Implications of Food Subsistence for Monetary Policy and Inflation

by Rafael Portillo, Luis-Felipe Zanna, Stephen O'Connell, and Richard Peck
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Abstract

We introduce subsistence requirements in food consumption into a simple new-Keynesian model with flexible food and sticky non-food prices. We study how the endogenous structural transformation that results from subsistence affects the dynamics of the economy, the design of monetary policy, and the properties of inflation at different levels of development. A calibrated version of the model encompasses both rich and poor countries and broadly replicates the properties of inflation across the development spectrum, including the dominant role played by changes in the relative price of food in poor countries. We derive a welfare-based loss function for the monetary authority and show that optimal policy calls for complete (in some cases near-complete) stabilization of sticky-price non-food inflation, despite the presence of a food-subsistence threshold. Subsistence amplifies the welfare losses of policy mistakes, however, raising the stakes for monetary policy at earlier stages of development.

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1 Introduction

Monetary policy in advanced and emerging economies underwent a radical transformation some thirty years ago, most notably with the advent of inflation targeting.\footnote{See Bernanke et al. (1999).} This resulted in greater focus on anchoring expectations, understanding the sources of inflation, and tailoring the policy response accordingly. The transformation was accompanied, although with a lag, by the emergence of the new-Keynesian macroeconomics literature, which helped clarify why the stabilization of inflation (and of which type) should be the objective of monetary policy.\footnote{See Clarida, Gali and Gertler (1999), Goodfriend and King (1997), and Woodford (2003).} The key insight is that variations in the inflation rate of those goods and services that have sticky prices are costly from a welfare perspective, since those nominal rigidities are the source of inefficient fluctuations in output. Instead, inflation that results from goods with flexible prices (e.g., food) need not be a source of concern and should actually be accommodated, as these movements are likely to reflect real shocks. Central banks should therefore concern themselves with stabilizing a narrower inflation measure consistent with the above distinction (“core”) rather than the broader consumer price index.\footnote{See Aoki (2001). Many caveats have emerged, e.g., stemming from the presence of nominal or real wage rigidities, or in the open economy context from potential trade externalities. But these have not fundamentally altered the above policy prescription.}

Low-income countries (LICs) are currently undergoing a similar process of policy modernization.\footnote{See Berg et al. (2015).} An open question is whether the lessons from the new-Keynesian literature also extend to these countries. Of particular interest is the role of food prices. These countries are often perceived to have particular structural characteristics that set them apart in this regard. The food sector (agriculture) represents a larger share of both production and consumption in LICs than in other countries. Furthermore, inflation in LICs is typically more volatile than in developed countries, with a larger share of that volatility reflecting movements in the relative price of food and arising from shocks to productivity in the agricultural sector. These shocks are very costly in these countries given the subsistence needs of the population. Moreover, the supply-side nature of inflation is also likely to imply a negative correlation between inflation and output. But where do these differences come from and what do they imply for the objectives of monetary policy?

In this paper we study the properties of inflation and the design of monetary policy—the nature of welfare-based loss functions and the design of targeting and instrument rules—in LICs. We do so by introducing a subsistence floor for food consumption into a simple new-Keynesian model with two
sectors (food and non-food). Key features of LICs emerge endogenously in the model and evolve as aggregate productivity increases. Consistent with the microeconomic evidence and a long tradition in development macroeconomics (e.g., Cardoso 1981), we assume that food prices are flexible and non-food prices are sticky. Our paper combines these elements in a fully articulated dynamic model with forward-looking expectations and an explicit treatment of monetary policy.

The presence of subsistence in the model leads to a version of what Chenery and Syrquin (1975) called the structural transformation. When the country is poor, i.e., it has a low level of aggregate productivity, it must allocate a larger fraction of labor (and capital) to help satisfy the subsistence need for food. As a result, the share of food in the CPI is large. As the country develops and moves away from subsistence, the economy is able to allocate a smaller fraction of total resources to the food sector, thus allowing for the relative expansion of the other sector (non-food, i.e., manufacturing and services). The share of food in the CPI declines as a result.

This process has important implications for monetary policy. First, because food is a flexible-price sector, structural transformation affects the aggregate importance of sticky prices in the economy. In addition, proximity to subsistence in the model reduces (increases) the income and price elasticities of demand in the food (non-food) sector, reduces the inter-temporal elasticity of substitution, and diminishes the effects of changes in food prices on household consumption. All of these features contribute to amplifying the effects of food-sector productivity shocks on the relative price of food and, therefore, on inflation, at earlier stages of development. The latter also implies that targeting alternative measures of inflation can have large welfare implications.

With the help of our model we present four key results. First, we simulate the model to assess whether it can replicate the main properties of inflation across the development spectrum. The calibration is such that the model encompasses the US and a group of African countries, by matching the pair of income per capita and food share in these countries. The economy is subject to a food productivity shock (the only real disturbance) and a shock to aggregate demand. Shocks are calibrated to replicate the properties of inflation in the US, at business cycle frequency. We then study the volatilities of these and other macro variables at earlier levels of development.

Our simple model replicates well some of the stylized facts of inflation across levels of development. About 50 percent of the volatility of inflation in LICs is accounted for by changes in the relative price of food (79 percent in the model), compared with 3 percent in the US (16 percent in the model). The model broadly generates the right co-movement between inflation and output: LICs tend to
have negative (or zero) inflation/output correlations; as countries develop, the correlation becomes increasingly positive. The model also generates inflation volatility in LICs that is about 160 percent higher than the volatility in the US. This falls short of the volatility observed in the data: inflation in African countries is about 300 percent more volatile than in the US. The model, however, under-(over-) predicts the volatility of changes in the relative price of food in LICs (the US). Overall, we interpret these results as broadly validating the use of our highly stylized model for monetary policy analysis in LICs.

Second, we analytically derive a welfare loss function based on a second-order approximation of the utility function of the representative agent. This welfare-theoretic approach allows us to determine the appropriate objectives of monetary policy. We show that the loss function can be expressed as a weighted sum of the variances of 3 variables: sticky-price non-food inflation, the aggregate output gap, and the gap of the relative price of food. We find that, despite food subsistence, optimal monetary policy, described as a targeting rule, calls for the complete stabilization of sticky-price non-food inflation, not headline inflation. This is sufficient to stabilize aggregate output and the relative price of food around their efficient levels. This finding is reminiscent of the results in Aoki (2001) and implies that a modified version of the “divine coincidence” of Blanchard and Galí (2007)—stabilizing inflation is sufficient to stabilize the output gap—still holds in our model with subsistence. In addition, the above result also applies to simple (non-optimal) instrument rules, so that the superiority of core inflation targeting is robust to alternative ways of modeling policy.

Third, we find that targeting headline inflation—thereby putting some weight on food inflation—implies greater welfare losses in poor countries than rich countries. This is because, in the presence of supply shocks, headline inflation stabilization requires larger adjustments in non-food inflation and non-food production in poor countries. Output volatility increases considerably as a result, which is welfare-reducing. In addition, we show that this effect is not solely due to the larger share of food in poor economies, but also to limited economy-wide substitutability in the presence of subsistence.

Fourth, we conclude by extending the model to study other features of LICs: limited asset market participation and the possibility that labor markets may be segmented. In this version of the model, the welfare-based loss function is now expressed as weighted sum of the variance of core inflation and an alternative measure of the output gap. The latter reflects the fact that the flexible price equilibrium of the model is no longer efficient. Though in principle a trade-off emerges, an analysis of targeting rules (which are more closely related to the objectives of policy) still yields near-perfect
core stabilization as preferable from a welfare perspective. The same does not hold for instrument rules however. We discuss why this is the case, and also argue against drawing definitive conclusions on policy objectives from the analysis of simple instrument rules.

The remainder of the paper is organized as follows. Section 2 discusses related literatures. Section 3 provides a description of stylized facts about food shares, food prices, inflation volatility and income. Section 4 introduces the model and presents some of its properties, while section 5 presents model simulations. Section 6 provides a welfare analysis of various policy rules, and section 7 discusses the extension to include limited asset market participation and segmented labor markets. Section 8 concludes.

2 Related Literature

In addition to the large literature on new-Keynesian macroeconomics in closed and open economies, our paper is related to two separate literatures. The first is the literature on structural transformation: Chenery and Syrquin (1975), Matsuyama (1992), Caselli and Coleman (2001), Kongsamut et al. (2001), Ngai and Pissarides (2007), and Rogerson (2008), among others. The second is a recent body of work that focuses on inflation in emerging markets and LICs, and the role of food: Walsh (2011), Adam et al. (2012), Anand and Prasad (2012), Anand et al. (2015), Andrle et al. (2013), Portillo and Zanna (2014), and Catao and Chang (2013 and 2015).

Our analysis follows the bulk of the structural transformation literature in assuming a closed economy (Herrendorf et al. 2014). For many LICs, this is perhaps not as restrictive as it first appears. Evidence from Gilbert (2011) on the pass-through of international grain prices into domestic prices suggests that domestic grain markets in LICs, particularly for rice, are indeed not strongly integrated with world markets. FAO et al. (2011) attribute this to a combination of restrictive trade policies and high transport and transaction costs. Gollin and Rogerson (2010, 2014) document the high costs of overland trade in Africa and argue that these costs can explain why the vast majority of the food consumed in many African countries does not enter international trade. If food is nontraded, then domestic supply and demand play a major role in determining its relative price regardless of whether or not non-food is traded (it is nontraded in our model).

Anand and Prasad (2012) and Anand et al. (2015) come closest to our specification, in a two-sector model with subsistence designed to study the appropriate inflation target for LICs. In contrast to our approach, they assume that some households are excluded from financial markets and that
there is perfect labor segmentation: labor is immobile across sectors, and the households that are excluded from (included in) financial markets supply labor exclusively to the food (non-food) sector. Anand et al. also include a traded good and shocks to non-food sector productivity and terms of trade. In such an environment they use numerical simulations to show that interest rate rules that respond to headline inflation are welfare-superior to rules that respond to core inflation. In section 7, we will briefly extend our model to include the two features above, and show that core inflation remains the main objective of policy. In that section, we clarify the apparent contradiction between their results and ours.5

3 Stylized Facts about Developed and Developing Countries

There are some key characteristics of developed and developing countries that we seek to reflect in our model. To elaborate on these characteristics, we collect a data set for the period 1995-2011, which comprises 28 OECD countries, 23 sub-Saharan African countries and 15 non-OECD countries, mostly emerging markets.6

(i) The share of food in the consumer price index falls as income rises.

The upper-left panel in Figure 1 plots the weight of food in the consumer price index against average income per capita in PPP dollars over the period 2001-2010.7 Income per capita for the US has been normalized to one. The relationship appears to be convex: the food share increases by more as income per capita decreases. This is captured by the good least-squares fit of the food shares to the log of GDP (the red dashed line). We also show the relation between income per capita and the share of food implied by the model (the black dashed line), which we derive below.

(ii) Food prices are more flexible than non-food prices.

Bils and Klenow (2004) were the first to examine the rates at which prices adjust, using the micro-data that the U.S. Bureau of Labor Statistics uses to calculate the monthly consumer price

---

5 While limited financial participation is a prominent feature of LIC economies, the assumption of segmented labor markets—implying complete labor immobility at business-cycle frequencies—is at odds with the informal and fluid nature of LIC labor markets (Fox 2015) and with our reading of the evidence on structural transformation in LICs (Gollin et al. 2013 and IMF 2012). We therefore allow for full labor mobility and focus on the role of subsistence per se for the bulk of our analysis.

6 The data for some countries, especially LICs, start in 2000.

7 GDP data are from the World Bank. Price indices are from the IMF. Food weights in the CPI come from several sources: OECD Stat Extracts for OECD countries and Haver Analytics for non-OECD non-African countries. Food weights for African countries come from central bank websites, a list of which is available upon request.
index. Since their work, there has been an explosion of papers using a similar strategy—examining prices underlying official CPI calculations—for dozens of countries. Table A.1 (in the Appendix) summarizes the average frequencies of price changes in each of these countries, for all products, food, and raw food, where reported. These papers show three patterns. First, food prices change more frequently than average. Second, unprocessed food prices change with markedly higher frequency than overall food prices. Finally, the difference in flexibility between food prices and overall prices is most pronounced in LICs, probably because a greater share of the food category is unprocessed in these countries. Our assumptions about price flexibility are therefore highly appropriate for LICs (like Sierra Leone, in the table). However, our model will understate the change in relative price stickiness as structural transformation occurs, because we do not model the shift towards more highly processed foods as income rises.

(iii) Inflation volatility falls as income rises.

The upper-right panel in Figure 1 shows the standard deviation of headline inflation (quarter-on-quarter) against income per capita. The focus here is on business-cycle frequency, and we use a band-pass filter that retains frequencies between 6 and 32 quarters.8 Note that there is also a decidedly negative relationship with real GDP per capita here: countries with lower income per capita have inflation rates that are considerably more volatile. The bottom-left panel shows that there is also a negative relationship between the volatility of changes in the relative price of food (in relation to the CPI) and income per capita.

(iv) The correlation between headline inflation and output increases with income.

The bottom-right panel in Figure 1 plots the correlation between headline inflation and output against income per capita at a business-cycle frequency. It reveals that there is a positive relationship between this variable and income per capita, starting from a negative value representing most of the LICs.

We now present a model consistent with these features.

