Chapter 6 Temporarily and Permanently Missing Prices and Quality Change

A. Introduction

6.1. Chapter 6 focuses on the treatment of temporarily and permanently missing varieties and their prices. While chapter 5 focuses on the collection of data, this chapter highlights the important role of the data collector in the context of the treatment of missing prices. First, the chapter provides an overview of the matched models method (MMM). While the MMM serves as the underlying method regarding the treatment of missing prices, the chapter describes how the MMM can potentially fail, the consequences of this failure, and how to deal with the effects of such failure on price measurement.

6.2. Next, temporarily missing prices and the methods used for the treatment of these missing varieties are reviewed. Third, the concept of quality is defined and discussed. Explicit (direct) and implicit (indirect) methods for quality adjustment are identified and described.

6.3. Some introductory notes are provided on general measurement issues including the use of additive versus multiplicative quality adjustments, price reference versus current period quality adjustment, short-term (S-T) versus long-term (L-T) comparisons, and geometric aggregation formula. Finally, the chapter considers the particular needs of price measurement in product markets with a rapid turnover of models, usually in the electronic and high-technology product markets.

B. Background

6.4. The measurement of changes in the level of consumer prices is complicated by the appearance and disappearance of new and old goods and services, as well as changes in the quality of existing ones. If there were no such complications, then a representative sample could be taken of the varieties\(^1\) of goods and services households consume in a reference period 0, their prices recorded and compared with the prices of the same matched varieties in subsequent periods. In this way the prices of like would be compared with like. In practice, such complications do exist. Varieties change in quality over time and replacements are of a different quality to the original. New and old models of varieties appear and disappear.

6.5. Changes in the quality of varieties should be treated as changes in the volume, as opposed to changes in the price, of the good/service provided. For example, increases over time in the concentration of a detergent (number of washes per 1kg. packet), faster internet service (megabits per second, Mbps), and inclusion of a warranty in the price of a dishwasher all contribute to effective decreases in price; consumers get more for their money. Similarly, quality decreases, for example less legroom in economy flights, when prices remain constant.

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\(^1\) As a departure from the nomenclature of the CPI Manual: Theory and Practice (2004), this Update uses the term “variety” instead of the previously used term “item” to refer to “An individual good or service in the sample of products selected for pricing.” Glossary, CPI Manual: Theory and Practice (2004, page 447). The term “model” of a good or service is also used in this chapter, mainly in the context of high-technology goods such as laptops, household appliances, automobiles. This use of the term “model” follows general usage and refers to a specific variety whose characteristics are updated regularly.
are effective increases in price. A volume change for an individual variety may be comprised of a quantity and quality change. The change in the variety’s nominal value of consumer expenditure would be the product of its price and volume change. It follows that the price change would be the change in value divided by the change in volume.

6.6. National Statistical Offices (NSOs) go to great lengths to ensure measured price changes are not influenced by changes in the quality of items. By measuring the price change of a fixed, constant-quality basket of goods and services, they use the matched-models method (MMM). When updating the basket, price collectors visit selected outlets with broad details of an item and identify the most popular, regularly stocked, varieties sold in each of the outlets. Next, they develop a detailed description of the variety that includes all price-determining characteristics (such as brand, size, etc.) and record the price. This specification must be sufficiently detailed to include all price-determining characteristics in order to define a unique, specific variety. The detailed specification allows the price-collector to easily identify the item in subsequent periods and record its matched price.

6.7. The measurement of changes in the level of consumer prices by the MMM is appropriate when variety prices are not missing. However, the use of the MMM is complicated by temporarily unavailable prices, for say one, two, or three months, due to a variety being out-of-stock and not yet replenished. A matched price is unavailable in these intervening months. The treatment of prices of temporarily missing varieties is considered in more detail below but typically requires that the missing variety’s price is imputed for the month(s) in which it is missing using the price changes of similar goods/services or price changes drawn from a higher level of aggregation. Actual prices are then compared with imputed prices for the measurement of the CPI.

6.8. Should price quotes become permanently unavailable a replacement variety is required that is preferably comparable in terms of the price-determining specifications of the missing variety. If the replacement is of a comparable quality (i.e., possesses the same price-determining characteristics), its price can be directly compared with the last actual or imputed price for the missing variety. If the replacement is non-comparable, say it is of a better quality, the improvement in quality has to be explicitly quantified in terms of its “worth” or its contribution to the price. Using this value, compilers make a price adjustment to reflect the difference in quality allowing the price of like be compared with like. If a reliable explicit quality-adjustment to the price is not possible, for data or resource reasons, implicit methods of quality adjustment are available. Details of explicit and implicit methods of quality adjustment are provided below and methods for dealing with new and disappearing goods and services illustrated in chapter 8.

6.9. Products that are “strongly” seasonal, that is, missing in particular months when out-of-season but expected to return in the next season could also be treated as temporarily missing and imputed. Chapter 11 describes in more detail the different options that can be used for the treatment of seasonal items. Strongly seasonal products include some fresh fruits, vegetables and clothing. Also considered in chapter 11 is the treatment of “weakly” seasonal products: available throughout the year but prices, sales and quality fluctuate throughout the year. The prices of weakly seasonal products are not missing and are treated differently from strongly seasonal ones, again as discussed in chapter 11.
6.10. Matching models facilitates the measurement of constant-quality price change. When the matching breaks down, due to missing prices, temporary imputations are necessary until the variety’s temporarily missing prices becomes available, or to introduce a replacement, thus serving to help update the sample. But what of product markets where the matching breaks down on a regular basis because of the high turnover in new models of different qualities to the old ones, for example, laptop computers? A failure to match and replace models would lead to a seriously depleted and unrepresentative sample. Yet, a continual process of linking-in new replacement variety has been found to lead to a bias in CPI measurement. Section .. of this chapter outlines an alternative approach making use of hedonic regressions.

C. Why the matched models method may fail

6.11. Three potential sources of error arise from the matched models approach: missing varieties, representativity of sample space, and new products.

1. Missing varieties

6.12. The first source of error, and the focus of this chapter, occurs when a variety is no longer available in the outlet. It may be temporarily out of stock, discontinued, or one or more of the price-determining characteristics have changed. Whatever the reason, the variety is effectively missing in the current period and a price cannot be collected. The variety’s price may be missing for other reasons. It may be a seasonal variety or one whose price does not need to be recorded so frequently, or it may be that the variety is a custom-made product or service, supplied each time to the customer’s specification.

6.13. It is necessary to distinguish between varieties that are permanently and temporarily missing. Varieties that are temporarily missing are varieties not available and not priced in the current period in question, but that are available and priced in subsequent periods. The treatment of varieties missing because demand and/or supply is seasonal, as is the case with some fruits and vegetables, is the subject of chapter 11.

6.14. The different methods for the treatment of missing prices, listed in Figure 6.3, will be discussed later in some detail, along with the assumptions implied by them. By definition, the prices of the unavailable items cannot be determined. The accuracy of some of the assumptions about their price changes, had they been available, is difficult to establish. What is stressed here is that the matching of prices of varieties allows for the measurement of price changes untainted by quality changes. When varieties are replaced with new ones of a different quality, then a quality-adjusted price is required. If the adjustment is inappropriate, there is an error, and if it is inappropriate in a systematic direction, there is a bias. Careful quality adjustment practices are required to avoid error and bias. Such adjustments are the subject of this chapter.

2. Sampling concerns

6.15. There are three sampling concerns. First, the MMM and the use of replacements is designed to meet the needs of constant-quality price measurement and while the sample of varieties priced might initially be designed to be representative of price changes of the population of all varieties, it is effectively following a static sample of varieties that, over time, can become increasingly unrepresentative. The matching of prices of identical varieties over
time, by its nature, is likely to lead to the monitoring of a sample of varieties increasingly unrepresentative of the population of transactions. The sample deteriorates over time because the MMM fails to incorporate new models/products into the sample except as replacements to obsolete ones. For example, substantial developments in telecommunication hardware and services, reflected in the growing variety of models, are excluded from the sample of models/services covered by the CPI. This omission would not be problematic were the (implicit) quality-adjusted price changes of the excluded varieties similar to those of the included matched model sample. However, this is unlikely to be the case. The (quality-adjusted) prices of old varieties being dropped may well be relatively low and the (quality-adjusted) prices of new ones relatively high as part of a sales strategy of dumping old models, at relatively low price to make way for the introduction of new models priced relatively high.

6.16. A second sampling concern relates to the timing of the substitution: when a replacement variety is chosen to replace an old one. In general, the prices of varieties continue to be monitored until they are no longer produced. This means that old varieties with limited sales continue to be monitored and included in the sample. Such varieties may exhibit unusual price changes as they near the end of their life cycle, because of the marketing strategies of firms. Firms typically identify gains to be made from different pricing strategies at different times in the life cycle of products, particularly at the introduction and end of their life cycle. The (implicit or otherwise) weight of end-of-cycle varieties in the index would thus remain relatively high, being based on their sales share when they were sampled. Furthermore, new unmatched varieties with possibly relatively large sales would be ignored. As a consequence, undue weight would be given to the unusual price changes of matched varieties at the end of their life cycle.

6.17. The final sampling problem results from the enumerator collecting prices until the variety is no longer available, thus forcing a replacement. Data collectors replace the missing discontinued variety with the most popular or typically-consumed variety. This improves the coverage and representativity of the sample. But it also makes reliable quality adjustments of prices between the old and new popular varieties more difficult. The differences in quality are likely to be beyond those that can be attributed to price differences in some overlap period, as one variety is in the final stages of its life cycle and the other in its first. Furthermore, the technical differences between the varieties are likely to be of a order that makes it more difficult to provide reliable, explicit estimates of the effect of quality differences on prices. Finally, the (quality-adjusted) price changes of very old and very new varieties are unlikely to meet assumptions of “similar price changes to existing varieties or classes of varieties”, as required by the imputation methods. Many of the methods of dealing with quality adjustment for unavailable varieties may be better served if the switch to a replacement variety is made earlier rather than later. Sampling issues are inextricably linked to quality adjustment methods. This will be taken up in Chapter 8 on variety selection and the need for an integrated approach to dealing with both representativity and quality-adjusted prices.

6.18. Throughout this chapter reference will be made to the need for permanently replacements to be undertaken for varieties with missing prices in order that the sample of varieties does not become unrepresentative. Samples of representative varieties and outlets are generally updated when an index is rebased. Where there is a lengthy period between rebasing, the sample can become seriously deteriorated. It is quite feasible to update/rotate the sample between periods of rebasing and chapter 7 outlines in the context of maintaining the
representativity of the sample. In this chapter we will refer to a need for regular rebasing to update the sample and bear in mind that this might also be achieved by sample rotation.

3. New products

6.19. A further potential source of error arises when something new is introduced into the marketplace. When a really new variety is introduced, there is an immediate gain in welfare or utility as demand switches from the old variety to the new variety. For example, the introduction of mobile telephones represented a completely new good that led to an initial gain in utility or welfare to consumers as they switched from the old (landlines) to the new technology. This gain from the introduction of mobile phones, and subsequently of increasingly smarter phones, would not be properly brought into the index by waiting until the index was rebased, or by waiting for at least two successive periods of prices for mobile phones and linking the new price comparison to the old index. Subsequent prices might be constant or even fall. The initial welfare gain would be calculated from a comparison between the price in the period of introduction and the hypothetical price in the preceding period, during which supply would be zero. The practical tools for estimating such a hypothetical price are neither well developed nor practical for CPI compilation, as outlined and referenced in more detail in chapter 7. For a CPI built on the concept of a base period and a fixed basket, there is, strictly speaking, no problem. The new product was not in the old basket and should be excluded. Although an index properly measuring an old fixed basket would be appropriate in a definitional sense, it would not be representative of what we buy. Such an index would thus be inappropriate. For a cost of living index concerned with measuring the change in expenditure necessary to maintain a constant level of utility, there is no doubt that it would be conceptually appropriate to include the new product and any welfare gain from its introduction, though as outlined in chapter 8, this is highly problematic in practice.

D. Some general points

1. Multiplicative versus additive adjustment

6.20. Explicit quality adjustments to prices can be made by adding a fixed amount or multiplication by a ratio. For example, consider \( m \), an old variety, and \( n \) its replacement and, for a price comparison over periods \( t, t + 1, \) and \( t + 2 \), the price of \( m \) is only available in periods \( t \) and \( t + 1 \) and \( n \) only available in periods \( t+1 \) and \( t + 2 \). The measurement of constant-quality price change between periods \( t \) and \( t + 1 \) is based on product \( m \), \( p_{m}^{t+1} / p_{m}^{t} \), and the price change between periods \( t + 1 \) and \( t + 2 \) on product \( n \), \( p_{n}^{t+2} / p_{n}^{t+1} \). Although not necessary for the compilation of the price index, the previous calculation can be elaborated in an equivalent, though more complex, form that enables the nature of the quality adjustment for \( m \) to \( n \) to be identified and the multiplicative formulation demonstrated.

6.21. A price relative over periods \( t, t + 1, \) and \( t + 2 \) requires an overlap ratio \( p_{n}^{t+1} / p_{m}^{t+1} \) to be used as a measure of the relative quality difference between the old variety and its replacement. This ratio could then be multiplied by the price of the old variety in period \( t, p_{m}^{t} \) to obtain the quality-adjusted prices \( p_{n}^{t} \) as outlined later in equation (6.5), but illustrated in the following table:
6.22. Such multiplicative formulations are generally advised, as the adjustment is invariant to the absolute value of the price. The overlap ratio equals, for example, 1.2; therefore, the new variety costs 20 percent more than the old. Yet there may be some varieties for which the worth of the constituent parts is not considered to be in proportion to the price. In other words, the constituent parts have their own, intrinsic, absolute, additive worth, which remains constant over time. Retailers selling over web sites may, for example, include free-shipping in the price. In some instances, the cost of shipping may remain the same in the short to medium-term irrespective of what happens to the price of the variety (exclusive of shipping). If the price no longer includes free-shipping, this fall in quality should be valued as a fixed additive sum.

2. Price reference versus current period adjustment

6.23. Two variants of the approaches to quality adjustment are to make the adjustment either to the price in the price reference period or to the price in the current period. For example, in the overlap method, described above, the implicit quality adjustment coefficient was used to adjust $p^t_m$ to $p^n_r$. An alternative procedure would have been to multiply the ratio $p^{t+1}_m / p^{t+1}_n$ by the price of the replacement variety $p^{t+2}_n$ to obtain the quality-adjusted price $p^{t+2}_m$, etc. The first approach is more straightforward since, once the base period price has been adjusted, no subsequent adjustments are required. Each new price of a replacement can be compared with the adjusted base period price. More importantly, the valuation of the quality differential that took place in period $t+1$ can be applied to period $t$ with more confidence than the ongoing periods $t+2, t+3, t+4$ and so forth.

3. Long-term versus short-term comparisons

6.24. Much of the analysis of quality adjustments in this manual has been undertaken by comparing prices between two adjacent periods, say, month-on-month period $t$ prices with those in a subsequent period $t+1$. For long-term comparisons the price reference period is taken as, for example, period $t$ and the index is compiled by comparing prices in period $t$ first with $t+1$; then $t$ with $t+2; t$ with $t+3$, etc. The short-term framework allows long-term comparisons built up as the product of links: $t$ first with $t+1$; then $t+1$ with $t+2; t+2$ with $t+3$, etc.; built up as a sequence of links joined together by successive multiplication. This chapter focuses on S-T comparisons, for reasons of their inherently better properties and for focus of exposition; chapter 9 provides detail, an illustration, and the relative merits of the use of the L-T as opposed to S-T methods. In particular, as will be outlined in this chapter, the S-T approach enables superior imputations to be made for temporarily missing prices and facilitates the incorporation of replacement varieties as and when an “old” variety’s price is permanently missing. The S-T approach is generally recommended.
Treatment of missing variety prices and quality adjustment within an elementary aggregate: two-stage (short-term) comparisons

6.25. This chapter uses a S-T framework of comparing period-on-period prices rather than a L-T framework of comparing the current period’s price with a fixed price reference period. The use of matching is particularly problematic for long-term (L-T) price comparisons. For L-T price comparisons a selection of representative models in period 0, say the year (or a month within) 2020, has their prices compared with those in January 2021; in February, for the 2020-February price relative; March, for the 2020-March price relative; continuing for what may be in some countries, several years. The sample is increasingly depleted over time as 2020 varieties become obsolete.

6.26. The S-T approach has a number of advantages. Illustration of the S-T as against the L-T approaches has also been delayed until chapter 7, to maintain a focus in this chapter on the methods for the treatment of missing prices, but also to avoid repetition. This chapter is concerned with temporarily and permanently missing variety prices. Temporarily missing variety prices return to the sample, as do seasonal ones, whose treatment is outlined in chapter 11. There is no issue here with maintaining the sample. However, permanently missing variety prices need a replacement variety otherwise, over time, the sample becomes increasingly depleted and degraded. Yet such one-on-one replacement is unlikely to be sufficient to maintain the representativity of the sample, something based on an initiation of variety selection at the last rebasing (chapter 9), or sample rotation, that may for some countries be many years ago. Since this initiation many newer varieties/products may have been introduced and old ones become obsolete. Maintaining the representativity of the sample is addressed in chapter 7 and a related use of web-based and scanner data sources in chapter 10. The treatment of permanently missing variety prices was set within the context of an elementary aggregate, where weights were neither available nor used. A two-stage Lowe price index number is used, though not explicit. We provide its formulas below and in more detail in chapter 7.

6.27. The recommended procedure in practical higher-level index number compilation is to use a two-stage or “modified” formulation, instead of basing the aggregation on long-run price changes, compiled in a single stage.

6.28. For example, consider a Lowe index in which, for each elementary aggregate the 2018 weights are price-updated to price reference period of 2019=100.0 as outlined in chapter 9, given here by \( w_i^{2019*} \). The January 2020 price index (2019=100.00) is the weighted \( w_i^{2019*} \) sum of each elementary aggregate’s price index, \( \frac{p_{i, \text{Jan}2020}}{p_i^{2019}} \), as shown by the first term below. The \( p_i^{\text{January}} \) here, for example, is a price index for each \( i \) elementary aggregate in January 2020.
6.29. The CPI is the sum of the weights times the associated price changes. In February, we start the two-stage procedure: first for each elementary aggregate \(i\) its weight is multiplied by the preceding month’s price change (for 2019 to January 2020) taken from the previous period’s calculation, to form an “uprated weight.” Second, the price change between the current month and its preceding month, January to February, is calculated and multiplied by the previous period’s up-rated weight, to form a new uprated weight. For March, the procedure is repeated: the “new” up-rated weight is taken from the previous period and multiplied by the February:March price change. In each month, all we need is the preceding period’s up-rated weights, readily available from last month’s calculation, and the current to preceding month’s price change. As can also be seen, the mean prices for each \(i\) in successive numerators cancel with those in the denominators to show this is equivalent to a direct long-run price index. This seeming equivalence masks two major advantages of the formula.

6.30. First, each S-T price relative for an elementary aggregate \(i\), for example, in April 2020, \(\frac{p_{\text{Apr}2020}^i}{p_{\text{Mar}2020}^i}\), are (geometric) mean prices in April 2020 compared with geometric mean prices in March of matched varieties. Table 6.9 might illustrate an individual elementary aggregate with geometric mean prices compiled from several outlets. If a variety is temporarily no longer available an imputation can be based on S-T month-on-month price relative, rather than a L-T price that might assume similar price movements over several years.

6.31. Similarly, for permanently missing prices where imputations are used to form an overlap comparison for the missing variety price and its replacement, assumptions based on similar S-T month-on-month price movements are more reasonable than the much less plausible ones based on L-T price movements. This mechanism facilitates the inclusion of new

\[
(6.21) \sum_i w_{i}^{2019^*} \frac{p_{\text{Jan}2020}^i}{p_{\text{Feb}2020}^i}
\]

\[
\sum_i \left[ w_{i}^{2019^*} \frac{p_{\text{Jan}2020}^i}{p_{\text{Feb}2020}^i} \right] p_{\text{Feb}2020}^i = \sum_i \left[ w_{i}^{2019^*} \frac{p_{\text{Feb}2020}^i}{p_{\text{Jan}2020}^i} \right]
\]

\[
\sum_i \left[ w_{i}^{2019^*} \frac{p_{\text{Feb}2020}^i}{p_{\text{Mar}2020}^i} \right] \left( \frac{p_{\text{Mar}2020}^i}{p_{\text{Feb}2020}^i} \right) = \sum_i \left[ w_{i}^{2019^*} \frac{p_{\text{Mar}2020}^i}{p_{\text{Feb}2020}^i} \right]
\]

\[
\sum_i \left[ w_{i}^{2019^*} \frac{p_{\text{Mar}2020}^i}{p_{\text{Apr}2020}^i} \right] \left( \frac{p_{\text{Apr}2020}^i}{p_{\text{Mar}2020}^i} \right) = \sum_i \left[ w_{i}^{2019^*} \frac{p_{\text{Apr}2020}^i}{p_{\text{Mar}2020}^i} \right]
\]

\[
\sum_i \left[ w_{i}^{2019^*} \frac{p_{t-1}^i}{p_{t-2}^i} \right] \frac{p_t^i}{p_{t-1}^i}
\]

\[2\] Use is made of the term “uprated” rather than “updated” since the new weights are not relative values updated to a more recent period, but the relative quantities in a reference period (2019 in the example) multiplied by the price increase from the reference period up to the preceding period of the S-T price relative, \(p_{t-1}^i\). It has no economic meaning as a value weight, but is a useful calculation device.
specifications when old specifications become obsolete and enables the index to better represent the dynamic changes taking place in consumer choice. A direct comparison between the price of a new replacement variety specification in April 2020 with its old specification in 2019 is likely to be fraught with difficulties given the quality differences between the two variety specifications over a long period.

6.32. The two-stage Lowe will differ—be improved—from its fixed base (L-T) counterpart since the monthly imputations on the left will differ to those on the right.

6.33. Second, the use of \( \frac{p_i^t}{p_i^{t-1}} \) in the compilation facilitates data verification since outliers in short-run changes are more readily identifiable than those in long-run ones. In practice, a computer routine for an index need only maintain as active files the previous month’s up-rated weights and the previous and current month’s prices and price change.

6.34. This chapter’s work has for the large part been concerned with S-T price relatives compiled within an elementary aggregate, \( \hat{i} \). The larger picture of weighted aggregation across elementary aggregates is for this context of missing prices and sample representativity, considered in chapters 7, with illustrative calculations of the aggregation formulas in chapter 8, and the introduction of new weights in chapter 9.

6.35.

4. Aggregation formula for elementary price indices

6.36. A ratio of geometric means—the Jevons price index number formula—is used to measure price changes at this un-weighted level below the elementary aggregate level. Alternative formulae include a ratio of arithmetic means—the Dutot price index number formula—and an arithmetic average of price ratios—the Carli price index number formula. The Jevons price index formula is used here for reasons of its better properties and focus of exposition. As with the previous section, chapter 9 provides detail, an illustration, and the relative merits of the use of the Dutot and Carli price index number formula.

6.37. It will be the case that with scanner and other such data, information on prices, expenditure values, and quality characteristics will often be available for the vast majority of individual models sold by major outlets. This availability of data on transaction values allows weights to be used at this detailed level of aggregation and thus the use of weighted price index formulas as outlined in chapters 9.

