WP/16/59

IMF Working Paper

Stochastic Trends, Debt Sustainability and Fiscal Policy

by Karim Barhoumi, Reda Cherif, and Nooman Rebei

INTERNATIONAL MONETARY FUND

 WP/16/59

IMF Working Paper

Institute for Capacity Development and African Department

Stochastic Trends, Debt Sustainability, and Fiscal Policy

Prepared by Karim Barhoumi, Reda Cherif, and Nooman Rebei[1]

Authorized for distribution by Ray Brooks and Joel Toujas-Bernate

March 2016

Abstract

We study empirically the reaction of fiscal policy to changes in the permanent and transitory components of GDP in a panel of countries. We find evidence that government spending tends to be counter-cyclical conditional on temporary shocks and pro-cyclical conditional on permanent shocks. We also find no evidence that developing countries are systematically different from developed ones in terms of fiscal policy. We present a theory featuring a fiscal reaction function to the output gap and a measure of debt sustainability. The fiscal impulse response to a permanent (temporary) shock to GDP is positive (negative) as the effect on debt sustainability (current output gap) dominates. The results are mostly sensitive to the relative weight of debt sustainability in the fiscal reaction function as well as to the extent of real rigidities in the economy.

JEL Classification Numbers: E2, E3

Keywords: Fiscal policy, Cyclicality, Public debt, Business cycles.

Author’s E-Mail Address: Kbarhoumi@imf.org, Acherif@imf.org, Nrebei@imf.org

[1] We are grateful to participants of the IMF Institute for Capacity Development seminar, Ruchir Agarwal, Tamim Bayoumi, Andrew Berg, Hafedh Bouakez, Ray Brooks, Kaddour Hadri, Fuad Hasanov, Andres Gonzales, Hali Edison, Daniel Leigh, Mariusz Jarmuzek, Dimitre Milkov, Racha Moussa, Carlos De Resende, Sampawende Tapsoba, and Dall Wendell for valuable suggestions.

Contents Page

Appendix

Tables

Figures

I. INTRODUCTION

The characterization of fiscal policy in a country as pro-cyclical or counter-cyclical is important for several reasons. To judge the performance of fiscal authorities over a period of time, it is necessary to gauge to what extent fiscal policy exaggerated or counteracted cyclical fluctuations. It also helps in making sense of the evolution of different macroeconomic aggregates. For example, public debt accumulation could be the result of a counter-cyclical policy during a recession, or fiscal profligacy during a period of economic boom. A priori, these examples should lead to very different policy recommendations in terms of changes in spending behavior. More importantly, governments should decide their budgets knowing what it entails in terms of pro- or counter-cyclical effects. There is a large and growing literature studying the cyclical nature of fiscal policies and comparing it across countries. The consensus is that developing countries tend to conduct pro-cyclical fiscal policies while developed ones conduct neutral or counter-cyclical fiscal policies.[1]

The premise of this paper is that fiscal policy reacts differently to temporary and permanent income shocks as a result of the tradeoff between stabilization in the short run and debt sustainability. Recognizing the existence of stochastic trends is a significant departure from the literature studying the cyclical behavior of fiscal policy in general. The standard approach consists in extracting the cyclical components of both government spending and income using filters a la Hodrick and Prescott (1997). The sign of the correlation between the two cyclical components would then tell us whether fiscal policy is pro- (when positive) or counter- (when negative) cyclical. Instead, we use a Beveridge-Nelson type decomposition of income to disentangle the stochastic trend and temporary components. There is growing evidence that stochastic trends should be taken into account to explain salient cross-country patterns of macroeconomic aggregates (e.g., Aguiar and Gopinath, 2007). Cherif and Hasanov (2013) in their study of commodity exporters show that the marginal propensity to consume out of permanent shocks is much larger than that of temporary shocks.[2] In addition, using an Hodrick-Prescott type of filter in the presence of permanent shocks could be misleading as shown by Cogley and Nason (1995) and Perron and Wada (2009). In our trend-cycle decom-

[1] See Gavin and Perotti (1997), Lane and Tornell (1999), Kaminsky, Reinhart, and Vegh (2005), Talvi and Vegh (2005), Mendoza and Oviedo (2006), Alesina, Campante, and Tabellini (2008) and Ilzetzki and Vegh (2008). Frankel, Vegh, and Vuletin (2013) stands out as they find evidence of developing economies "graduating" into counter-cyclical fiscal policy.

[2] Cherif and Hasanov (2012) study the effects of permanent and temporary income shocks on the saving-investment behavior in particular explaining why big savers invest relatively little.

position of countries' income we also take into account the occurrence of structural breaks. Perron and Wada (2009) shows that for the US, taking into account the change in the slope of the trend function leads to cycles that are close to the NBER chronology while ignoring structural breaks would lead to stark difference in cycles.[3] Based on mean-group type panel regressions, we find evidence that government spending tends to be counter-cyclical conditional on temporary shocks and pro-cyclical conditional on permanent shocks. We show that there is a relatively sizeable degree of dispersion in terms of country-specific coefficients both in magnitude and signs. However, we do not find any pattern based on income per capita in contrast with the common belief that developing economies conduct a different type of fiscal policy.

We propose a theory explaining the mechanism by which a government would react negatively to a temporary income shock and positively to a permanent shock as suggested by the data. The dilemma facing the government in dealing with economic fluctuations could be summed up in a simple tradeoff between the short run goal of stabilization and the medium to long run goal of controlling the level of debt. Faced with a temporary deviation from potential output, the government would focus on trying to close the output gap without worrying excessively about the effects on future debt. In contrast, if the deviation is long lasting or permanent, then worries about debt diverging from a long run target would dominate and the government would have to adjust spending in the same direction as the output deviation.

Our model has a standard dynamic stochastic general equilibrium setup featuring a fiscal reaction function to the output gap and a measure of debt sustainability, which is a function of future debt deviations from the long run level. The fiscal impulse response to a permanent (temporary) shock to GDP is positive (negative) as the effect on debt sustainability (current output gap) dominates. We show that a broad range of possible measures of debt sustainability would yield the same result as long as the deviation of debt of some year distant enough in the future would appear. We extend the model to account for potential reasons for the differences in the degree of cyclicality across countries. In particular, the degree of aversion to future debt levels, endogenous persistence—yielded by habit formation in consumption and costly adjustment of investment—, and the reaction of tax rates to government spending and debt fluctuations seem to be good candidates to understand the dispersion; however, learning

[3] One should note that they also show that in the presence of structural breaks, the HP and Beveridge-Nelson yield similar cycles. However, a priori the income of developing and emerging nations should have stochastic trends that are more variable compared to advanced ones implying that a Beveridge-Nelson type decomposition is still necessary which is confirmed in our results and also in Wada and Perron (2006) based on a sample of G7 countries.

about the source of the shock in an imperfect information environment tends to explain very little.

The paper is organized as follows: Section II tackles the empirical study both in terms of trend-cycle decomposition and panel regressions. An exposition of the model and the discussion of the theoretical results are detailed in Section III and Section IV, respectively. Section V concludes.

II. Empirical Analysis

A. Trend-Cycle decomposition

Since the pioneering work of Burns and Mitchell (1946), the literature on the measurement of business cycles evolved to include an array of different approaches. The standard approaches can be classified into two broad categories: methods that formally distinguish between permanent and transitory components, either based on an unconstrained ARIMA model following Beveridge and Nelson (1981) or based on latent variable models,[4] and those based on filters (e.g., Baxter and King, 1999; Hodrick and Prescott, 1997). In general, cross-country studies of the cyclicality of fiscal policy based on trend-cycle decompositions used solely the filters approach and in particular the Hodrick and Prescott (1997) filter. In contrast, our premise is that fiscal policy reacts differently to temporary and permanent shocks so we use a decomposition based on a model which explicitly distinguishes between temporary and permanent components. Moreover, we take into account the possibility of structural breaks in our trend-cycle decomposition. Perron and Wada (2009) demonstrated the importance of taking into account the change in the slope of the trend function in the measurement of the US business cycle. In particular, they show that the puzzling features of the different approaches stem from the neglect of the shift in the slope of the trend function. The issue of structural breaks is likely to be even more important in a cross-country study involving developing economies.

For a set of countries we use the logarithm of real GDP, on an annual frequency, which we note y_t evolving according to:[5, 6]

[4] See Morley, Nelson, and Zivot (2003) and Perron and Wada (2009) for an applications of both methods and their differences.

[5] Taken from the IMF World Economic Outlook database.

[6] As our model is close to Perron and Wada (2009) we retain their notation.

$$y_t = \tau_t + c_t$$

where τ_t is the permanent component and c_t is the temporary or cyclical component such that:

$$\tau_t = \mu + d1(t > T_b) + \tau_{t-1} + \eta_t$$

$$c_t = \rho c_{t-1} + e_t$$

The error terms η_t and e_t follow normal and independent iid distributions as it is the case in the standard unobserved component approach (e.g., Morley, Nelson, and Zivot, 2003). For each country in the dataset we first detect a structural break period T_b by using the procedure in Perron (2005).[7] The inclusion of two or more structural breaks could also be considered. However, the limitation in terms of times series data (yearly over 1970-2009) would be problematic as the tests to detect the years of the breaks would have low power. The translation of the model into a state-space form is straightforward where the state variables are τ_t and c_t. The estimation of the parameters and state variables is based on the Kalman filter algorithm and is performed on a large set of countries.[8]

B. Assessing the cyclicality of fiscal policy

Our approach to studying the cyclicality of fiscal policy is based on estimating the following equation where g_t is real fiscal expenditure in real terms (in log):[9]

$$g_t = \beta_0 + \beta_1 g_{t-1} + \beta_2 \Delta\tau_t + \beta_3 \Delta c_t + \varepsilon_t$$

---

[7] See Doan (2006) for an application in RATS where we used the additive outlier model (crash option and general to specific technique). Note that GDP series are typically non-stationary and thus one should rely on the type of procedures described in Perron (2005). Moreover, we are in the vein of Perron and Wada (2009) as we impose a unique break from outside the model.

[8] Real GDP series are taken from the published IMF World Economic Outlook database as of August 2014 and cover the period 1970-2011.

[9] The data on total expenditure were taken from Acosta-Ormaechea and Morozumi (2013) while deflators were taken from the IMF World Economic Outlook database.

The idea behind the equation is that the growth rate of fiscal spending, controlling for its lagged level, depends on a measure of the growth of the permanent component of real GDP as well as the growth of the temporary (or cyclical) component of GDP. [10] It is also possible to consider the explanatory variables as proxies for permanent and temporary income shocks. Our conjecture is that the signs of β_2 and β_3 could differ and therefore ignoring the permanent-temporary decomposition could be misleading in assessing the cyclicality of fiscal policy. According to our specification, the equivalent test of counter/pro-cyclicality hinges on whether β_3 is negative/positive. Meanwhile, the sign of β_2 indicates the reaction to permanent changes in income level. A positive β_2 could signal that fiscal sustainability is an issue and fiscal authorities are somehow forced to reflect changes in the permanent component of GDP. In the following section, we show the results of a series of panel regression techniques to estimate the parameters β_2 and β_3.

Past approaches could be classified in two broad “schools". The most widely used approach relies on filtering, typically using the HP or band pass filters, both fiscal spending and GDP to extract cyclical components. The study of procyclicality would focus then on the correlation between the cyclical components of fiscal expenditure and GDP. Another method is based on regressing the changes in the fiscal surplus as a share of GDP on the output gap (again typically based on HP and band pass filters).[11] Instead, we decompose GDP into its permanent and temporary components and we study the effect of changes in both component on the level of fiscal expenditure. In addition, we take into account structural breaks which we show have a sizeable effect on the estimation of the components of GDP. In doing so we are taking into accounts two salient features of developing and emerging economies in comparison to advanced ones: the presence of stochastic trends and the occurrence of structural breaks in their recent history.

