IMF Working Paper

Fiscal Implications of Interest Rate Normalization in the United States

by Huixin Bi, Wenyi Shen, and Shu-Chun Susan Yang

IMF Working Papers describe research in progress by the author(s) and are published to elicit comments and to encourage debate. The views expressed in IMF Working Papers are those of the author(s) and do not necessarily represent the views of the IMF, its Executive Board, or IMF management.
IMF Working Paper

Fiscal Affairs Department

Fiscal Implications of Interest Rate Normalization in the United States

Prepared by Huixin Bi, Wenyi Shen, and Shu-Chun Susan Yang*

Authorized for distribution by Catherine Pattillo

May 2019

Abstract

This paper studies the main channels through which interest rate normalization has fiscal implications in the United States. While unexpected inflation reduces the real value of government liabilities, a rising policy rate increases government financing needs because of higher interest payments and lower real bond prices. After an initial decline, the real government debt burden rises even with higher tax revenues in an expansion. Given the current net debt-to-GDP ratio at around 80 percent, interest rate normalization leads to a negligible increase in the sovereign default risk of the U.S. federal government, despite a much higher federal debt-to-GDP ratio than the post-war historical average.

JEL Classification Numbers: E43, E52, E62, E63, H30

Keywords: interest rate normalization, monetary and fiscal policy interaction, fiscal sustainability, non linear DSGE models, New Keynesian model

Author’s E-Mail Address: huixin.bi@kc.frb.org, wenyi.shen@okstate.edu, syang@imf.org.

*We thank Marco Arena, Peter Dohlman, Jean-Marc Fournier, Vitor Gaspar, Klaus Hellwig, Eric Leeper, Erik Lundback, Paulo Medas, Marialuz Moreno Badia, Catherine Pattillo, David Savitski, Ursula Wiriadinata, Jing Zhou, and participants in the FP division seminar and the FAD seminar for helpful comments. The views expressed here are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Kansas City, the Federal Reserve System, the IMF, or IMF policy.
Table of Contents

1. Introduction .............................................................................................................3
2. The Model Setup ....................................................................................................6
3. Calibration and Solution .......................................................................................11
4. Fiscal Limit Distributions .......................................................................................13
   4.1 The Baseline Distribution ..............................................................................14
   4.2 Alternative Distribution: Uncertain Future Fiscal Policies .........................16
5. Fiscal Implications of Rising Policy Rates ..........................................................17
   5.1 An Endogenous Policy Rate Increase: the Baseline Analysis .....................18
   5.2 An Endogenous Policy Rate Increase against a High-Debt Level ...............20
   5.3 Interactions with a More Active Interest Rate Policy ..................................21
   5.4 An Unexpected Acceleration in the Policy Rate Increase .........................22
6. Conclusion ..............................................................................................................23

Appendix A. The Equilibrium System .......................................................................25
Appendix B. The Numerical Solution Method ..........................................................26

Tables
Table 1. Baseline Calibration .....................................................................................30

Figures
Figure 1. Federal Government Debt, Interest Payments, and the Federal Funds Rate ...........................................................................................................31
Figure 2. Mandatory and Discretionary Spending of the Federal Government ..........32
Figure 3. Federal Income Tax Rates .......................................................................32
Figure 4. Fiscal Limit Distributions for the Federal Government .............................33
Figure 5. Responses of an Endogenous Rising Policy Rate to a Positive Investment Efficiency Shock .................................................................34
Figure 6. Federal Funds Rate vs. Real Interest Rates on Treasury Bonds ...............35
Figure 7. Responses of an Endogenous Rising Policy Rate to a Positive Investment Efficiency Shock: Different Monetary Policy Activeness .............36
Figure 8. Responses of an Endogenous Rising Policy Rate to a Positive Investment Efficiency Shock: Different Monetary Policy Activeness .........37
Figure 9. Responses to an Exogenous Monetary Policy Shock ................................38

References ..................................................................................................................39
1 Introduction

After three rounds of quantitative easing since the Great Recession in 2007–09, the U.S. economy is setting the stage for its longest expansion in history.\footnote{See the dates of business cycle expansions and contractions since the mid-1850s announced by \textit{National Bureau of Economic Research} (2018).} The focus of monetary policy has accordingly shifted to policy normalization, which involves raising the federal funds rate (the policy rate) and reducing the Federal Reserve’s holding of longer term securities \cite{Board of Governors of the Federal Reserve System (2018b)}.\footnote{The press release of the Federal Reserve Board on March 20, 2019 announces that the Federal Open Market Committee intends to slow the reduction of its holdings of Treasury securities in May 2019 and conclude the reduction of its aggregate securities holdings at the end of September 2019.} While normalization along both dimensions has fiscal implications, this paper focuses on the implications of interest rate normalization against a backdrop of elevated government debt.\footnote{An emerging literature focuses on fiscal implications of changing the size or portfolio composition of a central bank’s balance sheet. In particular, several papers study remittance transfers from a central bank to the Treasury from the income risk perspective, e.g., Carpenter \textit{et al.} (2015), Christensen \textit{et al.} (2015), Del Negro and Sims (2015) (reverse transfers—or fiscal support—from fiscal authorities to a central bank), and Hall and Reis (2015). See Cavallo \textit{et al.} (2018) for a literature survey.}

Interest rate normalization has direct fiscal implications through higher interest payments and lower real bond prices. A higher policy rate, propagating through financial markets, raises interest rates on government bonds. Figure 1 shows that the federal funds rate seems to play a more important role in government interest payments than government debt levels in the postwar U.S. history. The federal government’s net interest payments as a share of GDP have remained low despite a rapidly rising debt path after 2008. From 2008 to 2017, the average net interest payments were 1.4 percent of GDP, with an average federal debt (held by the public)-to-GDP ratio of 66.2 percent. The interest payments, however, were much higher at 2.4 percent of GDP from 1990 to 2007 with an average federal debt of 44.0 percent of GDP.\footnote{See Figure 1 for data description.} Moving forward, the Congressional Budget Office estimates that interest payments in 2038 would be three times that of the 2017 level (4.2 percent vs. 1.4 percent) even under the slowest debt growth projection,\footnote{Depending on fiscal policy assumptions, the federal debt held by the public is projected to reach 118–165 percent of GDP in 2038 \cite{Congressional Budget Office (2018a,b)}. The one with slowest debt growth projection is the extended baseline, which assumes that current law remains unchanged during the projection period, implying rising revenues as a share of GDP.} resulting from more debt accumulation and higher interest rates.

Despite a projected rising path of the federal debt-to-GDP ratio, U.S. Treasury debt is
generally perceived as risk-free, reflected in persistent low yields. Bohn (2008) concludes that historically U.S. debt satisfies a sufficient sustainability condition, as the primary surpluses respond positively to public debt fluctuations. Even combining expected rising interest rates with existing high debt and low growth, Blanchard and Zettelmeyer (2017) argue that fiscal crises are unlikely unless risky macroeconomic policies are pursued. From a long-term perspective, rising liabilities associated with Social Security, Medicare, and Medicaid are often seen as a threat to debt sustainability (e.g., Hagist and Kotlikoff (2008), Davig et al. (2010), Kotlikoff (2015), and Cao et al. (2018)). Elmendorf and Sheiner (2017) recognize that the federal budget is on an unsustainable path but argue that persistent low interest rates imply that federal debt and public investment should be higher and policy changes to bring federal debt on a sustainable trajectory should be delayed.\(^6\)

To study the fiscal implications of interest rate normalization amid mixed views on federal debt sustainability, we use a New Keynesian (NK) model with sovereign default risk. The model, calibrated to the U.S. economy, features a regime-switching transfer process between a stable and an unstable regime, as in Davig et al. (2010). It captures the most important factor in long-run debt sustainability for the federal government: an upward trend in the mandatory spending-to-GDP share (Figure 2) and uncertainty on transfer policy reform.

To allow for potentially rising sovereign default risk from interest rate normalization, we follow the approach in Bi (2012) to incorporate a fiscal limit distribution—a collection of the maximum sustainable debt—in a DSGE framework and conduct a fully non-linear analysis. The framework requires simulating fiscal limits (or “debt limits”) for the federal government. In this analytical framework, fiscal limits are state-dependent, accounting for the underlying economic structural and future policy uncertainties.\(^7\) As a result, fiscal limits are not represented by a number, but by a state-dependent distribution. Also, sovereign default risk premia arise endogenously as government debt approaches its fiscal limits, because agents

---

\(^6\)Blanchard (2019) also concludes that in an environment of low interest rates and persistent negative interest-growth rate differentials, the fiscal and welfare costs of debt issuance are likely to be low; hence, government debt levels need not be urgently reduced.

