Elements of Revenue Forecasting II: the Elasticity Approach and Projections of Revenue Components

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Lecture Outline

I. Basic concepts: Buoyancy and Elasticity
II. Estimating Tax Elasticity
III. From Mechanical Projection to Forecast

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I. Introduction and Basic Concepts

- **The elasticity approach**: one of the most commonly used "conditional" forecasting methods
- Like the effective tax rate approach, changes in tax revenue reflect mainly changes in the tax base *if policy is unchanged*
- What is different about the elasticity approach:
  - Revenues can rise faster or more slowly than changes in tax base ("non-linear response")
- **Forecast**: combines "mechanical" projection with estimated impact of any tax change and judgment (to reflect possible variations in compliance or other information on specific "shocks" to tax collection).

Buoyancy

- **Buoyancy of a tax** is the realized/observed relative variation in revenue collection or a specific revenue item compared to the relative change in the proxy tax base:

Buoyancy = $(\Delta T / T) / (\Delta Base/Base)$

- Thus, buoyancy is based on actual revenues and reflects all changes in the tax system, including the tax rates and brackets, the definition of the base, variations in enforcement/compliance, or other specific shocks.

- A tax is said to be buoyant if the tax revenues increase more than proportionately to a rise in output.

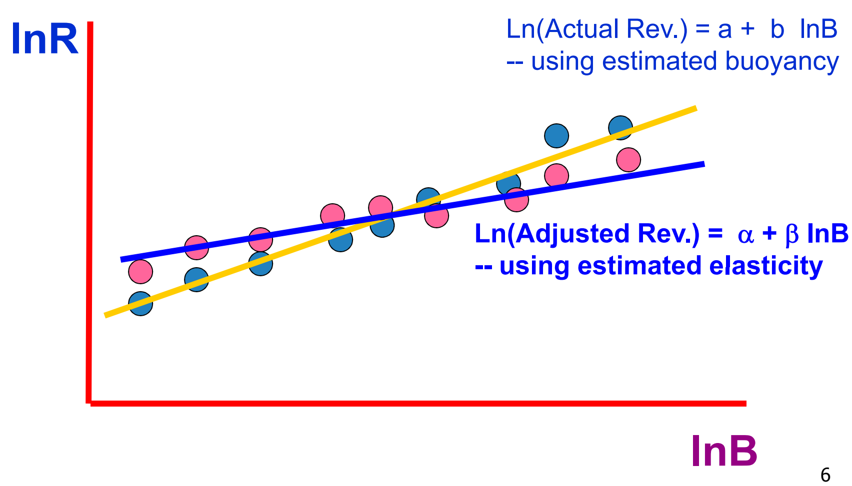
Elasticity

- The **elasticity** of a tax measures the **automatic** response of tax revenue to changes in the tax base.

  Elasticity = (ΔAT/ AT) / (ΔTB/TB)

- The elasticity excludes the effects of discretionary changes in the tax structure (tax rates, coverage, exemptions, and deductions) or administration, as well as the introduction of new taxes.
- Thus, the elasticity
  - can differ from the buoyancy (next slide)
  - is based on adjusted, rather than actual, revenues
- The elasticity must be estimated by "removing" the effects of discretionary changes from revenue data. This can be hard to do (need estimates of effects of these changes).

Forecasts with Buoyancies and Elasticities



Proportional Adjustment Method: An Example for Adjusting Revenue

Year	Actual Tax Collections	Discretionary Measure	Tax Collection Excluding New Measure	Share of Measure in Revenue	Adjusted Tax Receipts
	T	DS	T - DS	T/(T-DS)	T* Col 5
1	100	0			132.6
2	140	20	120	1.167	159.1
3	170				193.2
4	250	30	220	1.136	250.0
5	300				300.0

Elasticity

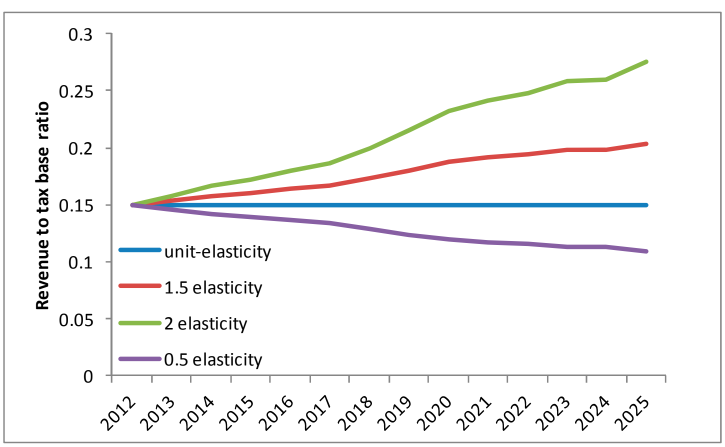
- **Golden rule**: the tax system is such that elasticity of tax revenues to the tax base should ideally be close to **1**.
- In practice, estimated elasticity can differ from **1.**
  - **Progressive income taxes can have an elasticity > 1**
  - **Lagged indexation** of tax brackets to inflation or wages raises elasticity.
  - **Proportional** (ad-valorem) taxes (VAT, payroll tax) more likely to exhibit estimated elasticity of 1.
- Elasticity **below 1** is possible for **specific taxes** (e.g., excise or stamp duties) if they are not indexed and/or collected promptly, or if consumers devote smaller shares of income to these items over time (e.g., tobacco products).

