Appendices to “Macroeconomic Effects of Reforms on Three Diverse Oil Exporters: Russia, Saudi Arabia and the UK”

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1 Appendix A: Model Solution

This appendix shows the non-linearized and linearized versions of the key optimality and market clearing conditions used in our analysis of the model's equilibrium dynamics. We denote by small letters with hat, $\hat{x}_{i,t}$, the deviation of a given variable, $X_{1,t}$, from its steady state value, while $(X_1)^{SS}$ stands for its steady state value. In what follows, for simplicity, we provide a general description of the UK model as the representative economy, and show differences for Saudi Arabia and Russia.

1.1 Country-Specific Relations

In what follows we derive the relations for the domestic country assuming that the same conditions apply to the foreign country because the model is symmetric.

1.1.1 Representative Household Maximization Problem

The representative household solves the following intertemporal problem:

$$E_t \left\{ \sum_{j=0}^{\infty} \beta_1^j \left[\begin{array}{c} \frac{1}{1-\sigma_1} \left(Z_{1,t}^c C_{1,t+j} - \kappa_1 C_{1,t+j-1} \right)^{1-\sigma_1} + \\ \frac{1}{1-\chi_1} \left(1 - L_{1,t+j} \right)^{1-\chi_1} \end{array} \right] \right\} \tag{A1}$$

subject to the budget constraint:

$$\begin{aligned} &\left(1+\tau_{1,t}^c\right) P_{1,t}^c C_{1,t} + P_{1,t}^i I_{1,t} + \left(R_{1,t}^b\right)^{-1} B_{1,t+1} + \frac{e_{1,t} \left(R_{2,t}^b\right)^{-1} B_{1,t+1}^f}{\phi_{1,t}^b} \\ &= \left(1 - \tau_{1,t}^l - \tau_{1,t}^{wh}\right) W_{1,t} L_{1,t} + R_{1,t}^k K_{1,t-1} + \left(1 - \tau_{1,t}^d\right) D_{1,t} \\ &+ \left(1 - \tau_{1,t}^{yo}\right) P_{1,t}^o Y_{1,t}^o + B_{1,t} + e_{1,t} B_{1,t}^f \end{aligned} \tag{A2}$$

and the capital accumulation equation:

$$K_{1,t} = (1-\delta_1) K_{1,t-1} + \left(1 - S\left(\frac{I_{1,t}}{I_{1,t-1}}\right)^2\right) Z_{1,t}^i I_{1,t} \tag{A3}$$

The first order condition for $C_{1,t}$ is:

$$\left(Z_{1,t}^c C_{1,t} - \kappa_1 C_{1,t-1}\right)^{-1} Z_{1,t}^c = \lambda_{1,t}^q \left(1+\tau_{1,t}^c\right) \frac{P_{1,t}^c}{P_{1,t}^d} \tag{A4}$$

where:

$$\lambda_{1,t}^q = \lambda_{1,t}^c P_{1,t}^d$$

and $\lambda^c_{1,t}$ is the Lagrange multiplier associated with the representative household budget constraint. The linearized equation is given by:

$$\begin{aligned}\frac{1}{1-\kappa_1}\left(\hat{c}_{1,t}+\hat{z}^c_{1,t}\right) = & \frac{\kappa_1}{1-\kappa_1}\hat{c}_{1,t-1}+\hat{z}^c_{1,t}-\hat{\lambda}^q_{1,t} \\ & -\frac{\left(\tau^c_1\right)^{SS}}{\left(1+\left(\tau^c_1\right)^{SS}\right)}\hat{\tau}^c_{1,t}-\left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t}\end{aligned} \tag{A5}$$

The first order condition for $L_{1,t}$ is:

$$\left(1-L_{1,t}\right)^{-\chi_1}=\lambda^q_{1,t}\left(1-\tau^l_{1,t}-\tau^{wh}_{1,t}\right)w^f_{1,t}$$

where:

$$w^f_{1,t}=\frac{W^f_{1,t}}{P^d_{1,t}} \tag{A6}$$

that is $w^f_{1,t}$ is the desired real wage expressed in terms of $P^d_{1,t}$. The linearized equation is given by:

$$\begin{aligned}\hat{w}^f_{1,t} = & \frac{\left(L_1\right)^{SS}}{1-\left(L_1\right)^{SS}}\chi_1\hat{l}_{1,t}+\frac{\left(\tau^l_1\right)^{SS}}{\left(1-\left(\tau^l_1\right)^{SS}-\left(\tau^{wh}_1\right)^{SS}\right)}\hat{\tau}^l_{1,t} \\ & +\frac{\left(\tau^{wh}_1\right)^{SS}}{\left(1-\left(\tau^l_1\right)^{SS}-\left(\tau^{wh}_1\right)^{SS}\right)}\hat{\tau}^{wh}_{1,t}-\hat{\lambda}^q_{1,t}\end{aligned} \tag{A7}$$

The first order condition for $B_{1,t+1}$ is:

$$\beta_1 R^b_{1,t}\left[\frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}}\frac{P^d_{1,t}}{P^d_{1,t+1}}\right]=1 \tag{A8}$$

The linearized equation is given by:

$$\hat{r}^b_{1,t}=-\left(\hat{\lambda}^q_{1,t+1}-\hat{\lambda}^q_{1,t}\right)+\hat{\pi}^d_{1,t+1} \tag{A9}$$

The first order condition for $B^f_{1,t+1}$ is:

$$\beta_1 R^b_{2,t}\left[\frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}}\frac{P^d_{1,t}}{P^d_{1,t+1}}\frac{e_{1,t+1}}{e_{1,t}}\right]\phi^b_{1,t}=1 \tag{A10}$$

where:

$$\phi^b_{1,t}=\phi^b_1 B^f_{1,t}$$

The linearized equation is given by:

$$\begin{aligned}\hat{r}^b_{2,t}= & -\left(\hat{\lambda}^q_{1,t+1}-\hat{\lambda}^q_{1,t}\right)+\hat{\pi}^d_{1,t+1} \\ & -\left(\hat{e}_{1,t+1}-\hat{e}_{1,t}\right)-\phi^b_1\hat{b}^f_{1,t}\end{aligned} \tag{A11}$$

1.1.2 Representative Household Maximization Problem (Russia)

The representative household solves the following intertemporal problem:

$$E_t \left\{ \sum_{j=0}^{\infty} \beta_1^j \left[\begin{array}{c} \frac{1}{1-\sigma_1} \left(Z_{1,t}^c C_{1,t+j} - \kappa_1 C_{1,t+j-1} \right)^{1-\sigma_1} + \\ \frac{1}{1-\chi_1} \left(1 - L_{1,t+j} \right)^{1-\chi_1} \end{array} \right] \right\} \tag{A12}$$

subject to the budget constraint:

$$\begin{aligned} & \left(1+\tau_{1,t}^c\right) P_{1,t}^c C_{1,t} + P_{1,t}^i I_{1,t} + \left(R_{1,t}^b\right)^{-1} B_{1,t+1} + \frac{e_{1,t}\left(R_{2,t}^b\right)^{-1} B_{1,t+1}^f}{\phi_{1,t}^b} \\ & = \left(1-\tau_{1,t}^l\right) W_{1,t} L_{1,t} + R_{1,t}^k K_{1,t-1} + \left(1-\tau_{1,t}^d\right) D_{1,t} \\ & + \left(1-\tau_{1,t}^{yo}\right) P_{1,t}^o Y_{1,t}^o + B_{1,t} + e_{1,t} B_{1,t}^f \end{aligned} \tag{A13}$$

and the private capital accumulation equation:

$$K_{1,t} = \left(1-\delta_1\right) K_{1,t-1} + \left(1 - S\left(\frac{I_{1,t}}{I_{1,t-1}}\right)^2\right) Z_{1,t}^i I_{1,t} \tag{A14}$$

The first order condition for $C_{1,t}$ is:

$$\left(Z_{1,t}^c C_{1,t} - \kappa_1 C_{1,t-1}\right)^{-1} Z_{1,t}^c = \lambda_{1,t}^q \left(1+\tau_{1,t}^c\right) \frac{P_{1,t}^c}{P_{1,t}^d} \tag{A15}$$

where:

$$\lambda_{1,t}^q = \lambda_{1,t}^c P_{1,t}^d$$

and $\lambda_{1,t}^c$ is the Lagrange multiplier associated with the representative household budget constraint. The linearized equation is given by:

$$\begin{aligned} \frac{1}{1-\kappa_1}\left(\hat{c}_{1,t} + \hat{z}_{1,t}^c\right) = & \frac{\kappa_1}{1-\kappa_1}\hat{c}_{1,t-1} + \hat{z}_{1,t}^c - \hat{\lambda}_{1,t}^q \\ & - \frac{\left(\tau_1^c\right)^{SS}}{\left(1+\left(\tau_1^c\right)^{SS}\right)}\hat{\tau}_{1,t}^c - \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} \end{aligned} \tag{A16}$$

The first order condition for $L_{1,t}$ is:

$$\left(1-L_{1,t}\right)^{-\chi_1} = \lambda_{1,t}^q \left(1-\tau_{1,t}^l\right) w_{1,t}^f$$

where:

$$w_{1,t}^f = \frac{W_{1,t}^f}{P_{1,t}^d} \tag{A17}$$

that is $w_{1,t}^f$ is the desired real wage expressed in terms of $P_{1,t}^d$. The linearized equation is given by:

$$\hat{w}_{1,t}^f = \frac{\left(L_1\right)^{SS}}{1-\left(L_1\right)^{SS}}\chi_1 \hat{l}_{1,t} + \frac{\left(\tau_1^l\right)^{SS}}{\left(1-\left(\tau_1^l\right)^{SS}\right)}\hat{\tau}_{1,t}^l - \hat{\lambda}_{1,t}^q \tag{A18}$$

The first order condition for $B_{1,t+1}$ is:

$$\beta_1 R^b_{1,t}\left[\frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}}\frac{P^d_{1,t}}{P^d_{1,t+1}}\right]=1 \tag{A19}$$

The linearized equation is given by:

$$\hat{r}^b_{1,t}=-\left(\hat{\lambda}^q_{1,t+1}-\hat{\lambda}^q_{1,t}\right)+\hat{\pi}^d_{1,t+1} \tag{A20}$$

The first order condition for $B^f_{1,t+1}$ is:

$$\beta_1 R^b_{2,t}\left[\frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}}\frac{P^d_{1,t}}{P^d_{1,t+1}}\frac{e_{1,t+1}}{e_{1,t}}\right]\phi^b_{1,t}=1 \tag{A21}$$

where:

$$\phi^b_{1,t}=\phi^b_1 B^f_{1,t}$$

The linearized equation is given by:

$$\begin{aligned}\hat{r}^b_{2,t}=&-\left(\hat{\lambda}^q_{1,t+1}-\hat{\lambda}^q_{1,t}\right)+\hat{\pi}^d_{1,t+1}\\&-\left(\hat{e}_{1,t+1}-\hat{e}_{1,t}\right)-\phi^b_1\hat{b}^f_{1,t}\end{aligned} \tag{A22}$$

1.1.3 Representative Household Maximization Problem (Saudi Arabia)

The representative household solves the following intertemporal problem:

$$E_t\left\{\sum_{j=0}^{\infty}\beta_1^j\left[\begin{array}{c}\frac{1}{1-\sigma_1}\left(Z^c_{1,t}C_{1,t+j}-\kappa_1 C_{1,t+j-1}\right)^{1-\sigma_1}+\\ \frac{1}{1-\chi_1}\left(1-L_{1,t+j}\right)^{1-\chi_1}\end{array}\right]\right\} \tag{A23}$$

subject to the budget constraint:

$$\begin{aligned}&\left(1+\tau^c_{1,t}\right)P^c_{1,t}C_{1,t}+P^i_{1,t}I_{1,t}+\left(R^b_{1,t}\right)^{-1}B_{1,t+1}+\frac{e_{1,t}\left(R^b_{2,t}\right)^{-1}B^f_{1,t+1}}{\phi^b_{1,t}}\\&=\left(1-\tau^{wh}_{1,t}\right)W_{1,t}L_{1,t}+R^k_{1,t}K_{1,t-1}+\left(1-\tau^d_{1,t}\right)D_{1,t}\\&+\left(1-\tau^{yo}_{1,t}\right)P^o_{1,t}Y^o_{1,t}+B_{1,t}+e_{1,t}B^f_{1,t}\end{aligned} \tag{A24}$$

and the private capital accumulation equation:

$$K_{1,t}=\left(1-\delta_1\right)K_{1,t-1}+\left(1-S\left(\frac{I_{1,t}}{I_{1,t-1}}\right)^2\right)Z^i_{1,t}I_{1,t} \tag{A25}$$

The first order condition for $C_{1,t}$ is:

$$\left(Z^c_{1,t}C_{1,t}-\kappa_1 C_{1,t-1}\right)^{-1}Z^c_{1,t}=\lambda^q_{1,t}\left(1+\tau^c_{1,t}\right)\frac{P^c_{1,t}}{P^d_{1,t}} \tag{A26}$$

where:

$$\lambda^q_{1,t} = \lambda^c_{1,t} P^d_{1,t}$$

and $\lambda^c_{1,t}$ is the Lagrange multiplier associated with the representative household budget constraint. The linearized equation is given by:

$$\begin{aligned}\frac{1}{1-\kappa_1}\left(\hat{c}_{1,t} + \hat{z}^c_{1,t}\right) = & \frac{\kappa_1}{1-\kappa_1}\hat{c}_{1,t-1} + \hat{z}^c_{1,t} - \hat{\lambda}^q_{1,t} \\ & - \frac{\left(\tau^c_1\right)^{SS}}{\left(1 + \left(\tau^c_1\right)^{SS}\right)}\hat{\tau}^c_{1,t} - \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t}\end{aligned} \tag{A27}$$

The first order condition for $L_{1,t}$ is:

$$(1 - L_{1,t})^{-\chi_1} = \lambda^q_{1,t}\left(1 - \tau^{wh}_{1,t}\right) w^f_{1,t}$$

where:

$$w^f_{1,t} = \frac{W^f_{1,t}}{P^d_{1,t}} \tag{A28}$$

that is $w^f_{1,t}$ is the desired real wage expressed in terms of $P^d_{1,t}$. The linearized equation is given by:

$$\hat{w}^f_{1,t} = \frac{(L_1)^{SS}}{1 - (L_1)^{SS}}\chi_1 \hat{l}_{1,t} + \frac{\left(\tau^{wh}_1\right)^{SS}}{\left(1 - \left(\tau^{wh}_1\right)^{SS}\right)}\hat{\tau}^{wh}_{1,t} - \hat{\lambda}^q_{1,t} \tag{A29}$$

The first order condition for $B_{1,t+1}$ is:

$$\beta_1 R^b_{1,t}\left[\frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}}\frac{P^d_{1,t}}{P^d_{1,t+1}}\right] = 1 \tag{A30}$$

The linearized equation is given by:

$$\hat{r}^b_{1,t} = -\left(\hat{\lambda}^q_{1,t+1} - \hat{\lambda}^q_{1,t}\right) + \hat{\pi}^d_{1,t+1} \tag{A31}$$

The first order condition for $B^f_{1,t+1}$ is:

$$\beta_1 R^b_{2,t}\left[\frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}}\frac{P^d_{1,t}}{P^d_{1,t+1}}\frac{e_{1,t+1}}{e_{1,t}}\right]\phi^b_{1,t} = 1 \tag{A32}$$

where:

$$\phi^b_{1,t} = \phi^b_1 B^f_{1,t}$$

The linearized equation is given by:

$$\begin{aligned}\hat{r}^b_{2,t} = & -\left(\hat{\lambda}^q_{1,t+1} - \hat{\lambda}^q_{1,t}\right) + \hat{\pi}^d_{1,t+1} \\ & - \left(\hat{e}_{1,t+1} - \hat{e}_{1,t}\right) - \phi^b_1 \hat{b}^f_{1,t}\end{aligned} \tag{A33}$$

1.1.4 Labour Supply Decision

If wages are flexible:

$$\hat{w}_{1,t} = \hat{w}_{1,t}^{f} + \frac{\left(\theta_1^w\right)^{SS}}{1+\left(\theta_1^w\right)^{SS}}\hat{\theta}_{1,t}^{w} \tag{A34}$$

If wages are sticky, the labour union solves the following maximization problem:

$$\max_{\{W_{1,t}(h)\}} E_t \sum_{j=0}^{\infty} \left(\xi_1^w\right)^j \psi_{1,t,t+j} \left[\begin{array}{c} \omega_{1,t,,j}^{l} W_{1,t}(h) L_{1,t+j}(h) - \\ W_{1,t+j}^{f} L_{1,t+j}(h) \end{array} \right] \tag{A35}$$

$$s.t : L_{1,t}(h) = \left(\frac{W_{1,t}(h)}{W_{1,t}}\right)^{-\frac{1+\theta_{1,t}^{w}}{\theta_{1,t}^{w}}} L_{1,t}^{d} \tag{A36}$$

$$and : \omega_{1,t,,j}^{l} = \prod_{s=1}^{j} \left\{ \left(\omega_{1,t-1+s}\right)^{\iota_1^w} \left(\pi_1^{SS}\right)^{1-\iota_1^w} \right\} \tag{A37}$$

the first order condition is given by:

$$E_t \sum_{j=0}^{\infty} \Xi_{1,t+j}^{w} \left[\begin{array}{c} \frac{W_{1,t}(h)}{W_{1,t}} \frac{1}{\theta_{1,t+j}^{w}} - \\ \frac{W_{1,t+j}^{f}}{P_{1,t+j}^{d}} \frac{P_{1,t+j}^{d}}{W_{1,t+j}} \left(\frac{W_{1,t}\omega_{1,t,j}^{l}}{W_{1,t+j}}\right)^{-1} \frac{1+\theta_{1,t+j}^{w}}{\theta_{1,t+j}^{w}} \end{array} \right] = 0 \tag{A38}$$

where:

$$\Xi_{1,t+j}^{w} = \left(\xi_1^w\right)^j \psi_{1,t,t+j} L_{1,t+j}(h)\, \omega_{1,t,j}^{l}$$

The linearized equation is given by:

$$\begin{aligned} &\frac{1}{\pi_1^{SS}}\left(\hat{\omega}_{1,t} - \iota_1^w \hat{\omega}_{1,t-1}\right) - \frac{\beta_1}{\pi_1^{SS}}\left(\hat{\omega}_{1,t+1} - \iota_1^w \hat{\omega}_{1,t}\right) \\ &= \frac{\left(1-\xi_1^w \beta_1\right)\left(1-\xi_1^w\right)}{\xi_1^w} \left(\hat{w}_{1,t+j}^{f} - \hat{w}_{1,t+j} + \frac{\left(\theta_1^w\right)^{SS}}{1+\left(\theta_1^w\right)^{SS}}\hat{\theta}_{1,t}^{w} \right) \end{aligned} \tag{A39}$$

where:

$$\frac{1}{\pi_1^{SS}}\left(\hat{\omega}_{1,t} - \iota_1^w \hat{\omega}_{1,t-1}\right) = \hat{\omega}_{1,t} - \iota_1^w \hat{\omega}_{1,t-1}$$

The wage inflation is:

$$\omega_{1,t} = \log\left(\frac{W_{1,t}}{W_{1,t-1}}\right) \tag{A40}$$

the linearized equation is given by:

$$\hat{\omega}_{1,t} = \hat{w}_{1,t} - \hat{w}_{1,t-1} + \hat{\pi}_{1,t}^{d} \tag{A41}$$

1.1.5 Private Capital Accumulation

The representative household solves the following intertemporal problem:

$$E_t\left\{\sum_{j=0}^{\infty}\beta_1^j\left[\begin{array}{c}\frac{1}{1-\sigma_1}\left(Z_{1,t}^c C_{1,t+j}-\kappa_1 C_{1,t+j-1}\right)^{1-\sigma_1}+\\ \frac{1}{1-\chi_1}\left(1-L_{1,t+j}\right)^{1-\chi_1}\end{array}\right]\right\} \tag{A42}$$

subject to the budget constraint:

$$\begin{aligned}
&\left(1+\tau_{1,t}^c\right)P_{1,t}^c C_{1,t}+P_{1,t}^i I_{1,t}+ \\
&\left(R_{1,t}^b\right)^{-1}B_{1,t+1}+\frac{e_{1,t}\left(R_{2,t}^b\right)^{-1}B_{1,t+1}^f}{\phi_{1,t}^b} \\
&=\left(1-\tau_{1,t}^l-\tau_{1,t}^{wh}\right)W_{1,t}L_{1,t}+R_{1,t}^k K_{1,t-1}+\left(1-\tau_{1,t}^d\right)D_{1,t} \\
&+P_{1,t}^o Y_{1,t}^o+B_{1,t}+e_{1,t}B_{1,t}^f
\end{aligned} \tag{A43}$$

and the capital accumulation equation:

$$K_{1,t}=\left(1-\delta_1\right)K_{1,t-1}+\left(1-S\left(\frac{I_{1,t}}{I_{1,t-1}}\right)^2\right)Z_{1,t}^i I_{1,t} \tag{A44}$$

The first order condition for $I_{1,t}$ is:

$$\begin{aligned}
0=1-q_{1,t}Z_{1,t}^i&\left[1-\frac{1}{2}\varphi_1^i\left(\frac{I_{1,t}}{I_{1,t-1}}-1\right)^2\right]+ \\
&q_{1,t}Z_{1,t}^i I_{1,t}\varphi_1^i\left(\frac{I_{1,t}}{I_{1,t-1}}-1\right)\frac{1}{I_{1,t-1}}- \\
&\beta_1\frac{P_{1,t+1}^i\lambda_{1,t+1}^c}{P_{1,t}^i\lambda_{1,t}^c}q_{1,t+1}Z_{1,t+1}^i I_{1,t+1}\varphi_1^i\left(\frac{I_{1,t+1}}{I_{1,t}}-1\right)\frac{I_{1,t+1}}{\left(I_{1,t}\right)^2}
\end{aligned} \tag{A45}$$

where:

$$q_{1,t}=\frac{Q_{1,t}}{P_{1,t}^i\lambda_{1,t}^c}$$

and $Q_{1,t}$ is the Lagrange multiplier associated with the capital accumulation equation. The linearized equation is given by:

$$\hat{q}_{1,t}=\varphi_1^i\left(\hat{\imath}_{1,t}-\hat{\imath}_{1,t-1}\right)-\varphi_1^i\beta_1\left(\hat{\imath}_{1,t+1}-\hat{\imath}_{1,t}\right)-\hat{z}_{1,t}^i \tag{A46}$$

The first order condition for $K_{1,t}$ is:

$$\begin{aligned}
q_{1,t}=\beta_1&\frac{\lambda_{1,t+1}^q}{\lambda_{1,t}^q}\frac{P_{1,t}^d}{P_{1,t}^i}\frac{R_{1,t+1}^k}{P_{1,t+1}^d} \\
&+\beta_1\left(1-\delta_1\right)q_{1,t+1}\frac{P_{1,t+1}^i}{P_{1,t+1}^d}\frac{P_{1,t}^d}{P_{1,t}^i}\frac{\lambda_{1,t+1}^q}{\lambda_{1,t}^q}
\end{aligned} \tag{A47}$$

The linearized equation is given by:

$$\hat{q}_{1,t} = \hat{\lambda}^q_{1,t+1} - \hat{\lambda}^q_{1,t} + (1-(1-\delta_1)\beta_1)\left(\hat{r}^k_{1,t+1} - \left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t}\right) \tag{A48}$$
$$+ (1-\delta_1\beta_1)\left(\hat{q}_{1,t+1}\left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t+1} - \left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t}\right)$$

The linearized capital accumulation equation is:

$$\hat{k}_{1,t} = (1-\delta_1)\hat{k}_{1,t-1} + \delta_1\left(\hat{z}^i_{1,t} + \hat{\imath}_{1,t}\right) \tag{A49}$$

1.1.6 Private Capital Accumulation (Russia)

The representative household solves the following intertemporal problem:

$$E_t\left\{\sum_{j=0}^{\infty}\beta_1^j\left[\begin{array}{c}\frac{1}{1-\sigma_1}\left(Z^c_{1,t}C_{1,t+j} - \kappa_1 C_{1,t+j-1}\right)^{1-\sigma_1} + \\ \frac{1}{1-\chi_1}\left(1-L_{1,t+j}\right)^{1-\chi_1}\end{array}\right]\right\} \tag{A50}$$

subject to the budget constraint:

$$\begin{aligned}&\left(1+\tau^c_{1,t}\right)P^c_{1,t}C_{1,t} + P^i_{1,t}I_{1,t} + \\ &\left(R^b_{1,t}\right)^{-1}B_{1,t+1} + \frac{e_{1,t}\left(R^b_{2,t}\right)^{-1}B^f_{1,t+1}}{\phi^b_{1,t}} \\ &= \left(1-\tau^l_{1,t}\right)W_{1,t}L_{1,t} + R^k_{1,t}K_{1,t-1} + \left(1-\tau^d_{1,t}\right)D_{1,t} \\ &+ P^o_{1,t}Y^o_{1,t} + B_{1,t} + e_{1,t}B^f_{1,t}\end{aligned} \tag{A51}$$

and the private capital accumulation equation:

$$K_{1,t} = (1-\delta_1)K_{1,t-1} + \left(1 - S\left(\frac{I_{1,t}}{I_{1,t-1}}\right)^2\right)Z^i_{1,t}I_{1,t} \tag{A52}$$

The first order condition for $I_{1,t}$ is:

$$\begin{aligned}0 = 1 - q_{1,t}Z^i_{1,t}&\left[1 - \frac{1}{2}\varphi^i_1\left(\frac{I_{1,t}}{I_{1,t-1}} - 1\right)^2\right] + \\ q_{1,t}Z^i_{1,t}I_{1,t}\varphi^i_1&\left(\frac{I_{1,t}}{I_{1,t-1}} - 1\right)\frac{1}{I_{1,t-1}} - \\ \beta_1\frac{P^i_{1,t+1}\lambda^c_{1,t+1}}{P^i_{1,t}\lambda^c_{1,t}}&q_{1,t+1}Z^i_{1,t+1}I_{1,t+1}\varphi^i_1\left(\frac{I_{1,t+1}}{I_{1,t}} - 1\right)\frac{I_{1,t+1}}{(I_{1,t})^2}\end{aligned} \tag{A53}$$

where:

$$q_{1,t} = \frac{Q_{1,t}}{P^i_{1,t}\lambda^c_{1,t}}$$

and $Q_{1,t}$ is the Lagrange multiplier associated with the capital accumulation equation. The linearized equation is given by:

$$\hat{q}_{1,t} = \varphi_1^i \left(\hat{\imath}_{1,t} - \hat{\imath}_{1,t-1}\right) - \varphi_1^i \beta_1 \left(\hat{\imath}_{1,t+1} - \hat{\imath}_{1,t}\right) - \hat{z}_{1,t}^i \tag{A54}$$

The first order condition for $K_{1,t}$ is:

$$\begin{aligned} q_{1,t} &= \beta_1 \frac{\lambda_{1,t+1}^q}{\lambda_{1,t}^q} \frac{P_{1,t}^d}{P_{1,t}^i} \frac{R_{1,t+1}^k}{P_{1,t+1}^d} \\ &+ \beta_1 \left(1 - \delta_1\right) q_{1,t+1} \frac{P_{1,t+1}^i}{P_{1,t+1}^d} \frac{P_{1,t}^d}{P_{1,t}^i} \frac{\lambda_{1,t+1}^q}{\lambda_{1,t}^q} \end{aligned} \tag{A55}$$

The linearized equation is given by:

$$\begin{aligned} \hat{q}_{1,t} &= \hat{\lambda}_{1,t+1}^q - \hat{\lambda}_{1,t}^q + \left(1 - \left(1 - \delta_1\right) \beta_1\right) \left(\hat{r}_{1,t+1}^k - \left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t} \right) \\ &+ \left(1 - \delta_1 \beta_1\right) \left(\hat{q}_{1,t+1} \left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t+1} - \left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t} \right) \end{aligned} \tag{A56}$$

The linearized private capital accumulation equation is:

$$\hat{k}_{1,t} = \left(1 - \delta_1\right) \hat{k}_{1,t-1} + \delta_1 \left(\hat{z}_{1,t}^i + \hat{\imath}_{1,t}\right) \tag{A57}$$

1.1.7 Private Capital Accumulation (Saudi Arabia)

The representative household solves the following intertemporal problem:

$$E_t \left\{ \sum_{j=0}^{\infty} \beta_1^j \left[\begin{array}{c} \frac{1}{1-\sigma_1} \left(Z_{1,t}^c C_{1,t+j} - \kappa_1 C_{1,t+j-1}\right)^{1-\sigma_1} + \\ \frac{1}{1-\chi_1} \left(1 - L_{1,t+j}\right)^{1-\chi_1} \end{array} \right] \right\} \tag{A58}$$

subject to the budget constraint:

$$\begin{aligned} &\left(1 + \tau_{1,t}^c\right) P_{1,t}^c C_{1,t} + P_{1,t}^i I_{1,t} + \\ &\left(R_{1,t}^b\right)^{-1} B_{1,t+1} + \frac{e_{1,t} \left(R_{2,t}^b\right)^{-1} B_{1,t+1}^f}{\phi_{1,t}^b} \\ &= \left(1 - \tau_{1,t}^{wh}\right) W_{1,t} L_{1,t} + R_{1,t}^k K_{1,t-1} + \left(1 - \tau_{1,t}^d\right) D_{1,t} \\ &+ P_{1,t}^o Y_{1,t}^o + B_{1,t} + e_{1,t} B_{1,t}^f \end{aligned} \tag{A59}$$

and the private capital accumulation equation:

$$K_{1,t} = \left(1 - \delta_1\right) K_{1,t-1} + \left(1 - S\left(\frac{I_{1,t}}{I_{1,t-1}}\right)^2\right) Z_{1,t}^i I_{1,t} \tag{A60}$$

The first order condition for $I_{1,t}$ is:

$$\begin{aligned}0 = 1 - q_{1,t}Z^i_{1,t}\left[1-\frac{1}{2}\varphi^i_1\left(\frac{I_{1,t}}{I_{1,t-1}}-1\right)^2\right]+ \\ q_{1,t}Z^i_{1,t}I_{1,t}\varphi^i_1\left(\frac{I_{1,t}}{I_{1,t-1}}-1\right)\frac{1}{I_{1,t-1}}- \\ \beta_1\frac{P^i_{1,t+1}\lambda^c_{1,t+1}}{P^i_{1,t}\lambda^c_{1,t}}q_{1,t+1}Z^i_{1,t+1}I_{1,t+1}\varphi^i_1\left(\frac{I_{1,t+1}}{I_{1,t}}-1\right)\frac{I_{1,t+1}}{(I_{1,t})^2}\end{aligned} \tag{A61}$$

where:

$$q_{1,t} = \frac{Q_{1,t}}{P^i_{1,t}\lambda^c_{1,t}}$$

and $Q_{1,t}$ is the Lagrange multiplier associated with the capital accumulation equation. The linearized equation is given by:

$$\hat{q}_{1,t} = \varphi^i_1\left(\hat{\imath}_{1,t}-\hat{\imath}_{1,t-1}\right)-\varphi^i_1\beta_1\left(\hat{\imath}_{1,t+1}-\hat{\imath}_{1,t}\right)-\hat{z}^i_{1,t} \tag{A62}$$

The first order condition for $K_{1,t}$ is:

$$\begin{aligned}q_{1,t} = \beta_1\frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}}\frac{P^d_{1,t}}{P^i_{1,t}}\frac{R^k_{1,t+1}}{P^d_{1,t+1}} \\ +\beta_1\left(1-\delta_1\right)q_{1,t+1}\frac{P^i_{1,t+1}}{P^d_{1,t+1}}\frac{P^d_{1,t}}{P^i_{1,t}}\frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}}\end{aligned} \tag{A63}$$

The linearized equation is given by:

$$\begin{aligned}\hat{q}_{1,t} = \hat{\lambda}^q_{1,t+1}-\hat{\lambda}^q_{1,t}+\left(1-\left(1-\delta_1\right)\beta_1\right)\left(\hat{r}^k_{1,t+1}-\left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t}\right) \\ +\left(1-\delta_1\beta_1\right)\left(\hat{q}_{1,t+1}\left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t+1}-\left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t}\right)\end{aligned} \tag{A64}$$

The linearized private capital accumulation equation is:

$$\hat{k}_{1,t} = \left(1-\delta_1\right)\hat{k}_{1,t-1}+\delta_1\left(\hat{z}^i_{1,t}+\hat{\imath}_{1,t}\right) \tag{A65}$$

1.1.8 Firms - Production of Consumption Goods

The cost minimization problem faced by the representative firm producing final consumption goods is given by:

$$\begin{aligned}
&\min_{\left\{C^d_{1,t},M^c_{1,t},C^{ne}_{1,t},O^c_{1,t}\right\}} P^d_{1,t}C^d_{1,t} + P^m_{1,t}M^c_{1,t} + \left(1+\tau^{oc}_{1,t}\right)P^o_{1,t}O^c_{1,t} \\
&s.t: C_{1,t} = \left(\left(\omega^{cc}_1\right)^{\frac{\rho^o_1}{1+\rho^o_1}}\left(C^{ne}_{1,t}\right)^{\frac{1}{1+\rho^o_1}} + \left(\omega^{oc}_1\right)^{\frac{\rho^o_1}{1+\rho^o_1}}\left(Z^o_{1,t}O^c_{1,t}\right)^{\frac{1}{1+\rho^o_1}}\right)^{1+\rho^o_1} \\
&and: C^{ne}_{1,t} = \left(\left(\omega^{c}_1\right)^{\frac{\rho^c_1}{1+\rho^c_1}}\left(C^{d}_{1,t}\right)^{\frac{1}{1+\rho^c_1}} + \left(\omega^{mc}_1\right)^{\frac{\rho^c_1}{1+\rho^c_1}}\left(Z^m_{1,t}M^c_{1,t}\right)^{\frac{1}{1+\rho^c_1}}\right)^{1+\rho^c_1}
\end{aligned}$$

The consumption basket equation is:

$$C_{1,t} = \left(\left(\omega^{cc}_1\right)^{\frac{\rho^o_1}{1+\rho^o_1}}\left(C^{ne}_{1,t}\right)^{\frac{1}{1+\rho^o_1}} + \left(\omega^{oc}_1\right)^{\frac{\rho^o_1}{1+\rho^o_1}}\left(Z^o_{1,t}O^c_{1,t}\right)^{\frac{1}{1+\rho^o_1}}\right)^{1+\rho^o_1} \tag{A66}$$

the linearized equation is given by:

$$\hat{c}_{1,t} = \omega^{cc}_1\hat{c}^{ne}_{1,t} + \omega^{oc}_1\left(\hat{o}^c_{1,t} + \hat{z}^o_{1,t}\right) \tag{A67}$$

The non-oil consumption aggregate equation is:

$$C^{ne}_{1,t} = \left(\left(\omega^{c}_1\right)^{\frac{\rho^c_1}{1+\rho^c_1}}\left(C^{d}_{1,t}\right)^{\frac{1}{1+\rho^c_1}} + \left(\omega^{mc}_1\right)^{\frac{\rho^c_1}{1+\rho^c_1}}\left(Z^m_{1,t}M^c_{1,t}\right)^{\frac{1}{1+\rho^c_1}}\right)^{1+\rho^c_1} \tag{A68}$$

the linearized equation is given by:

$$\hat{c}^{ne}_{1,t} = \omega^{c}_1\hat{c}^{d}_{1,t} + \omega^{mc}_1\left(\hat{m}^c_{1,t} + \hat{z}^m_{1,t}\right) \tag{A69}$$

The first order condition for $C^d_{1,t}$ in the consumption basket equation is:

$$\frac{P^c_{1,t}}{P^d_{1,t}}\left(\omega^{cc}_1\frac{C_{1,t}}{C^{ne}_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}}\left(\omega^{c}_1\frac{C^{ne}_{1,t}}{C^{d}_{1,t}}\right)^{\frac{\rho^c_1}{1+\rho^c_1}} = 1 \tag{A70}$$

the linearized equation is given by:

$$\left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} = -\frac{\rho^o_1}{1+\rho^o_1}\left(\hat{c}_{1,t} - \hat{c}^{ne}_{1,t}\right) - \frac{\rho^c_1}{1+\rho^c_1}\left(\hat{c}^{ne}_{1,t} - \hat{c}^{d}_{1,t}\right) \tag{A71}$$

The first order condition for $M^c_{1,t}$ in the consumption basket equation is:

$$\frac{P^m_{1,t}}{P^d_{1,t}} = \frac{P^c_{1,t}}{P^d_{1,t}}\left(\omega^{cc}_1\frac{C_{1,t}}{C^{ne}_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}}\left(\omega^{c}_1\frac{C^{ne}_{1,t}}{Z^m_{1,t}M^c_{1,t}}\right)^{\frac{\rho^c_1}{1+\rho^c_1}} Z^m_{1,t} \tag{A72}$$

the linearized equation is given by:

$$\left[\frac{\hat{P}^m}{\hat{P}^d}\right]_{1,t} = \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} + \frac{\rho_1^o}{1+\rho_1^o}\left(\hat{c}_{1,t} - \hat{c}_{1,t}^{ne}\right) + \frac{\rho_1^c}{1+\rho_1^c}\left(\hat{c}_{1,t}^{ne} - \hat{m}_{1,t}^c - \hat{z}_{1,t}^m\right) + \hat{z}_{1,t}^m \tag{A73}$$

The first order condition for $O_{1,t}^c$ in the consumption basket equation is:

$$\left(1+\tau_{1,t}^{oc}\right)\frac{P_{1,t}^o}{P_{1,t}^d} = \frac{P_{1,t}^c}{P_{1,t}^d}\left(\omega_1^{cc}\frac{C_{1,t}}{Z_{1,t}^o O_{1,t}^c}\right)^{\frac{\rho_1^o}{1+\rho_1^o}} Z_{1,t}^o \tag{A74}$$

the linearized equation is given by:

$$\left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} = -\frac{\left(\tau_1^{oc}\right)^{SS}}{\left(1+\left(\tau_1^{oc}\right)^{SS}\right)}\hat{\tau}_{1,t}^{oc} + \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} + \frac{\rho_1^o}{1+\rho_1^o}\left(\hat{c}_{1,t} - \hat{o}_{1,t}^c - \hat{z}_{1,t}^o\right) + \hat{z}_{1,t}^o \tag{A75}$$

1.1.9 Firms - Production of Consumption Goods (Russia and Saudi Arabia)

The cost minimization problem faced by the representative firm producing final consumption goods is given by:

$$\begin{aligned}
&\min_{\left\{C_{1,t}^d, M_{1,t}^c, C_{1,t}^{ne}, O_{1,t}^c\right\}} P_{1,t}^d C_{1,t}^d + P_{1,t}^m M_{1,t}^c + P_{1,t}^o O_{1,t}^c \\
&s.t: C_{1,t} = \left(\left(\omega_1^{cc}\right)^{\frac{\rho_1^o}{1+\rho_1^o}}\left(C_{1,t}^{ne}\right)^{\frac{1}{1+\rho_1^o}} + \left(\omega_1^{oc}\right)^{\frac{\rho_1^o}{1+\rho_1^o}}\left(Z_{1,t}^o O_{1,t}^c\right)^{\frac{1}{1+\rho_1^o}}\right)^{1+\rho_1^o} \\
&and: C_{1,t}^{ne} = \left(\left(\omega_1^{c}\right)^{\frac{\rho_1^c}{1+\rho_1^c}}\left(C_{1,t}^{d}\right)^{\frac{1}{1+\rho_1^c}} + \left(\omega_1^{mc}\right)^{\frac{\rho_1^c}{1+\rho_1^c}}\left(Z_{1,t}^m M_{1,t}^c\right)^{\frac{1}{1+\rho_1^c}}\right)^{1+\rho_1^c}
\end{aligned}$$

The private consumption basket equation is:

$$C_{1,t} = \left(\left(\omega_1^{cc}\right)^{\frac{\rho_1^o}{1+\rho_1^o}}\left(C_{1,t}^{ne}\right)^{\frac{1}{1+\rho_1^o}} + \left(\omega_1^{oc}\right)^{\frac{\rho_1^o}{1+\rho_1^o}}\left(Z_{1,t}^o O_{1,t}^c\right)^{\frac{1}{1+\rho_1^o}}\right)^{1+\rho_1^o} \tag{A76}$$

the linearized equation is given by:

$$\hat{c}_{1,t} = \omega_1^{cc}\hat{c}_{1,t}^{ne} + \omega_1^{oc}\left(\hat{o}_{1,t}^c + \hat{z}_{1,t}^o\right) \tag{A77}$$