8Lower-frequency movements in inflation are usually interpreted as changes in the explicit or implicit inflation target of the country, the choice of which is beyond the scope of our paper. We also drop higher-frequency movements in order to remove any noise or leftover seasonality.
4 The Model

4.1 The Consumer

The representative consumer chooses a consumption aggregate $c_t^*$, labor effort $n_t$ and holdings of a nominal bond $B_{t+1}$ to maximize lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln (c_t^*) - \frac{(n_t)^{1+\psi}}{1+\psi} \right],$$

subject to the budget constraint:

$$P_{F,t}c_{F,t} + P_{N,t}c_{N,t} + B_{t+1} = W_t n_t + \Pi_{F,t} + \Pi_{N,t} + R_{t-1}B_t,$$

and the composition of $c_t^*$:

$$c_t^* = Z (c_{F,t} - \bar{c}_F)^{\alpha_F} c_{N,t}^{1-\alpha_F}. \quad (1)$$

The pair $(c_{F,t}, c_{N,t})$ denotes consumption of food and non-food, valued at nominal prices $P_{F,t}$ and $P_{N,t}$. $W_t$ is the nominal wage, $\Pi_{F,t}$ and $\Pi_{N,t}$ are profits from food and non-food sectors, and $R_{t-1}$ is the gross nominal interest rate paid on bonds $B_t$. The parameter $\bar{c}_F$ indicates the subsistence level of food consumption, a threshold below which food consumption cannot decline. $Z$ is a scaling parameter that takes the value $(\alpha_F)^{-\alpha_F} (1 - \alpha_F)^{-(1-\alpha_F)}$ to simplify notation.

Utility maximization leads to the following first-order conditions:

$$(c_t^*)^{-1} = \beta E_t \left[ \frac{R_t}{\psi_{t+1}} (c_{t+1}^*)^{-1} \right], \quad (2)$$

$$(n_t)^{\psi} = w_t (c_t^*)^{-1}, \quad (3)$$

$$c_{F,t} = \bar{c}_F + \alpha_F \left( \frac{P_{F,t}}{P_t^*} \right)^{-1} c_t^* = \bar{c}_F + \alpha_F p_{F,t}^* c_t^*, \quad (4)$$

$$c_{N,t} = (1 - \alpha_F) \left( \frac{P_{N,t}}{P_t^*} \right)^{-1} c_t^* = (1 - \alpha_F) p_{N,t}^* c_t^*, \quad (5)$$

where $P_t^*$ is a price index that arises from utility maximization:

$$P_t^* = P_{F,t}^{\alpha_F} P_{N,t}^{1-\alpha_F}, \quad (6)$$

10
and \( \pi_t^* = P_t^* / P_{t-1}^* \) and \( w_t^* = W_t / P_t^* \) are the gross inflation rate and the real wage relative to that price index, respectively.

Note that \( c_t^* \) and \( P_t^* \) do not correspond to the aggregate consumption and the consumer price index that are actually measured. We define measured consumption \( c_t \) as follows:

\[
c_t = p_F c_{F,t} + p_N c_{N,t},
\]

where \( p_F = P_F / P \) and \( p_N = P_N / P \) denote the steady-state prices of food and non-food relative to the measured price index, given by:

\[
P_t = \left( \frac{p_{F,t} c_{F,t}}{c_t} \right) P_{F,t} + \left( \frac{p_{N,t} c_{N,t}}{c_t} \right) P_{N,t}.
\]

By now the choice of notation should be clear. Variables with an asterisk (\( c_t^* \), \( P_t^* \), \( \pi_t^* \), \( w_t^* \), \( p_{F,t}^* \), \( p_{N,t}^* \)) are relevant for consumer decisions but are not actually observed. We will refer to these as notional, in contrast with their observed counterparts (\( c_t \), \( P_t \), \( \pi_t \), \( w_t \), \( p_{F,t} \), \( p_{N,t} \)), where \( \pi_t = P_t / P_{t-1} \), and \( w_t = W_t / P_t \).

### 4.2 The Food Sector

The food sector features perfect competition and flexible prices. Production is given by:

\[
y_{F,t} = A_{F,t} (An_{F,t})^\alpha K_F^{1-\alpha},
\]

where \( K_F \) is the level of capital in the sector given the economy-wide level of labor augmenting productivity \( A \), \( n_{F,t} \) is the demand for labor in the food sector, \( \alpha \) is the labor share, and \( A_{F,t} \) is a productivity shock in agriculture. Our short-run analysis will take place around long-run equilibria (steady states) that correspond to different values for \( A \).

### 4.3 The Non-Food Sector

The non-food sector is composed of a continuum of monopolistic competitors, each providing a variety \( y_{N,t}(i) \), with \( i \in [0, 1] \). Varieties are combined by consumers into a Dixit-Stiglitz aggregate:

\[
y_{N,t} = \left[ \int y_{N,t}(i) \frac{1}{\alpha} \, di \right]^{\frac{1}{1-\alpha}},
\]
where $\epsilon$ is the elasticity of substitution between varieties. Cost minimization results in the following demand for variety $(i)$:

$$
y_{N,t}(i) = \frac{[P_{N,t}(i)]^{-\epsilon}}{P_{N,t}} y_{N,t},$$

where $P_{N,t}(i)$ is the price charged by firm “$i$” and $P_{N,t}$ is the price index for the entire sector:

$$
P_{N,t} = \left[ \int P_{N,t}(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}}.
$$

Production of non-food varieties is given by:

$$
y_{N,t}(i) = [A_n N, t(i)]^{\alpha} K_N^{1-\alpha}. \quad (11)
$$

As in Calvo (1983), firms are not allowed to change their prices unless they receive a random signal. The probability that a given price can be re-optimized in any particular period is constant and equal to $(1 - \theta)$. If firm $i$ gets the random signal at time $t$, it chooses a reset price $\bar{P}_{N,t}(i)$ to maximize its discounted stream of expected profits:

$$
Max \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} (\beta \theta)^j \lambda_{t+j} \left[ \frac{\bar{P}_{N,t}(i)}{P_{N,t+j}} \right]^{-\epsilon} y_{N,t+j} \left[ \bar{P}_{N,t}(i) - MC_{N,t+j}(i)(1 - \iota) \right] \right\},
$$

where $\lambda_{t+j}$ is the stochastic discount factor ($\lambda_{t+j} = c_t^i/c_{t+j}^i$), $\iota$ is an employment subsidy, and $MC_{N,t}(i)$ is firm $i$’s nominal marginal cost of producing one additional unit of variety $i$:

$$
MC_{N,t}(i) = \frac{W_t}{\alpha n N, t(i)^{\alpha-1} A^\alpha K_F^{1-\alpha}}. \quad (12)
$$

The aggregate price index in the non-food sector $P_{N,t}$ is the weighted sum of those prices that were reset (of which there is mass $1 - \theta$) and those that were not reset (of which there is mass $\theta$). Since the latter can be approximated with last period’s price index $P_{N,t-1}$, we get:

$$
P_{N,t} = \left[ (1 - \theta)\bar{P}_{N,t}^{1-\epsilon} + \theta P_{N,t-1}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \quad (13)
$$
4.4 Goods and Labor Market equilibrium

The description of the model is complete with the clearing conditions for food, non-food and labor markets:\(^9\)
\[ c_{F,t} = y_{F,t}, \quad c_{N,t} = y_{N,t}, \quad \text{and} \quad n_{F,t} + n_{N,t} = n_t, \]  
(14)

where \( n_{N,t} = \int n_{N,t}(i) \, di \).

We also define a real GDP measure \( y_t \), given by:
\[ y_t = p_F y_{F,t} + p_N y_{N,t}. \]  
(15)

4.5 The Steady State

The model’s structural transformation features are present at the steady state. We now analyse those features and some of their implications.

First, we assume \( \epsilon = 1/\epsilon \), which removes market power by monopolistic producers in the non-food sector. We set the relative price of food (\( p_F \)), the gross inflation rate (\( \pi \)), and food-sector specific total factor productivity (\( A_F \)) to 1:
\[ p_F = \pi = A_F = 1. \]

Setting \( p_F = 1 \) implies that all other relative prices (\( p_N, p^*_F, p^*_N \)) are also equal to one at steady state, and ensures that the notional and measured real wages are equal: \( w^* = w \). Setting \( \pi = 1 \) implies that gross notional inflation \( \pi^* \) is also equal to one.

The normalization of relative prices leads to a simple linear relation between \( c \) and \( c^* \), which is obtained by combining equations (4), (5) and (7):
\[ c = c_F + c^*. \]  
(16)

Firms choose steady-state values of capital by equating the marginal product of capital with the steady state rental rate \( 1/\beta - 1 \):
\[ (1 - \alpha)K_F^{-\alpha}A^\alpha n_F^\alpha = (1 - \alpha)K_N^{-\alpha}A^\alpha n_N^\alpha = (1/\beta - 1). \]  
(17)\footnote{For simplicity we assume the depreciation rate is zero, which implies there is no investment to keep track of in the model (including in the market clearing conditions).}
After some algebra we obtain a linear relation between real wages and aggregate labor productivity, as in the neoclassical growth model:

\[
    w = \left[ \frac{\alpha(1 - \alpha)^{1-\alpha}}{(1/\beta - 1)^{1-\alpha}} \right]^{1/\alpha} A = XA. \tag{18}
\]

Additional algebra also yields a relationship between measured consumption \(c\) and aggregate labor productivity \(A\):

\[
    \alpha^\psi c^\psi (c - \bar{c}_F) = X^{1+\psi} A^{1+\psi}.
\]

The presence of a subsistence threshold for food consumption \(\bar{c}_F\) makes the relationship between aggregate consumption (output) and economy-wide productivity nonlinear, with an elasticity that is below one but approaches one as labor productivity increases. In the background, the supply of labor satisfies:

\[
    n^\psi = X \left( \frac{A}{c - \bar{c}_F} \right).
\]

When consumption is close to subsistence, income effects dominate substitution effects in the supply of labor, and agents work more in order to satisfy their subsistence needs. As productivity and income increase, agents reduce their labor supply, which allows them to enjoy more leisure, at the cost of smaller increases in total consumption.

Changes in aggregate labor productivity also have implications for the share of expenditure and labor that is allocated to the food sector, which we denote as \(\gamma_F\):

\[
    \gamma_F = \frac{n_F}{n} = \frac{c_F}{c} = \frac{(1 - \alpha_F)\bar{c}_F + \alpha_F c}{c}, \quad \text{with} \quad \gamma_F > \alpha_F. \tag{19}
\]

When \(\bar{c}_F > 0\), \(\gamma_F\) converges to \(\alpha_F\) from above as steady-state consumption increases. Not surprisingly the relation between \(\gamma_F\) and \(c\) depends on the value of \(\bar{c}_F\). The higher the subsistence threshold, the greater the impact of income (consumption) on the food share. We will use this relationship to calibrate \(\bar{c}_F\) from the data.

Finally we define four new parameters that will be useful when presenting the log-linearized version of the model:

\[
    \xi = \frac{\gamma_F}{1 - \gamma_F} \geq \frac{\alpha_F}{1 - \alpha_F}, \quad \phi = \xi (1 - \alpha_F) - \alpha_F \geq 0,
\]
In the presence of subsistence, as steady-state consumption increases, $\xi$ converges toward $\alpha_F/(1-\alpha_F)$ from above, $\phi$ converges toward zero from above, and $\delta$ and $\sigma$ converge toward one, the former from below and the latter from above.

### 4.6 Log-linear Approximation to the Model

We now present the log-linearized version of the model. We focus on the features that are brought about by the existence of a subsistence threshold for food consumption, which in the log-linear version is captured by $\gamma_F > \alpha_F$ and the values of the related parameters ($\xi$, $\phi$, $\delta$, and $\sigma$). We also describe how these features change as the economy develops.

First is the forward-looking IS equation:

\[
\hat{y}_t = -\sigma^{-1}E_t \left( \hat{\rho}_t - \hat{\pi}_{t+1} + \phi \Delta \hat{p}_{F,t+1} \right) + \bar{E}_t \hat{y}_{t+1},
\]  

(20)

where a hat on top of a variable ($\hat{\ast}$) denotes percent deviations from steady state. The presence of subsistence introduces two modifications in this equation. First, the inter-temporal elasticity of substitution for output is given by $\sigma^{-1}$, which is less than one when $\gamma_F > \alpha_F$ ($\bar{c}_F > 0$). This is lower than the value that would obtain if $\bar{c} = 0$ (unity). This modification is related to the difference between the consumption aggregate that matters for private sector decisions ($c_t^*$) and measured consumption ($c_t$), with the former always smaller than the latter. The second difference concerns the presence of the expected change in relative food prices ($\Delta \hat{p}_{F,t+1}$). When $\gamma_F > \alpha_F$, there is a difference between the inflation rate that matters for private sector decisions ($\hat{\pi}_t^*$) and the measured headline inflation rate ($\hat{\pi}_t$); this difference equals $\phi \Delta \hat{p}_{F,t}$. As the economy develops, the inter-temporal elasticity of substitution converges to one, and the direct effect of changes in expected relative food prices in inter-temporal decisions falls to zero.

Second is the Frisch labor supply condition:

\[
\psi \hat{n}_t = \bar{w}_t + \phi \hat{p}_{F,t} - \sigma \hat{y}_t.
\]  

(21)

The presence of subsistence introduces the relative price of food as one of the direct determinants of
labor supply, in addition to the real wage and output. This reflects the fact that it is \( w^*_t \) that matters for households and not \( w_t \). This distinction lowers the substitution effect in labor supply relative to changes in \( \hat{\omega}_t \): changes in \( \hat{w}_t \) that are due to movements in \( \hat{p}_{F,t} \) will have a smaller effect on labor supply. Subsistence also raises the elasticity of labor supply with respect to changes in output (given by \( \sigma \)). As in the previous equation, the direct role of \( \hat{p}_{F,t} \) will tend to disappear as the economy develops.