\(^7\)Our approach differs from others, including the classic strategic sovereign default approach (e.g., Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008)) and the empirical approach in partial equilibrium frameworks such as Ghosh et al. (2013), Tanner (2013), and Collard et al. (2015).
take into account rising default probabilities when making saving decisions.

The baseline simulation for the U.S. federal government indicates that sovereign default risk is virtually zero if the Treasury debt held by the public is below 100 percent of GDP, as the baseline fiscal limit distribution has a long left tail. The position and the shape of the distribution, however, are subject to great uncertainties, such as the maximum income tax rates that can be imposed and trajectories of government transfers to people.

With the simulated fiscal limit distributions, we first analyze the effects of interest rate normalization as an endogenous response to an economic expansion, as interest rate normalization commenced in December 2015. To analyze the effect of an unexpected acceleration in raising the policy rate, we also simulate the effects for an exogenous policy rate shock.

For the analysis with an endogenous policy rate increase, an economic expansion triggered by a positive macroeconomic shock results in more economic activity, which expands the tax base and hence increases tax revenues. To the extent that inflation is unexpected from a positive macroeconomic shock, nominal government liabilities can be devalued. In spite of these two factors, the simulation finds that government debt as a share of output declines only in the short run. Over the longer horizon, higher interest payments together with lower real bond prices enlarge government financing needs, adding to debt burden. Between interest rate rules of different activeness in combating inflation, we find that an economy with a more active rule has smaller responses to a positive macroeconomic shock in both output and inflation. As a result, an initial smaller inflation response generates smaller debt devaluation effects and less positive output responses throughout the simulation periods, making the government debt-to-output ratio increase more than with a less active rule. For the sensible range of the response magnitudes to inflation, the effect difference in adding debt burden is nonetheless small.

In the analysis with an exogenous policy rate shock, we find that government debt burden as a share of output rises relative to the path without the shock, also because of higher interest payments and lower real bond prices. Unlike an endogenous policy rate increase that has an initial debt devaluation effect, reduced inflation from the positive policy rate shock enhances
the real value of existing nominal liabilities, contributing to the increase of government debt burden.

Overall, our analysis shows that government debt largely increases with interest rate normalization. The rising net debt-to-output ratio around 70–80 percent of GDP, however, does not increase sovereign risk premia because the debt level is distant from the simulated fiscal limits, implying almost zero default probability. The conclusion that a rising policy rate by the magnitude as observed since 2015 poses no risk on debt sustainability has to be interpreted with caution; it is conditional on an environment with moderate inflation, a current debt level relatively far away from fiscal limits, and fiscal expectations of future fiscal adjustment to debt growth. To see how a current debt level and fiscal limits can affect assessment on fiscal sustainability of federal government debt, we also conduct an alternative simulation assuming a current net debt-to-output ratio around 140–150 percent of GDP against a fiscal limit distribution centering at a lower debt level. The left-shifting of the baseline fiscal limit distribution is driven by the assumptions of a lower maximum implementable capital tax rate and transfers staying on a rising path as projected by Congressional Budget Office (2018b) for 20 years. The results suggest that at this higher initial debt range, the default probability increases to around 20 percent and the marginal increase in default probability from an endogenous increase in the policy rate is also more visible but remains small.

Our paper is closely related to Battistini et al. (2019), which studies monetary and fiscal policy interactions in a New Keynesian model with sovereign default risk. Different from our focus on interest rate normalization in an expansion, that study is interested in how constrained monetary policy at the zero lower bound and monetary policy activeness can affect fiscal limit distributions and government spending effects.

2 The Model Setup

We lay out a New Keynesian model with a regime-switching process for government transfers. Different from the existing papers that model sovereign default risk with fiscal limit distributions (e.g., Corsetti et al. (2013), Bi et al. (2013), and Battistini et al. (2019)), our model
includes capital, which allows interest rates to affect saving decisions between investment and government bonds.

A representative household chooses consumption ($c_t$), labor ($n_t$), investment ($i_t$), capital ($k_t$), and one-period nominal government bond ($B_t$) to maximize lifetime discounted utility:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma}}{1-\sigma} - \chi n_t^{1+\varphi} \right),$$

subject to the budget constraint:

$$P_t c_t + P_t i_t + \frac{B_t}{R_t} = (1 - \Delta_t) B_{t-1} + P_t \left[ (1 - \tau^l_t) w_t n_t + (1 - \tau^k_t) (r^k_t k_{t-1} + \Upsilon_t) \right] + P_t z_t,$$

where $P_t$ is the price level of the final goods, $w_t$ is the real wage rate, $r^k_t$ is the real turn to capital, $z_t$ is real government transfers, $\Upsilon_t$ is the real profits of the monopolistic competitive intermediate goods firms, and $\tau^l_t$ and $\tau^k_t$ are the tax rates on labor and capital income. At $t - 1$, the government sells $B_{t-1}$ units of nominal bonds at a price of $\frac{1}{R_{t-1}}$. At $t$, if the government does not default ($\Delta_t = 0$), it pays $B_{t-1}$ dollars; if the government defaults ($\Delta_t = \Delta > 0$), it only pays $(1 - \Delta) B_{t-1}$ of liabilities.

The law of motion for capital is

$$k_t = (1 - \delta) k_{t-1} + \nu_t \left[ i_t - \frac{\kappa}{2} \left( \frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1} \right],$$

where $\delta$ is the capital depreciation rate, $\kappa$ is the investment adjustment cost parameter, and $\nu_t$ is the investment efficiency shock that follows an AR(1) process:

$$\ln \frac{\nu_t}{\nu} = \rho \ln \frac{\nu_{t-1}}{\nu} + \varepsilon'_\nu,$$

where a variable without a time subscript indicates its steady-state value, and $\varepsilon'_\nu \sim N(0, \sigma^2_{\nu})$ is the investment efficiency shock (also called “investment-specific technology shock”) in the style of Greenwood et al. (1988). Investment efficiency shocks have been shown to be important for business cycle fluctuations (Greenwood et al. (2000) and Justiniano et al.
In the model, the investment efficiency shock generates business cycle fluctuations, which induce endogenous monetary policy rate changes.\(^8\)

The representative competitive final goods producer produces \(y_t\), using \(y_t(i)\) units of each intermediate goods \(i\) with the technology:

\[
y_t = \left[ \int_0^1 y_t(i) \frac{i}{\theta} \, di \right]^{\theta/\theta - 1}.
\]  

The final goods producer’s profit maximization yields the demand function for each intermediate good \(i\):

\[
y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} y_t,
\]  

where \(P_t(i)\) is the price for \(y_t(i)\).

Intermediate goods are produced by monopolistically competitive firms. Following Rotemberg (1982), each intermediate goods-producing firm \(i \in [0, 1]\) faces a quadratic cost to change its nominal price. Each period, the intermediate goods firm \(i\) chooses \(n_t(i), k_t(i)\), and \(P_t(i)\) to maximize its discounted total profit in units of current marginal utility for consumption, \(\lambda_t\):

\[
\max_{n_t(i), k_t(i), P_t(i)} E \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} y_t - w_t n_t(i) - \frac{k_t(i)}{\theta} n_t(i) - \frac{\psi}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 y_t \right],
\]  

subject to the demand function, (6), and the production function

\[
y_t(i) = a_t \left[ k_{t-1}(i) \right]^\alpha n_t(i)^{1-\alpha}.
\]  

---

\(^8\)In reality, business cycle fluctuations are caused by various structural and policy shocks. To minimize the number of state variables in solving the nonlinear model, we rely on a single macroeconomic shock to generate business cycles. One commonly used shock to explain business cycle fluctuation is the technology shock that affects total factor productivity. Since a positive technology shock generates an output increase with a price decline (due to an increase in goods supply), we do not use the technology shock to generate a boom as the endogenous monetary policy response would lower the policy rate.
transfers, and liabilities. The government’s flow budget constraint is

\[
\frac{B_t}{R_t} + P_t \left[ \tau_t w_t n_t + \tau^k_t (r^k_t k_{t-1} + \Upsilon_t) \right] = (1 - \Delta_t) B_{t-1} + P_t g_t + P_t z_t,
\]

(9)

where \( \Upsilon_t = y_t - mc_t y_t - \frac{\psi}{\pi} \left( \frac{\pi}{\pi} - 1 \right)^2 y_t \), and \( mc_t \) is the real marginal cost of intermediate goods production.