C. Results

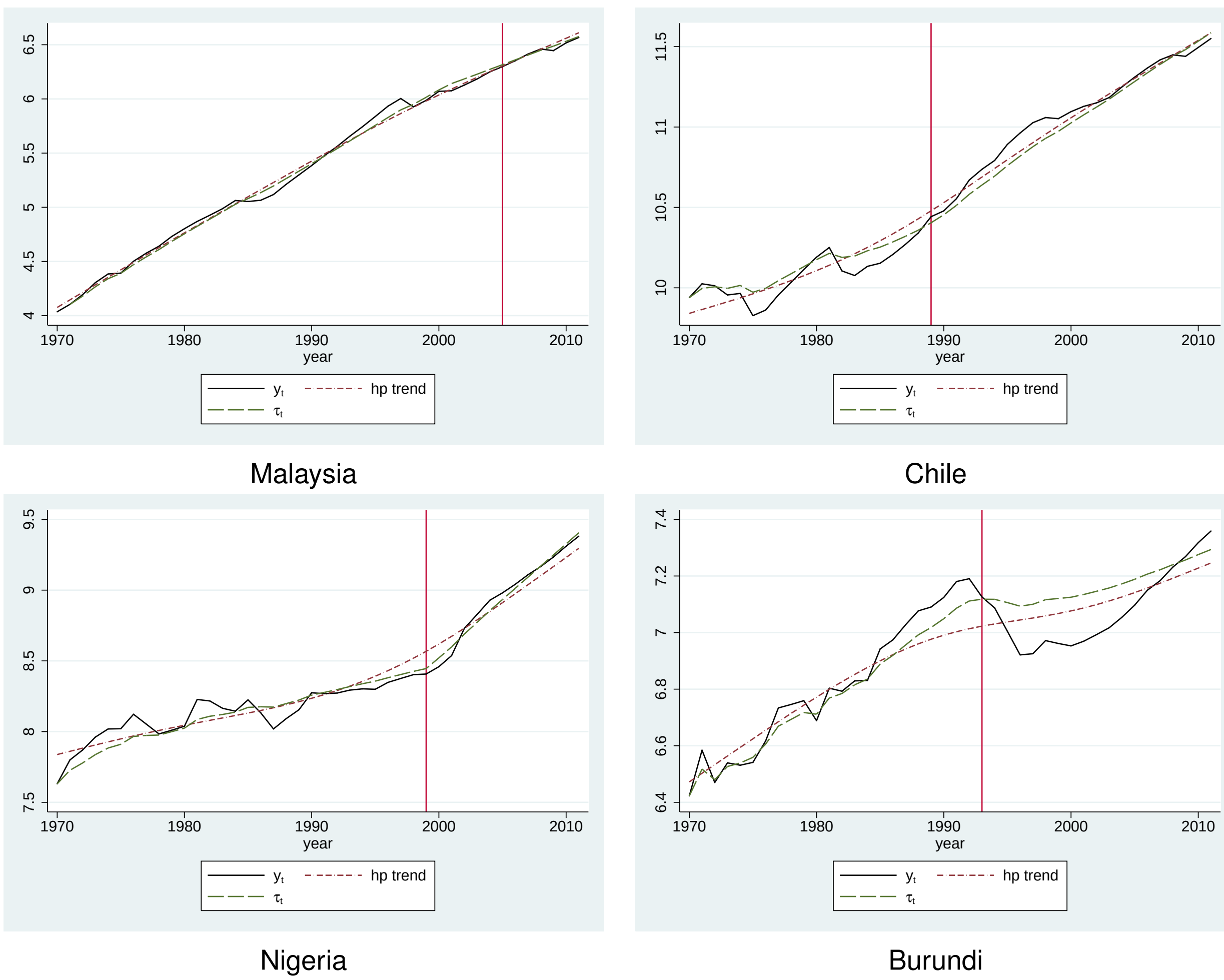
The first step in our study consists in performing the decomposition of GDP into its permanent and temporary components as shown in the previous section. We use the Kalman filter as the system corresponds to a state space model. Figure 1 shows the results for Malaysia, Chile, Nigeria and Burundi to illustrate the implications of the specification used. In all cases

[10] Although the left hand side of the equation is in level, it is implicitly a growth regression as one can substract the lag of spending from both sides.

[11] See Alesina, Campante, and Tabellini (2008).

there are visible differences between the HP-filter trend and the permanent component τ_t especially for Burundi. The results show that in general the use of the HP-filter, which ignores the presence of a stochastic trend, tends to exaggerate the magnitude of the cyclical component of GDP compared to our approach. In the case of Malaysia the differences are much less pronounced, although still noticeable, which confirms the intuition that the presence of more volatile stochastic trends should be taken into account especially for low income countries and commodity exporters.

Figure 1. GDP Trend-Cycle Decomposition in Selected Countries



In estimating our equation, we should take into account the fact that countries are likely to differ in terms of their fiscal reaction. Assuming that all countries share the same fiscal reaction would not be credible. Yet, we would like to draw the attention to patterns or more precisely to an average fiscal response. To do so we rely on a Mean Group (MG) technique, following

Pesaran and Smith (1995), the model of which allows for heterogeneous slope coefficients across group members. Table 1 shows the results of the regressions for different samples.[12] As we use the Klaman filter in our estimation it is recommended in general to discard some of the early observations.[13] In regression (1) we use the 1980-2009 samples (our fiscal data end in 2009), then in (2) we use 1985-2009 and in (3) 1990-2009.[14] All regressions indicate that $\beta_2 > 0$ and significant and $\beta_3 < 0$ and significant. Overall, the fiscal reaction to a permanent GDP shock is positive while the reaction to a temporary GDP shock is negative. In other words, fiscal policy is “counter-cyclical" when the nature of the shock is temporary while it is “pro-cyclical" when the nature of the shock is permanent.

Table 1. MG Regressions: Dependent Variable g_t

	(1)	(2)	(3)
Years	1980-2009	1985-2009	1990-2009
Variables			
g_{t-1}	0.863***	0.806***	0.719***
	(0.0239)	(0.0307)	(0.0542)
$\Delta\tau_t$	3.608**	6.637***	6.671**
	(1.579)	(2.183)	(3.057)
Δc_t	-0.969*	-1.824***	-2.629***
	(0.511)	(0.638)	(1.007)
Constant	0.826***	1.043***	1.260***
	(0.166)	(0.204)	(0.277)
Observations	1,330	1,092	838
Number of countries	64	61	60

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

As we decomposed the series of GDP separately, we could face issues related to cross-section dependence. In particular, permanent shocks could be correlated across countries. To tackle this issue we verify our results using the Pesaran (2006) common correlated effects mean

[12] We use outlier-robust means instead of unweighted means.

[13] This is done because the first estimates are in general sensitive to the initialization which in turn is typically arbitrary.

[14] Note that the sample contains 21 advanced economies and 12 emerging markets.

group (CCEMG) estimator. Table 2 summarizes the results. Regressions (1) and (2), based on the CCEMG method, shows that the coefficients β_2 and β_3 have the same signs, remain singificant and are relatively close in magnitude when compared to regressions (3) and (4) where we used the MG method. Overall, our results are broadly robust to the CCEMG method which corrects for cross-section dependence and to the inclusion of a time trend.

Table 2. CCEMG Regressions: Dependent Variable g_t 1985-2009

Model	(1) CCEG	(2) CCEG Trend	(3) MG	(4) MG Trend
g_{t-1}	0.817***	0.478***	0.806***	0.425***
	(0.0370)	(0.0561)	(0.0307)	(0.0506)
$\Delta\tau_t$	5.937***	3.971**	6.637***	7.893***
	(1.790)	(1.972)	(2.183)	(2.404)
Δc_t	-0.815*	-0.948*	-1.824***	-2.152***
	(0.447)	(0.499)	(0.638)	(0.661)
trend		0.0184***		0.0199***
		(0.00364)		(0.00294)
$\bar{g}_t$	0.799***	0.759***		
	(0.170)	(0.131)		
$\bar{g}_{t-1}$	-0.630***	-0.728***		
	(0.162)	(0.135)		
$\bar{\Delta}\tau_t$	9.180**	3.073		
	(4.585)	(3.608)		
$\bar{\Delta}c_t$	-2.119*	-1.769*		
	(1.179)	(0.981)		
Constant	-0.110	1.690***	1.043***	2.768***
	(0.388)	(0.410)	(0.204)	(0.422)
Observations	1,092	1,087	1,092	1,087
Number of countries	61	60	61	60

Standard errors in parentheses *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 3. Regressions with debt: Dependent Variable g_t 1985-2009

	(1)	(2)	(3)	(4)
Model	MG	MG	CCEG	CCEG Trend
g_{t-1}	0.843***	0.798***	0.827***	0.424***
	(0.0305)	(0.0453)	(0.0598)	(0.0851)
$\Delta\tau_t$	6.653***	7.915***	8.750***	9.192***
	(2.237)	(2.927)	(2.691)	(2.947)
Δc_t	-1.788***	-1.951**	-1.471*	-2.697***
	(0.659)	(0.903)	(0.767)	(0.990)
$debt_{t-1}$		-0.000106	0.000908	-0.000114
		(0.000495)	(0.000720)	(0.000851)
trend				0.0206***
				(0.00516)
$\bar{g}_t$			0.625***	0.663***
			(0.170)	(0.153)
$\bar{g}_{t-1}$			-0.601***	-0.593***
			(0.194)	(0.175)
$\bar{\Delta}\tau_t$			-1.048	2.244
			(3.458)	(3.678)
$\bar{\Delta}c_t$			0.165	-0.500
			(0.917)	(0.942)
$de\bar{b}t_{t-1}$			-0.00249**	-0.000287
			(0.00122)	(0.00107)
Constant	0.994***	1.073***	1.035**	2.554***
	(0.203)	(0.274)	(0.478)	(0.624)
Observations	1,130	1,097	1,097	1,091
Number of countries	53	52	52	51

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Government spending could be influenced by the ratio of public debt to GDP. Therefore, we control for this ratio in the regressions in table 3. It appears that adding debt to GDP does not alter our results: $\beta_2 > 0$ and significant and $\beta_3 < 0$ and significant in all the regressions (MG and CCEG, with and without time trend). Moreover, the coefficient of debt is never significant. The result should not be interpreted as showing that debt never matters. What they seem to suggest is that the debt to GDP ratio could be a poor proxy for debt sustainability once the temporary vs. permanent shocks decomposition is taken into account.

Finally, we are interested in studying whether the magnitudes and signs of the coefficients β_2 and β_3 depend on income per capita. As shown in figure 2 there is no clear tendency for relatively poorer countries to have: (i) a more or less pronounced reaction to permanent shocks i.e. β_2's are not in general more positive in poorer countries and (ii) β_3's are not systematically negative or positive conditional on income per capita.[15] In other words, there is no clear tendency for poorer countries to be counter-cyclical.[16] If the coefficients of the fiscal reaction function to permanent and temporary shocks are not determined by income then what could be their potential determinants? How can we justify theoretically the result that in general fiscal spending depends positively on a permanent shocks and negatively on a temporary shocks? The next section proposes a theory to tackle both questions.

Figure 2. β_2**,** β_3 **and GDP per capita**

III. The model

In order to account for the impact of output shocks on government spending and test for the pro-cyclicality of the latter, we propose a standard forward looking Real Business Cycle (RBC) model exhibiting an endogenous public spending rule (i.e., spending reversals). Three types of agents are considered in this framework: households, firms, and a government.

[15]Income per capita is measured by GDP per capita in PPP terms in 1995 taken from Penn World Table 8.0.

[16]Note that the great dispersion of the coefficients could stem from a variety of factors (e.g. access to financial markets) and is difficult to interpret a priori. In the next section we explore some of the most plausible factors which could affect the magnitude of the responses to shocks.

A. Standard RBC setup

At each period t, the representative household consumes goods and services, invests in physical capital, buys government bonds, and sells labor services. Formally, the representative household's optimization problem is:

$$\max_{\{C_t,L_t\}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \eta \frac{L_t^{1+\mu}}{1+\mu} \right),$$

subject to:

$$P_t C_t + P_t I_t + \frac{B_t}{R_t} \leq (1-\tau_t) \left[W_t L_t + R_t^k K_t \right] + B_{t-1}, \tag{1}$$

where C_t is consumption, L_t is the share of hours worked, I_t is investment, B_t is the risk free bonds issued by the government, K_t is the predetermined stock of capital, P_t is the final good price index, W_t is the nominal wage rate, R_t is the nominal interest rate on government bonds, R_t^k is the nominal interest on rented capital, and τ_t is the tax rate on labor and capital revenues.