Next is the demand equation for food:

\[
\hat{y}_{F,t} = -\delta \hat{p}_{F,t} + \delta \hat{y}_t. \tag{22}
\]

The parameter \( \delta \) now captures both the price and income elasticity of the demand for food. Subsistence reduces both elasticities, which would equal one if \( \tilde{c}_F = 0 \). Both parameters converge toward unity as steady-state consumption increases. Similar algebra yields the demand for non-food:

\[
\hat{y}_{N,t} = \delta \xi \hat{p}_{F,t} + \sigma \hat{y}_t \tag{23}
\]

The price elasticity of the demand for non-food (to the relative price of food) is greater in the presence of subsistence, and the income elasticity is higher than one.

The rest of the equations of the model are standard. Supply in both sectors is derived from profit maximization:

\[
\frac{1 - \alpha}{\alpha} \hat{y}_{F,t} = \hat{p}_{F,t} + \frac{1}{\alpha} \hat{A}_{F,t} - \hat{w}_t \tag{24}
\]

and

\[
\frac{1 - \alpha}{\alpha} \hat{y}_{N,t} = -\xi \hat{p}_{F,t} - \hat{\mu}_{N,t} - \hat{w}_t, \tag{25}
\]

where \( \hat{\mu}_{N,t} \) denotes changes in markups in the non-food sector. Inflation in the non-food sector is determined by a new-Keynesian Phillips curve:

\[
\hat{\pi}_{N,t} = \beta \hat{E}_t \hat{\pi}_{N,t+1} - \kappa \hat{\mu}_{N,t}, \tag{26}
\]

where \( \kappa \) is given by:

\[
\kappa = \frac{(1 - \theta \beta)(1 - \theta)\alpha}{\theta[\alpha + \epsilon(1 - \alpha)]}.
\]
Overall inflation is given by:
\[ \hat{\pi}_t = \hat{\pi}_{N,t} + \xi \Delta \hat{p}_{F,t}, \] (27)
and the definition of aggregate GDP and the relation between aggregate employment and output can be expressed as:

\[ \hat{y}_t = \gamma_F \hat{y}_{F,t} + (1 - \gamma_F) \hat{y}_{N,t} = \alpha \hat{n}_t + \gamma_F \hat{A}_{F,t}. \] (28)

We assume that technology in the food sector follows an autoregressive process of order 2: \(^{10}\)

\[ \hat{A}_{F,t} = (1 + \rho_A) \hat{A}_{F,t-1} - (\rho_A + q) \hat{A}_{F,t-2} + \varepsilon_{A_F,t}. \]

For the model simulations that attempt to match the aforementioned stylized facts, we describe monetary policy as the following rule:

\[ \hat{R}_t = (\hat{\omega}_{fex} + \xi \mathbb{E}_t \Delta \hat{p}_{F,t+1}^{fex}) + \xi \hat{\pi}_{N,t} + u_{MP,t}, \] (29)

where

\[ u_{MP,t} = \rho_{MP} u_{MP,t-1} + \varepsilon_{MP,t}. \]

When \( u_{MP,t} = 0 \), this rule ensures that core inflation is perfectly stabilized. Instead, a negative shock to \( \varepsilon_{MP,t} \) will generate a monetary policy loosening, which can be thought of as a shock to aggregate demand, and will affect core (and headline) inflation. This policy specification will therefore generate a simple dichotomy between supply and demand shocks.

For the welfare analysis, we focus instead on targeting rules because our interest is in understanding the “optimal” target of monetary policy. We consider the welfare implications from a more general policy that targets the inflation measure \( \hat{\pi}^\omega_t \) such that:

\[ \hat{\pi}^\omega_t = \omega \hat{\pi}_{F,t} + (1 - \omega) \hat{\pi}_{N,t} = 0 \quad \text{with} \quad \omega \in [0,1]. \] (30)

The targeting rule embeds the following specific cases: (i) non-food-inflation targeting, when \( \omega = 0 \); (ii) food-inflation targeting, when \( \omega = 1 \); and (iii) headline-inflation targeting, when \( \omega = \gamma_F \).

\(^{10}\)An AR(2) was found to provide a good empirical characterization of the process for international relative food prices. An AR(2) also allows for persistent effects on food inflation from shocks that have persistent but ultimately transitory effects on the relative price of food (AR(1) processes do not have this property).
in output that would hold if prices were flexible—potential output—and movements in output due to the presence of nominal rigidities—the output gap \( \hat{y}_t \)—with the latter directly related to inflationary pressures in the sticky-price sector.\(^\text{11}\) 

\[
\hat{y}_t = \hat{y}_{t}^{fex} + \hat{y}_{t}.
\]

This distinction can also be extended to other real variables such as the relative price of food:

\[
\hat{p}_{F,t} = \hat{p}_{F,t}^{fex} + \hat{p}_{F,t} \cdot \tag{32}
\]

5 Model Simulations

We now proceed to simulate the model to compare impulse response functions and inflation properties across the development spectrum.

5.1 Calibration

The calibration is summarized in Table 1. The choice of \((\alpha_F, \bar{c}_F)\) is such that the model encompasses the food share observed in the US and the median food share in a group of 16 African countries for which there are data, given their differences in income per capita. This can be seen by restating equation (19) for the US (rich country) and the median African country (poor):

\[
\gamma_{F,R} = (1 - \alpha_F)\bar{c}_F + \alpha_F \quad \text{and} \quad \gamma_{F,P} = \frac{(1 - \alpha_F)\bar{c}_F}{y_P} + \alpha_F,
\]

where we have normalized consumption (income) in the rich country to 1. Income per capita in this group of African countries over the period 2001-2010 is 2.9 percent that of the US \((c_P = y_P = 0.029)\), while the food shares \((\gamma_{F,R}, \gamma_{F,P})\) are \((0.08, 0.42)\), respectively.\(^\text{12}\) Given these values, the choice of \((\alpha_F, \bar{c}_F)\) ensures that the above relationship holds. The relation between food share and income generated by this calibration is shown in the upper-left panel of Figure 1. Note that the model does a reasonably good job of replicating the relation found in the data, though it tends to predict a lower food share for middle income countries than what is actually observed.

The choice of \((\alpha, \theta, \psi, \varsigma)\) is standard in the new-Keynesian literature when these models are

\(^{11}\)We present a thorough discussion of the flexible-price equilibrium and the gap representation of the model in the Appendix.

\(^{12}\)The African countries in the sample are Benin, Botswana, Burundi, Cape Verde, Cote d’Ivoire, Ethiopia, Kenya, Lesotho, Madagascar, Mauritius, Nigeria, Rwanda, South Africa, Tanzania, Uganda, and Zambia.
applied to the US.\textsuperscript{13} The parameters ($\rho_{MP}$, $\rho_A$) are chosen to match the observed persistence of the Federal Funds Rate and changes in the international relative price of food. Finally the standard deviations for the two shocks ($\sigma_{MP}$, $\sigma_{A'}$) and the value of $\varrho$ are chosen to match the volatilities of inflation and the relative price of food in the US.

5.2 Impulse Response Analysis

An exogenous monetary policy loosening ($\varepsilon_{MP,t} < 0$)

We first study the effect of an exogenous monetary policy loosening, which is captured by a negative shock to $\varepsilon_{MP,t}$. The effects of the shock for the poor and the rich country are shown in Figure 2. Unsurprisingly, the sector with flexible prices displays the biggest increase in prices following a monetary policy shock, producing an increase in the relative price of food. Relative food prices increase by more in the poor country than in the rich country, at roughly 2.5 and 2 percent, respectively. The poor country also experiences a larger increase in non-food inflation than the rich country, although the difference is small. Since the poor country has a much larger food share however, headline inflation increases by almost twice as much as in the rich country. Expansionary monetary policy, which would be neutral in the absence of price rigidities, results in an overall expansion of output. There are sectoral differences, however: the increase in the relative price of food translates into an expansion of the non-food sector and a contraction of the food sector, with a larger expansion (and smaller contraction) in the poor country.\textsuperscript{14} The overall expansion is higher in the rich country however, because of composition effects (larger non-food sector).

A negative shock to food production ($\varepsilon_{A',t} < 0$)

We now study the effect of a one percent decrease in productivity in the food sector $\varepsilon_{A',t}$ (Figure 3). Note that the decrease in productivity amplifies over time: food productivity is nearly 5 percent smaller after 20 quarters. Given the reduced substitutability in the economy—because of subsistence—the relative price of food increases by more in the poor country. This reduced substitutability is also reflected at the sectoral level: food production contracts by less, at the cost of a large contraction in the non-food sector. Non-food inflation does not increase in both cases due to the specification of the monetary policy rule. Because of the large food share however, headline inflation increases by more in the poor country.

\textsuperscript{13}See Gali (2008), among many others.
\textsuperscript{14}Output in the food sector declines because it is priced out of the labor market as non-food output expands. This lack of co-movement is typical of multisector new-Keynesian models.
A negative shock to food production ($\varepsilon_{A,F,t} < 0$) under headline inflation targeting

If monetary policy targets headline inflation ($\hat{\pi}_t = 0$, Figure A.1 in the Appendix), then the increase in the relative price of food described above must be compensated by a decrease in non-food inflation. In the presence of sticky prices, this can only come about via a demand-driven contraction in non-food production, which adds to the negative effects of the lower food productivity and results in a larger decrease in overall output.

In the case of the rich country, the effect is barely noticeable because of the small size of the food sector. A small decrease in non-food inflation is needed, requiring only a tiny contraction of non-food output. In the poor country, instead, the effect of targeting headline inflation is much larger because of the larger weight of food in the economy. Controlling headline inflation requires a larger decline in non-food prices and a larger decline in the non-food sector. The effect on aggregate output is therefore larger.

The lesson here is that the choice of inflation target is more important for output in the poor country than in the rich country, even though price stickiness is more relevant—because it affects a larger share of goods—in the rich country. We derive this analytically in section 6.

5.3 Second Order Moments

We now simulate the model and compare the model-generated second-order moments to those observed for the US and the median observation in our group of African countries. The data cover the period 1995:I to 2011:IV. We simulate the model for a period of 68 observations (as in the data), apply a bandpass filter to retain business-cycle frequency fluctuations, and then calculate the standard deviations for headline inflation, non-food inflation and changes in the relative price of food. We do this 1000 times and retain the average values along with the associated 95 percent confidence intervals (shown in brackets).

We calibrate the volatilities of the two shocks so that the model-generated volatilities of inflation and changes in the relative price of food for the rich country match up with the data for the US. We then adjust steady-state aggregate productivity to reproduce the food share observed in the poor economy, and compare the resulting model-generated volatilities with the volatilities we observe in Africa. We do this sequentially, looking first at food-sector shocks that reproduce the volatility of relative food prices in the US, and then at monetary-policy shocks that reproduce the volatility of non-food inflation. Finally, we combine both shocks.
Table 2 displays the results of the model when each type of shock is simulated separately, along with the standard deviations found in the data. First, it is worth stressing that headline and non-food inflation are considerably more volatile in the median African country in our group than in the US. The ratio between the two standard deviations is about 4 for headline and 2.2 for non-food inflation. Relative food prices are also more volatile, with a ratio of about 3. This is consistent with the cross-country evidence in Figure 1.

When only food productivity shocks are included, the model predicts that relative food prices should be 45 percent more volatile in the poor country than in the rich country. This is consistent with the analysis based on the impulse responses. With only monetary policy shocks, the model fails to generate increased volatility in either non-food inflation or relative food prices. However, different weights in the consumer price index imply that inflation in poor countries is twice as volatile. We infer from this result that while structural transformation amplifies the effects of food shocks on relative food prices in poor countries, it does not have the same effect under monetary policy shocks. In both cases however, structural transformation unequivocally increases the volatility of inflation in poor countries, although by less than what is observed in the data.

Table 3 shows the results of the simulation when both shocks are included. In this case, relative food prices and non-food inflation are less volatile in the poor country, which does not accord with the data. However, because of the weight of food in the economy, headline inflation is about 1.6 times more volatile. Looking at the decomposition of inflation into its two components, the model gets the relative importance of the components broadly right. There is one important difference between the model and the data, however. In the data, there is a slight negative correlation between $\hat{\pi}_{N,t}$ and $\Delta\hat{p}_{F,t}$, whereas the opposite is true in the model (not shown).

An additional finding is that, when hit with both shocks, the model broadly replicates the change in the correlation between output and inflation that is observed in the data (at annual frequency) for both the rich and the poor country. As inflation is driven to some extent by food supply shocks, and food prices account for a sizable share of the CPI, the correlation between output and inflation is low (and somewhat close to zero) for the median African country. In the US, instead, the correlation is higher and closer to one, as food price shocks play a smaller role in inflation dynamics and demand shocks dominate.

In sum, the model can help make sense of some of the properties of inflation that are observed in the data, although it falls short in others. There are many other reasons why inflation and relative
food prices are more volatile in poor countries, which we chose not to include in our model. First of all, central banks in developing countries have been less focused—at least until recently—on inflation stabilization than their counterparts in developed countries.\textsuperscript{15} It is precisely the transition towards more active regimes that motivates the analysis in this paper, so it is not surprising that the model generates less non-food inflation volatility than has been observed in recent history.

A different reason is that there are other aspects of structural transformation we have not analyzed. For example, technology adoption is also endogenous to the level of development. Countries at lower levels of development have production technologies in the food sector that are more vulnerable to exogenous factors such as the weather. For a given shock, the endogenous choice of technology will result in more volatile food prices. Another reason is that non-food prices may be more flexible in poor countries, while food prices may be more sticky in developed countries because of distribution costs or greater processing, as discussed earlier.\textsuperscript{16} Finally, less developed countries generally experience higher steady-state inflation on average, and this higher inflation has been shown to be associated with higher volatility.\textsuperscript{17} These additional features could push the simulations of the model toward the properties of the data. We leave the modelling of these issues for future research.