Following Bi (2012), a realized effective fiscal limit in real debt value, \( b_{t}^{\text{max}} \), is drawn from a fiscal limit distribution each period. If the government’s real debt liabilities at the end of \( t-1 \) (\( b_{t-1} = \frac{B_{t-1}}{R_{t-1}} \)) are less than \( b_{t}^{\text{max}} \), it fully repays its debt; otherwise, it defaults a fixed fraction of its liabilities. Specifically,

\[
\Delta_t = \begin{cases} 
0 & \text{if } b_{t-1} < b_{t}^{\text{max}}; \\
\Delta & \text{if } b_{t-1} \geq b_{t}^{\text{max}}.
\end{cases}
\]

(10)

The simulation of fiscal limit distributions is based on the government debt evaluation equation with some maximum tax rates imposed for computing the discounted sum of future primary surpluses (see Section 4 for details). In this framework, the debt threshold that triggers a default is uncertain. The uncertainty reflects complicated factors influencing sovereign default decisions in reality, such as institutional and policy making quality omitted here.\(^9\)

Although sovereign default is stochastic, default probabilities increase nonlinearly when government debt burden escalates, as observed in reality.

Following Davig et al. (2010), we assume that transfer policy fluctuates between a stable and an unstable regime:

\[
\zeta_t(i^z_t) = \begin{cases} 
(1 - \rho_z)z + \rho_z \zeta_{t-1} + \varepsilon^z_t, & \text{if } i^z_t = 1, \quad \rho_z < 1, \\
\mu \zeta_{t-1} + \varepsilon^z_t, & \text{if } i^z_t = 2, \quad \mu > 1.
\end{cases}
\]

(11)

\(^9\)Using probit regressions, Kraay and Nehru (2006) find that policy and institution quality is important for sovereign debt stress, aside from debt burden and economic growth shocks. Using a two-period model with strategic default, Qian (2012) shows institution setting in terms of the degree of government polarization is important in explaining sovereign default probabilities.
where \( \varepsilon_t^z \sim N(0, \sigma^2_z) \) is the transfer shock, and the regime index \( i_t^z \) evolves according to the transition matrix

\[
\begin{pmatrix}
    p^z_1 & 1 - p^z_1 \\
    1 - p^z_2 & p^z_2
\end{pmatrix},
\]

where \( p^z_1 \) (\( p^z_2 \)) is the probability of continuing to stay in the stable (unstable) regime each period, calibrated to be highly persistent. This modeling approach intends to capture the uncertainty in the timing of transfer policy reform.\(^{10}\)

We assume that income tax rates are the fiscal adjustment instruments used to stabilize debt:

\[
\tau^l_t = \tau^l + \gamma^l (b_{t-1} - b); \quad \tau^k_t = \tau^k + \gamma^k (b_{t-1} - b).
\]

Although government purchases, \( g_t \), can also serve as an adjustment instrument (e.g., in Bi et al. (2013)), we assume \( g_t = g \forall t \) for simplicity.\(^{11}\)

The central bank adjusts its policy interest rate to stabilize inflation. Following Bi et al. (2018), we distinguish between the nominal interest rate for risky government debt (\( R_t^f \)) and the monetary policy interest rate for risk-free debt (\( R_t^f \)). For simplicity, we assume that the risk-free debt in zero net supply.

\[
R_t^f = \max \left( R_{t-1}^f \right)^{R_t^f} \left[ R_t^f \left( \frac{\pi_t}{\pi} \right)^{\alpha_{\pi}} \left[ 1 - \rho R_t^f \varepsilon_t R_t^f, 1 \right] \right],
\]

where \( \pi_t = \frac{p_t}{R_{t-1}} \) is the inflation rate of final goods, and \( \alpha_{\pi} > 1 \) signals an active monetary policy to combat inflation, following Leeper (1991). As \( \alpha_{\pi} \) increases, the central bank responds more aggressively to stabilize inflation. In the quantitative analysis, we consider interest rate normalization either as an endogenous response to an economic expansion, or as an unexpected acceleration in raising the policy rate through the term of \( \varepsilon_t R_t^f \).

When making saving decisions on holding risky government debt, the household accounts

\(^{10}\)Social Security reform is an on-going policy agenda. The cost of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance (OASDI) is expected to exceed total income starting 2018, and the dollar level of the trust fund reserves will decline until reserves become depleted in 2022-2034 (The Board of Trustees, Federal OASDI Trust Funds (2018)).

\(^{11}\)In our model, \( g_t \) maps to discretionary spending and is stabilized as a share of GDP after mid-2020s. See the dashed line in Figure 2. If \( g_t \) is considered as a fiscal adjustment tool, then a distinction between current and capital spending can be important as reduced capital spending affects public capital accumulation, which has a negative effect on private productivity.
for default risk, and the optimality condition is

\[ \frac{1}{R_t} = \beta_t E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{(1 - \Delta_{t+1})}{\pi_{t+1}}, \]  

(15)

compared to the optimality condition for risk-free debt:

\[ \frac{1}{R_f t} = \beta_t E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}}. \]  

(16)

Equation (15) implies that, when default probabilities rise, households demand a higher return to hold government debt. Equations (15) and (16) also show that the two interest rates move together: As monetary authorities raise the policy rate, the no-arbitrage conditions imply that the return on government debt also increases to induce households to hold risky government debt.

Lastly, the aggregate resource constraint is

\[ y_t = c_t + g_t + \psi \left( \frac{\pi_t}{\pi} - 1 \right)^2 y_t. \]  

(17)

### 3 Calibration and Solution

We calibrate the model at the quarterly frequency. Table 1 summarizes the calibration and the steady-state values of the fiscal variables. We adopt the standard values in the literature. The capital depreciation rate is \( \delta = 0.025 \), and the capital income share is \( \alpha = 0.36 \). Taking the mean estimates in Smets and Wouters (2007), we calibrate the inverse of the intertemporal elasticity of substitution for consumption at \( \sigma = 1.38 \) and the inverse of the Frisch labor elasticity at \( \phi = 1.83 \). For the capital adjustment costs, we set \( \kappa = 1.7 \), as in Gourio (2012). To calibrate the market power of intermediate goods producing firms, we set \( \theta = 7.67 \), implying a markup of 15 percent, in line with the estimates for U.S. firms of 5–15 percent (Basu and Fernald (1995)). The degree of price stickiness is assumed to be one year (close to the Smets and Wouters’ (2007) estimate for the probability that firms can
optimally choose prices), implying $\psi = 78.2$.\(^\text{12}\) The steady-state inflation rate is assumed to be $\pi = 1$ for simplicity.

To calibrate the steady-state fiscal variables, we use the average values from 1970 to 2017 in the Historical Tables published by Office of Management and Budget (2018). Given the importance of debt servicing costs in the model, we calibrate the government interest payments as a share of output to be 0.02, which matches the 1970–2017 average of net interest outlays to GDP (Table 8.4). The discretionary outlays-to-output ratio ($\frac{z}{y}$) and the mandatory outlays-to-output ratio ($\frac{\xi}{y}$) are 0.08 and 0.11 (Table 8.4).\(^\text{13}\) To determine the real interest rate, we use the 1970–2007 average of the real long-term interest rate for the U.S. as constructed by Jordá et al. (2017) (long-term nominal interest less CPI inflation): $\frac{R_{f}}{\pi} = 1.008$, which implies the quarterly discount factor $\beta = 0.992$. Since there is no default risk in the steady state, $\Delta_t = 0$ and $R = R_f$ from (15) and (16). Given these calibrations, the government budget constraint implies that the steady-state debt-to-annual output is 0.605.

To calibrate the steady-state capital and labor income tax rates, we use NIPA data with Jones’s (2002) method for constructing average income tax rates. We set $\tau^l = 0.203$ and $\tau^k = 0.214$, the 1970–2017 average of the constructed series.\(^\text{14}\) For the response of income tax rates to debt, we set $\gamma^l = 0.02$ and $\gamma^k = 0.005$ for the baseline calibration. Since an increase in the capital income tax rate is more distorting than in the labor income tax rate, we assume that the government has labor income taxes bear most adjustments. The magnitudes of total adjustments are kept small, just sufficient to satisfy the transversality condition for government debt.

We follow Bi et al. (2016) to set $\rho_z = 0.96$, $\mu = 1.006$, $p_1^z = 0.9944$, and $p_2^z = 0.9875$ in the transfer process. From a long-run perspective, federal mandatory spending-to-GDP

\(^\text{12}\)See Ascari and Rossi (2012) for the equivalence of the first-order condition on the NK Phillips curve for the Rotemberg and Calvo specifications on price stickiness.

\(^\text{13}\)Federal discretionary outlays include national defense and non-defense outlays but exclude net interest payments. Federal mandatory outlays mainly include spending on Social Security, Medicare, Medicaid, and Children’s Health Insurance Program.