5. The role of price collectors

6.38. Price collectors have a critical role to play in the treatment of missing price observations. They observe and record that a price is missing; whether it is temporarily or permanently missing; if permanently missing, whether a comparable or non-comparable replacement is available, and in the latter case, the price and details of the replacement variety. As explained in chapter 5, on the sampling of periodic prices, the outlets are visited in a process referred to as initiation, to determine the detailed specifications of representative varieties sold. For example, for the general class of “large white bread, sliced,” the more detailed, “large loaf, white, unsliced, brand A, 800gm” may be selected and its details entered along with its price for subsequent periodic repricing. Ideally, price collectors should have in their possession a
checklist of these specifications when visiting outlets in subsequent periods. The detailed specifications serve many purposes including: (i) to help identify the variety to be priced; (ii) to review and verify the variety’s specification to ensure there have been no changes in the price determining characteristics; and (iii) if the variety is non-comparable, use the specifications to identify a replacement variety to be priced and record any changes in the price-determining specifications.

6.39. The price collector also plays an important role in determining whether the missing price should be treated as temporarily or permanently missing. A variety’s price can be considered to be temporarily missing if the same variety is likely to return to the market within a reasonable time period. On finding that the specified variety is not available for immediate sale, the price collector should check with the manager or informed member of staff whether it is temporarily or permanently missing. If temporarily missing, the expected duration: one, two, or more periods should be recorded along with the reason for it being unavailable and an indication of the likelihood of its return.

6.40. Temporarily missing varieties have their prices imputed; permanently missing ones require a replacement. As these are different issues requiring different treatments, it is important for the price collector to establish whether the unavailability of the variety is temporary or permanent. Consider the case of a monthly CPI. When the price is regarded as being temporarily missing, it should be imputed using an overall mean imputation, a targeted mean imputation, or a class-mean imputation. Some NSOs use a method referred to as carry-forward (repeating the last observed price). As stressed below, this is not recommended.

6.41. Permanent unavailability occurs when the variety is withdrawn from the market with no prospect of returning. In some instances, it might be absent the next month and confirmed by the outlet manager/senior staff that it is not going to be replaced. With such information the price collector should immediately look to collecting the price and specifications of a replacement variety. In other cases, if a variety is out of stock for, say, three consecutive months, the price collector should be instructed to choose a replacement which matches as closely as possible the missing variety’s specification. After being unavailable for three months and with no indication that the variety will return, it is treated as being permanently missing and a replacement variety is sought. There may be particular products/circumstances in which the three-month rule can be relaxed, such as a temporary withdrawal of products for health reasons, national emergencies, or the logistics of restocking where there is a sound basis for a belief that the product will return in the near future, but not within three-months.

6.42. Decisions on the treatment of missing prices are made by a desk officer based on information provided by the price collector and, in some instances, by a follow-up telephone/visit to the outlet.

6.43. Variety prices may be missing for products because they are seasonal and out of season. Products which are out-of-season, but expected to return in the next season, are treated differently to those considered here and their treatment is the subject of chapter 11.

6.44. As outlined in chapter 5, codes, such as those illustrated in Table 6.1 below, should be used to justify or explain each missing price in order to ensure proper treatment. Metadata should be collected on those product groups in which, for example, there is a high level of missing prices of different forms and the extent to which the prices are missing. Illustrative
variety codes are given below and NSOs should build on the detail required to meet their specific needs.

**Table 6.1 Illustrative variety codes for price collector for missing values**

<table>
<thead>
<tr>
<th>Collection code</th>
<th>Description</th>
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<tbody>
<tr>
<td>T</td>
<td>Temporarily Missing – The variety is unavailable but is expected to be available again in the near future.</td>
</tr>
<tr>
<td>P</td>
<td>Permanently Missing - The variety is no longer available and is unlikely to return.</td>
</tr>
<tr>
<td>S</td>
<td>Seasonally Missing – the variety (product group) is strongly seasonal and is out of season.</td>
</tr>
<tr>
<td>C</td>
<td>Comparable Replacement - A replacement variety that is comparable to the old variety in all major aspects.</td>
</tr>
<tr>
<td>NC</td>
<td>Non-comparable Replacement – A replacement variety that is not comparable to the old variety.</td>
</tr>
</tbody>
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### E. The Treatment of Temporarily and Permanently Missing Variety Prices

6.45. To measure aggregate price changes, a representative sample of varieties is selected from a sample of outlets, along with a host of details that define each variety, their specifications. For each selected variety, detailed item specifications, or descriptions, define a unique, specific variety that will be priced each period. The detailed specifications are included on the re-pricing form each period, and serve as a prompt to help ensure that the same varieties are being priced. Detailed checklists of variety descriptions should be used, as any lack of clarity in the specifications may lead to errors. Attention should also be devoted to ensuring that the specifications used are not just to identify the variety on a subsequent visit, for example its location in the outlet, but contain all pertinent, price-determining characteristics, otherwise it will not be possible to identify when possible changes in quality have occurred.

6.46. The MMM succeeds in ensuring the prices of like are compared with like. The detailed specifications facilitate this process, and the method ensures that the measurement of price change is not influenced by quality changes. However, when a price is missing there is the
potential for mis-measurement. The treatment of missing values depends on whether the variety is temporarily missing—the product will be available in the near future because it is out-of-stock—or permanently missing, the variety will no longer be available in the future. Each month, all imputed prices are used in the calculation of the index. The imputed March price would be compared with the actual price collected in February in order to calculate the price change from February to March. Should the variety return in April, the imputed March price is compared with its actual price collected in April to calculate the March to April price change. All permanently missing prices require a replacement.

6.47. When a variety is missing in a month, a number of approaches may be used. Details of each method are provided in separate sections of this chapter below. While terminology may differ between NSOs, the methods include:

- **imputation**: the price changes of all varieties in the product group, or targeted similar ones, are assumed to be the same as that for the missing variety. Such imputations can be used for temporarily missing varieties. Permanently missing varieties, however, require a comparable or non-comparable replacement;

- **direct comparison**: if a variety is permanently missing and a replacement variety is directly comparable, that is, it is so similar it can be assumed to have had more or less the same quality characteristics as the missing one, its price replaces the price of the unavailable variety. Any difference in price level between the new and the old is judged to be price change and not quality change;

- **explicit quality adjustment**: if a replacement variety is non-comparable—there are identifiable quality differences—estimates of the impact of the quality differences on the price enable quality-adjusted price comparisons to be made;

- **implicit quality adjustment: overlap**: if a replacement variety is non-comparable and no information is available, or resources too limited, to allow reasonable explicit estimates to be made of the impact of a quality change on the price, the price difference between the old variety and its replacement in the overlap period is then taken to be a measure of the quality differential.

6.48. Specific attention will need to be devoted to product areas with relatively high weights, where large proportions of varieties are turned over. Some of the methods are not straightforward and require a higher level of expertise and experience. Quality adjustment should be implemented by developing a methodical approach on a product-by-product basis. Such concerns should not be used as excuses for failing to attempt to estimate quality-adjusted prices. Ignoring quality change results in an implicit quality adjustment. This approach assumes that any difference in price is pure price change and not changes due to difference in quality. Such an implicit approach may not be appropriate, and may even be misleading.

6.49. The treatment of missing price quotes for varieties is divided into two types: temporarily and permanently missing variety prices. These two types cannot always be readily identified and treated accordingly. Some form of mechanism or rule is required to enable a transition from temporary to permanently missing variety prices. A price collector, supported by a desk officer, would regard a variety as permanently unavailable if verified by a senior member of the outlet’s staff or following a three-month period, the variety is no longer available and there is no evidence that it will reappear. A price index suffers from sample depletion if an
increasing number of temporary missing prices are imputed over a lengthy period of time. Once judged to be “permanently missing” a replacement is found. If non-comparable, a quality adjustment is made and imputations are no longer necessary.

6.50. However, it may the case that a replacement, comparable or otherwise, for example for a videocassette player when it became obsolete, is unavailable. This problem area of introducing new goods and services and removing obsolete ones is the subject of chapter 7. Alternatively, it may be that a price collector finds the variety to be permanently missing in the outlet visited, and that there is no comparable or non-comparable replacement, although the variety is sold in other outlets. A senior outlet manager may inform the price collector that the product line is no longer being stocked, for example, bicycles in a sports shop. A price index for bicycles could be continued by imputing the price change for this outlet using the price change in other outlets, as outlined below. Such a procedure depletes the sample and in the longer run this should be remedied, as outlined in chapter 7, by a forced outlet replacement, sample rotation, or rebasing.

6.51. Adjustments to prices is not a simple matter of applying routine methods to prices in specified product areas. A number of alternative approaches are suggested below. Some will be more appropriate than others for specific product areas. An understanding of the consumer market, technological features of the producing industry, and alternative data sources will all be required for the successful implementation of quality adjustments.

F. Temporarily missing price observations

1. Overall mean imputation

6.52. The overall mean imputation method uses the price changes of other similar varieties as estimates of the price change of the missing variety. Consider a Jevons elementary price index, i.e., a geometric mean of price relatives, equivalent to the ratio for geometric means of prices\(^3\). The price of the missing variety in the current period, say \(t + 1\), is imputed by multiplying its price in the immediately preceding period \(t\) by the geometric mean of the price relatives of the remaining matched varieties in the product group between these two periods. The method provides the same result as simply dropping the variety that is missing from both periods from the calculation. In practice, the series is continued in the database by including the imputed prices; this then forms a complete table of the variety prices in outlets A to F. The imputations are based on assumptions of similar price movements.

6.53. Consider the illustrative example in Table 6.2. A product with broad specifications is sold in six outlets, A to F, with different detailed store-specific specifications adopted for each outlet. The price reference period is December 2019 with successive prices collected for each outlet’s specification in each of January, February, March, April, May, June, and July 2020. The price collector finds the variety temporarily missing in outlet F’s price collection for March 2020, and likely to remain missing for the next month or so, but to return thereafter.

---

\(^3\) Chapter 9 provides further examples of imputations and replacements using the Jevons index, as compared with the Dutot (ratio of arithmetic means) and Carli indices (mean of the ratio of price relatives) and outlines the relative merits of the three indices.
Table 6.2 Temporarily missing price observations and imputed prices

<table>
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</thead>
<tbody>
<tr>
<td>A: supermarket</td>
<td></td>
<td>5.25</td>
<td>5.25</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
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</tr>
<tr>
<td>B: supermarket</td>
<td></td>
<td>5.10</td>
<td>5.10</td>
<td>5.10</td>
<td>5.25</td>
<td>5.25</td>
<td>5.25</td>
<td>5.25</td>
<td>5.25</td>
</tr>
<tr>
<td>C: supermarket</td>
<td></td>
<td>5.20</td>
<td>5.20</td>
<td>5.20</td>
<td>5.20</td>
<td>5.25</td>
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<td>5.25</td>
<td>5.25</td>
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<tr>
<td>D: independent trader</td>
<td></td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
</tr>
<tr>
<td>E: independent trader</td>
<td></td>
<td>5.99</td>
<td>6.50</td>
<td>6.50</td>
<td>6.90</td>
<td>6.90</td>
<td>6.90</td>
<td>6.90</td>
<td>7.00</td>
</tr>
</tbody>
</table>

Overall mean imputation: F Mar:May 20
Geo-mean: A:E
5.54 5.67 5.69 5.71
Geo-mean: A:F
5.49 5.57 5.61 5.74 5.76 5.78 5.79 5.84
Short-term (S-T) price relatives: A:F
1 1.014572 1.007181 1.023173 1.003484 1.003472 1.001730 1.008636
Long-term indices as product of S-T
100.00 101.46 102.19 104.55 104.92 105.28 105.46 106.38

Targeted imputation: indep traders
D: independent trader
5.49 5.49 5.49 5.65 5.75 5.80 5.80 6.00
E: independent trader
5.99 6.50 6.50 6.90 6.90 6.90 6.90 7.00
F: independent trader
Geo-mean: A:C & D:TF
5.49 5.57 5.61 5.76 5.79 5.81 5.79 5.84
S-T price relatives: A:F
1 1.014572 1.007181 1.023173 1.003484 1.003472 1.001730 1.008636
L-T indices as product of S-T
100.00 101.46 102.19 104.55 104.92 105.28 105.46 106.38

6.54. The Jevons index number formula is shown in equation (6.1) as a direct/long-term (L-T) index comparing, in its second last term, the geometric mean of the prices of each matched variety in the current month \( t \) with the geometric mean of the prices in the price reference period, hereafter 0, and in the last term, for the example in Table 6.2, July 2020 with the price reference period = 100.0 of December 2019.

\[
P_J(P^0, P^t) = \prod_{i=1}^{N} \left( \frac{P_i^t}{P_i^0} \right)^{\frac{1}{N}} = \prod_{i=1}^{N} \left( \frac{\prod_{t=1}^{t}(P_i^t)^{\frac{1}{N}}}{\prod_{i=1}^{N}(P_i^0)^{\frac{1}{N}}} \right)^{\frac{1}{N}}
\]

6.55. In practice the use of the Jevons formula in this L-T form is not advised. Instead a short-term (S-T) formulation is recommended as the product of month-on-month Jevons indices. The S-T cumulative Jevons index for December 2019=100 to July 2020 is:

\[
P_J(P^0, P^t) = \prod_{i=1}^{N} \left( \frac{P_i^{t-1}}{P_i^{t-2}} \right)^{\frac{1}{N}} \times \prod_{i=1}^{N} \left( \frac{P_i^{t-2}}{P_i^{t-3}} \right)^{\frac{1}{N}} \times \prod_{i=1}^{N} \left( \frac{P_i^{t-3}}{P_i^{t-4}} \right)^{\frac{1}{N}} \times \ldots \times \prod_{i=1}^{N} \left( \frac{P_i^{t-1}}{P_i^{t-N}} \right)^{\frac{1}{N}}
\]
6.56. The (L-T) and (S-T) approaches in equations (6.1) and (6.2) provide the same answer as the numerator of each term on the right-hand-side of equation (6.2) cancels with the denominator of the next. However, a major advantage of the S-T formulation is that when an individual price is missing, its price can be imputed using the month-on-month price changes of similar or higher-level aggregates rather than the L-T comparison of the current month to the price reference period, which may be many years ago.

6.57. The right-hand-side of equation (6.2) requires monthly geometric mean prices to be calculated for the numerators and denominators for each month-on-month comparison. Table 6.2 shows the geometric means for the price reference period, December 2019, January and February 2020, but then missing prices for March, April and May with prices returning in June and July. The first task is to impute the missing price for March 2020. This is undertaken using the ratio of geometric mean prices for each of the matched sample of outlets A to E in February and March to provide the short-term (S-T) price relative:

$$P^F_{Feb 20}, P^F_{Mar 20} = \prod_{i=1}^{E} \left( P^F_{i, Feb 20} \right)^{1/5} = \left( P^E_{i, Mar 20} \right)^{1/5} = \left( 5.49 \times 5.25 \times 5.20 \times 5.65 \times 6.90 \right)^{1/5} = 5.67 \approx 1.023173$$

This change in the (geometric) mean price for the matched prices A to E is from February to March. This increase of 1.023173 when multiplied by the February price of 5.99 yields an imputed March price of 6.13.

6.58. The price collector subsequently finds variety F’s April and May prices to be temporarily missing the respective imputed prices are: 1.003484 \times 6.13 = 6.15 and 1.003472 \times 6.15 = 6.17. The imputed prices are entered in Table 6.2, highlighted in bold red, providing a complete table of prices for outlets A to F over the reference price period and subsequent months.

6.59. The short-term month-on-month price relative all outlets A:F for February to March 2020 is given as:

$$P^F_{Feb 20}, P^F_{Mar 20} = \prod_{i=1}^{F} \left( P^F_{i, Feb 20} \right)^{1/6} = \left( P^E_{i, Mar 20} \right)^{1/6} = \left( 5.49 \times 5.10 \times 5.20 \times 5.49 \times 6.50 \times 6.90 \right)^{1/6} \approx 5.61 \approx 1.023173.$$ 

6.60. The S-T price relatives for outlets A:F calculated from imputed prices, as in equation (6.4), provides the same measure as the price relative calculated from outlets A:E since outlet F’s price is computed from the price change for outlets A:E. Other S-T price relatives are shown in Table 6.2. The L-T price index (December 2019 =100.00) to July 2020 is shown below in Table 6.3 and in Table 6.2 as the cumulative product of short-term relatives.

\[\exp \left[ \frac{1}{N} \left( \sum_{i=1}^{N} \ln P_i - \sum_{i=1}^{N} \ln P_0 \right) \right].\]
Table 6.3 Overall mean and targeted mean imputations

<table>
<thead>
<tr>
<th></th>
<th>Overall mean imputation</th>
<th>Targeted mean imputation</th>
</tr>
</thead>
<tbody>
<tr>
<td>December 2019</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>January 2020</td>
<td>100.00*1.014572=101.46</td>
<td>100.00*1.014572=101.46</td>
</tr>
<tr>
<td>February 2020</td>
<td>101.46*1.007181=102.19</td>
<td>101.46*1.007181=102.19</td>
</tr>
<tr>
<td>March 2020</td>
<td>102.19*1.023173=104.55</td>
<td>102.19*1.026738=104.92</td>
</tr>
<tr>
<td>April 2020</td>
<td>104.55*1.003484=104.92</td>
<td>104.92*1.005208=105.46</td>
</tr>
<tr>
<td>May 2020</td>
<td>104.92*1.003472=105.28</td>
<td>105.46*1.003454=105.83</td>
</tr>
<tr>
<td>June 2020</td>
<td>105.28*1.001730=105.46</td>
<td>105.83*0.996558=105.46</td>
</tr>
<tr>
<td>July 2020</td>
<td>105.46*1.008636=106.38</td>
<td>105.46*1.008636=106.38</td>
</tr>
</tbody>
</table>

2. Targeted mean imputation

6.61. The overall mean imputation is based on assuming the price change of the temporarily missing variety is the same as that of the overall price change at a higher level of aggregation. A targeted form of the method would use price movements of a cell or an aggregate of similar varieties that is, varieties expected to have similar S-T price changes. The sample of observations used for the targeting may be specific to a type of outlet and/or region and/or cluster of features, for example, “up-market” television sets. It would generally be a sub-set of varieties within a higher level of aggregation. Decisions as to whether to target the imputation using similar varieties in the sub-set or to use a wider higher level of aggregation will depend in part on the adequacy of the sample size for the sub-set of similar varieties and the homogeneity of the elementary aggregate at the higher level.

6.62. Imputed prices in the illustration in Table 6.2 for the missing variety-prices in outlet F for March, April, and May 2020 are based on adjusting the preceding period’s price by the price movements of the remaining matched pairs of prices at other independent traders, D and E, rather than all outlets. Changes in the geometric mean price relative as applied to adjust the preceding period’s price, are:

\[5.99 \times \left( \frac{5.65 \times 6.90}{5.49 \times 6.50} \right)^{\frac{1}{3}} = 5.99 \times 1.04522 = 6.26\] for March 2020; and

\[6.26 \times \left( \frac{5.75 \times 6.90}{5.65 \times 6.90} \right)^{\frac{1}{3}} = 6.26 \times 1.00881 = 6.32\] for April 2020; and

\[6.32 \times \left( \frac{5.80 \times 6.90}{5.75 \times 6.90} \right)^{\frac{1}{3}} = 6.32 \times 1.00434 = 6.34\] for May 2020.

6.63. The price index is compiled as the cumulative product of the S-T price relatives, as shown in Tables 6.2 and 6.3, again highlighted in bold red.
6.64. The higher levels used at this elementary stage of aggregation would be specific to a country in that it would follow the country’s CPI aggregation structure, as outlined in Chapter 9 (paragraphs 9.5–9.9) and Figure 9.1. The higher level might be a region and type of outlet, for example in Figure 9.1, Brand A of par-boiled long-grain white rice sold in supermarkets in the Northern region. Should there be an insufficient sample size for Brand A, similar Brands A and B, or all brands, might be used for all types of outlets in the region. Imputation of the missing price by the average change of the available prices may be applied for elementary aggregates where the prices can be expected to move in the same direction. The imputation can be made using all of the remaining prices in the elementary aggregate. This is numerically equivalent to omitting the variety for the immediate period, but it is necessary to make the imputation.

6.65. An imputed price should always be directly compared with the actual price on the variety’s return as this provides a self-correcting measure. For example, if the imputation was badly wrong and showed decreases in prices over the period, when in fact the price of the variety either sold elsewhere or, if not sold, being pent-up, was increasing, then a direct comparison between the last imputed and the returning actual price would bring the index back to its longer-term trend. The L-T price indices in Table 6.2 using an overall imputation for June and July 2020 are 105.46 and 106.38 respectively. These are the same results as those given in Table 6.2 for targeted imputations. Both series have self-corrected the imputations to return to the long-term price changes as properly measured using actual prices in outlet F. The overlap method described below, links-in a replacement variety’s price change and can be used for permanently missing varieties. The overlap method does not have this self-correcting feature and should not be used for temporarily missing varieties.

3. Carry forward imputation

6.66. Carrying forward the last observed price should be avoided and is acceptable only in the case of fixed or regulated prices. Special care needs to be taken in periods of high inflation or when markets are changing rapidly as a result of a high rate of innovation and product turnover. While simple to apply, carrying forward the last observed price biases the resulting index towards zero change. In addition, when the price of the missing variety is recorded again, there is likely to be a large compensating step-change in the index to return to its proper value. The adverse effect on the index will be increasingly severe if the variety remains un-priced for some length of time. In the illustration in Table 6.2, the missing prices in March 2020 would be imputed as the February 2020 price of 5.99 carried forward, as would be the imputed prices in April, and May 2020. However, on the variety’s return, in June there would be the step increase in price from May to June of 5.99 to 6.25. In general, carrying forward is not an acceptable procedure or solution to the problem. Exceptions may be well-established and well-advertised periodic increases of fixed or controlled prices and tariffs.

4. General considerations

6.67. As a general principle, temporarily missing prices require an explicit imputation entered into the data compilation. The overall mean imputation should, by default, be based on a higher-level of aggregation; however, it also may refer to a variety within a region or type of outlet. The higher-level of aggregation may comprise more than one variety, some with different price
changes. For example, if the missing price observation is for “canned tuna,” where the higher weighted level aggregate is “canned fish” which includes “canned tuna” and “canned salmon,” then subject to a sufficient sample size, the imputation should be based on price movements of “canned tuna.”

6.68. An overall mean imputation not only benefits from an automation of its implementation of imputations, but its transparency of methodology also serves the integrity of the index. By using an overall imputation, NSOs guard themselves against criticisms of influencing the CPI by their choice of “similar varieties,” particularly where there are missing values for heavily weighted elementary aggregates. However, such caution should not be exercised where there are strong a priori or empirical grounds to believe a targeted imputation to be superior. NSOs should have retrospective monthly price data available to them at higher levels of aggregation and be able to examine differences in S-T month-on-month price changes between the missing variety's price changes as against price changes of similar varieties and higher levels of aggregation and choose between an overall and targeted imputation accordingly. In the much-simplified illustration of Table 6.2 price changes of supermarkets are very different from independent traders and an imputation is illustrated for outlet F using independent traders. Outlet F might also have been imputed using the price index at a higher level of aggregation or even a single outlet’s price. Similar principles of aggregation apply.

6.69. The importance of software and the role of the price collector and desk officer is stressed. Typically, a price collector reports a variety price as temporarily missing; this is then passed to a desk officer for confirmation and then, perhaps, further at a higher level. If confirmed, a decision is made as to whether to use a targeted or overall imputation. If the overall mean imputation, an appropriate computer routine is applied which enters the imputed figure with a designation as being imputed into the data system. If a targeted imputation is used, the desk officer selects the variety rows regarded as likely to have similar price changes, and the imputation is applied accordingly. In all of this the computer routine should record the decisions made and tabulate the number of temporarily missing varieties by elementary aggregate and their treatment, for quality assurance.