The non-oil consumption aggregate equation is:

$$C_{1,t}^{ne} = \left(\left(\omega_1^{c}\right)^{\frac{\rho_1^c}{1+\rho_1^c}}\left(C_{1,t}^{d}\right)^{\frac{1}{1+\rho_1^c}} + \left(\omega_1^{mc}\right)^{\frac{\rho_1^c}{1+\rho_1^c}}\left(Z_{1,t}^m M_{1,t}^c\right)^{\frac{1}{1+\rho_1^c}}\right)^{1+\rho_1^c} \tag{A78}$$

the linearized equation is given by:

$$\hat{c}^{ne}_{1,t} = \omega^c_1 \hat{c}^d_{1,t} + \omega^{mc}_1 \left(\hat{m}^c_{1,t} + \hat{z}^m_{1,t}\right) \tag{A79}$$

The first order condition for $C^d_{1,t}$ in the private consumption basket equation is:

$$\frac{P^c_{1,t}}{P^d_{1,t}} \left(\omega^{cc}_1 \frac{C_{1,t}}{C^{ne}_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}} \left(\omega^c_1 \frac{C^{ne}_{1,t}}{C^d_{1,t}}\right)^{\frac{\rho^c_1}{1+\rho^c_1}} = 1 \tag{A80}$$

the linearized equation is given by:

$$\left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} = -\frac{\rho^o_1}{1+\rho^o_1}\left(\hat{c}_{1,t} - \hat{c}^{ne}_{1,t}\right) - \frac{\rho^c_1}{1+\rho^c_1}\left(\hat{c}^{ne}_{1,t} - \hat{c}^d_{1,t}\right) \tag{A81}$$

The first order condition for $M^c_{1,t}$ in the private consumption basket equation is:

$$\frac{P^m_{1,t}}{P^d_{1,t}} = \frac{P^c_{1,t}}{P^d_{1,t}} \left(\omega^{cc}_1 \frac{C_{1,t}}{C^{ne}_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}} \left(\omega^c_1 \frac{C^{ne}_{1,t}}{Z^m_{1,t} M^c_{1,t}}\right)^{\frac{\rho^c_1}{1+\rho^c_1}} Z^m_{1,t} \tag{A82}$$

the linearized equation is given by:

$$\begin{aligned}\left[\frac{\hat{P}^m}{\hat{P}^d}\right]_{1,t} = \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} &+ \frac{\rho^o_1}{1+\rho^o_1}\left(\hat{c}_{1,t} - \hat{c}^{ne}_{1,t}\right) \\ &+ \frac{\rho^c_1}{1+\rho^c_1}\left(\hat{c}^{ne}_{1,t} - \hat{m}^c_{1,t} - \hat{z}^m_{1,t}\right) + \hat{z}^m_{1,t}\end{aligned} \tag{A83}$$

The first order condition for $O^c_{1,t}$ in the private consumption basket equation is:

$$\frac{P^o_{1,t}}{P^d_{1,t}} = \frac{P^c_{1,t}}{P^d_{1,t}} \left(\omega^{cc}_1 \frac{C_{1,t}}{Z^o_{1,t} O^c_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}} Z^o_{1,t} \tag{A84}$$

the linearized equation is given by:

$$\left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} = \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} + \frac{\rho^o_1}{1+\rho^o_1}\left(\hat{c}_{1,t} - \hat{o}^c_{1,t} - \hat{z}^o_{1,t}\right) + \hat{z}^o_{1,t} \tag{A85}$$

1.1.10 Firms - Production of Investment Goods

We can express the cost minimization problem of typical firm producing investment goods as follows:

$$\begin{aligned}&\min_{\left\{I^d_{1,t}, M^i_{1,t}\right\}} P^d_{1,t} I^d_{1,t} + P^m_{1,t} M^i_{1,t} \\ &s.t: I_{1,t} = \left(\left(\omega^i_1\right)^{\frac{\rho^c_1}{1+\rho^c_1}} \left(I^d_{1,t}\right)^{\frac{1}{1+\rho^c_1}} + \left(\omega^{mi}_1\right)^{\frac{\rho^c_1}{1+\rho^c_1}} \left(Z^m_{1,t} M^i_{1,t}\right)^{\frac{1}{1+\rho^c_1}}\right)^{1+\rho^c_1}\end{aligned}$$

The investment basket equation is:

$$I_{1,t} = \left(\left(\omega_1^i\right)^{\frac{\rho_1^c}{1+\rho_1^c}} \left(I_{1,t}^d\right)^{\frac{1}{1+\rho_1^c}} + \left(\omega_1^{mi}\right)^{\frac{\rho_1^c}{1+\rho_1^c}} \left(Z_{1,t}^m M_{1,t}^i\right)^{\frac{1}{1+\rho_1^c}} \right)^{1+\rho_1^c} \tag{A86}$$

the linearized equation is given by:

$$\hat{\imath}_{1,t} = \omega_1^i \hat{\imath}_{1,t}^d + \omega_1^{mi} \left(\hat{m}_{1,t}^i + \hat{z}_{1,t}^m\right) \tag{A87}$$

The first order condition for $I_{1,t}^d$ in the investment basket equation is:

$$\frac{P_{1,t}^i}{P_{1,t}^d} \left(\omega_1^i \frac{I_{1,t}}{I_{1,t}^d}\right)^{\frac{\rho_1^c}{1+\rho_1^c}} = 1 \tag{A88}$$

the linearized equation is given by:

$$\left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t} = -\frac{\rho_1^c}{1+\rho_1^c} \left(\hat{\imath}_{1,t} - \hat{\imath}_{1,t}^d\right) \tag{A89}$$

The first order condition for $M_{1,t}^i$ in the investment basket equation is:

$$\frac{P_{1,t}^m}{P_{1,t}^d} = \frac{P_{1,t}^i}{P_{1,t}^d} \left(\omega_1^{mi} \frac{I_{1,t}}{Z_{1,t}^m M_{1,t}^i}\right)^{\frac{\rho_1^c}{1+\rho_1^c}} Z_{1,t}^m \tag{A90}$$

the linearized equation is given by:

$$\left[\frac{\hat{P}^m}{\hat{P}^d}\right]_{1,t} = \left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t} + \frac{\rho_1^c}{1+\rho_1^c} \left(\hat{\imath}_{1,t} - \hat{m}_{1,t}^i - \hat{z}_{1,t}^m\right) + \hat{z}_{1,t}^m \tag{A91}$$

1.1.11 Firms - Production of Domestic Intermediate Goods

The cost minimization problem of firm i that produces overall output $Y_{1,t}(i)$ can be expressed as:

$$\min_{\left\{K_{1,t-1}(i),L_{1,t}(i),O_{1,t}^y(i),V_{1,t}(i)\right\}} \begin{pmatrix} R_{1,t}^k K_{1,t-1}(i) + \\ \left(1+\tau_{1,t}^{wf}\right) W_{1,t} L_{1,t}(i) + \\ P_{1,t}^o O_{1,t}^y(i) \end{pmatrix}$$

$$s.t: Y_{1,t}(i) = \begin{pmatrix} \left(\omega_1^{vy}\right)^{\frac{\rho_1^o}{1+\rho_1^o}} \left(V_{1,t}(i)\right)^{\frac{1}{1+\rho_1^o}} + \\ \left(\omega_1^{oy}\right)^{\frac{\rho_1^o}{1+\rho_1^o}} \left(Z_{1,t}^o O_{1,t}^y(i)\right)^{\frac{1}{1+\rho_1^o}} \end{pmatrix}^{1+\rho_1^o}$$

$$and: V_{1,t}(i) = \begin{pmatrix} \left(\omega_1^k\right)^{\frac{\rho_1^v}{1+\rho_1^v}} \left(K_{1,t-1}(i)\right)^{\frac{1}{1+\rho_1^v}} + \\ \left(\omega_1^{kg}\right)^{\frac{\rho_1^v}{1+\rho_1^v}} \left(K_{1,t-1}^g\right)^{\frac{1}{1+\rho_1^v}} + \\ \left(\omega_1^l\right)^{\frac{\rho_1^v}{1+\rho_1^v}} \left(Z_{1,t} L_{1,t}(i)\right)^{\frac{1}{1+\rho_1^v}} \end{pmatrix}^{1+\rho_1^v}$$

The value added aggregate production function is:

$$V_{1,t}(i) = \begin{pmatrix} \left(\omega_1^k\right)^{\frac{\rho_1^v}{1+\rho_1^v}} \left(K_{1,t-1}\right)^{\frac{1}{1+\rho_1^v}} + \\ \left(\omega_1^{kg}\right)^{\frac{\rho_1^v}{1+\rho_1^v}} \left(K_{1,t-1}^g\right)^{\frac{1}{1+\rho_1^v}} + \\ \left(\omega_1^l\right)^{\frac{\rho_1^v}{1+\rho_1^v}} \left(Z_{1,t}L_{1,t}\right)^{\frac{1}{1+\rho_1^v}} \end{pmatrix}^{1+\rho_1^v} \tag{A92}$$

the linearized equation is given by:

$$\hat{v}_{1,t} = \phi_1^k \hat{k}_{1,t-1} + \phi_1^{kg} \hat{k}_{1,t-1}^g + \phi_1^l \left(\hat{l}_{1,t} + \hat{z}_{1,t}\right) \tag{A93}$$

where:

$$\phi_1^k = \left(\omega_1^k\right)^{\frac{\rho_1^v}{1+\rho_1^v}} \left(\frac{(K_1)^{SS}}{(V_1)^{SS}}\right)^{\frac{1}{1+\rho_1^v}}$$

$$\phi_1^{kg} = \left(\omega_1^{kg}\right)^{\frac{\rho_1^v}{1+\rho_1^v}} \left(\frac{(K_1^g)^{SS}}{(V_1)^{SS}}\right)^{\frac{1}{1+\rho_1^v}}$$

$$\phi_1^l = \left(\omega_1^l\right)^{\frac{\rho_1^v}{1+\rho_1^v}} \left(\frac{(L_1)^{SS}}{(V_1)^{SS}}\right)^{\frac{1}{1+\rho_1^v}}$$

$$\text{with: } \phi_1^k + \phi_1^l = 1$$
$$\text{and: } 0 < \phi_1^{kg} < 1$$

The gross output aggregate production function is:

$$Y_{1,t} = \left(\left(\omega_1^{vy}\right)^{\frac{\rho_1^o}{1+\rho_1^o}} \left(V_{1,t}\right)^{\frac{1}{1+\rho_1^o}} + \left(\omega_1^{oy}\right)^{\frac{\rho_1^o}{1+\rho_1^o}} \left(Z_{1,t}^o O_{1,t}^y\right)^{\frac{1}{1+\rho_1^o}}\right)^{1+\rho_1^o} \tag{A94}$$

the linearized equation is given by:

$$\hat{y}_{1,t} = \omega_1^{vy} \hat{v}_{1,t} + \omega_1^{oy} \left(\hat{o}_{1,t}^y + \hat{z}_{1,t}^o\right) \tag{A95}$$

The first order condition for $K_{1,t-1}$ in the output aggregator:

$$\frac{R_{1,t}^k}{P_{1,t}^d} = \frac{MC_{1,t}}{P_{1,t}^d} \left(\omega_1^{vy} \frac{Y_{1,t}}{V_{1,t}}\right)^{\frac{\rho_1^o}{1+\rho_1^o}} \left(\omega_1^k \frac{V_{1,t}}{K_{1,t-1}}\right)^{\frac{\rho_1^v}{1+\rho_1^v}} \tag{A96}$$

the linearized equation is given by:

$$r_{1,t}^k = m\hat{c}_{1,t} + \frac{\rho_1^o}{1+\rho_1^o} \left(\hat{y}_{1,t} - \hat{v}_{1,t}\right) - \frac{\rho_1^v}{1+\rho_1^v} \left(\hat{v}_{1,t} - \hat{k}_{1,t-1}\right) \tag{A97}$$

The first order condition for $K^g_{1,t-1}$ in the output aggregator:

$$\frac{P^{kg}_{1,t}}{P^d_{1,t}} = \frac{MC_{1,t}}{P^d_{1,t}} \left(\omega^{vy}_1 \frac{Y_{1,t}}{V_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}} \left(\omega^{kg}_1 \frac{V_{1,t}}{K^g_{1,t-1}}\right)^{\frac{\rho^v_1}{1+\rho^v_1}} \tag{A98}$$

the linearized equation is given by:

$$\hat{p}^{kg}_{1,t} = m\hat{c}_{1,t} + \frac{\rho^o_1}{1+\rho^o_1}\left(\hat{y}_{1,t} - \hat{v}_{1,t}\right) - \frac{\rho^v_1}{1+\rho^v_1}\left(\hat{v}_{1,t} - \hat{k}^g_{1,t-1}\right) \tag{A99}$$

The first order condition for $L_{1,t}$ in the output aggregator:

$$\left(1+\tau^{wf}_{1,t}\right)\frac{W_{1,t}}{P^d_{1,t}} = \frac{MC_{1,t}}{P^d_{1,t}} \left(\omega^{vy}_1 \frac{Y_{1,t}}{V_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}} \left(\omega^{l}_1 \frac{V_{1,t}}{Z_{1,t}L_{1,t}}\right)^{\frac{\rho^v_1}{1+\rho^v_1}} Z_{1,t} \tag{A100}$$

the linearized equation is given by:

$$\begin{aligned}\hat{w}_{1,t} = & -\frac{\left(\tau^{wf}_1\right)^{SS}}{\left(1+\left(\tau^{wf}_1\right)^{SS}\right)}\hat{\tau}^{wf}_{1,t} + m\hat{c}_{1,t} + \frac{\rho^o_1}{1+\rho^o_1}\left(\hat{y}_{1,t} - \hat{v}_{1,t}\right) \\ & + \frac{\rho^v_1}{1+\rho^v_1}\left(\hat{v}_{1,t} - \hat{l}_{1,t} - \hat{z}_{1,t}\right) + \hat{z}_{1,t}\end{aligned} \tag{A101}$$

The first order condition for $O^y_{1,t}$ in the output aggregator:

$$\frac{P^o_{1,t}}{P^d_{1,t}} = \frac{MC_{1,t}}{P^d_{1,t}} \left(\omega^{oy}_1 \frac{Y_{1,t}}{Z^o_{1,t}O^y_{1,t}}\right)^{\frac{\rho^o_1}{1+\rho^o_1}} Z^o_{1,t} \tag{A102}$$

the linearized equation is given by:

$$\left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} = m\hat{c}_{1,t} + \frac{\rho^o_1}{1+\rho^o_1}\left(\hat{y}_{1,t} - \hat{o}^y_{1,t} - \hat{z}^o_{1,t}\right) + \hat{z}^o_{1,t} \tag{A103}$$

The evolution equation for government capital is given by:

$$K^g_{1,t} = \left(1-\delta^g_1\right) K^g_{1,t-1} + A^g_{1,t} \tag{A104}$$

where $A^g_{1,t}$ denotes the authorized government investment or the stock of public investment. Expression (104) captures the time-to-build assumption. Its linearized equation is given by:

$$\hat{k}^g_{1,t} = \left(1-\delta^g_1\right)\hat{k}^g_{1,t-1} + \delta^g_1 \hat{a}^g_{1,t} \tag{A105}$$

Spending outlays authorized by appropriations bills typically occur over time. To capture this, let the sequence $\left\{\phi^{gi}_{1,0}, \phi^{gi}_{1,1}, \phi^{gi}_{1,2}, ..., \phi^{gi}_{1,N-1}\right\}$ denote the spending rates from the date the funding is authorised (0) to the period before project completion $(N-1)$. Thus, the implemented government investment at time t is given by:

$$I_{1,t}^{gd} = \sum_{n=0}^{N-1} \phi_{1,n}^{gi} A_{1,t-n}^{g} \tag{A106}$$

where $\sum_{n=0}^{N-1} \phi_{1,n}^{gi} = 1$. The linearized expression is given by:

$$\hat{\imath}_{1,t}^{gd} = \frac{1}{\sum_{n=0}^{N-1} \phi_{1,n}^{gi}} \left[\sum_{n=0}^{N-1} \phi_{1,n}^{gi} \hat{a}_{1,t-n}^{g} \right] \tag{A107}$$

For example, if we have 4 quarters of delay, equation (107) becomes:

$$\hat{g}_t^p = \left(\frac{1}{\phi_{1,0}^{gi} + \phi_{1,1}^{gi} + \phi_{1,2}^{gi} + \phi_{1,3}^{gi}} \right) \left[\sum_{n=0}^{3} \phi_{1,n}^{gi} \hat{a}_{t-n}^{gp} \right]$$

Instead of estimating the spending rates, for the UK economy we assume a version of the model with one quarter of delay ($N = 1$ and $\phi_{1,0}^{gi} = 1$) whereas for Russia and Saudi Arabia we assume a version of the model with two quarters of delay ($N = 2$, $\phi_{1,0}^{gi} = 0$ and $\phi_{1,1}^{gi} = 1$).

1.1.12 Evolution of the Marginal Cost

If prices are flexible:

$$\frac{MC_{1,t}}{P_{1,t}^d} = \frac{1}{1 + \theta_{1,t}^p} \tag{A108}$$

the linearized equation is given by:

$$m\hat{c}_{1,t} = -\frac{1}{1 + (\theta_1^p)^{SS}} \hat{\theta}_{1,t}^p \tag{A109}$$

If prices are sticky, the profit maximization problem of firms that are allowed to reoptimize their prices at time t can be expressed as:

$$\max_{\left\{P_{1,t}^d(i)\right\}} E_t \sum_{j=0}^{\infty} (\xi_1^p)^j \, \psi_{1,t,t+j} \left[\begin{array}{c} \pi_{1,t,,j}^d P_{1,t}^d(i) \, Y_{1,t+j}(i) - \\ MC_{1,t+j} Y_{1,t+j}(i) \end{array} \right] \tag{A110}$$

$$s.t: Y_{1,t}(i) = \left(\frac{P_{1,t}^d(i)}{P_{1,t}^d} \right)^{-\frac{1+\theta_1^p}{\theta_1^p}} Y_{1,t}^d \tag{A111}$$

$$and: \pi_{1,t,,j}^d = \prod_{s=1}^{j} \left\{ \left(\pi_{1,t-1+s}^d\right)^{\iota_1^p} \left(\pi_1^{SS}\right)^{1-\iota_1^p} \right\} \tag{A112}$$

the first order condition is given by:

$$E_t \sum_{j=0}^{\infty} \Xi^p_{1,t+j} \left[\begin{array}{c} \frac{P^d_{1,t}(i)}{P^d_{1,t}} \frac{1}{\theta^p_{1,t+j}} - \\ \frac{MC_{1,t+J}}{P^d_{1,t+J}} \left(\frac{P^d_{1,t}(i) \pi^d_{1,t,,j}}{P^d_{1,t+j}} \right) \frac{1+\theta^p_{1,t+j}}{\theta^p_{1,t+j}} \end{array} \right] = 0 \tag{A113}$$

where:

$$\Xi^p_{1,t+j} = (\xi^p_1 \beta_1)^j \frac{\lambda^c_{1,t+j}}{\lambda^c_{1,t}} \left(\frac{P^d_{1,t} \pi^d_{1,t,,j}}{P^d_{1,t+j}} \right)^{-\frac{1}{\theta^p_{1,t+j}}} \left(\frac{P^d_{1,t}(i)}{P^d_{1,t}} \right)^{-\frac{1+\theta^p_{1,t+j}}{\theta^p_{1,t+j}}} Y^d_{1,t+j} \tag{A114}$$

The linearized equation is given by:

$$\begin{aligned} \frac{1}{\pi^{SS}_1} \left(\hat{\pi}^d_{1,t} - \iota^p_1 \hat{\pi}^d_{1,t-1} \right) &= \frac{\beta_1}{\pi^{SS}_1} \left(\hat{\pi}^d_{1,t+1} - \iota^p_1 \hat{\pi}^d_{1,t} \right) \\ &+ \frac{(1-\xi^p_1 \beta_1)(1-\xi^p_1)}{\xi^p_1} \left(m\hat{c}_{1,t} + \frac{(\theta^p_1)^{SS}}{1+(\theta^p_1)^{SS}} \hat{\theta}^p_{1,t} \right) \end{aligned} \tag{A115}$$

where:

$$\frac{1}{\pi^{SS}_1} \left(\hat{\pi}^d_{1,t} - \iota^p_1 \hat{\pi}^d_{1,t-1} \right) = \hat{\pi}^d_{1,t} - \iota^p_1 \hat{\pi}^d_{1,t-1} \tag{A116}$$

1.1.13 Fiscal Sector

The government budget constraint is:

$$\begin{aligned} P^g_{1,t} G^d_{1,t} + P^{kg}_{1,t} I^{gd}_{1,t} + B_{1,t} = \tau^c_{1,t} P^c_{1,t} C_{1,t} + \left(\tau^l_{1,t} + \tau^{wh}_{1,t} + \tau^{wf}_{1,t} \right) W_{1,t} L_{1,t} + \\ \tau^d_{1,t} D_{1,t} + \tau^{oc}_{1,t} P^o_{1,t} O^c_{1,t} + \tau^{yo}_{1,t} P^o_{1,t} Y^o_{1,t} + \left(R^b_{1,t} \right)^{-1} B_{1,t+1} \end{aligned} \tag{A117}$$

the linearized equation is given by:

$$\hat{b}_{1,t+1} = \frac{1}{\beta} \left\{ \begin{array}{c} \hat{b}_{1,t} + \hat{g}^d_{1,t} + \hat{i}^{gd}_{1,t} - \left(\frac{P^c_1}{P^d_1} \right)^{SS} \frac{(C_1)^{SS}}{(Y^d_1)^{SS}} (\tau^c_1)^{SS} \left[\hat{\tau}^c_{1,t} + \left[\frac{\hat{P}^c}{\hat{P}^d} \right]_{1,t} + \hat{c}_{1,t} - \hat{y}^d_{1,t} \right] \\ - (w_1)^{SS} \frac{(L_1)^{SS}}{(Y^d_1)^{SS}} \left(\tau^l_1 \right)^{SS} \left[\hat{\tau}^l_{1,t} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}^d_{1,t} \right] \\ - (w_1)^{SS} \frac{(L_1)^{SS}}{(Y^d_1)^{SS}} \left(\tau^{wh}_1 \right)^{SS} \left[\hat{\tau}^{wh}_{1,t} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}^d_{1,t} \right] \\ - (w_1)^{SS} \frac{(L_1)^{SS}}{(Y^d_1)^{SS}} \left(\tau^{wf}_1 \right)^{SS} \left[\hat{\tau}^{wf}_{1,t} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}^d_{1,t} \right] \\ - \frac{(d_1)^{SS}}{(Y^d_1)^{SS}} \left(\tau^d_1 \right)^{SS} \left[\hat{\tau}^d_{1,t} + \hat{d}_{1,t} - \hat{y}^d_{1,t} \right] \\ - \left(\frac{P^o_1}{P^d_1} \right)^{SS} \frac{(O^c_1)^{SS}}{(Y^d_1)^{SS}} (\tau^{oc}_1)^{SS} \left[\hat{\tau}^{oc}_{1,t} + \left[\frac{\hat{P}^o}{\hat{P}^d} \right]_{1,t} + \hat{o}^c_{1,t} - \hat{y}^d_{1,t} \right] \\ - \left(\frac{P^o_1}{P^d_1} \right)^{SS} \frac{(Y^o_1)^{SS}}{(Y^d_1)^{SS}} (\tau^{yo}_1)^{SS} \left[\hat{\tau}^{yo}_{1,t} + \left[\frac{\hat{P}^o}{\hat{P}^d} \right]_{1,t} + \hat{y}^o_{1,t} - \hat{y}^d_{1,t} \right] \end{array} \right\} \tag{A118}$$

where the linearized equation for dividends is given by:

$$\hat{d}_{1,t} = \hat{y}^d_{1,t} - \frac{(MC_1)^{SS}}{1-(MC_1)^{SS}} m\hat{c}_{1,t} \tag{A119}$$

Moreover, we assume that the log-linearized expressions for the fiscal policy rules concerning the distortive taxes are:

$$\begin{aligned}
\hat{\tau}^c_{1,t} &= \psi^{cc}_1 \hat{c}_{1,t} \\
\hat{\tau}^l_{1,t} &= \psi^{ly}_1 \hat{y}^d_{1,t} + \psi^{lb}_1 \hat{b}_{1,t-4} \\
\hat{\tau}^{wh}_{1,t} &= \psi^{whb}_1 \hat{b}_{1,t-4} \\
\hat{\tau}^{oc}_{1,t} &= \psi^{ococ}_1 \hat{o}^c_{1,t} \\
\hat{\tau}^{wf}_{1,t} &= \psi^{wfb}_1 \hat{b}_{1,t-4} \\
\hat{\tau}^d_{1,t} &= \psi^{dd}_1 \hat{d}_{1,t} \\
\hat{\tau}^{yo}_{1,t} &= \psi^{yopo}_1 \left[\frac{\hat{P}^o}{\hat{P}^{GDP}}\right]_{1,t}
\end{aligned}$$

The oil revenues are:

$$OILREV_{1,t} = \tau^{yo}_{1,t} P^o_{1,t} Y^o_{1,t} \tag{A120}$$

the linearized equation is given by:

$$oi\hat{l}rev_{1,t} = \hat{\tau}^{yo}_{1,t} + \left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} + \hat{y}^o_{1,t} - \hat{y}^d_{1,t} \tag{A121}$$

The consumption tax (VAT) revenues are:

$$CONSTAXREV_{1,t} = \tau^c_{1,t} P^c_{1,t} C_{1,t} \tag{A122}$$

the linearized equation is given by:

$$cons\hat{t}axrev_{1,t} = \hat{\tau}^c_{1,t} + \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} + \hat{c}_{1,t} - \hat{y}^d_{1,t} \tag{A123}$$

The fuel duty revenues are:

$$FUELDUTYREV_{1,t} = \tau^{oc}_{1,t} P^o_{1,t} O^c_{1,t} \tag{A124}$$

the linearized equation is given by:

$$fueld\hat{u}tyrev_{1,t} = \hat{\tau}^{oc}_{1,t} + \left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} + \hat{o}^c_{1,t} - \hat{y}^d_{1,t} \tag{A125}$$

The dividends (corporation) tax revenues are:

$$CORTAXREV_{1,t} = \tau_{1,t}^{d} D_{1,t} \tag{A126}$$

the linearized equation is given by:

$$cort\hat{a}xrev_{1,t} = \hat{\tau}_{1,t}^{d} + \hat{d}_{1,t} - \hat{y}_{1,t}^{d} \tag{A127}$$

The labour income tax revenues are:

$$INCTAXREV_{1,t} = \tau_{1,t}^{l} W_{1,t} L_{1,t} \tag{A128}$$

the linearized equation is given by:

$$inct\hat{a}xrev_{1,t} = \hat{\tau}_{1,t}^{l} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}_{1,t}^{d} \tag{A129}$$

The households National Insurance Contribution revenues are:

$$NICHTAXREV_{1,t} = \tau_{1,t}^{wh} W_{1,t} L_{1,t} \tag{A130}$$

the linearized equation is given by:

$$nicht\hat{a}xrev_{1,t} = \hat{\tau}_{1,t}^{wh} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}_{1,t}^{d} \tag{A131}$$

The firms National Insurance Contribution revenues are:

$$NICFTAXREV_{1,t} = \tau_{1,t}^{wf} W_{1,t} L_{1,t} \tag{A132}$$

the linearized equation is given by:

$$nicft\hat{a}xrev_{1,t} = \hat{\tau}_{1,t}^{wf} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}_{1,t}^{d} \tag{A133}$$

1.1.14 Fiscal Sector (Russia)

The government budget constraint is:

$$\begin{aligned} P_{1,t}^{g} G_{1,t}^{d} + P_{1,t}^{kg} I_{1,t}^{gd} + B_{1,t} = \tau_{1,t}^{c} P_{1,t}^{c} C_{1,t} + \left(\tau_{1,t}^{l} + \tau_{1,t}^{wf}\right) W_{1,t} L_{1,t} + \\ \tau_{1,t}^{d} D_{1,t} + \tau_{1,t}^{yo} P_{1,t}^{o} Y_{1,t}^{o} + \left(R_{1,t}^{b}\right)^{-1} B_{1,t+1} \end{aligned} \tag{A134}$$

the linearized equation is given by:

$$\hat{b}_{1,t+1}=\frac{1}{\beta}\left\{\begin{array}{c}\hat{b}_{1,t}+\hat{g}^{d}_{1,t}+\hat{i}^{gd}_{1,t}-\left(\frac{P^{c}_{1}}{P^{d}_{1}}\right)^{SS}\frac{(C_{1})^{SS}}{\left(Y^{d}_{1}\right)^{SS}}\left(\tau^{c}_{1}\right)^{SS}\left[\hat{\tau}^{c}_{1,t}+\left[\frac{\hat{P}^{c}}{\hat{P}^{d}}\right]_{1,t}+\hat{c}_{1,t}-\hat{y}^{d}_{1,t}\right]\\-\left(w_{1}\right)^{SS}\frac{(L_{1})^{SS}}{\left(Y^{d}_{1}\right)^{SS}}\left(\tau^{l}_{1}\right)^{SS}\left[\hat{\tau}^{l}_{1,t}+\hat{w}_{1,t}+\hat{l}_{1,t}-\hat{y}^{d}_{1,t}\right]\\-\left(w_{1}\right)^{SS}\frac{(L_{1})^{SS}}{\left(Y^{d}_{1}\right)^{SS}}\left(\tau^{wf}_{1}\right)^{SS}\left[\hat{\tau}^{wf}_{1,t}+\hat{w}_{1,t}+\hat{l}_{1,t}-\hat{y}^{d}_{1,t}\right]\\-\frac{(d_{1})^{SS}}{\left(Y^{d}_{1}\right)^{SS}}\left(\tau^{d}_{1}\right)^{SS}\left[\hat{\tau}^{d}_{1,t}+\hat{d}_{1,t}-\hat{y}^{d}_{1,t}\right]\\-\left(\frac{P^{o}_{1}}{P^{d}_{1}}\right)^{SS}\frac{\left(Y^{o}_{1}\right)^{SS}}{\left(Y^{d}_{1}\right)^{SS}}\left(\tau^{yo}_{1}\right)^{SS}\left[\hat{\tau}^{yo}_{1,t}+\left[\frac{\hat{P}^{o}}{\hat{P}^{d}}\right]_{1,t}+\hat{y}^{o}_{1,t}-\hat{y}^{d}_{1,t}\right]\end{array}\right\} \tag{A135}$$

where the linearized equation for dividends is given by:

$$\hat{d}_{1,t}=\hat{y}^{d}_{1,t}-\frac{\left(MC_{1}\right)^{SS}}{1-\left(MC_{1}\right)^{SS}}m\hat{c}_{1,t} \tag{A136}$$

Moreover, we assume that the log-linearized expressions for the fiscal policy rules concerning the distortive taxes are:

$$\begin{aligned}\hat{\tau}^{c}_{1,t}&=\psi^{cc}_{1}\hat{c}_{1,t}\\\hat{\tau}^{l}_{1,t}&=\psi^{ly}_{1}\hat{y}^{d}_{1,t}+\psi^{lb}_{1}\hat{b}_{1,t-4}\\\hat{\tau}^{wf}_{1,t}&=\psi^{wfb}_{1}\hat{b}_{1,t-4}\\\hat{\tau}^{d}_{1,t}&=\psi^{dd}_{1}\hat{d}_{1,t}\\\hat{\tau}^{yo}_{1,t}&=\psi^{yopo}_{1}\left[\frac{\hat{P}^{o}}{\hat{P}^{GDP}}\right]_{1,t}\end{aligned}$$

The oil revenues are:

$$OILREV_{1,t}=\tau^{yo}_{1,t}P^{o}_{1,t}Y^{o}_{1,t} \tag{A137}$$

the linearized equation is given by:

$$oi\hat{l}rev_{1,t}=\hat{\tau}^{yo}_{1,t}+\left[\frac{\hat{P}^{o}}{\hat{P}^{d}}\right]_{1,t}+\hat{y}^{o}_{1,t}-\hat{y}^{d}_{1,t} \tag{A138}$$

The consumption tax (VAT) revenues are:

$$CONSTAXREV_{1,t}=\tau^{c}_{1,t}P^{c}_{1,t}C_{1,t} \tag{A139}$$

the linearized equation is given by:

$$cons\hat{t}axrev_{1,t}=\hat{\tau}^{c}_{1,t}+\left[\frac{\hat{P}^{c}}{\hat{P}^{d}}\right]_{1,t}+\hat{c}_{1,t}-\hat{y}^{d}_{1,t} \tag{A140}$$

The dividends (corporation) tax revenues are:

$$CORTAXREV_{1,t} = \tau_{1,t}^{d} D_{1,t} \tag{A141}$$

the linearized equation is given by:

$$cort\hat{a}xrev_{1,t} = \hat{\tau}_{1,t}^{d} + \hat{d}_{1,t} - \hat{y}_{1,t}^{d} \tag{A142}$$

The labour income tax revenues are:

$$INCTAXREV_{1,t} = \tau_{1,t}^{l} W_{1,t} L_{1,t} \tag{A143}$$

the linearized equation is given by:

$$inct\hat{a}xrev_{1,t} = \hat{\tau}_{1,t}^{l} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}_{1,t}^{d} \tag{A144}$$

The firms National Insurance Contribution revenues are:

$$NICFTAXREV_{1,t} = \tau_{1,t}^{wf} W_{1,t} L_{1,t} \tag{A145}$$

the linearized equation is given by:

$$nicft\hat{a}xrev_{1,t} = \hat{\tau}_{1,t}^{wf} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}_{1,t}^{d} \tag{A146}$$

1.1.15 Fiscal Sector (Saudi Arabia)

The government budget constraint is:

$$\begin{aligned} P_{1,t}^{g} G_{1,t}^{d} + P_{1,t}^{kg} I_{1,t}^{gd} + B_{1,t} = {} & \tau_{1,t}^{c} P_{1,t}^{c} C_{1,t} + \left(\tau_{1,t}^{wh} + \tau_{1,t}^{wf}\right) W_{1,t} L_{1,t} + \\ & \tau_{1,t}^{d} D_{1,t} + \tau_{1,t}^{yo} P_{1,t}^{o} Y_{1,t}^{o} + \left(R_{1,t}^{b}\right)^{-1} B_{1,t+1} \end{aligned} \tag{A147}$$

the linearized equation is given by:

$$\hat{b}_{1,t+1} = \frac{1}{\beta} \left\{ \begin{array}{c} \hat{b}_{1,t} + \hat{g}_{1,t}^{d} + \hat{i}_{1,t}^{gd} - \left(\frac{P_1^c}{P_1^d}\right)^{SS} \frac{(C_1)^{SS}}{\left(Y_1^d\right)^{SS}} \left(\tau_1^c\right)^{SS} \left[\hat{\tau}_{1,t}^{c} + \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} + \hat{c}_{1,t} - \hat{y}_{1,t}^{d}\right] \\ - \left(w_1\right)^{SS} \frac{(L_1)^{SS}}{\left(Y_1^d\right)^{SS}} \left(\tau_1^{wh}\right)^{SS} \left[\hat{\tau}_{1,t}^{wh} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}_{1,t}^{d}\right] \\ - \left(w_1\right)^{SS} \frac{(L_1)^{SS}}{\left(Y_1^d\right)^{SS}} \left(\tau_1^{wf}\right)^{SS} \left[\hat{\tau}_{1,t}^{wf} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}_{1,t}^{d}\right] \\ - \frac{(d_1)^{SS}}{\left(Y_1^d\right)^{SS}} \left(\tau_1^d\right)^{SS} \left[\hat{\tau}_{1,t}^{d} + \hat{d}_{1,t} - \hat{y}_{1,t}^{d}\right] \\ - \left(\frac{P_1^o}{P_1^d}\right)^{SS} \frac{\left(Y_1^o\right)^{SS}}{\left(Y_1^d\right)^{SS}} \left(\tau_1^{yo}\right)^{SS} \left[\hat{\tau}_{1,t}^{yo} + \left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} + \hat{y}_{1,t}^{o} - \hat{y}_{1,t}^{d}\right] \end{array} \right\} \tag{A148}$$

where the linearized equation for dividends is given by:

$$\hat{d}_{1,t} = \hat{y}^{d}_{1,t} - \frac{(MC_1)^{SS}}{1-(MC_1)^{SS}} m\hat{c}_{1,t} \tag{A149}$$

Moreover, we assume that the log-linearized expressions for the fiscal policy rules concerning the distortive taxes are:

$$\begin{aligned}
\hat{\tau}^{c}_{1,t} &= \psi^{cc}_{1}\hat{c}_{1,t} \\
\hat{\tau}^{wh}_{1,t} &= \psi^{whb}_{1}\hat{b}_{1,t-4} \\
\hat{\tau}^{wf}_{1,t} &= \psi^{wfb}_{1}\hat{b}_{1,t-4} \\
\hat{\tau}^{d}_{1,t} &= \psi^{dd}_{1}\hat{d}_{1,t} \\
\hat{\tau}^{yo}_{1,t} &= \psi^{yopo}_{1}\left[\frac{\hat{P}^{o}}{\hat{P}^{GDP}}\right]_{1,t}
\end{aligned}$$

The oil revenues are:

$$OILREV_{1,t} = \tau^{yo}_{1,t}P^{o}_{1,t}Y^{o}_{1,t} \tag{A150}$$

the linearized equation is given by:

$$oi\hat{l}rev_{1,t} = \hat{\tau}^{yo}_{1,t} + \left[\frac{\hat{P}^{o}}{\hat{P}^{d}}\right]_{1,t} + \hat{y}^{o}_{1,t} - \hat{y}^{d}_{1,t} \tag{A151}$$

The consumption tax (VAT) revenues are:

$$CONSTAXREV_{1,t} = \tau^{c}_{1,t}P^{c}_{1,t}C_{1,t} \tag{A152}$$

the linearized equation is given by:

$$cons\hat{t}axrev_{1,t} = \hat{\tau}^{c}_{1,t} + \left[\frac{\hat{P}^{c}}{\hat{P}^{d}}\right]_{1,t} + \hat{c}_{1,t} - \hat{y}^{d}_{1,t} \tag{A153}$$

The dividends (corporation) tax revenues are:

$$CORTAXREV_{1,t} = \tau^{d}_{1,t}D_{1,t} \tag{A154}$$

the linearized equation is given by:

$$cort\hat{a}xrev_{1,t} = \hat{\tau}^{d}_{1,t} + \hat{d}_{1,t} - \hat{y}^{d}_{1,t} \tag{A155}$$

The households National Insurance Contribution revenues are:

$$NICHTAXREV_{1,t} = \tau^{wh}_{1,t}W_{1,t}L_{1,t} \tag{A156}$$

the linearized equation is given by:

$$nicht\hat{a}xrev_{1,t} = \hat{\tau}^{wh}_{1,t} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}^{d}_{1,t} \tag{A157}$$

The firms National Insurance Contribution revenues are:

$$NICFTAXREV_{1,t} = \tau^{wf}_{1,t} W_{1,t} L_{1,t} \tag{A158}$$

the linearized equation is given by:

$$nicft\hat{a}xrev_{1,t} = \hat{\tau}^{wf}_{1,t} + \hat{w}_{1,t} + \hat{l}_{1,t} - \hat{y}^{d}_{1,t} \tag{A159}$$

1.1.16 Monetary policy

The linearized expression for the real interest rate is given by:

$$\hat{r}^{rb}_{1,t} = \hat{r}^{b}_{1,t} - \hat{\pi}^{d}_{1,t+1} = -\left(\hat{\lambda}^{q}_{1,t+1} - \hat{\lambda}^{q}_{1,t}\right) \tag{A160}$$

The Taylor rule is:

$$\begin{aligned} i_{1,t} = \bar{\imath}_1 + \gamma^{i}_{1}\left(i_{1,t-1} - \bar{\imath}_1\right) + \\ \left(1-\gamma^{i}_{1}\right)\left[\left(\pi^{core}_{1,t} - \bar{\pi}^{core}_{1}\right) + \gamma^{\pi^{core}}_{1}\left(\pi^{core}_{1,t} - \bar{\pi}^{core}_{1} - \bar{\pi}^{core}_{1,t}\right) + \gamma^{y}_{1} y^{gap}_{1,t}\right] \end{aligned} \tag{A161}$$

where:

$$i_{1,t} = R^{b}_{1,t} - 1 \tag{A162}$$

the linearized equation is given by:

$$\hat{r}^{b}_{1,t} = \gamma^{i}_{1}\hat{r}^{b}_{1,t-1} + \left(1-\gamma^{i}_{1}\right)\left[\hat{\pi}^{core}_{1,t} + \gamma^{\pi^{core}}_{1}\left(\hat{\pi}^{core}_{1,t} - \bar{\pi}^{core}_{1,t}\right) + \gamma^{y}_{1}\hat{y}^{gap}_{1,t}\right] \tag{A163}$$

1.1.17 Monetary Policy (Russia and Saudi Arabia)

The linearized expression for the real interest rate is given by:

$$\hat{r}^{rb}_{1,t} = \hat{r}^{b}_{1,t} - \hat{\pi}^{d}_{1,t+1} = -\left(\hat{\lambda}^{q}_{1,t+1} - \hat{\lambda}^{q}_{1,t}\right) \tag{A164}$$

The Taylor rule is:

$$i_{1,t} = \bar{\imath}_1 + \gamma^{i}_{1}\left(i_{1,t-1} - \bar{\imath}_1\right) + \left(1-\gamma^{i}_{1}\right)\left[\begin{array}{c} \left(\pi^{head}_{1,t} - \bar{\pi}^{head}_{1}\right) + \\ \gamma^{\pi^{head}}_{1}\left(\pi^{head}_{1,t} - \bar{\pi}^{head}_{1} - \bar{\pi}^{head}_{1,t}\right) + \\ \gamma^{y}_{1} y^{gap}_{1,t} + \gamma^{e}_{1}\left(rer_{1,t} - rer_{1,t-1}\right) \end{array}\right] \tag{A165}$$

where:

$$i_{1,t} = R^{b}_{1,t} - 1 \tag{A166}$$

the linearized equation is given by:

$$\hat{r}^{b}_{1,t} = \gamma^{i}_{1}\hat{r}^{b}_{1,t-1} + \left(1-\gamma^{i}_{1}\right)\left[\begin{array}{c} \hat{\pi}^{head}_{1,t} + \gamma^{\pi^{head}}_{1}\left(\hat{\pi}^{head}_{1,t} - \bar{\pi}^{head}_{1,t}\right) + \\ \gamma^{y}_{1}\hat{y}^{gap}_{1,t} + \gamma^{e}_{1}\left(r\hat{e}r_{1,t} - r\hat{e}r_{1,t-1}\right) \end{array}\right] \tag{A167}$$

1.1.18 Remaining Relations

The market clearing condition for the domestic non-oil goods market is:

$$Y_{1,t}^{d} = C_{1,t}^{d} + I_{1,t}^{d} + I_{1,t}^{gd} + G_{1,t}^{d} + X_{1,t} \tag{A168}$$

the linearized equation is given by:

$$\hat{y}_{1,t}^{d} = \frac{\left(C_1^d\right)^{SS}}{\left(Y_1^d\right)^{SS}}\hat{c}_{1,t}^{d} + \frac{\left(I_1^d\right)^{SS}}{\left(Y_1^d\right)^{SS}}\hat{i}_{1,t}^{d} + \frac{\left(G_1^d\right)^{SS}}{\left(Y_1^d\right)^{SS}}\hat{g}_{1,t}^{d} + \frac{\left(I_1^{gd}\right)^{SS}}{\left(Y_1^d\right)^{SS}}\hat{i}_{1,t}^{gd} + \frac{\left(X_1\right)^{SS}}{\left(Y_1^d\right)^{SS}}\hat{x}_{1,t} \tag{A169}$$

The oil demand equation is:

$$O_{1,t} = O_{1,t}^{y} + O_{1,t}^{c} \tag{A170}$$

the linearized equation is given by:

$$\hat{o}_{1,t} = \frac{\left(O_1^y\right)^{SS}}{\left(O_1\right)^{SS}}\hat{o}_{1,t}^{y} + \frac{\left(O_1^c\right)^{SS}}{\left(O_1\right)^{SS}}\hat{o}_{1,t}^{c} \tag{A171}$$

The core price level $P_{1,t}^{ne}$ is given by:

$$P_{1,t}^{ne} = P_{1,t}^{c}\left(\omega_1^{cc}\frac{C_{1,t}}{C_{1,t}^{ne}}\right)^{\frac{\rho_1^o}{1+\rho_1^o}} \tag{A172}$$

the linearized expression is given by:

$$\begin{aligned}\left[\frac{\hat{P}^{ne}}{\hat{P}^{d}}\right]_{1,t} &= \left[\frac{\hat{P}^{c}}{\hat{P}^{d}}\right]_{1,t} + \frac{\rho_1^o}{1+\rho_1^o}\left(\hat{c}_{1,t} - \hat{c}_{1,t}^{ne}\right) \\ &= \frac{1}{\omega_1^{cc}}\left[\frac{\hat{P}^{c}}{\hat{P}^{d}}\right]_{1,t} - \frac{\omega_1^{oc}}{\omega_1^{cc}}\left(\left[\frac{\hat{P}^{o}}{\hat{P}^{d}}\right]_{1,t} - \hat{z}_{1,t}^{o}\right)\end{aligned} \tag{A173}$$

As we can note from expression (A173) the shock to oil intensity enters since it affects the headline price $\hat{P}_{1,t}^{c}$.

