td>
<td>1.5</td>
<td>1.3</td>
</tr>
<tr>
<td>(\tau = 1)</td>
<td>3.3</td>
<td>2.5</td>
<td>2.0</td>
<td>1.7</td>
<td>1.4</td>
<td>1.3</td>
</tr>
<tr>
<td>(\psi = .20)</td>
<td>6.8</td>
<td>4.7</td>
<td>3.5</td>
<td>2.6</td>
<td>2.1</td>
<td>1.6</td>
</tr>
<tr>
<td>(\psi = 1.5)</td>
<td>5.3</td>
<td>3.8</td>
<td>2.8</td>
<td>2.2</td>
<td>1.8</td>
<td>1.5</td>
</tr>
<tr>
<td>(\theta_{nl} = \theta_{xl} = .67)</td>
<td>5.7</td>
<td>4.0</td>
<td>3.0</td>
<td>2.3</td>
<td>1.9</td>
<td>1.5</td>
</tr>
</tbody>
</table>

\(^1\) The upper bound is independent of the average length of a price quote in the nontradables sector, the degree of dollarization, and the elasticities of currency substitution and bond substitution.
Table 7: Values of $\alpha$ Compatible with Determinacy in a Flexible Exchange Rate System when the Central Bank Targets Core Inflation.\(^1\)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\sigma = .5$</th>
<th>$\sigma = 1$</th>
<th>$\sigma = 1.5$</th>
<th>$\sigma = 2$</th>
<th>$\sigma = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>1-3.48</td>
<td>1.52-4.10</td>
<td>1.76-4.54</td>
<td>1.87-4.86</td>
<td>1.94-5.31</td>
</tr>
<tr>
<td>$\tau = .20$(^2)</td>
<td>1.65-4.43</td>
<td>4.03-5.90</td>
<td>7.130-7.132</td>
<td>Null</td>
<td>Null</td>
</tr>
<tr>
<td>$\tau = .50$</td>
<td>1-2.98</td>
<td>1.14-3.29</td>
<td>1.25-3.49</td>
<td>1.30-3.64</td>
<td>1.35-3.82</td>
</tr>
<tr>
<td>$\tau = .75$</td>
<td>1-2.54</td>
<td>1-2.63</td>
<td>1-2.69</td>
<td>1-2.73</td>
<td>1-2.78</td>
</tr>
<tr>
<td>$\tau = 1$</td>
<td>1-2.30</td>
<td>1-2.63</td>
<td>1-2.69</td>
<td>1-2.73</td>
<td>1-2.78</td>
</tr>
<tr>
<td>$\omega = .25$</td>
<td>1.16-1.60</td>
<td>1.62-1.84</td>
<td>1.85-2.02</td>
<td>1.95-2.14</td>
<td>2.01-2.32</td>
</tr>
<tr>
<td>$\omega = .33$(^2)</td>
<td>1.14-1.95</td>
<td>1.60-2.27</td>
<td>1.83-2.49</td>
<td>1.94-2.65</td>
<td>1.99-2.88</td>
</tr>
<tr>
<td>$\omega = .67$</td>
<td>1-8.37</td>
<td>1-9.98</td>
<td>1.50-11.12</td>
<td>1.64-11.96</td>
<td>1.74-13.13</td>
</tr>
<tr>
<td>$\omega = .75$</td>
<td>U</td>
<td>U</td>
<td>U</td>
<td>U</td>
<td>U</td>
</tr>
<tr>
<td>$\tau = .50$ and $\omega = .67$</td>
<td>1-6.97</td>
<td>1-7.75</td>
<td>1-8.26</td>
<td>1-8.63</td>
<td>1-9.10</td>
</tr>
<tr>
<td>$\gamma = .30$</td>
<td>1-3.42</td>
<td>1.69-4.14</td>
<td>2.05-4.67</td>
<td>2.22-5.08</td>
<td>2.32-5.67</td>
</tr>
<tr>
<td>$\gamma = .60$</td>
<td>1-3.59</td>
<td>1.29-4.01</td>
<td>1.39-4.28</td>
<td>1.44-4.47</td>
<td>1.47-4.72</td>
</tr>
<tr>
<td>$\psi = .20$</td>
<td>1.19-2.39</td>
<td>1.96-2.90</td>
<td>2.45-3.28</td>
<td>2.68-3.58</td>
<td>2.79-4.02</td>
</tr>
<tr>
<td>$\tau = .20$ and $\omega = .33$</td>
<td>1.77-2.39</td>
<td>Null</td>
<td>Null</td>
<td>Null</td>
<td>Null</td>
</tr>
</tbody>
</table>