Parameters $\beta \in (0,1)$, $\sigma > 0$, $\eta > 0$, and $\mu > 0$ are the subjective discount factor, the inverse of the intertemporal elasticity of substitution, the weight on leisure in the utility function, and the inverse of the Frisch intertemporal elasticity of substitution in labor supply, respectively.

Investment, I_t, increases the household's stock of capital according to

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{2}$$

where $\delta \in (0,1)$ is the capital depreciation rate.

The first-order conditions associated with the optimal choices of C_t, L_t, K_{t+1}, and B_t are respectively given by:

$$\lambda_t = C_t^{-\sigma}, \tag{3}$$

$$\lambda_t = \frac{\eta L_t^{\mu}}{(1-\tau_t) w_t}, \tag{4}$$

$$\lambda_t = \beta \mathrm{E}_t \lambda_{t+1} \left[(1-\tau_{t+1}) r_{t+1}^k + 1 - \delta \right], \tag{5}$$

$$\lambda_t = \beta \mathrm{E}_t \lambda_{t+1} r_t, \tag{6}$$

where λ_t is the nonnegative Lagrange multiplier associated with the budget constraint, $r_t = \frac{R_t}{P_t}$, $r_t^k = \frac{R_t^k}{P_t}$, and $w_t = \frac{W_t}{P_t}$.

Final good's producers are perfectly competitive firms. the representative firm combines aggregate technology, A_t, K_t units of capital, and L_t units of labor to produce Y_t units of final goods according to a standard Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{7}$$

where $0 < \alpha < 1$ is the share of capital in the production function.

As in the empirical section, the technology, A_t, combines a permanent component, A_t^p, and a temporary component, A_t^t, with the form: $A_t = A_t^t \left(A_t^p\right)^{1-\alpha}$.[17] The two components follow the stochastic processes:

$$\log(A_t^p) = \log(A_{t-1}^p) + \varepsilon_t^p,$$

$$\log(A_t^t) = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}^t) + \varepsilon_t^t,$$

where $0 < \rho_A < 1$ and A is a constant. The serially uncorrelated shocks, ε_t^p and ε_t^t, are normally distributed with zero means and finite standard deviations σ^p and σ^t, respectively.

The first-order conditions of the firm's problem with respect to K_t and L_t are given by:

$$r_t^k = \alpha \frac{Y_t}{K_t}. \tag{8}$$

$$w_t = (1-\alpha)\frac{Y_t}{L_t}. \tag{9}$$

B. The government

We assume that the government is following an exogenous rule that stabilizes output fluctuations as well as the sum of future expected debt deviations from its historical average. In addition, we allow for some smoothing in government spending. Formally, this rule takes the form

$$\Delta\log(G_t) = \rho_g \Delta\log(G_{t-1}) - (1-\rho_g)\left[\rho_{g,y}\Delta\log(Y_t) + \rho_{g,b}\log\left(\frac{\zeta_t^b}{\bar{\zeta}_b}\right)\right] + \varepsilon_t^g, \tag{10}$$

where

$$\zeta_t^b = \mathrm{E}_t \sum_{i=0}^{\infty} \omega_{t+i} b_{t+i}. \tag{11}$$

[17]The permanent component is generally interpreted as labor productivity shocks. This form of aggregation implies a simple transformation of the variables of the model into stationary ones, which is necessary to finding the steady state and solving the dynamic model.

equation (11) corresponds to the expected weighted sum of future shares of debt over output, $b_{t+i} \equiv \frac{B_{t+i}}{Y_{t+i}}$. The serially uncorrelated shock, ε_t^g, is normally distributed with zero mean and finite standard deviation σ^g. Assuming that government spending reacts to the deviation of ζ_t^b from its steady-state value, $\bar{\zeta}_b$, reflects the unwillingness of the government to accumulate too much debt in the future. Note that the aversion to debt accumulation may have alternative patterns. For instance, we may assume that ω_{t+i} is declining over time—the government cares more about the near future levels of debt—or a bell-shaped function to possibly generate inflection points—the government cares more about the medium term levels of debt. Further, the assumed fiscal rule intends to reflect the fact that debt sustainability is an intertemporal phenomena as opposed to a rule that simply stabilizes a one period debt ratio.[18] At the same time, a deviation of the debt ratio from its steady state, which takes place far in the future, is reflected very mildly into the government spending decision rule.

Formally, we assume two alternative functional forms for ω_t:

- a declining function with respect to time, which takes the following form:

$$\omega_{1,t} = \beta_b^t, \tag{12}$$

  where the parameter β_b corresponds to the government's subjective discount factor of future levels of debt ratios.

- an asymmetric bell-shaped function with respect to time, which takes the following form:

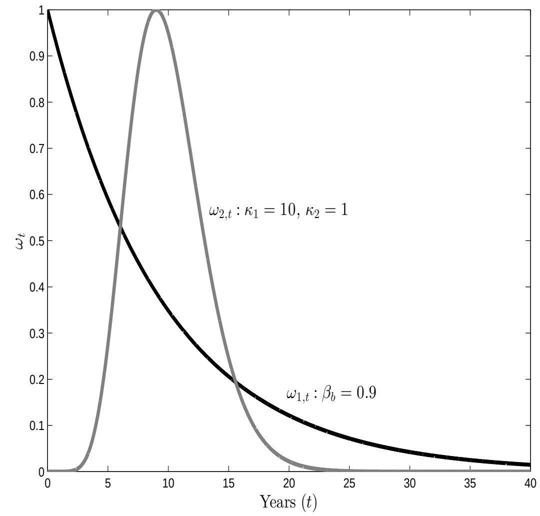
$$\omega_{2,t} = \frac{t^{\kappa_1 - 1} \exp\left(-\frac{t}{\kappa_2}\right)}{\kappa_2^{\kappa_1} \Gamma(\kappa_1)}, \tag{13}$$

  where $\Gamma(\kappa_1)$ is a gamma function evaluated at the parameter κ_1.[19]

[18]See Bohn (1998) for simple spending reversal rules

[19]The gamma Function is generally defined as $\int_0^\infty x^{\kappa_1 - 1} \exp(x) dx$, which corresponds to $(\kappa_1 - 1)!$ if κ_1 is a positive integer.

Figure 3. Future debt ratios weighting schemes



One can note that positive values for the parameter ρ_g^y reflect pro-cyclical government spending policies. Assuming ρ_g positive implies that the government is unwilling to face future hikes in its debt with respect to the initially identified target. A simple version of this debt-stabilizing spending rule is introduced by Corsetti, Meier, and Müller (2012) who define it as "spending reversals". In our specification debt spending reversal is introduced in an intertemporal form. In particular, the government cares about the discounted sum of the expected future debt deviation from the steady state in each period.[20] The rationale for adopting this specification is twofold. First, it captures the fact that future debt levels matter for a government; however, for obvious political reasons, a government puts higher weights on short- and medium-term levels of debt. Second, this specification is tractable in the sense that it yields an explicit solution for debt at the steady state and we can fully identify the sequence of debt at equilibrium given the set of state variables.

Finally, the budget constraint of the government is

$$\frac{B_t}{\kappa_t R_t} = B_{t-1} - \left(\tau_t Y_t - G_t\right). \tag{14}$$

To close the model we have to specify an additional tax policy feedback rule:

[20] The reaction function of public spending does not substitute the transversality condition, which is still imposed. Instead, as in Corsetti, Meier, and Müller (2012), this condition allows us to explicitly solve for the profile of debt at equilibrium.

$$\log\left(\frac{\tau_t}{\bar{\tau}}\right) = \gamma_{\tau,g}\log\left(\frac{g_t}{\bar{g}}\right) + \gamma_{\tau,b}\log\left(\frac{b_t}{\bar{b}}\right), \tag{15}$$

where g_t corresponds to government spending as a share of total output.

Given the stochastic processes for the structural shocks and the initial stocks of nominal debt, B_{t-1}, and capital, K_t, the equilibrium corresponds to a real price system $\{w_t, r_t, r_t^k\}_{t=0}^{\infty}$, an allocation $\left\{\frac{C_t}{A_t^p}, \frac{Y_t}{A_t^p}, L_t, \frac{I_t}{A_t^p}, \frac{K_{t+1}}{A_t^p}\right\}_{t=0}^{\infty}$, and a government policy $\left\{\frac{G_t}{A_t^p}, \tau_t, b_t\right\}_{t=0}^{\infty}$.[21]

IV. Theoretical Results

A. Calibration

The simulations are conducted based on an initial calibration of the structural parameters. We calibrate the deep parameters to values similar to those found in the literature by considering that a period corresponds to one year. Namely, the subjective discount rate, β, is set to 0.95, which implies that the annual real interest rate is equal to 5 percent. The elasticity of intertemporal substitution, σ, is assumed to be equal to 1 implying a log-normal utility function with respect to private consumption. the Frisch elasticity of labor supply is set to 0.75 based on the estimates from the microeconomic literature. The preference parameter μ is chosen so that the fraction of hours worked in the deterministic steady state is equal to 0.25. The depreciation rate, δ, is chosen to be 0.1 implying an average annual depreciation rate of capital equal to 10 percent. The share of physical capital in production, α, is assumed to be 0.36, which is standard in the literature. The steady-state shares of government spending and debt relative to total output are set to 0.2 and 0.4, respectively.[22]

[21] It is worth noting that solving the model with the discounting function equation (11) is done under a truncated presentation of the function $\omega_{2,t}$. In particular, we consider a horizon of 50 periods (years) only for this function, which is reasonable assuming that the government is adopting a debt sustainability problem over the same horizon. However, the model can be solved easily with the discounting functional form $\omega_{1,t}$, which allows ξ_t^b to be expressed in a simple recursive form under the log-linearized version.

[22] Tax rates are assumed to be equal to their steady-state level at each period. The steady-state tax rate is calibrated to satisfy the government budget constraint. The following section discusses the endogenous function of tax rates based on alternative calibrations of the parameters $\gamma_{\tau,g}$ and $\gamma_{\tau,b}$.

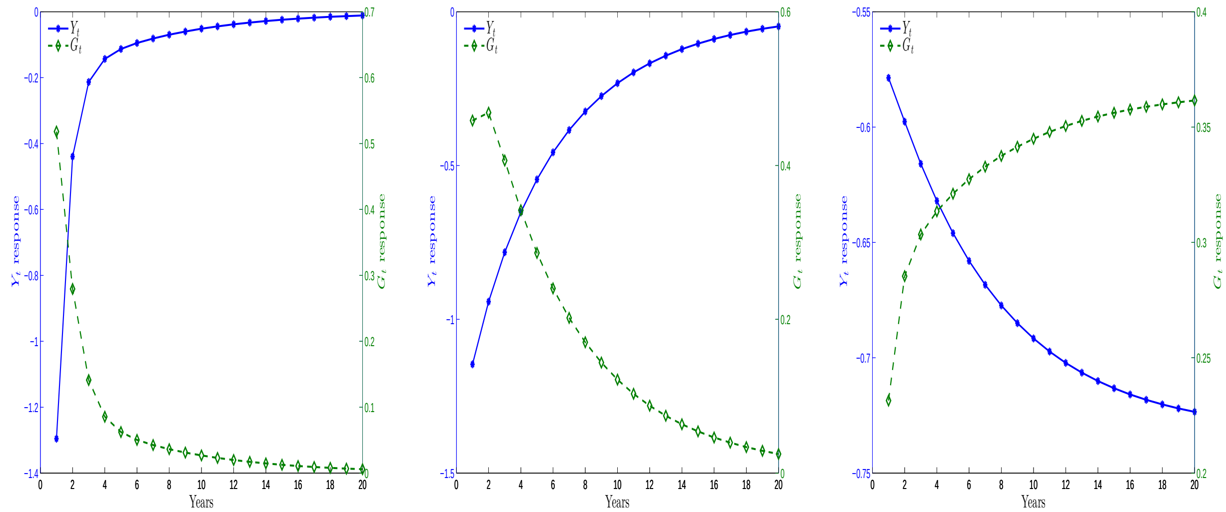
B. Impulse-response functions

The impulse responses portray the response of selected variables to a one percentage point decline in productivity for three different scenarios—based on alternative degrees of technology persistence—and two versions of the fiscal reaction function—with and without debt stabilization. Results are reported in Figures 4 to 7.