The inflation of domestic prices is given by:

$$\pi_{1,t}^{d} = \log\left(\frac{P_{1,t}^{d}}{P_{1,t-1}^{d}}\right) \tag{A174}$$

the linearized equation is given by:

$$\hat{\pi}_{1,t}^{d} = \hat{p}_{1,t}^{d} - \hat{p}_{1,t-1}^{d} \tag{A175}$$

The inflation of core prices is given by:

$$\pi_{1,t}^{core} = \log\left(\frac{P_{1,t}^{ne}}{P_{1,t-1}^{ne}}\right) \tag{A176}$$

the linearized equation is given by:

$$\hat{\pi}^{core}_{1,t} = \left[\frac{\hat{P}^{ne}}{\hat{P}^{d}}\right]_{1,t} - \left[\frac{\hat{P}^{ne}}{\hat{P}^{d}}\right]_{1,t-1} + \hat{\pi}^{d}_{1,t} \tag{A177}$$

The inflation of headline prices is given by:

$$\pi^{head}_{1,t} = \log\left(\frac{P^{c}_{1,t}}{P^{c}_{1,t-1}}\right) \tag{A178}$$

the linearized equation is given by:

$$\hat{\pi}^{head}_{1,t} = \left[\frac{\hat{P}^{c}}{\hat{P}^{d}}\right]_{1,t} - \left[\frac{\hat{P}^{c}}{\hat{P}^{d}}\right]_{1,t-1} + \hat{\pi}^{d}_{1,t} \tag{A179}$$

The equation for aggregate imports is given by:

$$M_{1,t} = \frac{P^{m}_{1,t}}{P^{d}_{1,t}} M^{c}_{1,t} + \frac{P^{m}_{1,t}}{P^{d}_{1,t}} M^{i}_{1,t} \tag{A180}$$

the linearized expression is given by:

$$\hat{m}_{1,t} = \frac{\left(M^{c}_{1}\right)^{SS}}{\left(M_{1}\right)^{SS}} \left(\left[\frac{\hat{P}^{m}}{\hat{P}^{d}}\right]_{1,t} + \hat{m}^{c}_{1,t}\right) + \frac{\left(M^{i}_{1}\right)^{SS}}{\left(M_{1}\right)^{SS}} \left(\left[\frac{\hat{P}^{m}}{\hat{P}^{d}}\right]_{1,t} + \hat{m}^{i}_{1,t}\right) \tag{A181}$$

because relative prices are assumed to be 1 in steady state.

The equation for aggregate exports is given by:

$$X_{1,t} = \frac{\zeta_2}{\zeta_1}\left(M^{c}_{2,t} + M^{i}_{2,t}\right) \tag{A182}$$

because country 1 real per capita exports, $X_{1,t}$, and country 2 real per capita imports, $M_{2,t}$, are related by the relative population weight, $\frac{\zeta_2}{\zeta_1}$.

The linearized expression is given by:

$$\hat{x}_{1,t} = \frac{\left(M^{c}_{2}\right)^{SS}}{\left(M^{c}_{2}\right)^{SS} + \left(M^{i}_{2}\right)^{SS}} \hat{m}^{c}_{2,t} + \frac{\left(M^{i}_{2}\right)^{SS}}{\left(M^{c}_{2}\right)^{SS} + \left(M^{i}_{2}\right)^{SS}} \hat{m}^{i}_{2,t} \tag{A183}$$

The ratio between total trade balance and gross output is given by:

$$\frac{T^{bal}_{1,t}}{P^{d}_{1,t} Y^{d}_{1,t}} = \frac{X_{1,t} - M_{1,t} + \frac{P^{o}_{1,t}}{P^{d}_{1,t}}\left(Y^{o}_{1,t} - O_{1,t}\right)}{Y^{d}_{1,t}} \tag{A184}$$

the linearized equation is given by:

$$\begin{aligned} \hat{t}^{bal}_{1,t} = {} & \frac{\left(X_{1}\right)^{SS}}{\left(Y^{d}_{1}\right)^{SS}} \hat{x}_{1,t} - \frac{\left(M_{1}\right)^{SS}}{\left(Y^{d}_{1}\right)^{SS}} \hat{m}_{1,t} + \frac{\left(P^{o}_{1}\right)^{SS}\left(Y^{o}_{1}\right)^{SS}}{\left(P^{d}_{1}\right)^{SS}\left(Y^{d}_{1}\right)^{SS}} \hat{y}^{o}_{1,t} \\ & - \frac{\left(P^{o}_{1}\right)^{SS}\left(O_{1}\right)^{SS}}{\left(P^{d}_{1}\right)^{SS}\left(Y^{d}_{1}\right)^{SS}} \hat{o}_{1,t} - \frac{\left(P^{o}_{1}\right)^{SS}\left[\left(O_{1}\right)^{SS} - \left(Y^{o}_{1}\right)^{SS}\right]}{\left(P^{d}_{1}\right)^{SS}\left(Y^{d}_{1}\right)^{SS}} \left[\frac{\hat{P}^{o}}{\hat{P}^{d}}\right]_{1,t} \end{aligned} \tag{A185}$$

The ratio between non-oil goods trade balance and gross output is given by:

$$\frac{G^{bal}_{1,t}}{P^d_{1,t} Y^d_{1,t}} = \frac{X_{1,t} - M_{1,t}}{Y^d_{1,t}} \tag{A186}$$

the linearized equation is given by:

$$\hat{g}^{bal}_{1,t} = \frac{(X_1)^{SS}}{\left(Y^d_1\right)^{SS}} \hat{x}_{1,t} - \frac{(M_1)^{SS}}{\left(Y^d_1\right)^{SS}} \hat{m}_{1,t} + \left[\frac{(X_1)^{SS}}{\left(Y^d_1\right)^{SS}} - \frac{(M_1)^{SS}}{\left(Y^d_1\right)^{SS}}\right] \hat{y}^d_{1,t} \tag{A187}$$

1.2 Bilateral Relations

For country 1, the relative import prices can be expressed as follows:

$$\frac{P^m_{1,t}}{P^d_{1,t}} = \frac{e_{1,t} P^c_{2,t}}{P^c_{1,t}} \frac{P^d_{2,t}}{P^c_{2,t}} \frac{P^c_{1,t}}{P^d_{1,t}} \tag{A188}$$

where $e_{1,t}$ is the nominal exchange rate. Considering that the consumption real exchange rate is:

$$rer_{1,t} = \frac{e_{1,t} P^c_{2,t}}{P^c_{1,t}} \tag{A189}$$

the linearized expression for (A188) is given by:

$$\left[\frac{\hat{P}^m}{\hat{P}^d}\right]_{1,t} = r\hat{e}r_{1,t} - \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{2,t} + \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} \tag{A190}$$

For country 2, the linearized expression for the relative import prices is given by:

$$\left[\frac{\hat{P}^m}{\hat{P}^d}\right]_{2,t} = -r\hat{e}r_{1,t} + \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{2,t} - \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} \tag{A191}$$

The uncovered interest rate parity condition is:

$$\frac{\lambda^q_{2,t+1}}{\lambda^q_{2,t}} \frac{P^d_{2,t}}{P^c_{2,t}} \frac{P^c_{2,t+1}}{P^d_{2,t+1}} = \phi^b_{1,t} \frac{rer_{1,t+1}}{rer_{1,t}} \frac{\lambda^q_{1,t+1}}{\lambda^q_{1,t}} \frac{P^d_{1,t}}{P^c_{1,t}} \frac{P^c_{1,t+1}}{P^d_{1,t+1}} \tag{A192}$$

the linearized equation is given by:

$$\begin{aligned}\left(\hat{\lambda}^q_{2,t+1} - \hat{\lambda}^q_{2,t}\right) = & \left(\hat{\lambda}^q_{1,t+1} - \hat{\lambda}^q_{1,t}\right) + \phi^b_1 \hat{b}^f_{1,t} + r\hat{e}r_{1,t+1} - r\hat{e}r_{1,t} \\ & - \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} + \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t+1} + \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{2,t} - \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{2,t+1}\end{aligned} \tag{A193}$$

The net foreign asset condition is:

$$\begin{aligned}\frac{e_{1,t} \left(R^b_{2,t}\right)^{-1} B^f_{1,t+1}}{\phi^b_{1,t}} = & \, e_{1,t} B^f_{1,t} + \frac{\zeta_2}{\zeta_1} e_{1,t} P^m_{2,t} \left(M^c_{2,t} + M^i_{2,t}\right) \\ & - P^m_{1,t} \left(M^c_{1,t} + M^i_{1,t}\right) + P^o_{1,t} \left(Y^o_{1,t} - O_{1,t}\right)\end{aligned} \tag{A194}$$

the linearized equation is given by:

$$\beta_1 \hat{b}^f_{1,t+1} = \hat{b}^f_{1,t} + \hat{t}^{bal}_{1,t} \tag{A195}$$

where:

$$\hat{b}^f_{1,t} = \frac{e_{1,t} B^f_{1,t}}{P^d_{1,t} Y^d_{1,t}} \tag{A196}$$

The oil market clearing condition is:

$$Y^o_{1,t} + \frac{\zeta_2}{\zeta_1} Y^o_{2,t} = O_{1,t} + \frac{\zeta_2}{\zeta_1} O_{2,t} \tag{A197}$$

the linearized equation is given by:

$$\begin{aligned} &\frac{\frac{\zeta_1}{\zeta_2}\left(Y^o_1\right)^{SS}}{\frac{\zeta_1}{\zeta_2}\left(Y^o_1\right)^{SS} + \left(Y^o_2\right)^{SS}} \hat{y}^o_{1,t} + \frac{\left(Y^o_2\right)^{SS}}{\frac{\zeta_1}{\zeta_2}\left(Y^o_1\right)^{SS} + \left(Y^o_2\right)^{SS}} \hat{y}^o_{2,t} \\ &= \frac{\frac{\zeta_1}{\zeta_2}\left(O_1\right)^{SS}}{\frac{\zeta_1}{\zeta_2}\left(O_1\right)^{SS} + \left(O_2\right)^{SS}} \hat{o}_{1,t} + \frac{\left(O_2\right)^{SS}}{\frac{\zeta_1}{\zeta_2}\left(O_1\right)^{SS} + \left(O_2\right)^{SS}} \hat{o}_{2,t} \end{aligned} \tag{A198}$$

The law of one price for oil is:

$$\frac{P^o_{1,t}}{P^d_{1,t}} = rer_{1,t} \frac{P^c_{1,t}}{P^d_{1,t}} \frac{P^d_{2,t}}{P^c_{2,t}} \frac{P^o_{2,t}}{P^d_{2,t}} \tag{A199}$$

the linearized equation is given by:

$$\left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} = r\hat{e}r_{1,t} + \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} - \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{2,t} + \left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{2,t} \tag{A200}$$

1.3 Exogenous Shocks

Table A1: Exogenous Processes

Shocks	Stochastic Process
Home Prod.	$\ln\left(Z_{1,t}\right)=\left(1+\rho_{1,1}^{z}-\rho_{2,1}^{z}\right)\ln\left(Z_{1,t-1}\right)-\rho_{1,1}^{z}\ln\left(Z_{1,t-2}\right)+\varepsilon_{1,t}^{z}$
Foreign Prod.	$\ln\left(Z_{2,t}\right)=\left(1+\rho_{1,2}^{z}-\rho_{2,2}^{z}\right)\ln\left(Z_{2,t-1}\right)-\rho_{1,2}^{z}\ln\left(Z_{2,t-2}\right)+\varepsilon_{2,t}^{z}$
Home Oil Sup.	$\ln\left(Y_{1,t}^{o}\right)=\left(1+\rho_{1,1}^{yo}-\rho_{2,1}^{yo}\right)\ln\left(Y_{1,t-1}^{o}\right)-\rho_{1,1}^{yo}\ln\left(Y_{1,t-2}^{o}\right)+\varepsilon_{1,t}^{yo}$
Foreign Oil Sup.	$\ln\left(Y_{2,t}^{o}\right)=\left(1+\rho_{1,2}^{yo}-\rho_{2,2}^{yo}\right)\ln\left(Y_{2,t-1}^{o}\right)-\rho_{1,2}^{yo}\ln\left(Y_{2,t-2}^{o}\right)+\varepsilon_{2,t}^{yo}$
Home Oil Int.	$\ln\left(Z_{1,t}^{o}\right)=\left(1+\rho_{1,1}^{zo}-\rho_{2,1}^{zo}\right)\ln\left(Z_{1,t-1}^{o}\right)-\rho_{1,1}^{zo}\ln\left(Z_{1,t-2}^{o}\right)+\varepsilon_{1,t}^{zo}$
Foreign Oil Int.	$\ln\left(Z_{2,t}^{o}\right)=\left(1+\rho_{1,2}^{zo}-\rho_{2,2}^{zo}\right)\ln\left(Z_{2,t-1}^{o}\right)-\rho_{1,2}^{zo}\ln\left(Z_{2,t-2}^{o}\right)+\varepsilon_{2,t}^{zo}$
Home Priv. Cons.	$\ln\left(Z_{1,t}^{c}\right)=\rho_{1,1}^{zc}\ln\left(Z_{1,t-1}^{c}\right)+\varepsilon_{1,t}^{c}$
Foreign Priv. Cons.	$\ln\left(Z_{2,t}^{c}\right)=\rho_{1,2}^{zc}\ln\left(Z_{2,t-1}^{c}\right)+\varepsilon_{2,t}^{c}$
Home Imp. Pref.	$\ln\left(Z_{1,t}^{m}\right)=\left(1+\rho_{1,1}^{zm}-\rho_{2,1}^{zm}\right)\ln\left(Z_{1,t-1}^{m}\right)-\rho_{1,1}^{zm}\ln\left(Z_{1,t-2}^{m}\right)+\varepsilon_{1,t}^{zm}$
Foreign Imp. Pref.	$\ln\left(Z_{2,t}^{m}\right)=\left(1+\rho_{1,2}^{zm}-\rho_{2,2}^{zm}\right)\ln\left(Z_{2,t-1}^{m}\right)-\rho_{1,2}^{zm}\ln\left(Z_{2,t-2}^{m}\right)+\varepsilon_{2,t}^{zm}$
Home Priv. Inv.	$\ln\left(Z_{1,t}^{i}\right)=\rho_{1,1}^{zi}\ln\left(Z_{1,t-1}^{i}\right)+\varepsilon_{1,t}^{i}$
Home Price Mar.	$\hat{\theta}_{1,t}^{p}=\rho_{1,1}^{p}\hat{\theta}_{1,t-1}^{p}+\varepsilon_{1,t}^{p}$
Home Wage Mar.	$\hat{\theta}_{1,t}^{w}=\rho_{1,1}^{w}\hat{\theta}_{1,t-1}^{w}+\varepsilon_{1,t}^{w}$
Home Infl. Target (UK Model)	$\bar{\pi}_{1,t}^{core}=\rho_{1,1}^{\pi}\bar{\pi}_{1,t-1}^{core}+\varepsilon_{1,t}^{\pi}$ -
Home Infl. Target (RU & SA Models)	$\bar{\pi}_{1,t}^{head}=\rho_{1,1}^{\pi}\bar{\pi}_{1,t-1}^{head}+\varepsilon_{1,t}^{\pi}$
Home Gov. Cons. Exp.	$\hat{g}_{1,t}^{d}=\rho_{1,1}^{g}\hat{g}_{1,t-1}^{d}+\varepsilon_{1,t}^{g}$
Home Auth. Gov. Inv. Exp.	$\hat{a}_{1,t}^{g}=\rho_{1,1}^{ag}\hat{a}_{1,t-1}^{g}+\varepsilon_{1,t}^{ag}$

1.4 Important Definitions

The definition of $GDP_{1,t}$ using Laspeyres index is:

$$GDP_{1,t}=GDP_{1,t-1}\frac{P_{1,t-1}^{d}Y_{1,t}-P_{1,t-1}^{o}O_{1,t}^{y}+P_{1,t-1}^{o}Y_{1,t}^{o}}{P_{1,t-1}^{d}Y_{1,t-1}-P_{1,t-1}^{o}O_{1,t-1}^{y}+P_{1,t-1}^{o}Y_{1,t-1}^{o}} \tag{A201}$$

the linearized equation is given by:

$$\begin{aligned}&\left(1-\frac{\left(P_{1}^{o}\right)^{SS}\left(O_{1}^{y}\right)^{SS}}{\left(P_{1}^{d}\right)^{SS}\left(Y_{1}^{d}\right)^{SS}}+\frac{\left(P_{1}^{o}\right)^{SS}\left(Y_{1}^{o}\right)^{SS}}{\left(P_{1}^{d}\right)^{SS}\left(Y_{1}^{d}\right)^{SS}}\right)\\&\left(g\hat{d}p_{1,t}-g\hat{d}p_{1,t-1}\right)-\left(\hat{y}_{1,t}-\hat{y}_{1,t-1}\right)\\&=-\left(\frac{\left(P_{1}^{o}\right)^{SS}\left(O_{1}^{y}\right)^{SS}}{\left(P_{1}^{d}\right)^{SS}\left(Y_{1}^{d}\right)^{SS}}\right)\left(\hat{o}_{1,t}^{y}-\hat{o}_{1,t-1}^{y}\right)\\&+\left(\frac{\left(P_{1}^{o}\right)^{SS}\left(Y_{1}^{o}\right)^{SS}}{\left(P_{1}^{d}\right)^{SS}\left(Y_{1}^{d}\right)^{SS}}\right)\left(\hat{y}_{1,t}^{o}-\hat{y}_{1,t-1}^{o}\right)\end{aligned} \tag{A202}$$

The ratio between nominal GDP and nominal gross output is:

$$\frac{NGDP_{1,t}}{P^d_{1,t}Y_{1,t}} = 1 - \frac{P^o_{1,t}O^y_{1,t}}{P^d_{1,t}Y_{1,t}} + \frac{P^o_{1,t}Y^o_{1,t}}{P^d_{1,t}Y_{1,t}} \tag{A203}$$

the linearized equation is given by:

$$\begin{aligned}
&\frac{(NGDP_1)^{SS}}{\left(P^d_1\right)^{SS}\left(Y^d_1\right)^{SS}}\left[\frac{NG\hat{D}P}{\hat{P}^d\hat{Y}}\right]_{1,t} \\
&= \left(\frac{(P^o_1)^{SS}(O^y_1)^{SS}}{\left(P^d_1\right)^{SS}\left(Y^d_1\right)^{SS}} - \frac{(P^o_1)^{SS}(Y^o_1)^{SS}}{\left(P^d_1\right)^{SS}\left(Y^d_1\right)^{SS}}\right)\left(\hat{y}_{1,t} - \left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t}\right) \\
&- \frac{(P^o_1)^{SS}(O^y_1)^{SS}}{\left(P^d_1\right)^{SS}\left(Y^d_1\right)^{SS}}\hat{o}^y_{1,t} + \frac{(P^o_1)^{SS}(O^y_1)^{SS}}{\left(P^d_1\right)^{SS}\left(Y^d_1\right)^{SS}}\hat{y}^o_{1,t}
\end{aligned} \tag{A204}$$

The equation for the oil price deflated by the GDP deflator is:

$$\frac{P^o_{1,t}}{P^{GDP}_{1,t}}\frac{P^{GDP}_{1,t}}{P^o_{1,t-1}} = \frac{P^o_{1,t}}{P^d_{1,t}}\frac{P^d_{1,t-1}}{P^o_{1,t-1}}\frac{\frac{NGDP_{1,t-1}}{P^d_{1,t-1}Y_{1,t-1}}}{\frac{NGDP_{1,t}}{P^d_{1,t}Y_{1,t}}}\frac{GDP_{1,t}}{GDP_{1,t-1}}\frac{Y_{1,t-1}}{Y_{1,t}} \tag{A205}$$

the linearized equation is given by:

$$\begin{aligned}
&\log\left(\left[\frac{P^o_{1,t}}{P^{GDP}_{1,t}}\right]^{obs}\right) - \log\left(\left[\frac{P^o_{1,t-1}}{P^{GDP}_{1,t-1}}\right]^{obs}\right) \\
&= -\left(\left[\frac{NG\hat{D}P}{\hat{P}^d\hat{Y}}\right]_{1,t} - \left[\frac{NG\hat{D}P}{\hat{P}^d\hat{Y}}\right]_{1,t-1}\right) - (\hat{y}_{1,t} - \hat{y}_{1,t-1}) \\
&+ \left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} - \left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t-1} + g\hat{d}p_{1,t} - g\hat{d}p_{1,t-1}
\end{aligned} \tag{A206}$$

The ratio between the total trade balance and nominal GDP is:

$$\frac{T^{bal}_{1,t}}{NGDP_{1,t}} = \frac{T^{bal}_{1,t}}{P^d_{1,t}Y_{1,t}}\frac{P^d_{1,t}Y_{1,t}}{NGDP_{1,t}} \tag{A207}$$

the linearized equation is given by:

$$\left[\frac{\hat{T}^{bal}}{NG\hat{D}P}\right]_{1,t} = \frac{\left(P^d_1\right)^{SS}\left(Y^d_1\right)^{SS}}{(NGDP_1)^{SS}}\hat{t}^{bal}_{1,t} \tag{A208}$$

The ratio between the non-oil goods trade balance and nominal GDP is:

$$\frac{G^{bal}_{1,t}}{NGDP_{1,t}} = \frac{1}{\frac{NGDP_{1,t}}{P^d_{1,t}Y_{1,t}}}\left(\frac{X_{1,t}}{Y_{1,t}} - \frac{P^d_{1,t}}{P^d_{1,t}}\frac{M_{1,t}}{Y_{1,t}}\right) \tag{A209}$$

the linearized equation is given by:

$$\left[\frac{\hat{G}^{bal}}{NG\hat{D}P}\right]_{1,t} = -\frac{1}{\frac{(NGDP_1)^{SS}}{\left(P_1^d\right)^{SS}(Y_1)^{SS}}}\left(\frac{(X_1)^{SS}}{\left(Y_1^d\right)^{SS}} - \frac{\left(P_1^d\right)^{SS}(M_1)^{SS}}{\left(P_1^d\right)^{SS}(Y_1)^{SS}}\right) \tag{A210}$$

$$\left[\frac{NG\hat{D}P}{\hat{P}^d\hat{Y}}\right]_{1,t} + \frac{1}{\frac{(NGDP_1)^{SS}}{\left(P_1^d\right)^{SS}(Y_1)^{SS}}}\hat{g}_{1,t}^{bal}$$

1.5 Observation Equations

In what follows we denote by $X_{i,t}^{obs}$ the observed data series associated with a given linearized variable, $\hat{x}_{i,t}$. We start listing the observation equations for country 1.

The observation equation for GDP is:

$$\log\left(GDP_{1,t}^{obs}\right) - \log\left(GDP_{1,t-1}^{obs}\right) = g\hat{d}p_{1,t} - g\hat{d}p_{1,t-1} \tag{A211}$$

The observation equation for oil production is:

$$\log\left(Y_{1,t}^{o,obs}\right) - \log\left(Y_{1,t-1}^{o,obs}\right) = \hat{y}_{1,t}^{o} - \hat{y}_{1,t-1}^{o} \tag{A212}$$

The observation equation for oil imports as a share of nominal GDP is:

$$\frac{OIL\ IMP_{1,t}^{obs}}{NGDP_{1,t}^{obs}} = \frac{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}}{(NGDP_1)^{SS}}\left(\frac{(O_1)^{SS}}{\left(Y_1^d\right)^{SS}} - \frac{\left(Y_1^o\right)^{SS}}{\left(Y_1^d\right)^{SS}}\right) \tag{A213}$$

$$\left[-\left[\frac{NG\hat{D}P}{\hat{P}^d\hat{Y}}\right]_{1,t} + \left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} - \hat{y}_{1,t}\right]$$

$$+\frac{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}}{(NGDP_1)^{SS}}\left(\frac{(O_1)^{SS}}{\left(Y_1^d\right)^{SS}}\hat{o}_{1,t} - \frac{\left(Y_1^o\right)^{SS}}{\left(Y_1^d\right)^{SS}}\hat{y}_{1,t}^{o}\right)$$

The observation equation for the real oil price is:

$$\log\left(\left[\frac{P_{1,t}^o}{P_{1,t}^{GDP}}\right]^{obs}\right) - \log\left(\left[\frac{P_{1,t-1}^o}{P_{1,t-1}^{GDP}}\right]^{obs}\right) = \left[\frac{\hat{P}^o}{\hat{P}^{GDP}}\right]_{1,t} - \left[\frac{\hat{P}^o}{\hat{P}^{GDP}}\right]_{1,t-1} \tag{A214}$$

The observation equation for non-oil imports as a share of nominal GDP is:

$$\frac{NONOIL\ IMP_{1,t}^{obs}}{NGDP_{1,t}^{obs}} = \frac{\left(P_1^d\right)^{SS}(M_1)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}}\frac{(NGDP_1)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \tag{A215}$$

$$\left(\hat{m}_{1,t} - \hat{y}_{1,t} - \left[\frac{NG\hat{D}P}{\hat{P}^d\hat{Y}}\right]_{1,t}\right)$$

The observation equation for non-oil exports as a share of nominal GDP is:

$$\frac{NONOIL\ EXP_{1,t}^{obs}}{NGDP_{1,t}^{obs}} = \frac{\left(P_1^d\right)^{SS}(X_1)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \frac{(NGDP_1)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \left(\hat{x}_{1,t} - \hat{y}_{1,t} - \left[\frac{NG\hat{D}P}{\hat{P}^d\hat{Y}}\right]_{1,t}\right) \tag{A216}$$

The observation equation for the real exchange rate is:

$$rer_{1,t}^{obs} = r\hat{e}r_{1,t} \tag{A217}$$

The observation equation for consumption as a share of nominal GDP is:

$$\frac{CONS_{1,t}^{obs}}{NGDP_{1,t}^{obs}} = \frac{\left(P_1^c\right)^{SS}(C_1)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \frac{(NGDP_1)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \left(\hat{c}_{1,t} + \left[\frac{\hat{P}^c}{\hat{P}^d}\right]_{1,t} - \hat{y}_{1,t} - \left[\frac{NG\hat{D}P}{\hat{P}^d\hat{Y}}\right]_{1,t}\right) \tag{A218}$$

The observation equation for total gross fixed capital formation as a share of nominal GDP is:

$$\frac{INV_{1,t}^{obs}}{NGDP_{1,t}^{obs}} = \frac{\left(P_1^i\right)^{SS}(I_1)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \frac{(NGDP_1)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \left(\hat{\imath}_{1,t} + \left[\frac{\hat{P}^i}{\hat{P}^d}\right]_{1,t} - \hat{y}_{1,t} - \left[\frac{NG\hat{D}P}{\hat{P}^d\hat{Y}}\right]_{1,t}\right) \tag{A219}$$

For the UK model, the observation equation for core price inflation is:

$$\pi_{1,t}^{core,obs} = \hat{\pi}_{1,t}^{core} \tag{A220}$$

For Russia and Saudi Arabia models, the observation equation for headline price inflation is:

$$\pi_{1,t}^{head,obs} = \hat{\pi}_{1,t}^{head} \tag{A221}$$

The observation equation for wage inflation is:

$$\omega_{1,t}^{obs} = \hat{\omega}_{1,t} \tag{A222}$$

The observation equation for nominal interest rate is:

$$r_{1,t}^{b,obs} = \hat{r}_{1,t}^{b} \tag{A223}$$

The observation equation for government debt as share of nominal GDP is:

$$\frac{GOV\ DEBT_{1,t}^{obs}}{NGDP_{1,t}^{obs}} = \frac{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}}{\left(NGDP_1\right)^{SS}}\hat{b}_{1,t} \tag{A224}$$

The observation equation for government investment as share of nominal GDP is:

$$\frac{GOV\ INV_{1,t}^{obs}}{NGDP_{1,t}^{obs}} = \frac{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}}{\left(NGDP_1\right)^{SS}}\hat{\imath}_{1,t}^{gd} \tag{A225}$$

For country 2 the observation equations are the following.

The observation equation for GDP is:

$$\log\left(GDP_{2,t}^{obs}\right) - \log\left(GDP_{2,t-1}^{obs}\right) = g\hat{d}p_{2,t} - g\hat{d}p_{2,t-1} \tag{A226}$$

The observation equation for oil production is:

$$\log\left(Y_{2,t}^{o,obs}\right) - \log\left(Y_{2,t-1}^{o,obs}\right) = \hat{y}_{2,t}^{o} - \hat{y}_{2,t-1}^{o} \tag{A227}$$

1.6 Decomposition of the Marginal Costs

As we derived above, from the profit maximization problem of firms producing intermediate domestic goods we have that:

$$\phi_1^k = \left(\omega_1^k\right)^{\frac{\rho_1^v}{1+\rho_1^v}}\left(\frac{\left(K_1\right)^{SS}}{\left(V_1\right)^{SS}}\right)^{\frac{1}{1+\rho_1^v}} \tag{A228}$$

$$\phi_1^{kg} = \left(\omega_1^{kg}\right)^{\frac{\rho_1^v}{1+\rho_1^v}}\left(\frac{\left(K_1^g\right)^{SS}}{\left(V_1\right)^{SS}}\right)^{\frac{1}{1+\rho_1^v}} \tag{A229}$$

$$\phi_1^l = \left(\omega_1^l\right)^{\frac{\rho_1^v}{1+\rho_1^v}}\left(\frac{\left(L_1\right)^{SS}}{\left(V_1\right)^{SS}}\right)^{\frac{1}{1+\rho_1^v}} \tag{A230}$$

$$\text{with: } \phi_1^k + \phi_1^l = 1$$

$$\text{and: } 0 < \phi_1^{kg} < 1$$

Thus, recasting (A228)-(A230):

$$\phi_1^k = \omega_1^k\left(\frac{shareky_1}{\omega_1^k sharevy_1}\right)^{\frac{1}{1+\rho_1^v}}$$

$$\phi_1^{kg} = \omega_1^{kg}\left(\frac{sharekgy_1}{\omega_1^k sharevy_1}\right)^{\frac{1}{1+\rho_1^v}}$$

$$\phi_1^l = \omega_1^l\left(\frac{sharely_1}{\omega_1^l sharevy_1}\right)^{\frac{1}{1+\rho_1^v}}$$

$$\text{with: } \phi_1^k + \phi_1^l = 1$$
$$\text{and: } 0 < \phi_1^{kg} < 1$$

The marginal products of oil, capital and labour are respectively:

$$\begin{aligned}
m\hat{p}o_{1,t} &= \frac{\rho_1^o}{1+\rho_1^o}\left(\hat{y}_{1,t} - \hat{o}_{1,t}^y - \hat{z}_{1,t}^o\right) + \hat{z}_{1,t}^o \\
m\hat{p}k_{1,t} &= \frac{\rho_1^o}{1+\rho_1^o}\left(\hat{y}_{1,t} - \hat{v}_{1,t}\right) + \frac{\rho_1^v}{1+\rho_1^v}\left(\hat{v}_{1,t} - \hat{k}_{1,t-1}\right) \\
m\hat{p}k_{1,t}^g &= \frac{\rho_1^o}{1+\rho_1^o}\left(\hat{y}_{1,t} - \hat{v}_{1,t}\right) + \frac{\rho_1^v}{1+\rho_1^v}\left(\hat{v}_{1,t} - \hat{k}_{1,t-1}^g\right) \\
m\hat{p}l_{1,t} &= \frac{\rho_1^o}{1+\rho_1^o}\left(\hat{y}_{1,t} - \hat{v}_{1,t}\right) \\
&\quad + \frac{\rho_1^v}{1+\rho_1^v}\left(\hat{v}_{1,t} - \hat{l}_{1,t} - \hat{z}_{1,t}\right) + \hat{z}_{1,t} \\
&\quad - \frac{\left(\tau_1^{wf}\right)^{SS}}{1+\left(\tau_1^{wf}\right)^{SS}}\hat{\tau}_{1,t}^{wf}
\end{aligned}$$

and from the first order conditions:

$$\left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} = m\hat{c}_{1,t} + m\hat{p}o_{1,t} \tag{A231}$$
$$\hat{r}_{1,t}^k = m\hat{c}_{1,t} + m\hat{p}k_{1,t} \tag{A232}$$
$$\hat{p}_{1,t}^{kg} = m\hat{c}_{1,t} + m\hat{p}k_{1,t}^g \tag{A233}$$
$$\hat{w}_{1,t} = m\hat{c}_{1,t} + m\hat{p}l_{1,t} \tag{A234}$$

Multiplying equation (A231) by ω_1^{oy}, equation (A232) by $\omega_1^{vy}\phi_1^k$, equation (A233) by $\omega_1^{vy}\phi_1^{kg}$ and equation (A234) by $\omega_1^{vy}\left(1-\phi_1^k\right)$, and summing up these three equations we have that:

$$\begin{aligned}
m\hat{c}_{1,t} = \omega_1^{oy}&\left(\left[\frac{\hat{P}^o}{\hat{P}^d}\right]_{1,t} - m\hat{p}o_{1,t}\right) \\
&+ \omega_1^{vy}\phi_1^k\left(\hat{r}_{1,t}^k - m\hat{p}k_{1,t}\right) \\
&+ \omega_1^{vy}\phi_1^{kg}\left(\hat{p}_{1,t}^{kg} - m\hat{p}k_{1,t}^g\right) \\
&+ \omega_1^{vy}\left(1-\phi_1^k\right)\left(\hat{w}_{1,t} - m\hat{p}l_{1,t}\right)
\end{aligned}$$

$$\text{where: } \omega_1^{oy} + \omega_1^{vy}\phi_1^k + \omega_1^{vy}\left(1-\phi_1^l\right) + \omega_1^{oy} = 1$$
$$\text{and: } 0 < \omega_1^{vy}\phi_1^k < 1$$

1.7 Calibrated Parameters

As we explained in the main body of the paper, some of the parameter values are taken from observed data means. In what follows we describe the relative expressions associated with these values.

The share of nominal oil demand on nominal gross output is:

$$shareoy_1 = \frac{(P_1^o)^{SS}(O_1)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} = \frac{(O_1)^{SS}}{\left(Y_1^d\right)^{SS}} \tag{A235}$$

because the real price of oil is assumed to be 1 in steady state.

The ratio between oil used in production and oil used in consumption is:

$$ratiooyoc_1 = \frac{(P_1^o)^{SS}(O_1^y)^{SS}}{\left(P_1^d\right)^{SS}(O_1^c)^{SS}} = \frac{(O_1^y)^{SS}}{(O_1^c)^{SS}} \tag{A236}$$

The share of investment on gross output is:

$$shareiy_1 = \frac{\left(I_1^d\right)^{SS}}{\left(Y_1^d\right)^{SS}} \tag{A237}$$

The share of government spending on gross output is:

$$sharegy_1 = \frac{\left(G_1^d\right)^{SS}}{\left(Y_1^d\right)^{SS}} \tag{A238}$$

The share of investment government spending on gross output is:

$$shareigy_1 = \frac{\left(I_1^{gd}\right)^{SS}}{\left(Y_1^d\right)^{SS}} \tag{A239}$$

The ratio between oil production and oil demand is:

$$ratioyoo_1 = \frac{(Y_1^o)^{SS}}{(O_1)^{SS}} \tag{A240}$$

The share of imports on gross output is:

$$sharemy_1 = \frac{(M_1)^{SS}}{\left(Y_1^d\right)^{SS}} \tag{A241}$$

The ratio between imports of investment goods and imports of consumption goods is:

$$ratioyoo_1 = \frac{(M_1^i)^{SS}}{(M_1^c)^{SS}} \tag{A242}$$

Finally, we assume that the weight of labour in the value added production function is:

$$\omega_1^l = 1 \tag{A243}$$

1.8 Composite Parameters

Given the parameter values taken from observed data means and the expressions listed above, we can derive the remaining parameters as follows.