Elasticity

- **Inflation** can raise or lower the elasticity.
  - Inflation can boost income tax elasticity if collection occurs shortly after assessment (progressivity and delayed indexation of brackets).
  - Inflation will depress elasticity of all taxes if significant collections lags → the real value of the tax dues is eroded (Oliveira-Tanzi effect), i.e., tax base is growing faster than revenues.
- For **practical purposes**:
  - Any estimated elasticity very different from 1 (e.g., 0.5 or 2) should raise questions: can you explain it?
  - More importantly: if you cannot estimate the elasticity (because data adjustment is hard), beware of highly unstable buoyancies from year to year

Elasticity

- The arithmetic of non-unitary elasticity.



Elasticity

- For medium-term forecast: safe to assume that outer-year elasticities are unlikely to stay very high or very low.
  - Tax brackets are ultimately indexed;
  - Limits to gains from better tax administration (compliance and enforcement,…).

- Translating other basic forecasting techniques in terms of elasticity:
  - Effective tax rate approach:
    - Keeping ETR constant = unit elasticity
    - Raising ETR: above 1 elasticity
  - Simple extrapolations of tax revenue:
    - Equivalent to unit elasticity if projected revenue growth is the same as the projected growth in the tax base

II. Estimating Tax Elasticity

Elasticity and regression analysis

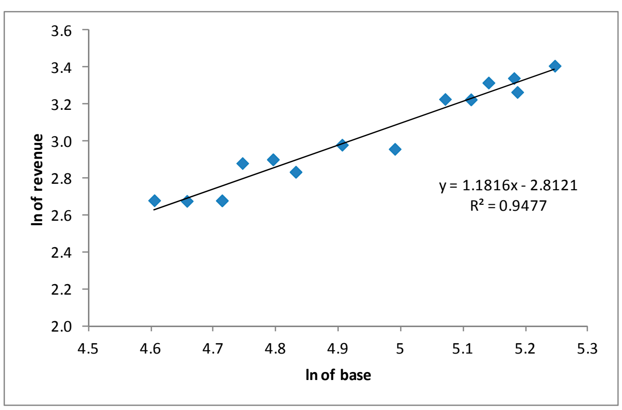
- Even with few data points, an elasticity can in principle be estimated using a simple linear ("OLS") regression model of the form:

$$Y = a_1 + a_2 X + \epsilon$$

where Y is the dependent variable (log of revenues), X is the explanatory variable (log of proxy tax base), and ε is an error term. We use logarithms because the estimate of a_2 is the estimated elasticity.

- We will have "n" values for Y and X, one for each observation in our estimation period.

Illustration: tax revenues vs. base (in logarithms) (Monte-Carlo simulation)

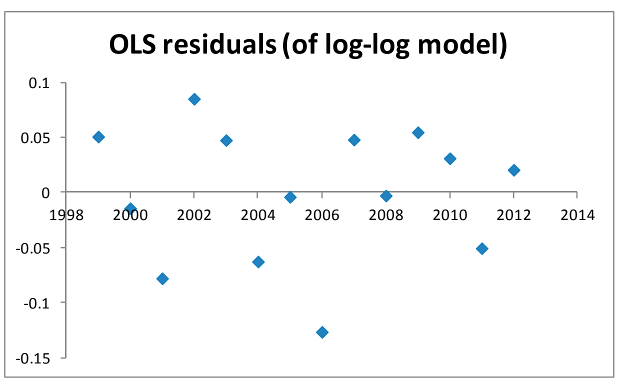


- Using logs, the estimated elasticity is the slope of the regression model.
- Even though data were generated with a model exhibiting unit elasticity as the truth, the estimated elasticity is 1.2.

Regression analysis

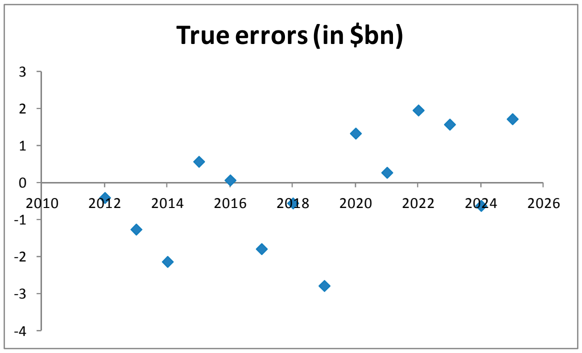
- The pattern of deviations (residuals) around this fitted line indicates how good a fit we have. The larger they are, the less our model is able to explain the evolution of Y by variations in X.
- If we have the forecast values for X, we can use the model to forecast values for Y.
- Because the model is imperfect, there will be errors around the forecast, which will depend on the distribution of the residuals.