\(^\text{14}\)Jones (2002) computes capital and labor income tax rates for all government levels. We apply the method for federal income taxes only. The main difference between the two tax rates is that federal capital income taxes do not have property taxes. The data of National Income and Product Account (NIPA) used for the calculation include: compensation of employees (NIPA Table 1.12, line 2), wages and salaries (NIPA Table 1.12, line 3), proprietors’ income with inventory valuation adjustment and capital consumption adjustment (NIPA 1.12, line 9), rental income of persons with capital consumption adjustment (NIPA Table 1.12, line 13), net interest and miscellaneous payments (NIPA Table 1.12, line 18), federal personal current taxes (NIPA Table 3.2, line 3), contributions for government social insurance (NIPA Table 3.2, line 10), and taxes on corporate income (NIPA Table 3.2, line 8).
ratio largely has an upward trend from the early 1960s, but the trend experienced different growth periods, during 1965 to 1985, 2000 to 2009, and 2018 to 2038 (see Figure 2). The calibration of \( p_2^z = 0.9875 \) gives an average length of an unstable regime of 20 years, and a sufficiently high enough \( p_1^z \) is required to maintain the stationarity of the equilibrium system.

Our default scheme assumes a constant haircut rate, \( \Delta \). Without default experience for the U.S. federal government, we use the haircut rate estimated from the emerging market economies. Bi (2012) uses the estimated haircut rates of sovereign debt restructurings in emerging market economies between 1998 and 2005 from Sturzenegger and Zettelmeyer (2008), and calculates that 90 percent of the annual haircut rates (as a share of all sovereign debt) fall below 0.3. Thus, we assume a constant annual haircut rate of 0.28, implying a quarterly rate of \( \Delta = 0.07 \).

Based on the common range for the response to inflation in a Taylor rule, we choose the baseline to be \( \alpha_\pi = 1.8 \).\(^{15}\) We also explore an alternative, more active rule, \( \alpha_\pi = 2.5 \), for comparison. For investment efficiency, we assume that its shock persistence parameter is \( \rho_\nu = 0.9 \) to generate a somewhat persistent expansion. When interest rate normalization is an unexpected acceleration in raising the policy rate, the interest rate persistence parameter is set to be \( \rho^{Rf} = 0.8 \).\(^{16}\)

We focus on a symmetric equilibrium in which all intermediate goods producers make the same decisions, so that \( y_t(i) = y_t \), \( n_t(i) = n_t \), \( k_t(i) \), and \( P_t(i) = P_t \). Appendix A lists equations that characterize the equilibrium system. We use the monotone mapping method of Coleman (1990) and Davig (2004) to obtain a fully-nonlinear solution. Appendix B describes the numerical solution method.

4 Fiscal Limit Distributions

Fiscal limits are defined as the expected sum of the discounted maximum primary fiscal surplus over an infinite horizon. By iterating (9) forward, imposing the tranversality conditions

\(^{15}\)Smets and Wouters (2007) fit an NK model to the U.S. data for 1966–2004 and obtain the 90-percent posterior range for \( \alpha_\pi \) of 1.7–2.3.

\(^{16}\)To minimize the state space in the solution method, when interest rate normalization is an endogenous response to an economic expansion, we set \( \rho^{Rf} = 0 \).
for government debt, and assuming no default at \( t \) (\( \Delta_t = 0 \)), we obtain the equilibrium debt valuation equation of government real debt liabilities at \( t \).

\[
\frac{b_{t-1}}{\pi_t} = \sum_{i=0}^{\infty} \beta_i^t E_t \left[ \frac{\lambda_{t+i}}{\lambda_t} \left( tax_{t+i} - g_{t+i} - z_{t+i} \right) \right],
\]

(18)

where \( b_{t-1} = \frac{B_{t-1}}{\pi_{t-1}} \) and \( \frac{b_{t-1}}{\pi_t} = \frac{B_{t-1}}{\pi_t} \) (the real value of government nominal liability at \( t \)). Fiscal limits are simulated based on (18), but all the variables are computed under \( \tau_{t+i}^l = \tau_{t+i}^{l,\text{max}} \) and \( \tau_{t+i}^k = \tau_{t+i}^{k,\text{max}} \), the maximum labor and capital income tax rates a government is willing and able to impose. Let the superscript “max” indicate a variable’s value computed under \( \tau_{t+i}^{l,\text{max}} \) and \( \tau_{t+i}^{k,\text{max}} \). Specifically, conditional on an initial state, \( S_t = \{\nu_t, k_{t-1}, z_t, i_t^z\} \), a fiscal limit distribution is

\[
\frac{b(S_t)}{\pi_t^{\text{max}}(S_t)} \sim \sum_{i=0}^{\infty} \beta_i^t E_t \left\{ \frac{\lambda_{t+i}^{\text{max}}(S_{t+i})}{\lambda_t^{\text{max}}(S_t)} \left[ tax_{t+i}^{\text{max}}(S_{t+i}) - g_{t+i}(S_{t+i}) - z_{t+i}(S_{t+i}) \right] \right\}.
\]

(19)

Equation (19) makes explicit the factors important for the maximum sustainable debt level at the beginning of \( t \), which include inflation, the stochastic discount factor \( (\beta_i^t E_t \frac{\lambda_{t+i}}{\lambda_t}) \), and expected fiscal primary surplus at the maximum tax rates. Ceteris paribus, 1) higher inflation (enhancing the debt devaluation effect on existing nominal liabilities), 2) higher expected stochastic discount factors (implying lower expected real interest rates, see (16)), or 3) higher future surplus, all contribute to higher fiscal limits.

4.1 The Baseline Distribution

To simulate fiscal limit distributions for the U.S. federal government, we assume \( g_{t+i} = g \) \( \forall i \). Federal discretionary spending (as a share of GDP) has a downward trend in postwar data and is projected to stabilize in the future (see Figure 2). Given a relatively low level of discretionary spending, only limited room exists for further reduction in raising future primary surplus. Thus, we also assume constant government purchases in the future. On the other hand, rising transfers as a share of GDP are the most alarming factor for federal debt sustainability. Conditional on a transfer regime index \( (i_{t+i}^z) \) and a transfer shock \( (\varepsilon_{t+i}^z) \)
each period, transfers evolve according to (11) and (12) in simulating fiscal limits.

To calibrate the maximum tax rates, we resort to historical data and set \( \tau^l_{\text{max}} = 0.378 \) and \( \tau^k_{\text{max}} = 0.256 \).\(^{17}\) The maximum labor income tax rate is the maximum average marginal federal tax rates on individual income plus Social Security payroll tax rates from 1912 to 2006, constructed by Barro and Redlick (2011). As pointed out, Barro and Redlick’s tax rates are weighted by income that is closer to the concept of labor income.\(^{18}\)

For the maximum capital income tax rate, we do not find a comparable series in the literature. Instead, we construct a series for the federal average tax rates on capital income (described in Section 2.2). Figure 3 shows that the federal capital income tax rate has two regimes: a downward trend from 1950 to 1979 and fluctuating between 0.150 and 0.256 from 1980 to 2018.\(^{19}\)

With the recent tax reform (the Tax Cuts and Jobs Act), which lowered the statutory corporate income tax rate substantially from 0.35 to 0.21—the lowest level in postwar history, the average capital income tax rate has dropped substantially, from 0.19 in 2017 to 0.15 in 2018. Future federal tax rates on capital income are likely to stay low and be politically difficult to return to the high levels before 1980. Hence, we set \( \tau^k_{\text{max}} = 0.256 \), the upper bound of the average federal tax rate on capital income since 1980.

With the above assumptions for the future fiscal variables, we simulate the baseline distribution conditional on a beginning state, \( S_t \), at the steady state. After period \( t \), the investment efficiency shock and transfers follow the stochastic processes in (4), (11), and (12). Appendix B.1 describes the procedures to simulate fiscal limit distributions. The solid line in Figure 4 plots the cumulative density function of the baseline fiscal limit distribution for the federal government. It shows that default probabilities are almost zero for debt

---

\(^{17}\)Instead of examining historical tax rates, the original approach proposed in Bi (2012) is to impose the peak of the model-implied Laffer curve. When simulating fiscal limits for a group of advanced economies and emerging market economies, Bi (2017) imposes the peak of Laffer curve, ranging from 0.5 to 0.7. For the U.S., this would imply that its government debt is risk free at levels below 350 percent of GDP. Our approach is similar to Collard et al. (2015); one of their methods is to use the maximum historical primary surplus to compute a country’s maximum sustainable debt.