The share of hours worked is:

$$labshare_1 = \frac{L_1^{SS}}{1 - L_1^{SS}} \tag{A244}$$

The weight of oil input in the overall output production function is:

$$\omega_1^{oy} = \frac{(O_1^y)^{SS}}{\left(Y_1^d\right)^{SS}} = shareoyy_1 \tag{A245}$$

The weight of value added input in the overall output production function is:

$$\omega_1^{vy} = 1 - \omega_1^{oy} = \frac{(V_1)^{SS}}{\left(Y_1^d\right)^{SS}} = sharevy_1 \tag{A246}$$

The real rental rate in steady state is:

$$\left(r_1^k\right)^{SS} = \frac{1}{\beta_1} - 1 + \delta_1 \tag{A247}$$

The share of capital on gross output is:

$$\frac{(K_1)^{SS}}{\left(Y_1^d\right)^{SS}} = \frac{1}{\delta_1} shareiy_1 = shareky_1 \tag{A248}$$

The weight of capital in the value added production function is:

$$\omega_1^k = \frac{1}{\delta_1}\left(\frac{1}{\beta_1} - 1 + \delta_1\right)^{\frac{1+\rho_1^v}{\rho_1^v}} \frac{shareiy_1}{sharevy_1} \tag{A249}$$

The weight of government capital in the value added production function is:

$$\omega_1^{kg} = 1 - \omega_1^k \tag{A250}$$

The share of government capital on gross output is:

$$\frac{(K_1^g)^{SS}}{\left(Y_1^d\right)^{SS}} = \frac{1}{\delta_1^g} shareigy_1 = sharekgy_1 \tag{A251}$$

The share of labour on gross output is:

$$\frac{(L_1)^{SS}}{\left(Y_1^d\right)^{SS}} = sharevy_1 \left(1 - \omega_1^k \left(\left(r_1^k\right)^{SS}\right)^{-\frac{1}{\rho_1^v}}\right)^{1+\rho_1^v} = sharely_1 \tag{A252}$$

The share of nominal consumption on nominal gross output is:

$$\frac{\left(P_1^c\right)^{SS}\left(C_1\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} = 1 - \frac{\left(P_1^d\right)^{SS}\left(I_1^d\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} - \frac{\left(P_1^d\right)^{SS}\left(G_1^d\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} - \tag{A253}$$

$$- \frac{\left(P_1^d\right)^{SS}\left(I_1^{gd}\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} + \frac{\left(P_1^o\right)^{SS}\left(Y_1^o\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}}$$

$$- \frac{\left(P_1^o\right)^{SS}\left(O_1^y\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}}$$

$$= sharecy_1$$

that is:

$$sharecy_1 = 1 - shareiy_1 - sharegy_1 - shareigy_1 + shareyoy_1 - shareoyy_1 \tag{A254}$$

The weights of oil in the production of consumption goods is:

$$\omega_1^{oc} = \frac{\left(O_1^c\right)^{SS}}{\left(C_1\right)^{SS}} = shareocc_1 \tag{A255}$$

where:

$$shareocc_1 = \frac{shareoy_1 - shareoyy_1}{sharecy_1} \tag{A256}$$

The weight of non-oil in the production of consumption goods is:

$$\omega_1^{cc} = 1 - \omega_1^{oc} = \frac{\left(C_1^{ne}\right)^{SS}}{\left(C_1\right)^{SS}} \tag{A257}$$

The weight of domestic goods in the production of consumption goods is:

$$\omega_1^{c} = 1 - \omega_1^{mc} = \frac{\left(C_1^{d}\right)^{SS}}{\left(C_1^{ne}\right)^{SS}} = sharecdcn_1 \tag{A258}$$

The weight of imported goods in the production of consumption goods is:

$$\omega_1^{mc} = \frac{\left(M_1^c\right)^{SS}}{\left(C_1^{ne}\right)^{SS}} = sharemccn_1 \tag{A259}$$

where:

$$sharemccn_1 = \frac{sharemy_1}{sharecy_1 \cdot sharecnc_1} \frac{1}{1 + ratiomimc_1} \tag{A260}$$

The weight of domestic goods in the production of investment goods is:

$$\omega_1^{i} = 1 - \omega_1^{mi} = \frac{\left(I_1^{d}\right)^{SS}}{\left(I_1\right)^{SS}} \tag{A261}$$

The weight of imported goods in the production of investment goods is:

$$\omega_1^{mi} = \frac{\left(M_1^i\right)^{SS}}{\left(I_1\right)^{SS}} = sharemii_1 \tag{A262}$$

where:

$$sharemii_1 = \frac{sharemy_1}{shareiy_1}\frac{ratiomimc_1}{1+ratiomimc_1} \tag{A263}$$

The share of exports on gross output of country 1 is:

$$\begin{aligned}\frac{\left(P_1^d\right)^{SS}\left(X_1\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} &= \frac{\left(P_1^d\right)^{SS}\left(M_1\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \\ &+ \frac{\left(P_1^o\right)^{SS}\left(O_1\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \\ &- \frac{\left(P_1^o\right)^{SS}\left(Y_1^o\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \\ &= sharexy_1\end{aligned} \tag{A264}$$

or:

$$sharexy_1 = sharemy_1 + shareoy_1 - shareyoy_1 \tag{A265}$$

The share of exports on gross output of country 2 is:

$$\frac{\left(X_2\right)^{SS}}{\left(Y_2^d\right)^{SS}} = \frac{\left(M_2\right)^{SS}}{\left(Y_2^d\right)^{SS}} - \left(\frac{\left(O_1\right)^{SS}}{\left(Y_1^d\right)^{SS}} + \frac{\left(Y_1^o\right)^{SS}}{\left(Y_1^d\right)^{SS}}\right)\frac{\left(Y_1^d\right)^{SS}}{\left(Y_2^d\right)^{SS}}\frac{1}{\zeta_2} \tag{A266}$$

or:

$$sharexy_2 = sharemy_2 - \left(shareoy_1 - shareyoy_1\right)\frac{\left(Y_1^d\right)^{SS}}{\left(Y_2^d\right)^{SS}}\frac{1}{\zeta_2} \tag{A267}$$

The share of imports on gross output of country 2 is:

$$\begin{aligned}&\frac{\left(P_1^d\right)^{SS}\left(M_1\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} + \frac{\left(P_1^o\right)^{SS}\left(O_1 - Y_1^o\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \\ &= \frac{\left(e_1\right)^{SS}\left(P_2^d\right)^{SS}\left(M_2\right)^{SS}}{\left(P_2^d\right)^{SS}\left(Y_2^d\right)^{SS}}\frac{\left(P_2^d\right)^{SS}}{\left(P_1^d\right)^{SS}}\frac{\left(Y_2^d\right)^{SS}}{\left(Y_1^d\right)^{SS}}\zeta_2 \\ &= sharemy_2\end{aligned} \tag{A268}$$

or:

$$sharemy_2 = \left(sharemy_1 + \left(shareoy_1 - shareyoy_1\right)\right)\frac{\left(Y_1^d\right)^{SS}}{\left(Y_2^d\right)^{SS}}\frac{1}{\zeta_2} \tag{A269}$$

The ratio between nominal GDP and nominal gross output is:

$$\frac{\left(NGDP_1\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} = 1 - \frac{\left(P_1^o\right)^{SS}\left(O_1^y\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} + \frac{\left(P_1^o\right)^{SS}\left(Y_1^o\right)^{SS}}{\left(P_1^d\right)^{SS}\left(Y_1^d\right)^{SS}} \tag{A270}$$

or:

$$sharengdpny_1 = 1 - shareoyy_1 + shareyoy_1 \tag{A271}$$

The ratio between gross outputs of country 1 and 2 is:

$$\frac{\left(Y_1^d\right)^{SS}}{\left(Y_2^d\right)^{SS}} = \frac{sharely_2}{sharely_1}\frac{(L_1)^{SS}}{(L_2)^{SS}} \tag{A272}$$

The share in world oil production for country 2 is:

$$\begin{aligned} &\frac{(Y_2^o)^{SS}}{\frac{\zeta_1}{\zeta_2}(Y_1^o)^{SS} + (Y_2^o)^{SS}} \\ &= \frac{\frac{\left(Y_2^o\right)^{SS}}{\left(Y_2^d\right)^{SS}}}{\frac{\zeta_1}{\zeta_2}\frac{\left(Y_1^o\right)^{SS}}{\left(Y_1^d\right)^{SS}}\frac{\left(Y_1^d\right)^{SS}}{\left(Y_2^d\right)^{SS}} + \frac{\left(Y_2^o\right)^{SS}}{\left(Y_2^d\right)^{SS}}} \\ &= shareoprod_2 \end{aligned} \tag{A273}$$

or:

$$shareoprod_2 = \frac{shareyoy_2}{\frac{\zeta_1}{\zeta_2}shareyoy_1\frac{\left(Y_1^d\right)^{SS}}{\left(Y_2^d\right)^{SS}} + shareyoy_2} \tag{A274}$$

The share in world oil consumption for country 2 is:

$$\begin{aligned} &\frac{(O_2)^{SS}}{\frac{\zeta_1}{\zeta_2}(O_1)^{SS} + (O_2)^{SS}} \\ &= \frac{\frac{(O_2)^{SS}}{\left(Y_2^d\right)^{SS}}}{\frac{\zeta_1}{\zeta_2}\frac{(O_1)^{SS}}{\left(Y_1^d\right)^{SS}}\frac{\left(Y_1^d\right)^{SS}}{\left(Y_2^d\right)^{SS}} + \frac{(O_2)^{SS}}{\left(Y_2^d\right)^{SS}}} \\ &= shareocons_2 \end{aligned} \tag{A275}$$

or:

$$shareocons_2 = \frac{shareoy_2}{\frac{\zeta_1}{\zeta_2}shareoy_1\frac{\left(Y_1^d\right)^{SS}}{\left(Y_2^d\right)^{SS}} + shareoy_2} \tag{A276}$$

The overall oil production as share of gross output is:

$$\frac{(Y_1^o)^{SS}}{\left(Y_1^d\right)^{SS}} = \frac{(Y_1^o)^{SS}}{(O_1)^{SS}}\frac{(O_1)^{SS}}{\left(Y_1^d\right)^{SS}} = shareyoy_1 \tag{A277}$$

or:

$$shareyoy_1 = ratioyoo_1 \cdot shareoy_1 \tag{A278}$$

The real wage in steady state is:

$$(w_1)^{SS} = \frac{1}{1+\left(\tau_1^{wf}\right)^{SS}} \frac{(MC_1)^{SS}}{\left(P_1^d\right)^{SS}} \left[(\omega_1^{vy}\omega_1^{vy})^{\frac{\rho_1^o}{1+\rho_1^o}} \left(\omega_1^{vy} \left(\phi_1^l\right)^{1+\rho_1^w} \right) \right] \tag{A279}$$

The ratio between real dividends and gross output is:

$$\frac{(d_1)^{SS}}{\left(Y_1^d\right)^{SS}} = 1 - \frac{(MC_1)^{SS}}{\left(P_1^d\right)^{SS}} \tag{A280}$$

where:

$$(MC_1)^{SS} = \frac{1}{1+(\theta_1^p)^{SS}} \tag{A281}$$

2 Appendix B: Data Construction and Sources

As we described in the main body of the paper, the data is quarterly and the model is estimated for the sample period 1996:Q1-2014:Q4 with a pre-sample 1995:Q1-1995:Q4. Here, we provide the sources and construction methods of the observed series. Unless otherwise noted, all original series are seasonally adjusted while some annual series below were converted to quarterly frequency using the Denton method (see, Di Fonzo and Marini, 2012).

2.1 UK Model

UK GDP. The UK GDP is the log of real UK GDP (code ABMI in ONS Quarterly National Accounts).

Foreign GDP. The foreign GDP is the log of trade-weighted foreign GDP. The data series for real GDPs of the foreign countries are taken from the OECD - Quarterly National Accounts. The countries are: the European Union, the United States, Switzerland, Japan, Norway and Canada. These are the most important trading partners of the United Kingdom for the period considered. We follow the paper of Loretan (2005) to construct the relative imports/exports weights.

UK crude oil production. The UK crude oil production is the log of the UK crude oil production taken from US Energy Information Administration - Monthly Energy Review - Table 11.1b.

Foreign crude oil production. The foreign crude oil production is the log of foreign crude oil production (calculated as world production net of UK production) taken from US Energy Information Administration - Monthly Energy Review - Table 11.1b.

Real oil price. The real oil price is the log of the Crude Oil Imported Acquisition Cost by Refiners from the US Energy Information Administration converted from US dollars to Sterling Pounds using the Quarterly average Spot Exchange Rate US$ into Sterling (code XUQAUSS in Bank of England Statistical Interactive Database) and deflated by the UK GDP deflator (code YBGB in ONS Quarterly National Accounts).

UK real effective exchange rate. The UK real effective exchange rate is the log of the Real Effective Exchange Rate for the Overall Economy (Index 2010) taken from the OECD Main Economic Indicators database.

UK private consumption expenditure. The UK private consumption expenditure is the Household Final Consumption Expenditure at market prices (code

ABJQ in ONS Quarterly National Accounts) and it is expressed as a share of UK GDP at Market Prices (code YBHA in ONS Quarterly National Accounts).

UK private total gross fixed capital formation. The UK private total gross fixed capital formation is the total gross fixed capital formation at market prices (code NPQS in ONS Quarterly National Accounts) and it is expressed as a share of UK GDP at Market Prices (code YBHA in ONS Quarterly National Accounts).

UK oil imports. The UK oil imports are the Crude Oil Imports (Table 3.1.1 in DUKES- UK DECC). Data at yearly frequency disaggregated into quarterly frequency, expressed as a share of UK GDP using the UK GDP at Market Prices (code YBHA in ONS Quarterly National Accounts).

UK non-oil goods imports. The UK non-oil goods imports are the Goods Imports at market prices (code BOKH in ONS Quarterly National Accounts) minus the UK oil imports, expressed as a share of UK GDP using the UK GDP at Market Prices (code YBHA in ONS Quarterly National Accounts).

UK non-oil goods exports. The UK non-oil goods exports are the Goods Exports at market prices (code BOKG in ONS Quarterly National Accounts) minus the UK oil exports, expressed as share of UK GDP using the UK GDP at Market Prices (code YBHA in ONS Quarterly National Accounts).

UK core inflation. The UK core inflation is the log change in the Consumer Price Index: All Items Excluding Food and Energy, Index 2010=100 and NSA (Organisation for Economic Co-operation and Development, Code: GBRCPICORMINMEI, retrieved from FRED, Federal Reserve Bank of St. Louis). The series is seasonally adjusted.

UK wage inflation. The UK wage inflation obtained from the log change in UK Total Compensation of Employees at Current Prices (Code DTWM in ONS - UK Output, Income and Expenditure Tables; and LF2G in ONS - LFS).

UK nominal interest rate. The UK nominal interest rate is the Bank Of England - Quarterly average rate of discount - 3 month Treasury bills (Code IUQAAJNB in Bank of England Statistical Interactive Database).

UK government debt. The UK government debt is Public Sector Finances - Net Debt at Current Prices and NSA (code RUTN in ONS Public Sector Finances). This series is seasonally adjusted and expressed as a share of UK GDP at Market Prices (code YBHA in ONS Quarterly National Accounts).

UK public total gross fixed capital formation. The UK public total gross fixed capital formation is the Central Government Total Gross Fixed Capital Formation at market prices (code RNCZ in ONS Quarterly National Accounts) and it is expressed as a share of UK GDP at Market Prices (code YBHA in ONS Quarterly National Accounts).

2.2 Russia Model

Russian GDP. The Russian GDP is the log of Gross Domestic Product at constant prices (World Economic Outlook - October 2017 Database).

Foreign GDP. The foreign GDP is the log of trade-weighted foreign GDP. The data series for real GDPs of the foreign countries are taken from the OECD - Quarterly National Accounts. The countries are: the European Union, the United States, Japan and Turkey. These are the most important trading partners of Saudi Arabia for the period considered. We follow Loretan (2005) to construct the relative imports/exports weights.

Russian crude oil production. The Russian crude oil production is the log of the Russian crude oil production taken from US Energy Information Administration - Monthly Energy Review - Table 11.1b.

Foreign crude oil production. The foreign crude oil production is the log of foreign crude oil production (calculated as world production net of Russian production) taken from US Energy Information Administration - Monthly Energy Review - Table 11.1b.

Real oil price. The real oil price is the log of the Crude Oil Imported Acquisition Cost by Refiners from the US Energy Information Administration converted from US dollars to Russian Rubles using the Spot Exchange Rate for the Russian Federation, US Dollar per National Currency provided by OECD (CCUSSP02RUM650N, retrieved from FRED, Federal Reserve Bank of St. Louis) and deflated by the Russian GDP deflator. The GDP deflator was obtained from the World Economic Outlook - October 2017 Database.

Russia real effective exchange rate. The Russian real effective exchange rate is the log of the Real Effective Exchange Rate, CPI based (Code: RECZF in International Financial Statistics - IMF).

Russian private consumption expenditure. The Russian private consumption expenditure is the Private Final Consumption at constant prices (World Economic Outlook - October 2017 Database) and is expressed as a share of Russian GDP at constant prices (World Economic Outlook - October 2017 Database).

Russian total gross fixed capital formation. The Russian private gross fixed capital formation is the Private Gross Fixed Capital Formation at constant prices (World Economic Outlook - October 2017 Database) and is expressed as a share of Russian GDP at constant prices (World Economic Outlook - October 2017 Database).

Russian total oil demand. The Russian total oil demand is the log of Total Oil Demand taken from the IEA - World Oil Statistics. Data at yearly frequency disaggregated into quarterly frequency.

Russian non-oil goods imports. The Russian non-oil goods imports is the Value of Non-Oil Imports at current prices (World Economic Outlook - October 2017 Database) and it is expressed as a share of Russian GDP at current prices (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Russian non-oil goods exports. The Russian non-oil goods exports is the Value of Non-Oil Exports at current prices (World Economic Outlook - October 2017 Database) and it is expressed as a share of Russian GDP at current prices (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Russian headline inflation. The Russian headline inflation is the log change in the Consumer Price Index (1997=100) period average (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Russian wage inflation. The Russian wage inflation was obtained from the Average Monthly Accrued Wages (taken from ROSSTAT). This series is seasonally adjusted. Before 2010 we applied the growth rate of the GDP deflator to obtain the series backward.

Russian nominal interest rate. The Russian nominal interest rate is the Short-Term Deposit Rate (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Russian government debt. The Russian government debt is the General Government Gross Debt as % of GDP (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Russian public gross fixed capital formation. The Russian public gross fixed capital formation is the Public Gross Fixed Capital Formation at current prices (World Economic Outlook - October 2017 Database) and it is expressed as a share of Russian GDP at current prices (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

2.3 Saudi Arabia Model

Saudi Arabia GDP. The Saudi Arabia GDP is the log of Gross Domestic Product at constant prices (World Economic Outlook - October 2017 Database). Data at yearly frequency disaggregated into quarterly frequency.

Foreign GDP. The foreign GDP is the log of trade-weighted foreign GDP. The data series for real GDPs of the foreign countries are taken from the OECD - Quarterly National Accounts. The countries are: the European Union, the United States, Japan, Korea and India. These are the most important trading partners of Saudi Arabia for the period considered. We follow Loretan (2005) to construct the relative imports/exports weights.

Saudi Arabia crude oil production. The Saudi Arabia crude oil production is the log of the Saudi Arabia crude oil production taken from US Energy Information Administration - Monthly Energy Review - Table 11.1b.

Foreign crude oil production. The foreign crude oil production is the log of foreign crude oil production (calculated as world production net of Saudi Arabia production) taken from US Energy Information Administration - Monthly Energy Review - Table 11.1b.

Real oil price. The real oil price is the log of the Crude Oil Imported Acquisition Cost by Refiners from the US Energy Information Administration converted from US dollars to Saudi Riyal using the Quarterly Average Spot Exchange Rate, US$ into Saudi Riyal (code XUQASRD in Bank of England Statistical Interactive Database) and deflated by the Saudi Arabia GDP deflator. The GDP deflator was obtained as Nominal GDP / Real GDP (both taken from World Economic Outlook - October 2017 Database). The GDP deflator at yearly frequency was disaggregated into quarterly frequency.

Saudi Arabia real effective exchange rate. The Saudi Arabia real effective exchange rate is the log of the Real Effective Exchange Rate, CPI based (International Financial Statistics - IMF).

Saudi Arabia private consumption expenditure. The Saudi Arabia private consumption expenditure is the Private Final Consumption at current prices (World Economic Outlook - October 2017 Database) and it is expressed as a share of Saudi Arabia GDP at current prices (World Economic Outlook - October 2017 Database).

Data at yearly frequency were disaggregated into quarterly frequency.

Saudi Arabia private gross fixed capital formation. The Saudi Arabia private gross fixed capital formation is the Private Gross Fixed Capital Formation at current prices (World Economic Outlook - October 2017 Database) and it is expressed as a share of Saudi Arabia GDP at current prices (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Saudi Arabia total oil demand. The Saudi Arabia total oil demand is the log of Total Oil Demand taken from the IEA - World Oil Statistics. Data at yearly frequency disaggregated into quarterly frequency.

Saudi Arabia non-oil goods imports. The Saudi Arabia non-oil goods imports is the Value of Non-Oil Imports at current prices (World Economic Outlook - October 2017 Database) and it is expressed as a share of Saudi Arabia GDP at current prices (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Saudi Arabia non-oil goods exports. The Saudi Arabia non-oil goods exports is the Value of Non-Oil Exports at current prices (World Economic Outlook - October 2017 Database) and it is expressed as a share of Saudi Arabia GDP at current prices (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Saudi Arabia headline inflation. The Saudi Arabia headline inflation is the log change in the Consumer Price Index (1997=100) period average (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Saudi Arabia wage inflation. The Saudi Arabia wage inflation obtained from the log change in Saudi Total Compensation of Employees at Current Prices (Total compensation of employees in IMF - World Economic Outlook - October 2017 Database; and Total Employed in IMF World Economic Outlook - October 2017 Database). Before 2012 we applied growth rate of GDP Deflator to obtain the series total compensation of employees. Data at yearly frequency were disaggregated into quarterly frequency.

Saudi Arabia nominal interest rate. The Saudi Arabia nominal interest rate is the Short-Term Deposit Rate (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Saudi Arabia government debt. The Saudi Arabia government debt is General

Government Gross Debt at current prices (World Economic Outlook - October 2017 Database) and it is expressed as a share of Saudi Arabia GDP at current prices (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

Saudi Arabia public gross fixed capital formation. The Saudi Arabia public gross fixed capital formation is the Public Gross Fixed Capital Formation at current prices (World Economic Outlook - October 2017 Database) and it is expressed as a share of Saudi Arabia GDP at current prices (World Economic Outlook - October 2017 Database). Data at yearly frequency were disaggregated into quarterly frequency.

3 Appendix C: Construction of the Tax Rates and Data Sources

This appendix describes the construction of the tax rates for the UK, Russian and Saudi economies and their related, additional data sources.

3.1 UK Model

We consider the sample period 1997-2014 and have collected all data series in nominal values.

Consumption Tax Revenues. The consumption tax revenues, T_1^c, are VAT revenues (code NZGF in ONS Public Sector Finances - Table PSF3D).

Consumption Tax Rate. The average consumption tax rate is defined as:

$$(\tau_1^c)^{SS} = \frac{T_1^c}{C_1 - T_1^c} \tag{C1}$$

Labour Income Tax Revenues. The labour income tax revenues, T_1^l, include self assessed income tax, paye IT, other income tax and miscellaneous revenues (codes LIBS, MS6W, MF6X, and MF6Z in ONS Public Sector Finances - Table PSF3D).

Labour Income Tax Rate. The average labour income tax rate is defined as:

$$\left(\tau_1^l\right)^{SS} = \frac{T_1^l}{TCE_1} \tag{C2}$$

where TCE_1 is the total compensation of employees (code DWTM in ONS UK Economic Accounts).

Social Security Tax Revenues (of Households and Firms). The social security tax revenues (of households and firms), T_1^w, are National Insurance Contributions revenues (code AIIH in ONS Public Sector Finances - Table PSF3D).

Social Security Tax Rates (of Households and Firms). As described in the main text, we assume the same tax rates for social security tax paid by households, $\left(\tau_1^{wh}\right)^{SS}$, and firms, $\left(\tau_1^{wf}\right)^{SS}$. The average social security tax rates paid by households and firms are defined as:

$$\left(\tau_1^{wh}\right)^{SS} = \left(\tau_1^{wf}\right)^{SS} = \frac{T_1^w}{TCE_1} \tag{C3}$$

Fuel Duty Tax Revenues. The fuel duty tax revenues, T_1^{oc}, are Fuel Duty revenues (code CUDG in ONS Public Sector Finances - Table PSF3D).

Fuel Duty Tax Rate. The average fuel duty tax rate is defined as:

$$(\tau_1^{oc})^{SS} = \frac{T_1^{oc}/2}{AFR_1} \tag{C4}$$

where AFR_1 is the series of total retail sales of automotive fuel (code IZ57 in ONS Retail Sales - Table ValSAT). As per numerator of (C4), we use the share of the automotive sector fuel consumption as share of total petroleum refined products (that is about 50%).

Corporation Tax Revenues. The corporation tax revenues, T_1^d, are Corporation Tax revenues (code ACCD in ONS Public Sector Finances - Table PSF3D).

Corporation Tax Rate. The average corporation tax rate is defined as:

$$\left(\tau_1^d\right)^{SS} = \frac{T_1^d - T_1^{dyo}}{GOSC_1} \tag{C5}$$

where T_1^{dyo} is the series of Total Corporation Tax coming from UK oil production (in ONS Statistics of Government Revenues from UK Oil and Gas Production - Table T11.11) and $GOSC_1$ is the series of gross operating surplus of corporations (code CGBY in ONS UK Economic Accounts) excluding the share of gross operating surplus of corporations involved in oil production.

Petroleum Revenue Tax Revenues. The petroleum revenue tax revenues, T_1^{yo}, are Petroleum Revenue Tax revenues (code ACCJ in ONS Public Sector Finances - Table PSF3D).

Petroleum Revenue Tax Rate. The average petroleum revenue tax rate is defined as:

$$(\tau_1^{yo})^{SS} = \frac{T_1^{yo} + T_1^{dyo} + R_1^{yo}}{RCOS_1} \tag{C6}$$

where R_1^{yo} is the series of Royalties coming from UK oil production (in ONS Statistics of Government Revenues from UK Oil and Gas Production - Table T11.11) and $RCOS_1$ is the series of revenue from crude oil sales (Table 4 in Scottish National Account Project - Oil and Gas Statistics).

3.2 Russia Model

Only constructed and estimated tax rates are shown below. All the other tax rates are shown in Table 2 in the main body of the paper.

Corporation Tax Revenues. The corporation tax revenues, T_1^d, are revenues from Taxes on Income, Profits, and Capital Gains (World Economic Outlook - October 2017 Database).

Corporation Tax Rate. The average corporation tax rate is defined as:

$$\left(\tau_1^d\right)^{SS} = \frac{T_1^d}{GOSC_1} \tag{C7}$$

where $GOSC_1$ is the series of gross operating surplus of firms (for 2010-2014, data are taken from ROSSTAT whereas, for the previous period, the series is obtained applying the growth rate of nominal GDP).

Petroleum Revenue Tax Revenues. The petroleum revenue tax revenues, T_1^{yo}, are Commodity-Related Revenues (World Economic Outlook - October 2017 Database).

Petroleum Revenue Tax Rate. The average petroleum revenue tax rate is defined as:

$$(\tau_1^{yo})^{SS} = \frac{T_1^{yo}}{OIL\ GDP_1} \tag{C8}$$

where $OIL\ GDP$ is estimated from the Russian total oil production (data taken from the IEA - World Oil Statistics).

3.3 Saudi Arabia Model

We use the sample period 1995-2014 and have collected all data series in nominal values.

Consumption Tax Revenues. The consumption tax revenues, T_1^c, are revenues form Taxes Goods & Services (World Economic Outlook - October 2017 Database).

Consumption Tax Rate. The average consumption tax rate is defined as:

$$(\tau_1^c)^{SS} = \frac{T_1^c}{C_1 - T_1^c} \tag{C9}$$

Social Security Tax Revenues (of Households and Firms). The social security tax revenues (of households and firms), T_1^w, are revenues from Social Benefits (World Economic Outlook - October 2017 Database).

Households Social Security Tax Rate. The average households social security tax rate is defined as:

$$\left(\tau_1^{wh}\right)^{SS} = \frac{T_1^w - T_1^{wf}}{TCE_1} \tag{C10}$$

where T_1^{wf} is Social Security Benefits (World Economic Outlook - October 2017 Database) TCE_1 is the total compensation of employees.

Firms Social Security Tax Rate. The average firm social security tax rate is defined as:

$$\left(\tau_1^{wf}\right)^{SS} = \frac{T_1^{wf}}{TCE_1} \tag{C11}$$

Corporation Tax Revenues. The corporation tax revenues, T_1^d, are revenues from Taxes on Income, Profits, and Capital Gains (World Economic Outlook - October 2017 Database).

Corporation Tax Rate. The average corporation tax rate is defined as:

$$\left(\tau_1^d\right)^{SS} = \frac{T_1^d}{GOSC_1} \tag{C12}$$

where $GOSC_1$ is the series of gross operating surplus of firms (for 2012-2014, data are taken from the Saudi Arabian Monetary Authority whereas, for the previous period, the series is obtained applying the growth rate of nominal GDP).

Petroleum Revenue Tax Revenues. The petroleum revenue tax revenues, T_1^{yo}, are Commodity-Related Revenues (World Economic Outlook - October 2017 Database).

Petroleum Revenue Tax Rate. The average petroleum revenue tax rate is defined as:

$$(\tau_1^{yo})^{SS} = \frac{T_1^{yo}}{OIL\ GDP_1} \tag{C13}$$

where $OIL\ GDP_1$ is the oil GDP (World Economic Outlook - October 2017 Database).

4 Appendix D: Parameters Values

4.1 UK Model

4.1.1 Average Ratios for the UK Economy

Unless otherwise noted, the average ratios are computed for the period 1995-2014.

The UK oil share to nominal gross output is defined as:

$$shareoy = \frac{GVA - GVA\ excl\ Oil\ and\ Gas}{GVA} \tag{D1}$$

where GVA is the total gross value added at basic prices (Code ABMM in ONS) and $GVA\ excl\ Oil\ and\ Gas$ is the gross value added excluding oil and gas at basic prices (Code KLS2 in ONS). Due to data availability issues, for these two series, the sample period is 1997-2014.

The ratio between oil used in production and oil used in consumption is defined as:

$$ratiooyoc = \frac{O^y}{O^c} \tag{D2}$$

where O^y is the total supply of products at purchasers' prices of coke and refined petroleum products (taken from ONS Input-Output Tables). O^c is the households final consumption expenditure of coke and refined petroleum products (taken ONS Input-Output Tables).

The share of investment to GDP is defined as:

$$shareiy = \frac{I}{Y} \tag{D3}$$

where I is the real total gross fixed capital formation (Code NPQT in ONS) and Y is the real gross domestic product (Code ABMI in ONS).

The share of consumption government spending to GDP is defined as:

$$sharegy = \frac{G}{Y} \tag{D4}$$

where G is the real general government final consumption expenditure (Code NMRY in ONS).

The share of investment government spending to GDP is defined as:

$$shareigy_1 = \frac{I^g}{Y} \tag{D5}$$

where I^g is the central government total gross fixed capital formation at current prices (Code RNCZ in ONS) and Y is the GDP at current prices (code YBHA in ONS Quarterly

National Accounts). Note that, due to data availability issues, to obtain this ratio we had to use nominal vararaibles.

The ratio of oil production to oil demand is defined as:

$$ratioyoo = \frac{Y^o}{O} \tag{D6}$$

where Y^o is the crude oil production of the United Kingdom (taken from the US Energy Information Administration - Monthly Energy Review - Table 11.1b) and O is the petroleum consumption of United Kingdom (taken from the US Energy Information Administration - Monthly Energy Review - Table 11.2).

The share of imports to GDP is defined as:

$$sharemy = \frac{M}{Y} \tag{D7}$$

where M are the real imports of goods and services (Code IKBL in ONS).

The ratio of imports of investment goods to imports of consumption goods is defined as:

$$ratioyoo = \frac{M^i}{M^c} \tag{D8}$$

where M^i are total imports of services (Code IKBC in ONS) and M^c are total imports of goods (Code IKBI in ONS).

Finally, ζ_1 is the average between total oil production and consumption of the UK (IEA - World Oil Statistics).

4.1.2 Average Ratios for the Foreign Bloc

In order to aggregate data for the foreign bloc we use the Loretan (2005) technique. In particular, we proceed as follows. Firstly, from the Direction of Trade Statistics database of the International Monetary Fund we compute the average of imports/exports between the UK and foreign countries for the period 1995-2014. Secondly, we select the major UK trading partners. Specifically, they are the European Union, the United States, China, Switzerland, Japan, Norway and Canada. The average of imports/exports of the UK with this group of countries corresponds to the 80% of total UK trade. Thirdly, we compute the average ratios for the foreign bloc by aggregating their data series through the weighted average for the period 1995-2014.

Unless otherwise noted, the average ratios are computed for the period 1995-2014.

Due to data inavailability the construction of the foreign oil share only considers the European Union and the United States. The European Union and the US oil shares on their nominal gross outputs are defined as:

$$shareoy = \frac{Petroleum\ Items\ Exp}{GDP} \tag{D9}$$

For the Europaen Union, *Petroleum Items Exp* is the sum of crude petroleum and natural gas expenditures and coke, refined petroleum products and nuclear fuels expenditures (taken from Eurostat Input-Output Tables). *GDP* is the nominal gross domestic product (taken from the Eurostat interactive database).

For the US, *Petroleum Items Exp* is the sum of natural gas expenditures and petroleum expenditures (taken from the Annual Energy Outlook of US EIA). *GDP* is the nominal gross domestic product (taken from FRED).

Again, due to data inavailability, the construction of the foreign oil share considers only the European Union and the United States. The ratio between oil used in production and oil used in consumption is defined as:

$$ratiooyoc = \frac{O^y}{O^c} \tag{D10}$$

For the European Union, O^y is the total use at basic price of coke, refined petroleum products and nuclear fuel (taken from the Eurostat Input-Output Tables). O^c is the final consumption expenditure at basic price of coke, refined petroleum products and nuclear fuel (taken from the Eurostat Input-Output Tables). Due to data availability issues of these two series we consider only the sample period 2000-2014.

For the US, O^y is the total commodity output of petroleum and coal products (taken from the Bureau of Economic Analysis Input-Output Tables). O^c is the personal consumption expenditures of petroleum and coal products (taken from Bureau of Economic Analysis Input-Output Tables). Due to the availability issues of these two series we consider the sample period 1998-2014.

The share of investment to GDP for foreign countries is defined as:

$$shareiy = \frac{I}{Y} \tag{D11}$$

For the European Union, the United States, Switzerland, Japan, Norway and Canada, I is the gross fixed capital formation (taken from OECD - Quarterly National Accounts) whereas Y is the gross domestic product (taken from OECD - Quarterly National Accounts).

For China (sample period 1998-2014) I is the gross fixed capital formation (Code 93E.ZF in International Financial Statistics - IMF) whereas Y is the gross domestic product (Code 99B.ZF in International Financial Statistics - IMF).

The share of government spending to GDP for foreign countries is defined as:

$$sharegy = \frac{G}{Y} \tag{D12}$$

For the European Union, the United States, Switzerland, Japan, Norway and Canada, G is the general government final consumption expenditure (taken from OECD - Quarterly National Accounts) whereas Y is the gross domestic product (taken from OECD - Quarterly National Accounts).

For China (sample period 1998-2014), G is the government consumption expenditure (Code 91F.ZF in International Financial Statistics - IMF) whereas Y is the gross domestic product (Code 99B.ZF in International Financial Statistics - IMF).

The ratio between oil production and oil demand for theforeign bloc is defined as:

$$ratioyoo = \frac{Y^o}{O} \tag{D13}$$

For the European Union, the United States, China, Switzerland, Japan, Norway and Canada, Y^o is petroleum production (taken from the US Energy Information Administration - International Energy Statistics) whereas O is the petroleum consumption (taken from the US Energy Information Administration - International Energy Statistics).

The ratio of imports of investment goods to imports of consumption goods for the foreign bloc is defined as:

$$ratioyoo = \frac{M^i}{M^c} \tag{D14}$$

For the European Union, the United States, Switzerland, Japan, Norway and Canada, M^i is the series of imports of goods (taken from OECD - Quarterly National Accounts) whereas M^c is the series of imports of services (taken from OECD - Quarterly National Accounts).

Finally, ζ_2 is the average of total oil production to consumption of the rest of the world minus the UK (IEA - World Oil Statistics).

4.2 Russia Model

4.2.1 Average Ratios for the Russian Economy

Unless otherwise noted, the average ratios are computed for the period 1995-2014.

The Russian oil share to nominal gross output is defined as:

$$shareoy = \frac{OIL\ GDP}{Total\ GDP} \tag{D15}$$

where $OIL\ GDP$ is estimated from Russian total oil production (data taken from the IEA - World Oil Statistics) and $Total\ GDP$ is the Gross Domestic Product at constant prices (World Economic Outlook - October 2017 Database).

The ratio of oil used in production to oil used in consumption is defined as:

$$ratiooyoc = \frac{O^y}{O^c} \tag{D16}$$

where O^y and O^c are estimated from the Russian total oil production and demand (data taken from the IEA - World Oil Statistics).

The share of investment to GDP is defined as:

$$shareiy = \frac{I}{Y} \tag{D17}$$

where I is the Gross Fixed Capital Formation at constant prices (World Economic Outlook - October 2017 Database) and Y is Gross Domestic Product at constant prices (World Economic Outlook - October 2017 Database).

The share of government spending to GDP is defined as:

$$sharegy = \frac{G}{Y} \tag{D18}$$

where G is Final Public Consumption Expenditure at constant prices (World Economic Outlook - October 2017 Database).

The share of investment government spending to GDP is defined as:

$$shareigy_1 = \frac{I^g}{Y} \tag{D19}$$

where I^g is the Public Gross fixed capital formation at constant prices (World Economic Outlook - October 2017 Database).

The ratio of oil production to oil demand is defined as:

$$ratioyoo = \frac{Y^o}{O} \tag{D20}$$

where Y^o is the oil production of Russia (taken from the taken from the IEA - World Oil Statistics) and O is the oil demand of Russia (taken from the taken from the IEA - World Oil Statistics).

The share of imports to GDP is defined as:

$$sharemy = \frac{M}{Y} \tag{D21}$$

where M are the Imports of Goods and Services at constant prices (World Economic Outlook - October 2017 Database).

The ratio of imports of investment goods to imports of consumption goods is defined as:

$$ratioyoo = \frac{M^i}{M^c} \tag{D22}$$

where M^i are the Imports of Services at constant prices (World Economic Outlook - October 2017 Database) and M^c are Imports of Goods at constant prices (World Economic Outlook - October 2017 Database).

Finally, ζ_1 is the average between total oil production and consumption of Russia (IEA - World Oil Statistics)

4.2.2 Average Ratios for the Foreign Bloc

As before, in order to aggregate data for the foreign countries we use the Loretan (2005) technique. In particular, we proceed as follows. Firstly, from the Direction of Trade Statistics database of the International Monetary Fund we compute the average of imports/exports between the Russia and foreign countries for the period 1995-2014. Secondly, we select the major Russian trading partners. Specifically, they are the European Union, China, Ukraine, Turkey, Belarus, the United States and Japan. The average of imports/exports of Russia with this group of countries corresponds to the 78% of total Russia trade. Thirdly, we compute the average ratios for the foreign bloc by aggregating their data series through the weighted average for the period 1995-2014.

Unless otherwise noted, the average ratios are computed for the period 1995-2014.

Due to data availability issues to construct foreign oil share we only consider the European Union and the United States. The European Union and the US oil shares on their nominal gross outputs are defined as:

$$shareoy = \frac{Petroleum\ Items\ Exp}{GDP} \tag{D23}$$

For the European Union, $Petroleum\ Items\ Exp$ is the sum of crude petroleum and natural gas expenditures and coke, refined petroleum products and nuclear fuels expenditures (taken from Eurostat Input-Output Tables). GDP is the nominal gross domestic product (taken from the Eurostat interactive database). Due to data availability the sample period is 2000-2014.

For the US, *Petroleum Items Exp* is the sum of natural gas expenditures and petroleum expenditures (taken from the Annual Energy Outlook of US EIA). GDP is the nominal gross domestic product (taken from FRED).