\(^1\) U = unique is entered when the lower bound equals unity and the upper bound exceeds fifteen.

\(^2\) The determinacy region includes $\alpha = 1-1.02$ when $\sigma = .5-1.5$. 
Figure 1: Equilibrium Determinacy Regions under Inflation Targeting and a Pure Float.
Figure 1 (continued): Equilibrium Determinacy Regions under Inflation Targeting and a Pure Float.

Figure 2: Equilibrium Determinacy Regions when Targeting the Exchange Rate in the Interest Rate Rule.
Figure 3: Equilibrium Determinacy Regions when Targeting the Exchange Rate with a Second Rule (Managed Floats).
Figure 4: Equilibrium Determinacy Regions under Flexible Inflation Targeting.
Figure 5: Contemporaneous IT with a Flexible Exchange Rate vs. NERA-IT (red, dash).
Scenario A with $\chi$ equal to one (first row) or five in the Managed Float.

Scenario B with $\chi$ equal to one (first row) or five in the Managed Float.

Scenario C with $\chi$ equal to one (first row) or five in the Managed Float.

Figure 6: Managed Float vs. NERA-IT (red, dash).
Figure 7: CPI IT with a Flexible Exchange Rate vs. NERA-IT (red, dash) when $\tau = .5$, $\sigma = 1$, $\psi = .2$, and Labor is Intersectorally Mobile.
Appendix A

In this appendix we derive the core dynamic system that determines the stability properties of the economy when the exchange rate floats and the central bank targets either headline inflation or core inflation.

Intermediate Solutions

The core dynamic system consists of six first-order difference equations in $C$, $P_n$, $\pi_n$, $\pi_e$, $X$, and $b^*$. The first long step in solving the system is to derive intermediate solutions that relate the paths of various endogenous variables to the paths of these six variables.

We start with the intermediate solutions for labor demand, marginal cost, and sectoral output. From (3), (14), and (16)-(18),
\begin{align*}
\dot{Q}_{n,t} &= \hat{C}_t - (1 - \gamma) \hat{P}_{n,t}, \\
\hat{H}_t &= \hat{w}_t + \frac{1 - \theta_n}{\theta_n} [\hat{C}_t - (1 - \gamma) \hat{P}_{n,t}], \\
\hat{L}_{n,t} &= \frac{\hat{C}_t - (1 - \gamma) \hat{P}_{n,t}}{\theta_n}, \\
\hat{L}_{x,t} &= -\frac{\hat{w}_{x,t}}{1 - \theta_x}, \\
\text{and} \\
\dot{Q}_{x,t} &= -\frac{\theta_x}{1 - \theta_x} \hat{w}_{x,t}.
\end{align*}

Solving for the change in the market-clearing wage requires the solutions for sectoral labor supply. From (10),
\begin{equation}
\dot{L}_{s_{i,t}} = \psi(\hat{w}_{i,t} - \gamma \hat{P}_{n,t} - \hat{C}_t/\tau), \quad \text{for} \quad i = n, x.
\end{equation}

Equations (A3), (A4), (A6) and
\begin{equation}
L_{i,t} = L_{s_{i,t}} \quad \text{for} \quad i = n, x,
\end{equation}
can be solved for $L_n$, $L_x$, $w_n$, and $w_x$. The solutions for $w_n$ and $w_x$ are
\begin{equation}
\hat{w}_{x,t} = m_1 \hat{P}_{n,t} + m_2 \hat{C}_t.
\end{equation}
\[ \dot{w}_{n,t} = m_4 \hat{P}_{n,t} + m_3 \hat{C}_t, \]  