\(^{18}\)Individual income taxes also apply to capital income, though the average share of capital income in individual income from 1948Q1 to 2018Q1 is only about 16 percent. See footnote 14 for the data used in this calculation, and we assume that half of the proprietors’ income is attributable to capital. Other individual capital income includes rental income and interest income.

\(^{19}\)Figure 3 also compares average federal income tax rates (based on capital income and labor income taxes) with the average marginal income tax rates we use to calibrate \( \tau^l_{\text{max}} \) and the average income tax rates constructed using Jones’s (2002) method. As expected, the average marginal rates are higher but the patterns are similar to the average income tax rates.
below 100 percent of steady-state annual output. Although the probability rises with the
debt level, it does not rise significantly until debt exceeds 200 percent of output. While the
current federal debt held by the public—around 80 percent of GDP—is high for postwar
history, it remains almost risk free as judged by the baseline fiscal limit distribution.

4.2 Alternative Distribution: Uncertain Future Fiscal Policies

Expected future fiscal policies play a key role in determining the level and the shape of fiscal
limit distributions, which in turn affects the assessment of debt sustainability. There are large
uncertainties surrounding future fiscal parameters and unobserved maximum implementable
tax rates for the U.S., a country without default experience at the federal level. Therefore,
we simulate an alternative distribution with lower maximum average tax rates and transfer
trajectories without reforms for the next 20 years. The dashed line in Figure 4 presents this
alternative distribution.

The alternative distribution represents a less optimistic view for the future fiscal policy
relative to the baseline: the federal government would encounter resistance in raising income
tax rates and reforming social security or health care programs. It assumes that maximum
tax rates on capital and labor income are 0.240 and 0.292 (compared to 0.256 and 0.378
in the baseline). To set the alternative $\tau^{k,\text{max}}$, we conjecture that tax competition motive
would set an upper-bound for the U.S. capital income tax rates. The Congressional Budget
Office (2017) estimates that, among the G-7 excluding U.S., the country with the highest
average corporate income tax rate is Japan at 0.28. This arguably sets an upper bound for
the U.S. federal average capital tax rates as concern on tax competition to attract investment
and profit allocation may prevent the government from adopting higher capital income rates.
After subtracting an average 0.04 state and local corporate tax rate, this implies a maximum
average federal rate of 0.24. For the maximum labor tax rate, we adopt the mean of the

For transfer policy in the alternative distribution, we assume that transfers follow the
extended baseline projection from 2018 to 2038 by the Congressional Budget Office (2018a),
which assumes no changes in the current law, as shown in Figure 2. After twenty years, transfers revert to the regime-switching process as in (11) and (12).

Compared to the baseline, the alternative distribution is narrower—driven by an extended period of a rising path of transfer-to-output ratios, and has much lower fiscal limits—driven by lower maximum income tax rates. The default probability starts rising quickly when federal debt held by the public exceeds 120 percent of output. Both lower maximum tax rates and higher transfer spending reduce expected future surplus, increasing the probability that debt is unsustainable at lower levels. The baseline and the alternative distributions illustrate that for a given debt level, assessment of debt sustainability crucially depends on the expected future fiscal policies.

5 Fiscal Implications of Rising Policy Rates

We approach our main analysis—fiscal implications of interest rate normalization—from two perspectives. The first one is to analyze an endogenous response of the policy rate to an economic expansion, driven by a positive macroeconomic shock. The second one is to analyze an unexpected, exogenous acceleration of raising the policy rate, driven by a positive monetary policy shock.

To proceed with the first analysis, we begin by simulating a fiscal state at $t = 0$ using the transfer process (11), and select those with the government debt level on par with the current federal debt level held by the public. Then a positive macroeconomic shock—an investment efficiency shock—is used to generate an economic expansion at $t = 0$. Specifically, the economy starts from $t = -161$ at the steady state with the debt-to-annual output ratio at 0.605. It is subject to the stochastic processes of (4) for investment efficiency and (11) and (12) for transfers. We perform 20,000 simulations. Since the current net federal debt level is around 77 percent of GDP (at the end of 2017), we retain 2,690 simulations that have a debt-to-annual steady state output ratio between 70–80 percent at $t = 0$. Next, the economy is injected with a 1 percent investment efficiency shock (the macroeconomic shock in the model) at $t = 0$. When computing impulse responses we take the differences
between the path with and without the macroeconomic shock for each variable and for all 2,690 simulations.

5.1 An Endogenous Policy Rate Increase: the Baseline Analysis

Figure 5 plots the impulse responses to a 1 percent investment efficiency shock. With an AR(1) coefficient of $\rho_\nu = 0.9$, this has a somewhat persistent effect on output, generating an expansion over seven years. (The x-axis in Figure 5 is in quarters.) The shock drives up inflation by 0.125 percent (or 0.5 percentage points at an annualized rate) on impact, and the policy rate increase averages to about 130 basis points in the annualized rate each year for the first five years. Although our analysis is monetary policy responses to a one-time macro shock, the cumulative magnitude of policy rate increases is comparable to those in an expansion cycle observed in recent history.  

20 We use the fiscal limit distribution from the baseline case when solving the full-nonlinear model.21 The responses for the macroeconomic variables are the median differences in percent of steady-state values; tax rates and fiscal variables in shares of output are the median level differences in percentage points; and interest rates are the median level differences in basis points.

The positive investment efficiency shock increases inflation, leading the central bank to raise the policy rate. The unexpected increase in inflation at $t = 0$ reduces the real value of existing debt ($B_{t+1}/P_t = \frac{b_{t+1}}{\pi_t}$), and real interest payments ($B_{t+1}P_t - B_{t+1}R_{t-1}P_{t-1} = \frac{b_{t+1}}{\pi_t} - \frac{b_{t+1}}{R_{t-1}}$). As shown in Figure 5, the government debt as a share of output falls initially relative to the path without the endogenous policy rate increase, both because of the debt devaluation effect and higher output from the positive investment efficiency shock.

Although the debt devaluation effect works to lower government financing needs, a rising policy rate increases the interest rate on government debt through the no-arbitrage conditions

---

20 In the last expansion, the federal funds rate was on an upward cycle from June 2004 to September 2007, and the total increase was 125 basis points, roughly 140 basis points per year. In the current expansion, the Federal Reserve raised the federal funds rate by 50 basis points in 2016, 75 basis points in 2017, and 100 basis points in 2018.

21 We could use the fiscal limit distribution simulated from the state with the same macroeconomic shock that triggers the increase in the policy rate. The conditional distribution on an initial investment efficiency shock almost overlaps with the baseline distribution conditional on an initial state at the steady state. The reason is that the positive initial shock on the one hand increases tax revenues and, on the other hand, drives up the real interest rate, reducing the discount factor. The two effects roughly cancel, making the fiscal limit distribution move little.
of (15) and (16), driving up real interest payments after the initial decline from higher inflation. Also, a higher interest rate decreases the real bond price (the inverse of the real interest rate, $\frac{1}{r_t} = \frac{1}{r_t/E_t\pi_{t+1}}$), which increases the real cost to roll over existing debt and to issue new debt. Real debt liabilities decline initially, but government debt as a share of output largely increases in later periods, despite more tax revenues from increased labor and capital income due to the positive macroeconomic shock.

In addition to these direct consequences, rising government debt has indirect effects through expected higher future income tax rates to stabilize debt growth. Expecting a higher capital tax rate offsets some of the positive current investment responses. Expecting a higher labor tax rate, on the other hand, amplifies the positive labor response to the macroeconomic shock because households work harder to smooth future consumption loss from higher taxes. The effects from fiscal adjustments are small as our calibration assumes very small fiscal adjustment speeds.\(^{22}\)

Among the channels analyzed above, higher interest rates—affecting both debt servicing costs and real bond prices—are the main cause for driving up government debt burden, and has been a focus for discussion on monetary policy normalization. Beck and Wieland (2017) argue that raising policy rates together with winding down sovereign debt purchases by the European Central Bank can decrease bond prices quickly due to higher interest rates demanded by investors.\(^{23}\) Also, Faria-e Castro and Bharadwaj (2018) point out that increases in the federal funds rate would directly affect the interest rates of short-term Treasuries. Figure 6 plots the federal funds rate and the real rates for 1-year and 5-year Treasury bonds; it confirms that the real rate of short-term Treasury bonds closely follow the federal funds rate, in particular for the 1-year bond. As the majority (about 70 percent) of federal government debt has a maturity below 5 years,\(^{24}\) this suggests that government debt servicing costs would increase without much delay, as shown by the beginning of an

---

\(^{22}\)Given a progressive income tax system in the U.S., an economic boom should induce an automatic increase in the tax rates (not modeled here), which could dampen some output and revenue increases in an economic expansion, making the government debt rise slightly more than what we simulate here.