Due to data availability issues, in order to construct the foreign oil share we only consider the European Union and the United States. The ratio between oil used in production and oil used in consumption is defined as:

$$ratiooyoc = \frac{O^y}{O^c} \tag{D24}$$

For the European Union, O^y is the total use at basic price of coke, refined petroleum products and nuclear fuel (taken from the Eurostat Input-Output Tables). O^c is the final consumption expenditure at basic price of coke, refined petroleum products and nuclear fuel (taken from the Eurostat Input-Output Tables). Due to the availability of these two series we consider the sample period 2000-2014.

For the US, O^y is the total commodity output of petroleum and coal products (taken from the Bureau of Economic Analysis Input-Output Tables). O^c is the personal consumption expenditures of petroleum and coal products (taken from Bureau of Economic Analysis Input-Output Tables). Due to the availability of these two series we consider the sample period 1998-2014.

The share of investment to GDP for foreign countries is defined as:

$$shareiy = \frac{I}{Y} \tag{D25}$$

For the European Union, the United States and Japan, I is the gross fixed capital formation (taken from OECD - Quarterly National Accounts) whereas Y is the gross domestic product (taken from OECD - Quarterly National Accounts).

For China (sample period 1998-2005), Ukraine, Turkey and Belarus, I is the gross fixed capital formation (Code 93E.ZF in International Financial Statistics - IMF) whereas Y is the gross domestic product (Code 99B.ZF in International Financial Statistics - IMF).

The share of government spending to GDP for foreign countries is defined as:

$$sharegy = \frac{G}{Y} \tag{D26}$$

For the European Union, the United States and Japan G is the general government final consumption expenditure (taken from OECD - Quarterly National Accounts) whereas Y is the gross domestic product (taken from OECD - Quarterly National Accounts).

For China (sample period 1998-2005), Ukraine, Turkey and Belarus, G is the government consumption expenditure (Code 91F.ZF in International Financial Statistics - IMF) whereas Y is the gross domestic product (Code 99B.ZF in International Financial Statistics - IMF).

The ratio between oil production and oil demand for theforeign bloc is defined as:

$$ratioyoo = \frac{Y^o}{O} \tag{D27}$$

For the European Union, China, Ukraine, Turkey, Belarus, the United States and Japan, Y^o is petroleum production (taken from the US Energy Information Administration - International Energy Statistics) whereas O is the petroleum consumption (taken from the US Energy Information Administration - International Energy Statistics).

The ratio between imports of investment goods and imports of consumption goods for the foreign bloc is defined as:

$$ratioyoo = \frac{M^i}{M^c} \tag{D28}$$

For the European Union, the United States and Japan, M^i is the series of imports of goods (taken from OECD - Quarterly National Accounts) whereas M^c is the series of imports of services (taken from OECD - Quarterly National Accounts).

Finally, ζ_2 is the average between total oil production and consumption of the rest of the world minus Russia (IEA - World Oil Statistics).

4.3 Saudi Arabia Model

4.3.1 Average Ratios for Saudi Arabia

Unless otherwise noted, the average ratios are computed for the period 1995-2014.

The Saudi Arabia oil share to nominal gross output is defined as:

$$shareoy = \frac{OIL\ GDP}{Total\ GDP} \tag{D29}$$

where $OIL\ GDP$ is the Oil Gross Domestic Product at constant prices (World Economic Outlook - October 2017 Database) and $Total\ GDP$ is the Gross Domestic Product at constant prices (World Economic Outlook - October 2017 Database).

The ratio between oil used in production and oil used in consumption is defined as:

$$ratiooyoc = \frac{O^y}{O^c} \tag{D30}$$

where O^y and O^c are estimated from the Saudi Arabia total oil production and demand (data taken from the IEA - World Oil Statistics).

The share of investment to GDP is defined as:

$$shareiy = \frac{I}{Y} \tag{D31}$$

where I is the Gross Fixed Capital Formation at constant prices (World Economic Outlook - October 2017 Database) and Y is Gross Domestic Product at constant prices (World Economic Outlook - October 2017 Database).

The share of consumption government spending to GDP is defined as:

$$sharegy = \frac{G}{Y} \tag{D32}$$

where G is Final Public Consumption Expenditure at constant prices (World Economic Outlook - October 2017 Database).

The share of investment government spending to GDP is defined as:

$$shareigy_1 = \frac{I^g}{Y} \tag{D33}$$

where I^g is the Public Gross fixed capital formation at constant prices (World Economic Outlook - October 2017 Database).

The ratio between oil production and oil demand is defined as:

$$ratioyoo = \frac{Y^o}{O} \tag{D34}$$

where Y^o is the oil production of Saudi Arabia (taken from the taken from the IEA - World Oil Statistics) and O is the oil demand of Saudi Arabia (taken from the taken from the IEA - World Oil Statistics).

The share of imports to GDP is defined as:

$$sharemy = \frac{M}{Y} \tag{D35}$$

where M are the Imports of Goods and Services at constant prices (World Economic Outlook - October 2017 Database).

The ratio between imports of investment goods and imports of consumption goods is defined as:

$$ratioyoo = \frac{M^i}{M^c} \tag{D36}$$

where M^i are the Imports of Services at constant prices (World Economic Outlook - October 2017 Database) and M^c are Imports of Goods at constant prices (World Economic Outlook - October 2017 Database).

Finally, ζ_1 is the average between total oil production and consumption of Saudi Arabia (IEA - World Oil Statistics).

4.4 Average Ratios for the Foreign Bloc

As before, in order to aggregate data for the foreign countries we use the Loretan (2005) technique. In particular, we proceed as follows. Firstly, from the Direction of Trade Statistics database of the International Monetary Fund we compute the average of imports/exports between the Saudi Arabia and foreign countries for the period 1995-2014. Secondly, we select the major Saudi Arabia trading partners. Specifically, they are the European Union, the United States, Japan, China, Korea, India and United Arab Emirates. The average of imports/exports of Saudi Arabia with this group of countries corresponds to the 70% of total Saudi Arabia trade. Thirdly, we compute the average ratios for the foreign bloc by aggregating their data series through the weighted average for the period 1995-2014.

Unless otherwise noted, the average ratios are computed for the period 1995-2014.

Due to data availability issues, again, to construct foreign oil share we only consider the European Union and the United States. The European Union and the US oil shares on their nominal gross outputs are defined as:

$$shareoy = \frac{Petroleum\ Items\ Exp}{GDP} \tag{D37}$$

For the European Union, *Petroleum Items Exp* is the sum of crude petroleum and natural gas expenditures and coke, refined petroleum products and nuclear fuels expenditures (taken from Eurostat Input-Output Tables). GDP is the nominal gross domestic product (taken from the Eurostat interactive database). Due to data availability the sample period is 2000-2014.

For the US, *Petroleum Items Exp* is the sum of natural gas expenditures and petroleum expenditures (taken from the Annual Energy Outlook of US EIA). GDP is the nominal gross domestic product (taken from FRED).

Due to data availability, in order to construct foreign oil share we only consider the European Union and the United States. The ratio between oil used in production and oil used in consumption is defined as:

$$ratiooyoc = \frac{O^y}{O^c} \tag{D38}$$

For the European Union, O^y is the total use at basic price of coke, refined petroleum products and nuclear fuel (taken from the Eurostat Input-Output Tables). O^c is the final consumption expenditure at basic price of coke, refined petroleum products and nuclear fuel (taken from the Eurostat Input-Output Tables). Due to the availability of these two series we consider the sample period 2000-2014.

For the US, O^y is the total commodity output of petroleum and coal products (taken from the Bureau of Economic Analysis Input-Output Tables). O^c is the personal consumption expenditures of petroleum and coal products (taken from Bureau of Economic Analysis Input-Output Tables). Due to the availability of these two series we consider the sample period 1998-2014.

The share of investment to GDP for foreign countries is defined as:

$$shareiy = \frac{I}{Y} \tag{D39}$$

For the European Union, the United States and Japan I is the gross fixed capital formation (taken from OECD - Quarterly National Accounts) whereas Y is the gross domestic product (taken from OECD - Quarterly National Accounts).

For China (sample period 1998-2014), Korea, India and the United Arab Emirates, I is the gross fixed capital formation (Code 93E.ZF in International Financial Statistics - IMF) whereas Y is the gross domestic product (Code 99B.ZF in International Financial Statistics - IMF).

The share of government spending to GDP for foreign countries is defined as:

$$sharegy = \frac{G}{Y} \tag{D40}$$

For the European Union, the United States and Japan, G is the general government final consumption expenditure (taken from OECD - Quarterly National Accounts) whereas Y is the gross domestic product (taken from OECD - Quarterly National Accounts).

For China (sample period 1998-2014), Korea, India and the United Arab Emirates, G is the government consumption expenditure (Code 91F.ZF in International Financial Statistics - IMF) whereas Y is the gross domestic product (Code 99B.ZF in International Financial Statistics - IMF).

The ratio between oil production and oil demand for theforeign bloc is defined as:

$$ratioyoo = \frac{Y^o}{O} \tag{D41}$$

For the European Union, the United States, Japan, China, Korea, India and United Arab Emirates, Y^o is petroleum production (taken from the US Energy Information Administration - International Energy Statistics) whereas O is the petroleum consumption (taken from the US Energy Information Administration - International Energy Statistics).

The ratio between imports of investment goods and imports of consumption goods for the foreign bloc is defined as:

$$ratioyoo = \frac{M^i}{M^c} \tag{D42}$$

For the European Union, the United States and Japan,, M^i is the series of imports of goods (taken from OECD - Quarterly National Accounts) whereas M^c is the series of imports of services (taken from OECD - Quarterly National Accounts).

Finally, ζ_2 is the average between total oil production and consumption of the rest of the world minus Saudi Arabia (IEA - World Oil Statistics).

5 Appendix E: Priors and posteriors values for shocks parameters of the three models

Table A1: Priors and Posteriors for the Persistences of Shock Processes Parameters for the three models

Par.	Description	Priors									Posteriors								
		Russia			*Saudi Arabia*			*UK*			*Russia*			*Saudi Arabia*			*UK*		
		Distr.	Mean	St. Dev.	Distr.	Mean	St. Dev.	Distr.	Mean	St. Dev.	Mean	Conf. Inter.		Mean	Conf. Inter.		Mean	Conf. Inter.	
$\rho^{z}_{1,1}$	Home Productivity	B	0.0100	0.0050	B	0.0100	0.0050	B	0.0100	0.0050	0.0102	0.0023	0.0178	0.0121	0.0028	0.0207	0.0090	0.0022	0.0156
$\rho^{z}_{2,1}$	Home Productivity	B	0.0045	0.0040	B	0.0045	0.0040	B	0.0045	0.0040	0.0169	0.0031	0.0301	0.0021	0.0001	0.0039	0.0069	0.0001	0.0139
$\rho^{yo}_{1,1}$	Home Oil Supply	B	0.1000	0.0010	B	0.3000	0.0050	B	0.9000	0.0500	0.1000	0.0983	0.1016	0.3200	0.2919	0.3083	0.6750	0.5170	0.8387
$\rho^{yo}_{2,1}$	Home Oil Supply	B	0.0010	0.0005	B	0.0045	0.0040	B	0.0045	0.0040	0.0010	0.0002	0.0017	0.0033	0.0003	0.0059	0.0087	0.0007	0.0161
$\rho^{yo}_{1,2}$	Foreign Oil Supply	B	0.1000	0.0010	B	0.3000	0.0050	B	0.9000	0.0500	0.1000	0.0983	0.1016	0.2998	0.2917	0.3080	0.7460	0.6073	0.8900
$\rho^{yo}_{2,2}$	Foreign Oil Supply	B	0.0010	0.0005	B	0.0045	0.0040	B	0.0045	0.0040	0.0010	0.0002	0.0017	0.0059	0.0001	0.0123	0.0050	0.0001	0.0111
$\rho^{zo}_{1,1}$	Home Oil Intensity	B	0.9000	0.0500	B	0.0010	0.0005	B	0.9000	0.0500	0.4603	0.3943	0.5121	0.0010	0.0002	0.0017	0.6340	0.4937	0.7778
$\rho^{zo}_{2,1}$	Home Oil Intensity	B	0.0010	0.0001	B	0.0045	0.0020	B	0.0045	0.0020	0.0005	0.0003	0.0007	0.0061	0.0024	0.0098	0.0054	0.0018	0.0090
$\rho^{zc}_{1,1}$	Home Priv. Consumption	B	0.7000	0.2000	B	0.7000	0.2000	B	0.7000	0.2000	0.2700	0.1153	0.4156	0.8125	0.6862	0.9448	0.3797	0.1321	0.6156
$\rho^{zi}_{1,1}$	UK Priv. Investment	B	0.7000	0.2000	B	0.7000	0.2000	B	0.7000	0.2000	0.4163	0.2114	0.6197	0.4500	0.2661	0.6290	0.2365	0.0798	0.3840
$\rho^{zm}_{1,1}$	UK Import Preferences	B	0.0010	0.0005	B	0.0010	0.0005	B	0.9000	0.0500	0.0010	0.0002	0.0017	0.0010	0.0002	0.0017	0.5393	0.4301	0.6446
$\rho^{zm}_{2,1}$	UK Import Preferences	B	0.0045	0.0040	B	0.0045	0.0040	B	0.0045	0.0040	0.0064	0.0001	0.0138	0.0037	0.0001	0.0083	0.0055	0.0005	0.0094
$\rho^{p}_{1,1}$	UK Price Markup	B	0.7000	0.2000	B	0.7000	0.2000	B	0.7000	0.2000	0.7566	0.6477	0.8743	0.7094	0.4179	0.9978	0.3178	0.1655	0.4672
$\rho^{w}_{1,1}$	UK Wage Markup:	B	0.7000	0.2000	B	0.7000	0.2000	B	0.7000	0.2000	0.6982	0.4032	0.9986	0.7037	0.4156	0.9992	0.6974	0.4064	0.9987
$\rho^{\pi}_{1,1}$	UK Inflation Target	B	0.7000	0.2000	B	0.7000	0.2000	B	0.7000	0.2000	0.9882	0.9757	0.9999	0.9786	0.9537	0.9999	0.9582	0.9388	0.9930
$\rho^{g}_{1,1}$	UK Gov. Cons. Expenditure	B	0.7000	0.2000	B	0.7000	0.2000	B	0.7000	0.2000	0.9351	0.8819	0.9838	0.8470	0.7636	0.9385	0.5313	0.3775	0.6815
$\rho^{ag}_{1,1}$	UK Authorized Gov. Inv. Expenditure	B	0.7000	0.2000	B	0.7000	0.2000	B	0.7000	0.2000	0.8870	0.8092	0.9696	0.8521	0.7682	0.9367	0.1309	0.0267	0.2288

Table A2: Priors and posteriors for the standard deviations of shock processes parameters for the three models

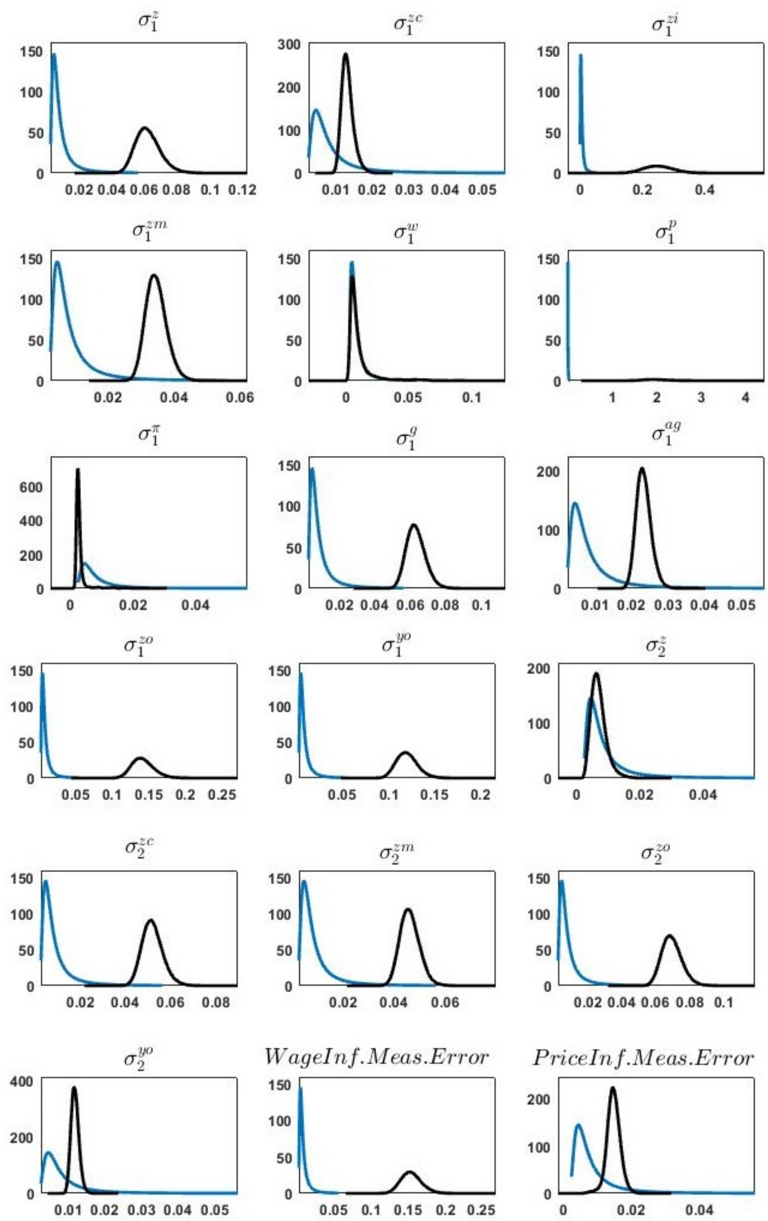
Par.	Description	Priors									Posteriors								
		Russia			*Saudi Arabia*			*UK*			*Russia*			*Saudi Arabia*			*UK*		
		Distr.	Mean	St. Dev.	Distr.	Mean	St. Dev.	Distr.	Mean	St. Dev.	Mean	Conf. Inter.		Mean	Conf. Inter.		Mean	Conf. Inter.	
σ_1^z	Home Productivity	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0707	0.0584	0.0827	0.0971	0.0821	0.1120	0.0621	0.0499	0.0740
σ_2^z	Foreign Productivity	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0630	0.0493	0.0766	0.0455	0.0368	0.541	0.0071	0.0034	0.0106
σ_1^{yo}	Home Oil Supply	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0099	0.0085	0.0112	0.0349	0.0301	0.0396	0.1200	0.1013	0.1382
σ_2^{yo}	Foreign Oil Supply	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0113	0.0098	0.0128	0.0105	0.0091	0.0119	0.0119	0.0101	0.0136
σ_1^{zo}	Home Oil Intensity	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0757	0.0633	0.0883	0.0455	0.0382	0.0525	0.1417	0.1171	0.1654
σ_2^{zo}	Foreign Oil Intensity	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0257	0.0209	0.0305	0.0747	0.0636	0.0855	0.0694	0.0597	0.0788
σ_1^{zc}	Home Priv. Consumption	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0164	0.0139	0.0189	0.0608	0.0366	0.0838	0.0132	0.0107	0.0157
σ_2^{zc}	Foreign Priv. Consumption	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0603	0.0476	0.0729	0.0388	0.0318	0.0457	0.0519	0.0445	0.0592
σ_1^{zm}	Home Import Preferences	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0669	0.0561	0.0774	0.0268	0.0226	0.0308	0.0346	0.0295	0.0396
σ_2^{zm}	Foreign Import Preferences	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.1080	0.0932	0.1224	0.0725	0.0625	0.0823	0.0461	0.0399	0.0522
σ_1^{zi}	Home Priv. Investment	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.2437	0.1725	0.3113	0.1052	0.0701	0.1390	0.2532	0.1787	0.3279
σ_1^p	Home Price Markup	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	1.2694	0.9096	1.6098	0.0092	0.0023	0.0169	1.9982	1.5321	2.4521
σ_1^w	Home Wage Markup	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0097	0.0022	0.0184	0.0102	0.0022	0.0213	0.0102	0.0022	0.0193
σ_1^π	Home Inflation Target	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0073	0.0044	0.0103	0.0276	0.0211	0.0340	0.0032	0.0017	0.0040
σ_1^g	Home Gov. Cons. Expenditure	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0952	0.0821	0.1082	0.1149	0.0994	0.1306	0.0632	0.0546	0.0718
σ_1^{ag}	Home Authorized Gov. Inv. Expenditure	I-G	0.0100	Inf	I-G	0.0100	Inf	I-G	0.0100	Inf	0.0071	0.0061	0.0080	0.0745	0.0645	0.0844	0.0233	0.0200	0.0265

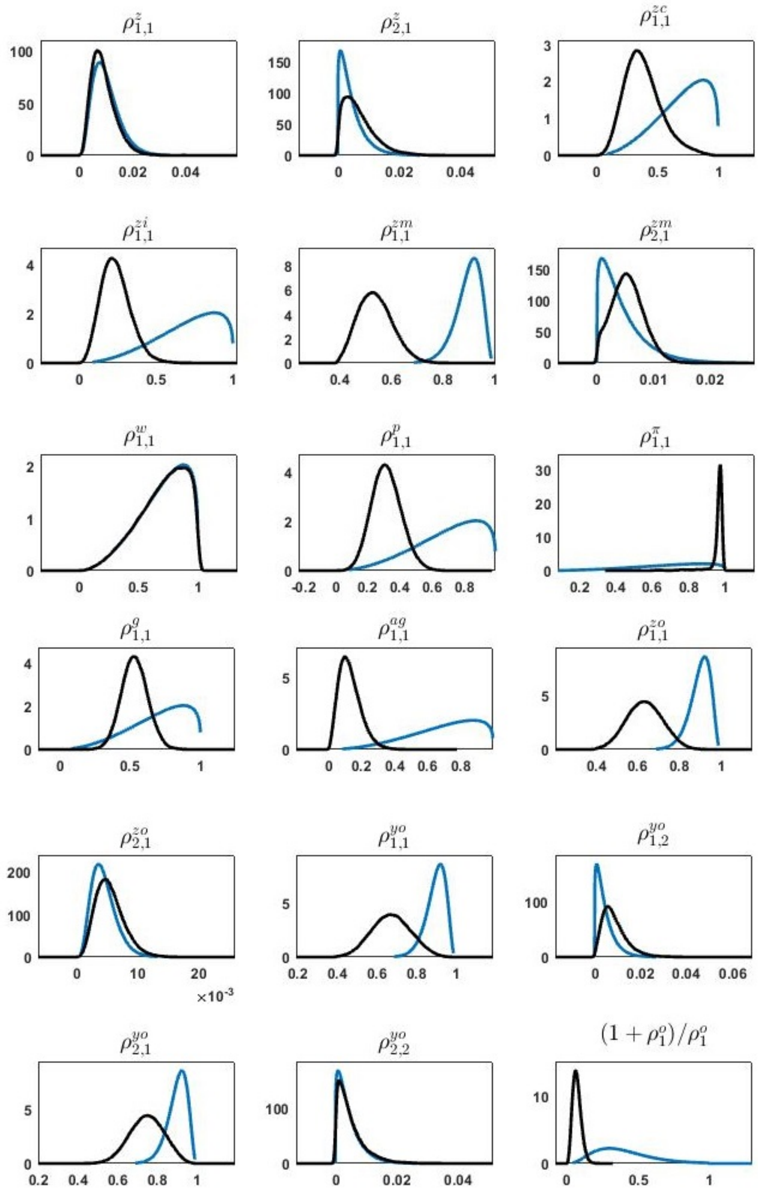
6 Appendix F: Diagnostic Tests

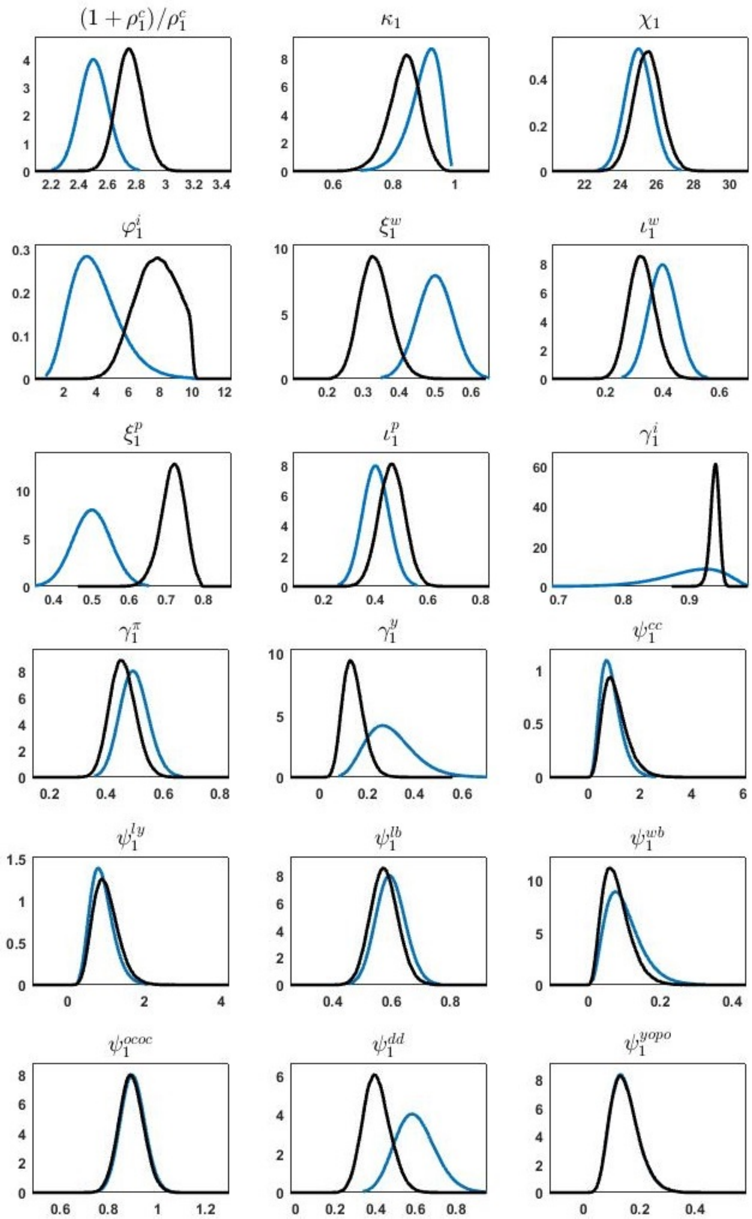
In the graphs below, the blue lines represent the prior distributions while the black lines correspond to the posterior distributions.

6.1 Priors and posteriors distributions for parameters of the three models

6.1.1 UK Model

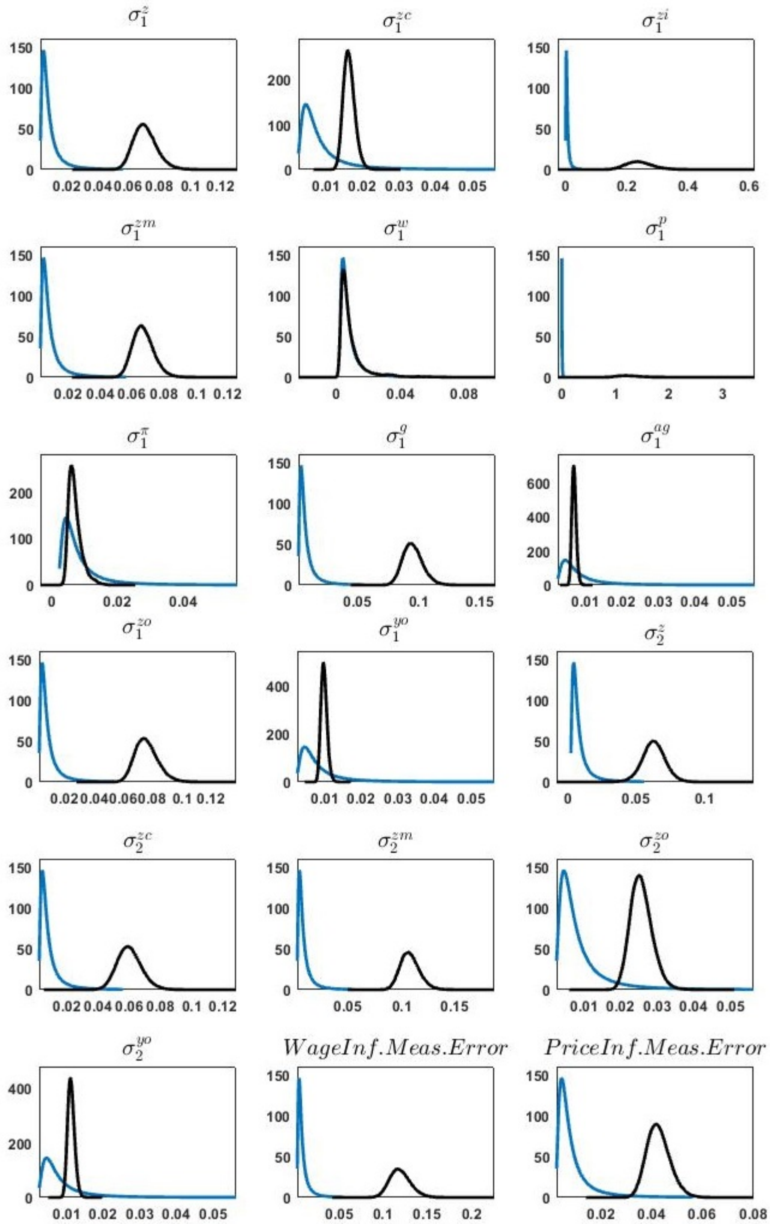


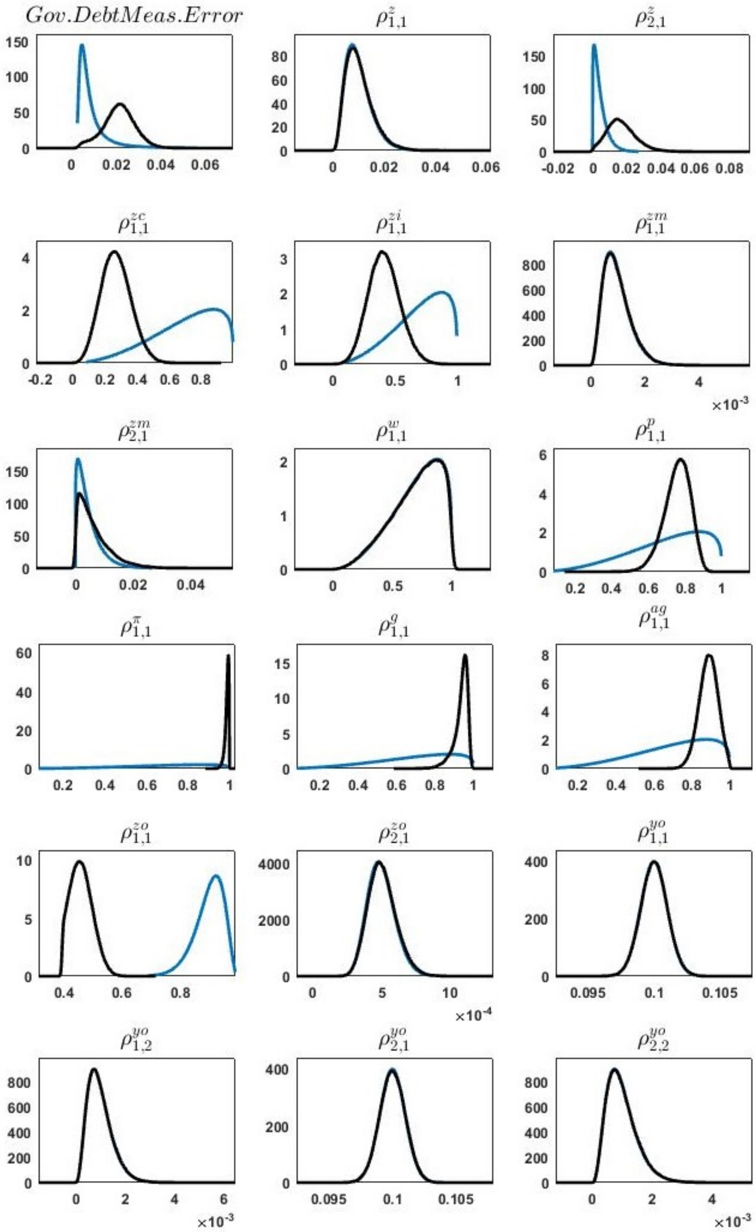

$\rho^{z}_{1,1}$
$\rho^{z}_{2,1}$
$\rho^{zc}_{1,1}$
$\rho^{zi}_{1,1}$
$\rho^{zm}_{1,1}$
$\rho^{zm}_{2,1}$
$\rho^{w}_{1,1}$
$\rho^{p}_{1,1}$
$\rho^{\pi}_{1,1}$
$\rho^{g}_{1,1}$
$\rho^{ag}_{1,1}$
$\rho^{zo}_{1,1}$
$\rho^{zo}_{2,1}$
$\rho^{yo}_{1,1}$
$\rho^{yo}_{1,2}$
$\rho^{yo}_{2,1}$
$\rho^{yo}_{2,2}$
$(1+\rho^{o}_{1})/\rho^{o}_{1}$
$\times 10^{-3}$



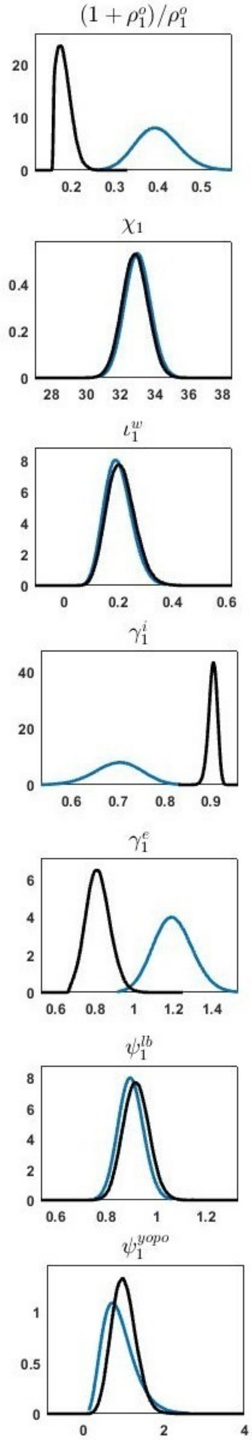
$(1+\rho_1^c)/\rho_1^c$
κ_1
χ_1
φ_1^i
ξ_1^w
ι_1^w
ξ_1^p
ι_1^p
γ_1^i
γ_1^π
γ_1^y
ψ_1^{cc}
ψ_1^{ly}
ψ_1^{lb}
ψ_1^{wb}
ψ_1^{ococ}
ψ_1^{dd}
ψ_1^{yopo}

6.1.2 Russia Model

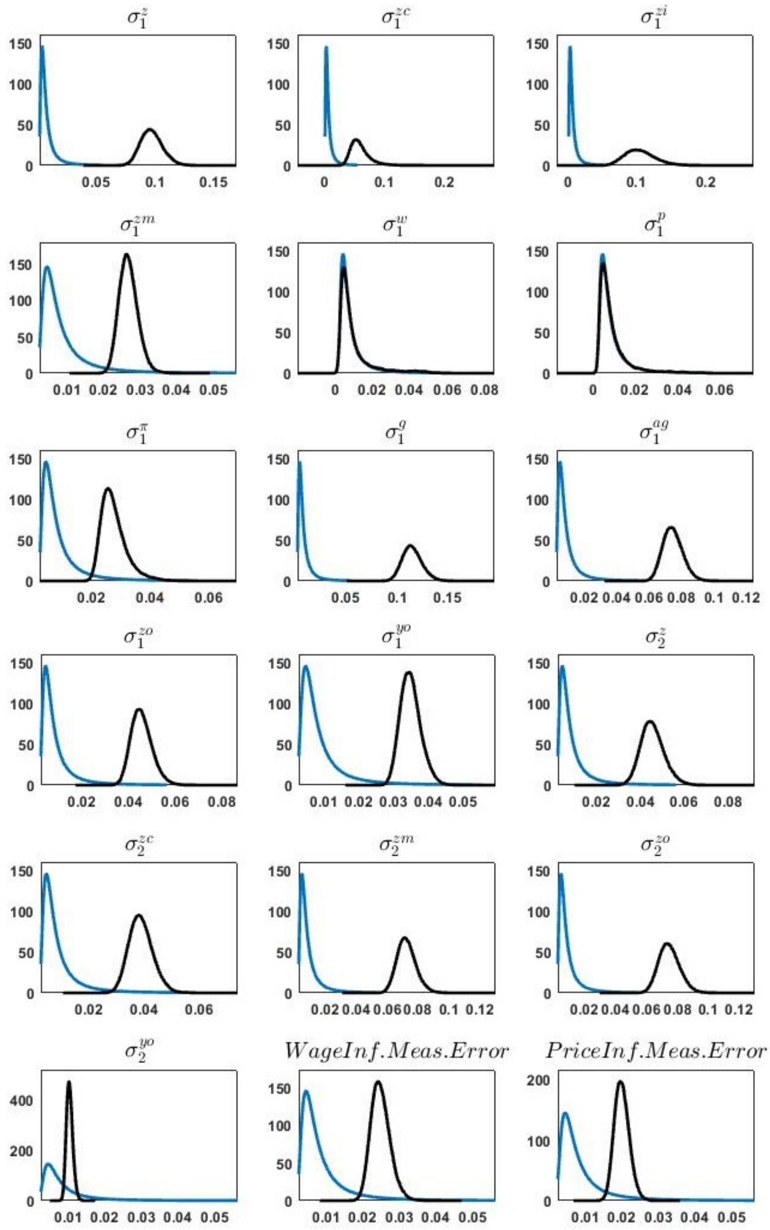


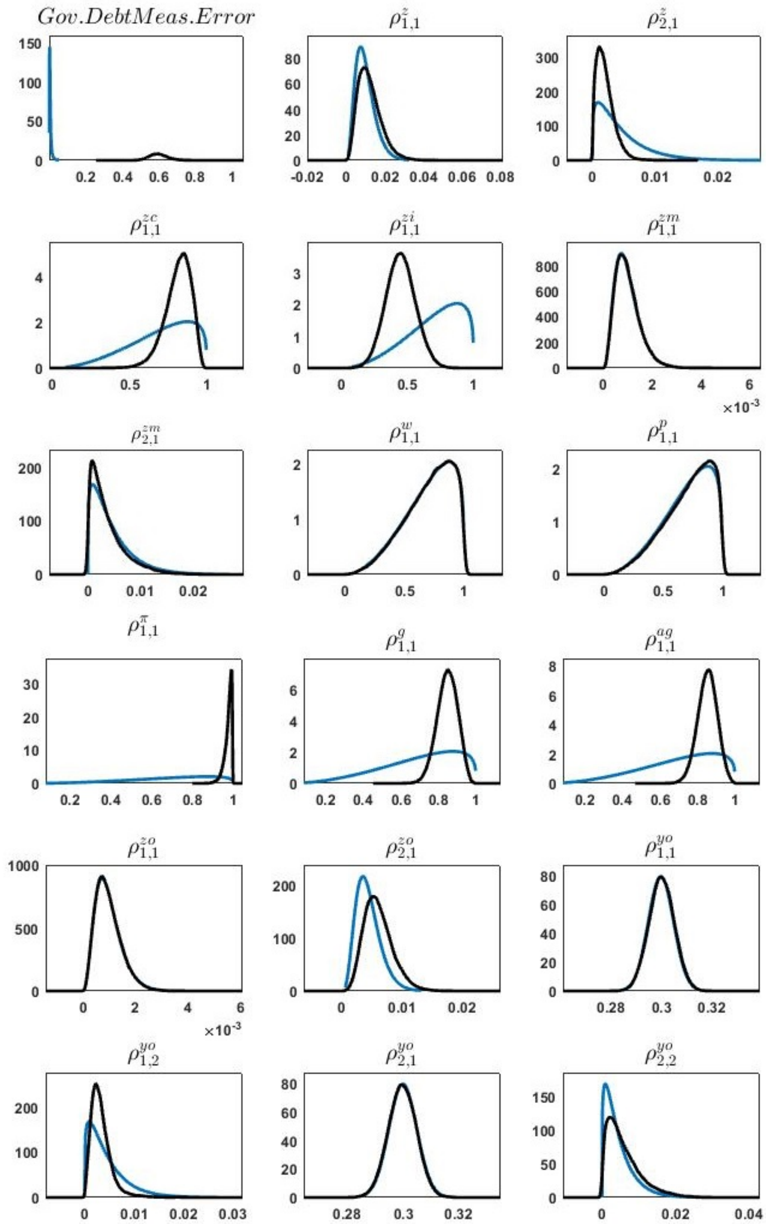


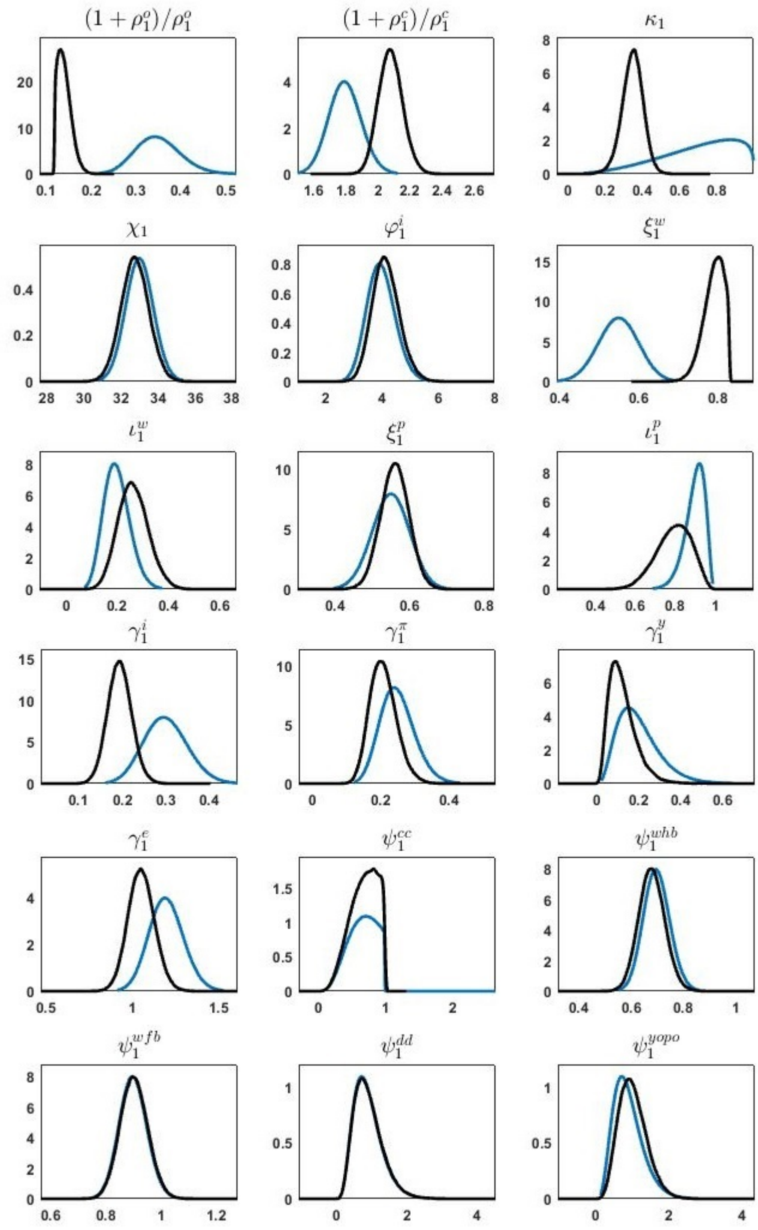
$Gov.DebtMeas.Error$
$\rho^{z}_{1,1}$
$\rho^{z}_{2,1}$
$\rho^{zc}_{1,1}$
$\rho^{zi}_{1,1}$
$\rho^{zm}_{1,1}$
$\rho^{zm}_{2,1}$
$\rho^{w}_{1,1}$
$\rho^{p}_{1,1}$
$\rho^{\pi}_{1,1}$
$\rho^{g}_{1,1}$
$\rho^{ag}_{1,1}$
$\rho^{zo}_{1,1}$
$\rho^{zo}_{2,1}$
$\rho^{yo}_{1,1}$
$\rho^{yo}_{1,2}$
$\rho^{yo}_{2,1}$
$\rho^{yo}_{2,2}$