Transitory productivity shocks yield short lived blips in output as opposed to persistent shocks (see Figure 4). In this version of the model intertemporal substitution would lead households to increase their consumption a little and work more. Under the permanent income assumption, households would increase investment in an attempt to smooth out the shock. A permanent shock implies a less pronounced response of output on impact. The underlying mechanism is that once the shock is persistent enough, households prefer to initially switch their resources away from consumption and leisure owing to the low productivity of labor in the medium term. Figure 4 also shows that regardless of the degree of persistence, a negative technology shock yields opposite signs of impulse-response functions for output and government spending. Further, as the shock becomes less persistent, the conditional correlation between government spending and output approaches -1. The negative reaction of production is mitigated on impact by the increase in government demand. In a Ricardian environment consumption falls following higher government spending through the wealth effect and the interest rate channel. This moderates the multiplier effect of the counter-cyclical fiscal policy.

Figure 4. Dynamic responses to -1 percent technology shock: Without debt

(a) $\rho_{A^t} = 0.25$ (b) $\rho_{A^t} = 0.75$ (c) Permanent



Our analysis suggests that accounting for the unwillingness of a government to accumulate high levels of debt, as ratios of output, has non-trivial implications besides stabilizing borrowing. Figures 5 to 7 report the impact of the technology shock on output, government spending, and debt ratio for different degree of persistence while all other parameters remain constant at their baseline calibration. Assuming a mild persistence of technology, the impulse-response function of output is virtually the same as in the case without reaction to debt. The contemporaneous decline in output generates an instantaneous sharp increase of debt ratio. Moreover, the positive reaction of a counter-cyclical government through spending combined with the decline in government revenues drastically reduce the primary surplus and, thus, exacerbate the increase in sovereign debt. As a result, output declines by about 1.4 percent and debt overshoots by 2.5 percent on impact.

Consider now the transmission mechanism under the scenario of high persistence of technology. This feature contributes to switching the repones of government spending from positive to negative. The rationale for this result is straightforward; when the shock is persistent output decline is persistent and its impact on debt is twofold. Everything else equal, the debt ratio over total output increases proportionally, in absolute term, to the decline of output. Further, as tax rates remain at the steady state, tax revenues decline pushing fiscal authorities to increase their borrowing to keep their spending constant. The latter prevails despite a significant reaction of public expenditures to output deviations. It is worth noting that output reacts less aggressively to the persistent shock in the case with debt sensitive government rule. In fact, spending reversal helps increasing output because future reduction in spending entails positive wealth effects which enhances private consumption. This second round effect, although moderate, partially offsets the initial fall of output.

Figure 5. Dynamic responses to -1 percent technology shock: With debt (assuming $\omega_{1,t}$)

(a) $\rho_{A^t} = 0.25$ (b) $\rho_{A^t} = 0.75$ (c) Permanent

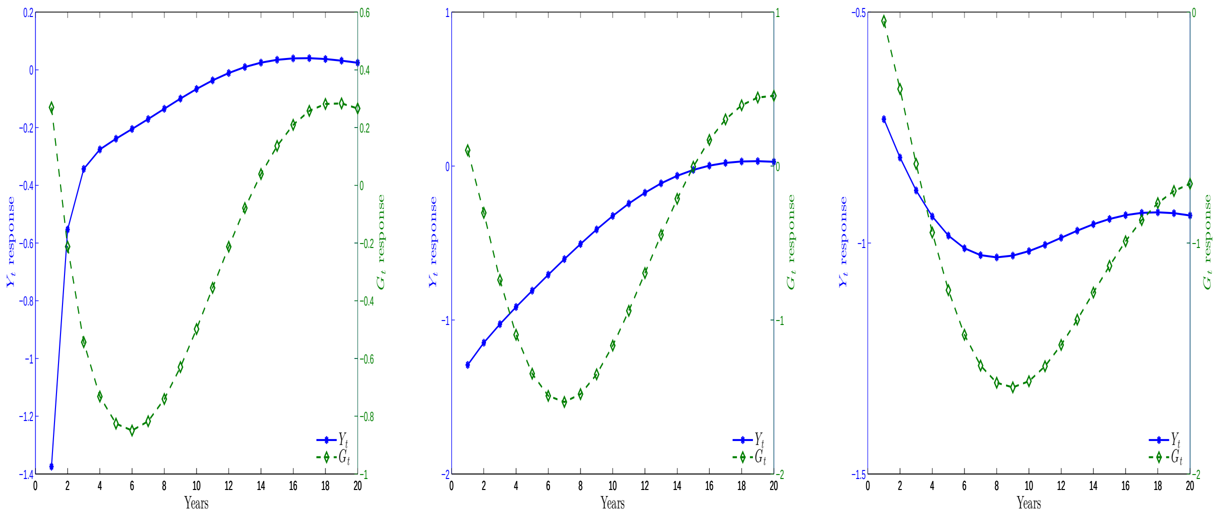


Figure 6. Dynamic responses to -1 percent technology shock: With debt (assuming $\omega_{2,t}$)

(a) $\rho_{A^t} = 0.25$ (b) $\rho_{A^t} = 0.75$ (c) Permanent

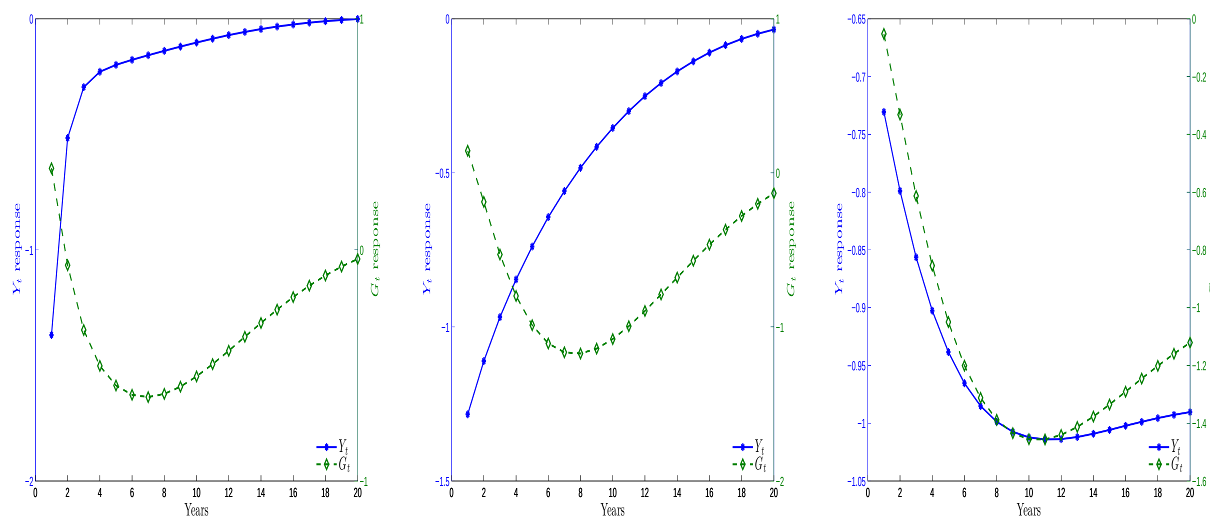
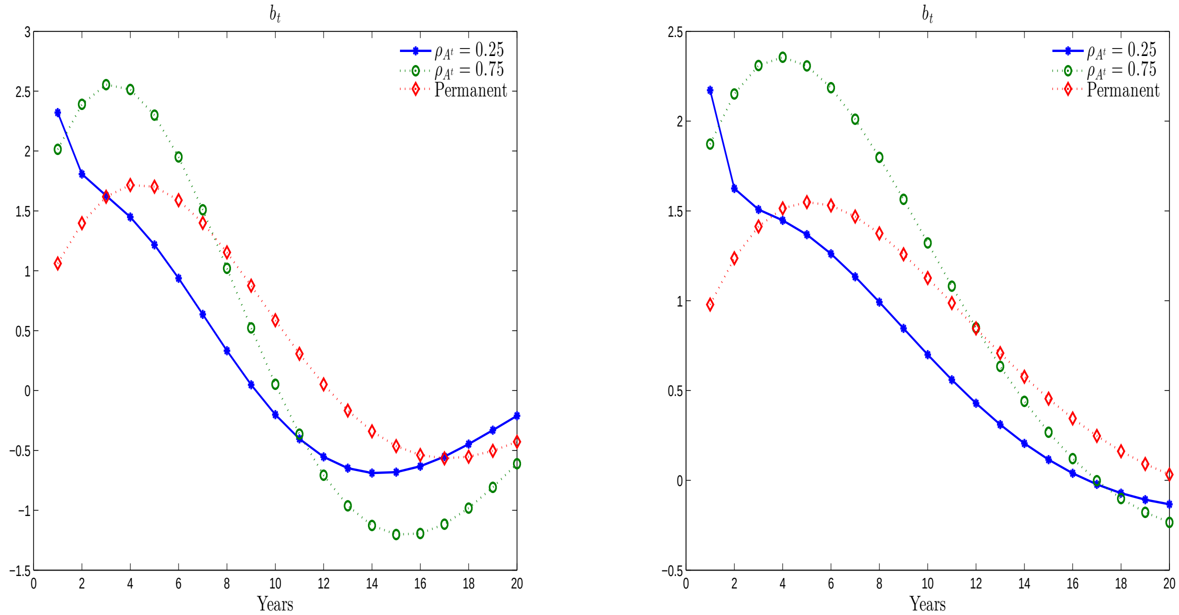


Figure 7. Dynamic responses of the debt ratio

(a) Assuming $\omega_{1,t}$ (b) Assuming $\omega_{2,t}$



The discounting scheme a government is adopting for future expected debt somehow matters for the responses to productivity shocks. Figures 5 and 6 show that a gradual discount, $\omega_{1,t}$, produces lower counter-cyclicality compared with the alternative discount factor, $\omega_{2,t}$. The reason is that a persistent negative technology shock yields an immediate overshooting of debt, which has a negative impact on spending as the government cares more about the near future levels of debt. Besides, following a permanent shock, government spending seem to be more correlated with output as the discount function allows less aversion to future debt expectations.

Another way of interpreting the result is through examining the theoretical conditional correlations between government spending and output generated by the calibrated model. Table 4 shows that the model is solely able to generate counter-cyclicality if the spending rule reacts uniquely to output fluctuations, regardless of the nature of shocks and the their persistence.