6 Welfare Analysis

6.1 Optimal Monetary Policy under Subsistence

In this section, we show analytically that despite the presence of food subsistence, optimal monetary policy requires complete stabilization of sticky-price non-food inflation. This turns out to be sufficient to stabilize both aggregate output and the relative price of food around their efficient levels.

\textsuperscript{15}Goncalves and Salles (2008) use difference-in-differences to assess the impact of adopting inflation targeting in a sample of 36 emerging-market economies. They find that inflation targeting significantly reduces inflation and inflation variability. While very few LICs have adopted formal inflation targeting, many are taking on elements of inflation targeting (Berg et al. 2015). It is not obvious, therefore, that our assumption of inflation targeting at all levels of income misses an important source of differential inflation volatility between low- and high-income countries.

\textsuperscript{16}Bowdler and Malik (2005) observe that trade openness is negatively correlated with the volatilities of reserve money growth and inflation. They posit two mechanisms for this relationship. First, international trade increases competition among producers of tradable goods. As a consequence, welfare losses due to inflation volatility are higher and governments work harder to avoid it. Second, trade openness supports a transition to production as well as consumption of high-value added products. Prices for such products are less volatile than less processed goods, and hence inflation volatility is lower.

\textsuperscript{17}Ball (1992) argues that when the level of inflation is higher, inflation volatility increases because the monetary authority vacillates between tolerating high inflation and accepting the high costs of disinflation. Gagnon (2009) investigates a different channel, looking at episodes of low and high inflation in Mexico between 1994 and 2002. He finds that the frequency of price changes is positively correlated with the inflation rate, particularly in periods of high inflation, a finding he attributes to menu costs.
To obtain these analytical results, we make the simplifying assumption that the labor share equals one—i.e., $\alpha = 1$—and derive a loss function using a second-order approximation to the utility losses faced by the representative agent due to deviations from the efficient equilibrium. The following proposition makes this loss function explicit.

**Proposition 1** Consider the model with food subsistence, described above, and assume that $\alpha = 1$. The average welfare loss per period is given by the following linear function:

$$L = \frac{1}{2} \sigma \left[ (1 - \gamma_F) \frac{\epsilon}{\kappa} \text{var}(\hat{\pi}_{N,t}) + (\psi + \sigma) \text{var}(\hat{y}_t) + \frac{\alpha_F}{1 - \gamma_F} \text{var}(\hat{p}_{F,t}) \right],$$

where $\sigma = \frac{1 - \alpha_F}{1 - \gamma_F}$ and $\kappa = \frac{(1 - \theta)\beta(1 - \theta)}{\theta}$. 

**Proof.** See Appendix.

Proposition 1 states that the welfare loss can be expressed as the weighted sum of the variances of sticky-price non-food inflation ($\hat{\pi}_{N,t}$), the aggregate output gap ($\hat{y}_t$), and the gap of the relative price of food ($\hat{p}_{F,t}$). Note that the weights are functions not only of the preference parameters ($\alpha_F$, $\psi$, $\epsilon$, $\beta$) and the degree of price stickiness ($\theta$), but also of the share of expenditures allocated to food ($\gamma_F$) and the related parameter $\sigma$. The latter parameters reflect subsistence and play an important role in determining the relative weights that the central bank gives to the variances of the aggregate output gap and the gap of the relative price of food, relative to sticky-price non-food inflation.

Figure A.2 in the Appendix plots these relative weights for the loss function (33)—i.e., $\frac{(\psi + \sigma)\kappa}{(1 - \gamma_F)^{\epsilon}}$ and $\frac{\alpha_F\kappa}{(1 - \gamma_F)^{\epsilon}}$—and shows that both relative weights are increasing in the degree of subsistence. The slope of the relative weight on the output gap is much steeper than that on the relative price of food. In particular, holding everything else constant, a poor country ($\gamma_F = 0.42$) should assign almost twice the weight a rich country should to the objective of stabilizing the output gap ($\gamma_F = 0.08$).

Although stabilizing aggregate output and the relative price of food around their efficient levels are appropriate goals for monetary policy—as implied by the loss function (33)—optimal policy is still characterized as a strict inflation-targeting regime. More specifically, despite food subsistence, optimal monetary policy corresponds to the complete stabilization of a core inflation measure, as in Aoki (2001). The appropriate core measure in our model is sticky-price non-food inflation. The following corollary formalizes this result.
Corollary 1  The welfare loss (33) can be rewritten as

\[
L = \frac{1}{2} \sigma \left\{ (1 - \gamma_F) \frac{\epsilon}{\kappa} \text{var}(\hat{\pi}_{N,t}) + \frac{1}{\kappa \kappa_g} \left[ (\psi + \sigma) + \frac{\alpha_F}{1 - \gamma_F} \left( \frac{\psi + \sigma}{\sigma} \right)^2 \right] \text{var}(\hat{\pi}_N - \beta \mathbb{E}_t \hat{\pi}_{N,t+1}) \right\}, \quad (34)
\]

and therefore optimal monetary policy corresponds to strict targeting of sticky-price non-food inflation, as implemented by setting \( \hat{\pi}_{N,t} = 0 \) for every \( t \).

**Proof.** See Appendix.

Corollary 1 implies that strict targeting of sticky-price non-food inflation maximizes social welfare. This approach completely stabilizes aggregate output and the relative price of food around their efficient levels. The “divine coincidence” of Blanchard and Galí (2007) therefore holds in our model: stabilizing inflation is equivalent to stabilizing the welfare-relevant output gap. The straightforward caveat, as in Aoki (2001), is that the monetary authority should target core inflation (\( \hat{\pi}_{N,t} \)) rather than headline inflation (\( \hat{\pi}_t = \gamma_F \hat{\pi}_{F,t} + (1 - \gamma_F) \hat{\pi}_{N,t} \)) to achieve that social optimum.

Food subsistence does not overturn the optimal policy result of strictly targeting core (sticky-price) inflation, but it raises the stakes for monetary stabilization policy. In particular, targeting headline inflation instead of core inflation is more costly in terms of welfare losses for countries that are closer to the subsistence threshold. Using the previously-described calibration, Table 4 calculates the welfare losses for poor and rich countries of targeting headline versus core inflation, when the economies are hit by a negative shock to productivity in the food sector. Similar results in terms of the ranking of policies can be found for Taylor rules that respond to non-food inflation versus Taylor rules that respond to headline inflation.

As expected, when both countries implement the optimal policy of targeting core inflation, standard deviations and welfare losses are equal to zero. Adopting headline inflation targeting increases the volatility of these economies and reduces welfare; and the welfare loss for the poor country is much greater than that of the rich country. In a poor country that faces a negative productivity shock in the food sector (which increases the relative price of food), keeping broad measures of inflation stable implies engineering large decreases in non-food inflation. These decreases are bigger in poor countries than in a rich countries, given the larger weight of food in the poor economy. And, because of sticky prices in the non-food sector, these drops are also accompanied by bigger contrac-

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18 Similar results in terms of the ranking of policies can be found for Taylor rules that respond to non-food inflation versus Taylor rules that respond to headline inflation.

19 This is consistent with the impulse response analysis of Figures 3 and A.1.
tions in non-food output and overall output in a poor country. Furthermore, the resulting increase in the volatility of output receives a larger weight in the poor country (Figure A.2 in the Appendix), inducing bigger welfare losses.

6.2 Subsistence is More Than a Higher Food Share

It is tempting to conclude that the importance of subsistence stems simply from generating a higher food share at lower levels of development. An argument could then be made that all that is necessary to analyze developing countries is the standard model without subsistence but with higher food share, i.e., \( \alpha_F = \gamma_F \approx \text{large} \). We now show that this is not the case, by comparing the welfare costs of targeting various measures of inflation (according to equation (30)), in a poor economy with food subsistence \((\alpha_F = 0.07, \gamma_F = 0.42)\) to the same economy without subsistence \((\alpha_F = \gamma_F = 0.42)\).

Figure 4 shows the standard deviation of the output gap and the welfare loss as \(\omega\), which is the weight on food inflation in the measure of inflation that is targeted by the central bank, goes from zero (no weight on food inflation) to one (only food inflation is stabilized), for the two economies mentioned above. For any positive weight on food inflation \((\omega > 0)\), both the volatility of aggregate output and the welfare losses are bigger for the poor country with subsistence, and are increasing in that weight. The volatilities of non-food inflation and the gap of the relative price of food (not shown) are broadly similar, so most of the variation in welfare stem from the impact on output. But what accounts for the higher output volatility and higher welfare costs?

A poor economy with subsistence is an economy with less substitutability relative to food, especially in response to negative shocks to productivity in that sector. Relocation away from agriculture is hampered by the need to maintain a certain level of food consumption. The limited economy-wide factor reallocation implies that supply-side shocks have bigger aggregate effects. The corollary is that equilibrium relative food prices will be more volatile under subsistence. In this context, targeting the wrong price level is particularly costly because the associated real adjustment shown in Figure A.1 in the Appendix increases with the level of relative food price volatility.

In addition to generating a more volatile output gap if policy is suboptimal, the economy with subsistence assigns a larger weight to output volatility in its loss function. This can be seen by comparing equation (33) to the welfare loss when there is no subsistence:

\[
\bar{L} = \frac{1}{2} \left[ (1 - \gamma_F) \frac{\epsilon}{\kappa} \text{var}(\hat{\pi}_{N,t}) + (\psi + 1) \text{var}(\hat{y}_t) + \frac{\gamma_F}{1 - \gamma_F} \text{var}(\hat{p}_{F,t}) \right].
\]  

(35)
With subsistence, the weight on output volatility corresponds to $\frac{1}{2}\sigma(\psi + \sigma)$ with $\sigma \geq 1$, while absent subsistence, it reduces to $\frac{1}{2}(\psi + 1)$. In sum, this exercise reinforces our view that subsistence raises the stakes for monetary policy at earlier stages of development.

7 Model Extension

We have shown that, despite the presence of subsistence, welfare continues to depend on the stability of inflation in the sticky-price sector, which is directly proportional to the output gap. Subsistence therefore does not overturn the “divine coincidence” result of Blanchard and Galí (2007) although, as we have discussed, it raises the stakes for stabilization policy. This result contrasts with recent findings by Anand and Prasad (2012) and Anand et al. (2015) (APZ).\footnote{APZ features an open economy model with imported goods, whereas Anand and Prasad’s specification is closer to ours as it assumes a closed-economy setting.} In a model that features subsistence but also limited asset market participation (LAMP) and segmented labor markets (SLM), these authors find that targeting headline inflation is superior to targeting core inflation from a welfare perspective. In this section, we briefly extend our model to include LAMP and SLM and revisit the design of monetary policy in this context (the model is presented in more detail in the Appendix).\footnote{For simplicity we continue to assume here that the labor share ($\alpha$) is 1.}

As in Anand and Prasad’s framework, we now introduce two types of agent, with one type providing labor services exclusively to the non-food sector (urban agents, which make up a share $\lambda^*$ of total agents) and the second type providing labor exclusively to the food sector (rural agents, with share $(1 - \lambda^*)$). The resulting labor market segmentation implies that wages do not equalize across sectors. In addition, again similarly to Anand and Prasad, we assume that rural agents do not have access to financial assets, which implies that their consumption is not sensitive to movements in interest rates.\footnote{Although one type of agent has access to financial assets and the other type does not, the consumption of each type is given by their income. This is because, in a closed economy setting such as ours, the net supply of assets is zero so that access to financial markets does not result in consumption smoothing or risk sharing unless there is heterogeneity within the set of agents with access to financial markets. This point is often overlooked in the discussion of models with limited asset market participation.}

The interaction of subsistence, LAMP and SLM affects the economy-wide response to food productivity shocks in important ways. We will focus on a negative shock to food productivity for illustration. First, SLM prevents the reallocation of labor across sectors, which amplifies the effects of the shock on sectoral production and leads to larger increase in relative food prices. Second, food
productivity shocks now have large and opposing effects on the income of the two types of agent. By lowering the price elasticity in the food sector, subsistence ensures that the negative shock has positive effects on real income from the food sector, as the increase in relative food prices more than compensates for the decrease in food production. This decreases labor supply from rural agents, which adds to the contraction in food production. The opposite holds for the non-food sector, so that negative food productivity shocks lower the level of income of urban agents and result in an increase in labor supply that contributes to an expansion in the non-food sector. Depending on how close the economy is to subsistence, the latter effect can dominate, to the point where total output increases in response to a negative food supply shock. This is the case in our model when the food share corresponds to the average in African countries.

What are the implications of the above discussion for monetary policy? To answer this question, we derive the welfare-based loss function for this version of the model, focusing on the case in which \( \lambda^* = (1 - \gamma_F) \).\(^{23}\)

**Proposition 2** Consider the model with food subsistence, limited asset market participation and segmented labor markets, and assume that \( \alpha = 1 \) and \( \lambda^* = (1 - \gamma_F) \). The average welfare loss per period, derived using a weighted sum of the utility of urban and rural agents, is given by the following function:

\[
L = \frac{1}{2} \left[ (1 - \gamma_F) \frac{\epsilon}{\kappa} \text{var}(\tilde{\pi}_{N,t}) + \frac{(\psi + \sigma)}{(1 - \gamma_F)} \text{var}(\tilde{y}_t) \right],
\]

where \( \tilde{\phi}_t = \hat{y}_t - \hat{\gamma}_{alt}^t \), where \( \hat{\gamma}_{alt}^t \neq \hat{\gamma}^{flex}_t \).