\(^{23}\)Beck and Wieland (2017) do not quantify the effects of monetary policy normalization; they mainly discuss the strategies of policy normalization in the euro area and various concerns, including on public finance sustainability arising from monetary policy normalization.

\(^{24}\)Department of the Treasury (2018) reports that the average share of Treasury debt with maturity below 5 years to total debt held by the public is 0.694 for the first quarter of 2018.
upward trend of interest payments in 2016 (see the bottom plot of Figure 1).

A caveat related to short-term debt is that our model only futures a one-period government bond. In reality, with a debt maturity structure, the reaction of real bond prices to a rising policy rate is likely to be slower than what we have presented here. Future research can incorporate both short- and long-term bonds in the model to better capture real bond price dynamics to a rising policy rate.

5.2 An Endogenous Policy Rate Increase Against a High-Debt Level

The above analysis shows that interest rate normalization increases government debt burden except in the very short run. Sovereign default probabilities, however, are unlikely to rise, because the current debt level (77 percent of GDP) is far from the simulated fiscal limits judged by all distributions simulated in Section 4. To see how the debt sustainability assessment can change if the economy is at a much higher level, we conduct an alternative simulation assuming that the current debt level is at 140–150 percent of steady-state annual output. Moreover, we assume that the relevant fiscal limit distribution is the alternative distribution with lower maximum tax rates and a longer period of transfers staying in an unstable regime (the dashed line in Figure 4). The dashed lines in Figure 7 plot the median responses in this high-debt scenario.25

In the baseline where the debt level is around 70–80 percent of output (solid lines), both the level and change of default probabilities due to the endogenous policy rate increase is virtually zero. When government debt rises to 140–150 percent of output, the default probability increases to about 25 percent at the peak. The marginal probability increase due to the rising policy rate is also more visible but remains small, at about 1 percentage point at the peak. Higher default probabilities make forward-looking households demand higher risk premia on sovereign debt. The increase in the interest payment-to-output ratio is higher and also more persistent than the scenario with a debt level of 70–80 percent of output. Because of higher interest payments and lower real bond prices, the debt-to-annual output

25 Among the 20,000 simulations, 3.2 percent or 640 simulations have debt falling into the range of 140–150 percent of annual output.
ratio increases by 1.78 percentage points after five years, compared to only 0.05 percentage points in the baseline.

Against a higher debt level, the responses of key macroeconomic variables to a positive investment efficiency shock remain qualitatively similar. Output, labor, and investment all rise relative to the paths without the investment efficiency shock, but the magnitudes are smaller than those in the baseline with a lower debt level. A higher debt level induces more fiscal adjustments in terms of higher current and expected future income tax rates, which have more negative effects on current investment and labor.

5.3 Interactions with a More Active Interest Rate Policy

We now turn to the role of interest policy activeness on the fiscal implications of interest rate normalization. Relative to the baseline analysis ($\alpha_\pi = 1.8$), we simulate the effects under a more active rule ($\alpha_\pi = 2.5$), while keeping all other aspects of the simulation the same. The comparison is plotted in Figure 8.

The most noticeable difference between the two monetary policy rules is that the economy with a more active rule has a smaller increase in inflation. With a more active rule, inflation expectation is better anchored and households expect less future inflation. This generates a lower goods demand in the current period and hence a smaller increase in firms’ labor demand and real wage. Consequently, output responds less positively to the same macroeconomic shock than in the baseline.

On the nominal side, a smaller increase in the real wage rate with a more active rule leads to a smaller increase in the price level and hence a smaller increase in the policy rate. Thus, an economy with a more active rule brings greater macroeconomic stability in reduced output and inflation fluctuation to a positive macroeconomic shock. This result is in line with the view that a stronger anchor for inflation expectation reduces the volatility of macroeconomic variables, which in turn requires less interest policy intervention to achieve price stability (e.g., Woodford (2003) and Mishkin (2010)).\(^\text{26}\)

\(\text{26}\)Some empirical studies provide support for the proposition that some monetary policy targets, such as inflation targeting reduce both inflation and output volatility (e.g., Fatás et al. (2007) and Mishkin and Schmidt-Hebbel (2007)), while others do not (e.g., Ball and Sheridan (2004)).
With a smaller increase in inflation, a more active rule generates a smaller initial debt devaluation effect than the baseline. Moreover, a more positive output responses with $\alpha = 1.8$ also generate more revenue. Both contribute to a smaller increase in government debt than with $\alpha = 2.5$. The default probability remains about zero with a slightly higher debt burden under a more active rule.\footnote{Also using a New Keynesian model (but without capital), Battistini et al. (2019) conclude that a more active interest rule improves debt sustainability. Their conclusion is based on the comparison of fiscal limit distributions under different values of interest rate parameters, starting from the steady state. They find that a more active monetary policy brings less price fluctuation, which reduces firms’ price adjustment costs and hence profits, increasing tax revenues and shifting fiscal limits to the right. Our analysis, instead, focuses on the debt changes from an endogenous increase in the policy rate to a macroeconomic shock. The important driver underlying our result is the weaker debt valuation effect under a more active rule from unexpected inflation due to a macroeconomic shock, which is not analyzed in their simulated fiscal limit distributions.}

5.4 An Unexpected Acceleration in the Policy Rate Increase

The above analysis focuses on the monetary policy responses embedded in a Taylor-type rule to control inflation. The Federal Reserve may accelerate the pace of interest rate normalization unexpectedly, which is captured by the term of $\varepsilon_t^{Rf}$ in (14).\footnote{In reality, this is less likely to occur as the Federal Reserve often signals the directions and pace of future policy changes to enhance transparency in monetary policy. This simulation, however, can illuminate the fiscal effects of an exogenous policy rate increase without mixing the effects due to macroeconomic shocks.}

Figure 9 plots the impulse responses to an exogenous monetary policy shock of 25 basis points at the quarterly rate. As the contractionary monetary policy shock lowers inflation, the policy rate increases by only 7 basis points on impact, following the monetary policy rule of (14). Like earlier analysis, we conduct simulations with an initial debt state of 70–80 percent of output against the baseline fiscal limit distribution (solid lines) and a high-debt state of 140–150 percent of output against the alternative distribution with lower fiscal limits (dashed lines). The responses are the median responses based on 20,000 simulations that have initial debt levels in the debt ranges of interest.

The figure shows that an exogenous increase in the monetary policy rate lowers output, labor, and investment relative to the case without the monetary policy shock. The decline magnitudes are bigger with a high-debt state, because of larger fiscal adjustments. Consistent with the earlier analysis on an endogenous rate increase, a rising policy rate induced by an exogenous monetary policy shock drives up the interest rate on government bonds and lowers real bond prices, both contributing to increase of the government debt burden as a share
of output. The key difference against an endogenous rate increase (Figure 5) is that the current scenario does not have the initial debt devaluation effect from unexpected inflation from the positive macroeconomic shock. Instead, an exogenous increase in the policy rate lowers inflation, increasing the real value of government debt from the initial period. Default probabilities and risk premia increase more because of higher debt levels and lower fiscal limits.

Overall our results show that rising policy rates increase government debt burden, but sovereign default risk is subdued whether with an endogenous or exogenous policy rate increase. One potential concern for this benign conclusion is that the loss of bond holders is only associated with a constant haircut rate. When the loss can be bigger, risk premia could be higher and its impact on default probabilities from interest rate normalization could be larger.\textsuperscript{29}

6 Conclusion

We study the fiscal implications of interest rate normalization using a New Keynesian model that is solved fully nonlinearly. Interest rate normalization through an endogenous or exogenous increase in the monetary policy rate adds to government debt accumulation, mainly through higher interest payments and lower real bond prices. The increased debt burden is, however, unlikely to threaten debt sustainability of the U.S. federal government at the recent pace of rate increases in the near term.

Also, we find that a more active monetary policy rule increases government debt more. The economy with a more active rule produces less inflation for a given positive macroeconomic shock, generating a smaller debt devaluation effect. In addition, expecting less future inflation decreases goods demand today and leads to a smaller output response and a less tax revenue increase to the macroeconomic shock. Both effects contribute to a larger increase in the debt-to-output ratio for a more active monetary policy rule than a less active one.