The two last rows of Table 4 report the conditional correlations when the future levels of debt are considered in the government spending rule. The two profiles of discounting debt expectations are alternatively adopted. One can notice that the degree of counter-cyclicality considerably falls proportionally to the persistence of the technology shock. Interestingly, when the shock is persistent, the conditional correlation turns positive as motivated in the

Table 4. Conditional correlations

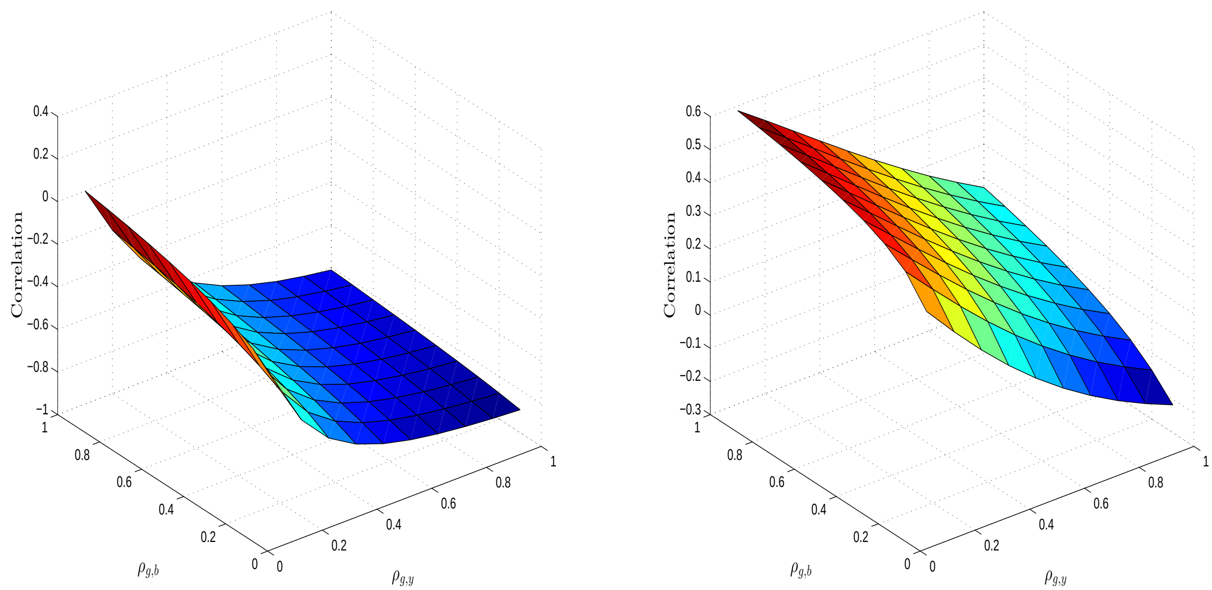
	$Corr(\Delta g_t, \Delta y_t \mid \varepsilon_t^t)$		$Corr(\Delta g_t, \Delta y_t \mid \varepsilon_t^p)$
	$\rho_{A^t} = 0.25$	$\rho_{A^t} = 0.75$	
Without debt	-0.983	-0.980	-0.979
With debt			
$\omega_{1,t}$	-0.655	-0.157	0.229
$\omega_{2,t}$	-0.837	-0.350	0.242

discussion of the impulse responses. As expected, the pro-cyclical behavior, owing to debt expectations in the fiscal rule, depends on the time profile of the discounting function, $\omega_{\{1,2\},t}$. Results indicate that under the same elasticity between government spending and debt expectations, when the government is highly discounting debt ratios in the near future—using $\omega_{1,t}$ function—spending become more stabilizing.

We conduct the sensitivity analysis of our model with respect to the main parameters of the fiscal spending rule to show how each component of its specification matters for our results. We examine the change in the conditional correlation between growth rates of government spending and output under different values of the elasticities $\rho_{g,y}$ and $\rho_{g,b}$. Results are reported in Figure 8.

Figure 8. Conditional correlations: Debt versus output stabilization

(a) $Corr(\Delta g_t, \Delta y_t \mid \varepsilon_t^t)$ (b) $Corr(\Delta g_t, \Delta y_t \mid \varepsilon_t^p)$



As described in the discussion of the impulse-response function, introducing debt in the rule dampens the correlation between spending and output conditional on a temporary shock; and this is particularly true when $\rho_{g,y}$ is low (see the left panel in Figure 8). The same happens for the permanent shock conditional correlation. For extreme values of the elasticities the correlations may shift sign, but for reasonable values government spending may become counter- or pro-cyclical conditional on persistent and permanent technology shocks, respectively.

C. Standard data filtering and the mismeasurement of cyclicality

Some of the key findings of the existing literature regarding the cyclicality of public spending are likely to hinge critically on the definition of trends and the statistical tool used to define the cycles. We evaluate in this section whether some purely statistical filters are capable of generating the same degree of government spending cyclicality using the model-based generated data.

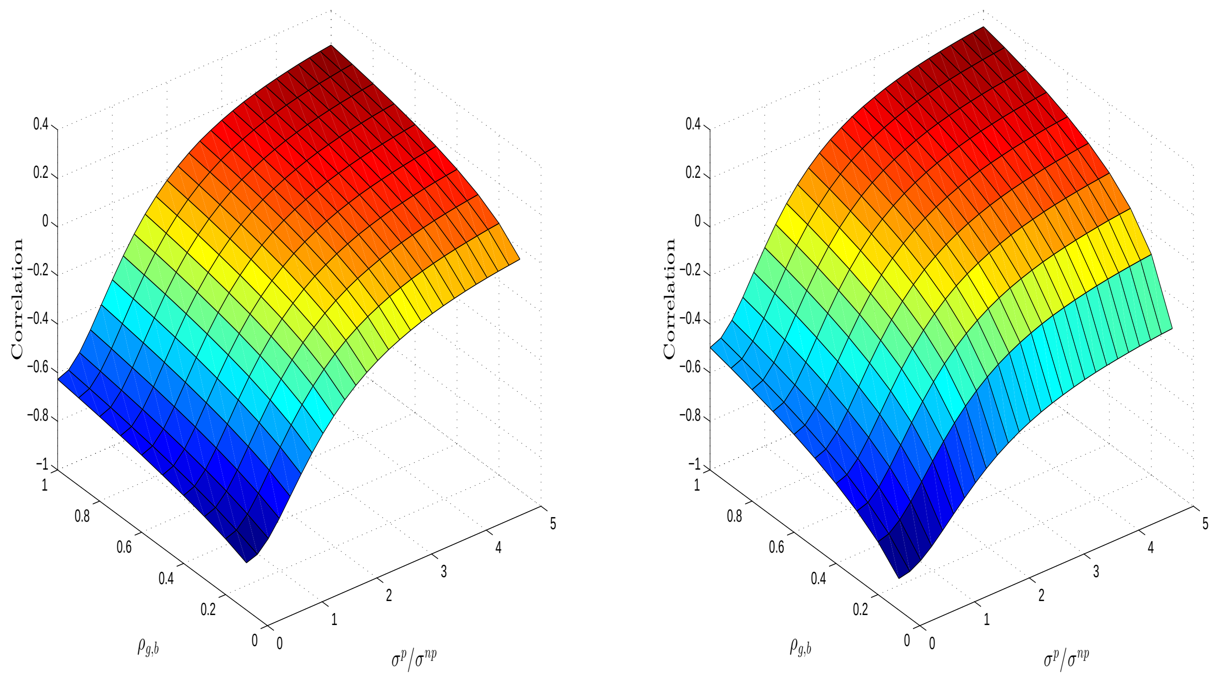
Considering the calibration used so far, and regardless of the values of the standard deviations of shocks, one should unambiguously observe counter-cyclical spending as dictated by the present model—the data generating process. In other words, the model by construction generates a negative correlation between spending and output conditional to transitory shocks when considering positive values for $\rho_{g,y}$. In the following, we check this premise by generating theoretical moments from the filtered non-stationary variables of the model, namely, real output and real government spending. In particular, we calculate the unconditional correlations between the two variables based on their growth rates and HP-filter transformations. We conduct this exercise under different calibrations of two key parameters (while keeping the others unchanged): the relative volatilities of shocks to the trend and those to the cycle, σ^p/σ^t, and the degree of aversion to future debt deviations, $\rho_{g,b}$.[23]

Figure 9 shows that filtering the data generated by the benchmark model may produce mismeasured cycles. Interestingly, the HP-filter- and the growth-rate-based unconditional correlations largely deviate from the model-based ones in the region where trend shocks are more volatile. The filters seem to dampen the effect of the reaction to deviations of the discounted sum of future debt ratios. For relatively strong shocks on the permanent component, the filters tend to produce pro-cyclical co-movements between government spending and output as opposed to the true measure of cyclicality.

[23] As reported in Figure 8, the model consistent conditional correlation between government spending and output should remain between -0.6 and -0.8, based on the calibration of the structural parameters and $\rho_{g,b}$ between 0 and 1.

Figure 9. Output and government spending unconditional correlations

(a) Growth rates (b) HP filter ($\lambda = 100$)



Consequently, once a country is hit more frequently with shocks to the trend, as in the context of a developing country, one should expect to mistakenly observe a pro-cyclical behavior of a government based of HP filtered data or growth rates.

As shown in the empirical analysis, the degree of cyclicality—captured by the correlation between government growth rates and the cyclical component of output—is quite variable across countries. The results also show that the heterogeneity cannot be directly attributed to the level of revenue per country. In Appendix A we discuss potential ways of altering the co-movements between the cyclical components of output and government spending. In particular, we investigate the role of: (i) the learning process about the source of supply shocks; (ii) the degree of aversion to future debt deviations; (iii) other sources of endogenous persistence, namely, habit formation in consumption and costly adjustments of investment; and (iv) endogenous tax rules.

V. CONCLUSION

This paper contributes to the study of the relationships between government spending and the business cycle. Based on a latent variable type trend-cycle decomposition we disentangle the effects of changes in the permanent and temporary components of income on government spending. Mean group panel regressions showed that government spending reacts positively to a permanent shock and negatively to a temporary shock. There is a relatively sizeable degree of dispersion in terms of country-specific “cyclicality"; i.e., the signs and magnitudes of the coefficients of the temporary and permanent shocks. Meanwhile, these coefficients do not seem to be determined by income per capita in contrast with the common belief that developing economies conduct a different type of fiscal policy.

To make sense of the empirical result we propose a theory combining stochastic trend shocks to productivity and a fiscal reaction function featuring the need to stabilize future debt levels for fiscal sustainability considerations. We established that an endogenous spending rule encompassing reactions to output and debt is important to explain the degree of cyclicality in the data. Extensions of the model shed light on some potential sources of the observed dispersion of cyclicality in the data. In particular, the degree of aversion to future debt levels, endogenous persistence, and the determinants of tax rates seem to be good candidates to understand the dispersion; however, learning about the source of the shock in an imperfect information environment tends to explain very little.

References

Abbas, S. M. Ali, Belhocine Nazim, Asmaa A ElGanainy, and Mark A. Horton, 2010, "A Historical Public Debt Database," IMF Working Papers 10/245, International Monetary Fund.

Acosta-Ormaechea, Santiago, and Atsuyoshi Morozumi, 2013, "Can a Government Enhance Long-Run Growth by Changing the Composition of Public Expenditure?" IMF Working Papers 13/162, International Monetary Fund.

Aguiar, Mark, and Gita Gopinath, 2007, "Emerging Market Business Cycles: The Cycle Is the Trend," *Journal of Political Economy*, Vol. 115, pp. 69–102.

Alesina, Alberto, Filipe R. Campante, and Guido Tabellini, 2008, "Why is Fiscal Policy Often Procyclical?" *Journal of the European Economic Association*, Vol. 6, No. 5, pp. 1006–1036.

Baxter, Marianne, and Robert G. King, 1999, "Measuring Business Cycles: Approximate Band-Pass Filters For Economic Time Series," *The Review of Economics and Statistics*, Vol. 81, No. 4, pp. 575–593.

Beveridge, Stephen, and Charles R. Nelson, 1981, "A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle'," *Journal of Monetary Economics*, Vol. 7, No. 2, pp. 151–174.

Bohn, Henning, 1998, "The Behavior Of U.S. Public Debt And Deficits," *The Quarterly Journal of Economics*, Vol. 113, No. 3, pp. 949–963.

Boz, Emine, Christian Daude, and C. Bora Durdu, 2011, "Emerging Market Business Cycles: Learning about the Trend," *Journal of Monetary Economics*, Vol. 58, pp. 616–631.

Burns, Arthur F., and Wesley C. Mitchell, 1946, *Measuring Business Cycles* (NBER).

Cherif, Reda, and Fuad Hasanov, 2012, "The Volatility Trap; Precautionary Saving, Investment, and Aggregate Risk," IMF Working Papers 12/134, International Monetary Fund.

———, 2013, "Oil Exporters' Dilemma: How Much to Save and How Much to Invest," *World Development*, Vol. 52, No. C, pp. 120–131.

Cogley, Timothy, and James M. Nason, 1995, "Effects of the Hodrick-Prescott filter on trend and difference stationary time series Implications for business cycle research," *Journal of Economic Dynamics and Control*, Vol. 19, No. 1-2, pp. 253–278.

Corsetti, Giancarlo, André Meier, and Gernot J. Müller, 2012, "Fiscal policy transmission with spending reversals," *Review of Economics and Statistics*, Vol. forthcoming.

Doan, Tom, 2006, "Perronbreaks: RATS procedure to compute various unit root tests with breaks," .