Illustration: tax revenues vs. base (in log) (Monte-Carlo simulation)



- These OLS residuals are very well behaved: the estimator of the elasticity is unbiased and efficient, even though it is WRONG.
- That was expected: underlying errors were generated randomly, assuming a normal distribution. But why is OLS estimator different from the truth?

Illustration: tax revenues vs. base (in log) (Monte-Carlo simulation)



- Even though they were randomly generated (true!), this particular draw of error terms is such that there seems to be an (unintended) trend.
- This trend will be captured by the OLS estimation and explains why the estimated coefficient higher than the true model (1.2 > 1) → there is always a confidence interval around the estimated elasticity.

Risks with Simple Estimation

- Our simple regression may yield "good" results because tax revenues and the tax base are both increasing. In technical language, we are dealing with "non-stationary" series.
- To be sure of our relationship, we need to use stationary series, meaning that there is no systematic tendency for the data to rise or fall over time.

Developing Stationary Series

- One way to "solve" the problem is to estimate an equation using the change in the log of revenues (rather than the log of revenues) and the change in the log of the tax base (rather than the log of the tax base). This usually provides stationary series.
- Another approach is to use "cointegration" analysis. Econometric software offers ways to do this. IMF course on Macroeconomic Forecasting teaches how to test for stationarity and do cointegration .

What If You Can't Do Cointegration?

- **Cointegration approach:** combines long-term relationship with short-term and estimates adjustment dynamics from ST to LT. Hard to do in data constrained environments.

- **Second best:**
  - Focus the econometrics on short-term relationship: we deliberately ignore the long-term and the implied dynamics. But at least, the trends will not "pollute" the simple regression estimation.
  - For outer years of projections: assume convergence of coefficient towards an "intuitive" long-term value.
  - Beware of results if using actual rather than adjusted data: will estimate buoyancy rather than elasticity if tax law or administration have changed noticeably during the period

Conclusion on estimation

- Simple model linking revenues to the tax base can be useful.
- Elasticity (buoyancy) can be estimated simply.
- But:
  - Keep in mind presumed long-term value for elasticity (ideally around 1).
  - The estimated elasticity will not necessarily be the “true” one!
  - If data are actuals, estimate may be a buoyancy

III. From Projection to Forecast

- Use **estimated elasticity cautiously**.
  - Does the overall value make sense? Large deviations from 1 must be substantiated.
  - Compare with past realizations (point elasticities);
  - Look for evidence of an emerging break in the estimated model (e.g. systematically positive or negative forecast errors over the last few years).
  - Always plot data on charts, check residuals.
- **Revenue Forecast**: projection + adjustments

$$R_{t+1}^{f} = R_t \times \left(1 + \left(g_{base} \times elasticity\right)\right) + Adjustments$$

Adjustment factors

- Adjustments factors form the **"art" dimension of forecasting.** As it involves **judgment**, **transparency** is essential for the credibility of the forecast.

- The most common reasons for adjusting mechanical projections are:
  - **Temporary** shocks have occurred in the past: base effect must be corrected (otherwise implicit assumption that the shock is permanent).
  - Role of non-quantifiable/qualitative information about **impending shocks**.
  - Assumption about **compliance/enforcement/tax administration** reforms must be clearly spelled out.
  - Systematic revenue increases (decreases) during the period will bias estimated elasticity up (down)

Short-term and longer term

- **Short-term nature** of the econometric relationship during a time of economic transition:
  - Likelihood of unstable relationship (the past may not be a good predictor of the future)
  - Be explicit about how you see long-term trends for outer-year (t+3 and beyond):
    - Historical averages?
    - Other assumptions?

When to Use Elasticity Approach

- Elasticity approach can be valuable when data suggest a non-linear response of revenue to the proxy tax base: elasticity different from 1
- If estimated elasticity equals 1, result is same as effective tax rate approach
- Risk arises if buoyancies or elasticities vary a lot from year to year. If so, using estimated elasticity may give poor forecast to revenue.
  - Example: if tax base rises but revenue falls, buoyancy is negative. Is decline due to policy change? Can impact be estimated and data adjusted to show elasticity?

Summary and Conclusion

- Elasticity approach allows revenues to grow faster or more slowly than the tax base
- Compare buoyancy and elasticity:
  - Buoyancy is relationship between changes in actual revenues and changes in proxy tax base
  - Elasticity reflects automatic change in revenues from change in tax base; represents an "average" response
- Buoyancies and elasticities can be estimated using regression approaches; note risks
- Beware of frequent year-to-year changes in buoyancies or elasticities