This paper conducts the analysis assuming that fiscal and monetary policy interaction is

\textsuperscript{29}To address this concern, a sensitivity analysis with a haircut rate of 0.15 (vs. 0.07 in the baseline calibration) is pursued with the endogenous policy rate increase, which doesn’t change our key messages. This result is available upon request.
conditioned on a regime of active monetary policy and passive fiscal policy (i.e., monetary authorities raising policy rates sufficiently to control inflation and fiscal authorities raising fiscal surplus sufficiently to stabilize debt). Alternatively, as nominal debt burden continues to rise, monetary authorities could switch to a passive regime and fiscal authorities to an active regime. The result of this combination is a rise in the price level (as implied by the fiscal theory of price determination, see e.g., Leeper and Leith (2016)), which devalues government nominal liabilities. Allowing for this policy regime switching may provide a different perspective on debt sustainability of highly indebted economies. This analysis can be of interest for future research.
Appendix A  The Equilibrium System

Equations (A.1)-(A.20) below plus (10) and (11) in Section 2 characterize the equilibrium system. When simulating the fiscal limit distributions, the labor tax rate rule, (A.16), and capital tax rate rule, (A.17), are replaced by \( \tau_l^t = \tau_l^{l,\text{max}} \), \( \tau_k^t = \tau_k^{k,\text{max}} \), and \( \Delta_t = 0 \).

\[
\lambda_t = (c_t)^{-\sigma} \quad (A.1)
\]

\[
\chi n_t^2 = \lambda_t (1 - \tau_l^t) w_t \quad (A.2)
\]

\[
\frac{1}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1 - \Delta_{t+1}}{\sigma_{t+1}} \quad (A.3)
\]

\[
\frac{1}{R_t^f} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\sigma_{t+1}} \quad (A.4)
\]

\[
\nu_t q_t = \left( 1 - \kappa \frac{i_t}{k_{t-1} - \delta} \right)^{-1} \quad (A.5)
\]

where \( q_t \) is the Tobin’s q.

\[
q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ (1 - \tau_k^{k+1}) \frac{r_{t+1}}{k_{t+1}} + \frac{q_{t+1}}{n_{t+1}} \left( 1 - \delta - \nu_{t+1} \kappa^2 \left( \frac{i_{t+1}}{k_{t+1} - \delta} \right)^2 + \nu_{t+1} \kappa \left( \frac{i_{t+1}}{k_{t+1} - \delta} \right) \left( \frac{i_{t+1}}{k_{t+1}} \right) \right) \right] \quad (A.6)
\]

\[
\psi \left( \frac{\pi_t}{\pi} - 1 \right) = 1 - \theta + \theta m c_t + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \left[ \frac{\pi_{t+1}}{\pi} - 1 \right] \frac{y_{t+1} \pi_{t+1}}{y_t \pi} \right) \quad (A.7)
\]

\[
w_t = (1 - \alpha) m c_t a_t (k_{t-1})^\alpha n_t^{-\alpha} = (1 - \alpha) m c_t \frac{y_t}{n_t} \quad (A.8)
\]

\[
r_t^k = \alpha m c_t a_t (k_{t-1})^\alpha n_t^{-\alpha} = \alpha m c_t \frac{y_t}{k_{t-1}} \quad (A.9)
\]

\[
y_t = a_t \kappa^\alpha n_t^{1-\alpha} \quad (A.10)
\]

\[
k_t = (1 - \delta) k_{t-1} + \nu_t \left[ i_t - \kappa^2 \left( \frac{i_t}{k_{t-1} - \delta} \right)^2 k_{t-1} \right] \quad (A.11)
\]

\[
\gamma_t = \left[ 1 - m c_t - \psi^2 \left( \frac{\pi_t}{\pi} - 1 \right)^2 \right] y_t \quad (A.12)
\]

\[
tax_t = \tau_l^t w_t n_t + \tau_k^t (r_t^k k_{t-1} + \gamma_t) \quad (A.13)
\]
\[ \frac{b_t}{R_t} + tax_t = \frac{(1 - \Delta_t) b_{t-1}}{\pi_t} + g_t + z_t(i_t^z) \]  
(A.14)

\[ z_t(i_t^z) = \begin{cases} 
(1 - \rho_z)z + \rho_z z_{t-1} + \varepsilon_t^z, & \text{if } i_t^z = 1, \quad \rho_z < 1 \\
\mu z_{t-1} + \varepsilon_t^z, & \text{if } i_t^z = 2, \quad \mu > 1 
\end{cases} \]  
(A.15)

\[ \tau_t^l = \tau^l + \gamma^l (b_{t-1} - b) \]  
(A.16)

\[ \tau_t^k = \tau^k + \gamma^k (b_{t-1} - b) \]  
(A.17)

\[ R_t^f = \max \left( \left( R_{t-1}^f \right)^{\rho R_f} \left[ R^f \left( \frac{\pi_t}{\pi} \right)^{\alpha_n} \right]^{1 - \rho R_f} e^{\varepsilon_t^R}, 1 \right) \]  
(A.18)

\[ y_t = c_t + g_t + \frac{\psi}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 y_t \]  
(A.19)

\[ \ln \frac{\nu_t}{\nu} = \rho_n \ln \frac{\nu_{t-1}}{\nu} + \varepsilon_t^\nu \]  
(A.20)

**Appendix B  The Numerical Solution Method**

The method discretizes the state space and finds a fixed point in decision rules for each point in the state space. The solutions converge to functions that map the minimum set of state variables into values for the endogenous variables.

**Appendix B.1 Simulating Fiscal Limit Distributions**

Since the fiscal limits are the maximum level of debt that can be supported without default, when simulating fiscal limits, we set \( \Delta_t = 0 \ \forall \ t \). For simulating fiscal limit distributions, the minimum set of state variables is \( S_t = \{ \nu_t, k_{t-1}, z_t, i_t^z \} \). Define the decision rules for hours as \( n_t = f^n(S_t) \), inflation as \( \pi_t = f^\pi(S_t) \), and consumption as \( c_t = f^c(S_t) \).

1. Define the grid points by discretizing the state space. Make initial guesses for \( f^n_0, f^\pi_0, \) and \( f^c_0 \) over the state space.

2. Under the maximum tax rates \( (\tau_t^{l,\text{max}} \text{ and } \tau_t^{k,\text{max}}) \), at each grid point, solve the nonlinear model using the given rules \( f^n_{j-1}, f^\pi_{j-1}, \) and \( f^c_{j-1} \), and obtain the updated rules \( f^n_j, f^\pi_j, \)
and $f^c_j$. Specifically:

(a) Derive $\lambda_t$ and $w_t$ in terms of $c_t$, $n_t$, and $\tau^{l,max}$ using (A.1) and (A.2).

(b) Compute $m_{ct}$ and $r^k_t$ from (A.8) and (A.9). Derive $y_t$ in terms of $a_t$ and $n_t$ using (A.10).

(c) From (A.5), (A.11), and (A.19), derive $i_t$, $q_t$, and $k_t$.

(d) Given $\pi_t$, obtain the policy rate, $R^f_t$, from equation (A.18). If $R^f_t < 1$, set $R^f_t = 1$.

(e) Use linear interpolation to obtain $f^n_{t-1}(S_{t+1})$, $f^\pi_{t-1}(S_{t+1})$, and $f^c_{t-1}(S_{t+1})$. Then follow the above steps to solve $\lambda_{t+1}$, $r^k_{t+1}$, $q_{t+1}$, $i_{t+1}$, and $y_{t+1}$.

(f) Update the decision rules $f^n_j$, $f^\pi_j$, and $f^c_j$, using (A.4), (A.6), and (A.7). The integral in expectation terms is evaluated using numerical quadrature.

3. Check convergence of the decision rules. If $|f^n_j - f^n_{j-1}|$, $|f^\pi_j - f^\pi_{j-1}|$, or $|f^c_j - f^c_{j-1}|$ is above the desired tolerance (set to $1e-6$), go back to step 2. Otherwise, $f^n_j$, $f^\pi_j$, and $f^c_j$ are the decision rules. Use the converged rules—$f^n_j$, $f^\pi_j$, and $f^c_j$—to compute the decision rules for $f_T^j$ and $f^\lambda_j$, where $f_T^j$ is the rule for maximum tax revenue.

After we obtain the decisions rules, $f_T^j$, $f^\lambda_j$, and $f^\pi_j$, a fiscal limits distribution is simulated using Markov Chain Monte Carlo methods, described below.