**Proof.** See Appendix. \( \blacksquare \)

As in the baseline version of the model, welfare continues to depend on core (and not headline) inflation volatility, although it no longer depends on the volatility of the gap in relative food prices. However, it now depends on an alternative measure of the output gap (\( \tilde{y}_t \)). This measure reflects the fact that the economy’s response to food productivity shocks is inefficiently low (because of the offsetting effect coming from non-food production), with the inefficiency stemming from the interaction of the three features mentioned above (subsistence, LAMP and SLM).\(^{24}\)

As a result of this inefficiency, there is now, at least in principle, a trade-off between non-food

\(^{23}\)This implies that the share of urban agents corresponds to the share of the non-food sector in the economy. This equality will arise endogenously if migration between sectors is allowed in the steady state.

\(^{24}\)Additional derivations (not shown) confirm that any two combinations of these two features are not enough to generate this result.
inflation and output stabilization. The divine coincidence breaks down: perfectly stabilizing non-food inflation is no longer optimal. In particular, optimal policy now calls for some degree of tightening when there is a negative food supply shock, so that the economy approaches the more efficient level of output, though at the cost of a decline in non-food inflation. Including food prices in the measure of inflation that is targeted is an indirect way of approaching the optimal policy prescription, as it will elicit the policy tightening mentioned above.

To assess whether this matters quantitatively, we evaluate targeting rules as in equation (30). These are shown in Figure 5, with each of the three panels showing results for a different degree of persistence in food productivity. Although perfect core (non-food) inflation stabilization is no longer optimal because of the trade-off mentioned above, it is still very close to optimal. In all three cases considered, the optimal weight on food inflation (indicated by the black vertical line) is minimal, between zero and 2 percent. This is much lower than the weight on food inflation in the CPI (at 0.42, as indicated by the dashed grey line), and it implies near-perfect core inflation stabilization. We conclude from this exercise that (near-perfect) core inflation stabilization remains the main objective of policy even in the presence of these additional features.

Finally, how can we reconcile these findings with those of APZ? These authors obtained different results using a standard interest rate rule:

\[
\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left( \phi_{\pi} \hat{\pi}_t^\omega + \phi_y \hat{y}_t \right),
\]

where \( \hat{\pi}_t^\omega \) is given by the first equality in equation (30). Figure 5 also includes a welfare evaluation of their rule, for the case in which \( \rho_R = 0.67, \phi_{\pi} = 1.5, \phi_y = 0.5 \). Unlike targeting rules, the optimal weight in their rule (given by the dashed black line) depends to a large extent on the persistence of the shock. Under a persistent AR(2) process like the one used throughout this paper, the optimal weight on food inflation is near zero. As the persistence decreases, the optimal weight on food inflation increases, and begins to approximate the weight in the CPI. With the persistence used in their paper (the middle panel), the optimal weight jumps to 15 percent; with i.i.d. shocks it increases further to 29 percent. The results in the two bottom panels are broadly consistent with the findings

\footnote{This is not surprising, as new-Keynesian models with Calvo prices tend to favor inflation stabilization over output.}

\footnote{These calculations use the concept of unconditional welfare, which we have used throughout the paper. Anand and Prasad (2012) and Anand et al. (2015), on the other hand, use the concept of conditional welfare. Results regarding the optimal weight, however, are very similar regardless of the welfare concept. Conditional welfare results are available upon request.}
in APZ, and would seem to suggest that central banks should strive to stabilize other measures of inflation than core (and with some non-zero weight on food).

Though the difference between interest rate rules and targeting rules appears striking, the above results stem from the coincidence of certain structural features of their model with the use of a conventional Taylor-type instrument rule. Under these particular conditions targeting headline inflation (along with output) can help stabilize core more successfully than targeting core itself! But, unlike in the case of targeting rules, this is not robust to various features of the model or the specification of the instrument rule. This is a well known issue in the design of monetary policy, discussed for example in Svensson (2003) and Svensson and Woodford (2005). Targeting rules are more closely related to the objectives of monetary policy and are therefore more robust to the parameters of the model, as the discussion in this section illustrates. Instrument rules are instead much more sensitive to the structure of the model and are not as directly related to the policy objectives. In the case of interest rates, near optimality is given by the ability of the rule to replicate movements in the natural rate of interest. As discussed in the Appendix, the combination of subsistence, LAMP and SLM can lead to an increase in real rates when there is a negative food productivity shock, and the size of the increase depends on the persistence of the shock. The less persistent the shock, the greater the increase in equilibrium real rates. In this case, assigning greater weight to food inflation can help generate the desired increase in real interest rates, but for reasons unrelated to the deeper policy objectives of the monetary authority.

8 Conclusions

We have studied the implications of food subsistence for monetary policy design and the properties of inflation across the development spectrum. Four key findings emerge from our analysis, which is based on a two-sector model in which a subsistence threshold induces a structural transformation of the economy as aggregate productivity rises. First, incorporating food subsistence into a simple new-Keynesian model can help make sense of the relatively higher volatility of inflation in poor countries and the tendency in these countries for inflation and output to be negatively correlated over the business cycle. Second, adding subsistence to an otherwise conventional two-sector model does not overturn the Aoki (2001) result that the monetary authority should seek to stabilize core inflation rather than headline inflation in order to maximize aggregate welfare. Third, a food-subsistence threshold raises the stakes for monetary policy in poor countries, because targeting the
wrong measure of inflation induces larger welfare losses the closer the economy is to the subsistence threshold. Finally, additional LIC features (limited asset market participation and segmented labor markets) create the possibility of a trade-off between core inflation stability and a measure of output, but near-perfect core inflation stabilization remains optimal.

Our second and fourth results are based on an explicit derivation of the central bank’s loss function, both in our baseline model with subsistence and in a variant that incorporates limited asset market participation and segmented labor markets. The explicit loss function allows us to illustrate the difference between targeting rules, defined broadly to accommodate any systematic attack on the elements of the loss function, and instrument rules, defined as simple feedback rules that determine the interest rate as a function of an inflation measure and a measure of real activity. We show that there are versions of our model—with very low persistence of the food-sector shock, limited asset-market participation, and zero labor mobility—in which headline inflation out-performs core inflation within a particular class of Taylor-type instrument rules (as in Anand et al. 2015). But in all such cases, it is core rather than headline inflation that the monetary authority cares about, and a targeting rule that keeps core inflation at zero is superior to the “best” of these instrument rules.

There are a variety of promising directions for extending this work, including the derivation of optimal targeting rules—for example, rules that incorporate the alternative output gap that appears in the central bank’s loss function—and the incorporation of additional features of the structural transformation.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{c}_F$</td>
<td>Subsistence level of food consumption</td>
<td>0.0099</td>
</tr>
<tr>
<td>$\alpha_F$</td>
<td>Non-subsistence food consumption share</td>
<td>0.0701</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labor income share</td>
<td>0.7</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Probability of not being able to reset price</td>
<td>0.75</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Response coefficient to non-food inflation in the rule</td>
<td>1.5</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Inverse of Frisch elasticity of labor supply</td>
<td>5</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of substitution between different varieties</td>
<td>6</td>
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<tr>
<td>$\rho_A$</td>
<td>Parameter in the AR(2) process for food productivity shocks</td>
<td>0.8</td>
</tr>
<tr>
<td>$\varrho$</td>
<td>Parameter in the AR(2) process for food productivity shocks</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>Standard deviation of food productivity shocks</td>
<td>0.6</td>
</tr>
<tr>
<td>$\rho_{MP}$</td>
<td>Persistence in the AR(1) process for monetary policy shocks</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sigma_{MP}$</td>
<td>Standard deviation of monetary policy shocks</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 1: Calibration.
<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th></th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food share in CPI (percentage)</td>
<td>8.0</td>
<td>39.4</td>
<td></td>
</tr>
<tr>
<td>GDP per capita</td>
<td>41.7</td>
<td>1.2</td>
<td>2.9%</td>
</tr>
<tr>
<td>PPP (thousands of dollars, avg 2001-2010)</td>
<td></td>
<td></td>
<td>0.3 (\sigma_{tf}) = 0.60</td>
</tr>
<tr>
<td>Volatility (standard deviations, BP filtered quarterly data, 1995:I-2011:IV))</td>
<td></td>
<td></td>
<td>0.26 (\sigma_{dev}) = 0.60</td>
</tr>
<tr>
<td>Headline inflation</td>
<td>0.29</td>
<td>1.18</td>
<td>4.06</td>
</tr>
<tr>
<td>Non Food inflation</td>
<td>0.34</td>
<td>0.74</td>
<td>2.20</td>
</tr>
<tr>
<td>Changes in the relative price of food</td>
<td>0.56</td>
<td>1.75</td>
<td>3.14</td>
</tr>
</tbody>
</table>

Table 2: Second Order Moments, Data and Model.
<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>(\sigma(t_f) = 0.60, \sigma({\text{dev}}) = 0.60)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>US</td>
<td>Med. Afr</td>
<td>Rich</td>
</tr>
<tr>
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**Volatility (standard deviations, BP filtered quarterly data, 1995:i-2011:iv))**

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<thead>
<tr>
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<td>3.14</td>
</tr>
</tbody>
</table>

**Inflation decomposition**

<table>
<thead>
<tr>
<th></th>
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<th>Model</th>
<th>(\sigma(t_f) = 0.60, \sigma({\text{dev}}) = 0.60)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>US</td>
<td>Med. Afr</td>
<td>Rich</td>
</tr>
<tr>
<td>Non food</td>
<td>1.11</td>
<td>0.75</td>
<td>0.64</td>
</tr>
<tr>
<td>Food weight*changes in the relative pri</td>
<td>0.02</td>
<td>0.52</td>
<td>0.17</td>
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**Correlation (BP filtered, annual data, 1995-2011)**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>(\sigma(t_f) = 0.60, \sigma({\text{dev}}) = 0.60)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US</td>
<td>Med. Afr</td>
<td>Rich</td>
</tr>
<tr>
<td>Output/Inflation</td>
<td>0.79</td>
<td>0.08</td>
<td>0.89</td>
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</table>

Table 3: Second Order Moments, Data and Model.
### Targeting Rules

<table>
<thead>
<tr>
<th></th>
<th>Poor Country</th>
<th>Rich Country</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Headline Inflation</td>
<td>Non-Food Inflation</td>
</tr>
<tr>
<td>( \sqrt{\text{var}(\hat{\pi}_{N,t})} )</td>
<td>0.374</td>
<td>0</td>
</tr>
<tr>
<td>( \sqrt{\text{var}(\hat{y}_t)} )</td>
<td>0.103</td>
<td>0</td>
</tr>
<tr>
<td>( \sqrt{\text{var}(\hat{p}_{F,t})} )</td>
<td>0.425</td>
<td>0</td>
</tr>
<tr>
<td>Welfare Loss</td>
<td>4.630</td>
<td>0</td>
</tr>
</tbody>
</table>

1/ For a poor country \( \gamma_F = 0.42 \), while for a rich country \( \gamma_F = 0.08 \).
2/ Headline inflation targeting: \( \gamma_F \hat{\pi}_{F,t} + (1 - \gamma_F) \hat{\pi}_{N,t} = 0 \).
3/ Core (non-food) inflation targeting: \( \hat{\pi}_{N,t} = 0 \).

Table 4: Welfare Losses from Alternative Targeting Rules, Rich and Poor Countries.
Figure 1: Stylized Facts.
Figure 2: A Monetary Policy Shock, $\varepsilon_{MP} < 0$. 

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Figure 3: A Shock to Food Sector Productivity, $\varepsilon_{AF} < 0$. 

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Figure 4: Standard Deviation of Output Gap and Welfare Loss, with and without Subsistence.
Figure 5: Welfare Losses at Alternative Food-Inflation Weights.
A Appendix

In this appendix we provide: (1) the flexible-price solution and gap representation of the model, (2) the proofs for Proposition 1 and Corollary 1, (3) an extension of our model to introduce limited asset participation and segmented labor markets, as in Anand and Prasad (2012), including the derivation of the loss function in such an environment, and (4) supplementary figures and tables.

A.1 The Flexible-Price Solution and the Gap Representation

We first solve for \( \hat{y}_t^{flex} \) and \( \hat{p}_{F,t}^{flex} \), by using the system (21)-(25) and (28) and imposing \( \hat{\mu}_{N,t} = 0 \):

\[
\hat{y}_t^{flex} = \Phi_y \hat{A}_{F,t},
\]

\[
\hat{p}_{F,t}^{flex} = -\Phi_{pf} \hat{A}_{F,t},
\]

where:

\[
\Phi_y = \gamma_F \phi + \left(1 + \frac{1-\alpha}{\alpha}\right) \xi \gamma_F (1+\psi) \\
\Phi_{pf} = \frac{2 \xi}{\alpha} \left[ \Upsilon - (\sigma - 1) \frac{1-\alpha}{\alpha} (1 + \psi) \right]
\]

and

\[
\Upsilon = \psi + 1 - \alpha + \sigma.
\]

Proximity to subsistence increases the response of real variables to food supply shocks: \( \Phi_y \) and \( \Phi_{pf} \) are both increasing functions of \( \gamma_F \). Note that when \( \bar{c}_F = 0 \), these two terms reduce to \( \Phi_y = \alpha_F \) and \( \Phi_{pf} = (1 - \alpha_F) \).