6.70. Of note is that imputations are preferable to simply omitting the missing price observation for the calculation of the index. First, for example, consider a variety priced at 4 in January, temporarily missing in February, and returning at 6 in March. Say the price change between January and February for the remaining varieties in the elementary aggregate was 25 percent, a price relative of 1.25. The imputed variety price for February would be $1.25 \times 4 = 5$. Of course, we do not know the actual price, but the imputation serves as a benchmark without impairing the 25 percent price index measure based on observed matched prices. The February to March price relative for the missing variety is now $6 / 5 = 1.20$. This is referred to as a self-correcting imputation; it self-corrects in the sense that the long-run February to March calculation would allow the index to return to its appropriate level: $5 / 4 \times 6 / 5 = 6 / 4 = 1.5$. Similar principles apply for the treatment of seasonal goods and services as outlined in chapter 11. Simply omitting the February price, and basing the February to March price relative on matched prices in these two months, would not have this self-correcting property.

6.71. Second, omitting a missing variety’s price from the calculation is equivalent to an overall imputation using the other matched prices in the elementary aggregate. The entering of an imputed value allows for the flexibility of using a targeted imputation, if desired.
Third, the designation of an imputed value by a tab or color clearly shows in the prices database the extent to which imputations are used, the length of their runs, and product codes where they are over-used. This Manual recommends that summary counts of imputed prices be monitored by type, product code, frequency, and duration as part of quality assurance. The need for this is particularly strong to identify if/where temporarily missing variety prices have continued imputed values well over a three-month run, with the possibility of continuing sample depletion as other variety prices go missing.

G. The nature of quality change

This section defines what is meant by quality change. To understand the meaning of quality change requires a conceptual and theoretical platform, so that adjustments to prices for quality differences are made against a well-considered framework.

Over time the quality of what is produced changes. As one example, new automobiles for the large part have an increasing number of options, become more reliable, durable, safer, powerful, and economical. Another example is smart phones. Each new model includes faster processing speeds and power, more memory, improved screen resolution, and other technological advances such as facial recognition. In matching the prices of a sample of models selected in a price reference period with the same models in subsequent periods, the quality mix remains constant in an attempt to avoid contaminating the price measurement through quality differences. As will be seen later, however, the resulting sample of models is one that gives less emphasis to newer models which have benefited from more recent technological change and have different price changes given the quality of services they provide.

Observed changes in prices arise in theory from a number of sources, including quality changes, changes in tastes and preferences, and changes in the technology of producers. Price differences of similar products are often taken to be measures of differences in quality. Yet observed differences in prices are often observed for varieties of the same quality. This may arise from a number of reasons: (i) some consumers may be unaware of the availability of the same varieties being available at lower prices, as there may be “search costs” to exploring the market to discover lower priced varieties; (ii) there may be price discrimination because the seller is able to charge different prices to different categories of consumers, such as movie tickets for children and senior citizens; (iii) prices may be sticky with some retailers changing their prices infrequently to avoid the costs of doing so, including adverse customer reaction, or as strategic competitive behavior, such as loss leaders. Different retailers charging prices at different times leads to price variation for the same variety; and (iv) where there are parallel markets, an official market subject to government or official control at which products are rationed and an unofficial unregulated market unregulated one. The unofficial market may be at lower price because it avoids taxes and regulations, or at higher price since the official price is a subsidized one, but has limited, possibly varying, quantities available for sale (SNA 2008 paragraphs 15.64-15.75).

The changing mix of the observed characteristics of varieties is not the only concern. There is also the practical problem of not always being able to observe or quantify characteristics such as the style, reliability, and ease of use and safety of what is produced. The
same good provided at a different and more convenient location may command a higher price and be considered to be of a higher quality. Furthermore, different times of the day or periods of the year may also give rise to quality differences: For example, electricity or transport provided at peak times must be treated as being of higher quality than the same amount of electricity or transport provided at off-peak times. The fact that peaks exist shows that purchasers or users attach greater utility to the services at these times and/or reflect supply-side pressure. Other differences, including the conditions of sale and circumstances or environment in which the goods or services are supplied or delivered, can make an important contribution to differences in quality. A retailer, for example, may attract customers by providing free delivery, financing, or better variety, by being more accessible, by offering shorter order times, smaller tailor-made orders, clearer labeling, better support and advice, more convenient car parking or a wider range of brands, or simply by operating in a more pleasant or fashionable environment. Although these sorts of benefits are not always specified in the variety description, such quality improvements should conceptually not be outside the scope of the index.

6.77. To consider how to adjust prices for quality changes, it is first necessary to ask what is meant by quality. While there may be an intuition as to whether a variety consumed in one period is better than its counterpart in the next, a theoretical framework will help in establishing the basis for such comparisons. For example, a variety of clothing is sampled and, after a few periods, it is missing. One option is to replace it with a similar variety. The most comparable option may have more cloth in it, or have a lining, be a different color, have different buttons, have better stitching or be considered to be better styled in some fashionable sense. There is a need to put a price estimate on the difference in quality between the old and new varieties so that the price of like can be compared with like. To propose or criticize a quality adjustment procedure requires some concept of what is ideally required and how the procedure stands up to this. Although such a discussion takes us away from the practicalities of the procedures for a while, its use will become apparent in subsequent sections.

6.78. In Chapter 3 of Consumer Price Index Theory a cost of living index (COLI) is defined as the ratio of the minimum expenditures in the base and current period required to achieve a given standard of living or “utility”. Quality adjustments to prices involve attempting to measure the price change for a product that has exhibited some change in its characteristics from an earlier period that provides a different level of utility to the consumer. Equating of the value of a quality change with the change in utility derived by the consumer, while falling naturally under a COLI framework, is not exclusive to it. A cost of a fixed basket of goods index (COGI) can also benefit from regarding quality in this way. While a COGI requires the pricing of a fixed basket of products, some varieties will become unavailable and the replacement varieties selected to maintain the sample may not be of the same quality. A COGI based on a fixed basket concept has the pragmatic need to adjust for quality differences when an variety becomes unavailable, and there is nothing in the definition of a fixed basket index that precludes differences in utility being used as a guideline. If variety A is better than its old version, variety B, it is because it delivers something more to the consumer who in turn is willing to pay more. That “thing” is called utility.

6.79. Note that the definition of a quality change is based on equating some change in characteristics to a different level of utility provided. Consider an example in which a new, improved quality variety is substituted for an old one in period \( t \), the consumer having to choose
between the two. Suppose that after the new quality variety appeared, both qualities were offered to a consumer at the same price, say \( p' = 100 \). The consumer was then asked to choose between them and naturally preferred the new quality. Say the price of the old quality was then progressively reduced until it reached a point \( p^* = 75 \), at which the consumer was indifferent as regards the choice between purchasing the old quality at \( p^* = 75 \) and the new quality at \( p' = 100 \). The consumer might then select the old quality at 75 or the new one at 100. Either way, the consumer would obtain the same utility, because of being indifferent as to which to choose. Any further decrease below \( p^* = 75 \) would cause the consumer to switch back to the old quality.

6.80. The difference between \( p' \) and \( p^* \) would be a measure of the additional utility that the consumer placed on the new quality as compared with the old quality. It would measure the maximum amount that the consumer was prepared to pay for the new quality over and above the price of the old quality. In economic theory, if consumers (or households) are indifferent between two purchases, the utility derived from them is the same. The difference between 75 and 100 must therefore arise from the consumers’ valuation of the difference in utility they derive from the two varieties: their quality difference. The definition serves as a conceptual framework.

6.81. The utility-based framework provides insights into the question of how consumers might choose between varieties of different qualities. Consumers (or households) derive more utility from a variety of higher quality than from an variety of lower quality, and thus they prefer it. But this does not explain why consumers buy one variety rather than the other. For this, it is also necessary to know the relative price of one variety with respect to the other, since if the lower-quality variety is cheaper, it may still be purchased. The above thought experiment to determine the price below which the old quality would be purchased, \( p^* \leq 75 \), serves this purpose.

**H. Permanently missing price observations**

6.82. Tables 6.4a, 6.4b, and 6.4c illustrate the treatment of permanently missing price observations. In these tables, prices are observed for Outlets A to E over the seven months of December 2019 to June 2020. However, from May 2020 the price observations for the variety in outlet F are reported as permanently missing. The price collector has to find a replacement variety for the missing variety in outlet F. The use of replacement varieties is not only to maintain the original sample sizes at the last rebasing, but also to maintain the representativity of the varieties selected. Senior store staff should be asked to confirm that the missing variety is permanently missing and indeed help identify a best-selling replacement variety, its specifications, and how these specifications differ from those of the old variety. For logistical reasons the selected replacement variety would be expected to have high sales for the foreseeable future. A desk officer at the NSO would confirm or reject the choice of replacement variety.
1. Comparable replacement

6.83. The price collector should use the existing variety’s specification and identify a comparable variety with the same specifications: a washing machine with the same spin-speed, capacity, brand or cluster of equivalent brands. If a comparable replacement exists its detailed specifications should be confirmed by the price collector against the existing specifications. Any changes in the specification deemed to be not sufficiently price-determining should be noted for the desk officer to confirm, say color, trim, and so forth.

Table 6.4a Illustration of treatment of comparable replacements

<table>
<thead>
<tr>
<th>Outlets</th>
<th>Dec-19</th>
<th>Jan-20</th>
<th>Feb-20</th>
<th>Mar-20</th>
<th>Apr-20</th>
<th>May-20</th>
<th>Jun-20</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: supermarket</td>
<td>5.25</td>
<td>5.25</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
</tr>
<tr>
<td>B: supermarket</td>
<td>5.10</td>
<td>5.10</td>
<td>5.10</td>
<td>5.25</td>
<td>5.25</td>
<td>5.25</td>
<td>5.25</td>
</tr>
<tr>
<td>C: supermarket</td>
<td>5.20</td>
<td>5.20</td>
<td>5.20</td>
<td>5.20</td>
<td>5.20</td>
<td>5.25</td>
<td>5.25</td>
</tr>
<tr>
<td>D: independent trader</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.65</td>
<td>5.75</td>
<td>5.80</td>
<td>5.80</td>
</tr>
<tr>
<td>E: independent trader</td>
<td>5.99</td>
<td>6.50</td>
<td>6.50</td>
<td>6.90</td>
<td>6.90</td>
<td>6.90</td>
<td>6.90</td>
</tr>
</tbody>
</table>

Permanent missing: F, May, June

<table>
<thead>
<tr>
<th>F1: comparable replacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geo-mean: A:F</td>
</tr>
<tr>
<td>S-T price relatives: A:F</td>
</tr>
<tr>
<td>L-T indices as product of S-T relatives</td>
</tr>
</tbody>
</table>

6.84. The comparable replacement method requires the price collector to make a judgment that the replacement is of a similar quality to the old variety and any price changes are untainted by changes due to quality differences. In the illustration in Table 6.4a there is a comparable variety F1 to replace variety F, both from outlet F. The replacement variety is considered by the price collector and confirmed by the desk officer as being directly comparable and its price in May and June respectively, 6.20 and 6.25, are entered into the data system as a continuation of the outlet F series. The price index is calculated using S-T price relatives built into a L-T price index. The price index as at July 2020 (December 2019=100.00) is 105.52, a 5.52 percent increase over this period. The price index remains as a constant-quality index since in May and June 2020 the prices of like quality varieties continue to be compared with like.

6.85. A common practice of manufacturers of electronic goods, such as televisions, household appliances, computers and computer-related hardware and software, and of automobiles is to have major quality changes in some years but relatively minor ones in other years. A new “comparable” model would have a new model number with a new production run, though nothing much physically has changed. The method of comparable replacement relies on the efficacy of the price collectors, desk officers and, in turn, on the completeness of the specifications used as a description of the varieties. NSOs may tend towards designating replacements as comparable since they are wary of sample sizes being reduced by dropping varieties. They may also wary of the intensive use of resources to introduce non-comparable
replacements or make explicit estimates of quality differences as outlined below. The use of varieties of a comparable specification has practical advantages. If the quality of varieties is improving, however, the preceding variety will be inferior to the current one. Continually ignoring small changes in the quality of replacements can lead to an upward bias in the index. The extent of the problem will depend on the proportion of such occurrences, the extent to which comparable replacements are accepted as being so despite quality differences, and the weight attached to those varieties. Proposals in Chapter 8 to monitor types of quality adjustment methods by product area provide a basis for a strategy for applying explicit adjustments where they are most needed.

2. Non-comparable replacements

6.86. Non-comparable replacements result when the price determining characteristics of the replacement variety are different from those of the old variety. This means that the collected price of the replacement variety cannot be compared directly to the price of the old variety because the difference in these prices reflects not only pure price change, but also differences due to changes in quality. Non-comparable replacements require some form of quality adjustment.

6.87. Methods of quality adjustment for prices are generally classified into implicit/imputed (or indirect) quality adjustment methods and explicit (or direct) methods. Implicit and explicit methods are discussed below. Both decompose the price change between the old item and its replacement into quality and pure price changes.

6.88. The main implicit method is the overlap method. The replacement’s price change is linked onto the old variety’s price change using an overlap period that includes both the old and replacement variety’s price. Where an overlap price for the replacement variety does not exist, it might be imputed.

6.89. For explicit adjustments, however, an explicit estimate is made of the quality difference, usually on the basis of external information, and the pure price effect is identified as a remainder. Explicit methods include quantity adjustments, option/feature costs, and “patched” hedonic regression methods. These implicit and explicit methods are considered in turn.

I. Implicit methods of quality adjustment

6.90. This section discusses the following implicit methods for obtaining adjusting for quality differences: overlap method; class mean imputation; and link to show no change.

1. Overlap method

The use of an overlap price

6.91. Consider for purposes of illustration, Table 6.4b where in outlet F there is a pre-existing “old” model F up to April 2020 and a non-comparable “new” replacement model for this missing variety in outlet F in May and June 2020, that is F2, with actual prices of 5.25 and 5.25 respectively. The prices are lower than would be expected from the prices of F, but this is a non-comparable replacement: it is a replacement variety with a major share of the market that is expected to remain on the market for the foreseeable future.
Table 6.4b Illustration of treatment using the overlap method, non-comparable replacements: actual preceding period price

<table>
<thead>
<tr>
<th>Outlets</th>
<th>Dec-19</th>
<th>Jan-20</th>
<th>Feb-20</th>
<th>Mar-20</th>
<th>Apr-20</th>
<th>May-20</th>
<th>Jun-20</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: supermarket</td>
<td>5.25</td>
<td>5.25</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
</tr>
<tr>
<td>B: supermarket</td>
<td>5.10</td>
<td>5.10</td>
<td>5.10</td>
<td>5.25</td>
<td>5.25</td>
<td>5.25</td>
<td>5.25</td>
</tr>
<tr>
<td>C: supermarket</td>
<td>5.20</td>
<td>5.20</td>
<td>5.20</td>
<td>5.20</td>
<td>5.20</td>
<td>5.20</td>
<td>5.20</td>
</tr>
<tr>
<td>D: independent trader</td>
<td>5.49</td>
<td>5.49</td>
<td>5.49</td>
<td>5.65</td>
<td>5.75</td>
<td>5.80</td>
<td>5.80</td>
</tr>
<tr>
<td>E: independent trader</td>
<td>5.99</td>
<td>6.50</td>
<td>6.50</td>
<td>6.90</td>
<td>6.90</td>
<td>6.90</td>
<td>6.90</td>
</tr>
<tr>
<td>F : independent trader</td>
<td>5.99</td>
<td>5.99</td>
<td>5.99</td>
<td>6.13</td>
<td>6.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2: non-comparable replacement:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.25</td>
</tr>
<tr>
<td>Geo-mean: April, May, June: A:E,F2</td>
<td>5.61</td>
<td>5.63</td>
<td>5.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-T price relatives</td>
<td>100.00</td>
<td>1.0137</td>
<td>1.0075</td>
<td>1.0237</td>
<td>1.0035</td>
<td>1.0030</td>
<td>1.0000</td>
</tr>
<tr>
<td>Long-term indices as product of S-T</td>
<td>100.00</td>
<td>101.37</td>
<td>102.13</td>
<td>104.55</td>
<td>104.91</td>
<td>105.23</td>
<td>105.23</td>
</tr>
</tbody>
</table>

6.92. The overlap method requires a price for both “old” model F, and the “new” model F2 in an overlap period: F exists up to and including April, F2 exists in May and June, and thereafter. The problem is how to determine an overlap price? One source of information for this overlap price of 5.25 in May is the price collector.

Use the actual price of the replacement in the preceding period, if it exists

6.93. The price collector may have anticipated falling sales and a switch of consumers to a new model, brand, or variety and recorded the overlap prices for the replacement prior to its adoption, in May rather than June. Price collectors should be trained to anticipate such changes, to corroborate them with senior outlet staff, and relay the information back to the desk officer for possible action, that is, in our example: in April F may be seen by the price collector to have dwindling sales and a poorer positioning for display in the outlet and it is apparent that it is to be replaced by F2. Senior outlet staff confirm that F2 is to effectively replace F as a model aimed at that the same segment of the market. The price for F2 and its quality characteristics should be recorded alongside that of F to provide an overlap April price to facilitate the introduction of F2 in May. As a general principle, the replacement of models is best not undertaken when the old model has limited sales and is at the end of its life cycle.

6.94. Alternatively, the price collector may have asked senior outlet staff in May whether the new model was sold in the previous month to obtain an overlap price for April, or if sold in other outlets, whether there is a pricing agreement with the supplier that this outlet would have kept to had it been supplied to them, and what would have been the price. The desk officer should confirm such details by visit or telephone with senior staff of outlet F. Table 6.4b shows 5.25 to be entered as an estimated price in May 2020 for the new model, to provide an overlap price in April for the old and replacement models, F and F2.

6.95. The price index is measured through and including April 2020 using the prices of the old model: from Table 6.4a the geometric mean of the prices of the old model in April 2020 is 5.76, and in March 5.74, with a S-T price relative of 5.76/5.74=1.0035. The price index to April
2020 is the cumulated product of the old index’s price relatives: for April 2020 it is \(104.91=(104.55 \times 1.0035) \times 100\) (December 2019=100.00).

6.96. In Table 6.4b the index from May onwards no longer uses F, but switches to F2. For this there is a need for the overlap prices: average prices up to and including April using the old F and for May and June and onwards, using the new F2. The prices for outlet F in May and June are 5.25 and 5.25 respectively, are based on F2. The overlap in May for the new model F2 is 5.25. The S-T price relative for May to June in outlet F is 5.25/5.25=1.00000. This completes the price table. The geometric means are calculated as before, and their ratios form the S-T price relatives, and, in turn, the cumulative product of the price relatives, commencing from December 2019 is the price index, at 15.23 for June 2020 (December 2019=100.00).

Imputed overlap prices

6.97. The new replacement variety may not have existed in April or the price collector may not have been able to obtain a reliable estimate of its price. The overlap price in April can be imputed. The validity of this imputation is critical to the quality-adjustment methodology. In April 2020 in Table 6.4b there was a price of 6.15 for the old model and 5.25 for the new model. As will be shown in equation (6.5), the method implicitly attributes this price difference in the common April overlap period as an indicator of their quality difference.

Table 6.4c: Illustration of treatment using the overlap method, non-comparable replacements: imputed succeeding period price

<table>
<thead>
<tr>
<th>Outlets</th>
<th>Price reference period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dec-19</td>
</tr>
<tr>
<td>A: supermarket</td>
<td>5.25</td>
</tr>
<tr>
<td>B: supermarket</td>
<td>5.10</td>
</tr>
<tr>
<td>C: supermarket</td>
<td>5.20</td>
</tr>
<tr>
<td>D: independent trader</td>
<td>5.49</td>
</tr>
<tr>
<td>E: independent trader</td>
<td>5.99</td>
</tr>
</tbody>
</table>

**F3: non-comparable replacement:**
- Geo-mean: May, June: A:F
  - 5.76
- Geo-mean: May, June: A:E,F3
  - 5.63

S-T price relatives
- 100.00
- 101.37
- 102.13
- 104.55
- 104.91
- 105.30
- 105.30

6.98. If the new model was not sold in April, an imputation for the May (June) price of the old model can be made to provide an overlap price. The imputation may be an overall-mean or targeted imputation following the principles outlined for temporarily missing variety prices as illustrated in Table 6.2. The imputation is illustrated in Table 6.4c. In Table 6.4c the imputation is a *forward* imputation of the old model’s price to provide an estimate of the price in May 2020 had it existed then. The imputed price is given in Table 6.4c as 6.17. It is calculated by taking the ratio (relative) of the geometric mean of the May prices of outlets A to E to the geometric mean of the April prices, 5.78/5.76=1.0037, and multiplying this by the old variety’s
price in April, \(1.0037 \times 6.15 = 6.17\). The index in Table 6.4c is calculated by multiplying the S-T May to June price relative for the new replacement F3, \(5.63/5.63=1.0000\), by the value of the L-T index for May, 105.30. The 5.63 are the geometric means of the prices in outlets A to E and that of F3, the replacement.

**Forward versus backwards imputation**

6.99. It should be noted that in Table 6.4b the overlap took place in April, while in Table 6.4c, it was in May. In Table 6.4b an actual price was sought and found for the new (replacement) variety in April. There would be no equivalent May price for the old permanently missing old variety. Given that an actual price is preferred to an imputation based on the price movements of varieties priced in other outlets, the use of a *backwards* price to provide an overlap in April is fully justified. In Table 6.4c we use a *forward* imputation for the old variety’s price in May. But one could apply a *backwards imputation* for the new (replacement) variety’s price in April to provide an overlap in April akin to that in Table 6.4c. It is relatively straightforward to demonstrate algebraically that the index would be the same if the index is calculated either way. Both methods impute the missing price using the price movements of varieties in the outlets A to E for which prices exist in each April and May. The backwards imputation is simply the inverse of the forward one. The ratio of prices in the overlap periods, which as will be shown by equation (6.5), is the implicit measure of the quality differential between the old variety and the replacement variety, will be numerically the same for backward and forward imputations.

**Class-mean imputation**

6.100. The overall imputation method has many advantages resource-wise. It can be automated as a default measure to readily link-in replacement varieties keeping the sample up-to-date. Yet the (forward) imputation assumes that the price movements of existing continuing varieties would be the same as that for the old variety, had it continued to exist. A backwards imputation assumes the price movement of the new (replacement) variety would have the same price movements as existing continuing varieties had the new variety existed in the period prior to its introduction. Such assumptions are unlikely to be valid for high-technology goods being replaced at the end of their life-cycle. An alternative imputation procedure designed to mitigate this problem is the class-mean imputation which in principle, is more suited to this context of imputing replacement variety prices for permanently missing varieties, as opposed to temporarily missing ones.

6.101. *The class-mean imputation method* is a specifically designed targeted imputation to use to introduce a replacement when a variety’s price is permanently missing. The class-mean method of implicit quality adjustment to prices arose from concerns that unusual prices were charged at the start and end of a model’s life-cycle. Thus, the price movement of continuing varieties appears to be a flawed proxy for the pure price component of the difference between old and replacement varieties. A class-mean imputation is mainly considered as a means of quality adjustment where there is a relatively high rate of frequent replacements, such as different models of automobiles launched each year.

6.102. The class-mean method is a form of targeted imputation. The “target” is measured price changes of replacements for permanently missing products. Only the price changes of “comparable” replacements are used to impute the overlap price, the replacements being limited to those that have exactly the same price-determining characteristics, or those varieties
with replacements that have been declared comparable after review or have already been quality-adjusted through one of the "explicit" methods. A discussion of explicit quality adjustments follows. For example, when the arrival of a new model of a particular make of motor vehicle forces price collectors to find replacements, some of the replacements will be of comparable quality, others can be made comparable with explicit quality adjustments, but the remaining ones will need imputed prices for an overlap month. Class mean imputation calculates imputed price relatives using only the prices of comparable and, where appropriate, explicitly quality-adjusted varieties or models. In general, it does not use the prices for the varieties or models that were not replaced, because these are likely to be different from those of new models. The prices of old models tend to fall as they become obsolete, while the new models (represented by the replacements) tend to have a higher price before falling.