1. For each simulation $l = \{1, 2, \ldots, 10000\}$, we randomly draw a sequence of investment efficiency shocks ($\varepsilon^{\nu}_{t+i}$), transfers shocks ($\varepsilon^z_{t+i}$), and the transfer regime ($i^z_{t+i}$) for 1000 periods ($i = \{1, 2, \ldots, 1000\}$), conditional on the starting state $S_t = \{\nu_t, k_{t-1}, z_t, i^z_t\}$. As labor and capital tax rates are set at the maximum rates ($\tau^{l,max}$ and $\tau^{k,max}$), we obtain $T_{t+i}^{max}$, $\lambda^{max}_{t+i}$, and $z^{max}_{t+i}$, for $i = \{1, 2, \ldots, 1000\}$. Then, the expected discounted maximum fiscal surplus for period $t+i$ is computed as

$$
\pi^{max}_t(S_t) \beta_t E_t \left\{ \frac{\lambda^{max}_{t+i}(S_{t+i})}{\lambda^{max}_t(S_t)} \left( tax^{max}_{t+i}(S_{t+i}) - g - z_{t+i}(S_{t+i}) \right) \right\}, \quad (B.1)
$$
for $i = \{1, 2, \ldots, 1000\}$. The maximum sustainable debt is

$$b_{t}^{\text{max}}(S_{t}) = \pi_{t}^{\text{max}}(S_{t}) E_t \sum_{i=0}^{1000} \beta_i E_t \left\{ \frac{\lambda_{t+i}^{\text{max}}(S_{t+i})}{\lambda_{t}^{\text{max}}(S_{t})} \left( t\alpha x_{t+i}(S_{t+i}) - g - z_{t+i}(S_{t+i}) \right) \right\},$$

(B.2)

2. Repeat the simulation for 10000 times to generate $\{b_{t}^{\text{max}, l}\}_{l=1}^{10000}$, which forms the distribution of $b(S_t)$ in (19).

**Appendix B.2 Solving the Nonlinear Model**

When solving the nonlinear model, the minimum set of state variables is denoted by $S_t = \{\nu_t, b_{t-1}^d, k_{t-1}, z_t, i_t^z\}$. Define the decision rules for hours as $n_t = f^n(S_t)$, inflation as $\pi_t = f^\pi(S_t)$, consumption as $c_t = f^c(S_t)$, and debt as $b_t = f^b(S_t)$. The decision rules are solved as follows.

1. Define the grid points by discretizing the state space. Make initial guesses for $f^n_0$, $f^\pi_0$, $f^c_0$, and $f^b_0$ over the state space.

2. At each grid point, solve the nonlinear model and obtain the updated rules $f^n_t$, $f^\pi_t$, $f^c_t$, and $f^b_t$ using the given rules $f^n_{t-1}$, $f^\pi_{t-1}$, $f^c_{t-1}$, and $f^b_{t-1}$. Specifically:

(a) Derive $\tau_t^l$ and $\tau_t^k$ using (A.16) and (A.17).

(b) Derive $\lambda_t$ and $w_t$ in terms of $c_t$, $n_t$, and $\tau_t^l$ using (A.1) and (A.2).

(c) Compute $mc_t$ and $y_t^k$ from (A.8) and (A.9). Derive $y_t$ in terms of $a_t$ and $n_t$ using (A.10).

(d) From (A.5), (A.11), and (A.19), we can derive $i_t$, $q_t$, and $k_t$.

(e) Given $\pi_t$, obtain the risk free nominal interest rate, $R_t^f$, from equation (A.18). If $R_t^f < 1$, set $R_t^f = 1$ as the nominal interest rate.

(f) Given $b_t$, solve the risky rate $R_t$ using (A.14).

(g) Use linear interpolation to obtain $f^n_{t-1}(S_{t+1})$, $f^\pi_{t-1}(S_{t+1})$, and $f^c_{t-1}(S_{t+1})$, where the state vector is $S_{t+1} = \{\nu_{t+1}, b_{t+1}^d, k_{t+1}, z_{t+1}, i_{t+1}^z\}$. Then follow the above steps to solve
\[ \lambda_{t+1}, r_{t+1}^k, q_{t+1}, i_{t+1}, \text{ and } y_{t+1}. \]

(h) Update the decision rules \( f^n_i, f^\pi_i, f^c_i, \text{ and } f^b_i \), using (A.3), (A.4), (A.6), and (A.7).

The integral in expectation terms is evaluated using numerical quadrature.

3. Check convergence of the decision rules. If \( |f^n_i - f^n_{i-1}|, |f^\pi_i - f^\pi_{i-1}|, |f^c_i - f^c_{i-1}|, \text{ or } |f^b_i - f^b_{i-1}| \) is above the desired tolerance (set to \( 1e^{-6} \)), go back to step 2. Otherwise, \( f^n_i, f^\pi_i, f^c_i, \) and \( f^b_i \) are the decision rules.
<table>
<thead>
<tr>
<th>parameters or steady-state variables</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.992</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.38</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.83</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.7</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>1</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1</td>
</tr>
<tr>
<td>$\psi$</td>
<td>7.67</td>
</tr>
<tr>
<td>$\pi$</td>
<td>78.2</td>
</tr>
<tr>
<td>$R, R^f$</td>
<td>1.008</td>
</tr>
<tr>
<td>$\tau = \frac{R}{\pi}$</td>
<td>1.008</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>0.07</td>
</tr>
<tr>
<td>$\alpha_\tau$</td>
<td>1.8</td>
</tr>
<tr>
<td>$\tau^l$</td>
<td>0.203</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>0.214</td>
</tr>
<tr>
<td>$\frac{b}{y}$</td>
<td>0.605</td>
</tr>
<tr>
<td>$\gamma^l$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\gamma^k$</td>
<td>0.005</td>
</tr>
<tr>
<td>$\frac{g}{y}$</td>
<td>0.083</td>
</tr>
<tr>
<td>$\frac{z}{y}$</td>
<td>0.106</td>
</tr>
<tr>
<td>$p^1_z$</td>
<td>0.9944</td>
</tr>
<tr>
<td>$p^2_z$</td>
<td>0.9875</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.96</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.006</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.012</td>
</tr>
<tr>
<td>$\rho_\nu$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\sigma_\nu$</td>
<td>0.001</td>
</tr>
<tr>
<td>$\rho_{R^f}$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 1: Baseline calibration.
Figure 1: **Federal government debt, interest payments, and the federal funds rate.** All four series are plotted in the annual frequency. The federal funds rate are the averages of the monthly effective federal funds rate (Board of Governors of the Federal Reserve System (2018a)). Federal debt held by the public and net interest payments data are taken from Historical Tables 7.1 and 8.4 of Office of Management and Budget (2018), respectively. The market value of gross federal debt is published by the Federal Reserve Bank of Dallas (Federal Reserve Bank of Dallas (2018)).
Figure 2: **Mandatory and discretionary spending of the federal government.** Mandatory spending includes spending on Social Security, health care programs (such as Medicare and Medicaid), income security, veterans’ programs, etc. Historical spending data are taken from Table 8.4 of the Historical Tables in *Office of Management and Budget* (2018). Projection from 2018 is the extended baseline scenario by *Congressional Budget Office* (2018a).

Figure 3: **Federal income tax rates.** The annual average capital tax rates are calculated based on Jones’s (2002) method; see footnote 14 for detail. The average (overall) income tax rate is calculated by dividing total federal income tax revenue (the sum of capital and labor tax revenues) with total income (the sum capital and labor income). The average marginal income tax rate is the sum of the rates for federal individual income tax and Social Security payroll tax as reported in Table 1 of *Barro and Redlick* (2011). The 2018 values for the average capital income tax and (overall) income tax are based on the 2018Q4 data.
Figure 4: Fiscal limit distributions for the federal government: different assumptions on maximum tax rates.
Figure 5: **Responses of an endogenous rising policy rate to a positive investment efficiency shock.** The responses (for those without a parenthesis) are plotted as the differences in percent of stochastic steady-state levels between the paths with and without a 1 percent investment efficiency shock. The two tax rates, the debt-to-annual output ratio, the interest payment-to-output ratio, and default probability are level differences in percent between the paths with and without the shock.
Figure 6: **Federal funds rate vs. real interest rates on Treasury bonds.** All series are at the monthly frequency from daily averages. The federal funds rate and the nominal interest rates of Treasury bonds are from Board of Governors of the Federal Reserve System (2018a). Real Treasury rates are computed by subtracting annualized monthly CPI inflation (constructed from the CPI published by Bureau of Labor Statistics (2018)) from the nominal Treasury constant maturity rates for 1-year and 5-year bonds.
Figure 7: Responses of an endogenous rising policy rate to a positive investment efficiency shock: different initial debt levels. See Figure 5 for units of y-axes.
Figure 8: Responses of an endogenous rising policy rate to a positive investment efficiency shock: different monetary policy activeness. See Figure 5 for units of y-axes.
Figure 9: **Responses to an exogenous monetary policy shock of 25 basis points.** See Figure 5 for units of y-axes.
References


The Board of Trustees, Federal OASDI Trust Funds (2018). The 2018 annual report of the board of trustees of the federal old-age and survivors insurance and federal disability insurance trust funds. Social Security Administration, the U.S. Federal Government.