Francis, N., and V. A. Ramey, 2005, "Is the Technology-driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited," *Journal of Monetary*

Economics, Vol. 50, pp. 1379–1399.

Frankel, Jeffrey A., Carlos A. Vegh, and Guillermo Vuletin, 2013, “On graduation from fiscal procyclicality,” *Journal of Development Economics*, Vol. 100, No. 1, pp. 32–47.

Galí, Jordi, J.D. López-Salido, and J. Vallés, 2007, “Understanding the effects of government spending on consumption,” *Journal of the European Economic Association*, Vol. 5, pp. 227–270.

Gavin, Michael, and Roberto Perotti, 1997, “Fiscal Policy in Latin America,” in *NBER Macroeconomics Annual 1997, Volume 12*, NBER Chapters, pp. 11–72 (National Bureau of Economic Research, Inc).

Harvey, A. C., 1989, *Time Series Models and the Kalman Filter* (Cambridge University Press).

Hodrick, Robert J, and Edward C Prescott, 1997, “Postwar U.S. Business Cycles: An Empirical Investigation,” *Journal of Money, Credit and Banking*, Vol. 29, No. 1, pp. 1–16.

Ilzetzki, Ethan, and Carlos A. Vegh, 2008, “Procyclical Fiscal Policy in Developing Countries: Truth or Fiction?” NBER Working Papers 14191, National Bureau of Economic Research, Inc.

Kaminsky, Graciela L., Carmen M. Reinhart, and Carlos A. Vegh, 2005, “When It Rains, It Pours: Procyclical Capital Flows and Macroeconomic Policies,” in *NBER Macroeconomics Annual 2004, Volume 19*, NBER Chapters, pp. 11–82 (National Bureau of Economic Research, Inc).

Lane, Philip R., and Aaron Tornell, 1999, “The Voracity Effect,” *American Economic Review*, Vol. 89, No. 1, pp. 22–46.

Mendoza, Enrique G., and P. Marcelo Oviedo, 2006, “Fiscal Policy and Macroeconomic Uncertainty in Developing Countries: The Tale of the Tormented Insurer,” NBER Working Papers 12586, National Bureau of Economic Research, Inc.

Morley, James, Charles Nelson, and Eric Zivot, 2003, “Why are Beveridge-Nelson and Unobserved-component decompositions of GDP so Different?” Working Papers UWEC-2002-18-P, University of Washington, Department of Economics.

Perron, Pierre, 2005, “Dealing with Structural Breaks,” Boston University - Department of Economics - Working Papers Series WP2005-017, Boston University - Department of Economics.

Perron, Pierre, and Tatsuma Wada, 2009, “Let’s take a break: Trends and cycles in US real GDP,” *Journal of Monetary Economics*, Vol. 56, No. 6, pp. 749–765.

Pesaran, M. Hashem, 2006, “Estimation and Inference in Large Heterogeneous Panels with a Multifactor Error Structure,” *Econometrica*, Vol. 74, No. 4, pp. 967–1012.

Pesaran, M. Hashem, and Ron Smith, 1995, “Estimating long-run relationships from dynamic heterogeneous panels,” *Journal of Econometrics*, Vol. 68, No. 1, pp. 79–113.

Rebei, Nooman, 2014, "What (really) accounts for the fall in hours after a technology shock?" *Journal of Economic Dynamics and Control*, Vol. 45, pp. 330–352.

Schorfheide, F., 2000, "Function-Based Evaluation of DSGE Models," *Journal of Applied Econometrics*, Vol. 15, pp. 645–670.

Smets, F., and R. Wouters, 2003, "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, Vol. 1, pp. 1123–1175.

Talvi, Ernesto, and Carlos A. Vegh, 2005, "Tax base variability and procyclical fiscal policy in developing countries," *Journal of Development Economics*, Vol. 78, No. 1, pp. 156–190.

Wada, Tatsuma, and Pierre Perron, 2006, "State Space Model with Mixtures of Normals: Specifications and Applications to International Data," Boston University - Department of Economics - Working Papers Series WP2006-029, Boston University - Department of Economics.

APPENDIX A. SOURCES OF HETEROGENEITY IN THE DEGREE OF COUNTER-CYCLICALITY

A.1. Learning

So far we assumed that agents are fully informed about the source of a technology shock temporary versus permanent. One can argue that agents only observe a composite measure of productivity processes that corresponds to $Z_t = \log(A_t) - \log(A_{t-1}) + \varepsilon_t^z$, which can be expressed as follows:

$$Z_t = (1-\alpha)\varepsilon_t^p + \log(A_t^t) - \log(A_{t-1}^t) + \varepsilon_t^z,$$

where $\varepsilon_t^z \sim N(0, \sigma^z)$ is a noise term. As described by Boz, Daude, and Durdu (2011), for very low fluctuations ($\sigma^z \to 0$), Z_t fully reveals the components of productivity and the model features a perfect information environment.

Using the Kalman filter to conduct signal extraction exercises, agents are allowed to update their beliefs about the source of the productivity shock. We can map the signal extraction problem into the Kalman ÂĚfilter framework; i.e., state-space model (see Harvey, 1989). We assume that agents do not directly observe the source of the productivity shock and instead must form a belief and update this belief about the source of the productivity shock. They observe the composite productivity shock Z_t when forming forecasts of the underlying processes ε_t^p, $\log(A_t^t)$, and $\log(A_{t-1}^t)$.

In the state-space form, the measurement equation corresponds to:

$$Z_t = \underbrace{\begin{bmatrix} 1-\alpha & 1 & -1 & 1 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} \varepsilon_t^p \\ \log(A_t^t) \\ \log(A_{t-1}^t) \\ \varepsilon_t^z \end{bmatrix}}_{s_t}$$

The transition equation is given by

$$\begin{bmatrix} \varepsilon_t^p \\ \log(A_t^t) \\ \log(A_{t-1}^t) \\ \varepsilon_t^z \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho_{A^t} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{N} \begin{bmatrix} \varepsilon_{t-1}^p \\ \log(A_{t-1}^t) \\ \log(A_{t-2}^t) \\ \varepsilon_{t-1}^z \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{O} \underbrace{\begin{bmatrix} \varepsilon_t^p \\ \varepsilon_t^t \\ \varepsilon_t^z \end{bmatrix}}_{\varepsilon_t} \quad (16)$$

Assuming $\hat{s}_t$ the optimal estimator for s_t based on the set of information $\mathscr{I}_t$ (i.e., $\hat{s}_t \equiv E[s_t \mid \mathscr{I}_t]$), the covariance matrix estimation error is given by $V_t \equiv E\left[(s_t - \hat{s}_t)(s_t - \hat{s}_t)'\right]$. Harvey (1989) shows that the error covariance matrix would converge to a steady state, V, which can be derived from the solution of the following algebraic Riccati equation:

$$V = NVN' - NVM'(MVM')^{-1}MVN' + O\Omega O',$$

where Ω is the covariance matrix of ε_t.

Using the set of information, $\mathscr{I}_t$, and the transition equation (16) yield the one-step ahead forecast

$$a_{t|t-1} = Na_{t-1}.$$

The updating rule sets the posteriors a_t to be a combination of prior beliefs $a_{t|t-1}$ and the new signal Z_t

$$a_t = \left[I - VM'(MVM')^{-1}M\right] a_{t|t-1} + \left[VM'(MVM')^{-1}\right] Z_t,$$

where I is an identity matrix of size 4×4. Finally, the two last equations fully define the learning process.[24]

[24]The coefficient $\left[VM'(MVM')^{-1}\right]$ corresponds to the Kalman gain.

Figure 10. The effect of learning on the impulse-response functions

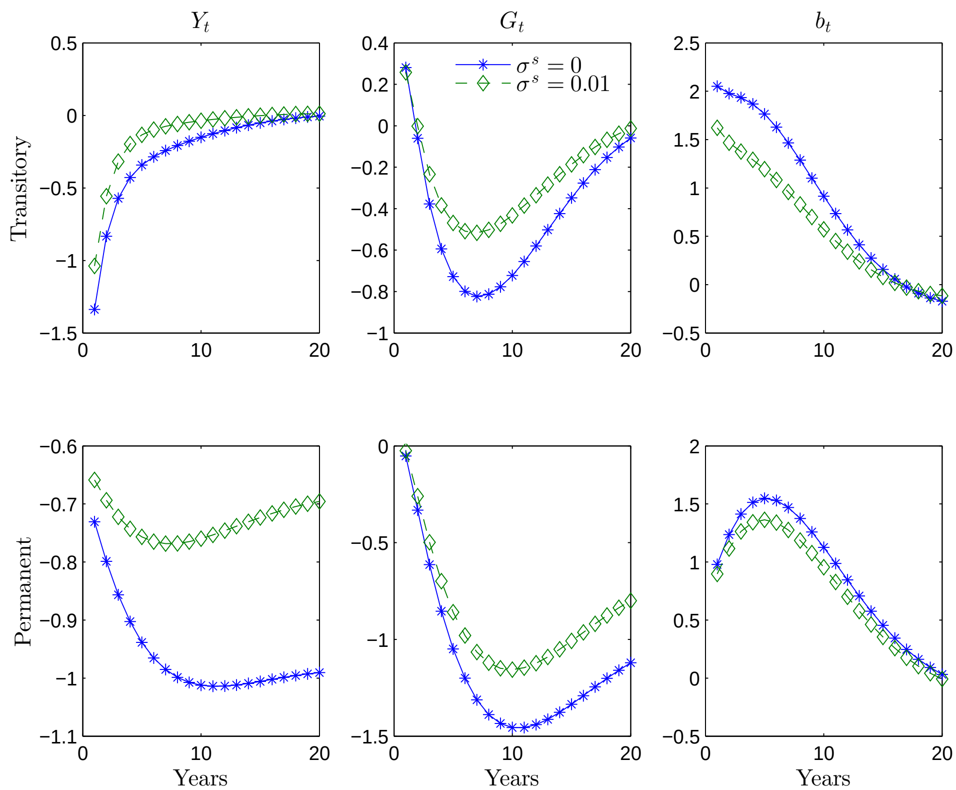
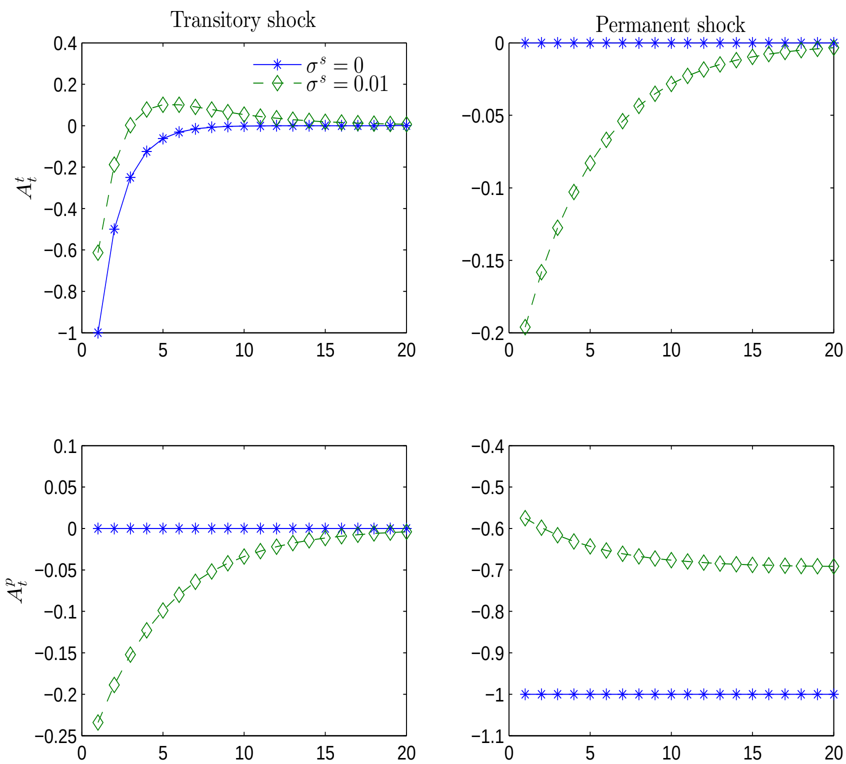


Figure 10 plots the impulse responses to a -1 percent transitory and permanent shocks, respectively. The figure also displays the impulse-response functions for the perfect and the imperfect information cases. The results show that the introduction of this particular learning mechanism into the benchmark model adds little to generate change in the degree of cyclicality. In the case of a transitory shock, the impulse responses are qualitatively similar under the two informational environments.