We then use the system (20)-(28) and the two flexible-price solutions to reduce the model to a system of two equations (the forward-looking IS curve and the new-Keynesian Phillips curve) and two unknowns (the output gap and the non-food inflation rate):

\[
\tilde{y}_t = -\Theta \mathbb{E}_t \left( \hat{R}_t - \tilde{\pi}_{N,t+1} - \hat{y}_t^{flex} \right) + \mathbb{E}_t \tilde{y}_{t+1}, \quad \text{(A.1)}
\]

\[
\hat{\pi}_{N,t} = \beta \mathbb{E}_t \tilde{\pi}_{N,t+1} + \kappa_y \tilde{y}_t, \quad \text{(A.2)}
\]
where:
\[
\Theta = \frac{\sigma^{-1}}{1 + \sigma^{-1} \phi \Omega},
\]
\[
\kappa_y = \kappa \left[ \frac{1 - \alpha (\sigma - 1) \phi + (1 + \frac{1 - \alpha}{\alpha} \delta) \xi \Upsilon}{\gamma_F \phi + \left( 1 + \frac{1 - \alpha}{\alpha} \delta \right) \xi (1 - \gamma_F)} \right] = \kappa \Gamma,
\]
and
\[
\Omega = \Gamma \left[ \frac{\gamma_F \Upsilon - (\sigma - 1) \frac{1 - \alpha}{\alpha} (1 - \gamma_F)}{\xi \left( 1 + \frac{1 - \alpha}{\alpha} \delta \right) \Upsilon + (\sigma - 1) \frac{1 - \alpha}{\alpha} \phi} \right].
\]

The rate $\hat{\rho}_{flex}$ is the natural rate of interest, the interest rate that would hold under flexible prices. It is given by:
\[
\hat{\rho}_{flex} = \Phi_{r,1} \hat{A}_{F,t} + \Phi_{r,2} \hat{A}_{F,t-1},
\]
where $\Phi_{r,1}$ and $\Phi_{r,2}$ have been derived by assuming that technology in the food sector follows an autoregressive process of order 2 and satisfy
\[
\Phi_{r,1} = \rho_A \left( \sigma \Phi_y + \phi \Phi_{p_f} \right) \quad \text{and} \quad \Phi_{r,2} = - (\rho_A + \phi) \left( \sigma \Phi_y + \phi \Phi_{p_f} \right).
\]

Having solved for the flexible-price equilibrium and the gap presentation, we can explain headline inflation as a combination of movements in non-food inflation, movements in the gap component of relative food prices, and movements in the flexible-price component of the latter variable:
\[
\hat{\pi}_t = \hat{\pi}_{N,t} + \xi \left( \Delta \hat{p}_{F,t}^{gap} + \Delta \hat{p}_{F,t}^{flex} \right),
\]
with $\hat{p}_{F,t}$ related to movements in the output gap as follows:
\[
\hat{p}_{F,t} = \Omega \hat{y}_t, \quad (A.3)
\]
where this last expression can be derived using equations (21), (28), (31) and (32).
A.2 Proofs of Proposition 1 and Corollary 1

A.2.1 Proof of Proposition 1

**Proof.** To prove this proposition we derive a second-order approximation to the household’s welfare. We define time $t$ utility $U_t$ as $U_t = \ln \left[ Z (c_{F,t} - \bar{c}_F)^{\alpha F} c_{1:0}^{1-\alpha F} \right] - (1 + \psi)^{-1} n_t^{1+\psi}$. A second-order Taylor expansion of $U_t$ around the steady state yields:

$$U_t - U \approx \alpha_F \frac{c_F}{c_F - \bar{c}_F} \left( \frac{c_{F,t} - c_F}{c_F} \right) + \left( 1 - \alpha_F \right) \left( \frac{c_{N,t} - c_N}{c_N} \right) - \frac{\alpha_F}{2} \left( \frac{c_F}{c_F - \bar{c}_F} \right)^2 \left( \frac{c_{F,t} - c_F}{c_F} \right)^2 \ldots$$

$$- \left( 1 - \alpha_F \right) \frac{c_{N,t} - c_N}{c_N} - n_t^{1+\psi} \left[ \frac{n_F}{n} \left( \frac{n_{F,t} - n_F}{n_F} \right) + \frac{n_N}{n} \left( \frac{n_{N,t} - n_N}{n_N} \right) \right] \ldots$$

$$- \frac{\psi}{2} n_t^{1+\psi} \left( \frac{n_t - n}{n} \right)^2.$$

We follow Galí (2008) and use the second-order approximation of relative deviations in terms of log deviations $(\frac{\hat{z}_2 - \hat{z}_1}{\hat{z}_1} = \hat{z}_t + \frac{1}{2} \hat{z}_t^2)$. We also use the following steady-state identities and definitions: (i) $\alpha_F = (c_F - \bar{c}_F)/c^*$, (ii) $\gamma_F = n_F/n = c_F/c$, (iii) $\sigma = (1 - \alpha_F)/(1 - \gamma_F) = c/c^*$, (iv) $\delta = \alpha_F/\gamma_F$, and (v) $n_t^{1+\psi} = \sigma$ to simplify the above approximation as follows:

$$U_t - U \approx \sigma \left[ \gamma_F \hat{c}_{F,t} + (1 - \gamma_F) \hat{c}_{N,t} - \frac{1}{2} \gamma_F (\sigma \delta^{-1} - 1) \hat{c}_{F,t}^2 \ldots \right.$$  

$$- \gamma_F \left( \hat{n}_{F,t} + \frac{1}{2} \hat{n}_{F,t}^2 \right) - (1 - \gamma_F) \left( \hat{n}_{N,t} + \frac{1}{2} \hat{n}_{N,t}^2 \right) \frac{1}{2} \psi \hat{n}_t^2 \right].$$

We now assume that $\alpha = 1$ and make use of (i) $\hat{c}_{F,t}^i = \hat{y}_{F,t}^i$, (ii) $\hat{c}_{N,t}^i = \hat{y}_{N,t}^i$, (iii) $\hat{y}_t = \gamma_F \hat{y}_{F,t} + (1 - \gamma_N) \hat{y}_{N,t}$, (iv) $\hat{n}_t = \gamma_F \hat{n}_{F,t} + (1 - \gamma_N) \hat{n}_{N,t}$, (v) $\hat{n}_{F,t} = \hat{y}_{F,t} - \hat{A}_{F,t}$, and (vi) $\hat{n}_{N,t} = \hat{y}_{N,t} + d_{N,t}$, where $d_{N,t} = \log \frac{1}{\int_0^1 \frac{P_{F,t}(i)}{F_{F,t}^i} - \epsilon} di$, to rewrite the approximation as:

$$U_t - U \approx - \frac{1}{2} \sigma \left[ 2(1 - \gamma_F) d_{N,t} + \gamma_F (\delta^{-1} \sigma - 1) \hat{y}_{F,t}^2 + \psi (\hat{y}_t - \gamma_F \hat{A}_{F,t})^2 \ldots \right.$$  

$$+ \gamma_F (\hat{y}_{F,t} - \hat{A}_{F,t})^2 + (1 - \gamma_F) \hat{y}_{N,t}^2 \right] + t.i.p.,$$

where the term $t.i.p.$ stands for “terms independent of policy” ($t.i.p. = \sigma \gamma_F \hat{A}_{F,t}$). To further simplify the welfare approximation, we first focus on the term $B = -2 \gamma_F \left( \psi \hat{y}_F \hat{A}_{F,t} + \hat{y}_{F,t} \hat{A}_{F,t} \right)$, which includes two of the terms in $\psi (\hat{y}_{F,t}^2 + \gamma_F \hat{A}_{F,t})^2 + \gamma_F (\hat{y}_{F,t} - \hat{A}_{F,t})^2$. We use the facts that $\hat{y}_{F,t}^{\text{flex}} = \frac{\delta + \psi}{\sigma + \psi} \gamma_F \hat{A}_{F,t}$,
\[ \delta^{-1}\sigma\hat{y}_{F,t} - \hat{y}_{N,t} = -\frac{1}{1 - \gamma_F}\hat{p}_{F,t} \], and \( \hat{p}_{F,t}^{\text{flex}} = -(1 - \gamma_F)\hat{A}_{F,t} \) to derive:

\[
B = -2\gamma_F \left[ (\psi + \delta)\hat{y}_t\hat{A}_{F,t} + \hat{y}_{F,t}\hat{A}_{F,t} - \delta\hat{y}_t\hat{A}_{F,t} \right] \\
= -2(\psi + \sigma)\hat{y}_t\hat{y}_{F,t}^{\text{flex}} - 2\gamma_F [\hat{y}_{F,t} - \delta (\gamma_F\hat{y}_{F,t} + (1 - \gamma_F)\hat{y}_{N,t})] \hat{A}_{F,t} \\
= -2(\psi + \sigma)\hat{y}_t\hat{y}_{F,t}^{\text{flex}} - 2[\gamma_F(1 - \alpha_F)\hat{y}_{F,t} - \alpha_F(1 - \gamma_F)\hat{y}_{N,t}] \hat{A}_{F,t} \\
= -2(\psi + \sigma)\hat{y}_t\hat{y}_{F,t}^{\text{flex}} - 2\alpha_F(1 - \gamma_F) [\delta^{-1}\sigma\hat{y}_{F,t} - \hat{y}_{N,t}] \hat{A}_{F,t} \\
= -2(\psi + \sigma)\hat{y}_t\hat{y}_{F,t}^{\text{flex}} - 2\frac{\alpha_F}{(1 - \gamma_F)}\hat{p}_{F,t}\hat{p}_{F,t}^{\text{flex}}. \\
\]

We return to the welfare approximation:

\[
U_t - U \simeq \frac{1}{2} \sigma \left[ 2(1 - \gamma_F)\hat{d}_{N,t} + \psi\hat{y}_t^2 + \gamma_F\delta^{-1}\sigma\hat{y}_{F,t}^2 + (1 - \gamma_F)\hat{y}_{N,t}^2 + B \right] + \text{t.i.p.} \\
U_t - U \simeq \frac{1}{2} \sigma \left[ 2(1 - \gamma_F)\hat{d}_{N,t} + (\psi + \sigma)\hat{y}_t^2 + \gamma_F\delta^{-1}\sigma\hat{y}_{F,t}^2 + (1 - \gamma_F)\hat{y}_{N,t}^2 - \sigma\hat{y}_t^2 + B \right] + \text{t.i.p.,} \\
\]

where \( \text{t.i.p.} = \sigma\gamma_F\hat{A}_{F,t} - 1/2\sigma (\psi\gamma_F^2 + \gamma_F^*) \hat{A}_{F,t}^2 \), and \( C \) can be rearranged as:

\[
C = \gamma_F\delta^{-1}\sigma\hat{y}_{F,t}^2 + (1 - \gamma_F)\hat{y}_{N,t}^2 - \sigma\hat{y}_t^2 \\
= \gamma_F\delta^{-1}\sigma\hat{y}_{F,t}^2 + (1 - \gamma_F)\hat{y}_{N,t}^2 - \sigma\hat{y}_{F,t}^2 - \sigma(1 - \gamma_F)^2\hat{y}_{N,t}^2 - 2\sigma\gamma_F(1 - \gamma_F)\hat{y}_{F,t}\hat{y}_{N,t} \\
= \gamma_F\sigma(\delta^{-1} - \gamma_F)\hat{y}_{F,t}^2 + (1 - \gamma_F)(1 - \sigma(1 - \gamma_F))\hat{y}_{N,t}^2 - 2\sigma\gamma_F(1 - \gamma_F)\hat{y}_{F,t}\hat{y}_{N,t} \\
= \gamma_F\delta^{-1}\sigma(1 - \alpha_F)\hat{y}_{F,t}^2 + (1 - \gamma_F)\alpha_F\hat{y}_{N,t}^2 - 2\gamma_F(1 - \alpha_F)\hat{y}_{F,t}\hat{y}_{N,t} \\
= \alpha_F(1 - \gamma_F) [(\delta^{-1}\sigma)^2\hat{y}_{F,t}^2 + \hat{y}_{N,t}^2 - 2\delta^{-1}\sigma\hat{y}_{F,t}\hat{y}_{N,t}] \\
= \alpha_F(1 - \gamma_F) [(\delta^{-1}\sigma)\hat{y}_{F,t} - \hat{y}_{N,t}]^2 = \frac{\alpha_F}{1 - \gamma_F}\hat{p}_{F,t}^2. \\
\]

Terms \( B \) and \( C \) can now be combined in the above equation (A.4), which yields:

\[
U_t - U \simeq -\frac{1}{2} \sigma \left[ 2(1 - \gamma_F)\hat{d}_{N,t} + (\psi + \sigma)\hat{y}_t^2 + \frac{\alpha_F}{1 - \gamma_F}\hat{p}_{F,t}^2 \right] + \text{t.i.p.,} \\
\]

where \( \hat{z}_t = \hat{z}_t^{\text{flex}}, \text{t.i.p.} = \sigma\gamma_F\hat{A}_{F,t} - 1/2\gamma_F\hat{A}_{F,t}^2, \) and \( D \) is given by:

\[
D = \sigma \left\{ \gamma_F^2 \left[ \frac{\psi + \delta}{(\psi + \sigma)^2} \right] + \gamma_F - \alpha_F(1 - \gamma_F) \right\} > 0. \\
\]
To complete the derivation, we follow Galí (2008) and reexpress $\delta_{\tau}$ as

$$\delta_{\tau} = \frac{1}{2\kappa} \left[ 1 + \beta \mu_{\tau} \{ \pi_{\tau}(\tau) \} \right],$$

where $\mu_{\tau} \{ \pi_{\tau}(\tau) \}$ denotes the dispersion of relative prices in the non-food sector. We also follow Galí (2008) in making use of the lemma in Woodford (2003):

$$\sum_{t=0}^{\infty} \beta^t \delta_{\tau} = \frac{1}{\kappa} \sum_{t=0}^{\infty} \beta^t \pi_{\tau, t},$$

where $\kappa = \frac{(1-\theta)(1-\theta)}{\theta}$. Excluding terms independent of policy, the welfare loss function can now be stated as:

$$W = \frac{1}{2} \sigma E_0 \sum_{t=0}^{\infty} \beta^t \left( (1 - \gamma_F) \frac{\epsilon}{\kappa} \pi_{\tau, t} + (\psi + \sigma) \hat{y}_t^2 + \frac{\alpha_F}{1 - \gamma_F} \hat{p}_{F, t}^2 \right).$$