6.103. Class mean-imputations rely on other explicit quality adjustments and comparable replacements. The other explicit quality adjustments may be from available option or feature prices and may be limited in nature, covering only some of the differences in product attributes, available for only a small proportion of unrepresentative model changes, and the availability of comparable replacements limited. Given a substantial churn in the market and difficulties with such imputations and estimates an alternative recommended approach is that of hedonic indices, as outlined below.

6.104. It may be the case, however, that sufficiently large samples of comparable substitutes or directly quality-adjusted varieties are unavailable. Or it may be that the quality adjustments and selection of comparable varieties are not deemed sufficiently reliable. In that case, a targeted imputation, outlined above (see section F.2), might be considered. The targeted mean is less ambitious in that it seeks only to capture price changes of similar varieties, irrespective of their point in the life cycle. Yet it is an improvement on the overall mean imputation, as long as sufficiently large sample sizes are used.

The implicit assumptions behind and possible concerns about using the overlap method

6.105. The overlap method is only as good as the validity of its underlying assumptions which are identified in this section. Let $p_{m}^t$ and $p_{m}^{t+1}$ denote the prices of an old variety $m$ in periods $t$ and $t+1$; $p_{n}^{t+2}$ is the price of a new replacement variety $n$ in period $t + 2$; and an overlap can be made by imputing a price for the new replacement in period $t+1$, $p_{n}^{t+1*}$. Now variety $n$ replaces $m$, but is of a different quality. The measured price index between periods $t$ and $t+2$ shown by the right-hand-side expression in equation (6.5) is the price change of the old to new varieties, between these two periods, multiplied by (adjusted for) the price overlap for $m$ to $n$ in period $t+1$ which the method implicitly takes to be a measure of the quality differential.

6.106. A forward imputation is used here for the price of the old variety in period $t+1$. What equation (6.5) also clearly shows for this forward imputation is that not only does the overlap method depend on the validity of the relative difference in the prices of the old and the new varieties in period $t + 1$ to be a measure of the quality differential between varieties $m$ and $n$, but also on the reliability of the imputed price, designated with an asterisk(*), of the old variety $m$ in period $t+1$, $p_{m}^{t+1*}$, as an estimate of $p_{m}^{t+1}$.

$$ I^{t,t+2} = \frac{p_{m}^{t+1*}}{p_{m}^{t}} \times \frac{p_{n}^{t+2}}{p_{n}^{t+1}} = \frac{p_{n}^{t+2}}{p_{m}^{t}} \times \frac{p_{m}^{t+1*}}{p_{n}^{t+1}} $$(6.5)

Table 6.5: Introducing a non-comparable replacement via an overlap
<table>
<thead>
<tr>
<th>Model</th>
<th>January</th>
<th>February</th>
<th>March</th>
<th>April</th>
<th>May</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old ((m))</td>
<td>25</td>
<td>28</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New ((n))</td>
<td>35</td>
<td>38</td>
<td>40</td>
<td>41</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.107. For example, in Table 6.5, with an old model \(m\) permanently missing in March \((t+1)\) and replaced by a new model \(n\) in April, with an overlap in March \((t+1)\), the price index for February \((t+1)\) to April \((t+2)\) using the overlap method is given by the first expression in equation (6.5) as the product of the old model’s price change between February and March and the new model’s price change between March and April. This is equivalent to the second expression in equation (6.5) which is a direct price comparison between the new and old models between February and April with a quality adjustment as the value of the relative prices in the overlap month March \((t+1)\). The overlap method implicitly values the quality difference as the ratio of the two prices in the overlap period.

\[
(6.6) \quad I_{Feb, Apr} = \frac{p_{Mar}^{m}}{p_{Mar}^{m}} \times \frac{p_{Apr}^{n}}{p_{Mar}^{n}} = \frac{p_{n}}{p_{Mar}^{m}} \times \frac{p_{Mar}^{m}}{p_{Mar}^{m}} = \frac{30}{28} \times \frac{38}{35} = \frac{38}{28} \times \frac{30}{35} = 1.163
\]

6.108. Moreover, for a longer-term price comparison, say January to June, the valuation of the quality difference remains as that of the price ratio at March, the time of the splice.

\[
(6.7) \quad I_{Feb, Apr} = \frac{p_{Mar}}{p_{Mar}} \times \frac{p_{Mar}}{p_{Mar}} \times \left( \frac{p_{n}}{p_{n}} \times \frac{p_{Mar}}{p_{Mar}} \times \frac{p_{n}}{p_{n}} \right) = \frac{p_{n}}{p_{n}} \times \frac{p_{Mar}}{p_{Mar}} = \frac{41}{25} \times \frac{30}{35} = 1.4057
\]

6.109. Of note is that the price of a missing good is, by definition, not usually observed at the same time period as the price of the replacement variety since the choice of replacing the variety is only made after it has disappeared. Indeed, the list of specifications is not always comprehensive since their main aim is to identify the product in the shop rather than comparing the products. However, it may be that the replacement variety was on sale in the previous period and senior store staff has a record of its price.

6.110. The assumption is that the quality difference in any period equates to the price difference at the time of the splice. The timing of the switch from the old variety \(m\) to the new variety \(n\) is thus crucial. Unfortunately, price collectors usually hang onto a variety so that the switch may take place at an unusual period of pricing, near the end of variety \(m\)’s life cycle and the start of variety \(n\)’s life cycle. The analysis is more formally given in Annex 1.

6.111. Relative prices may not always reflect quality differences. For example, a new replacement model or brand of an improved quality may be stocked and sold at the same price as the old model. The outlet competes in the market in part by changing the quality of what is sold, as opposed to the price. Retailers may also reflect unusual pricing policies aimed at minority segments of the market. For example, the ratio of prices in an overlap period of a generic and a branded pharmaceutical drug may reflect the (perceived or otherwise) needs of
two different market segments, rather than quality. The overlap method can be used with a judicious choice of the overlap period. If possible, it should be a period before the use of the replacement since in such periods the pricing may reflect a strategy to dump the old model to make way for the new one.

6.112. The overlap method has at its roots a basis in the law of one price: that when a price difference is observed it must arise from some difference in quality or similar factors for which consumers are willing to pay a premium, such as the timing of the sale, location, convenience or conditions. Economic theory would dictate that such price differences would not persist, given markets made up of rational producers and consumers. However, as outlined in the section on “The nature of quality change” in Chapter 15 of 2008 SNA (paragraphs 15.70–15.72, pages 303–304) there are many reasons why identical varieties can be sold at different prices, and the law-of-one-price does not hold in practice. These include lack of information due to search costs, price discrimination, and the existence of parallel markets.

6.113. The overlap method is commonly used as a default procedure for introducing replacement varieties when they are permanently missing. NSOs that re-base their CPI less frequently, at least every 5 years, may also experience sample deterioration with many varieties becoming permanently missing and, without replacements, the sample becoming increasingly composed of imputed prices. Replacements serve to maintain the sample composition and update the representativity of the varieties being priced.

6.114. Yet only non-comparable replacements—those of a different quality—of varieties with missing prices may be available. In these cases, an explicit adjustment to the price to account for the quality difference should be made in order to compare the prices of these non-comparable replacements in one period with the price of the now missing varieties in a previous period. The widely used overlap method has a major advantage of it not requiring an explicit quality adjustment. Explicit quality adjustments are considered below and are more resource intensive than the implicit overlap method. Further, it is recommended that the overlap method be automated with information from the price collector fed through to the desk officer and then to a computational routine. The recommendation is that in accepting the use of the overlap method the desk officer should ensure that relative prices at the time of the overlap reflect quality differences. For example, a much-improved new model of a smart phone may be launched at the same price as the old model, and the relative price would not reflect the differences in quality. Where the relative price does not reflect quality differences and resources allow, an explicit quality-adjustment method should be used.

6.115. The overlap method is implicitly employed when samples of varieties are rotated. That is, the old sample of varieties is used to compute the category index price change between periods $t-1$ and $t$, and the new sample is used between $t$ and $t+1$. The “splicing” together of these index movements is justified by the assumption that – on a group-to-group rather than variety-to-variety level – differences in price levels at a common point in time accurately reflect differences in qualities.

6.116. The bias in using the overlap method within an elementary aggregate depends on (i) the ratio of missing to total observations and (ii) the difference between the mean of price changes for existing varieties and the mean of quality-adjusted replacement price changes. The bias decreases as either of these terms decrease. A formal analysis is given in Annex 2.
Link-to-show-no-price-change

Table 6.5a: Introducing a non-comparable replacement to illustrate link-to-show-no-price-change

<table>
<thead>
<tr>
<th>Model</th>
<th>January</th>
<th>February</th>
<th>March</th>
<th>April</th>
<th>May</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old ((m))</td>
<td>25</td>
<td>28</td>
<td>28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New ((n))</td>
<td></td>
<td>35</td>
<td>38</td>
<td>40</td>
<td>41</td>
<td></td>
</tr>
</tbody>
</table>

6.117. Returning to the example in Table 6.5, reproduced in Table 6.5a above, the new replacement variety is non-comparable, i.e. of a different quality. The link-to-show-no-price-change method imputes the price in March for the old model to be the same as its February price, 28. The new variety is linked-in to show no price change in the period of replacement: February to March. It should be noted that this method is used for the treatment of a non-comparable replacement variety and should not be confused with carrying forward a previous period price for the treatment of temporarily missing prices (described above). This method biases the index downward when prices are rising and biases them upward when (true, quality-adjusted) prices are falling. The link-to-show-no-price-change method attributes the price difference between the new and old model in February to quality difference. The new model is of a better quality, valued at being worth an additional 7, from 28 to 35. The (quality-adjusted) price is therefore constant between February and March. The February to April price change is:

\[
\frac{28}{28} \times \frac{38}{35} = 1.0857, \text{ an 8.57 percent increase in price.}
\]

In equation (6.5) \( p_{t}^{t+1} = p_{n}^{t} \), \( p_{m}^{t+1} = p_{t}^{t} \) and

\( p_{n}^{t+1} = p_{m}^{t+1} \) \( \times \) \( \frac{p_{n}^{t+2}}{p_{n}^{t+1}} = \frac{p_{n}^{t+2}}{p_{m}^{t}} \)

That is, a (quality-adjusted) price change between February (period \( t \)) and April (period \( t+2 \)) is measured as that for the new model between March \( (t+1) \) and April \( (t+2) \).

6.118. The bias is perpetuated through subsequent periods of measurement. The February to June index, for example, would still have its price change measured as the product of month-on-month price changes which would include the February to March link that would show no price change.

6.119. As with carry forward, the method is particularly harmful since it can be readily incorporated into a regular automatic compilation routine and simply not noticed: the price of the replacement is automatically imputed to form the overlap price and the index compiled. The link-to-show-no-price-change should not be used.

J. Explicit methods of quality adjustment

6.120. The methods described above do not rely on explicit information on the value of the change in quality. This section discusses the following methods that rely on obtaining an
explicit valuation of the quality difference: quantity adjustment; differences in production or option costs; and the hedonic approach.

1. Quantity adjustment

6.121. Quantity adjustment is one of the most straightforward explicit adjustments to undertake. It is applicable when the size of the replacement variety differs from that of the available variety. Any change in quantity is considered a change in quality. In some situations, there is a readily available quantity measure that can be used to compare the varieties. Examples are the number of units in a package (e.g., paper plates or vitamin pills) and the size or weight of a container (e.g., kilogram of flour, liter of cooking oil). Quantity adjustment to prices can be accomplished by scaling the price of the old or new variety by the ratio of quantities. The index calculation system may do this scaling adjustment automatically, by converting all prices in the category to a price per unit of size, weight or number. Scaling is important. For example, if the weight of a candy bar is 450 grams in the current period, 500 grams in the previous period, and the price remains unchanged; an adjustment is needed so that the index reflects the implicit price increase.

6.122. The specification of a variety is often to a specific size, for example, a 1-kilogram packet of flour. If only 2-kilogram packets are now sold in a specific outlet, the price collector should choose a representative 2-kilogram packet, but mark the new specification as such and, after confirmation by the desk officer, prices should continue to be collected for the 2-kilogram packet. In this example, an adjustment would be needed to ensure that the index reflects only pure price change and not changes due to differences in quality (i.e. the change in size). This is particularly important where price variances are computed, or use is made of a Dutot price index number formula—an elementary index formula sensitive to the homogeneity of the varieties used.5

6.123. Changes in the size of varieties sold can be dealt with similarly, however, there are some caveats. In the pharmaceutical context, for example, prices of bottles of pills of different sizes differ. A bottle of 100 pills, each having 50 milligrams of a drug, is not the same as a bottle of 50 pills of 100 milligrams, even though both bottles contain 5,000 milligrams of the same drug. If there is a change, for example, to a larger size container, and a unit price decrease of 2 per cent accompanies this change, then it should not be regarded as a price fall of 2 per cent if consumers gain less utility from the larger and more inconvenient containers. In practice, it will be difficult to determine what proportion of the price fall is attributable to quality and what proportion to price. A general policy is not to automatically interpret unit price changes arising from packaging size changes as pure price changes, if contrary information is available.

Table 6.6 Example of size, price and unit price of bags of flour

<table>
<thead>
<tr>
<th>Size (kilograms)</th>
<th>First price</th>
<th>First unit price</th>
<th>Second price</th>
<th>Second unit price</th>
</tr>
</thead>
</table>

5 A Dutot elementary price index number formula is a ratio of arithmetic means of matched prices. More weight is given to price changes with higher prices in the reference period. Thus, if the price of a container in the price reference period doubles, while others remain the same, so too will the implicit weight given to its price change, see chapter 9.
Consider another example: a branded bag of flour previously available in a 0.5-kilogram bag priced at 1.5 is replaced by a 0.75-kilogram bag priced at 2.25. The main concern here is with rescaling the quantities.

The method would use the relative quantities of flour in each bag for the adjustment. The price may have increased by \([(2.25/1.5) \times 100 = 150]\) 50\% but the quality-adjusted price (i.e., price adjusted by size) has remained constant \([(2.25/0.75) = (1.5/0.5) = 3]\); 3 per kilogram. The approach can be outlined in a more elaborate manner as illustrated by Figure 6.1. The concern here is with the part of the unbroken line between the (price, quantity) coordinates \((1.5, 0.5)\) and \((2.25, 0.75)\), both of which have *unit* prices of 3 (price = 1.5/0.5 and 2.25/0.75). There should be no change in quality-adjusted price. The symbol \(\Delta\) denotes a change. The slope of the line is \(\beta\) which is \(\Delta\text{price}/\Delta\text{size} = (2.25-1.5)/(0.75-0.50)=3\), i.e., the change in price arising from a unit (kilogram) change in size. The quality- (size-) adjusted price in period \(t-1\) of the old \(m\) bag, to make it equivalent to the new bag, \(n\), is:

\[
\hat{p}^n_{t-1} = p^m_{t-1} + \beta \Delta \text{size} = 1.5 + 3(0.75 - 0.5) = 2.25
\]

6.125. The quality-adjusted price change shows no change, as before:

\[
\frac{p^n_t}{\hat{p}_n^{t-1}} = \frac{2.25}{2.25} = 1.00
\]

6.126. The approach is outlined in this form so that it can be seen as a special case of the hedonic approach (discussed below), where price is related to a number of quality characteristics of which size may be only one.

**Figure 6.1 Quality adjustments for different sized varieties**

6.127. Now assume that the 0.5-kilogram bag was missing and a 0.25-kilogram replacement packet was used priced at 0.75, as shown by the continuation to the coordinate \((0.75, 0.25)\) of
the unbroken line in Figure 6.1 and in Table 6.6; the quality-adjusted prices would again not change. Assume, however, that the unit (kilogram) prices were 5, 3 and 3 for the 0.25, 0.5- and 0.75-kilogram bags, respectively, as shown in Table 6.6 and in Figure 6.1 (including the broken line). Then the measure of quality-adjusted price change would depend on whether the 0.5-kilogram bag was replaced by the 0.25 kilogram one (a 67 percent increase) or the 0.75 kilogram one (no change). This is not satisfactory because the choice of replacement size is arbitrary. The rationale behind the quality adjustment process is to separate pure price change from changes due to differences in quality (in this case, quantity changes).

2. Differences in feature/option costs

6.128. Consider an example of the price of an option being used to adjust for quality. Let the prices for a variety in periods $t-1$ and $t$ be 10,000 and 10,500, respectively, but assume the price in period $t$ is for the variety with a new feature, as standard, that previously in period $t-1$ had to be purchased as an “option” for an additional 300. Then between periods $t-1$ and $t$ that include the feature in both periods the price change would be $10,500/10,300=1.01942$ or 1.942 percent.

6.129. Option costs are thus useful in situations in which the old and new varieties differ by quantifiable characteristics that can be valued in monetary terms by reference to market prices. The valuation of a quantifiable product feature may be readily available from the comparison of different product prices. This is especially, and conveniently, so for some goods and services sold on the Internet which can be identified by their brands and price-determining characteristics.

6.130. Consider the addition of a feature to a product – say an automatic icemaker in the door of a refrigerator. Refrigerators for a particular brand may be sold as standard or with a door-installed automatic icemaker. The price collector may always have collected prices on the standard model, but this may no longer be in production, being replaced by a model with an installed automatic icemaker. The cost of the option is thus known from before and a continuing series developed by simply adjusting the old price in the price reference period to include the option price. Even this process may have its problems. First, the cost of producing something as standard may be lower than when it was an option, say all new refrigerators now have the door-installed automatic icemaker. This saving may be passed on, at least in part, to the consumer. The option cost method would thus understate the price increase. Further, by including something as standard the consumer’s valuation of the option may fall since buyers cannot refuse it. Some consumers may attribute little value to the option. The overall effect would be that the estimate of the option cost, priced for those who choose it, is likely to be higher than the implicit average price consumers would pay for it as standard. Estimates of the effect on price of this discrepancy should in principle be made, though in practice are difficult to quantify.

6.131. Quality differences are not necessarily positive; an airline may charge for a second piece of baggage when previously it did not. Again, there will be an option price available for the additional piece of baggage so that the price of like –two pieces of baggage – is compared with like.

6.132. Option cost adjustments can be seen to be similar to quantity adjustments, except that instead of size being the additional quality feature of the replacement, the added quality can be any other individual option/feature. The comparison is: $p_a^t / p_m^{t-1}$ where $p_m^{t-1} = p_m^{t-1} + \beta \Delta z$.
for an individual $z$ characteristic where $\Delta z = (z'_m - z'^{-1}_m)$. The characteristics may be the size of the memory (RAM) of a personal computer (PC) when a specific model of PC is replaced by a model that is identical except for the amount of RAM it possesses. For example, the web pages of sellers of laptops allow buyers to customize their purchase, an extra 4-gigabyte (GB) of RAM, from 8 to 12 GB, for a specific brand and model of a laptop may cost an additional 70. Say the standard laptop used for CPI measurement has 8GB of memory, costs 899.99 and is not available in the next period. The new standard model in period $t$ has 12 GB but costs the same 899.99, $p'_m$. We want to compare the (constant-quality) price of the new model with the old model in $t-1$, but the latter should have its price adjusted to include an extra 4 GB of RAM. The price of an additional GB of memory for this brand/model in period $t-1$ is $70/4 = 17.5$, and its quality-adjusted price in period $t-1$ is $\hat{p}^t_{m^{-1}} = 899.99 + 17.5 (12 - 8) = 969.99$. The period $t$ (unchanged) price of 899.99 is now compared with its comparable period $t-1$ price to yield a constant-quality price change of $899.99/969.99 = 0.9278$, which is a price fall of 7.22 percent, while the package price is constant.

6.133. Again, this phrasing of the calculation is more complex than required: the adjustment is to simply add 70 to the old price, $70 + 899.9 = 969.99$. However, it serves to demonstrate some limitations of these approaches as special cases of the hedonic method, as outlined in the next section.

6.134. This calculation conveniently makes the quality adjustment to the old model’s price in period $t-1$ so that the new model’s price in future months can be directly compared with the quality-adjusted old price for the life of the new specification. However, the required information on the value of an extra 4 GB may only be available in period $t$ and not be applicable to a period $t-1$ adjustment. NSOs should ideally keep a record of, say, web customizations of specified varieties along with comparable/non-comparable replacements especially for products with a high degree of technical change and turnover of models and maintain good relations with senior outlet staff.

6.135. If the relationship between price and RAM is linear, the above formulation is appropriate. Many web pages give the price of additional RAM as being independent of other features of PCs, and a linear adjustment is appropriate. Bear in mind that a linear formulation values the worth of an additional fixed additional amount of RAM to be the same, irrespective of the amount of RAM the machine possesses or a number of other features.

6.136. The relationship between price and the product features may be non-linear. Denote the price-determining characteristics as $z$, and assume there are $k$ of them. The change in $z$ is intended to reflect the service flow, but the non-linearity in the price–$z$ relationship may reflect consumer’s decreasing marginal utility to the scale of the provision. The price a customer is willing to pay per GB falls as increasing amounts of GB are purchased. For some features there will be economies of scale: supplying much more of a feature makes the price fall, possibly substantially; while for others it may become technically difficult, and more expensive, to compress higher amounts of a feature into the available space. The data should reveal some of this relationship and caution against applying linear relationships outside of the range in which they are warranted. Further, it should give some insight into the required adjustments for such non-linear relationships, though this may be better estimated using a regression formulation and non-linear specification, as considered in the next section.
6.137. The similarity between the quantity adjustment and the option cost approaches is apparent since both relate price to some dimension of quality: the size or the option. The option cost approach can be extended to more than one quality dimension. Both approaches rely on the acquisition of estimates of the change in price resulting from a unit change in the option or size: the \( \beta \) slope estimates. In the case of the quantity adjustment, this was taken from a variety identical to the one being replaced, aside from the fact that it was of a different size. The \( \beta \) slope estimate in this case was perfectly identified from the two pieces of information. It is as if the nature of the experiment controlled for changes in the other quality factors by comparing prices of what is essentially the same thing except for the quantity (size) change.

6.138. The same reasoning applies to option costs. There may be, for example, two varieties, identical but for the possession of a single feature. Their difference in price allows the value of the feature to be determined. Yet sometimes the value of a feature or option has to be extracted from a much larger data set. This may be because the quality dimension takes a relatively large range of possible numerical values without an immediately obvious consistent valuation. Consider the simple example of only one feature varying for a product, the speed of a PC. It is not a straightforward matter to determine the value of an additional unit of speed. To complicate matters, there may be several quality dimensions to the varieties and not all combinations of these may exist as varieties in the market in any one period. Furthermore, the combinations existing in the second period being compared may be quite different to those in the first. Considering these aspects leads to a more general framework, known as the hedonic approach.