(A8)

where

\[ m_1 \equiv \frac{(1 - \theta_x)\psi \gamma}{1 + (1 - \theta_x)\psi}, \quad m_2 \equiv \frac{(1 - \theta_x)\psi}{1 + (1 - \theta_x)\psi} \tau, \quad m_3 \equiv \frac{\tau + \theta_n \psi}{\theta_n \psi \tau}, \quad \text{and} \quad m_4 \equiv \frac{\gamma(\theta_n \psi + 1) - 1}{\theta_n \psi}. \]

Substituting for \( \hat{w}_t \) in (A2) and (A5) now delivers

\[ \hat{H}_t = \left( m_3 + \frac{1 - \theta_n}{\theta_n} \right) \hat{C}_t + \left[ m_4 - \frac{1 - \theta_n}{\theta_n} (1 - \gamma) \right] \hat{P}_{n,t}, \]  

(A9)

and

\[ \hat{Q}_{x,t} = -\frac{\theta_x}{1 - \theta_x} (m_1 \hat{P}_{n,t} + m_2 \hat{C}_t). \]  

(A10)

In addition to (A9) and (A10), we need the intermediate solution for \( \pi_{e,t+1} \). This is obtained from the first-order conditions for domestic and foreign currency, which read

\[ G_F \left( \frac{m_t}{P_t}, \frac{F_t}{P_t} \right) = \beta C_{t+1}^{-1/\tau} \frac{P_t}{\pi_{e,t+1}} \left( \frac{R_t}{\pi_{e,t+1}} - 1 \right) \]

and

\[ G_m \left( \frac{m_t}{F_t}, \frac{C_t}{P_t} \right) = \beta C_{t+1}^{-1/\tau} \frac{P_t}{\pi_{e,t+1}} \frac{R_t - 1}{\pi_{e,t+1}}. \]

where\(^{45}\)

\[ G_F = \left[ k_2^{1/\sigma} \left( m_t/P_t \right)^{(1-\sigma)/\sigma} + (1 - k_2)^{1/\sigma} \left( F_t/P_t \right)^{(1-\sigma)/\sigma} \right]^{(\tau-\sigma)/\tau(\sigma-1)} k_0 (1 - k_2)^{1/\sigma} \left( F_t/P_t \right)^{-1/\sigma} \]

and

\[ G_m = \left[ k_2^{1/\sigma} \left( m_t/P_t \right)^{(1-\sigma)/\sigma} + (1 - k_2)^{1/\sigma} \left( F_t/P_t \right)^{(1-\sigma)/\sigma} \right]^{(\tau-\sigma)/\tau(\sigma-1)} k_0 k_2^{1/\sigma} \left( m_t/P_t \right)^{-1/\sigma}. \]

Equations (11) and (12) can be solved for \( m_t \) and \( \pi_{e,t+1} \) as a function of \( C_{t+1}, P_{n,t+1}, P_{n,t}, F_t, \) and \( \pi_{n,t+1} \) after substituting for \( \hat{R}_t \) from the Taylor rule [i.e., \( \hat{R}_t = \alpha \hat{\pi}_{t+1} = \alpha (1 - \gamma) \hat{\pi}_{e,t+1} + \alpha \gamma \hat{\pi}_{n,t+1} \)].

\(^{45}\) \( G_m \) and \( G_F \) are partial derivatives with respect to the first and second arguments of \( G(\bullet) \). (That is, \( G_F \equiv \partial G/\partial (F/P) \) and \( G_m \equiv \partial G/\partial (m/P) \).)
Straightforward algebra gives
\[ \hat{\pi}_{e,t+1} = n_1 \hat{P}_{n,t} - n_2 \hat{\pi}_{n,t+1} + n_3 \hat{C}_{t+1} + n_4 \hat{P}_{n,t+1} - n_3 \hat{F}_t, \]  
(A11)

