



IMF INSTITUTE 
FOR CAPACITY DEVELOPMENT



Fiscal and Debt Sustainability

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Outline

- I. Introduction
- II. Fiscal Sustainability: Maintaining Solvency
- III. Debt Dynamics
- IV. Debt Sustainability Analysis

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Introduction—Defining Fiscal Sustainability

The government is able to achieve a fiscal stance that allows it to service public debt in the short, medium and long run ...

... without the need to undertake policy adjustments that are implausible from an economic or political standpoint ...

... without debt default or renegotiation...

... given financing costs and conditions it faces

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Introduction—Fiscal Sustainability Concepts

- **Solvency:** refers to an entity's ability to pay its debt/meet its long-term financial obligations
 - Fiscal and debt sustainability is (mostly) about maintaining solvency for the government
 - We will discuss in more detail what this requires
- **Liquidity:** An entity is (il)liquid if, regardless of whether it satisfies the solvency condition, its liquid assets and available financing are (in)sufficient to meet or roll-over its maturing liabilities
- **Vulnerability:** Risk that the liquidity or solvency conditions are violated and the borrower enters a crisis

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Introduction—Fiscal Sustainability Coverage

- Focus on public debt:
 - Domestic
 - External
- Awareness of contingent liabilities:
 - Debt guarantees
 - Sub-national governments
 - State-owned enterprises
 - Spillovers from financial institutions
 - Public-private partnerships

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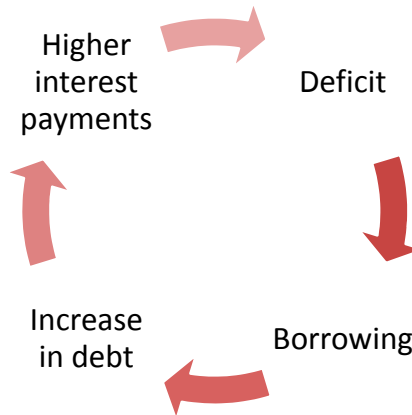
Fiscal Sustainability: Why Worry?

- Excessive debt ([debt overhang](#)) is bad for growth and development.
- A rising share of revenues devoted to debt-service payments weakens a government's ability to implement desired policies.
- Heavy debt service obligations make a country more vulnerable to interruption of commercial or official flows (sudden stops, shift in aid policies).
- Debt restructuring can be highly disruptive to economic activity and undermines the development of a credit culture.

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Fiscal Unsustainability: Adverse Consequences

A potentially vicious circle:



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Fiscal Sustainability: Maintaining Solvency

What does maintaining solvency require? Condition 1:

Debt limit: debt-to-GDP ratio will never exceed a certain threshold.

What is this threshold?

We don't know!

There are many rule of thumbs but no inviolable threshold.

Practical implication: aim for a stable or declining debt-to-GDP ratio; failing that, keep debt ratio below a ceiling

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Fiscal Sustainability: Maintaining Solvency

What does maintaining solvency require? Condition 2:

No-Ponzi game: the government does not service its debt by issuing new debt on a regular basis.

Debt and interest payments cannot be postponed forever!

Practical implication: debt and interest are not rolled over systematically

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Fiscal Sustainability: Maintaining Solvency

What does maintaining solvency require? Condition 3:

Solvency: the government has enough resources in the future to service the debt accumulated from the past.

Existing debt, including accumulated interest, is eventually paid in full through future fiscal surpluses.

Practical implication: eventually the budget will have to aim for (primary) surpluses.

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Fiscal Sustainability: Maintaining Solvency

What does maintaining solvency require?

Putting it all together:

- Debt and interest are not rolled over systematically.
- Existing debt, including accumulated interest, is eventually paid in full through future fiscal surpluses.
- The debt ratio is kept below a ceiling.