3. Differences in production costs

6.139. An alternative approach to quality adjustment is to adjust the price of an old variety by an amount equal to the resource costs of the additional features of the new variety. An important source of such data is the manufacturers. They would be asked to provide data on direct and indirect production costs for the embodied quality change which would include research and development costs, assembly and installation associated with the change, the manufacturer’s established mark-up, the retail margin, and associated indirect taxes. This method is similar to estimating market-equivalent market option prices in the absence of market prices. This approach is most practicable in markets where there is a relatively small number of manufacturers, and where updates of models are infrequent and predictable. It only works if there is good communication between manufacturers and the NSO staff. It is particularly suitable when the quality adjustments are also being undertaken to calculate the producer price index (PPI) and export and import price indices (XMPIs). One NSO uses production cost estimates to value quality-adjustments arising in from model changes in new vehicles. Allowable product changes for the purpose of quality adjustments include reliability, durability, safety, fuel economy, maneuverability, speed, acceleration/deceleration, carrying capacity, and related changes and/or additional parts required to accommodate the principal change in a component. Excluded are any characteristics or features that do not affect or impact the price. Only those characteristics that impact or affect the price (i.e. price-determining characteristics) are included. Also excluded for the CPI, unlike the PPI, are changes mandated by government that provide no direct benefit to the purchaser, including modifications to meet air pollution standards. A new model of a specified automobile is introduced, its changes in
quality components identified, valued, and added to the price of the old model so that the price of the old and new models can be compared on a like to like basis.6

6.140. A critical feature of the method is its reliance on estimates of the retail margin for the new components. With the option cost approach, a consumer’s valuation of the new feature was available. If only production cost data are available then estimates of the retail mark-up must consider the (average) age of the models under consideration. Mark-ups will decrease as models come to the end of their life cycles. Therefore, retail mark-ups based on models at the start of their life cycle should not be applied to the production costs of models during their life-cycle, and particularly at the end. Moreover, estimates of the retail margin of a component may well not be available. A pragmatic practice in one NSI is to use the proportionate retail mark-up on the vehicle, from the price charged by the manufacturer to the dealer for the identical vehicle to the manufacturer’s suggested retail delivered price for the equipped vehicle.

4. Hedonic approach: patching

6.141. The hedonic approach is an extension of the two preceding approaches in that, first, the change in price arising from a unit change in quality—the quantity or option/feature—is now estimated from a data set comprising prices and quality characteristic values of a larger number of varieties. Second, the quality characteristic set is extended to cover, in principle, all major characteristics that might determine price, rather than just the quantity or option/feature adjustment.

6.142. The hedonic approach is particularly useful when the market does not reveal the price of the quality characteristics required for the adjustment. Markets reveal prices of varieties, not quality characteristics, so it is useful to consider varieties as tied bundles of characteristics. A sufficiently large data set of varieties with their characteristics and sufficient variability in the mix of characteristics between the varieties allows the hedonic regression to provide estimates of the implicit prices of the characteristics. For example, the price of (clothes) washing machines will be listed, though a new (replacement) model for a brand may have a (cotton) capacity load size not previously available, say 12 kilograms (kgs.), instead of the preceding model’s 10 kgs. To make an explicit quality adjustment we require the price of the additional 2 kgs. The regression approach using a dataset of many models’ prices and characteristics can estimate the price of additional kgs. of capacity from data for models of washing machines on their price, capacity, year (age of model), color, running cost, and so forth.

6.143. Under the matched models method, each price collector needs to select a representative variety, record its price and specifications, and re-pricing the same variety in subsequent periods. The extension required in the hedonic approach is that the prices and price-determining characteristics should be collected for a large sample of, if not all, models. The method is particularly suitable when there are no immediately apparent comparable replacements and the non-comparable ones vary in their characteristics over more than one variable. A new model of car, household appliance, computer or related hardware and software, telecommunication equipment and much more, can differ from the old model in many respects, yet there is only a single price for each new and old model. This approach is particularly necessary when there is

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a frequent turnover of varieties in the market, where new models with quite different values for their characteristics are frequently replacing old ones

6.144. The requirement that data be collected on the prices and specifications of a large sample if not all models is not as demanding as it might appear. Extensive data on prices and characteristics of models of consumer goods and services are generally readily available on web sites—many comparing prices and salient characteristics—that can be copied with relative ease, and indeed automated using web-scraping. Such detailed information is also available as scanner data, see chapter 10.

6.145. Figure 6.2 is a scatter diagram relating the price (£ sterling) to the (cotton) capacity (kgs.) of models of washing machines sold in the United Kingdom. Data are from the Which? Magazine. It is apparent that washing machines with larger capacities command higher prices—a positive relationship. It is also apparent from Figure 6.2 that there are several models of washing machine with the same capacity but quite different prices, resulting from the fact that other things differ. For example, 12 kgs. capacity machines’ prices range from £754 to £1,349.

Figure 6.2: Scatter diagram of price against capacity: washing machine data

![Scatter diagram](image)

Price = -436.23 + 117.3Capacity
$R^2 = 0.65565$

---


8 *Which?* is a brand name used by the Consumers’ Association, a registered charity based in the United Kingdom. It exists to promote informed consumer choice in the purchase of goods and services by testing products, highlighting inferior products or services, raising awareness of consumer rights and offering independent advice. Data for this illustration was taken from the December 2017 website: [https://www.which.co.uk/reviews/washing-machines](https://www.which.co.uk/reviews/washing-machines). The example here is for illustrative purposes only.
To estimate the value given to additional units of capacity an estimate of the slope of the line that best fits the data is required. The equation of a straight line is: \( \text{Price} = \hat{\beta}_0 + \hat{\beta}_1 z_1 \).

The slope \( \hat{\beta}_1 \) is a measure of the change in \( \text{Price} \) that arises from a one-unit change in the characteristic, \( z_1, \text{Capacity} \). The \(^\wedge\) (hat) above \( \hat{\beta}_1 \) denotes that it is estimated from the data. The estimated slope is from the equation of a line that best fits the data: that best represents the underlying pattern of the relationship. In Figure 6.2 the equation of the line that best fits the data was derived using ordinary least squares (OLS) regression. The intercept and slope of the line that best fits the data are estimated as ones that minimizes the sum of the squared differences between the individual prices and their counterpart prices predicted by the line: the least squares criterion. Facilities for regression are available on standard statistical and econometric software, as well as spreadsheets.\(^9\) The estimated (linear) equation in this instance is:

<table>
<thead>
<tr>
<th></th>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-436.2293196</td>
<td>86.30954526</td>
<td>-5.054241896</td>
<td>3.09227E-06</td>
</tr>
<tr>
<td>Capacity</td>
<td>117.2976003</td>
<td>9.949378608</td>
<td>11.78943981</td>
<td>1.43937E-18</td>
</tr>
</tbody>
</table>

(6.10) \( \text{Price} = -436.229 + 117.298 \text{ Capacity} \). \( R^2 = 0.65 \)

Formula (6.10) is the estimated regression equation of \( \text{Price} \) on \( \text{Capacity} \); there are of course many other price-determining variables and this regression equation only includes \( \text{Capacity} \), for illustration. In Table 6.7 the regression model is expanded to include other variables.

The coefficient on \( \text{Capacity} \) is the estimated slope of the line: the change in price (£117.30) resulting from a 1 kg. change in \( \text{Capacity} \). This can be used to estimate quality-adjusted price changes for washing machines of different capacities. The value of \( R^2 \) is 0.65; this indicates that 65 per cent of price variation is explained by variation in \( \text{Capacity} \). A t-statistic to test the null hypothesis of the coefficient being zero was found to be 11.789: recourse to standard tables on t-statistics found the null hypothesis was rejected with a \( p\)-value of 1.43937E-18: the decimal point is moved to the left 18 digits (zeros). The fact that the estimated coefficient differs from zero cannot be attributed to sampling errors at this level of significance. There is a miniscule probability that the test has wrongly rejected the null hypothesis.

Hedonic regressions should generally be conducted using a semi-logarithmic formulation. The dependent variable is the (natural) logarithm of the price, but the variables on the right-hand side of the equation are kept in their normal units, hence the semi-logarithmic formulation. A double-logarithmic formulation would also take logarithms of the right-hand side price-determining characteristic variables. However, if any of these variables are dummy variables which take the value of zero in some instances, the double-logarithmic formulation

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\(^9\) The illustrative empirical work in this section was undertaken using R, though would be equally applicable with any such standard statistical software including EViews, SAS, and STATA.
would break down because logarithms of zero cannot be taken. The focus is thus on the semi-logarithmic form. The estimated (semi-logarithmic) regression equation in this instance is:

\[
(6.11) \log(Price) = 4.77611 + 0.17374 \text{Capacity}. \quad R^2 = 0.61
\]

6.151. The coefficient of 0.17374 has a useful direct interpretation: when multiplied by 100 it is the percentage change in price arising from a 1 unit (kg.) change in capacity. There is an estimated 17.374 per cent change in price for each additional kg. of capacity.

6.152. The range of prices for a given capacity was noted to be substantial which suggests that other quality characteristics may be involved. Table 6.7 provides the results of a regression equation that relates price to a number of quality characteristics as listed in the first column.\(^{10}\) While the results are given for both linear and semi-logarithmic regression specifications, the focus here is on the latter functional form.

### Table 6.7 Illustrative hedonic regression estimates for washing machines

<table>
<thead>
<tr>
<th></th>
<th>Linear</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Std. Error</td>
<td>(p)-value</td>
<td>Coefficient</td>
<td>Std. Error</td>
</tr>
<tr>
<td>(Intercept)</td>
<td>-206.939</td>
<td>112.3</td>
<td>0.06986+</td>
<td>5.21691</td>
<td>0.165379</td>
</tr>
<tr>
<td>Age</td>
<td>-1.579</td>
<td>2.1</td>
<td>0.44971</td>
<td>-0.005997</td>
<td>0.003059</td>
</tr>
<tr>
<td>Capacity</td>
<td>81.024</td>
<td>13.2</td>
<td>5.32e-08***</td>
<td>0.108452</td>
<td>0.019404</td>
</tr>
<tr>
<td>Warranty</td>
<td>-138.651</td>
<td>48.7</td>
<td>0.00592**</td>
<td>-0.264562</td>
<td>0.071761</td>
</tr>
<tr>
<td>Ststeel</td>
<td>144.036</td>
<td>74.0</td>
<td>0.05608+</td>
<td>0.265767</td>
<td>0.109074</td>
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<tr>
<td>Energycost</td>
<td>10.103</td>
<td>3.4</td>
<td>0.00430**</td>
<td>0.018969</td>
<td>0.00503</td>
</tr>
<tr>
<td>LG</td>
<td>-115.816</td>
<td>43.9</td>
<td>0.01044*</td>
<td>-0.219743</td>
<td>0.064687</td>
</tr>
<tr>
<td>Steam</td>
<td>191.196</td>
<td>92.3</td>
<td>0.04233*</td>
<td>0.257177</td>
<td>0.135987</td>
</tr>
<tr>
<td>Hyg_AllergyP</td>
<td>63.627</td>
<td>40.2</td>
<td>0.11842</td>
<td>0.152722</td>
<td>0.05923</td>
</tr>
<tr>
<td>LEDdisplay</td>
<td>49.409</td>
<td>52.9</td>
<td>0.35391</td>
<td>0.166143</td>
<td>0.077946</td>
</tr>
</tbody>
</table>

\[ R^2 \quad 0.701 \quad 0.721 \]

\[ \text{F-statistic} \quad 20.25 \quad 22.22 \]

\[ p\text{-value} \quad 1.06E-15 \quad 2.20E-16 \]

\(**, **, * and + denote statistically significant at 0.1, 1, 5, and 10 percent levels, respectively."

\(^{10}\) These include Age (Months since model was launched); Capacity (for cottons, kg.); Warranty (if 5 years, benchmark: 2 years); Steel (Stainless steel outer); (Annual) Energy cost, £; LG (manufactured, benchmarked on Samsung); Steam (wash/refresh); Hygiene/Allergy program; LED display (benchmarked on LCD); and price for 74 models as advertised on November 2017 in *Which*? for three up-market brands: Bosch, LG, and Samsung. A much larger general regression model with more variables was first estimated but reduced to this smaller specific model using standard econometric principles and practice. The White (studentized) Breusch-Pagan test for homoscedastic residuals was not rejected at conventional significance levels with a \(p\)-value of 0.2197.
A multivariate semi-logarithmic hedonic regression model is given by:

\begin{equation}
\text{Price} = \beta_0 + \beta_1 \ln z_1 + \beta_2 \ln z_2 + \ldots + \beta_n \ln z_n + \varepsilon,
\end{equation}

\begin{equation}
\ln \text{Price} = \ln \beta_0 + z_1 \ln \beta_1 + z_2 \ln \beta_2 + \ldots + z_n \ln \beta_n + \ln \varepsilon,
\end{equation}

where \( \varepsilon \) is an error term assumed to have the usual properties to satisfy OLS assumptions. Note that for this semi-logarithmic form, logarithms are taken only of the left-hand-side variable, i.e., \( \text{Price} \). Each of the \( z \) characteristics enters the regression without having logarithms taken. This has the advantage of allowing dummy variables for the possession or otherwise of a feature to be included on the right-hand-side. Such dummy variables take the value of one if the variety possesses the feature and zero otherwise. The taking of logarithms of the first equation in (6.12) allows it to be transformed in the second equation to a linear form. This allows the use of a conventional ordinary least squares (OLS) estimator to yield estimates of the logarithms of the coefficients. These are given as the coefficients for the Semi-logarithmic model in Table 6.7.

The estimated coefficients in Table 6.7 are based on a multivariate model: for \( \text{Capacity} \), for example, the estimated coefficient of 0.108452 is of the effect of a unit change in capacity on price, having controlled for the effect of other variables in the equation.

When dummy variables are used, the coefficients, when multiplied by 100, are estimates of the percentage change in price, given by \( (e^{\beta_1} - 1)100 \). For example, from Table 6.7, LG models have a \( (e^{-0.219743} - 1)100 = 19.73 \) percent lower price than their benchmarked Samsung counterpart, having controlled for other differences in their price-determining characteristics as specified in the regression equation.\(^{11}\)

The value \( \hat{R}^2 = 0.721 \) is the proportion of variation in \( \ln(\text{price}) \) explained by the estimated equation.\(^{12}\) A high value of \( \hat{R}^2 \) can be misleading for the purpose of quality adjustment. Although, such values indicate that the explanatory variables account for much of the price variation over a relatively large number of varieties of goods in the period concerned. This, of course, is not the same as implying a high degree of prediction for an adjustment to a replacement variety of a single brand in a subsequent time period. Predicted values depend for their accuracy not just on the fit of the equation, but also on how far the characteristics of the variety whose price is to be predicted are from the means of the sample. The more unusual the variety, the higher the prediction probability interval. Second, the value \( \hat{R}^2 \) indicates the

\[^{11}\text{There is some bias in these coefficients; and in the (semi-) logarithmic equation, half the variance of each coefficient should be added to the coefficient before using it. For the LG coefficient the standard error from Table 6.7 is 0.064687, its variance is 0.064687^2 = 0.00418: the adjustment is to add 0.00418/2 to -0.219743, giving -0.21765; a lower price of }-(e^{-0.21765} - 1)100 = 19.56 \text{ percent; see Triplett (2006) for further details.}\]

\[^{12}\text{More formally, where } p_i^t \text{ is the price of washing machine } i \text{ in period } t, \hat{R}^2 = 1 - \text{the ratio of the variance of the residuals, } \sum_{i=1}^{N} (p_i^t - \hat{p}_i^t)^2 / N \text{, of the equation to the variance of prices, } \sum_{i=1}^{N} (p_i^t - \overline{p}_t)^2 / N \text{. The bar on the term } \hat{R}^2 \text{ denotes that an appropriate adjustment for degrees of freedom is made to this expression, which is necessary when comparing equations with different numbers of explanatory variables.}\]
proportion of variation in prices explained by the estimated equation. It may be that 0.90 is explained while 0.10 is not explained. If the dispersion in prices is very large, this still leaves a large absolute margin of prices unexplained. A high $R^2$ is a necessary, though not sufficient, condition for the use of hedonic adjustments.

The interpretation of estimated hedonic coefficients

6.156. Some mention should be made of the interpretation of the coefficients from hedonic regressions. There used to be an erroneous perception that the coefficients from hedonic methods represented estimates of user value as opposed to resource cost. The former is the relevant concept in constructing a consumer price index (CPI), while for producer price index compilation it is the latter. Yet hedonic coefficients may reflect both user value and resource cost – both supply and demand influences. There is what is referred to in econometrics as an identification problem; the observed data do not permit the estimation of the underlying demand and supply parameters. What is being estimated is the actual locus of intersection of the demand curves of different consumers with varying tastes and the supply curves of different producers with possible varying technologies of production.

6.157. In many cases the implicit quality adjustment to prices arising from the use of the overlap method, as described above, may be inappropriate because the implicit assumptions are unlikely to be valid. In such instances, the practical needs of reliable economic statistics require explicit quality adjustments. However, the use of the hedonic approach may only be warranted, due to the cost of implementing the method, when the weight, churn and extent of the quality adjustment is substantial.

6.158. The proper use of hedonic regression requires an examination of the coefficients of the estimated equations to see if they make sense. It might be argued that the very multitude of distributions of tastes and technologies, along with the interplay of supply and demand, that determine the estimated coefficients make it unlikely that “reasonable” estimates will arise from such regressions. A firm may, for example, cut a profit margin relating to a characteristic for reasons related to long-run strategic plans; this may yield a coefficient on a desirable characteristic that may even be negative. This does not negate the usefulness of examining hedonic coefficients as part of a strategy for evaluating estimated hedonic equations. First, there has been extensive empirical work in this field and the results for individual coefficients are, for the most part, quite reasonable. Over time, individual coefficients can show quite sensible patterns. Unreasonable coefficients on estimated equations are the exception and should be treated with some caution. Second, one can have more faith in an estimated equation whose coefficients make sense and which makes good predictions, than one which may also predict well but whose coefficients do not make sense. Third, if a coefficient for a characteristic does not make sense, it may be due to multicollinearity, a data problem, and should be examined, say, using variance inflation factors, to see if this is the case.

The implementation of a hedonic quality adjustment

6.159. The implementation of hedonic methods to estimate quality adjustments for matched non-comparable replacements can take two forms. The first is what we refer to as “patching”: undertaking a quality adjustment to the price of the old model to make it comparable with the new model. For many varieties it can be seen as a one-off process for individual varieties within the lifetime of rebasing a sample. The second is the more wholesale process for rapidly
changing high-technology products whose changes in quality are substantial within relatively short periods; this is considered below.

6.160. Patching is the term used here for introducing non-comparable replacements, that is replacements of a different quality, via hedonic regression estimates. Consider varieties $l$, $m$ and $n$ in Table 6.8a where variety $l$ is available in all periods, the “old” variety $m$ is only available in periods $t$, $t+1$, and $t+2$ and the replacement variety $n$ only in period $t+3$ and subsequently. The varieties are defined by their $z$ quality characteristics; for variety $m$, for example, in period $t$ these are $z_{m}^t$ and the price of variety $m$ is $p_m^t$. There is no problem with comparing the prices of matched variety $l$ with characteristics $z_l$, for they have the same quality characteristics. But there is a problem when comparing varieties $m$ and $n$. Variety $m$’s replacement $n$ is non-comparable so $p_m^{t+2}$ cannot be directly compared with $p_n^{t+3}$. What is required is an imputed price in order that there are prices for both the old and new varieties in the same period. This could be achieved by imputing the price of the new variety $n$ in period $t+2$ to form an overlap in this period with the actual price of the old variety $m$, in that period, as illustrated in Table 6.4c above. This is a backwards imputation. In this case, as illustrated in Table 6.8a, the overlap period is period $t+2$. However, variety $n$ does not have a recorded price in period $t+2$, indeed it may not have been sold then. The backwards hedonic imputation approach would predict the price of variety $n$ in period $t+2$ using a hedonic regression estimated in period $t+2$ and the characteristics of the new variety $n$, taken from period $t+3$, i.e. the predicted price of variety $n$ in period $t+2$, $\hat{p}_n^{t+2}$ — the hat over the price, “ $\hat{\cdot}$ ”, denotes a predicted value from the regression. The predicted prices are for the characteristics of the replacement variety $n$. It is an estimate of what the characteristics of the new replacement variety would have been priced at had it been sold in period $t+2$.

### Table 6.8a: hedonic regression imputation of new variety’s price

<table>
<thead>
<tr>
<th>Variety/period</th>
<th>$t$</th>
<th>$t+1$</th>
<th>$t+2$</th>
<th>$t+3$</th>
<th>$t+4$</th>
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<td>$m$</td>
<td>$p_m^t$</td>
<td>$p_m^{t+1}$</td>
<td>$p_m^{t+2}$</td>
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<tr>
<td>$n$</td>
<td>$\hat{p}_n^{t+2}$</td>
<td>$\hat{p}_n^{t+3}$</td>
<td>$\hat{p}_n^{t+4}$</td>
<td>$\hat{p}_n^{t+4}$</td>
<td>$\hat{p}_n^{t+4}$</td>
</tr>
</tbody>
</table>

6.161. For short-term comparisons an overlap method is used with a price relative for $t+2$ compared with $t+1$ given by $p_m^{t+2}/p_m^{t+1}$ and for $t+3$ compared with $t+2$ given by $p_n^{t+3}/\hat{p}_n^{t+2}$ and subsequently, without the need for an imputation, by $p_n^{t+4}/\hat{p}_n^{t+3}$.

6.162. The simple example outlined above using data on washing machines sold in the United Kingdom is used here to illustrate the methodology. Assume the linear regression equation (6.10) was estimated using period $t+2$ data, the old model $m$ had a capacity of 10kg., but the new model $n$ in period $t+3$ to have a capacity of 12 kg., model $n$’s price in period $t+2$ could be predicted as the predicted price, $\hat{p} = 117.3 \times 12 - 436.23 = 971.37$. The ratio of actual price of model $m$ in period $t+2$, say (£750) to predicted price in period 2 is the quality adjustment shown
for the overlap method in equation (6.5), though for period $t+2$ in this example, $\frac{p_{m,t+2}}{p_{n,t+2}}$, that is:

$$\frac{750}{971.37} = 0.7721.$$ The models are not comparable. The new model in period $t+2$ is more expensive even when its superior quality, its capacity, has been considered.

6.163. Given we have an estimate of the worth of an extra unit of capacity, an alternative approach would be to simply add 2 times 117.3 to the period $t+2$ price of $m$, rather than use predicted prices. Such use of individual coefficients is not recommended. In practice a hedonic regression will include several explanatory price-determining variables. These may be linearly related and thus not strictly independent; larger (higher capacity) washing machines may also have higher spin-speeds, be more likely to have a steam feature, and so forth. The estimated coefficient of each such multicollinear variable would be imprecise, though the predicted price of a regression equation that includes them would be unbiased.

6.164. With the option cost example, the quality adjustment might be for a single characteristic and an explicit valuation of the price of further units of this characteristic, such as a GB of storage, available from another source. Hedonic regressions are used where the market does not reveal the implicit shadow prices of individual characteristics; these shadow prices have to be estimated from price data for many varieties with differing bundled sets characteristics.

6.165. The method makes use of short-term month-on-month comparisons: predicting the price of variety $n$ in period $t+2$, had it been on sale then, is only for this one-off period as the new variety replaces the old, with a quality adjustment. Variety $n$’s characteristics are held constant for month-on-month comparisons from $t+2$ onwards, and variety $m$’s characteristics are held constant for month-on-month comparisons from period $t$ up to, and including, period $t+2$.

6.166. Alternatively, a forward imputation might have been used, a procedure akin to that adopted in Table 6.4c. The price of variety $m$ might have its price predicted from a hedonic regression run on period $t+3$ data, $\hat{p}_{m,t+3}$. As with the preceding methodology, a predicted price is only required for the overlap period, after which the replacement variety forms the continuing index. It is not obvious which of the two approaches, predicting prices for $m$ or $n$, is preferred. Resources permitting, a geometric mean of the two would be defensible, as would a clear rule from the outset as to the method applied based on some retrospective research on the outcome of using either method for particular product groups.

6.167. Table 6.4c, the backward and forward imputation methods yield the same answer when the imputation is based, for both methods, on the price movements of varieties available in all periods. However, in this case the backwards prediction is based on a hedonic regression run in period $t+2$ and the forward imputation on a hedonic regression run in period $t+3$. The practical advantage of running a hedonic regression in a prior period argues for a backward imputation, as in Table 6.8a, as the most feasible procedure.