$(1+\rho_1^o)/\rho_1^o$
$(1+\rho_1^c)/\rho_1^c$
κ_1
χ_1
φ_1^i
ξ_1^w
ι_1^w
ξ_1^p
ι_1^p
γ_1^i
γ_1^π
γ_1^y
γ_1^e
ψ_1^{cc}
ψ_1^{ly}
ψ_1^{lb}
ψ_1^{wfb}
ψ_1^{dd}
ψ_1^{yopo}

6.1.3 Saudi Arabia Model



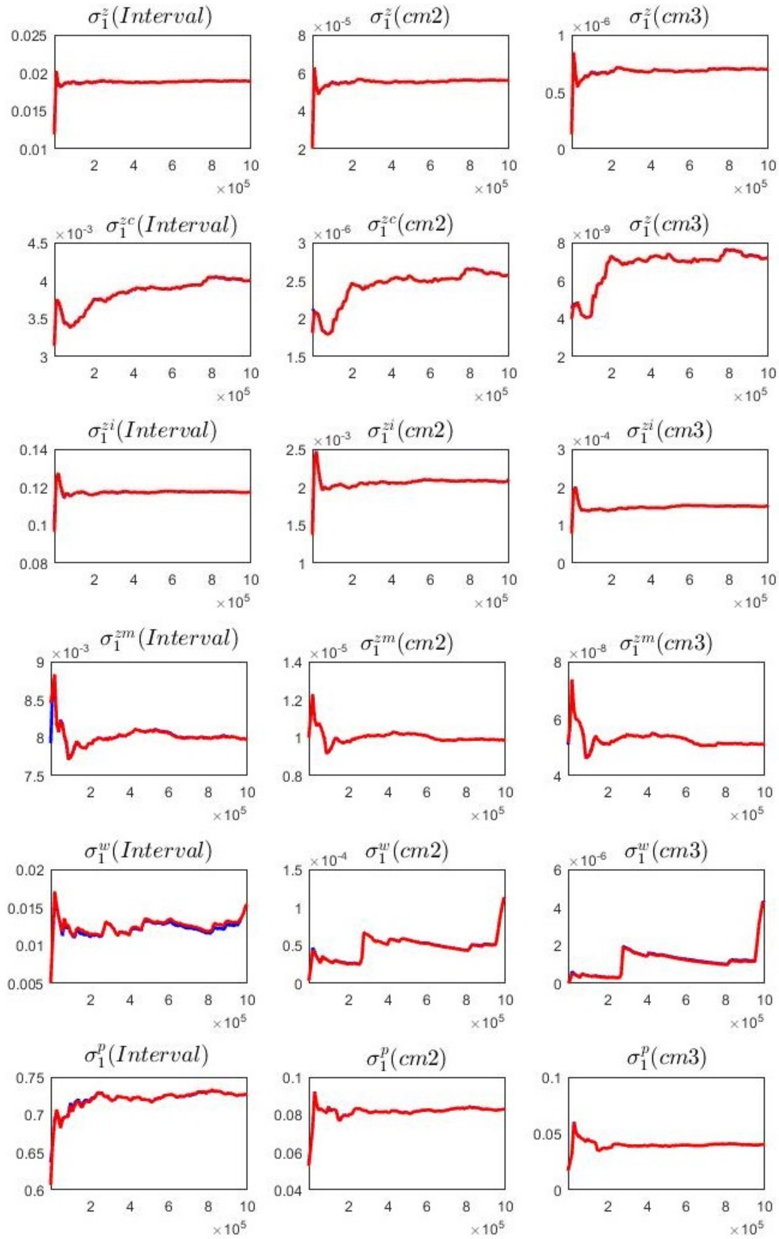

$Gov.DebtMeas.Error$
$\rho^{z}_{1,1}$
$\rho^{z}_{2,1}$
$\rho^{zc}_{1,1}$
$\rho^{zi}_{1,1}$
$\rho^{zm}_{1,1}$
$\times 10^{-3}$
$\rho^{zm}_{2,1}$
$\rho^{w}_{1,1}$
$\rho^{p}_{1,1}$
$\rho^{\pi}_{1,1}$
$\rho^{g}_{1,1}$
$\rho^{ag}_{1,1}$
$\rho^{zo}_{1,1}$
$\times 10^{-3}$
$\rho^{zo}_{2,1}$
$\rho^{yo}_{1,1}$
$\rho^{yo}_{1,2}$
$\rho^{yo}_{2,1}$
$\rho^{yo}_{2,2}$


$(1+\rho_1^o)/\rho_1^o$
0 10 20
0.1 0.2 0.3 0.4 0.5
$(1+\rho_1^c)/\rho_1^c$
0 2 4
1.6 1.8 2 2.2 2.4 2.6
κ_1
0 2 4 6 8
0 0.2 0.4 0.6 0.8
χ_1
0 0.2 0.4
28 30 32 34 36 38
φ_1^i
0 0.2 0.4 0.6 0.8
2 4 6 8
ξ_1^w
0 5 10 15
0.4 0.6 0.8
ι_1^w
0 2 4 6 8
0 0.2 0.4 0.6
ξ_1^p
0 5 10
0.4 0.6 0.8
ι_1^p
0 2 4 6 8
0.4 0.6 0.8 1
γ_1^i
0 5 10 15
0.1 0.2 0.3 0.4
γ_1^π
0 5 10
0 0.2 0.4
γ_1^y
0 2 4 6
0 0.2 0.4 0.6
γ_1^e
0 2 4
0.5 1 1.5
ψ_1^{cc}
0 0.5 1 1.5
0 1 2
ψ_1^{whb}
0 2 4 6 8
0.4 0.6 0.8 1
ψ_1^{wfb}
0 2 4 6 8
0.6 0.8 1 1.2
ψ_1^{dd}
0 0.5 1
0 2 4
ψ_1^{yopo}
0 0.5 1
0 2 4

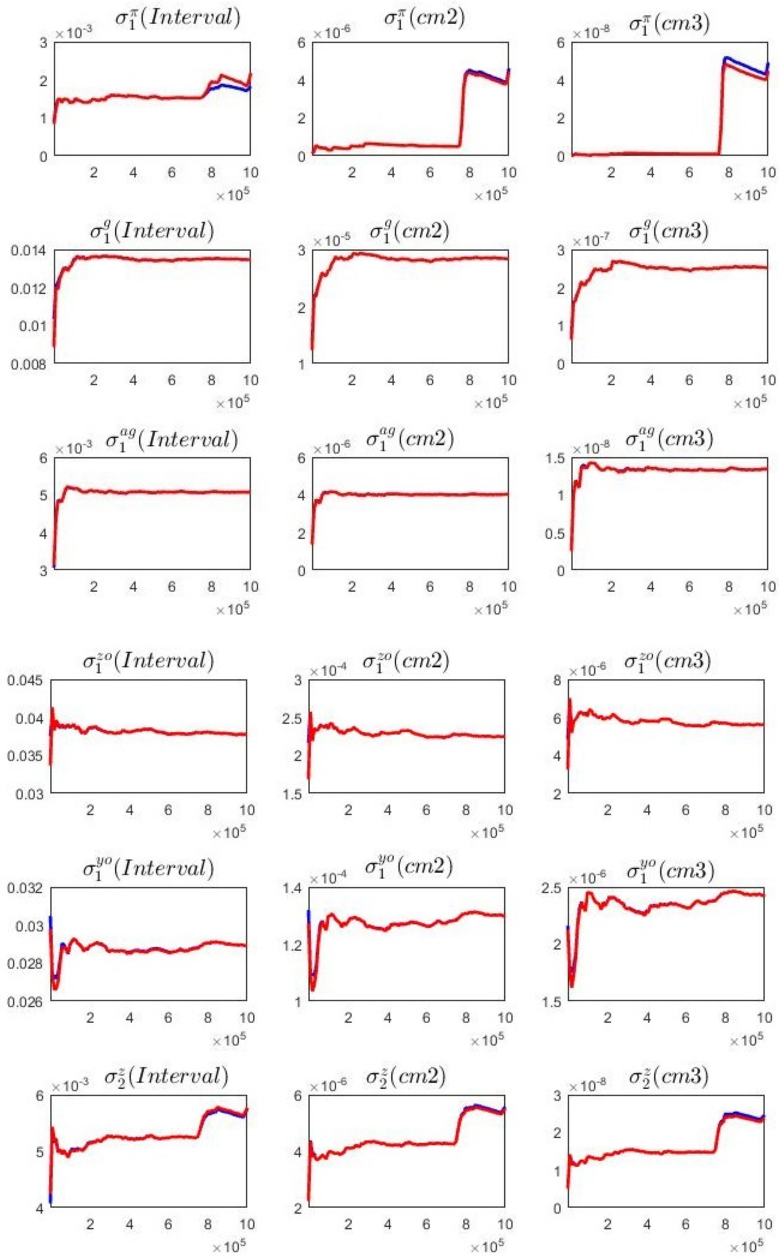
6.2 Monte Carlo Markov Chain Univariate Diagnostics

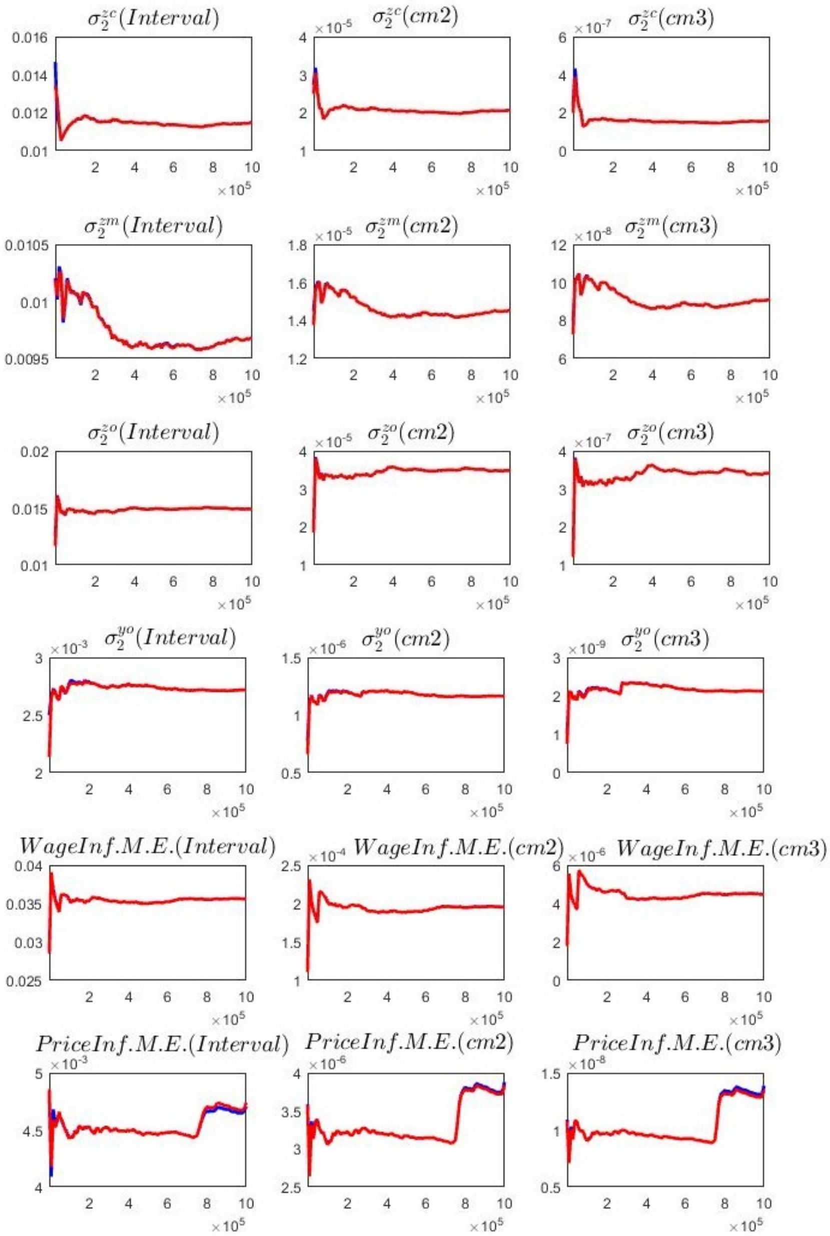
In the graphs below, the first column with the label "Interval" shows the Brooks and Gelman (1998) convergence diagnostics for the 80% interval. The blue line represents the 80% interval range based on the pooled draws from all sequences, whereas the red line indicates the mean interval based on the draws of the individual sequences. The second and the third column with labels "CM2" and "CM3" denote an estimate of the same statistics for the second and third central moments.

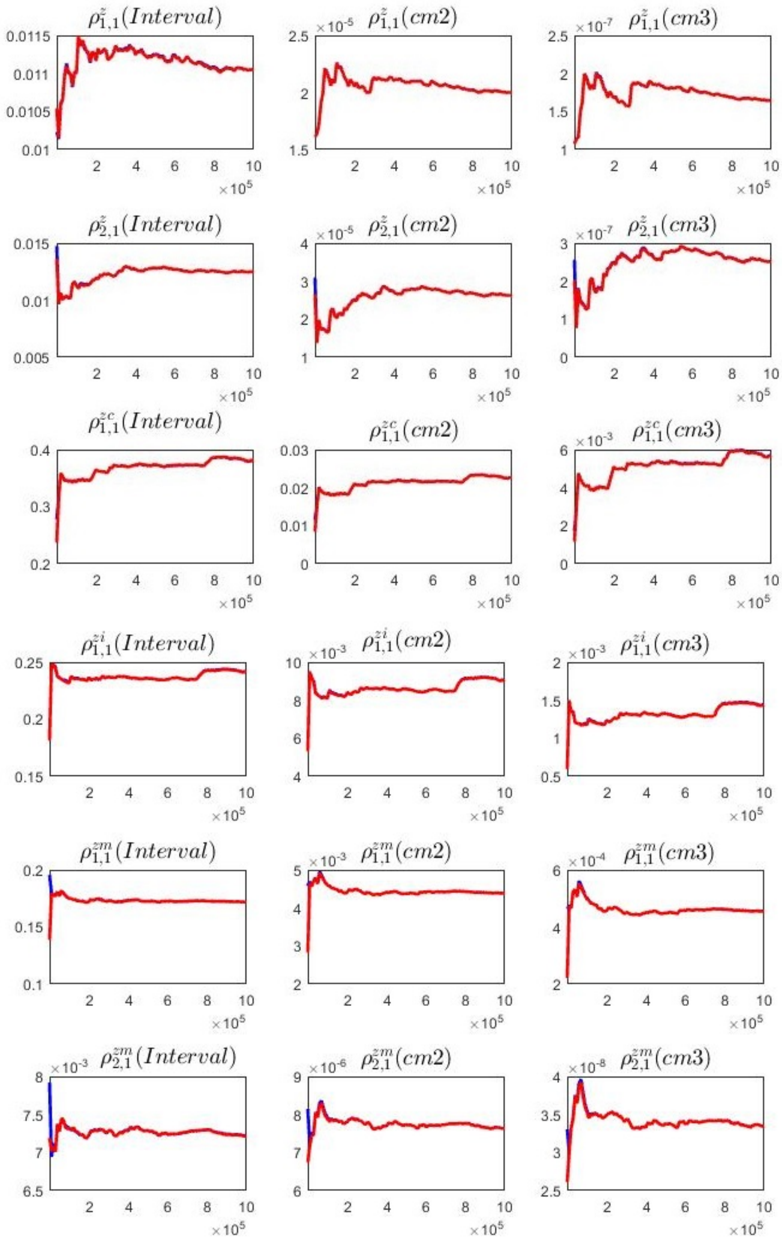
6.2.1 UK Model

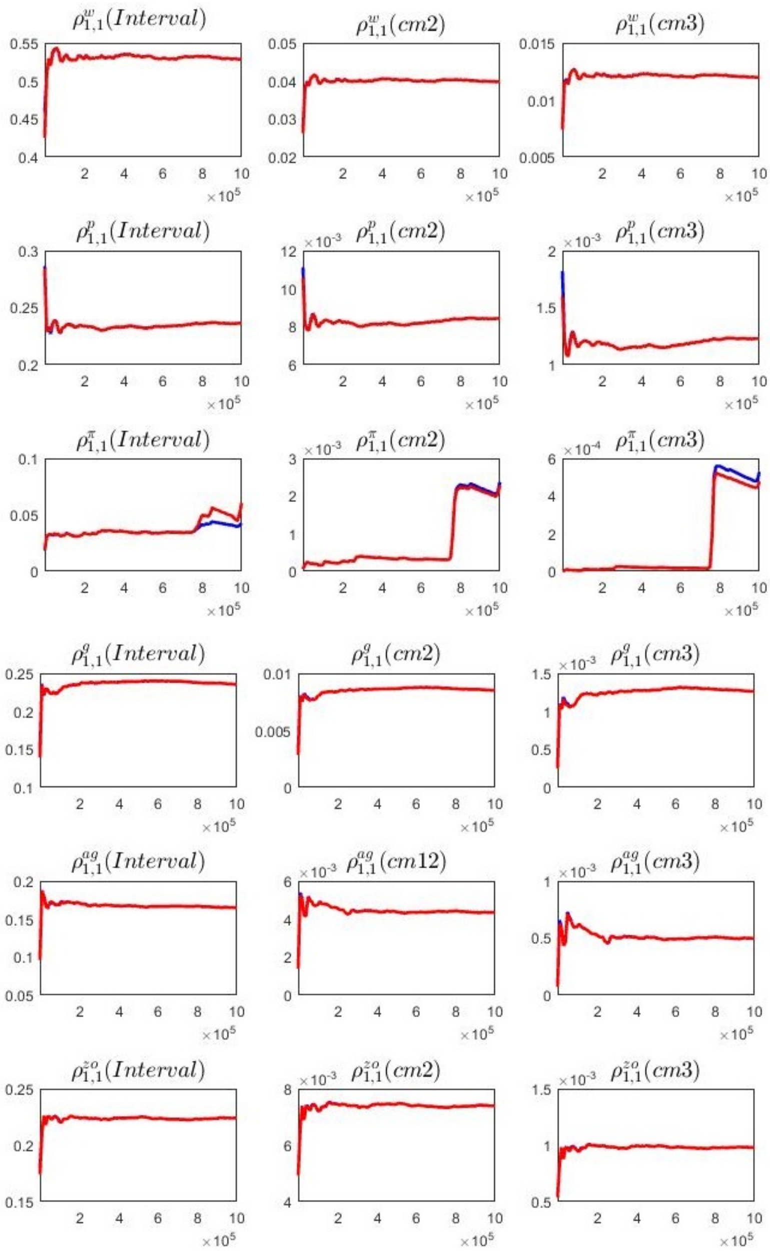


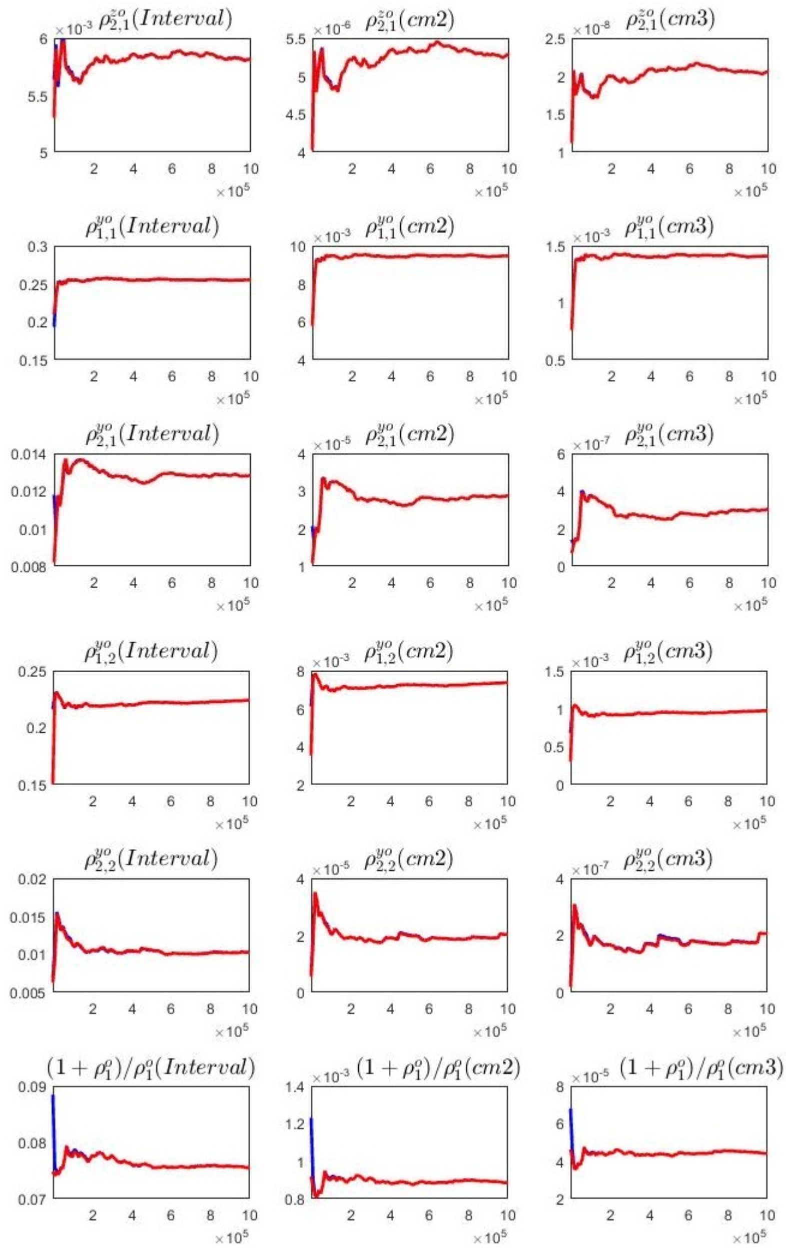
$\sigma_1^z(Interval)$
$\sigma_1^z(cm2)$
$\sigma_1^z(cm3)$
$\sigma_1^{zc}(Interval)$
$\sigma_1^{zc}(cm2)$
$\sigma_1^{z}(cm3)$
$\sigma_1^{zi}(Interval)$
$\sigma_1^{zi}(cm2)$
$\sigma_1^{zi}(cm3)$
$\sigma_1^{zm}(Interval)$
$\sigma_1^{zm}(cm2)$
$\sigma_1^{zm}(cm3)$
$\sigma_1^w(Interval)$
$\sigma_1^w(cm2)$
$\sigma_1^w(cm3)$
$\sigma_1^p(Interval)$
$\sigma_1^p(cm2)$
$\sigma_1^p(cm3)$

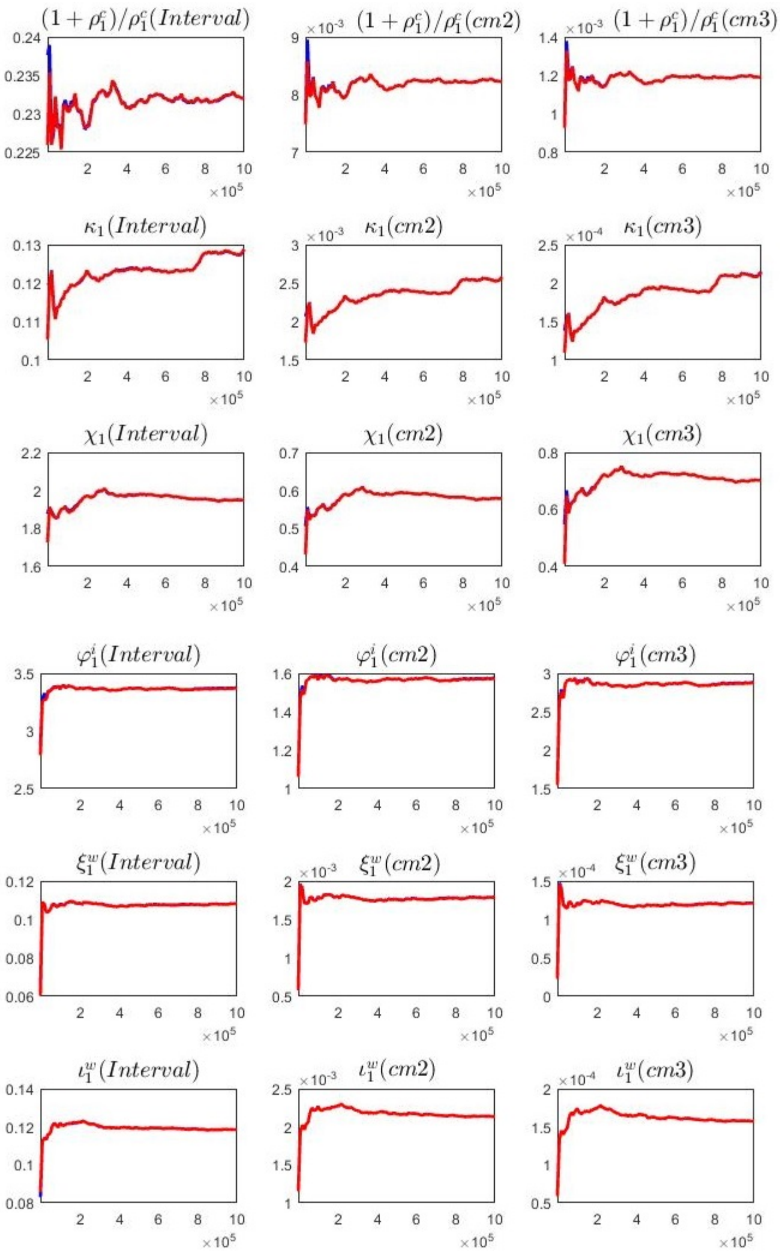

$\sigma_1^{\pi}(Interval)$
$\sigma_1^{\pi}(cm2)$
$\sigma_1^{\pi}(cm3)$
$\sigma_1^{g}(Interval)$
$\sigma_1^{g}(cm2)$
$\sigma_1^{g}(cm3)$
$\sigma_1^{ag}(Interval)$
$\sigma_1^{ag}(cm2)$
$\sigma_1^{ag}(cm3)$
$\sigma_1^{zo}(Interval)$
$\sigma_1^{zo}(cm2)$
$\sigma_1^{zo}(cm3)$
$\sigma_1^{yo}(Interval)$
$\sigma_1^{yo}(cm2)$
$\sigma_1^{yo}(cm3)$
$\sigma_2^{z}(Interval)$
$\sigma_2^{z}(cm2)$
$\sigma_2^{z}(cm3)$

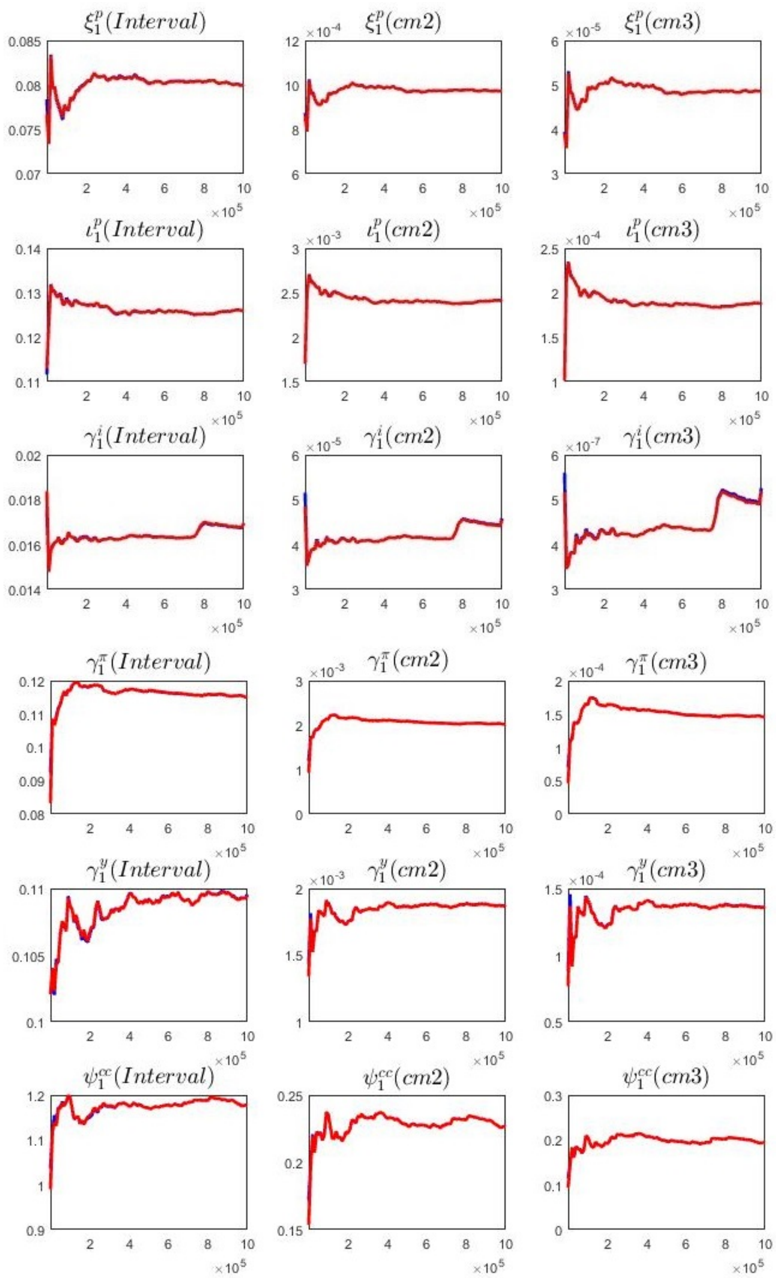

$\sigma_2^{zc}(Interval)$
$\sigma_2^{zc}(cm2)$
$\sigma_2^{zc}(cm3)$
$\sigma_2^{zm}(Interval)$
$\sigma_2^{zm}(cm2)$
$\sigma_2^{zm}(cm3)$
$\sigma_2^{zo}(Interval)$
$\sigma_2^{zo}(cm2)$
$\sigma_2^{zo}(cm3)$
$\sigma_2^{yo}(Interval)$
$\sigma_2^{yo}(cm2)$
$\sigma_2^{yo}(cm3)$
$WageInf.M.E.(Interval)$
$WageInf.M.E.(cm2)$
$WageInf.M.E.(cm3)$
$PriceInf.M.E.(Interval)$
$PriceInf.M.E.(cm2)$
$PriceInf.M.E.(cm3)$


$\rho^{z}_{1,1}(Interval)$
$\rho^{z}_{1,1}(cm2)$
$\rho^{z}_{1,1}(cm3)$
$\rho^{z}_{2,1}(Interval)$
$\rho^{z}_{2,1}(cm2)$
$\rho^{z}_{2,1}(cm3)$
$\rho^{zc}_{1,1}(Interval)$
$\rho^{zc}_{1,1}(cm2)$
$\rho^{zc}_{1,1}(cm3)$
$\rho^{zi}_{1,1}(Interval)$
$\rho^{zi}_{1,1}(cm2)$
$\rho^{zi}_{1,1}(cm3)$
$\rho^{zm}_{1,1}(Interval)$
$\rho^{zm}_{1,1}(cm2)$
$\rho^{zm}_{1,1}(cm3)$
$\rho^{zm}_{2,1}(Interval)$
$\rho^{zm}_{2,1}(cm2)$
$\rho^{zm}_{2,1}(cm3)$

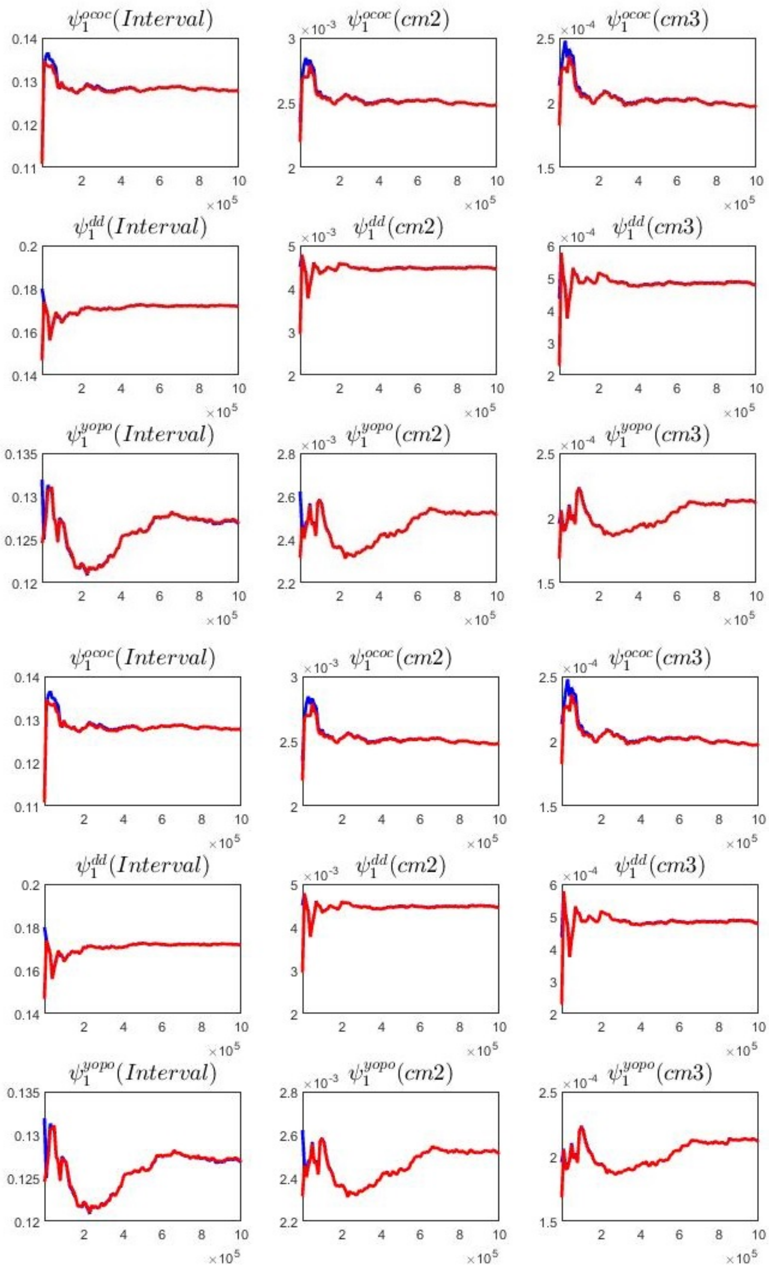

$\rho^{w}_{1,1}(Interval)$
$\rho^{w}_{1,1}(cm2)$
$\rho^{w}_{1,1}(cm3)$
$\rho^{p}_{1,1}(Interval)$
$\rho^{p}_{1,1}(cm2)$
$\rho^{p}_{1,1}(cm3)$
$\rho^{\pi}_{1,1}(Interval)$
$\rho^{\pi}_{1,1}(cm2)$
$\rho^{\pi}_{1,1}(cm3)$
$\rho^{q}_{1,1}(Interval)$
$\rho^{q}_{1,1}(cm2)$
$\rho^{q}_{1,1}(cm3)$
$\rho^{ag}_{1,1}(Interval)$
$\rho^{ag}_{1,1}(cm12)$
$\rho^{ag}_{1,1}(cm3)$
$\rho^{zo}_{1,1}(Interval)$
$\rho^{zo}_{1,1}(cm2)$
$\rho^{zo}_{1,1}(cm3)$