These rules do not have to be followed strictly each year → they are a guide for fiscal policy in the long run

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Debt Dynamics

Overall balance and debt:

Total revenues and grants

- if $OB > 0$ assets \uparrow or debt \downarrow :
net debt \downarrow

Total expenditures

= if $OB < 0$ assets \downarrow or debt \uparrow :
net debt \uparrow

Overall balance

$$D_t = D_{t-1} - OB_t$$

Financing:

$$D_t - D_{t-1} = -OB_t$$

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Exercise

Develop a path for the overall balance that reduces the public debt stock to approximately zero after 50 years!

Also plot the overall balance and the debt stock: what is the relationship between the two?



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Debt Dynamics

Primary balance and debt:

$$D_t = D_{t-1} - OB_t$$

$$D_t = D_{t-1} - (R_t - NIE_t - I_t)$$

$$D_t = D_{t-1} + I_t - (R_t - NIE_t)$$

Primary balance, PB

$$D_t = D_{t-1} + iD_{t-1} - PB_t$$

Nominal interest rate

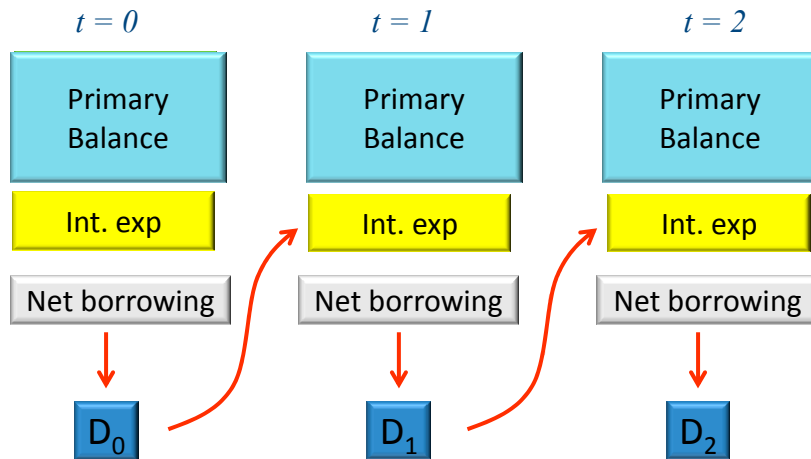
$$D_t = (1 + i)D_{t-1} - PB_t$$

Autonomous component of debt dynamic

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Debt Dynamics

Primary balance, interest and debt:



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Exercise

Now develop a path for the primary balance for an approximately zero public debt stock after 50 years!

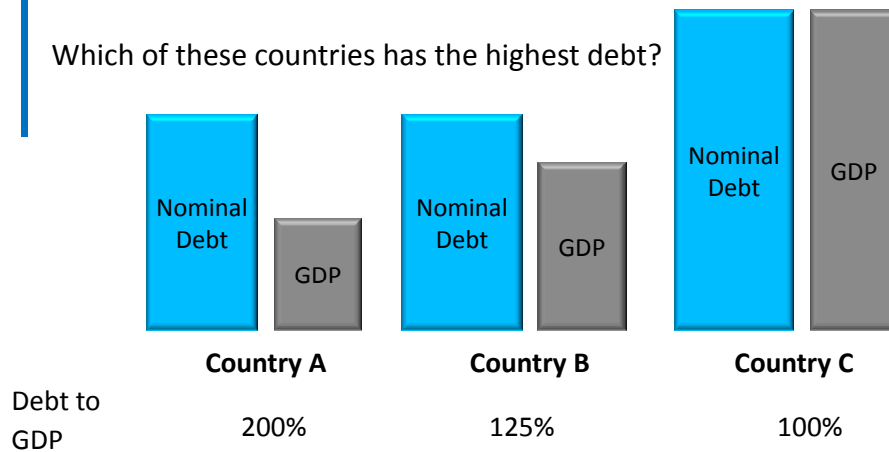


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Debt Dynamics

Debt dynamics in relation to GDP

Which of these countries has the highest debt?