**Table 6.8b hedonic regression imputation of old variety’s price**

<table>
<thead>
<tr>
<th>Variety/period</th>
<th>$t$</th>
<th>$t+1$</th>
<th>$t+2$</th>
<th>$t+3$</th>
<th>$t+4$</th>
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<tbody>
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</tr>
</tbody>
</table>
A refinement to these approaches is to use predicted values for, say varieties $m$ and $n$, in the overlap period, for example, $\hat{p}_m^{t+3}/\hat{p}_m^{t+2}$. For this purpose, consider a misspecification problem in the hedonic equation. For example, there may be an interaction effect between a brand dummy and a characteristic. Possessions of a characteristic for a particular brand may be priced higher than all other brands, say a 5 percent premium. The use of $\hat{p}_m^{t+3}/\hat{p}_m^{t+2}$ would be misleading since the actual price in the denominator would incorporate the premium, while the one predicted from the hedonic regression would not. It is stressed that, in adopting this approach, a recorded actual price is being replaced by an imputation. This is not desirable, but neither is the omitted variable (interaction term) bias. The dual imputation approach is preferred whenever there are concerns about the suitability of the regression equation’s specification to fully model prices, as would generally be the case.

A further approach would be to not use a replacement variety. Variety $m$’s characteristics would be held constant in the comparison from period $t+2$ onwards. However, this would require a hedonic regression being run for each subsequent period, $\hat{p}_m^{t+3}, \hat{p}_m^{t+4}$. It would also lead to a continuing degradation of the sample as an obsolete old variety $m$ has its characteristics repeatedly priced into the future, rather than being replaced by a new variety. For this reason, this method is not recommended.

In the above examples short-term (S-T) price comparisons are used and are preferable to long-term (L-T) ones. A L-T equivalent of Table 6.8a is shown in Table 6.8c. A predicted price for any replacement variety $n$ in its month of introduction is estimated for the reference period $t$ using a hedonic regression using that period’s data. The regression is estimated using period $t$ prices and characteristics, but the predicted prices are for the characteristics of the replacement variety $n$ in $t+3$ and subsequently. It is an estimate of what the characteristics of the new replacement variety would have been priced at had it been sold in period $t$. The L-T method has the significant advantage of only requiring a hedonic regression to be estimated in the single reference period. For periods $t+3$ and $t+4$ the price relatives are $p_n^{t+3}/\hat{p}_n^{t}$ and $p_n^{t+4}/\hat{p}_n^{t}$ respectively. However, as time passes, such comparisons become less meaningful, for example, comparing the actual price this current month of a model of a laptop with one predicted say 18 months ago using the hedonic approach, will estimate market valuations of each characteristic which is then applied to the characteristic set of a laptop sold now. Indeed, the need for a double imputation becomes more important as time passes by, yet a double imputation requires monthly estimation of hedonic regressions that forestalls the very advantage of this approach. If hedonic regressions are to be used on this L-T basis it is important that the regressions are re-estimated regularly at a rate that will depend on the rate of the technological innovations, and changes in consumer preference specific to that product. For example, it may be that consumer’s valuations of characteristics of washing machines, including spin-speed, front-loaders, capacity, number and types of wash programs and so forth, are fairly constant over time, even if the technology is itself changing rapidly. Frequent, say monthly, updating of estimated hedonic regression equations is not required. Prior empirical studies on the stability
over time in hedonic characteristics would be valuable in this respect. As a general principle, S-T hedonic imputations are preferred to L-T ones.

**Table 6.8c: Hedonic regression imputation of new variety’s price**

<table>
<thead>
<tr>
<th>Variety/period</th>
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</tr>
</tbody>
</table>

**Limitations of the hedonic approach**

6.171. The limitations and challenges of implementing the hedonic approach should be considered. First, the approach requires statistical expertise for the estimation and maintenance of the hedonic regression equations. The availability of user-friendly statistical/econometric software with regression facilities makes this less problematic. Yet staff must possess sufficient expertise and understanding of statistical regression methodology as applied to hedonic regression equations and the interpretation of the results from, and diagnostic statistics of, the regression models. Statistical and econometric software carry a range of diagnostic tests to help judge if the final formulation of the model is satisfactory. These include $R^2$ as a measure of the overall explanatory power of the equation, $F$-test and $t$-test statistics to enable tests to be conducted as to whether the differences between the estimated coefficients of the explanatory (price-determining) variables are jointly and individually different from zero at specified levels of statistical significance. These statistics make use of the errors from the estimated regression equation. The regression equation can be used to predict prices for each variety by inserting the values of the characteristics of the varieties against the estimated coefficients of the explanatory variables. The differences between the actual prices and these predicted results are the residuals. Statistical/econometric software calculate predicted values and residuals as a matter of routine. A hedonic regression equation estimated using ordinary least squares (OLS) requires assumptions as to the nature of the distribution of these residual errors. These include: (i) the error term has a constant variance. When this assumption is violated, the errors are heteroscedastic; standard tests of statistical significance can be biased and unreliable; (ii) that explanatory variable(s) are not correlated with the error term, they are not endogenous. This is particularly important when explanatory price-determining characteristics are omitted from the hedonic regression. If an omitted variable is correlated with an included one, the estimated coefficient on the included one is biased; (iii) the price-determining explanatory independent variables are not truly independent, but correlated with each other—multicollinearity. The coefficient estimates and their tests become sensitive to change in the model and/or data. While the estimated coefficients are imprecise, the predicted prices in a hedonic regression would be unbiased. A full account of all OLS assumptions, consequences, means of detection of violation, and treatment, that may involve use of alternative (to OLS) estimators, can be found in any introductory econometrics/statistical text. Modern software provides the appropriate tests for, and means of surmounting, breaches of these assumptions and thus, validation of the hedonic model used. It is recommended that a background paper by
the NSO responsible for the CPI be published on the hedonic regression model used and its supporting diagnostic statistics to demonstrate the validity of the model and satisfy the need for transparency.

6.172. Second, the estimated coefficients require regular updating. Say the predicted price is for the new model in a reference period, as in Table 6.8c. There is, at first sight, no need to update the estimated coefficients each period. Yet the valuation of characteristics in the price reference period may be quite out of line with their valuation in the new period. Quite dramatic falls in the price of storage and processing speed, among other attributes, of computers make the valuation of additional MBs or GBs of a new model, introduced a few years after the hedonic regression was estimated, a less meaningful exercise. Continuing to use the coefficients from some far-off period to adjust prices in the current period is akin to using out-of-date reference period weights. The comparison may be well defined, but have little meaning. There is a need to update the hedonic regression estimates if they are considered to be out of date, say because of changing tastes or technology, and splice the new estimated comparisons onto the old. The regular updating of hedonic estimates when using imputations or adjustments is thus recommended, especially when there is evidence of instability in the parameter estimates of the hedonic regression over time.

6.173. Third, the sample of prices and characteristics used for the hedonic adjustments should be suitable for the purpose. If they are taken from a particular outlet or outlet type, trade source or web page and then used to adjust non-comparable prices for varieties sold in quite different outlets, then there must at least be an intuition that the marginal price differences for characteristics are similar between the outlets. A similar principle applies for the brands of varieties used in the sample for the hedonic regression. It should be borne in mind that high $R^2$ statistics do not alone ensure reliable results. Such high values arise from regressions in periods prior to their application and indicate the proportion of variation in prices across many varieties and brands. They are not in themselves a measure of the prediction error for a particular variety, sold in a specific outlet, of a given brand in a subsequent period, though they can be an important part of this.

6.174. Fourth, there is the issue of functional form and the choice of variables to include in the model. Simple functional forms generally work well, though there is a class of more complex flexible-functional forms. These include linear, semi-logarithmic (logarithm of the left-hand side) and double-logarithmic (logarithms of both sides) forms. Semi-logarithmic models are often employed since many of the price-determining explanatory variables are binary, 1 or 0, depending on whether or not a model has a particular feature. The specification of a model should include all price-determining characteristics. Typically, a study would start with a large number of explanatory variables and a general econometric model of the relationship, while the final model would be more specific, having dropped a number of variables. The dropping of variables would depend on the result of experimenting with different formulations, and seeing their effects on diagnostic test statistics, including the overall fit of the model and the accordance of signs and magnitudes of coefficients with prior expectations.

6.175. Fifth, there is the important issue of resource requirements for hedonic regression requirements. Hedonic regressions require data on prices and price-determining characteristics for varieties (models) sold. Fortunately, this is not as intensive an exercise as previously considered. Extensive data sets may be readily available on the internet or from scanner data.
containing all pertinent price-determining characteristics either from the web-sites of individual retailers or specialist web sites comparing prices and features of models of laptops, household appliances, and many other such goods and services. The data used for the above example of (patched) hedonic explicit quality adjustments for washing machines was taken from a web-site and was copied and pasted in a matter of hours. Web-scraping software can reduce even this workload substantially.

6.176. Finally, while data and software may not be problematic, staff resources will be required in devising the specification and estimation and validation of the estimated hedonic model particular to each product. Such hedonic models should be estimated regularly prior to their use for the CPI and the results made available in a methodological (working) paper for the purpose of transparency and feedback. In this regard, the resource requirements can be substantial compared with a say implicit overlap method. At least at first, hedonic methods should be applied only to products with a relatively high weight and profile for which the implicit assumptions of alternative methods are found to be invalid and badly distort the results, especially if they provide a reputational risk to the NSO.

6.177. Hedonic methods may also improve quality adjustment procedures in the CPI by indicating which product attributes do not appear to have material impacts on the prices. That is, if a replacement variety differs from the old variety only in characteristics that have been rejected as price-determining variables in a hedonic study, this would support a decision to treat the varieties as comparable. Care has to be exercised in such analysis because a feature of multicollinearity in regression estimates is an imprecision of the estimated parameter estimates. This may give rise to statistical tests that do not reject null hypotheses that are false. However, econometric/statistical software provides the tools to explore the nature and extent of multicollinearity; these include variance inflation factors (VIF). The results from VIFs provide valuable information on the nature and extent to which different explanatory variables (characteristics) are inter-related and this in turn can help in the selection of replacement varieties. The results from hedonic regressions thus have a role to play in identifying price-determining characteristics and may be useful in the design of quality checklists in price collection.

5. Choice between quality adjustment methods

6.178. Choice of method for quality adjustments is not straightforward. The analyst must consider the technology and market for each commodity and devise appropriate methods. This is not to say the methods selected for one product area will be independent of those selected for other areas. Expertise built up using one method may encourage its use elsewhere, and intensive use of resources for one commodity may lead to less resource-intensive methods for others. The methods adopted for individual product areas may vary between countries as access to data, relationships with the outlet managers, resources, expertise and features of the production, and market for the product vary. Guidelines on choice of method arise directly from the features of the methods outlined above. A good understanding of the methods, and their implicit and explicit assumptions, is essential to the choice of an appropriate method.

6.179. Figure 6.3 provides a guide to the decision-making process. Assume that the matched models method is being used. If the variety is matched for re-pricing in a subsequent period,
there is no change in the specifications and no quality adjustment is required. This is the simplest of procedures. However, a caveat applies. If the variety belongs to a product group where model replacement is rapid, and replacements non-comparable, the matched sample may become unrepresentative of the universe of transactions. Continued long-term matching would deplete the sample. This a matter for the frequent re-basing and maintenance of the sample, Chapter 7.

**Figure 6.3: Guide to treatment of missing prices**

6.180. Consider a variety found to be *temporarily missing*. Were it a seasonal good its treatment would follow the principles and practices outlined in Chapter 11. If it was temporarily missing but not a seasonal good, a price imputation is required, and if subsequently determined to be permanently missing—either from information from a senior outlet staff member or use of a three-month rule—a replacement needs to be found. *Overall or targeted price imputations* for temporarily missing prices may be used; the carry forward method is not recommended unless for controlled or regulated prices.

6.181. For *permanently missing* variety prices, the selection of a comparable variety is preferred and the use of its price as a *comparable replacement* price to be directly compared with the preceding variety price. This direct price comparison would require that none of the price difference between the comparable replacement and the previous variety is attributable to quality and confidence that all price-determining factors are included in the specification. In practice, varieties may be taken to be deemed comparable if there are limited price-determining differences, as might be the case with styling, color, even some more substantial technical
changes including performance and reliability that may not be immediately apparent to the consumer. A decision as to the comparability or otherwise of a replacement must be made by a desk officer with appropriate information on product differences supplied by the price collector. A comparable replacement variety should also be representative and account for a reasonable proportion of sales. Caution is required when replacing near obsolete varieties with unusual pricing at the end of their life cycles with similar ones that account for relatively low sales, or with ones that have quite substantial sales but are at different points in their cycle. Strategies for ameliorating such effects are discussed below and in chapter 7, including early substitutions before pricing strategies become dissimilar. With comparable replacements the price of the old variety is directly compared with the price in the next period of the comparable replacement.

6.182. Figure 6.3 considers the case where non-comparable replacements are only available. If explicit estimates of the price dimension of the quality differences are unavailable, and no replacement varieties are deemed comparable, implicit estimates might be used. One such method is that the use of imputations as applied to temporarily missing varieties is continued. Such use is not recommended as a default procedure.\textsuperscript{13} It may be used to extend the period of search for a replacement, though the absence of the old variety and the unavailability of a replacement should indicate to the desk officer that the weight for that variety might be better attributed to a quite different variety. Such changes naturally take place on re-basing an index, chapter 7.

6.183. If the old and replacement varieties are available simultaneously, and if the quality difference cannot be quantified, an implicit approach can be used whereby the price difference between the old and replacement varieties in a period in which they both exist is assumed to be attributable to quality. This overlap method, in replacing the old variety by a new one, takes the ratio of prices in a period to be a measure of their quality difference. It is implicitly used when new samples of varieties are taken. The assumption of relative prices equating to quality differences at the time of the splice is unlikely to hold if the old and replacement varieties are at different stages in their life cycles and different pricing strategies are used at these stages. For example, there may be deep discounting of the old variety to clear inventories, and price skimming of market segments that will purchase new models at relatively high prices. As with comparable replacements, early substitutions are advised so that the overlap is at a time when varieties are at similar stages in their life cycles. It may well be the case that overlap prices are unavailable. In such cases a range of imputation approaches are available to estimate an overlap price.

\textsuperscript{13} The use of imputations has much to commend it resource-wise. It is relatively easy to employ. It requires no judgment (unless targeted) and is therefore objective. Targeted mean imputation is preferred to overall mean imputation as long as the sample size upon which the target is based is adequate. Yet the bias from using imputations for permanently missing variety prices is directly related to the proportion of missing varieties and the difference between quality-adjusted prices of available matched varieties and the quality-adjusted prices of unavailable ones. The nature and extent of the bias depends on whether short-term or long-term imputations are being used (the former being preferred) and on market conditions. Imputation, in practical terms, produces the same result as deletion of the variety for an elementary aggregate. The inclusion of imputed prices may give the illusion of larger sample sizes. Imputation should by no means be the overall catch-all strategy, and NSOs are strongly advised against its use as a default device which may lead to serious sample degradation.
6.184. The quality differences between the replacement and missing variety may be explicitly quantified. *Explicit estimates* of quality differences are generally considered to be more reliable, although they are also more resource intensive, at least initially. Once an appropriate methodology has been developed, they can often be easily replicated. General guidelines are more difficult here as the choice depends on the host of factors discussed above, which are likely to make the estimates more reliable in each situation. Central to all of this is the quality of the data upon which the estimates are based. Estimates based on objective data are preferred. A relatively straightforward quality adjustment is when the quantity differs. The standardization of quantity units sold across outlets to say price-per-kilogram is relatively straightforward, though a change in the quantity of a variety included in the price—a quantity adjustment—may be more complicated, as explained below.

6.185. The replacement variety may differ from the old one by its *possession of a feature*. Often it is the *price collector* who is best placed to provide an estimate of the price difference in quality of a non-comparable replacement. Say a specified brand of a bottle of tomato ketchup used for pricing is missing in the current period. However, a non-comparable replacement of the same brand is available, though the bottle has been restyled to now stand on its head, and label reversed. The price collector might note that other brands have both sizes on sales with a say 25cent price margin for the new one. The price collector in selecting a non-comparable replacement might also provide the basis for the desk officer to make an explicit quality adjustment. A desk officer might also make use of the Internet to identify the percentage markup for a quality characteristic, for example, for additional memory for a computer, bluetooth technology in their automobile, and so forth. The *option cost* approach is applicable when a new feature is first sold as an option and then becomes a standard component included in the basic price. This requires that the old and new varieties differ by easily identifiable characteristics that are or have been separately priced as options. The use of production cost estimates critically relies on the availability of suitable estimates for the price cost margin.

6.186. The use of *hedonic regressions* for patching price changes due to quality differences is most appropriate where data on price and characteristics are available for a range of models and where the characteristics are found to predict and explain price variability well in terms of *a priori* reasoning and econometric terms. Their use is appropriate where the cost of an option or change in characteristics cannot be separately identified and has to be gleaned from the prices of varieties sold with different specifications in the market. The estimated regression coefficients are the estimate of the contribution to price of a unit change in a characteristic, having controlled for the effects of variations in the quantities of other characteristics. The estimates are particularly suited to valuing changes in the quality of a variety when only a given set of characteristics changes and the valuation is required for changes in these characteristics only. The results from hedonic regressions may be used to target the salient characteristics for variety selection. The synergy between the selection of prices according to characteristics defined as price determining by the hedonic regression, and their subsequent use for quality adjustment, should reap rewards. The method should be applied where there are high ratios of non-comparable replacements, though not a frequent churn as outlined below, and where the differences between the old and new varieties can be well defined by its characteristics.

6.187. For the reasons discussed, the use of the *link-to-show-no-price-change* method for permanently and the *carry-forward* method for temporarily missing variety prices are not generally advised for making quality adjustment and imputations.
6.188. While Figure 6.3 is appropriate for the treatment of temporarily and permanently missing prices in the routine compilation of a CPI, there is a context in which a quite different strategy is required. The context is where there is a rapid turnover or “churn” in the models of varieties sold. For example, a television sets are sold by several manufacturers each having a range of models with different features. Over time many new phases of technological development have occurred including the cathode ray tube (CRTs), color TVs, wireless remotes, plasma, liquid-crystal-display televisions (LCD), digital, high definition (HD), larger screens, smart functions, 3D, light-emitting diodes (LEDs), Ultra HD resolution, OLED (Organic Light Emitting Diode), and roll-up OLED. New features and restyling extend the life cycle of each model in each phase. As with automobiles, computers, computer-related hardware and software, telecommunication equipment, household appliances and much more, the product-market is characterized by different manufacturers producing several varieties (models) each of different quality, say screen size for a computer or television set, aimed at different segments of the market. These will, over time, usually have a rapid turnover in their quality characteristics. The methods outlined above if applied to these markets may lead to a biased CPI. Figure 6.3 notes that matching, class-mean imputations and hedonic price indices may be used, though there may be severe bias in the use of the former. The next section considers CPI measurement for these product markets.

K. High-technology and Other Sectors with a Rapid Turnover of Models

6.189. The measurement of price changes of varieties unaffected by quality changes is primarily achieved by matching models; however, when the matching breaks down the implicit or explicit methods can be used. But what of industries where the matching breaks down on a regular basis because of the high turnover in new models of different qualities to the old ones? The matching of prices of identical models over time, by its nature, is likely to lead to a seriously depleted sample. There is both a dynamic universe of all varieties consumed and a static universe of the varieties selected for re-pricing. If, for example, the sample is initiated in December, by the subsequent May, for a L-T price comparison, the static universe will be matching prices of those varieties available in the static universe in both December and May, but will omit the unmatched new varieties introduced in January, February, March, April and May, and the unmatched old ones available in December but unavailable in May. For December to May cumulative month-on-month S-T comparisons, similar considerations apply. Although there will be improved imputations for temporarily missing variety prices and an improved timelier introduction of replacements, the replacements only borrow from the dynamic universe of new models on a one-on-one basis. The example here is for a December to January matched price comparison. For many countries matching may effectively continue for many years until the CPI is rebased leaving an extremely degraded sample on rebasing. Two empirical questions show whether there will be any significant bias. First, is sample depletion substantial? Substantial depletion of the sample is a necessary condition for such bias. Second, are the unmatched new and unmatched old varieties likely to have quality-adjusted prices that substantially differ from those of the matched varieties in the current and the base periods?

6.190. The matching of prices of identical models over time may lead to the monitoring of a sample of models that is increasingly unrepresentative of the population of transactions. Some of the old models that existed when the sample was drawn are not available in the current period; and new models that enter the sample are not available in the base period. It may be
that the models that are going out have relatively low prices, while the entrants have relatively high ones. By ignoring these prices, a bias is being introduced. Using old low-priced varieties and ignoring new high-priced ones has the effect of biasing the index downwards. In some industries, the new variety may be introduced at a relatively low price though the old one may continue at a relatively high price, serving a minority segment of the market. In this case, the bias would take the opposite direction. The nature of the bias will depend on the pricing strategies of firms for new and old varieties. Some strategies for the introduction of new models, and implications for CPI measurement, are considered in Annex 3.

6.191. This sampling bias exists for most products. Our concern here, however, is with product markets where the NSOs are finding the frequency of new variety introductions and old variety obsolescence sufficiently high that they may have little confidence in their results. Three procedures will be considered: an extensive use of the matched model (overlap) technique, class-mean imputation, and the use of hedonic price indices (as opposed to the partial, hedonic patching discussed above).

1. Matching and the overlap method for markets with rapid turnover of models

Table 6.9 Illustration of rapid model turnover

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6.192. This first approach is simply a more extensive use of the overlap approach outlined above for permanently missing prices. Its adoption here is for permanently missing varieties that occur frequently as is usual for changes in models of electronic goods and automobiles. Matching prices of a few representative varieties becomes less feasible in this context. A backwards imputation is illustrated here though, as outlined above in the illustration for Table 6.4c, a forward imputation can be equally justified and indeed, as outlined in the text following Table 6.4c, both methods provide the same results when the imputations are based on the price changes of overlapping matched samples.

6.193. Consider Model 1 in table 6.9: in November there is no overlap price for the new model 1R, so its price is imputed “backwards” by using the ratio of geometric means of the December to November prices but only including those for which matched models exist, that is models 2R, 4R, and 5. These are all constant-quality price comparisons; of like with like.
and its imputed price \( 0.974 \times 30 = 29.2 \).

6.194. The imputed price for the replacement model 2R in June is based on the price changes of matched models 1, 3, and 5 for June and July, that is:

\[
\left( \frac{37 \times 30 \times 29}{40 \times 30 \times 29} \right)^{\frac{1}{3}} = \left( \frac{37}{40} \times \frac{30}{30} \times \frac{29}{29} \right) = 0.974
\]

and its imputed price: \( 1.00 \times 30 = 30 \). The imputed prices for 3R in November, and 4R in June are 32.15 (rounded to 32.2, for simplicity of exposition) and 30.0 respectively.