(A.5)

Finally, taking unconditional expectations of (A.5) and letting $\beta \to 1$, the average welfare loss per period is given by equation (33). ■

A.2.2 Proof of Corollary 1

**Proof.** To derive (34), solve for $\hat{y}_t$ and $\hat{p}_{F, t}$ in terms of $\pi_{N, t}$ and $E_t \hat{\pi}_{N, t+1}$ using equations (A.2) and (A.3), and recalling that $\alpha = 1$. In this case $\Omega = \frac{\psi + \sigma}{\sigma}$ and $\kappa_{\hat{y}} = \kappa = \frac{(1-\theta)(1-\theta)}{\theta}$. Then replace the resulting expressions in (33). The welfare loss (34) attains its theoretical minimum when $\hat{\pi}_{N, t} = 0$ for every $t$, implying that complete stabilization of sticky-price non-food inflation minimizes welfare losses. ■

A.3 The Model with Limited Asset Market Participation and Segmented Labor Markets

We assume that there are now two types of agent: those with access to financial markets, denoted with superscript ($^o$) and representing a fraction $\lambda^*$ of the population; and those who do not have access, denoted with superscript ($^l$) and representing the remaining share $(1-\lambda^*)$. The representative agent in the first group maximizes lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_{t}^{o*}) - \varphi^{o}(n_{t}^{o})^{1+\psi} \right],$$

subject to the following budget constraint:

$$P_{F,t}c_{F,t}^{o} + P_{N,t}c_{N,t}^{o} + B_{t+1}^{o} = W_{N,tn_{t}^{o}} + \Pi_{F,t} + \Pi_{N,t} + R_{t-1}B_{t}^{o}.$$
and the composition of \( c_t^* \) similar to \( c_t^1 \) in equation (1). Relative to the baseline version of the model, there is labor market segmentation: agents that participate in financial markets supply labor exclusively to the non-food sector, and are paid a wage rate \( W_{N,t} \). These agents also own firms in both sectors and therefore receive profits \( \Pi_{F,t} + \Pi_{N,t} \). Utility maximization results in a condition similar to equation (2), a revised labor supply curve:

\[
\varphi^o (n_t^p) = w_{N,t} (c_t^*)^{-1}, \tag{A.6}
\]

an Euler equation for \( c_t^* \)

\[
(c_t^*)^{-1} = \beta \mathbb{E}_t \left[ \frac{R_t}{\pi_{t+1}^*} (c_{t+1}^*)^{-1} \right], \tag{A.7}
\]

and demand conditions (4) and (5).

Agents that have no access to financial markets maximize a similar utility function, but with parameter \( \varphi^r \) instead of \( \varphi^o \), subject to a static budget constraint:

\[
P_{F,t} c_{F,t} + P_{N,t} c_{N,t} = W_{F,t} n_t^f. \tag{A.8}
\]

\( c_t^* \) is also similar to \( c_t^1 \) in equation (1). Given labor market segmentation, they supply labor exclusively to the food sector, and are paid a wage rate \( W_{F,t} \). In addition to demand conditions (4) and (5), utility maximization results in a labor supply curve:

\[
\varphi^r (n_t^p) = w_{F,t} (c_t^*)^{-1}, \tag{A.9}
\]

With two types of agent, aggregate consumption is now given by:

\[
c_t = \lambda^* c_t^o + (1 - \lambda^*) c_t^r. \tag{A.10}
\]

The rest of the model is the same as before, with two exceptions: equilibrium in the non-food labor market is given by \( \lambda^* n_t^p = n_{N,t} \), while labor market equilibrium in the food sector is given by \( (1 - \lambda^*) n_t^f = n_{F,t} \). To simplify matters further, we assume the labor share is one (\( \alpha = 1 \)).

**Steady State**

To facilitate comparisons, we focus on an aggregate steady state that is the same as in the model.
with only one representative agent. Moreover, we concentrate on a steady state where consumption
is the same across the two types of agent—it will follow under these assumptions that the supply of
labor is also the same for these agents. To ensure this, we set \(1 - \lambda^* = \gamma_F\), so that the share of agents
not participating in financial markets is the same as the share of expenditure allocated to food.\(^{27}\) At
the steady state, \(p_F = \pi = A_F = 1\) and, since \(\alpha = 1\), \(w_N^* = w_F^* = w_N = w_F = A\). Using equations
(A.6), (A.9), (A.10), \(c^\phi = w_N n^\phi\), and \(c^\tau = w_F n^\tau\), it is simple to demonstrate that \(c^\phi = c^\tau = c\) and
\(n^\tau = n^\phi = c/A\), with \(c\) satisfying \((c - \bar{c}_F)c^\psi = A^{1+\psi}\).

**Log-linearization**

The linearized model features equations (22), (23), (26), (27), and the first identity in (28). Output is given by log-linearized versions of equations (9) and (11) with \(\alpha = 1\):

\[
\hat{y}_{F,t} = \hat{n}_{F,t} + \hat{A}_{F,t} \tag{A.11}
\]

and

\[
\hat{y}_{N,t} = \hat{n}_{N,t}. \tag{A.12}
\]

Labor demand in both sectors is different in that wages are not equalized:

\[
\hat{w}_{F,t} = \hat{p}_{F,t} + \hat{A}_{F,t} \tag{A.13}
\]

and\(^{28}\)

\[
\hat{w}_{N,t} = -\xi \hat{p}_{F,t} - \hat{\mu}_{N,t}, \tag{A.14}
\]

where \(\hat{w}_{F,t}\) and \(\hat{w}_{N,t}\) are wages in the food and non-food sector, respectively. Labor supply is also
differentiated across sectors, and using the equilibrium conditions \(\hat{n}_{F,t} = \hat{n}_{F,t}^\tau\) and \(\hat{n}_{N,t} = \hat{n}_{N,t}^\phi\), we obtain:

\[
\psi \hat{n}_{F,t} = \hat{w}_{F,t} + \phi \hat{p}_{F,t} - \sigma \hat{c}_{F}^\tau, \tag{A.15}
\]

and

\[
\psi \hat{n}_{N,t} = \hat{w}_{N,t} + \phi \hat{p}_{F,t} - \sigma \hat{c}_{N}^\phi, \tag{A.16}
\]

\(^{27}\) We also set \(\varphi^\phi = \varphi^\tau = 1\).

\(^{28}\) Here we use the fact that \(\hat{p}_{N,t} = -\xi \hat{p}_{F,t}\).
where consumptions of the two types of agent are given by:

$$\hat{c}^r_t = \hat{p}_{F,t} + \hat{y}_{F,t}. \quad (A.17)$$

and

$$\hat{c}^o_t = -\xi\hat{p}_{F,t} + \hat{y}_{N,t}. \quad (A.18)$$

Log-linearizing equation (A.7) and combining it with equations (A.17) and (A.18), and the following relations:

$$\hat{c}^o_t = \sigma \hat{c}^r_t,$$

$$\hat{\pi}^s_t = \hat{\pi}_t - \phi \Delta \hat{p}_{F,t},$$

and

$$\hat{y}_t = \hat{c}_t = (1 - \gamma_F)\hat{c}^o_t + \gamma_F \hat{c}^r_t,$$

we derive the following linearized aggregate IS curve for this extended model:

$$\hat{y}_t = -(1 - \gamma_F)(\sigma)^{-1} \mathbb{E}_t \left( \hat{R}_t - \hat{\pi}_{t+1} + \phi \Delta \hat{p}_{F,t+1} \right) + \mathbb{E}_t \hat{y}_{t+1} - \gamma_F \mathbb{E}_t (\Delta \hat{p}_{F,t+1} + \Delta \hat{y}_{F,t+1}). \quad (A.19)$$

Note that with limited asset participation and segmented labor markets variations in interest rates have smaller effects on aggregate demand, and changes in income in the food sector have a direct impact on aggregate demand. To see the role of these two features, we can use equation (22) to restate movements in food income as the weighted sum of changes in relative food prices and changes in overall income:

$$\hat{p}_{F,t} + \hat{y}_{F,t} = (1 - \delta)\hat{p}_{F,t} + \delta \hat{y}_t. \quad (A.20)$$

We then introduce equation (A.20) into equation (A.19), and solve forward to obtain the following equation:

$$\hat{y}_t = -\frac{1}{\sigma^2} \mathbb{E}_t \sum_{\tau=0}^{\infty} \left( \hat{R}_{t+\tau} - \hat{\pi}_{t+\tau+1} \right) + \frac{\phi}{\sigma^2} (1 + \sigma) \hat{p}_{F,t}. \quad (A.21)$$

Instead if we solve forward equation (20) from our model we obtain:

$$\hat{y}_t = -\frac{1}{\sigma} \mathbb{E}_t \sum_{\tau=0}^{\infty} \left( \hat{R}_{t+\tau} - \hat{\pi}_{t+\tau+1} \right) + \frac{\phi}{\sigma} \hat{p}_{F,t}. \quad (A.22)$$
Comparing these two equations we can observe that, since $\sigma \geq 1$ and $\phi \geq 0$, the model with subsistence, limited asset market participation, and segmented labor markets has a higher sensitivity of aggregate demand to changes in the relative price of food than our model just with subsistence. Any shock that raises the relative price of food, including negative shocks to productivity in the food sector, will have a positive effect on aggregate demand and therefore on sticky-price inflation.

For the derivation of the welfare-based loss function, it is also useful to derive the flexible-price solution for aggregate and sectoral outputs. To do so, combine equations (A.11), (A.13), (A.15), and (A.17) to obtain:

$$\hat{y}_{F,t} = \hat{y}^{f_{\text{lex}}}_{F,t} = \left(\frac{\psi + 1}{\psi + \sigma}\right) \hat{A}_{F,t},$$

(A.21)

where the first equality follows from the fact that this derivation is independent of the markup $\hat{\mu}_{N,t}$. As a result, the gap $\hat{y}_{F,t} = \hat{y}_{F,t} - \hat{y}^{f_{\text{lex}}}_{F,t} = 0$.

In addition, setting $\hat{\mu}_{N,t} = 0$ and combining equations (A.12), (A.14), (A.16), and (A.18) yields:

$$\hat{y}^{f_{\text{lex}}}_{N,t} = \left[\frac{(\sigma - 1)\xi + \phi}{\psi + \sigma}\right] \hat{p}^{f_{\text{lex}}}_{F,t}.$$  

(A.22)

Multiplying equation (A.21) by $(1 - \gamma_F)$ and (A.22) by $\gamma_F$, adding up the resulting equations and rearranging terms gives:

$$\hat{y}^{f_{\text{lex}}}_{t} = \gamma_F \left(\frac{\psi + 1}{\psi + \sigma}\right) \hat{A}_{F,t} + \left(\frac{\phi}{\psi + \sigma}\right) \hat{p}^{f_{\text{lex}}}_{F,t}.$$  

(A.23)

On the other hand, combining (22) and (A.21), we obtain:

$$\hat{y}^{f_{\text{lex}}}_{t} = \delta^{-1} \left(\frac{\psi + 1}{\psi + \sigma}\right) \hat{A}_{F,t} + \hat{p}^{f_{\text{lex}}}_{F,t},$$  

(A.24)

which together with equation (A.23) can be used to derive:

$$\hat{p}^{f_{\text{lex}}}_{F,t} = -\delta^{-1} \sigma (1 - \gamma_F) \hat{A}_{F,t}.$$  

(A.25)

Note that, compared to our original specification in which $\hat{p}^{f_{\text{lex}}}_{F,t} = -(1 - \gamma_F) \hat{A}_{F,t}$, the combination of segmented labor markets and limited financial participation amplifies the impact of supply shocks on relative prices, since $\delta^{-1} \sigma > 1$. This is because the overall ability of the economy to adjust, by shifting factors of production from one sector to the other, has been reduced.
Finally, if we insert the solution for $\hat{\phi}^{\text{flex}}$ from equation (A.25) into equations (A.22) and (A.24) we get:

$$\hat{y}_{N,t}^{\text{flex}} = -\left[\frac{\delta^{-1}\sigma(\sigma - 1)}{\sigma + \psi}\right] \hat{A}_{F,t} \tag{A.26}$$

and

$$\hat{y}_t^{\text{flex}} = \gamma F \left[\frac{\psi + 1 - \sigma(\delta^{-1} - 1)}{\psi + \sigma}\right] \hat{A}_{F,t}. \tag{A.27}$$

Note that, in contrast to the baseline version of the model, in this extended version positive food supply shocks lead to a decline in non-food production. This is because the effect on non-food production is dominated by the (positive) wealth effects of the shock on households that have access to financial markets and the corresponding reduction in their labor supply. In the baseline version, these shocks lead to an increase in non-food production, since the overall increase in demand (and switch toward non-subsistence goods) dominates.