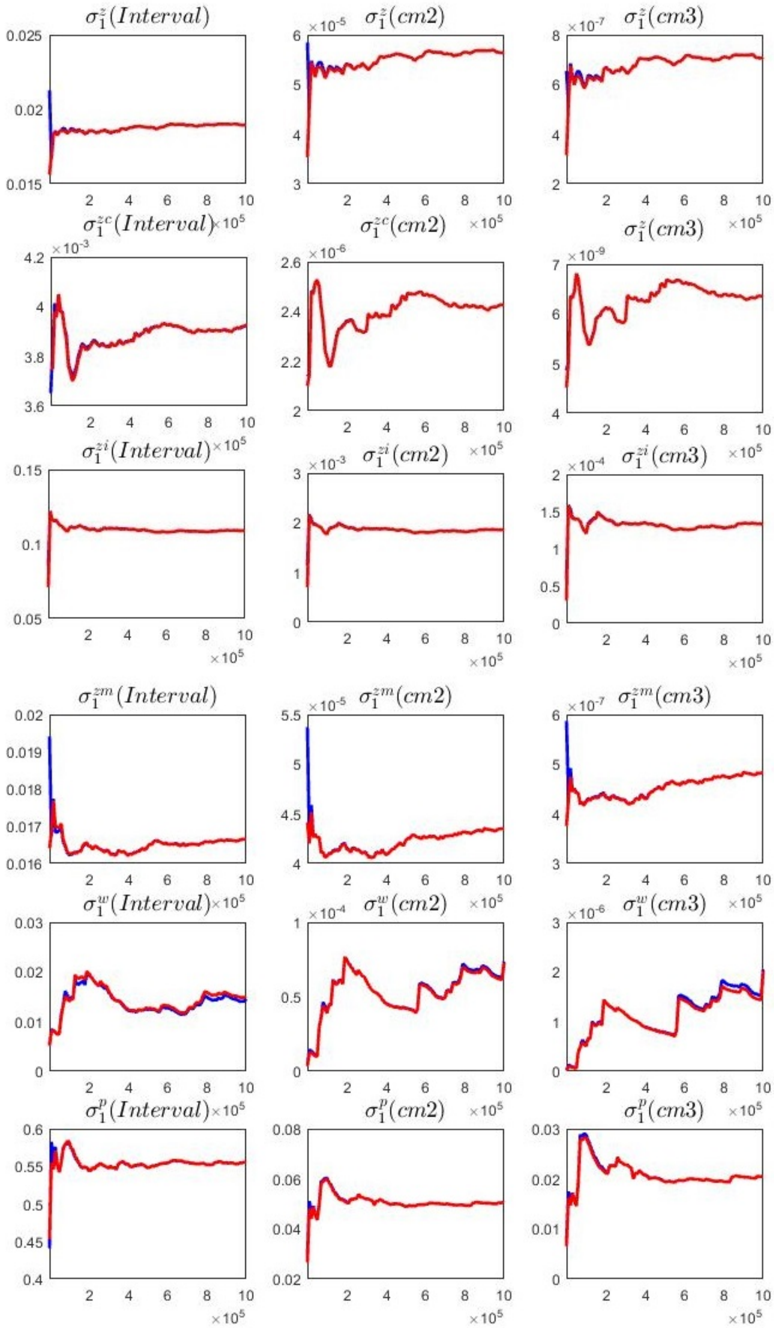

$(1+\rho_1^c)/\rho_1^c(Interval)$
$(1+\rho_1^c)/\rho_1^c(cm2)$
$(1+\rho_1^c)/\rho_1^c(cm3)$
$\kappa_1(Interval)$
$\kappa_1(cm2)$
$\kappa_1(cm3)$
$\chi_1(Interval)$
$\chi_1(cm2)$
$\chi_1(cm3)$
$\varphi_1^i(Interval)$
$\varphi_1^i(cm2)$
$\varphi_1^i(cm3)$
$\xi_1^w(Interval)$
$\xi_1^w(cm2)$
$\xi_1^w(cm3)$
$\iota_1^w(Interval)$
$\iota_1^w(cm2)$
$\iota_1^w(cm3)$

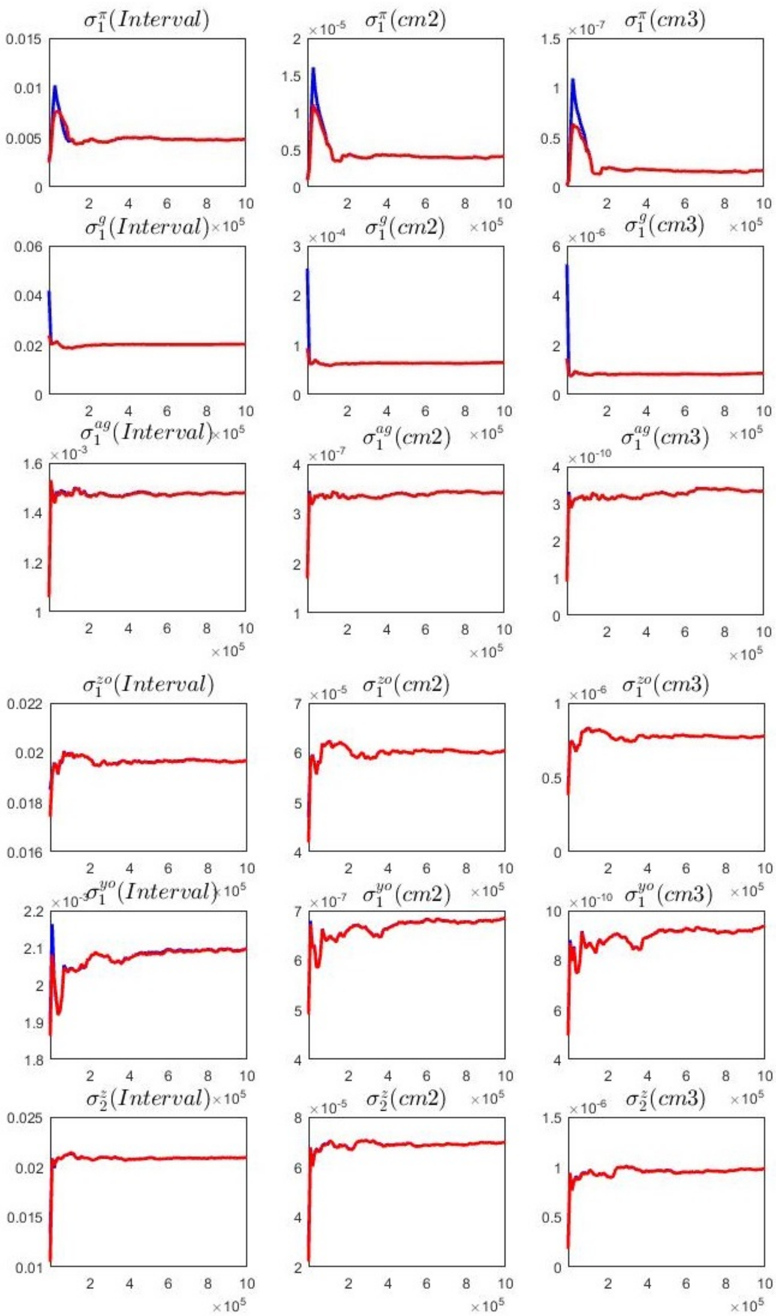


$\xi_1^p(Interval)$
$\xi_1^p(cm2)$
$\xi_1^p(cm3)$
$\iota_1^p(Interval)$
$\iota_1^p(cm2)$
$\iota_1^p(cm3)$
$\gamma_1^i(Interval)$
$\gamma_1^i(cm2)$
$\gamma_1^i(cm3)$
$\gamma_1^\pi(Interval)$
$\gamma_1^\pi(cm2)$
$\gamma_1^\pi(cm3)$
$\gamma_1^y(Interval)$
$\gamma_1^y(cm2)$
$\gamma_1^y(cm3)$
$\psi_1^{cc}(Interval)$
$\psi_1^{cc}(cm2)$
$\psi_1^{cc}(cm3)$

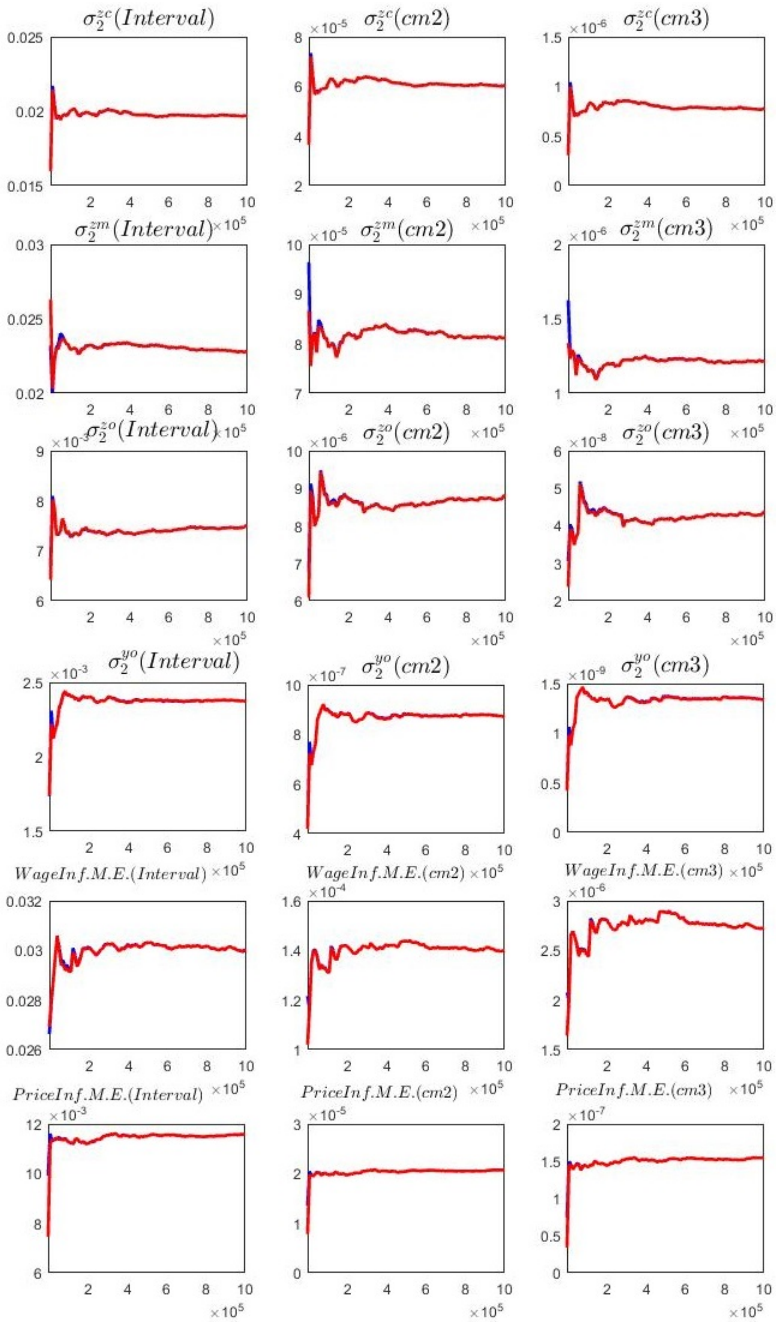

$\psi_1^{ococ}(Interval)$
$\psi_1^{ococ}(cm2)$
$\psi_1^{ococ}(cm3)$
$\psi_1^{dd}(Interval)$
$\psi_1^{dd}(cm2)$
$\psi_1^{dd}(cm3)$
$\psi_1^{yopo}(Interval)$
$\psi_1^{yopo}(cm2)$
$\psi_1^{yopo}(cm3)$
$\psi_1^{ococ}(Interval)$
$\psi_1^{ococ}(cm2)$
$\psi_1^{ococ}(cm3)$
$\psi_1^{dd}(Interval)$
$\psi_1^{dd}(cm2)$
$\psi_1^{dd}(cm3)$
$\psi_1^{yopo}(Interval)$
$\psi_1^{yopo}(cm2)$
$\psi_1^{yopo}(cm3)$

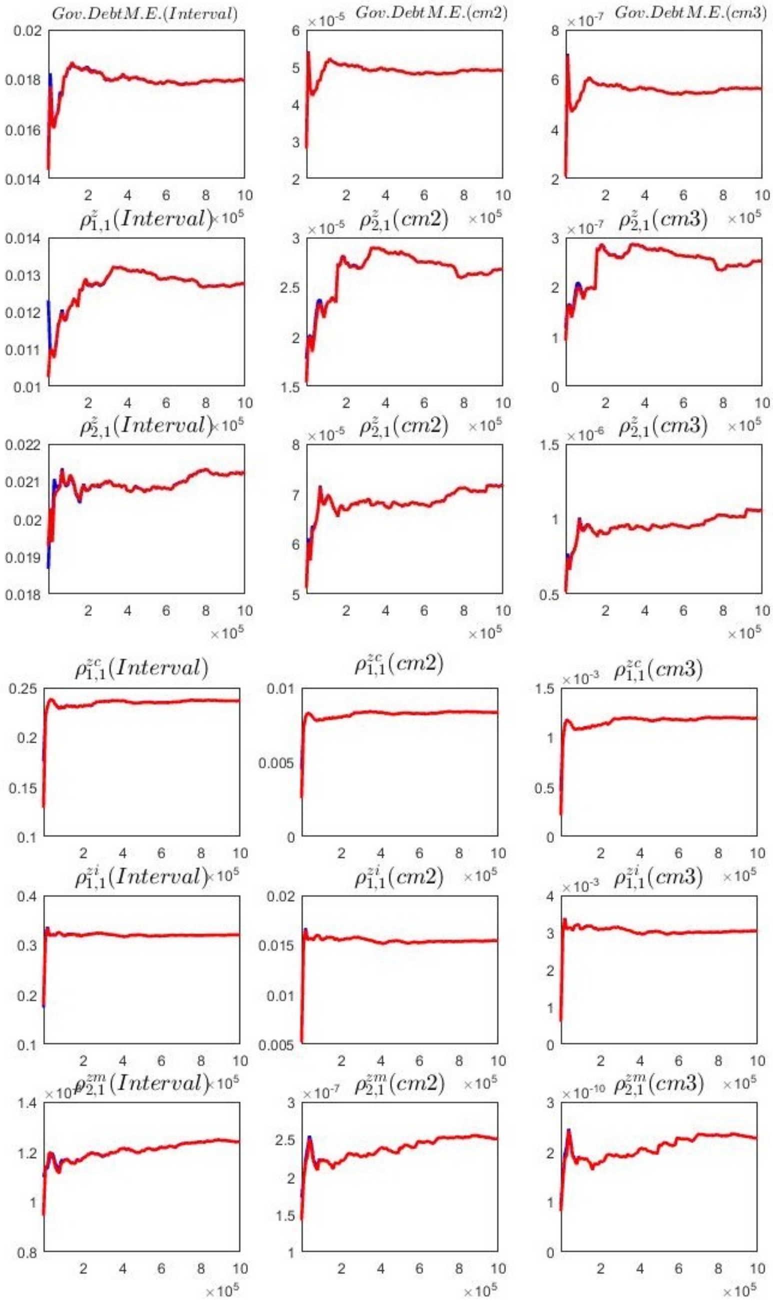
6.2.2 Russia Model



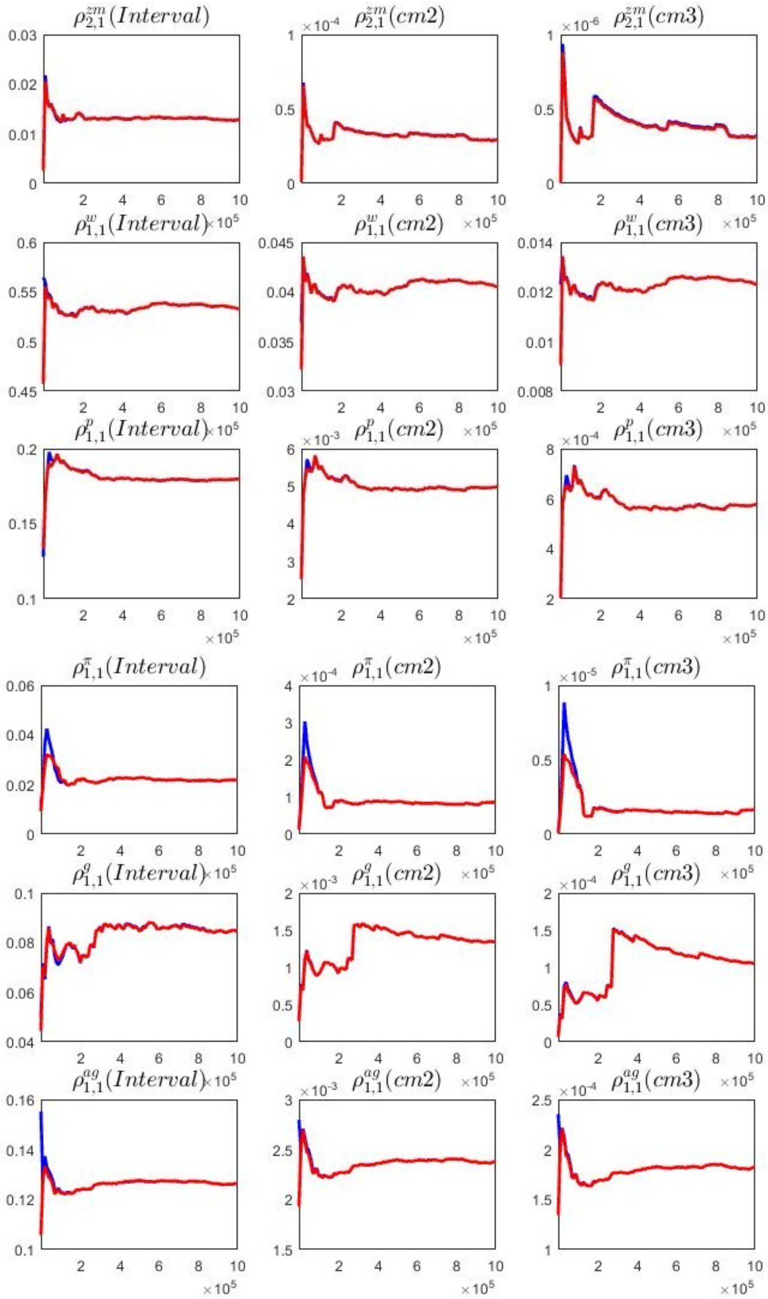


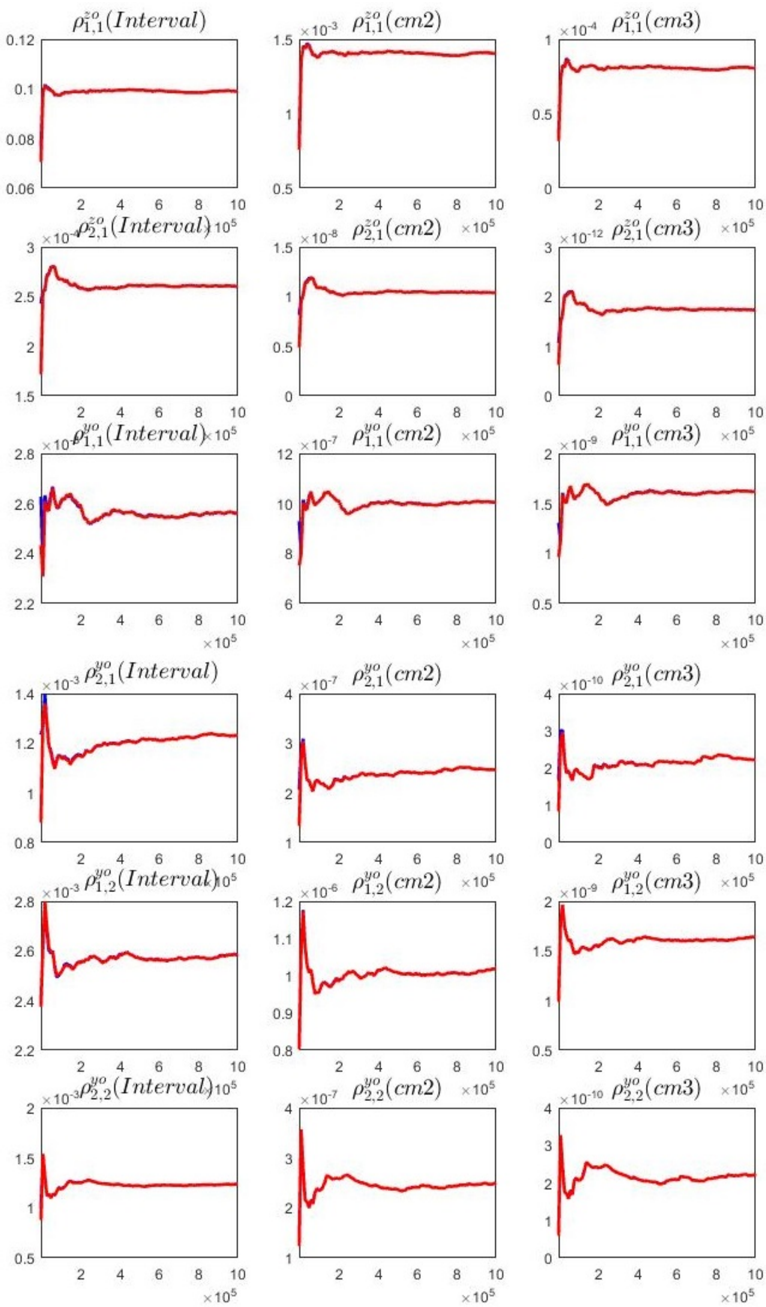
$\sigma_1^{\pi}(Interval)$
$\sigma_1^{\pi}(cm2)$
$\sigma_1^{\pi}(cm3)$
$\sigma_1^{g}(Interval)$
$\sigma_1^{g}(cm2)$
$\sigma_1^{g}(cm3)$
$\sigma_1^{ag}(Interval)$
$\sigma_1^{ag}(cm2)$
$\sigma_1^{ag}(cm3)$
$\sigma_1^{zo}(Interval)$
$\sigma_1^{zo}(cm2)$
$\sigma_1^{zo}(cm3)$
$\sigma_1^{yo}(Interval)$
$\sigma_1^{yo}(cm2)$
$\sigma_1^{yo}(cm3)$
$\sigma_2^{z}(Interval)$
$\sigma_2^{z}(cm2)$
$\sigma_2^{z}(cm3)$
$\times 10^5$


$\sigma_2^{zc}(Interval)$
$\sigma_2^{zc}(cm2)$
$\sigma_2^{zc}(cm3)$
$\sigma_2^{zm}(Interval)$
$\sigma_2^{zm}(cm2)$
$\sigma_2^{zm}(cm3)$
$\sigma_2^{zo}(Interval)$
$\sigma_2^{zo}(cm2)$
$\sigma_2^{zo}(cm3)$
$\sigma_2^{yo}(Interval)$
$\sigma_2^{yo}(cm2)$
$\sigma_2^{yo}(cm3)$
$WageInf.M.E.(Interval)$
$WageInf.M.E.(cm2)$
$WageInf.M.E.(cm3)$
$PriceInf.M.E.(Interval)$
$PriceInf.M.E.(cm2)$
$PriceInf.M.E.(cm3)$

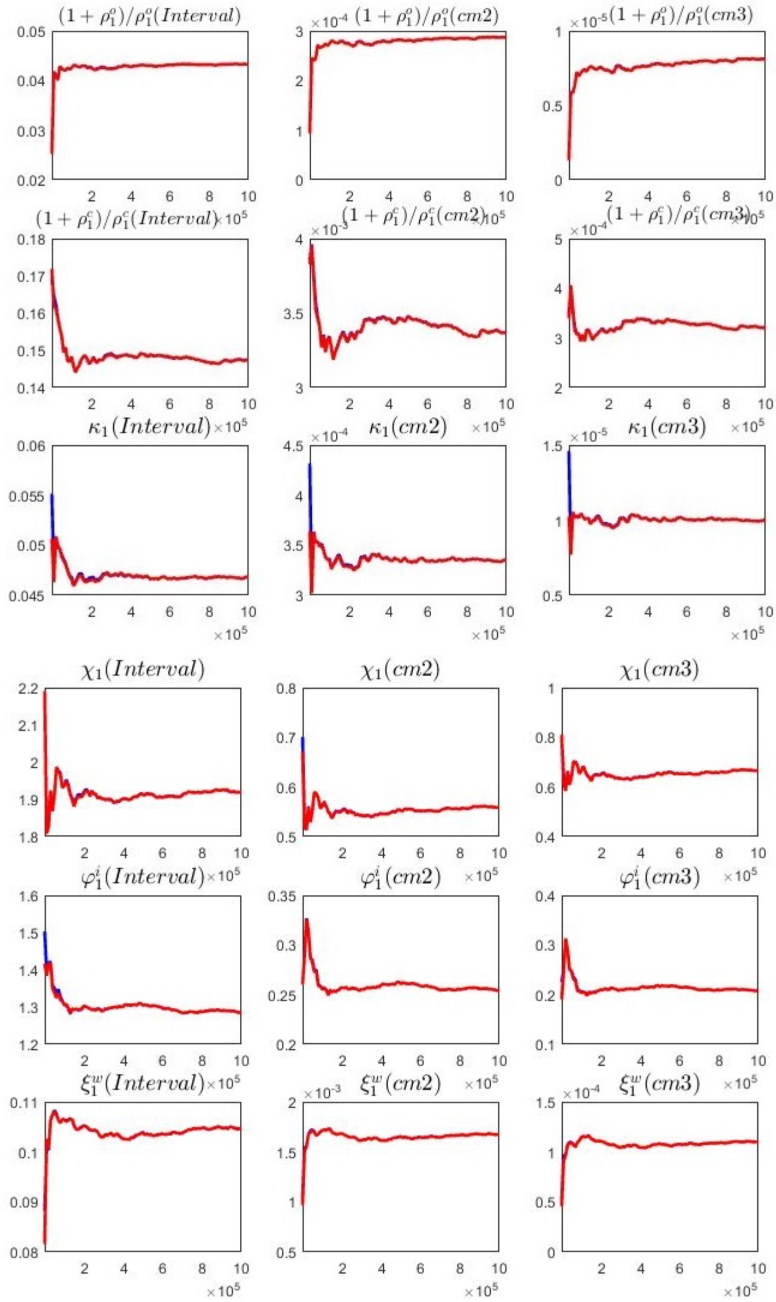


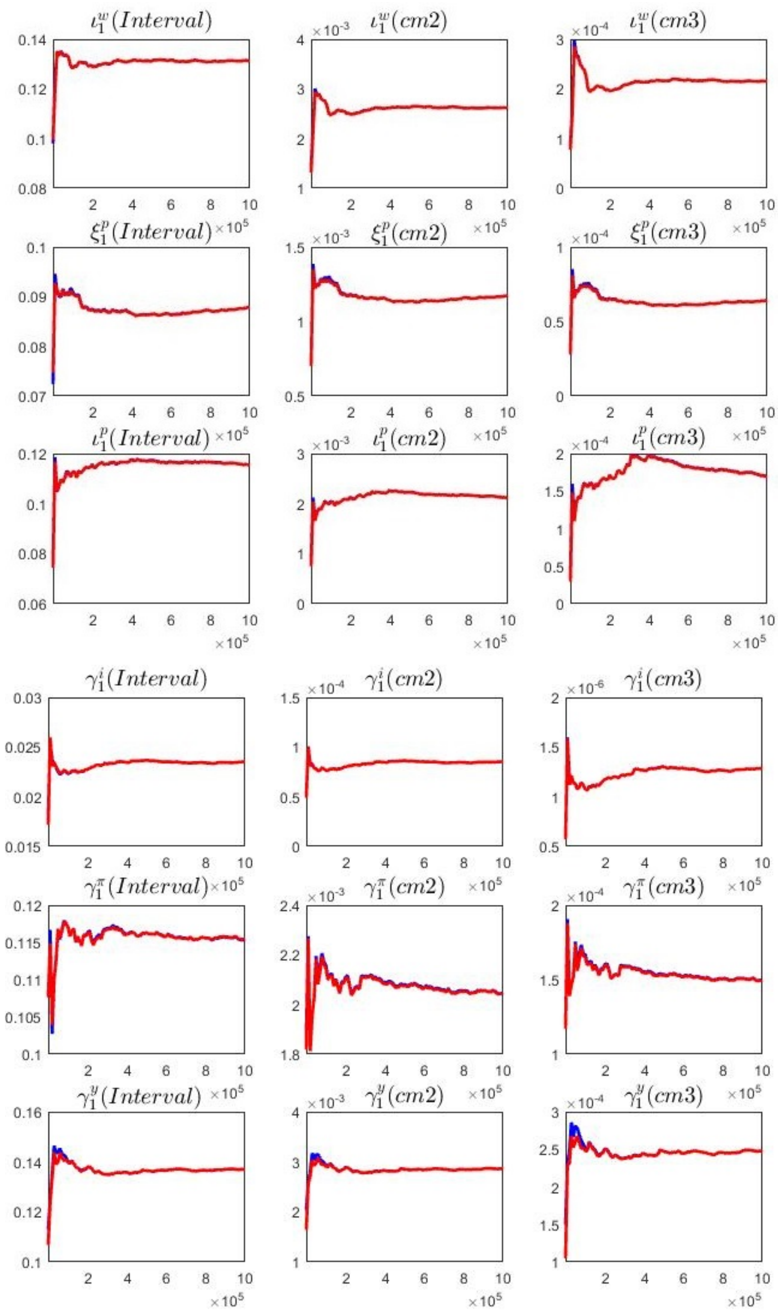
$Gov.Debt\,M.E.(Interval)$
$Gov.Debt\,M.E.(cm2)$
$Gov.Debt\,M.E.(cm3)$
$\rho^{z}_{1,1}(Interval)$
$\rho^{z}_{2,1}(cm2)$
$\rho^{z}_{2,1}(cm3)$
$\rho^{z}_{2,1}(Interval)$
$\rho^{z}_{2,1}(cm2)$
$\rho^{z}_{2,1}(cm3)$
$\rho^{zc}_{1,1}(Interval)$
$\rho^{zc}_{1,1}(cm2)$
$\rho^{zc}_{1,1}(cm3)$
$\rho^{zi}_{1,1}(Interval)$
$\rho^{zi}_{1,1}(cm2)$
$\rho^{zi}_{1,1}(cm3)$
$\rho^{zm}_{2,1}(Interval)$
$\rho^{zm}_{2,1}(cm2)$
$\rho^{zm}_{2,1}(cm3)$


$\rho^{zm}_{2,1}(Interval)$
$\rho^{zm}_{2,1}(cm2)$
$\rho^{zm}_{2,1}(cm3)$
$\rho^{w}_{1,1}(Interval)$
$\rho^{w}_{1,1}(cm2)$
$\rho^{w}_{1,1}(cm3)$
$\rho^{p}_{1,1}(Interval)$
$\rho^{p}_{1,1}(cm2)$
$\rho^{p}_{1,1}(cm3)$
$\rho^{\pi}_{1,1}(Interval)$
$\rho^{\pi}_{1,1}(cm2)$
$\rho^{\pi}_{1,1}(cm3)$
$\rho^{g}_{1,1}(Interval)$
$\rho^{g}_{1,1}(cm2)$
$\rho^{g}_{1,1}(cm3)$
$\rho^{ag}_{1,1}(Interval)$
$\rho^{ag}_{1,1}(cm2)$
$\rho^{ag}_{1,1}(cm3)$

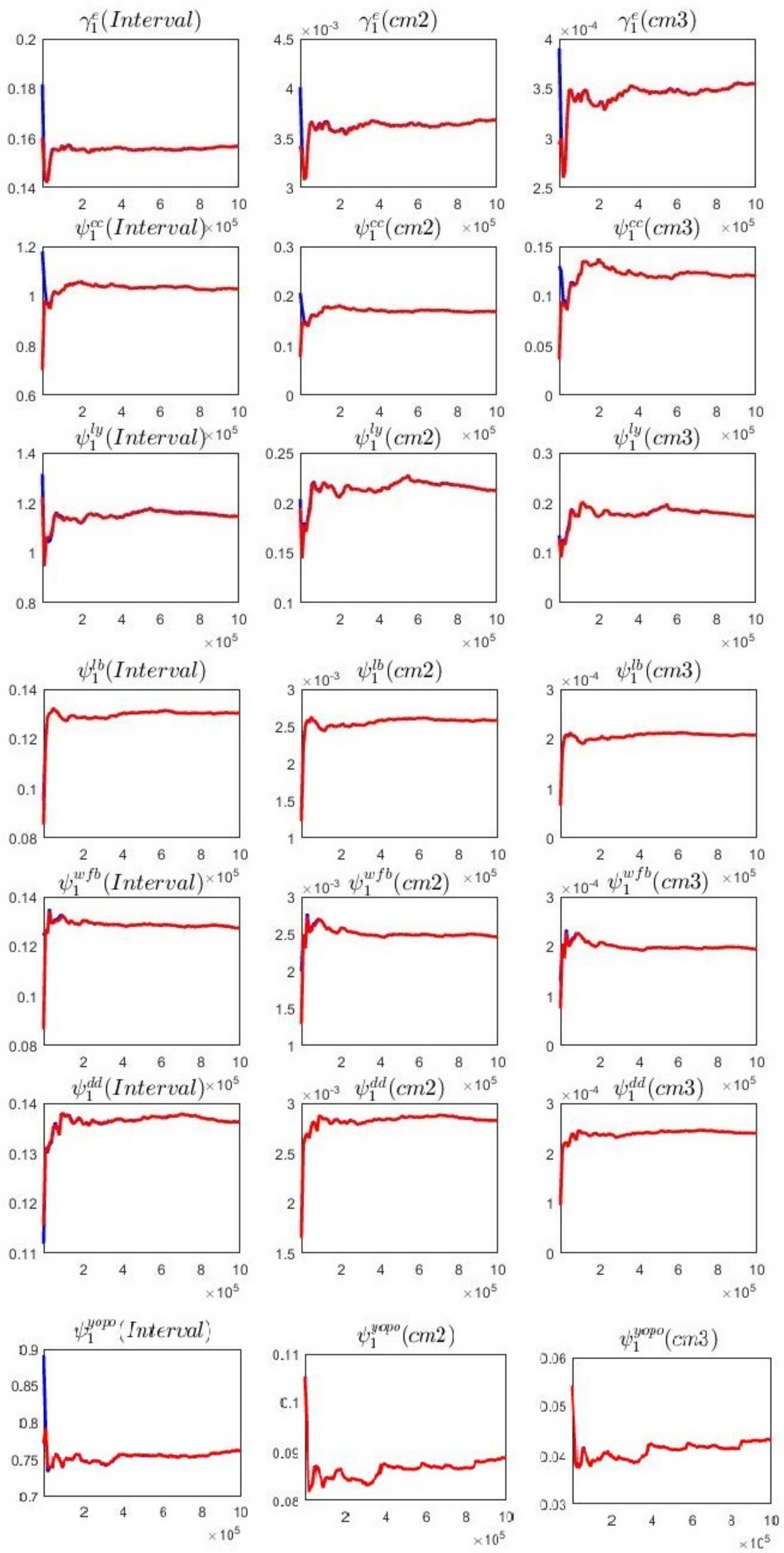


$\rho^{zo}_{1,1}(Interval)$
$\rho^{zo}_{1,1}(cm2)$
$\rho^{zo}_{1,1}(cm3)$
$\rho^{zo}_{2,1}(Interval)$
$\rho^{zo}_{2,1}(cm2)$
$\rho^{zo}_{2,1}(cm3)$
$\rho^{yo}_{1,1}(Interval)$
$\rho^{yo}_{1,1}(cm2)$
$\rho^{yo}_{1,1}(cm3)$
$\rho^{yo}_{2,1}(Interval)$
$\rho^{yo}_{2,1}(cm2)$
$\rho^{yo}_{2,1}(cm3)$
$\rho^{yo}_{1,2}(Interval)$
$\rho^{yo}_{1,2}(cm2)$
$\rho^{yo}_{1,2}(cm3)$
$\rho^{yo}_{2,2}(Interval)$
$\rho^{yo}_{2,2}(cm2)$
$\rho^{yo}_{2,2}(cm3)$


(1 + ρ_1^o)/ρ_1^o(Interval)
(1 + ρ_1^o)/ρ_1^o(cm2)
(1 + ρ_1^o)/ρ_1^o(cm3)
(1 + ρ_1^c)/ρ_1^c(Interval)
(1 + ρ_1^c)/ρ_1^c(cm2)
(1 + ρ_1^c)/ρ_1^c(cm3)
κ_1(Interval)
κ_1(cm2)
κ_1(cm3)
χ_1(Interval)
χ_1(cm2)
χ_1(cm3)
φ_1^i(Interval)
φ_1^i(cm2)
φ_1^i(cm3)
ξ_1^w(Interval)
ξ_1^w(cm2)
ξ_1^w(cm3)
×10^5

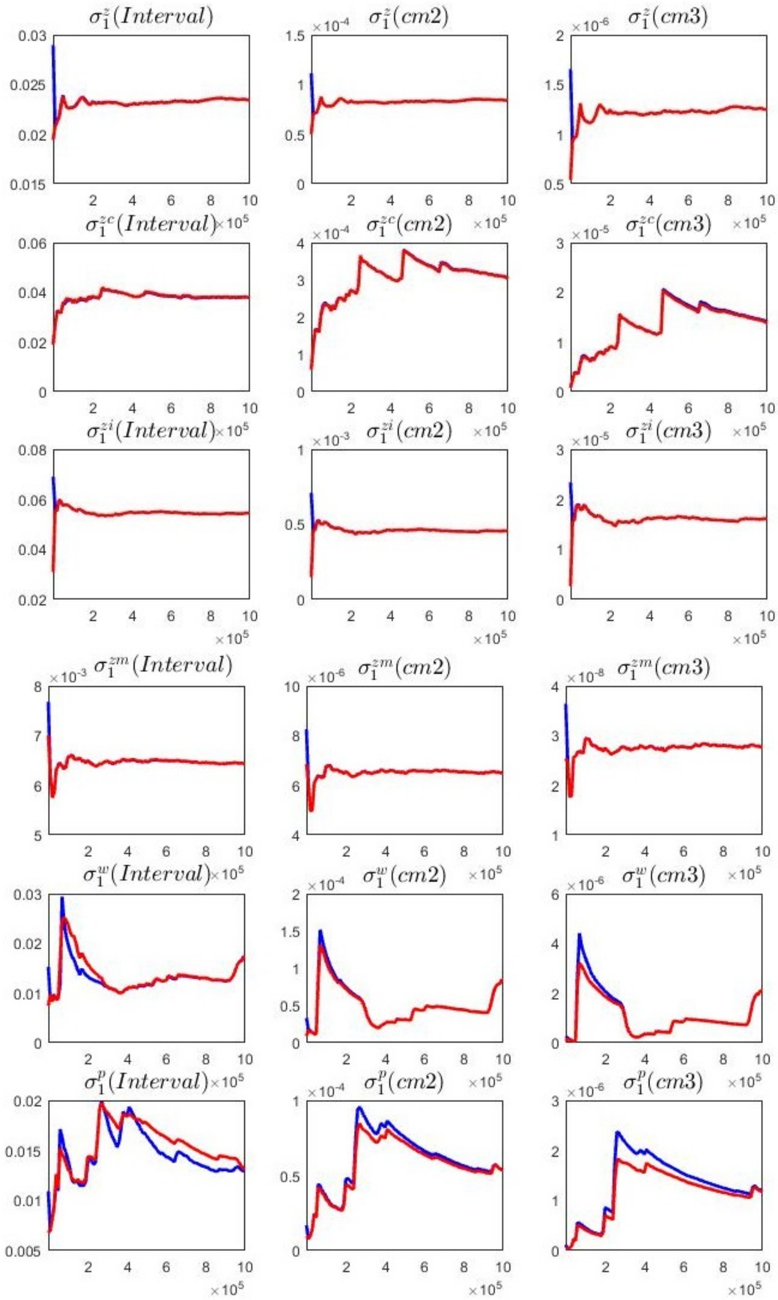


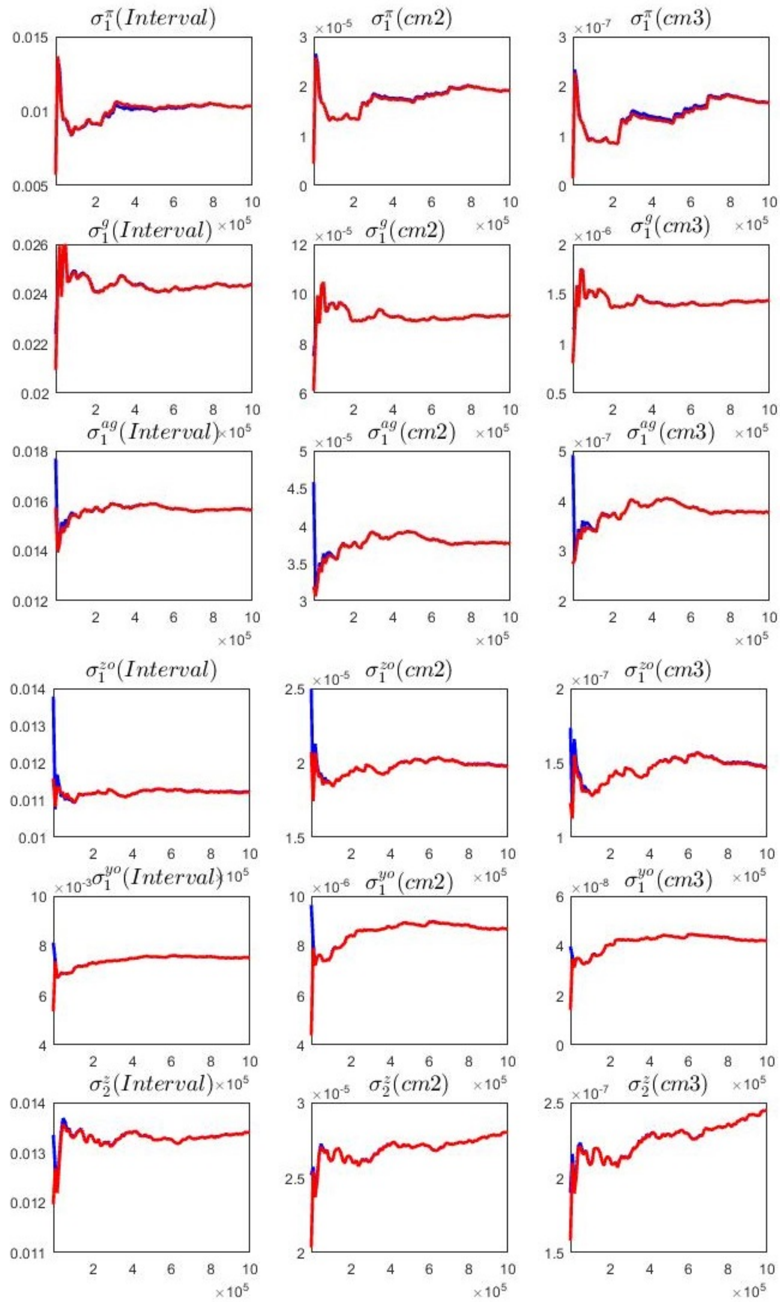
$\iota_1^w(Interval)$
$\iota_1^w(cm2)$
$\iota_1^w(cm3)$
$\xi_1^p(Interval)$
$\xi_1^p(cm2)$
$\xi_1^p(cm3)$
$\iota_1^p(Interval)$
$\iota_1^p(cm2)$
$\iota_1^p(cm3)$
$\gamma_1^i(Interval)$
$\gamma_1^i(cm2)$
$\gamma_1^i(cm3)$
$\gamma_1^\pi(Interval)$
$\gamma_1^\pi(cm2)$
$\gamma_1^\pi(cm3)$
$\gamma_1^y(Interval)$
$\gamma_1^y(cm2)$
$\gamma_1^y(cm3)$
$\times 10^5$