Figure 11. The effect of learning on beliefs about transitory and permanent components



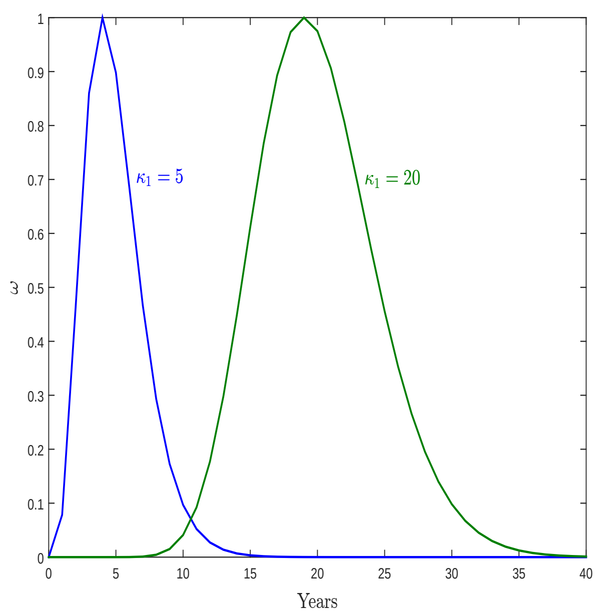
Following a trend shock the impact of learning on the response of output is notorious. When the shock occurs, agents also put some weight on the shock being temporary and respond by a smaller amount in the long term (see Figure 11). The reason is that part of the initial permanent shock is interpreted as temporary and the impact on the latter in the long term would vanish. Only the amount of the initial shock attributed to the permanent component of technology will persist resulting in a smaller reaction of output as well as government spending and debt. Similarly, the effect of a temporary shock is mildly lower on impact. The reason is that the genuine shock is partly interpreted as permanent driving output to react slowly in the short term. Surprisingly, government spending reacts as in the full-information case owing to the smaller expected effect of debt in the following periods. Hence, one should expect an exacerbation of counter-cyclicality under imperfect information, although very limited.

The learning process does not seem to be a sufficient candidate to interpret the heterogeneous counter-cyclicality across countries given that the impulse-response functions are stable following a temporary shock and the conditional correlation between output and government spending is moderately affected.

A.2. Government degree of impatience

Now, consider the responses in Figure 13 when the parameter capturing the degree of government patience with respect to future debt fluctuations, κ_1, takes the values 5 and 20—less versus more patient government (see Figure 12). A negative persistent productivity shock leads to an overshooting of government spending when $\kappa_1 = 20$ since the government cares less about the medium term levels of debt. At the same time, the adjustment path of debt is delayed since the degree of high debt aversion of the government is relatively low. It follows that countries exhibiting high levels of κ_1 are expected to witness high degree of countercyclical behaviors.[25]

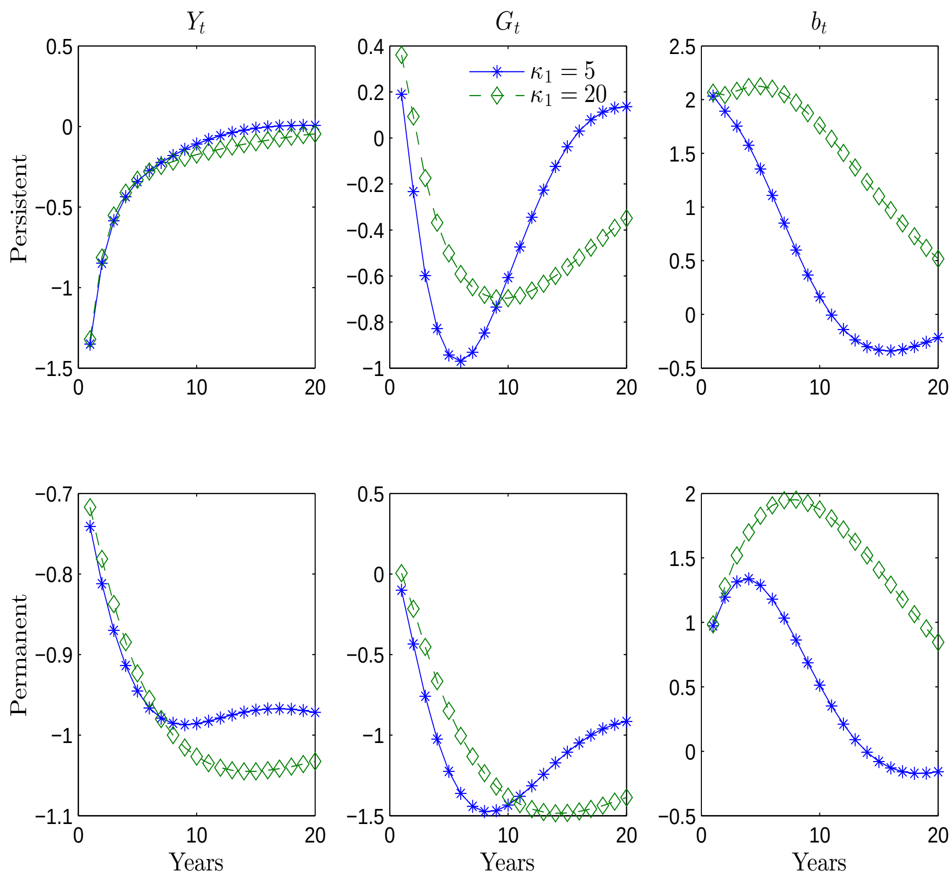
Figure 12. The profiles of the discount function



In the case of a permanent negative productivity shock the impulse-response functions of output and government spending are not very sensitive to the parameter κ_1 especially in the short term. However, a higher value of κ_1 generates the intuitive result corresponding to a higher persistence in the model.

[25]Although we do not explicitly model a risk premium in this model, an impatient government in this context may reflect this assumption.

Figure 13. The effect of government aversion on the impulse-response functions



A.3. Endogenous persistence

We discuss in this section the effect of real rigidities on the degree of counter-cyclicality. More specifically, we introduce two sources of real endogenous rigidities in the form of habit formation in consumption and costly capital adjustments. The functional form of period utility is now given by:

$$u(t) = \frac{(C_t - \psi C_{t-1})^{1-\sigma}}{1-\sigma} - \eta \frac{L_t^{1+\mu}}{1+\mu}$$

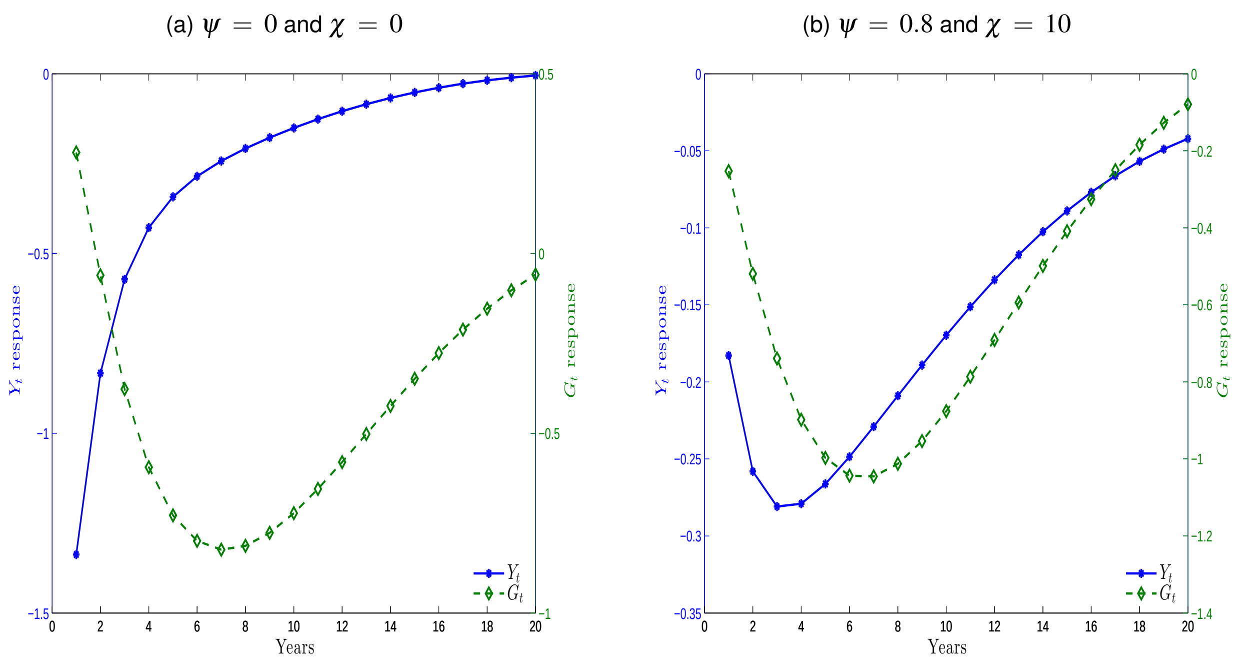
where the parameter ψ is the degree of habit formation for a typical household.

Further, Investment is subject to convex adjustment costs, and the transformed equation of accumulation of the following form:

$$K_{t+1} = (1-\delta)K_t + \left[1 - \Gamma\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{17}$$

and $\Gamma(\cdot)$ is an incurred cost when investment is changed over time. We restrict the investment adjustment cost function, Γ, to satisfy the following properties: $\Gamma(1) = 0$, $\Gamma'(1) = 0$, and $\chi = \Gamma''(1) > 0$.

Figure 14. Real rigidities and government spending cyclicality



As discussed by Francis and Ramey (2005) and Rebei (2014), combining habit formation and investment adjustment amplifies the sluggishness of the real variables responses. Following a transitory shock on productivity, households would like to smooth the shock by increasing investment, but since it is very costly to undertake new investment, they instead choose to consume more leisure in the short term. The latter generates a reversed hump-shaped response of output with a mild reaction on impact. Similar to the outcome of a permanent productivity shock, real rigidities may raise government concerns about debt sustainability, which discourages her from undertaking counter-cyclical spending policies following a transitory shock. Figure 14 shows that assuming reasonably high values for the habit and adjustment cost parameters yields a positive conditional correlation between output and government spending in contrast with the baseline calibration.[26]

[26] The parameters of habit formation and investment adjustment cost are set to 0.8 and 10, respectively; which are slightly in the upper side of the usual calibration or estimation found in the literature.

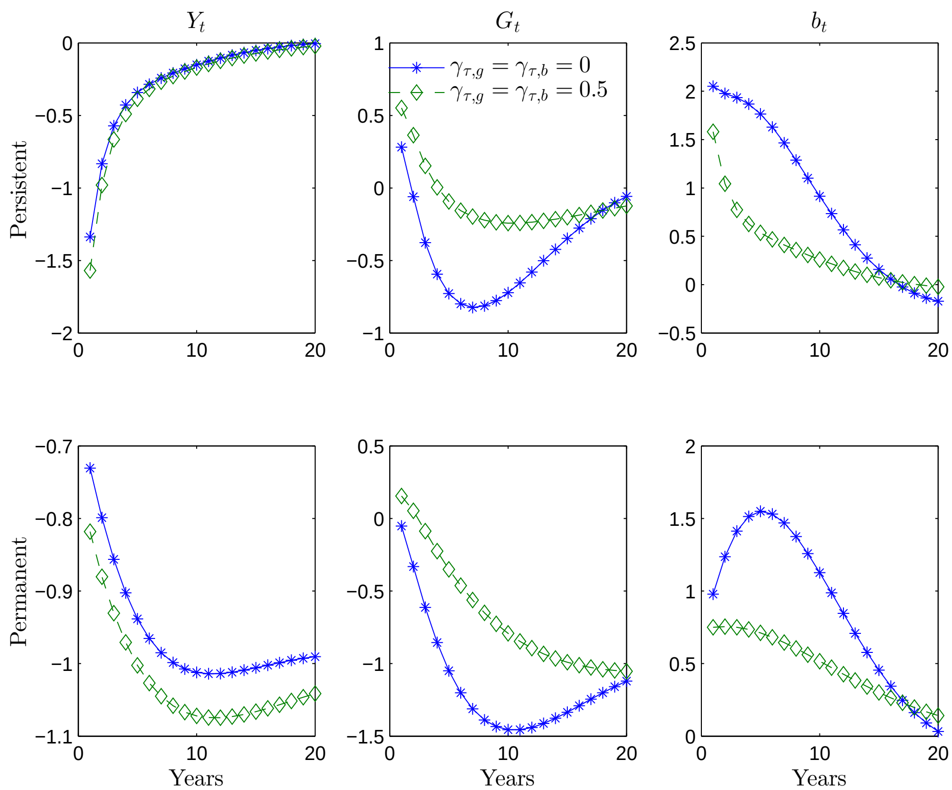
Clearly, our benchmark model is highly stylized, however, our extended model with additional sources of endogenous propagation mechanisms can explain the heterogeneity in the degree of cyclicality observed in the empirical analysis.