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Debt Dynamics

Debt dynamics in terms of GDP ratios: Overall Balance

$$D_t = D_{t-1} - OB_t$$

$$\frac{D_t}{GDP_t} = \frac{D_{t-1}}{GDP_t} - \frac{OB_t}{GDP_t}$$

$$\frac{D_t}{GDP_t} = \frac{GDP_{t-1}}{GDP_t} \frac{D_{t-1}}{GDP_{t-1}} - \frac{OB_t}{GDP_t} \quad \frac{GDP_t}{GDP_{t-1}} = 1 + n$$

$$\frac{D_t}{GDP_t} = \frac{1}{1+n} \frac{D_{t-1}}{GDP_{t-1}} - \frac{OB_t}{GDP_t}$$

Evolution of debt ratio depends on:

- Overall balance ratio
- (Nominal) GDP growth

$$d_t = \frac{1}{1+n} d_{t-1} - ob_t$$

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Exercise

Derive debt dynamics for primary balance in terms of GDP ratios:

- First, take a piece of paper and a pen
- Second, start with the following relationship:

$$D_t = (1+i_t) D_{t-1} - PB_t$$

- Third, proceed as before



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Debt Dynamics

Debt dynamics in terms of GDP ratios: Primary Balance

$$d_t = \frac{1+i}{1+n} d_{t-1} - pb_t$$

$$1+i = (1+r)(1+\pi)$$

$$1+n = (1+g)(1+\pi)$$

r is real interest rate
 g is real growth rate
 π is GDP deflator inflation

$$d_t = \frac{1+r}{1+g} d_{t-1} - pb_t$$

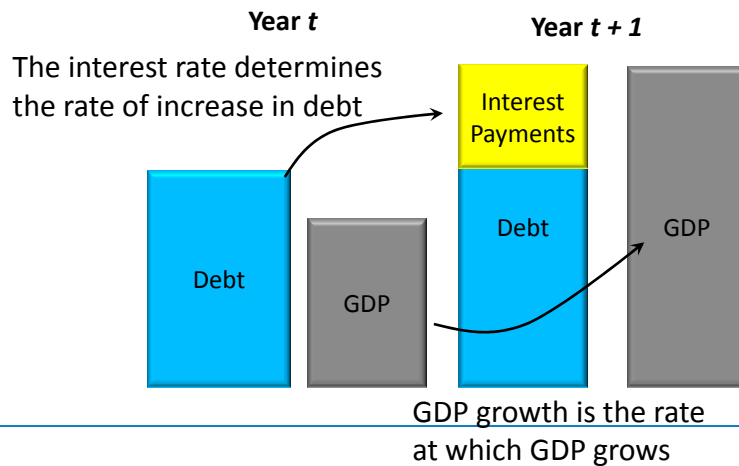
Autonomous
component of debt
dynamic

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Debt Dynamics

Debt dynamics in terms of GDP ratios: Autonomous component

Consider the case where $pb = 0$ (i.e., we abstract from fiscal policy):



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Debt Dynamics

Debt dynamics in terms of GDP ratios: Autonomous component

Think of debt-to-GDP as an aerostatic balloon:



Interest rate is like hot air: it pushes debt-to-GDP up

GDP growth is like the sand bags: it helps bring the debt-to-GDP down

If interest rate $>$ GDP growth, debt-to-GDP tend to \uparrow

If interest rate $<$ GDP growth, debt-to-GDP tend to \downarrow

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Exercise

Consider the role of autonomous component for debt dynamics using numerical examples:

- What happens when $r > g$?
- Assuming that the $r-g$ differential is 2 percentage points, what is the primary balance that would stabilize the debt ratio?



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Exercise (continued)

Consider the role of autonomous component for debt dynamics using numerical examples:

- What happens when $r < g$?
- Assuming that the $r-g$ differential is -2 percentage points, what is the primary balance that would stabilize the debt ratio?