6.195. The overall price relatives for each model, and its linked-in replacement, can now be computed as the product of S-T month-on-month price changes, that is model 1 and its replacement 1R, for January to January, using the overlap month of November:

\[
(6.14a) \quad \frac{p_1^{Feb}}{p_1^{Jan}} \times \frac{p_1^{Mar}}{p_1^{Feb}} \times \frac{p_1^{Apr}}{p_1^{Mar}} \times \frac{p_1^{May}}{p_1^{Apr}} \times \frac{p_1^{Jun}}{p_1^{May}} \times \frac{p_1^{Jul}}{p_1^{Jun}} \times \frac{p_1^{Aug}}{p_1^{Jul}} \times \frac{p_1^{Sep}}{p_1^{Aug}} \times \frac{p_1^{Oct}}{p_1^{Sep}} \times \frac{p_1^{Nov}}{p_1^{Oct}} \times \frac{p_1^{Dec}}{p_1^{Nov}} \times \frac{p_{1R}^{Dec}}{p_{1R}^{Jan}}
\]

\[
= \frac{p_{1R}^{Dec}}{p_{1R}^{Jan}}
\]

\[
(6.14b) = \frac{25}{25} \times \frac{25}{25} \times \frac{25}{25} \times \frac{25}{25} \times \frac{25}{25} \times \frac{25}{25} \times \frac{25}{25} \times \frac{20}{25} \times \frac{30}{30} \times \frac{30}{30} \times \frac{30}{30}
\]

\[
= \frac{20}{25} \times \frac{30}{29.2} \times \frac{30}{30} = 0.8219
\]

6.196. The price relative for model 1 from January in the preceding year to January in the current year shows a 17.81 percent price decrease. It is clear from Table 6.9 that the price for model 1 has been constant up to October, there was a price fall in November, but this was to clear the market for the replacement. There should be a counteracting November-December price increase for the old model 1 to the new replacement 1R that reflects that part of the price difference between mode 1 and model 1R was not due to quality differences. But the imputation is based on the constant price movements of models 4R and 5 and a coincidental price increase in model 2R; it assumes that S-T price movements of matched pairs will proxy the price change of model 1. However, in this context, the constant price changes of matched models are an inappropriate proxy badly biasing the measured price change downwards. At fault are first, the use of the unrepresentative price for model 1 at the end of its life cycle in November, and second, the inappropriate imputation for the replacement variety.

6.197. The January-January price decreases for models 2, 3, and 4, using replacements, and 5 are respectively: 1.2, 6.3, and 3.4 percent and no change for model 5. With a substantial churn
of models, possibly more frequently than annual, the bias from using overlaps can be substantial.

6.198. In its favor, the method is simply an extension of the linking-in of new products and can be readily applied by a NSO, especially one with limited resources. Yet in basing the imputations on price changes of matched varieties not subject to the price changes that occur on the replacement of a model, it biases the CPI.

6.199. The overlap method, outlined above, may be subject to bias if applied to where there is substantial churn in the product market and an active policy by the supplier of introducing upgraded replacement models. The nature and extent of the bias depends on the pricing strategy. Table 6.9 illustrated a policy of lower pricing at the end of the life cycle and a higher price at the start. Importantly, the example had no price change for other matched models, from which the imputation was drawn, and thus a biased imputation at this critical overlap period. The bias was substantial and downwards. Alternative pricing strategies are given in Annex 3 along with their implication for bias from using the overlap matching. That the method can introduce substantial bias under quite reasonable conditions has been demonstrated in empirical studies. That the nature and extent of the bias depends on business pricing strategies that may change over time and are unpredictable, is worrying for CPI compilation in this important product area. The overlap method is thus not recommended for product markets with a high rate of model churn.

2. Use of a class-mean imputation

6.200. It was shown above that an imputation based on price movements of other matched models not at the end of their life cycle, could introduce a bias. An alternative, though more resource intensive method, is to base the imputations not on price changes of matched varieties but use, where possible, explicit quality adjustments for linked-in non-comparable replacement. For example, internet web pages of prices of similar products may show the difference in characteristics and prices of the old models and the replacement models. At its simplest, the replacement may simply have a higher value of some performance characteristic or feature for which the price is available as an option. If a sufficient number of explicit quality adjustments can be made, imputations might be better made on the basis of only those models that have had explicit quality adjustments to their price.

6.201. However, the very nature of the high frequency of replacements makes the procedure resource intensive and, in some instances, not viable due to the absence of explicit information on prices of features or options. However, should sufficient models have an explicit adjustment, an average of their price change could be used to impute the price change of other models being replaced. This is the basis of using class-mean imputations. The method requires care that the linking-in of replacement models does not take place at the end of the model’s life cycle, when pricing might be abnormally low for a variety relatively few are purchasing. This not only has a detrimental effect on the quality adjustment methodology, but also on the representativity of the models upon which the prices change measurement is based.

6.202. The class-mean imputation method was outlined above. It is similar in procedure to the overall mean imputation and is a form of targeted imputation. The “target” is measured price changes of replacements for permanently missing products. Only the price changes of “comparable” replacements are used to impute the overlap price, the replacements being
limited to those that have exactly the same price-determining characteristics, or those varieties with replacements that have been declared comparable after review and/or have already been quality-adjusted through one of the "explicit" methods. For example, when the arrival of a new model of a particular make of automobile forces price collectors to find replacements, some of the replacements will be of comparable quality, others can be made comparable with explicit quality adjustments, but the remaining ones will need imputed prices for an overlap month.

6.203. Class mean-imputations rely on other explicit quality adjustments and comparable replacements. The other explicit quality adjustments may be derived from available option or feature prices and may be limited in nature, covering only some of the differences in product attributes, available for only a small proportion of unrepresentative model changes, and the availability of comparable replacements is thus limited. Given a substantial churn in the market and difficulties with such imputations and estimates an alternative recommended approach is that of hedonic indices.

3. Hedonic price indices

6.204. It is important to distinguish between the use of hedonic regressions for patching and their use in their own right as hedonic price indices, which are measures of quality-adjusted price changes. Patching adjusts individual item prices for quality differences when a non-comparable substitute is used while hedonic price indices are measures of quality-adjusted price changes. Hedonic price indices are suitable when the pace and scale of replacements of varieties are substantial because, first, an extensive use of these overlap quality adjustments may lead to bias and, second, the sampling will be from a static matched/replacement universe likely to be biased. With new models being continually introduced and old ones disappearing, the coverage of a matched sample may deteriorate and bias may be introduced as the price changes of new/old models differ from those of the matched ones. What is required is a sample to be drawn in each month and price indices constructed; but instead of controlling for quality differences by matching, they will be controlled for, or “partialled out”, in the hedonic regression. Note that all the indices described below use a fresh sample of the data available in each period. If there is a new variety in a period, it is included in the data set and its quality differences controlled for by the regression. Similarly, if old varieties drop out, they are still included in the data for the indices in the periods in which they exist. Paragraphs 6.110 to 6.115 stress the need for caution in the use of hedonic regressions for quality adjustments.

6.205.

6.206. Consider a price comparison between two adjacent time periods, say periods \( t \) and \( t+1 \). The models sampled do not have to be matched. They may simply be all recorded models on sale in the two periods, and they comprise a different mix of qualities. The hedonic formulation regresses the price of model \( i, p_i \), on the \( k=2,\ldots,K \) characteristics of the varieties \( z_{ki} \). A single regression is estimated on the data in the two time periods compared, the equation also including a dummy variable \( D_{t+1} \) being 1 in period \( t+1 \), zero otherwise.

The time dummy variable approach

6.207. A single hedonic regression equation is estimated with observations across models over adjacent time periods, including the reference period 0 and a subsequent periods \( t \). (The logarithm of) prices of individual models are regressed on their characteristics and a dummy
variable for time, taking the values of $D_i = 1$ if the model is sold in period 1 and 0 otherwise. A log-linear specification is given by:

\[ (6.15) \ln p_i^t = \ln b_i^0 + \sum_{k=1}^{K} \gamma_{k,i} \ln b_i^k + \ln e_i^t \]

6.208. The $\gamma_i$ are estimates of the proportionate change in price arising from a change between the excluded reference period $t=0$ and successive periods $t=1,T$ having controlled for changes in the quality characteristics via the term $\sum_{k=1}^{K} \gamma_{k,i} \ln b_i^k$.

6.209. In principle, the index $100 \times \exp(\delta)$ requires an adjustment for it to be a consistent (and almost unbiased) approximation of the proportionate impact of the time dummy, see Triplett (2006). In practice, it usually has little effect.

6.210. The method implicitly restricts the coefficients on the quality characteristics to be constant over time: for example, for an adjacent period January and February regression, for $k=1,\ldots,K$ characteristics and where period 0 and $t$ are January and February respectively,

$$\beta_k^t = \beta_k^{Jan} = \beta_k^{Feb}.$$  The (relative) valuation of a characteristic, for example for a washing machine with an additional 100 rpm spin speed, is the same in January as in February. The index, $100 \times \exp(\delta^{Feb})$, is an estimate of the quality-adjusted price change for February (January=100).

**The characteristics/repricing approach**

6.211. A hedonic regression is run to determine the price-determining characteristics of models in a say reference period 0. The average model in period 0 can then be defined as a tied bundle of the averages of each price-determining characteristic, for example for washing machines: Spin-speed: 1,375 rpm; Capacity (cotton load): 8.5 kg.; Annual energy cost: £36.5 (pounds sterling); Steam facility: 4 percent; LG brand: 15 percent; Warranty period: 5.4 years; Run-time (cotton): 18.8 mins; and so forth. These are the $\overline{z}_k$ averages for each of the $k$ price-determining characteristics.

6.212. These average values of each characteristic are held constant in each period but valued in turn using period 0 and period $t$ hedonic regressions. One form of the (average) characteristics approach is as a measure of the price change of a set of average period 0 characteristics valued first, at period $t$ hedonic valuations, and second, at period 0 hedonic valuations. A ratio of the results is a constant (period 0 characteristics) quality price index. The numerator, the period $t$ hedonic valuation, provides an answer to a counterfactual question: what would be the estimated transaction price of a model with period 0 average characteristics, were it on the market in period $t$?

6.213. A constant-quality hedonic geometric mean characteristics (HGMC) price index from a log-linear hedonic regression equation is a ratio of geometric means with average characteristics held constant in the reference period 0, $\overline{z}_k^0$.

\[ (6.16) \prod_{k=0}^{K} \left( \hat{b}_k^0 \right)^{\overline{z}_k^0} = \frac{\prod_{k=0}^{K} \left( \hat{b}_k^t \right)^{\overline{z}_k^0}}{\prod_{k=0}^{K} \left( \hat{b}_k^0 \right)^{\overline{z}_k^0}} = \exp \left( \sum_{k=0}^{K} \overline{z}_k^0 \ln \hat{b}_k^t \right) \exp \left( \sum_{k=0}^{K} \overline{z}_k^0 \ln \hat{b}_k^0 \right) \quad \text{where} \quad \overline{z}_k^0 = \frac{1}{N_0^0} \sum_{i=0}^{N_0^0} z_{i,k}^0 \]
6.214. Equation (6.16) holds the (quality) characteristics constant in period 0, though a similar
index could be equally justified by valuing in each period a constant period \( t \) average quality
set:

\[
\text{(6.17) } \frac{P_{\text{hmc}}}{P_{\text{hmc}}} = \frac{\prod_{k=0}^{K} (b'_k)^{z'_i}}{\prod_{k=0}^{K} (b'_k)^{z'_i}} \exp\left(\sum_{k=0}^{K} z'_i \ln \hat{b}'_k \right) \exp\left(\sum_{k=0}^{K} z'_i \ln \hat{b}^0_k \right)
\]

where \( z'_i = \frac{1}{N'} \sum_{i \in N'} z'_{i,k} \).

\( N^0 = N' \) the number of matched observations (varieties) in the sample. Neither a period 0
constant-characteristics index nor a period \( t \) constant-characteristic quantity basket can be
considered to be superior, both acting as bounds for their theoretical counterparts. Some
average or compromise solution is required. An index making symmetric use of period 0 and
period \( t \) characteristics values is intuitive:

\[
\text{(6.18) } \frac{P_{\text{hmc}}}{P_{\text{hmc}}} = \frac{\prod_{k=0}^{K} (b'_k)^{z'_i}}{\prod_{k=0}^{K} (b'_k)^{z'_i}} \exp\left(\sum_{k=0}^{K} z'_i \ln \hat{b}'_k \right) \exp\left(\sum_{k=0}^{K} z'_i \ln \hat{b}^0_k \right)
\]

6.215. Note that equations (6.16), (6.17), and (6.18) all use predicted prices in both the
denominator and numerator. This follows the advice to use dual imputations for reasons
outlined below.

6.216. However, it also entails running hedonic regressions in each period. Yet a fortuitous
result is that a feature of the OLS estimator is that the mean of actual prices is equal to the mean
of predicted prices:

\[
\frac{1}{N^0} \sum_{i \in N^0} \ln \hat{p}^0_{i,t} = \frac{1}{N^0} \sum_{i \in N^0} \ln p^0_i \quad \text{and} \quad \frac{1}{N^t} \sum_{i \in N^t} \ln \hat{p}_i = \frac{1}{N^t} \sum_{i \in N^t} \ln p'_i.
\]

Thus, while the numerator of equations (6.16) and denominators of equation (6.17) must be
counterfactual—the are valuing period 0 (\( t \)) average characteristics at period \( t \) (0) prices—the
denominator of equations (6.16) and numerator of (6.17) can use actual prices, since the means
are the same for an OLS estimator. This leaves us with the important results that equation (6.17)
does not require a hedonic regression to be estimated in every current period \( t \), only in the price
reference period 0. This is an important result since, it aids the practical work of compilers who
do not have to estimate a hedonic regression equation in each period, but maybe once every
one or two years, depending on the amount of churn in the market and shifting technologies
and preferences. The hedonic indices from one regression can be chained to its preceding
hedonic indices, and so forth, using successive multiplication.

**The hedonic imputation approach**

6.217. In contrast to the characteristics approach, the *imputation* approach works at the level
of individual varieties/models, rather than the average values of their characteristics. The
rational for the imputation approach lies in the matched model method. Consider a set of
models transacted in period 0. We want to compare their period 0 prices with the prices of the
same matched models in period \( t \). In this way there is no contamination of the measure of price
change by changes in the quality-mix of models transacted. However, for goods and services
with a high model turnover, not all of the period 0 models were sold in period \( t \)—there is no
corresponding period \( t \) price in many cases. The solution—in the numerator of equation (6.19)—is to predict the period \( t \) price of each \( i \) period 0 model, \( \hat{p}_{0|z_{0}^{0}}^{i} \).

6.218. A constant-quality hedonic geometric mean imputation (HGMI) price index from a log-linear hedonic regression equation is a ratio of geometric means with characteristics held constant in the reference period 0, \( z_{k}^{0} \):

\[
(6.19) \quad P_{HGMI:z_{t}^{t}}^{0} = \frac{\prod_{i \in N^{t}} \left( \hat{p}_{0|z_{t}^{t}}^{i} \right)^{\frac{1}{N_{t}}} \exp \left( \frac{1}{N_{t}} \sum_{i \in N^{t}} \ln \hat{p}_{0|z_{t}^{t}}^{i} \right)}{\prod_{i \in N^{0}} \left( \hat{p}_{0|z_{i}^{0}}^{i} \right)^{\frac{1}{N_{0}}} \exp \left( \frac{1}{N_{0}} \sum_{i \in N^{0}} \ln \hat{p}_{0|z_{i}^{0}}^{i} \right)}
\]

6.219. Alternatively, the value in the numerator of equation (6.19) is the geometric mean of the period \( t \) price of period \( t \) price-determining characteristics, \( z_{i,k}^{t} \). This is compared, in the denominator, with the geometric mean of the period 0 predicted price of the same period \( t \) price-determining characteristics, \( z_{i,k}^{0} \). For each model, the quantities of characteristics are held constant in period \( t \), \( z_{i,k}^{t} \); only the characteristic prices change.

\[
(6.20) \quad P_{HGMI:z_{t}^{t}}^{0} = \frac{\prod_{i \in N^{t}} \left( \hat{p}_{0|z_{t}^{t}}^{i} \right)^{\frac{1}{N^{t}}} \exp \left( \frac{1}{N^{t}} \sum_{i \in N^{t}} \ln \hat{p}_{0|z_{t}^{t}}^{i} \right)}{\prod_{i \in N^{0}} \left( \hat{p}_{0|z_{i}^{0}}^{i} \right)^{\frac{1}{N^{0}}} \exp \left( \frac{1}{N^{0}} \sum_{i \in N^{0}} \ln \hat{p}_{0|z_{i}^{0}}^{i} \right)}
\]

6.220. As with the characteristics approach, a compromise solution as to whether period 0 or period \( t \) constant characteristics should be used is to apply an average of the two. However, as with the characteristics approach, equation (6.20) has the advantage of only requiring a single hedonic regression to be estimated in the price reference period 0. Should this be used, the regression should be re-estimated every year or so, the frequency being determined by the turnover of products.

6.221. The three approaches have different, yet valid, intuitions. Yet as long as the functional form of the aggregator is aligned to the hedonic regression in the manner shown in Table 6.10 below, the imputation and characteristics approaches yield the same result. This consolidation not only markedly narrows down the choice between approaches, but also validates the measure as one resulting from quite different intuitions.

<table>
<thead>
<tr>
<th>Hedonic regression: functional form</th>
<th>Characteristics approach: form of average of characteristics</th>
<th>Imputation approach: Form of average of predicted prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Arithmetic mean</td>
<td>Arithmetic mean</td>
</tr>
<tr>
<td>Log-linear</td>
<td>Arithmetic mean</td>
<td>Geometric mean</td>
</tr>
<tr>
<td>Log-log</td>
<td>Geometric mean</td>
<td>Geometric mean</td>
</tr>
</tbody>
</table>
6.222. For a log-linear functional form of a hedonic regression, the requirements are that (i) for the characteristics approach, $z_{t,k}$ and $z_{0,k}$ are arithmetic means of characteristic’s values, the right-hand-side (RHS) of the hedonic regression, and (ii) for the imputation approach, the ratio of average predicted prices is a ratio of geometric means, the left-hand-side (LHS).

6.223. The important feature of these hedonic indices is that they require no matching of individual models in the periods compared. Matching is required so that the price of a model in period 0 can be compared with that in period $t$, without a concern that the price change is affected by changes in quality. Such matching restricts the sample and, importantly in this context of a high level of churn in models and where prices change when models change, can lead to bias. This was illustrated using the example in Table 6.9. The price comparison of matched models effectively removes from the sample price changes in the important period of a price comparison when models change. The imputation for November to December for model 1 in Table 6.9 is based on matched prices only. Hedonic indices adjust for quality change not by any painstaking matching and, for that matter, identification of replacements, but by applying a hedonic regression to value constant-quality characteristics.

6.224. Hedonic indices use data on matched and unmatched observations and, again importantly, can naturally be applied to large monthly data sets, such as scanner and web-scraped data, as opposed to a small sample of what may have been in some long-past reference period, a representative variety.

6.225.

6.226. An advantage of the imputation approach over the dummy variable approach is that explicit weighting systems can be more readily, accurately, and intuitively applied at this elementary level. For example, equation (6.20) may be defined for models $i$ over a set of models of television sets sold in period $t$. The formula gives equal weight to each model sold. A major improvement would be to apply to each model’s quality-adjusted price change the weight of that price change, that is, the individual model’s share of transaction expenditure values, say from scanner data, for example. Silver (2018) outlines the methodology for the imputation approach, again in the context of house price indices, to include quasi-superlative and superlative formulations. The weighted imputation approach also has a correspondence to a weighted characteristics approach, and the more intuitive application of weights, if formulated as in Table 6.10.

6.227. A final issue to note is that hedonic indices are particularly well suited for large data sets of models, say web-scraped or scanner data (see chapter 10) for which there is no matching of varieties. It is at initiation that a price collector selects a representative variety and matches its characteristics in subsequent period in order to track the price of this same variety. In doing so the sample of prices collected is highly restricted to what may be a single price. With hedonic indices it is the varying values of the characteristics of the models that enable a constant quality price change. There may be datasets in which accurately matched sampled prices form part of the sampled data. In such a case there would be no need for predicted prices to be used for constant quality price change. The overall measure for this data set would contain: (i) actual price changes for the matched sample; (ii) hedonic price changes for the period 0 models not sold in period $t$ (as, for the hedonic imputation approach, in equation (6.19)); and (iii) hedonic price changes for the period $t$ models not sold in period 0 (as, for the hedonic imputation approach, in equation (6.20)). Each of these terms would be weighted by their relative
expenditure shares, if available. It is from the aforementioned measure of all three components that the difference between the matched models method and hedonic indices becomes apparent.

4. The difference between hedonic indices and matched indices

6.228. An advantage of hedonic indices over matched comparisons was the inclusion by the former of un-matched data. Consider a data set of prices and characteristics over two successive time periods, say periods 0 and \( t \). Assume there are \( m \) matched models in both periods 0 and \( t \), \( o \) old models in period 0, but disappearing thereafter, and \( n \) new models appearing in period \( t \), and subsequently.

<table>
<thead>
<tr>
<th></th>
<th>Period 0</th>
<th>Period ( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matched models ((m))</td>
<td>( m )</td>
<td>( m )</td>
</tr>
<tr>
<td>Old model ((o))</td>
<td>( o )</td>
<td></td>
</tr>
<tr>
<td>New model ((n))</td>
<td></td>
<td>( n )</td>
</tr>
</tbody>
</table>

6.229. A constant-quality, period 0 to \( t \), price index, from a hedonic imputation approach, is made up of three terms:

- *The change in the geometric mean price of the \( m \) matched models*, with no need for quality adjustment because they are matched;

- *The change in the constant-quality geometric mean price of the old models with actual prices in period 0 and counterfactual ones in period \( t \).* The counterfactual constant-quality price in period \( t \) has to be estimated since there is only a price in period 0. A prediction is required of what each old model’s price in period 0 would have been had it been sold in period \( t \). A period \( t \) hedonic regression is estimated and a predicted price estimated for each model by inserting its period 0 characteristic values into the right-hand-side of the estimated regression equation. A geometric mean is compiled of these predicted values, \( \prod_{i=0}^{o} \left( \hat{p}_i^{0} \right)^{\frac{1}{o}} \), and compared with the period 0 geometric mean, \( \prod_{i=0}^{o} \left( p_i^{0} \right)^{\frac{1}{o}} \), as in equation (6.17); and

- *The change in the constant-quality geometric mean price of the new model in period \( t \).* The counterfactual constant-quality price in period 0 has to be estimated since there is only a price in period \( t \). A prediction is required of what each new model’s price in period \( t \) would have been had it been sold in period 0. A period 0 hedonic regression is estimated and a predicted price estimated for each model by inserting its period 0 characteristics values into the right-hand-side of the estimated regression. A geometric mean is compiled of these predicted values, \( \prod_{i=n}^{n} \left( \hat{p}_i^{0} \right)^{\frac{1}{n}} \), and compared with the period \( t \) geometric mean, \( \prod_{i=n}^{n} \left( p_i^{t} \right)^{\frac{1}{n}} \), as in equation (6.18).
6.230. The overall index can be phrased as a weighted average of these three elements with the matched comparison having a weight of \(2Nm/(2Nm+ No+ Nn)\), the old of \(No/(2Nm+ No+ Nn)\), and the new, \(Nn/(2Nm+ No+ Nn)\), though preferably the weights should be expenditure shares rather than the numbers of each model.

6.231. The matched model method effectively ignores the last two elements of the bullet points. This procedure would result in no bias if the imputed quality-adjusted price change of new and old varieties were the same as that for matched models. However, as illustrated in Table 6.9, there may be substantial differences for goods and services where there is a great deal of churn in the models bought and sold. The matched model method might be appropriate if the number of new and number of old models—or their expenditure weights—is small relative to matched models. This would be the case for the hedonic patching of permanently missing model prices outlined above, but not for this context where there is a high and frequent turnover in models.

6.232. Even if the matched model method is used with replacements, something of the dynamic universe of models is brought into the measure, but only insofar as there is a one-on-one variety replacement. Further, hedonic indices employ a consistent basis for the explicit quality adjustment for non-comparable replacements.