where
\[ \Delta \equiv \frac{\tau \theta_f + \sigma \theta_m}{1 - \beta} \left[ \alpha(1 - \gamma) - 1 \right] + \theta_m(\tau - \sigma)(1 - \gamma) \left( \frac{R \alpha}{R - 1} - 1 \right), \]
\[ n_1 \equiv \frac{\gamma}{\Delta} \left[ 1 - \tau \theta_F - \sigma \theta_m \right], \quad n_2 \equiv \frac{\gamma}{\Delta} \left[ (\tau \theta_f + \sigma \theta_m) \frac{\alpha}{1 - \beta} + \theta_m(\tau - \sigma) \left( \frac{R \alpha}{R - 1} - 1 \right) \right], \]
\[ n_3 \equiv 1/\Delta, \quad n_4 \equiv \frac{\gamma}{\Delta} (\tau \theta_f + \sigma \theta_m), \]

and
\[ \theta_m \equiv \frac{G_{mm}}{G_{mm} + G_{F F}} = \frac{im}{im + (R - \pi^*)F} \quad \text{and} \quad \theta_F \equiv 1 - \theta_m \]

are the shares of liquidity services supplied by domestic and foreign currency, with \( i = R - 1 \) denoting the nominal interest rate. Substitute \( \hat{P}_{n,t+1} = \hat{P}_{n,t} + \hat{\pi}_{n,t+1} - \hat{\pi}_{e,t+1} \) and \( \hat{F}_t = (X/F) \hat{X}_t - (b^*/F) \hat{b}_t^* \) into (A11) and collect terms. After simplification, there emerges
\[ \hat{\pi}_{e,t+1} = n_5 \hat{P}_{n,t} + n_6 \hat{\pi}_{n,t+1} + n_7 \hat{C}_{t+1} - n_7 (X/F) \hat{X}_t + n_7 (b^*/F) \hat{b}_t^*, \]  
(A12)

where
\[ n_5 \equiv \frac{\gamma}{1 + n_4}, \quad n_6 \equiv \frac{\gamma}{\Delta (1 + n_4)} \left[ (\tau \theta_f + \sigma \theta_m) \left( 1 - \frac{\alpha}{1 - \beta} \right) + \theta_m(\sigma - \tau) \left( \frac{R \alpha}{R - 1} - 1 \right) \right], \]

and
\[ n_7 \equiv \frac{n_3}{\Delta (1 + n_4)}. \]

**The Log-Linearized Solution for the Core Dynamic System**

We are now in a position to quickly derive the solution for the core dynamic system. From (9) and the Taylor rule,
\[ \hat{C}_t = \hat{C}_{t+1} - \tau (\alpha - 1)[\gamma \hat{\pi}_{n,t+1} + (1 - \gamma) \hat{\pi}_{e,t+1}], \]

then
\[ \hat{C}_t = \hat{C}_{t+1} - v_1 \hat{\pi}_{n,t+1} - v_2 \hat{\pi}_{e,t+1}, \]  
(A13)
where $v_1 = \tau (\alpha - 1) \gamma$ and $v_2 = \tau (\alpha - 1) (1 - \gamma)$.

Turn next to equation (23) and the law of motion for $X$. After making use of the solutions for $Q_n$ and $Q_x$ in (A1) and (A10), we have

$$\tilde{X}_t = f_1 \tilde{C}_t - f_2 P_{n,t} + \tilde{X}_{t-1} + f_3 \tilde{b}_{t-1},$$  \hspace{1cm} (A14)

where

$$f_1 \equiv X^{-1} \left( P_n Q_n - \frac{Q_x \theta_x m_2}{1 - \theta_x} - PC \right), \quad f_2 \equiv X^{-1} \left[ P_n Q_n (1 - \gamma) + \frac{Q_x \theta_x m_1}{1 - \theta_x} \right],$$

and

$$f_3 \equiv (R^* - 1) b^*/X.$$

The solution for nontradables inflation follows directly from (19) and (A9):

$$\hat{\pi}_{n,t+1} + f_5 \hat{P}_{n,t} = \frac{\hat{\pi}_{n,t}}{\beta} - f_4 \hat{C}_t,$$  \hspace{1cm} (A15)

where

$$f_4 \equiv \frac{\kappa \theta_n m_3 + 1 - \theta_n}{\beta \theta_n} = \kappa \left[ 1 + \psi (1 - \theta_n) \right] + \theta_n \psi > 0,$$

and

$$f_5 \equiv \kappa \theta_n m_4 + \gamma (1 - \theta_n) - 1 = -\kappa \frac{(1 - \gamma) (1 + \psi)}{\beta \theta_n \psi} < 0.$$