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Debt Dynamics—Useful Formulas

Decomposition of debt dynamics:

From $d_t = \frac{1+r}{1+g}d_{t-1} - pb_t$ the change in debt becomes

$$d_t - d_{t-1} = \frac{r-g}{1+g}d_{t-1} - pb_t$$

One can attribute the change in debt-to-GDP to:

- Interest rates $\frac{r}{1+g}d_{t-1}$
- Growth $-\frac{g}{1+g}d_{t-1}$
- Fiscal policy $-pb_t$

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Exercise

Add the formulas for the debt decomposition to the debt projection spreadsheet



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Debt Dynamics—Useful Formulas

The debt-stabilizing primary balance:

If government wants to keep debt stable so that $d_t = d_{t-1}$ then:

$$d_{t-1} = \frac{1+r}{1+g} d_{t-1} - pb_t \quad \longrightarrow \quad pb_t = \frac{1+r}{1+g} d_{t-1} - d_{t-1}$$

$$pb_t = \frac{r-g}{1+g} d_{t-1}$$

- The larger the difference between r and g and the larger the initial debt, the greater the primary surplus/deficit needed to stabilize the debt
- One can revert the formula to compute at which level debt will stabilize if a certain balance was kept forever

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Exercise

Compute the debt-stabilizing primary balance for our previous exercise—does it look familiar?



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Debt Dynamics—Useful Formulas

Deriving the primary balance for targeting a debt level:

Consider the formula of the debt dynamic:

$$d_t = \frac{1+r}{1+g} d_{t-1} - pb_t \quad d_t = \beta d_{t-1} - pb_t$$

Starting from d_0 : $d_1 = \beta d_0 - pb_1$ $d_2 = \beta^2 d_0 - \beta pb_1 - pb_2$

$$d_3 = \beta^3 d_0 - \beta pb_1 - \beta pb_2 - pb_3 \quad \dots \quad d_t = \beta^t d_0 - \sum_{i=1}^t \beta^{t-i} pb_i$$

Given d_0 : What will debt be if the pb is kept constant forever?

Given d_0 : What is the constant pb that allows reaching a target debt to GDP d^* in T years?

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Debt Dynamics—Useful Formulas

Deriving the primary balance for targeting a debt level:

From $d_t = \beta^t d_0 - \sum_{i=1}^t \beta^{t-i} pb_i$ it follows that

$$d_t = \beta^t d_0 - pb \frac{1 - \beta^t}{1 - \beta}$$

If $\beta < 1$ then debt will eventually converge to

$$d^* = -pb \frac{1}{1 - \beta}$$

The constant pb that allows reaching a target debt to GDP d^* in T years is

$$pb = \frac{1 - \beta}{1 - \beta^t} (\beta^t d_0 - d^*)$$

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Debt Dynamics—Useful Formulas

Adding foreign currency/external debt:

Suppose part of debt is denominated in foreign currency. Let:

- e_t be the exchange rate at time t (units of local currency per 1 unit of foreign currency)
- i^F be the interest rate on foreign currency denominated debt, including risk premium
- D_t be total debt, D_t^N be debt denominated in local currency, and D_t^F be the local currency value of the debt denominated in foreign currency

With this notation, the debt stock comprising domestic and foreign-currency debt can be written as:

$$D_t = D_t^N + e_t D_t^F$$

D^N = Local currency-denominated debt
 D^F = Foreign currency-denominated debt
 e = Nominal exchange rate (local currency per \$)

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Debt Dynamics—Useful Formulas

Adding foreign currency/external debt (continued):

Let ε_t be the exchange rate depreciation and α_t be the share of debt denominated in foreign currency to total debt

$$D_t = (1+i)(1-\alpha_{t-1})D_{t-1} + (1+i^F)(1+\varepsilon_t)\alpha_{t-1}D_{t-1} - PB_t$$

Expressing all nominal variables in terms of GDP this becomes:

$$d_t = \left[(1+i)(1-\alpha_{t-1}) + (1+i^F)(1+\varepsilon_t)\alpha_{t-1} \right] \frac{d_{t-1}}{1+n} - pb_t$$

$$1 + i(1-\alpha_{t-1}) + i^F \alpha_{t-1} + (1+i^F)\varepsilon_t \alpha_t$$

Average interest rate (i^*)

$$d_t = \frac{1+i^*}{1+n} d_{t-1} - pb_t + \frac{(1+i^F)}{1+n} \varepsilon_t \alpha_{t-1} d_{t-1}$$