A.4. Endogenous tax rules

This section focuses on policy feedback rules for revenue taxes. As described in Section III, we assume the tax rates are given by a policy feedback rule, which is a function of the levels of government spending and debt—defined as shares of total output as in Galí, López-Salido, and Vallés (2007). Figure 15 shows the impulse-response functions under the two assumptions: exogenous and endogenous tax rates; while the remaining parameters are kept at their baseline values.

A clear result emerges from this figure. The impact of a negative transitory shock on output is exacerbated owing to the increase in tax rates as a reaction to higher government spending and debt immediate reactions. Consequently, an increase in the government spending becomes more persistent when taxes are debt stabilizing. The conditional correlation between government spending and total output should be higher as the tax rates become reactive to the other fiscal variables. The same results is obtained following a permanent shock.

Figure 15. The effect of learning on the impulse-response functions



As a conclusion, a negative comovement of government spending and total output requires a sufficiently high response of taxes to debt; and we expect that countries exhibiting more counter-cyclicality may have a higher coefficient on debt in the tax rule.

A.5. Model-based country comparison

In this section we estimate the model based on data for two countries exhibiting different levels of public spending cyclicality. Then, we identify in the context of our model the sources of the differences between the two countries. Namely, we consider the US and Morocco with relatively high and low degree of counter-cyclicality, respectively, as shown in the empirical analysis (see Figure 2).

The model is estimated using Bayesian techniques that update prior distributions for the deep parameters, which are defined according to a reasonable calibration. The estimation is done using recursive simulation methods—the Metropolis-Hastings algorithm—, which has been applied to estimate similar dynamic stochastic general-equilibrium models in the literature (e.g., Schorfheide, 2000; Smets and Wouters, 2003). Once the model is log-linearized and solved, its state-space representation can be derived and the likelihood function can be evaluated using the Kalman filter. The Bayesian approach places a prior distribution on parameters and updates the prior through the likelihood function. Markov Chain Monte Carlo methods are used to generate the draws from the posterior distribution and based on the posterior draws one can make inference on the parameters.

To identify the shock processes during the estimation, we need to use at most the same number of actual series. The three shocks in the different versions of the model are: a persistent productivity shock, a permanent productivity shock, and a government spending shock. In addition, measurement errors in each of the observable variables are added. We consider the same variables used in the empirical section (i.e., growth rates of the real output and the growth rates of the real government spending); further, we include an additional variable capturing movements of the sock of debt as a ratio to output in the set of observed variables. We use annual series for the U.S. data for the period extending from 1970 until 2008.[27]

Before estimation, some parameters are fixed to avoid identification issues. As in other similar studies, we calibrate some parameters in order to match important stylized facts in the data. The subjective discount factor is calibrated to 0.95. The average annual rate of capital destruction, δ, has a value of 10 percent. The weight of leisure in utility, μ, is adjusted in each iteration so that the fraction of hours worked in the deterministic steady state is equal to 0.25. The inverse of Frisch elasticity, η, is set equal to 0.75. The remaining parameters are estimated.

The priors for the exogenous variables' autocorrelations are set using the beta distribution with a mean of 0.5 and a standard deviation of 0.15, and the priors for the standard deviations of the shock innovations are set using the inverse gamma distribution with a mean of 0.005 and a standard deviation of 4. For the government spending rule, we did not have strong priors. The degree of government spending growth reactions to output and discounted future debt ratios are assumed to have a uniform distribution with zero mean and one standard deviation, which produce rather uninformative priors. Similarly, we assumed zero mean priors for the coefficients of the distortionary tax rule; while the standard deviations for these priors were set at 0.25, producing reasonably loose priors. The average prior of the habit formation parameter is assumed to be equal to 0.5 with a standard deviation of 0.15 . Finally, a normal distribution was considered for the investment adjustment cost parameter, with prior mean and standard deviation set at 10 and 5, respectively.

[27]Gross government debt-to-GDP ratios are taken from the Abbas and others (2010) database.

The likelihood function is evaluated using the Kalman filter. Draws from the posterior distribution of the model parameters are generated with the Metropolis-Hastings algorithm.[28] The estimation results are shown in Table 5 as well as the prior distributions.

[28]In the estimation procedure, 200,000 draws are generated, and the first 40,000 draws are discarded. The scale factor for the jumping distribution in the Metropolis-Hastings algorithm is adjusted so that the acceptance rate is about 25 percent.

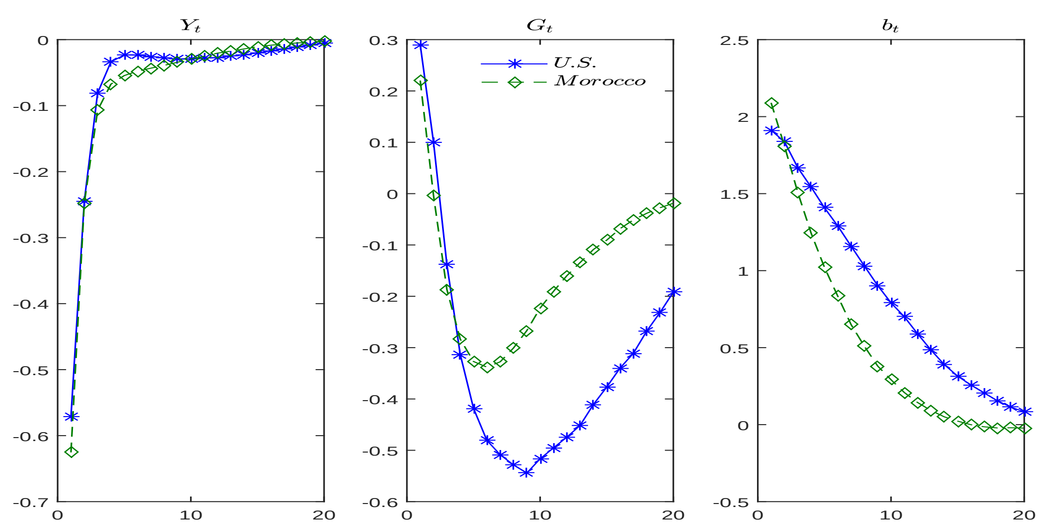
Table 5. Parameter Estimation Results

Parameters		**Prior**			**Posterior**							
					Unites States				**Morocco**			
					Benchmark model		Exogenous spending		Benchmark model		Exogenous spending	
		shape	mean	s.d.	mean	90 percent	mean	90 percent	mean	90 percent		
Real rigidity parameters												
ψ	habit formation in consumption	Beta	0.5	0.15	0.2656	[0.111, 0.413]	0.199	[0.083, 0.318]	0.242	[0.123, 0.368]	0.195	[0.091, 0.300]
χ	capital adjustment cost	Normal	10	5	13.870	[5.326, 22.155]	13.677	[5.036, 21.81]	11.532	[3.975, 18.842]	11.931	[3.915, 19.508]
Fiscal policy parameters												
$\rho_{g,y}$	gov. spending reaction to output	Uniform	0	1	1.251	[0.784, 1.732]	—	—	0.820	[0.081, 1.731]	—	—
$\rho_{g,b}$	gov. spending reaction to debt	Uniform	0	1	0.636	[0.035, 1.314]	—	—	1.251	[0.206, 2.327]	—	—
κ_1	debt discount function	Uniform	10	8	14.462	[6.051, 23.844]	—	—	11.435	[5.230, 17.704]	—	—
$\gamma_{\tau,g}$	elast. of taxes to gov. spending	Normal	0	0.25	0.559	[0.317, 0.797]	0.505	[0.325, 0.685]	0.220	[-0.090, 0.511]	0.145	[-0.132, 0.441]
$\gamma_{\tau,b}$	elast. of taxes to debt ratio	Normal	0	0.25	0.245	[0.060, 0.423]	—	—	0.362	[0.167, 0.543]	—	—
Exogenous processes parameters												
ρ_{A^t}	AR term in temp. technology	Beta	0.5	0.15	0.299	[0.165, 0.431]	0.271	[0.138, 0.393]	0.219	[0.108, 0.334]	0.230	[0.113, 0.343]
ρ_g	AR term in gov. spending	Beta	0.5	0.15	0.395	[0.222, 0.562]	0.379	[0.220, 0.553]	0.339	[0.155, 0.514]	0.253	[0.109, 0.397]
σ^p	σ perm. technology shock	Inv. gamma	0.01	4	0.059	[0.044, 0.073]	0.058	[0.045, 0.071]	0.134	[0.103, 0.166]	0.137	[0.106, 0.167]
σ^t	σ temp. technology shock	Inv. gamma	0.01	4	0.020	[0.015, 0.025]	0.019	[0.015, 0.024]	0.042	[0.032, 0.052]	0.045	[0.035, 0.056]
σ^g	σ gov. spending shock	Inv. gamma	0.01	4	0.030	[0.023, 0.036]	0.027	[0.022, 0.032]	0.106	[0.080, 0.129]	0.084	[0.068, 0.099]
σ^s	σ technology signal noise	Inv. gamma	0.005	4	0.006	[0.001, 0.013]	0.005	[0.001, 0.009]	0.005	[0.001, 0.010]	0.005	[0.001, 0.011]
Log-likelihood					208.733		205.287		120.008		116.187	

We conduct a formal test to identify whether government spending are endogenous as assumed in the benchmark model. From the results in Table 5, we notice that the model embedding an endogenous behavior of government spending is preferable under the assumed priors. The Bayes factor is largely in favor of this model regardless of the considered country. The marginal data density of the benchmark model is 3.45 and 3.82 higher on a log-scale for the US and Morocco, respectively; which translates into a posterior odds ratio largely in favor of the model with the government spending rule.

Table 5 reveals that our estimates of most standard parameters are consistent with those in the literature. We shall highlight some of the estimates affecting the behavior of government variables. First, the reaction of government spending to output is highly counter-cyclical in the US compared with Morocco—1.28 and 0.82, respectively. Second, the posterior mode of the parameter capturing the aversion to high future debt is larger in Morocco than in the US (1.25 versus 0.64). This result is important because a large (low) absolute elasticity between government spending and output (debt) is needed for this model to explain a high counter-cyclical fiscal policy. Further, the posterior mode for the parameter κ_1 is estimated to be larger in the US exhibiting a higher patience from the government regarding an increase in the share of debt over output—the government highly cares about debt changes after more than 14 years following the materialization of the shock compared to only 11 years for Morocco.

Figure 16. Transitory shock: U.S. versus Morocco



Our estimated model helps to distinguish the source of differences across countries in terms of the degree of cyclicality. In particular, compared to the government in Morocco, the US government is estimated to be: (i) more reactive to output fluctuations; (ii) less reactive to debt fluctuations; and (iii) less concerned with short to medium term debt. The left and middle panels of Figure 16 show that government spending is more negatively correlated with output in the case of the US than in Morocco, where the government is more concerned with bringing debt to its pre-shock steady state (see the right panel). This is reflected in the posterior mode correlation between government spending an output of -0.6 versus -0.4 in the US and Morocco, respectively.