In the presence of subsistence, limited asset market participation, and segmented labor markets, and depending on their persistence, supply shocks will tend to raise equilibrium real interest rates. This can be seen by introducing equation (A.20) into equation (A.19) when prices are perfectly flexible, and solving for the resulting real interest rate:

$$\hat{\rho}_t^{\text{flex}} = -\sigma^2 \left(\hat{y}_t^{\text{flex}} - E_t \hat{y}_{t+1}^{\text{flex}}\right) + \phi(1 + \sigma) \left(\hat{p}_t^{\text{flex}} - E_t \hat{p}_{t+1}^{\text{flex}}\right). \tag{A.28}$$

Consider the flexible-price solutions in (A.25) and (A.27) and rewrite them as $\hat{p}_t^{\text{flex}} = -\vartheta_p \hat{A}_{F,t}$ with $\vartheta_p > 0$ and $\hat{y}_t^{\text{flex}} = \vartheta_y \hat{A}_{F,t}$ with $\vartheta_y \geq 0$. For simplicity, assume that $\hat{A}_{F,t}$ follows an AR(1) with persistence $\rho_A \in [0,1)$. Using these expressions, we can rewrite equation (A.28) as

$$\hat{r}_t^{\text{flex}} = -\left[\phi(1 + \sigma) \vartheta_p + \sigma^2 \vartheta_y\right] (1 - \rho_A) \hat{A}_{F,t}. \tag{A.29}$$

Clearly since $\sigma \geq 1, \phi \geq 0, \rho_A \in [0,1), \vartheta_p > 0$ and $\vartheta_y \geq 0$, then a negative food productivity shock can increase the equilibrium real interest rates, and the size of this increase depends on the persistence $\rho_A$. The less persistent the shock, the greater can be the increase in equilibrium real rates. This result underpins the evaluation of interest rate rules in Figure 5.

We are now ready to derive the welfare-based loss function in the presence of subsistence, limited asset market participation and segmented labor markets. The results are summarized in the Proposition 2 of the main text that we reproduce here and provide the proof.
**Proposition 2** Consider the model with food subsistence, limited asset market participation and segmented labor markets and assume that $\alpha = 1$ and $\lambda^* = 1 - \gamma_F$. The average welfare loss per period, derived using a weighted sum of the utility functions of both types of agent, is given by the following linear function:

$$ L = \frac{1}{2} \sigma \left[ (1 - \gamma_F) \frac{\epsilon}{\kappa} \text{var} (\hat{\pi}_{N,t}) + \frac{(\psi + \sigma)}{(1 - \gamma_F)} \text{var} (\hat{\eta}) \right], \quad (A.29) $$

where $\tilde{\eta}_t = \hat{\eta}_t - \hat{\eta}^{\text{alt}}_t$, where $\hat{\eta}^{\text{alt}}_t \neq \hat{\eta}^{\text{flex}}_t$.

**Proof.** Recall that $\varphi^r = \varphi^o = 1$ and that the steady state is symmetric in the sense that:

$$ x^o = x^r = x \text{ for } x = (c, c_F, c_N, n). $$

Then define time $t$ aggregate utility $U_t$ as:

$$ U_t = \gamma_F \left[ \ln (c^r_t) - \left( \frac{n^r_t}{1 + \psi} \right) \right] + (1 - \gamma_F) \left[ \ln (c^o_t) - \left( \frac{n^o_t}{1 + \psi} \right) \right]; $$

A second-order Taylor expansion of $U_t$ around the steady state yields:

$$ U_t - U \approx \gamma_F \left[ \alpha_F \frac{c_F}{c_F - \bar{c}_F} \left( \frac{c^0_{F,t} - c_F}{c_F} \right) + (1 - \alpha_F) \left( \frac{c^0_{N,t} - c_N}{c_N} \right) - \frac{1}{2} \alpha_F \left( \frac{c_F}{c_F - \bar{c}_F} \right)^2 \left( \frac{c^0_{F,t} - c_F}{c_F} \right)^2 \right. $$

$$ \left. \cdots \right. $$

$$ \cdots + (1 - \gamma_F) \left[ \alpha_F \frac{c_F}{c_F - \bar{c}_F} \left( \frac{c^0_{F,t} - c_F}{c_F} \right) - \frac{1}{2} \alpha_F \left( \frac{c_F}{c_F - \bar{c}_F} \right)^2 \left( \frac{c^0_{F,t} - c_F}{c_F} \right)^2 \right. $$

$$ \cdots $$

$$ \cdots - n^{1+\psi} \left( \frac{n^r_t - n}{n} \right) - \frac{1}{2} \psi n^{1+\psi} \left( \frac{n^r_t - n}{n} \right)^2 \right]. $$

We follow Galí (2008) and use the second-order approximation of relative deviations in terms of log deviations ($\tilde{z} = \tilde{\pi} + \frac{1}{2} \bar{d}^2 \tilde{\tau}$). Then we use the steady-state identities and definitions (i) $\alpha_F = (c_F - \bar{c}_F)/c^*$, (ii) $\sigma = (1 - \alpha_F)/(1 - \gamma_F) = c/c^*$, (iii) $\delta = \alpha_F/\gamma_F$, (iv) $n^{1+\psi} = \sigma$, (v) identities $\gamma_F \tilde{c}_{i,t} = \gamma_F c_{i,t} + (1 - \gamma_F) c^*_i$, for $i = (F, N)$, and labor market equilibria $\tilde{n}_t^F = \hat{n}_{N,t}$ and $\tilde{n}_t^r = \hat{n}_{F,t}$, to
simplify the above approximation as follows:

$$U_t - U \simeq \sigma \left[ \gamma_F \hat{c}_{F,t} + (1 - \gamma_F) \hat{c}_{N,t} - \frac{1}{2} \gamma_F (\sigma \delta^{-1} - 1) \left( \gamma_F \hat{c}_{F,t}^2 + (1 - \gamma_F) \hat{c}_{F,t}^2 \right) \right]$$

$$... - \gamma_F \hat{n}_{F,t} - (1 - \gamma_F) \hat{n}_{N,t} - \frac{1}{2} (1 + \psi) \left( \gamma_F \hat{n}_{F,t}^2 + (1 - \gamma_F) \hat{n}_{N,t}^2 \right) \right] .$$

We now make use of the identities (i) \( \hat{c}_t^F = \hat{y}_t^F \), (ii) \( \hat{c}_t^N = \hat{y}_t^N \), (iii) \( \hat{c}_{F,t} = \gamma_F \hat{c}_{F,t}^r + (1 - \gamma_F) \hat{c}_{F,t}^o \), (iv) \( \hat{n}_{F,t} = \hat{y}_{F,t} - \hat{A}_{F,t} \), and (v) \( \hat{n}_{N,t} = \hat{y}_{N,t} + d_{N,t} \), where \( d_{N,t} = \log \int_0^1 \frac{y_{N,t}(i)}{P_{N,t}(i)} \, di \), to simplify the welfare approximation further to:

$$U_t - U \simeq \frac{1}{2} \sigma \left( 2(1 - \gamma_F) d_{N,t} + \gamma_F (\delta^{-1} - 1) \left[ \gamma_F \hat{c}_{F,t}^2 + (1 - \gamma_F) \hat{c}_{F,t}^2 \right] \right.$$  

$$... + (1 + \psi) \left[ \gamma_F (\hat{y}_{F,t} - \hat{A}_{F,t})^2 + (1 - \gamma_F) \hat{y}_{N,t}^2 \right] \} + t.i.p.$$  

The term \( t.i.p. \) stands for “terms independent of policy” and, in this case, corresponds to \( t.i.p. = \sigma \gamma_F \hat{A}_{F,t} \). We focus on the term \( \left[ \gamma_F \hat{c}_{F,t}^2 + (1 - \gamma_F) \hat{c}_{F,t}^2 \right] \), in this last expression, and rewrite it as:

$$\left[ \gamma_F \hat{c}_{F,t}^2 + (1 - \gamma_F) \hat{c}_{F,t}^2 \right] = \left[ \gamma_F \hat{c}_{F,t}^2 + (1 - \gamma_F) \hat{c}_{F,t}^2 \right] + \hat{c}_{F,t}^2 - \hat{c}_{F,t}^2$$

$$= \hat{c}_{F,t}^2 + \left[ \gamma_F \hat{c}_{F,t}^2 + (1 - \gamma_F) \hat{c}_{F,t}^2 \right] - \left[ \gamma_F \hat{c}_{F,t}^2 + (1 - \gamma_F) \hat{c}_{F,t}^2 \right]^2$$

$$= \gamma_F (1 - \gamma_F) \left( \hat{c}_{F,t}^2 + \hat{c}_{F,t}^2 - 2 \hat{c}_{F,t} \right)$$

$$= \gamma_F (1 - \gamma_F) \left( \hat{c}_{F,t}^2 - \hat{c}_{F,t} \right)^2 .$$

Using \( \hat{c}_{F,t} = \delta(-\hat{p}_{F,t} + \hat{c}_t^r) \), for \( i = r, o \), and equations (22), (23), (A.17) and (A.18), we can derive that:

$$\left( \hat{c}_{F,t}^r - \hat{c}_{F,t}^o \right)^2 = \delta^2 [\hat{c}_t^r - \hat{c}_t^o]^2 = \delta^2 [\hat{c}_t^r - \hat{c}_t^o]^2$$

$$= \delta^2 (\delta^{-1} - 1)^2 \hat{y}_{F,t}^2, \quad (A.31)$$

which together (A.30) implies that \( \left[ \gamma_F \hat{c}_{F,t}^2 + (1 - \gamma_F) \hat{c}_{F,t}^2 \right] = B \hat{y}_{F,t}^2 \), where \( B = 1 + \gamma_F (1 - \gamma_F) (\sigma - \delta)^2 \).

Next recall equation (A.21). Then all square terms related to food output are independent of policy, which by (A.31) implies that the term \( (\hat{c}_{F,t}^r - \hat{c}_{F,t}^o)^2 \) is also independent of policy, as it depends only on \( \hat{y}_{F,t}^2 \). Using this, the welfare-based loss function relevant for policy now reduces to:

$$U_t - U \simeq \frac{1}{2} \sigma \left[ 2(1 - \gamma_F) d_{N,t} + (1 + \psi) (1 - \gamma_F) \hat{y}_{N,t}^2 \right] + t.i.p,$$
where \( t.i.p. = \sigma \gamma_F \hat{A}_{F,t} - \gamma_F (\delta^{-1} \sigma - 1) B \hat{y}_{F,t}^2 - (1 + \psi) \gamma_F (\hat{y}_{F,t} - \hat{A}_{F,t})^2 \). We define an alternative welfare relevant measure of output in the non-traded sector \( \hat{y}_{alt} = 0 \). When combined with the solution for \( \hat{y}_{F,t} \) in equation (A.21), we obtain the following alternative measure of aggregate output:

\[
\hat{y}_t^{alt} = \gamma_F \left( \frac{1 + \psi}{\sigma + \psi} \right) \hat{A}_{F,t},
\]

which clearly differs from the flex-price solution (A.27). The loss function can then be restated using \( \hat{y}_t^{alt} \) as:

\[
U_t - U \simeq -\frac{1}{2} \sigma \left[ 2(1 - \gamma_F) d_{N,t} + \frac{1 + \psi}{1 - \gamma_F} \hat{y}_t^2 \right] + t.i.p,
\]

where \( \hat{y}_t^2 = \hat{y}_t - \hat{y}_t^{alt} \).

To conclude the derivation, we follow Galí (2008) and Woodford (2003), who show that:

\[
d_{N,t} = \frac{\epsilon}{2} \text{var}_i \{ p_{N,t}(i) \} \quad \text{and} \quad \sum_{t=0}^{\infty} \beta^t \text{var}_i \{ p_{N,t}(i) \} = \frac{1}{\kappa} \sum_{t=0}^{\infty} \beta^t \bar{\pi}^2_{N,t},
\]

where \( \text{var}_i \{ p_{N,t}(i) \} \) denotes the dispersion of relative prices in the non-food sector and \( \kappa = \frac{(1-\theta \beta)(1+\theta)}{\theta} \).

Using these, the inter-temporal loss function can now be stated as:

\[
\mathbb{W} = -\frac{1}{2} \sigma \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ (1 - \gamma_F) \frac{\epsilon}{\kappa} \bar{\pi}^2_{N,t} + \frac{1 + \psi}{1 - \gamma_F} \hat{y}_t^2 \right]. \tag{A.32}
\]

Finally, taking unconditional expectations of (A.32) and letting \( \beta \to 1 \), the average welfare loss per period is thus given by equation (36).

### A.4 Additional Figures and Tables

Here we present Table A.1 and Figures A.1 and A.2 that were referred to in the main text of the paper.
<table>
<thead>
<tr>
<th>Country</th>
<th>Paper</th>
<th>Sample</th>
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<th>Mean Frequency</th>
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<tr>
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<td>Medina et al. (2009)</td>
<td>1999-2005</td>
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<td>99.5</td>
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<td>1997-2005</td>
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<td>Klenow and Kryvtsov (2008)</td>
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</table>

Notes. Jonker et al. (2004) splits the sample into a high inflation period (1975-1989) and a low inflation period (1990-2004). The latter period is reported in parentheses. Wulfsberg (2009), Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008) also report frequencies excluding temporary sales. We include temporary sales here for the sake of consistent comparison.

Table A.1: Mean Frequency of Price Changes Across Goods and Countries.
Figure A.1: A Shock to Food Sector Productivity, $\varepsilon_{AF} < 0$, under Headline Inflation Targeting ($\pi_t = 0$).
Figure A.2: Relative Welfare Weights of the Output Gap and Relative Food Price Gap.
References


Catao, L. and R. Chang, 2013, “World Food Prices, the Terms of Trade-Real Exchange Rate Nexus, and Monetary Policy,” Manuscript, Rutgers University.