Equation similar to previous, except for effect of ε on interest payment and stock of debt

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Debt Dynamics

Adding foreign currency/external debt (continued):

Remembering that we could rewrite

$$d_t = \frac{1+i}{1+n} d_{t-1} - pb_t \quad \text{as} \quad d_t = \frac{1+r}{1+g} d_{t-1} - pb_t$$

We can express the corresponding formula with foreign currency debt as

$$d_t = \frac{1+r^*}{1+g} d_{t-1} - pb_t + \frac{(1+i^F)}{1+n} \varepsilon_t \alpha_{t-1} d_{t-1}$$

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Debt Sustainability Analysis

Basic approach:

Project debt in the medium and long term under:

- Current fiscal policy, including announced changes that will credibly be implemented
- Projections on long-term interest rates, growth rates, exchange rate, composition of debt, etc.

Consider also the major fiscal and macroeconomic vulnerabilities and project debt under different shocks scenarios:

- Increase interest rates
- Decrease growth
- Depreciation
- Materialize contingent liabilities
- Worse fiscal balances

Is debt sustainable under current policies? Does it explode? Will it stay below or breach comfortable levels under shocks?

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Debt Sustainability Analysis

Debt thresholds in IMF/WB DSA for LICs:

Countries operating in a weaker institutional and policy environment are likely to experience debt distress at significantly lower debt ratios.

Debt-burden indicators for external public debt	Assessment of institutional strength and quality of policies		
	Weak	Medium	Strong
1. PV of Debt/GDP	30	40	50
2. PV of Debt/Exports	100	150	200
3. PV of Debt/Revenue	200	250	300
4. Debt service/Exports	15	20	25
5. Debt service/Revenue	25	30	35

Source: IMF Staff Guidance Note on the Application of the Joint Fund-Bank Debt Sustainability Framework for LICs (2010)

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Debt Sustainability Analysis

What are Present Values (PVs)?

- Debt stock can be thought of as the sum of all future principal (amortization) payments
- Present value is the sum of all future *discounted debt service* payments
- Why are PVs used? Because they capture the concessionality of debt:
 - ✓ A loan that is repaid in 40 years, at a low interest rate, is much less burdensome than a loan that has to be repaid next year at a high interest rate.
 - ✓ The nominal debt stock computed today is indifferent between these two loans if the outstanding debt is the same, but the PV captures that the first loan is less burdensome.

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Exercise

Let's explore the PV of debt with some examples ...



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Exercise

You have encountered today (most) of the concepts used by the IMF for its LIC DSAs:

Take a look at the IMF/WB DSAs for your respective countries!



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Debt Sustainability Analysis

Tools for debt sustainability analysis—references:

- Debt sustainability analysis for market-access countries:
<http://www.imf.org/external/pubs/ft/dsa/mac.htm>
- Debt sustainability analysis for low-income countries:
<http://www.imf.org/external/pubs/ft/dsa/lic.aspx>
- DSAx -- Online course on debt sustainability analysis:
 - <http://imf.smartcatalogiq.com/en/current/Catalog/Online-Learning>
 - <https://www.edx.org/school/imfx>

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Appendix

Extra slides ...

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Debt Dynamics

Debt dynamics in terms of GDP ratios: Primary Balance

$$D_t = (1 + i) D_{t-1} - PB_t$$

$$\frac{D_t}{GDP_t} = (1 + i) \frac{D_{t-1}}{GDP_t} - \frac{PB_t}{GDP_t}$$

$$\frac{D_t}{GDP_t} = (1 + i) \frac{GDP_{t-1}}{GDP_t} \frac{D_{t-1}}{GDP_{t-1}} - \frac{PB_t}{GDP_t} \quad \frac{GDP_t}{GDP_{t-1}} = 1 + n$$

$$\frac{D_t}{GDP_t} = \frac{1 + i}{1 + n} \frac{D_{t-1}}{GDP_{t-1}} - \frac{PB_t}{GDP_t}$$

$$d_t = \frac{1 + i}{1 + n} d_{t-1} - pb_t$$

Autonomous
component of debt
dynamic

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