Finally, equations (13) and (24) give

$$\Lambda b^* \tilde{b}_t = [1 - \alpha (1 - \gamma)] \hat{\pi}_{e,t+1} - \alpha \gamma \hat{\pi}_{n,t+1},$$  \hspace{1cm} (A16)

and

$$\hat{P}_{n,t} = \hat{P}_{n,t-1} + \hat{\pi}_{n,t} - \hat{\pi}_{e,t}.$$  \hspace{1cm} (A17)
Equations (A12) - (A17) form the matrix system

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -v_1 & 0 & -v_2 & 0 \\
f_5 & 0 & 1 & 0 & 0 & 0 \\
f_2 & 0 & 0 & 1 & 0 & 0 \\
n_5 & n_7 & n_6 & -n_7 X/F & -1 & n_7 b^*/F \\
0 & 0 & -\alpha \gamma & 0 & v_3 & -\Lambda b^*
\end{bmatrix}
\begin{bmatrix}
P_{n,t} \\
\hat{C}_{t+1} \\
\hat{\pi}_{n,t+1} \\
\hat{X}_t \\
\hat{\pi}_{e,t+1} \\
\hat{b}_t^*
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 1 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & -f_4 & 1/\beta & 0 & 0 & 0 \\
0 & f_1 & 0 & 1 & 0 & f_3 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
P_{n,t-1} \\
\hat{C}_t \\
\hat{\pi}_{n,t} \\
\hat{X}_{t-1} \\
\hat{\pi}_{e,t} \\
\hat{b}_{t-1}^*
\end{bmatrix}
\]

(A30)

where \(v_3 = 1 - \alpha(1 - \gamma)\). Stability of the system depends on the eigenvalues of \(A^{-1}J\).

**B Appendix B**

In section 4.2 the central bank pegs either the real exchange rate or the rate of crawl of the nominal exchange rate. Below we derive the core dynamic systems for the two pegs and prove Propositions 1 and 2.

**Pegging the Real Exchange Rate**

When the central bank pegs the real exchange rate \((1/P_n)\), \(\hat{P}_{n,t} = 0\) and \(\hat{\pi}_{e,t} = \hat{\pi}_{n,t}\). More importantly, the difference equations for \(C_t\) and \(\hat{\pi}_{n,t}\) comprise a subsystem. Equations (A13) and (A15) simplify to

\[
\begin{bmatrix}
\hat{C}_{t+1} \\
\hat{\pi}_{n,t+1}
\end{bmatrix}
= \begin{bmatrix}
1 - \tau(\alpha - 1) & \tau(\alpha - 1)/\beta \\
-f_4 & 1/\beta
\end{bmatrix}
\begin{bmatrix}
\hat{C}_t \\
\hat{\pi}_{n,t}
\end{bmatrix}
\]

or

\(\hat{C}_t = \hat{C}_{t+1} - \tau(\alpha - 1)\hat{\pi}_{n,t+1}\) (A13')

and

\(\hat{\pi}_{n,t} = \beta \hat{\pi}_{n,t+1} + \beta f_4 \hat{C}_t\) (A15')

or

\[
\begin{bmatrix}
\hat{C}_{t+1} \\
\hat{\pi}_{n,t+1}
\end{bmatrix}
= \begin{bmatrix}
1 - \tau(\alpha - 1)f_4 & \tau(\alpha - 1)/\beta \\
-f_4 & 1/\beta
\end{bmatrix}
\begin{bmatrix}
\hat{C}_t \\
\hat{\pi}_{n,t}
\end{bmatrix}
\]  

(B1)
$C$ and $\pi_n$ are both jump variables. Consequently, the subsystem in (B1) is saddle point stable iff both of the system's eigenvalues lie outside the unit circle. This requires (Woodford, 2003)

(a) $\det > 1$,  
(b) $1 + tr + \det > 0$,  
(c) $1 - tr + \det > 0$,  

or

(d) $\det - tr < -1$,  
(e) $\det + tr < -1$.