6.233. The deficiency of the matched model method against a hedonic index has been shown above in terms of the hedonic imputation approach, though similar considerations apply to a time dummy variable approach. Consider an adjacent period time dummy variable hedonic index of the form of equation (6.15), with the index change captured by the coefficient on the dummy variable for time. A sample of models of washing machines for periods \(t\) and \(t+1\) would have in the regression the (log of) price on the left and price-determining characteristics on the right-hand-side (RHS). On this RHS, a dummy variable would also denote whether the observation is drawn from period \(t\) or \(t+1\). The hedonic regression includes matched, new and old models and the quality adjustment is achieved through the term \(\sum z_{k,j} \ln b_i^t\) in equation (6.15). A matched model measure of price change would again only measure the price change for the more limited sample of matched models, but would not require a quality adjustment. The hedonic dummy variable approach, in its inclusion of unmatched old and new observations will likely differ from a geometric mean of matched prices changes, the extent of any difference depending, in this unweighted formulation, on the proportions of old and new varieties leaving and entering the sample and on the price changes of old and new varieties relative to those of matched ones. If the market for products is one in which old quality-adjusted prices are unusually low while new quality-adjusted prices are unusually high, then the matched index will understate price changes. Different market behavior will lead to different forms of bias, see Annex 3.

The use of the geometric mean

6.234.

L. Summary

6.235. Chapter 6 focused on the treatment of temporarily and permanently missing prices. All temporarily missing prices should be imputed using one of the imputation methods described in the chapter. The imputation of temporarily missing prices is especially important when using
the two-stage (modified) Laspeyres (short-term) formula. Imputations, which are self-correcting, avoid introducing any bias into the index. The carry-forward method should not be used, except for fixed or controlled prices. NSOs should define a period of time during which non-seasonal products can be considered temporarily missing. While this threshold varies from country to country, the most commonly used threshold is three months. If a price continues to be missing in month four, the variety should be considered as permanently missing and a replacement variety should be selected.

6.236. Permanently missing prices require a replacement variety. Quality change refers to changes in the price-determining characteristics when one variety replaces another. If these differences are judged to be comparable (that is, they are deemed to be similar), the price of the old and the new variety can be compared directly and any difference in price is reflected as price change. Should the differences be such that the old and the new variety are deemed to be non-comparable, a quality adjustment is needed. Quality adjustments ensure that the index reflects only pure price change and not changes due to differences in quality. Explicit or direct quality adjustments are preferred. They include quantity adjustment due to changes in size or quantity, changes in option costs, differences in production costs, and hedonics. Quantity adjustments are straightforward, and many countries apply this method for changes in size. The other explicit methods require data and experience making explicit quality adjustments. Implicit or indirect quality adjustments are the second-best approach; however, they could be preferred given a lack of data and expertise required for the explicit methods. Implicit methods include overlap pricing and imputation. Each of the explicit and implicit methods are described in detail in the chapter.

6.237. The rapid turnover in the models of varieties sold of select products (e.g. televisions, computers, telecommunications, appliances, etc) requires a different strategy. Over time, these items usually have a rapid turnover in their quality characteristics. While the matched models method, class-mean imputations, and hedonic price indices may be used, the chapter notes that the matched models methods may lead to significant bias.
Annex 1 Overall mean (or targeted) imputation

6.238. Consider \( i=1...m \) varieties in period \( t \) and \( p_i^t \) is the price of variety \( i \) in period \( t \). All varieties continue into period \( t+1 \) with the exception of the single variety \( m \) which is replaced by variety \( n \). \( p_{n}^{t+1} \) is the price of a replacement variety \( n \) in period \( t+1 \). Now \( n \) replaces \( m \), but is of a different quality. There are \( (m-1) \) matched prices between periods \( t \) and \( t+1 \) and a single replacement price such that \( m=(m-1)+1 \). Let \( A(z) \) be the quality adjustment to \( p_{n}^{t+1} \) which equates its quality services or utility to \( p_{n}^{t+1} \), had it existed, such that the quality-adjusted price \( p_{n}^{t+1}=A(z) \ p_{n}^{t+1} \). For the imputation method to work, the average price change of the \( i=1...,m \) varieties, including the quality-adjusted price \( p_{n}^{t+1} \), given on the left-hand side of equation (A6.1.1), must equal the average price change from just using the overall mean of the rest of the \( i=1...,m \) varieties, on the right-hand side of equation (A6.1.1). The discrepancy or bias from the method is the balancing term \( Q \). It is the implicit adjustment that allows the method to work. The arithmetic formulation is given here is based on Triplett (2006), though a similar geometric one can be readily formulated. The equation for one unavailable variety is given by:

\[
(A6.1.1) \quad \frac{1}{m} \left[ \frac{p_{n}^{t+1}}{p_{m}^{t}} + \sum_{i=1}^{m-1} \frac{p_{i}^{t+1}}{p_{i}^{t}} \right] = \left[ \frac{1}{(m-1)} \sum_{i=1}^{m-1} \frac{p_{i}^{t+1}}{p_{i}^{t}} \right] + Q
\]

\[
Q = \frac{1}{m} \frac{p_{n}^{t+1}}{p_{m}^{t}} - \frac{1}{m(m-1)} \sum_{i=1}^{m-1} \frac{p_{i}^{t+1}}{p_{i}^{t}}
\]

and for \( x \) unavailable varieties by:

\[
(A6.1.2) \quad Q = \frac{1}{m} \sum_{i=m-x+1}^{m} \frac{p_{n}^{t+1}}{p_{m}^{t}} - \frac{x}{m(m-x)} \sum_{i=1}^{m-x} \frac{p_{i}^{t+1}}{p_{i}^{t}}
\]

6.239. The relationships are readily visualized if \( r_{1} \) is defined as the arithmetic mean of price changes of varieties that continue to be recorded and \( r_{2} \) of quality-adjusted unavailable varieties. For the arithmetic case, where

\[
(A6.1.3) \quad r_{1} = \left[ \sum_{i=1}^{m-x} \frac{p_{i}^{t+1}}{p_{i}^{t}} \right] \div (m-x) \quad \text{and} \quad r_{2} = \left[ \sum_{i=m-x+1}^{m} \frac{p_{i}^{t+1}}{p_{i}^{t}} \right] \div x
\]

then the bias of arithmetic mean of ratios from substituting equations (A6.1.3) in (A6.1.2) is:

\[
(A6.1.4) \quad Q = \frac{x}{m} (r_{2} - r_{1})
\]

which equals zero when \( r_{1} = r_{2} \). The bias depends on the ratio of unavailable values and the difference between the mean of price changes for existing varieties and the mean of quality-adjusted replacement price changes. The bias decreases as either \( (x/m) \) or the difference between \( r_{1} \) and \( r_{2} \) decreases. Furthermore, the method is reliant on a comparison between price changes for existing varieties and quality-adjusted price changes for the replacement or unavailable comparison. This is more likely to be justified than a comparison without the quality adjustment to prices. For example, suppose there were \( m = 3 \) varieties, each with a price of 100 in period \( t \). Let the \( t + 1 \) prices be 120 for two varieties, but assume the third, i.e., \( x = 1 \), is unavailable and is replaced by a variety with a price of 140, of which 20 is attributable to quality differences. Then the arithmetic bias as given in equations (A6.1.3) and (A6.1.4), where \( x = 1 \) and \( m = 3 \), is
6.240. Had the bias depended on the unadjusted price of 140 compared with 100, the imputation would be prone to serious error. In this calculation, the direction of the bias is given by \((r_2 - r_1)\) and does not depend on whether quality is improving or deteriorating, in other words whether \(A(z)<1\) or \(A(z)>1\). If \(A(z)<1\), a quality improvement, it is still possible that \(r_2 < r_1\) and for the bias to be negative.

6.241. The analysis here is framed in terms of a short-term price change framework. That is, the short-term price changes between the prices in a period and those in the preceding period are used for the imputation. This is different from the long-term imputation where a base period price is compared with prices in subsequent months, and where the implicit assumptions are more restrictive.

6.242. Table A6.1.1 provides an illustration in which the (mean) price change of varieties that continue to exist, \(r_1\), is allowed to vary for values between 1.00 and 1.5 – corresponding to a variation between no price change and a 50 per cent increase. The (mean) price change of the quality-adjusted new varieties compared with the varieties they are replacing is assumed not to change, i.e., \(r_2 = 1.00\). The bias is given for ratios of missing values of 0.01, 0.05, 0.1, 0.25 and 0.5, both for arithmetic means and geometric means. For example, if 50 per cent of price quotes are missing and the missing quality-adjusted prices do not change, but the prices of existing varieties increase by 5 per cent (\(r_1=1.05\)), then the bias for the geometric mean is represented by the proportional factor 0.9759; i.e., instead of 1.05, the index would be 0.9759 \times 1.05 = 1.0247. For an arithmetic mean, the bias is –0.025; instead of 1.05 it should be 1.025.

6.243. Equation (A6.1.4) shows that the ratio \(x/m\) and the difference between \(r_1\) and \(r_2\) determine the bias. Table A6.1.1 shows that the bias can be quite substantial when \(x/m\) is relatively large. For example, for \(x/m = 0.25\), an inflation rate of 5 per cent for existing varieties translate to an index change of 3.73 per cent and 3.75 per cent for the geometric and arithmetic formulations, respectively, when \(r_2 = 1.00\), i.e., when quality-adjusted prices of unavailable varieties are constant. Instead of being 1.0373 or 1.0375, ignoring the unavailable varieties would give a result of 1.05. Even with 10 per cent missing (\(x/m = 0.1\)), an inflation rate of 5 per cent for existing varieties translate to 4.45 per cent and 4.5 per cent for the geometric and arithmetic formulations, respectively, when \(r_2 = 1.00\). Considering a fairly low ratio of \(x/m\), say 0.05, then even when \(r_2 = 1.00\) and \(r_1 = 1.20\), Table A6.1.1 shows that the corrected rates of inflation should be 18.9 per cent and 19 per cent for the geometric and arithmetic formulations, respectively. In competitive markets, \(r_1\) and \(r_2\) are unlikely to differ by substantial amounts since \(r_2\) is a price comparison between the new variety and the old variety after adjusting for quality differences. If \(r_1\) and \(r_2\) are the same, then there would be no bias from the method even if \(x/m = 0.9\). There may, however, be more sampling error. It should be borne in mind that it is not appropriate to compare bias between the arithmetic and geometric means, at least in the form they take in Table A6.1.1 The latter would have a lower mean, rendering comparisons of bias meaningless.

Table A6.1.1: Example of the bias from implicit quality adjustment when the (mean) price change of quality-adjusted new varieties compared with the varieties they are replacing is assumed not to change (\(r_2=1.00\))
<table>
<thead>
<tr>
<th>$r_1$</th>
<th>0.01</th>
<th>0.05</th>
<th>0.1</th>
<th>0.25</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>0.999901</td>
<td>0.999503</td>
<td>0.999005</td>
<td>0.997516</td>
<td>0.995037</td>
</tr>
<tr>
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<td>0.999802</td>
<td>0.999001</td>
<td>0.998222</td>
<td>0.995062</td>
<td>0.990148</td>
</tr>
<tr>
<td>1.03</td>
<td>0.999704</td>
<td>0.998523</td>
<td>0.997048</td>
<td>0.992638</td>
<td>0.985329</td>
</tr>
<tr>
<td>1.04</td>
<td>0.999608</td>
<td>0.998041</td>
<td>0.996086</td>
<td>0.990243</td>
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</tr>
<tr>
<td>1.05</td>
<td>0.999512</td>
<td>0.997563</td>
<td>0.995133</td>
<td>0.987877</td>
<td>0.9759</td>
</tr>
<tr>
<td>1.1</td>
<td>0.999047</td>
<td>0.995246</td>
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<td>0.976454</td>
<td>0.953463</td>
</tr>
<tr>
<td>1.15</td>
<td>0.998603</td>
<td>0.993036</td>
<td>0.986121</td>
<td>0.965663</td>
<td>0.932505</td>
</tr>
<tr>
<td>1.2</td>
<td>0.998178</td>
<td>0.990925</td>
<td>0.981933</td>
<td>0.955443</td>
<td>0.912871</td>
</tr>
<tr>
<td>1.3</td>
<td>0.99738</td>
<td>0.986967</td>
<td>0.974105</td>
<td>0.936514</td>
<td>0.877058</td>
</tr>
<tr>
<td>1.5</td>
<td>0.995954</td>
<td>0.979931</td>
<td>0.960265</td>
<td>0.903602</td>
<td>0.816497</td>
</tr>
</tbody>
</table>

$r_1 =$ (mean) price change for varieties that continue to exist.

6.244. An awareness of the market conditions relating to the commodities concerned is instructive in understanding likely differences between $r_1$ and $r_2$. The concern here is when prices vary over the life cycle of the varieties. Thus, for example, at the introduction of a new model, the price change may be quite different from price changes of other existing varieties. Thus, assumptions of similar price changes, even with quality adjustment, might be inappropriate. For example, if new computers enter the market at prices equal to, or lower than, prices of previous models, but with greater speed and capability, an assumption that $r_1 = r_2$ could not be justified. Or if new clothing enters the market at relatively high quality-adjusted prices, while old, end-of-season or out-of-style clothes are being discounted. Again, there will be bias, as $r_1$ differs from $r_2$.

6.245. Some of these differences arise because markets are composed of different segments of consumers. Indeed, the very training of consumer marketers involves consideration of developing different market segments and ascribing to each appropriate pricing, product quality, promotion and place (method of distribution) – the 4Ps of the marketing mix as taught in introductory marketing. In addition, consumer marketers are taught to plan the marketing mix for the life cycle of varieties. Such planning allows for different inputs of each of these marketing mix variables at different points in the life cycle. This includes “price skimming” during the period of introduction, when higher prices are charged to skim off the surplus from segments of consumers willing to pay more. The economic theory of price discrimination would also predict such behavior. Thus, the quality-adjusted price change of an old variety compared with a new replacement variety may be higher than price changes of other varieties in the product group. After the introduction of the new variety its prices may fall relative to others in the group. There may be no law of one price change for differentiated varieties within a market.

6.246. There is thus little in economic or marketing theory to support any expectation of similar (quality-adjusted) price changes for new and replacement varieties, as compared to other varieties in the product group. Some knowledge of the realities of the particular market under study would be helpful when considering the suitability of this approach. Two aspects need to be considered in any decision to use the imputation approach. The first is the proportion of replacements; Table A6.1.1 provides guidance here. The second is the expected difference
between \( r_1 \) and \( r_2 \). It is clear from the above discussion that there are markets in which they are unlikely to be similar. This is not to say the method should not be used. It is a simple and expedient approach. What arguably should not happen is that it is used by default, without any prior evaluation of expected price changes and the timing of the switch. Furthermore, its use should be targeted, by selecting varieties expected to have similar price changes. The selection of such varieties, however, should take account of the need to include a sufficiently large sample so that the estimate is not subject to undue sampling error.

6.247. The manner in which these calculations are undertaken is also worth considering. In its simplest form, the pro forma setting for the calculations, say on a spreadsheet, would usually have each variety description and its prices recorded on a monthly basis. The imputed prices of the missing varieties are inserted into the spreadsheet, and are highlighted to show that they are imputed. The need to highlight such prices is, first, because they should not be used in subsequent imputations as if they were actual prices. Second, the inclusion of imputed values may give a false impression of a larger sample size than actually exists. Care should be taken in any audit of the number of prices used in the compilation of the index to code such observations as “imputed”.

6.248. The method described above is an illustration of a short-term imputation. As is discussed in chapter 8, there is a strong case for using short-term imputations as against long-term ones.
Annex 2 Quality adjustment using a replacement and price overlap

6.249. Consider \( p^t_m \) as the price of variety \( m \) in period \( t \), \( p^{t+1}_n \), the price of a replacement variety \( n \) in period \( t + 1 \); \( n \) replaces \( m \), but is of a different quality. Let there be overlap prices for \( m \) and \( n \) in period \( t \) and let \( A(z^{t+1}) \) be the quality adjustment to \( p^{t+1}_n \) which equates its quality to \( p^t_m \) such that the quality-adjusted price \( p^{t+1}_n = A(z^{t+1})p^{t+1}_n \). The index for the variety in question over the period \( t-1 \) to \( t + 1 \) is: Now the quality adjustment to prices in period \( t + 1 \) is defined which is the adjustment to \( p_n \) in period \( t + 1 \) which equates it to \( p^t_m \) in period \( t + 1 \) (had it existed then).

6.250. A desired measure of price changes between periods \( t-1 \) and \( t + 1 \) is thus:

\[
\left( \frac{p^{t+1}_m}{p^t_m} \right)
\]

(A6.2.1)

6.251. The overlap formulation equals this when:

\[
\frac{p^{t+1}_n}{p^t_m} = A(z^{t+1}) \frac{p^{t+1}_n}{p^t_m} = \frac{p^{t+1}_n}{p^t_m} \times \frac{p^t_m}{p^{t+1}_m}
\]

\[
A(z^{t+1}) = \frac{p^t_m}{p^t_n} \text{ and similarly for future periods of the series}
\]

\[
A(z^{t+i}) = \frac{p^{t+i}_n}{p^t_m} \text{ for } i = 2,...,T
\]

6.252. But what if the assumption does not hold? What if the relative prices in period \( t \), \( R^t = p^t_m / p^t_n \), do not equal \( A(z^t) \) in some future period, say \( A(z^{t+i}) = \alpha_i R^t \)? If \( \alpha_i = \alpha \), the comparisons of prices between future successive periods, say between \( t + 3 \) and \( t + 4 \), are unaffected, as would be expected, since variety \( n \) is effectively being compared with itself,

\[
\frac{p^{t+4}_m}{p^{t+3}_m} \frac{p^{t+3}_m}{p^{t+1}_m} = \frac{\alpha R^t}{p^{t+4}_n} \frac{\alpha R^t}{p^{t+3}_n} = \frac{\alpha R^t}{p^{t+2}_n} \frac{\alpha R^t}{p^{t+1}_n}
\]

(A6.2.3)

6.253. However, if differences in the relative prices of the old and replacement varieties vary over time, then:

\[
\frac{p^{t+4}_m}{p^{t+3}_m} \frac{p^{t+3}_m}{p^{t+1}_m} = \frac{\alpha R^t}{p^{t+4}_n} \frac{\alpha R^t}{p^{t+3}_n} = \frac{\alpha R^t}{p^{t+2}_n} \frac{\alpha R^t}{p^{t+1}_n}
\]

(A6.2.4)

6.254. Note that the quality difference here is not related to the technical specifications or resource costs, but to the relative prices consumers pay.
Annex 3: The nature and extent of the index number bias if only matched varieties are used?

6.255. Sample degradation and differences in the (quality-adjusted) prices of unmatched new, unmatched old and matched models can lead to bias in matched models price indices. The nature and extent of such bias depends on the frequency with which manufacturers turn over their models and the pricing strategy retailers employ over the life cycle of the models. If, say, the quality-adjusted prices of new unmatched models in period $t=2$ are higher than their matched counterparts in period 2, and if the quality-adjusted prices of old unmatched models in period 1 are lower than their matched counterparts in period 1, then there will be a larger fall in the matched models index between periods 1 and 2 compared with a hedonic index that uses all of the data. Similarly, if the quality-adjusted prices of unmatched new models, are below matched ones in period 2, and the quality-adjusted prices of old models above matched ones in period 1, there will be a smaller fall in the matched models index compared with a hedonic index that uses all of the data. The nature and extent of the matched models index bias thus depends on the pricing strategy adopted for new and old models. Indeed, if hedonic-adjusted prices are consistently above or consistently below average prices for unmatched new and old models some of the bias will cancel.

6.256. The case for old unmatched models having below average quality-adjusted prices is based on an inventory-clearing argument. For an old model near or at the end of its life cycle retailers want to clear out the remaining inventory from both their warehouses and store shelves so they have room to stock and display the replacement model. They do not wish the old model to cannibalize some of the sales of the new model which may well have a higher price (profit) margin. The extent of any such cannibalization will depend on the cross-price elasticities between the new and old models. This inventory clearing argument is noted in quadrant IV of Figure A6.3.1.

6.257. New and old models may coexist for some time and the case for existing models having their post-entry prices increased following the introduction of a new model is of interest. In principle, our focus is on unmatched old varieties no longer available for the matched price comparison, and hence they have no post-entry prices. However, the logic behind such post-entry pricing applies to a pricing strategy for a multi-product firm that anticipates the introduction of a new model. A multi-product monopolist can increase the prices of existing models because some of the demand for existing models that would usually be lost to competitors, due to the price increase, will now not go to the competitor’s products (an assumption upon which the existing prices were set) but will go instead to the firm’s new model. The new model will cannibalize some of the existing model’s sales that would otherwise be lost due to the price increase in the existing model (Figure A6.3.1, quadrant II). However, it has been argued that any such effect may be outweighed by the need to cut the prices of the existing models to prevent the existing model’s sales cannibalizing sales of the new, more profitable, model (Figure A6.3.1, quadrant IV). Old, branded, pharmaceutical drugs can increase after the expiration of a patent and introduction of new generic models. This is because of price discrimination with some market segments remaining with particularly strong preferences for the old models willing to pay higher prices (Figure A6.3.1, quadrant IV). A study of computer processors and disk drives found that with the introduction of products embodying new technologies the prices for older products decline rapidly to permit the older
technology to compete with the newer one for a limited time, but the old technology is eventually driven out.

6.258. New models may have *above* average quality-adjusted prices in their period of introduction because firms ‘price-skim’ market segments willing to pay a premium for the new model over and above that due to its improved quality (Figure A6.3.1, quadrant I). Indeed, marketing texts advocate price-skimming as one of two ‘new product’ pricing strategies. The alternative strategy is ‘market-penetration’ pricing for which a low initial price is set for a new model to attract a large number of buyers quickly to win market share and take advantage of falling costs due to scale economies. Such pricing may initially be possible because the new model is based on new, lower cost components that can provide a feature set that is comparable to existing models, but at a lower price point. In either event quality-adjusted prices of new models may have *below* average prices (Figure A6.3.1, quadrant III).

6.259. Figure A6.3.1 summarizes these positions. The combination of above-average prices for new models in quadrant I and below-average prices for old models in quadrant III leads to an overall net upwards bias. Similarly pricing in quadrants II and III lead to match models indices which are biased downwards. However, pricing in quadrants I and II lead to an indeterminate bias, with countervailing positive bias from the new above average priced models and negative bias from old average priced models. The bias from pricing in quadrants III and IV is also indeterminate, positive bias from the new above average priced old models and negative bias from the old below average priced models.

6.260. It is possible to say something about the likely pricing strategies of different products. Consider the case of digital cameras compared to film-based cameras. Given the current differential in product costs for the two technologies, and where the two categories are in their respective life-cycles, we can speculate that relatively greater effort is likely to be placed on R&D in new models which reduce unit costs for digital cameras compared to R&D in film-based camera models that reduce unit costs. Products in a mature stage of their category life cycle, where R&D development is relatively small and product enhancing, as opposed to cost reducing, may be more likely to have above average quality-adjusted prices for new models.

6.261. The nature and the extent of bias from using matched models is dictated by the pricing and production strategies of the retailers and manufacturers.

### Figure A6.1.3: Matched models price index bias and pricing strategies

<table>
<thead>
<tr>
<th>Quality-adjusted prices</th>
<th>New unmatched models</th>
<th>Old unmatched models</th>
<th>Pricing</th>
<th>Matched models price index bias</th>
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<tr>
<td><strong>above</strong> matched models prices</td>
<td>Market skimming [quadrant I]</td>
<td>Multiproduct monopoly pricing strategy; price discrimination to segments with sticky downwards pricing; old technologies reduce prices. [quadrant II]</td>
<td>I and IV Upwards</td>
<td>II and III downwards</td>
</tr>
<tr>
<td><strong>below</strong></td>
<td>Market penetration pricing; low unit</td>
<td>Inventory clearing.</td>
<td>I and II</td>
<td>Countervailing</td>
</tr>
<tr>
<td>matched models prices</td>
<td>costs, new producing technology</td>
<td>quadrant III</td>
<td>quadrant IV</td>
<td>III and IV</td>
</tr>
<tr>
<td>-----------------------</td>
<td>-------------------------------</td>
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