The second set of conditions do not apply in the present case. In the first set, $\det = 1/\beta > 1$. So condition (a) holds. Condition (c) holds for $\alpha > 1$, while condition (b) requires

$$\tau(\alpha - 1) < \frac{2(1 + \beta)}{\beta f_4},$$

or

$$\alpha < 1 + \frac{2(1 + \beta)\theta_n\psi}{\kappa\{\tau[1 + \psi(1 - \theta_n)] + \theta_n\psi\}}.$$  \hspace{1cm} (B2)

after substituting for $f_4$. Hence the necessary and sufficient condition for a unique equilibrium is

$$1 < \alpha < 1 + \frac{2(1 + \beta)\theta_n\psi}{\kappa\{\tau[1 + \psi(1 - \theta_n)] + \theta_n\psi\}}.$$  \hspace{1cm} (B3)

**Pegging the Rate of Crawl**

Intervening to peg the rate of crawl at the target inflation rate ($\hat{\pi}_{e,t} = 0$) eliminates $\pi_e$ from the core dynamic system. This still leaves a cumbersome system of five first-order difference equations in $C_t$, $\pi_{n,t}$, $P_{n,t}$, $X_t$ and $b^*_t$. Equations (A13), (A15), and (A17), however, form a 3x3 subsystem

$$\begin{bmatrix}
\hat{C}_{t+1} \\
\hat{\pi}_{n,t+1} \\
\hat{P}_{n,t}
\end{bmatrix} = \begin{bmatrix}
1 - v_1f_4 & v_1(1/\beta - f_5) & -v_1f_5 \\
-f_4 & 1/\beta - f_5 & -f_5 \\
0 & 1 & 1
\end{bmatrix} \begin{bmatrix}
\hat{C}_t \\
\hat{\pi}_{n,t} \\
\hat{P}_{n,t-1}
\end{bmatrix}.  \hspace{1cm} (B4)$$
The characteristic polynomial is

$$(1 - \chi)[\chi^2 - (1 + 1/\beta - v_1f_4 - f_5)\chi + 1/\beta] = 0. \quad (B5)$$

The system has a unit root. Since $P_n$ is the only state variable, saddle point stability requires the quadratic equation

$$\chi^2 - (1 + 1/\beta - v_1f_4 - f_5)\chi + 1/\beta = 0$$

to deliver one root inside the unit circle and one root outside the circle. This is the case if either

(a) $1 - tr + \det < 0$,  
(b) $1 + tr + \det > 0$,  

or

(c) $1 - tr + \det > 0$,  
(d) $1 + tr + \det < 0$.  

Conditions (a) and (b) give

$$\tau\gamma(\alpha - 1)f_4 + f_5 < 0 \quad (B6)$$

and

$$\frac{2(1 + \beta)}{\beta} - \tau\gamma(\alpha - 1)f_4 - f_5 > 0. \quad (B7)$$

Both (B6) and (B7) hold if

$$\alpha < 1 - \frac{f_5}{\tau\gamma f_4} = 1 + \frac{(1 - \gamma)(1 + \psi)}{\gamma[\tau(1 + \psi(1 - \theta_n)] + \theta_n\psi}. \quad (B8)$$

This result is stated in Proposition 2 in the text.

Conditions (c) and (d) yield another result. The equilibrium is also unique when

$$\alpha > 1 + \frac{2(1 + \beta)/\beta - f_5}{\tau\gamma f_4}.$$  

We decided to ignore this result because it requires an absurdly large value of $\alpha$ (typically $> 20$).
References


International Monetary Fund, 2011, “Foreign Exchange Intervention: How Good a Defense Against Appreciation Winds?” Chapter 3 in Regional Economic Outlook, Western Hemisphere (Washington, International Monetary Fund).