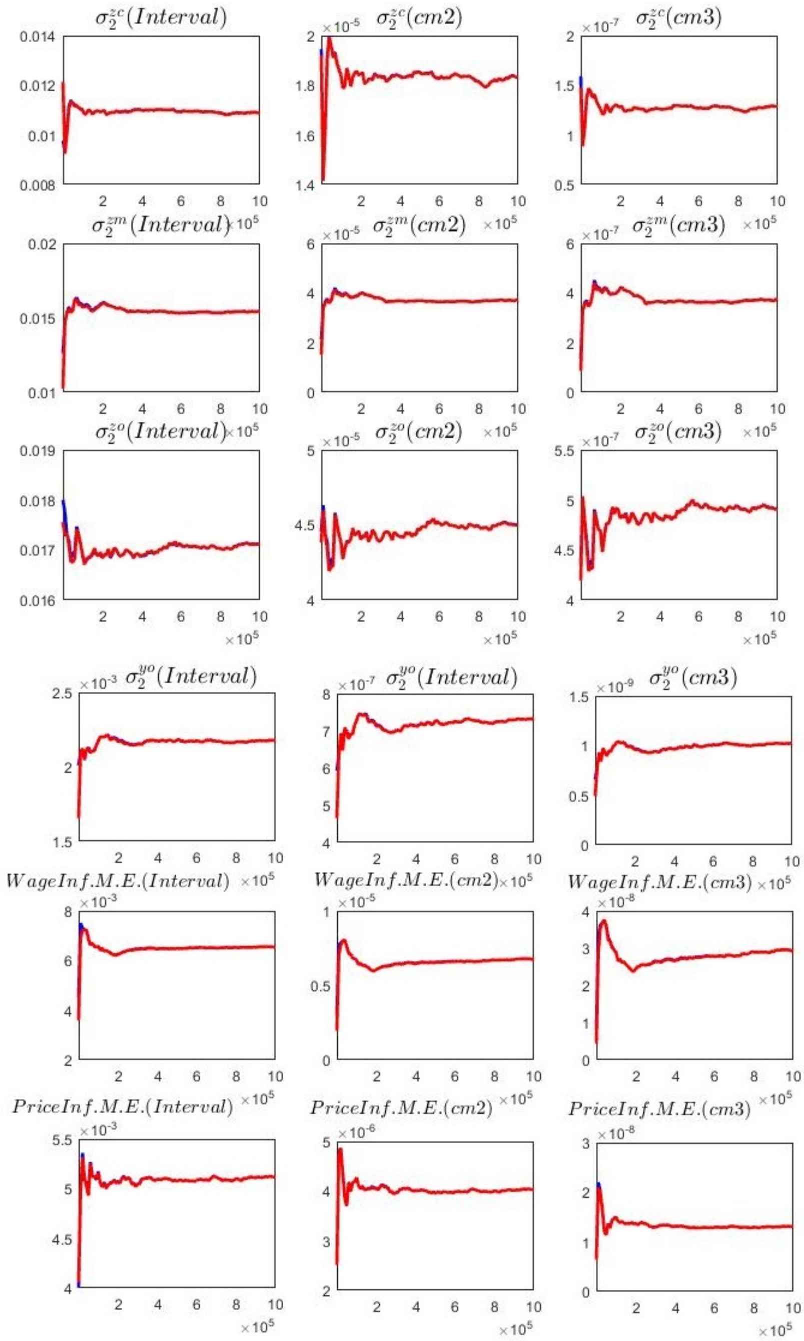
$\gamma_1^e(Interval)$
$\gamma_1^e(cm2)$
$\gamma_1^e(cm3)$
$\psi_1^{cc}(Interval)$
$\psi_1^{cc}(cm2)$
$\psi_1^{cc}(cm3)$
$\psi_1^{ly}(Interval)$
$\psi_1^{ly}(cm2)$
$\psi_1^{ly}(cm3)$
$\psi_1^{lb}(Interval)$
$\psi_1^{lb}(cm2)$
$\psi_1^{lb}(cm3)$
$\psi_1^{wfb}(Interval)$
$\psi_1^{wfb}(cm2)$
$\psi_1^{wfb}(cm3)$
$\psi_1^{dd}(Interval)$
$\psi_1^{dd}(cm2)$
$\psi_1^{dd}(cm3)$
$\psi_1^{yopo}(Interval)$
$\psi_1^{yopo}(cm2)$
$\psi_1^{yopo}(cm3)$

6.2.3 Saudi Arabia Model

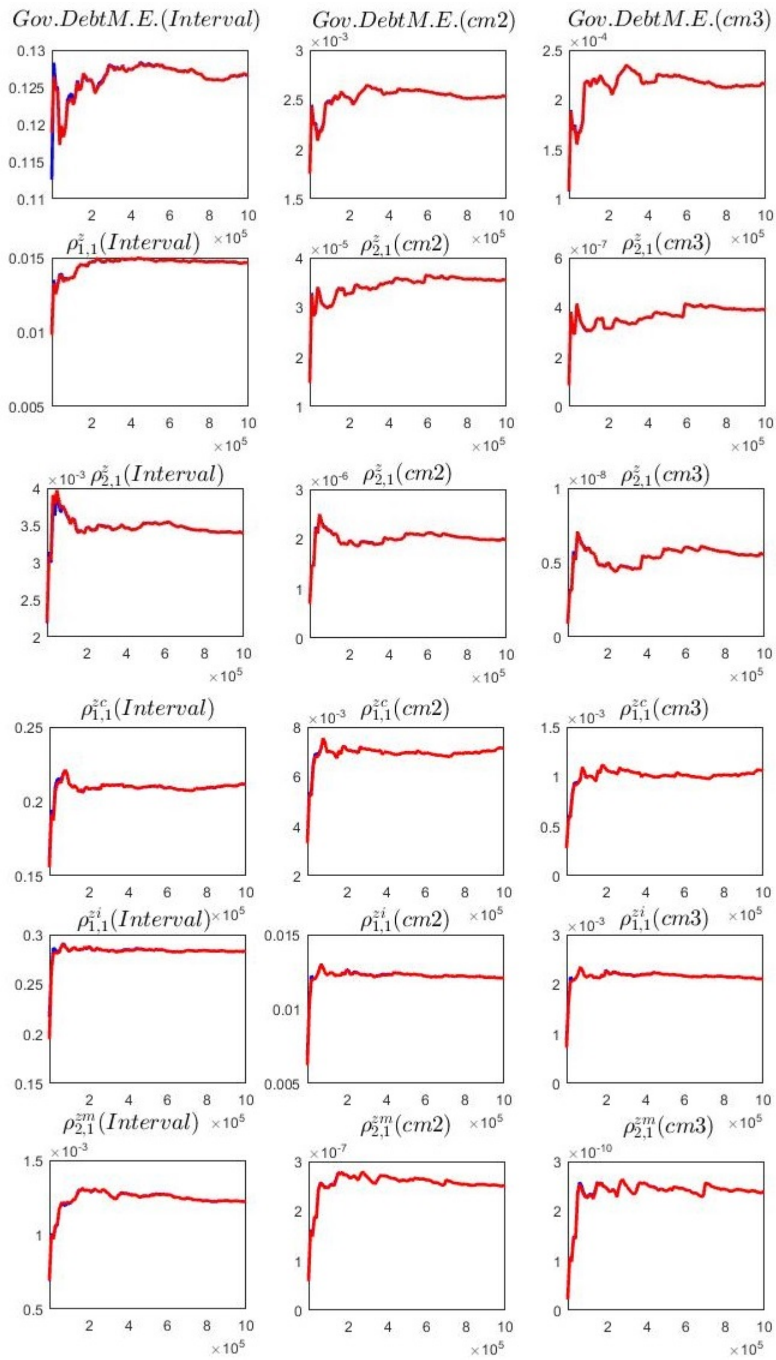




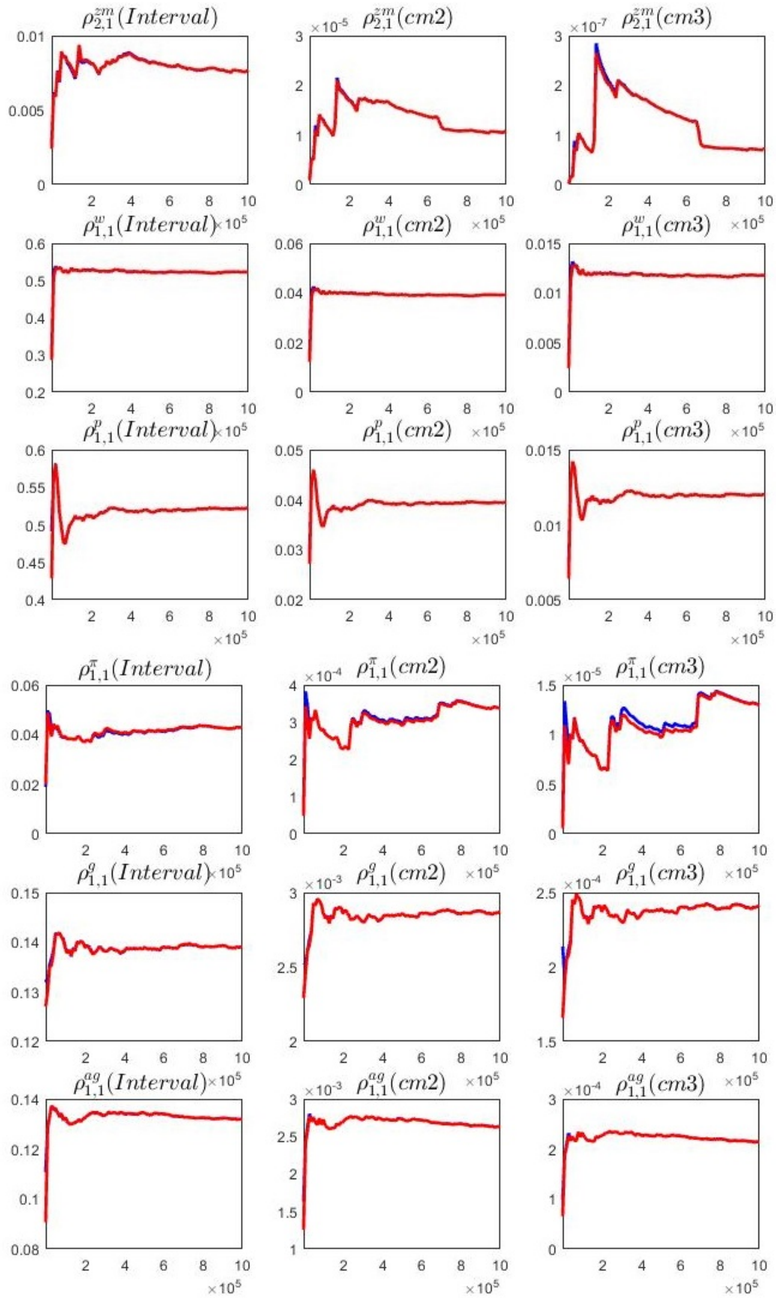
$\sigma_1^{\pi}(Interval)$
$\sigma_1^{\pi}(cm2)$
$\sigma_1^{\pi}(cm3)$
$\sigma_1^{g}(Interval)$
$\sigma_1^{g}(cm2)$
$\sigma_1^{g}(cm3)$
$\sigma_1^{ag}(Interval)$
$\sigma_1^{ag}(cm2)$
$\sigma_1^{ag}(cm3)$
$\sigma_1^{zo}(Interval)$
$\sigma_1^{zo}(cm2)$
$\sigma_1^{zo}(cm3)$
$\sigma_1^{yo}(Interval)$
$\sigma_1^{yo}(cm2)$
$\sigma_1^{yo}(cm3)$
$\sigma_2^{z}(Interval)$
$\sigma_2^{z}(cm2)$
$\sigma_2^{z}(cm3)$
$\times 10^5$



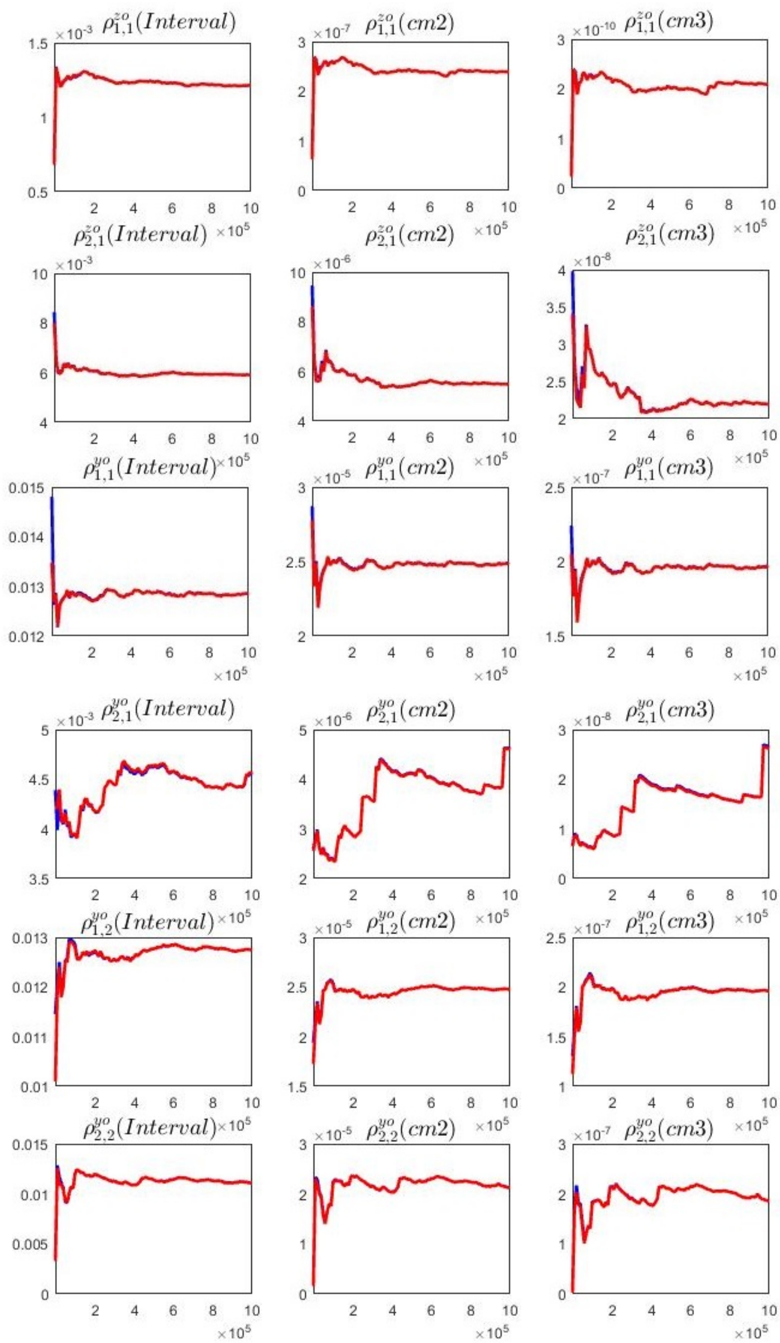
$\sigma_2^{zc}(Interval)$
$\sigma_2^{zc}(cm2)$
$\sigma_2^{zc}(cm3)$
$\sigma_2^{zm}(Interval)$
$\sigma_2^{zm}(cm2)$
$\sigma_2^{zm}(cm3)$
$\sigma_2^{zo}(Interval)$
$\sigma_2^{zo}(cm2)$
$\sigma_2^{zo}(cm3)$
$\sigma_2^{yo}(Interval)$
$\sigma_2^{yo}(Interval)$
$\sigma_2^{yo}(cm3)$
$WageInf.M.E.(Interval)$
$WageInf.M.E.(cm2)$
$WageInf.M.E.(cm3)$
$PriceInf.M.E.(Interval)$
$PriceInf.M.E.(cm2)$
$PriceInf.M.E.(cm3)$
$\times 10^5$



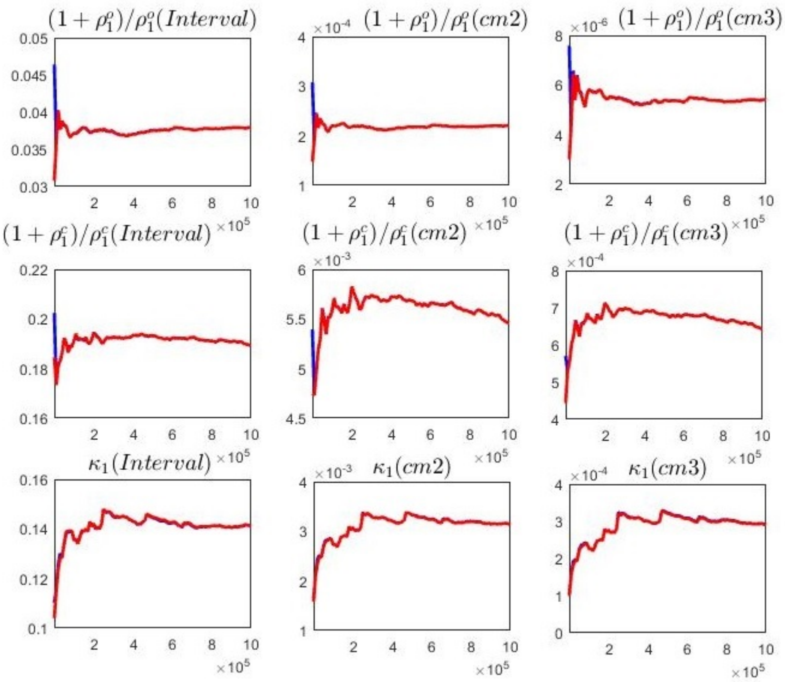
$Gov.DebtM.E.(Interval)$
$Gov.DebtM.E.(cm2)$
$Gov.DebtM.E.(cm3)$
$\rho^{z}_{1,1}(Interval)$
$\rho^{z}_{2,1}(cm2)$
$\rho^{z}_{2,1}(cm3)$
$\rho^{z}_{2,1}(Interval)$
$\rho^{z}_{2,1}(cm2)$
$\rho^{z}_{2,1}(cm3)$
$\rho^{zc}_{1,1}(Interval)$
$\rho^{zc}_{1,1}(cm2)$
$\rho^{zc}_{1,1}(cm3)$
$\rho^{zi}_{1,1}(Interval)$
$\rho^{zi}_{1,1}(cm2)$
$\rho^{zi}_{1,1}(cm3)$
$\rho^{zm}_{2,1}(Interval)$
$\rho^{zm}_{2,1}(cm2)$
$\rho^{zm}_{2,1}(cm3)$



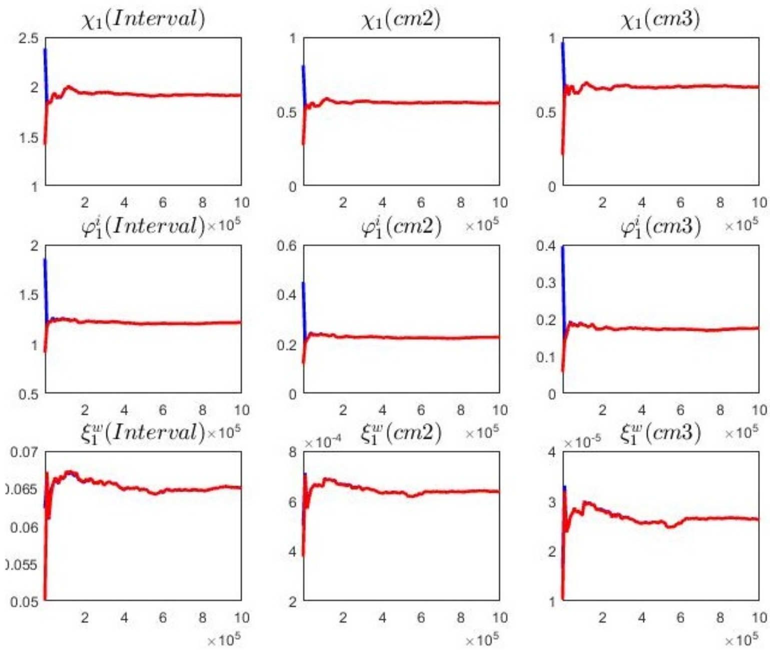
$\rho^{zm}_{2,1}(Interval)$
$\rho^{zm}_{2,1}(cm2)$
$\rho^{zm}_{2,1}(cm3)$
$\rho^{w}_{1,1}(Interval)$
$\rho^{w}_{1,1}(cm2)$
$\rho^{w}_{1,1}(cm3)$
$\rho^{p}_{1,1}(Interval)$
$\rho^{p}_{1,1}(cm2)$
$\rho^{p}_{1,1}(cm3)$
$\rho^{\pi}_{1,1}(Interval)$
$\rho^{\pi}_{1,1}(cm2)$
$\rho^{\pi}_{1,1}(cm3)$
$\rho^{g}_{1,1}(Interval)$
$\rho^{g}_{1,1}(cm2)$
$\rho^{g}_{1,1}(cm3)$
$\rho^{ag}_{1,1}(Interval)$
$\rho^{ag}_{1,1}(cm2)$
$\rho^{ag}_{1,1}(cm3)$
$\times 10^5$



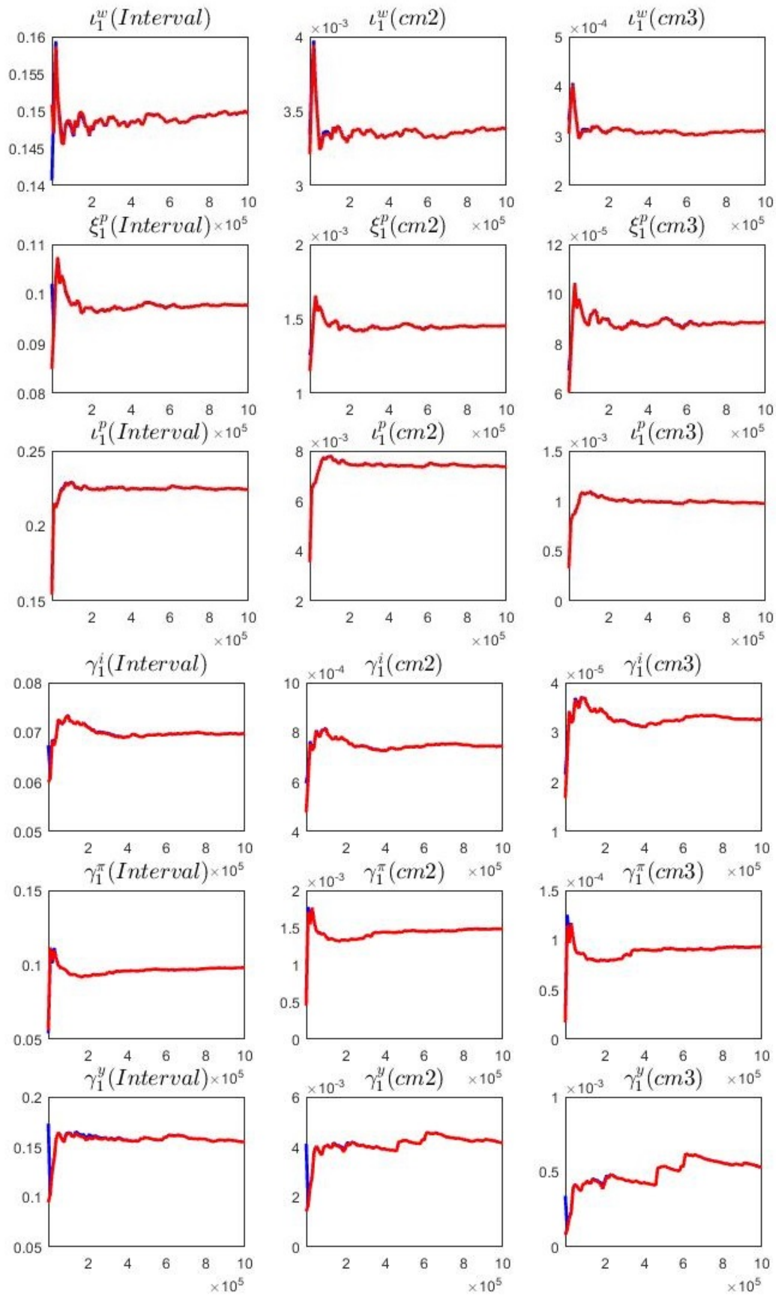
$\rho^{zo}_{1,1}(Interval)$
$\rho^{zo}_{1,1}(cm2)$
$\rho^{zo}_{1,1}(cm3)$
$\rho^{zo}_{2,1}(Interval)$
$\rho^{zo}_{2,1}(cm2)$
$\rho^{zo}_{2,1}(cm3)$
$\rho^{yo}_{1,1}(Interval)$
$\rho^{yo}_{1,1}(cm2)$
$\rho^{yo}_{1,1}(cm3)$
$\rho^{yo}_{2,1}(Interval)$
$\rho^{yo}_{2,1}(cm2)$
$\rho^{yo}_{2,1}(cm3)$
$\rho^{yo}_{1,2}(Interval)$
$\rho^{yo}_{1,2}(cm2)$
$\rho^{yo}_{1,2}(cm3)$
$\rho^{yo}_{2,2}(Interval)$
$\rho^{yo}_{2,2}(cm2)$
$\rho^{yo}_{2,2}(cm3)$
$\times 10^5$



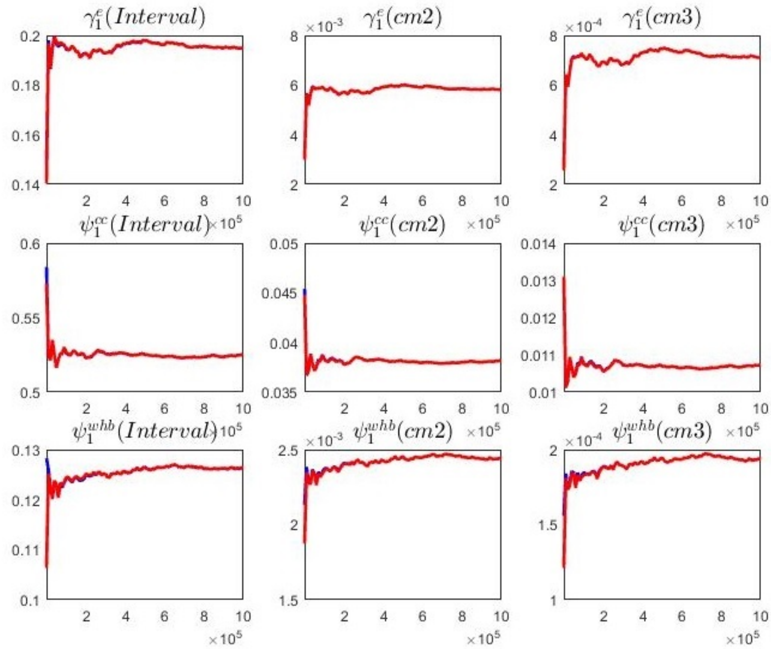
$(1+\rho_1^o)/\rho_1^o(Interval)$
$(1+\rho_1^o)/\rho_1^o(cm2)$
$(1+\rho_1^o)/\rho_1^o(cm3)$
$(1+\rho_1^c)/\rho_1^c(Interval)$
$(1+\rho_1^c)/\rho_1^c(cm2)$
$(1+\rho_1^c)/\rho_1^c(cm3)$
$\kappa_1(Interval)$
$\kappa_1(cm2)$
$\kappa_1(cm3)$

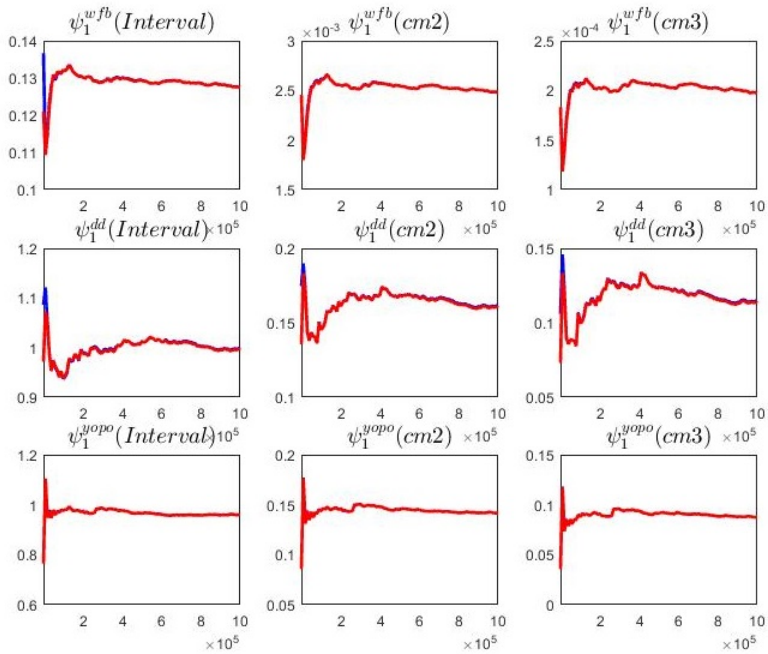


$\chi_1(Interval)$
$\chi_1(cm2)$
$\chi_1(cm3)$
$\varphi_1^i(Interval)$
$\varphi_1^i(cm2)$
$\varphi_1^i(cm3)$
$\xi_1^w(Interval)$
$\xi_1^w(cm2)$
$\xi_1^w(cm3)$



$\iota_1^w(Interval)$
$\iota_1^w(cm2)$
$\iota_1^w(cm3)$
$\xi_1^p(Interval)$
$\xi_1^p(cm2)$
$\xi_1^p(cm3)$
$\iota_1^p(Interval)$
$\iota_1^p(cm2)$
$\iota_1^p(cm3)$
$\gamma_1^i(Interval)$
$\gamma_1^i(cm2)$
$\gamma_1^i(cm3)$
$\gamma_1^\pi(Interval)$
$\gamma_1^\pi(cm2)$
$\gamma_1^\pi(cm3)$
$\gamma_1^y(Interval)$
$\gamma_1^y(cm2)$
$\gamma_1^y(cm3)$
$\times 10^{-3}$
$\times 10^{-4}$
$\times 10^{-5}$
$\times 10^{5}$

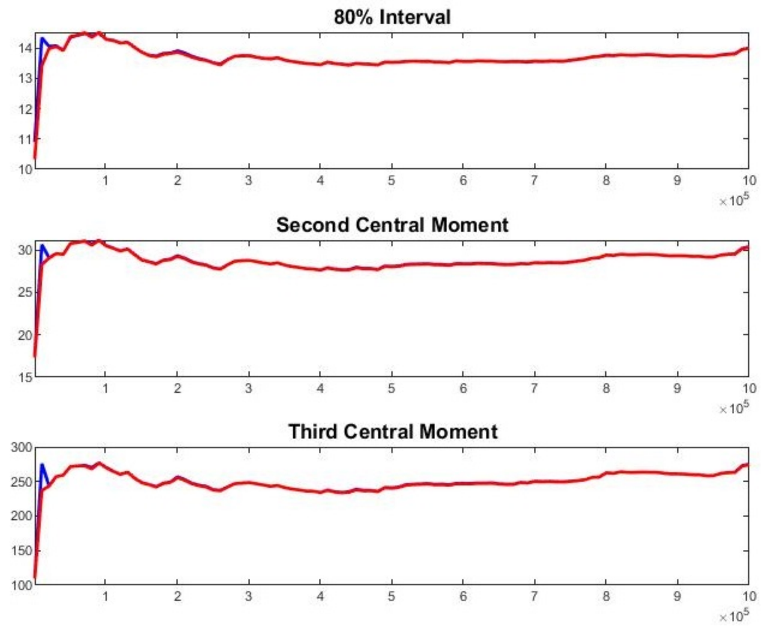

$\gamma_1^e(Interval)$
$\gamma_1^e(cm2)$
$\gamma_1^e(cm3)$
$\psi_1^{cc}(Interval)$
$\psi_1^{cc}(cm2)$
$\psi_1^{cc}(cm3)$
$\psi_1^{whb}(Interval)$
$\psi_1^{whb}(cm2)$
$\psi_1^{whb}(cm3)$


$\psi_1^{wfb}(Interval)$
$\psi_1^{wfb}(cm2)$
$\psi_1^{wfb}(cm3)$
$\psi_1^{dd}(Interval)$
$\psi_1^{dd}(cm2)$
$\psi_1^{dd}(cm3)$
$\psi_1^{yopo}(Interval)$
$\psi_1^{yopo}(cm2)$
$\psi_1^{yopo}(cm3)$

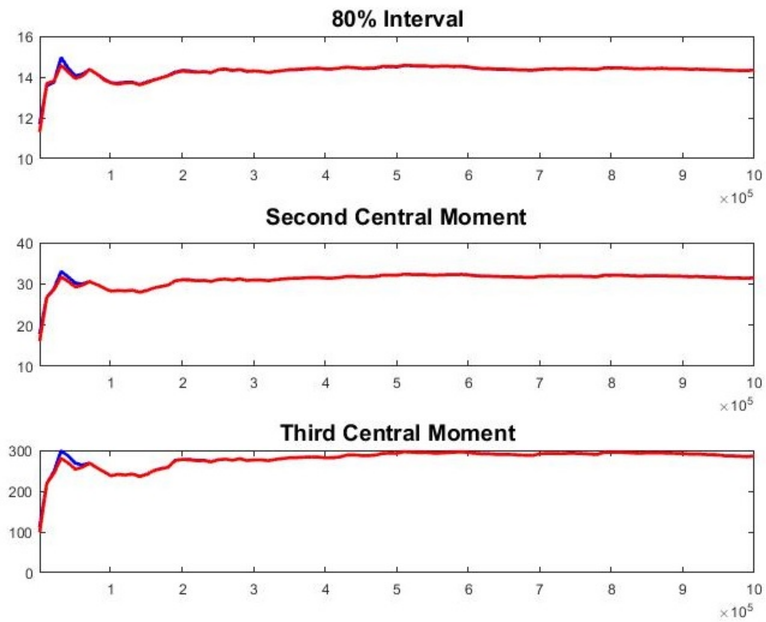
6.3 Multivariate Convergence Diagnostics

In the graphs below, the diagnostics is based on the range of the posterior likelihood function. The posterior kernel is used to aggregate the parameters.

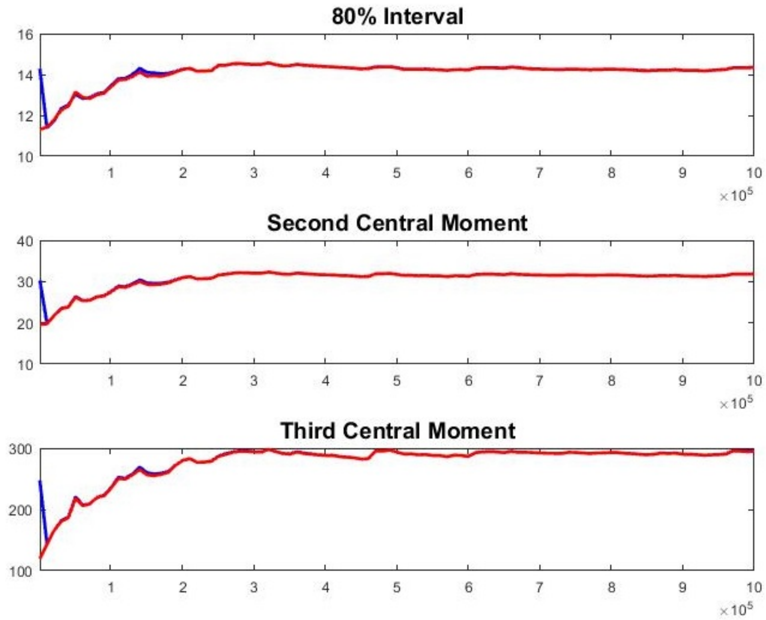
6.3.1 UK Model



6.3.2 Russia Model



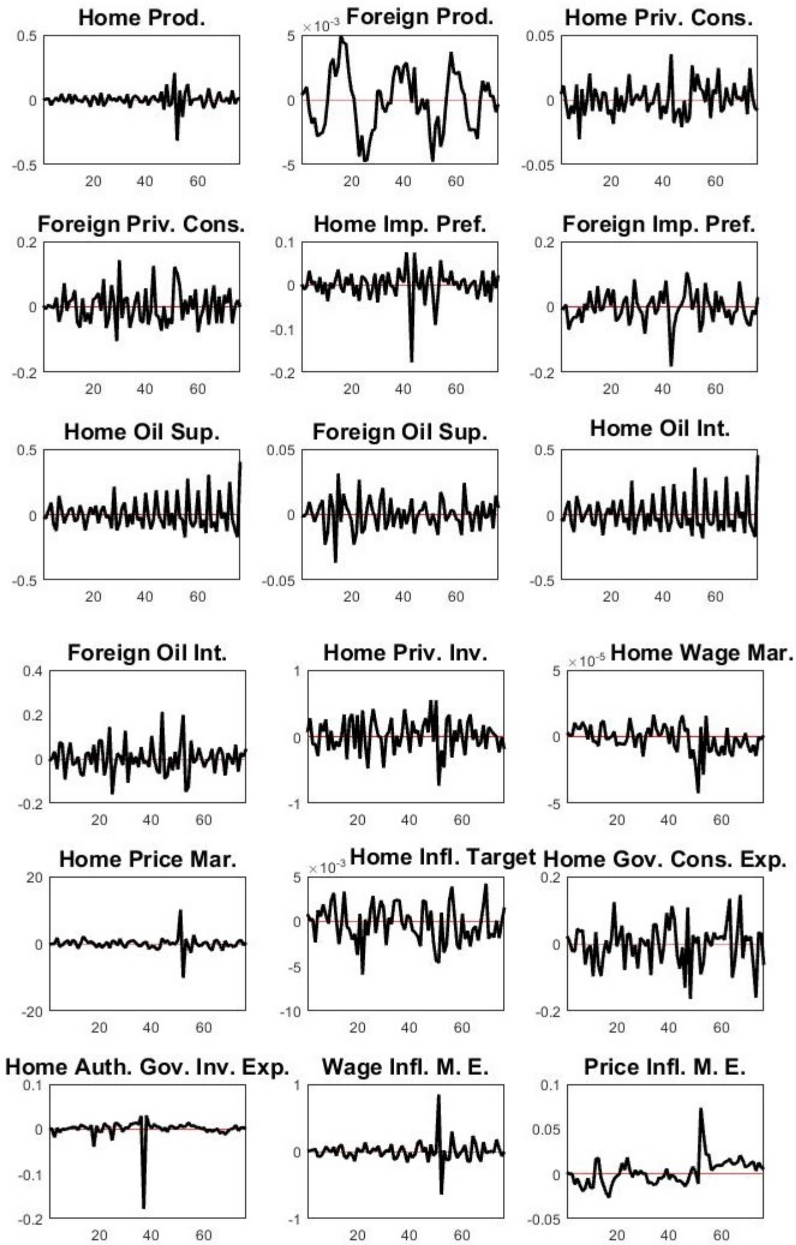
6.3.3 Saudi Arabia Model



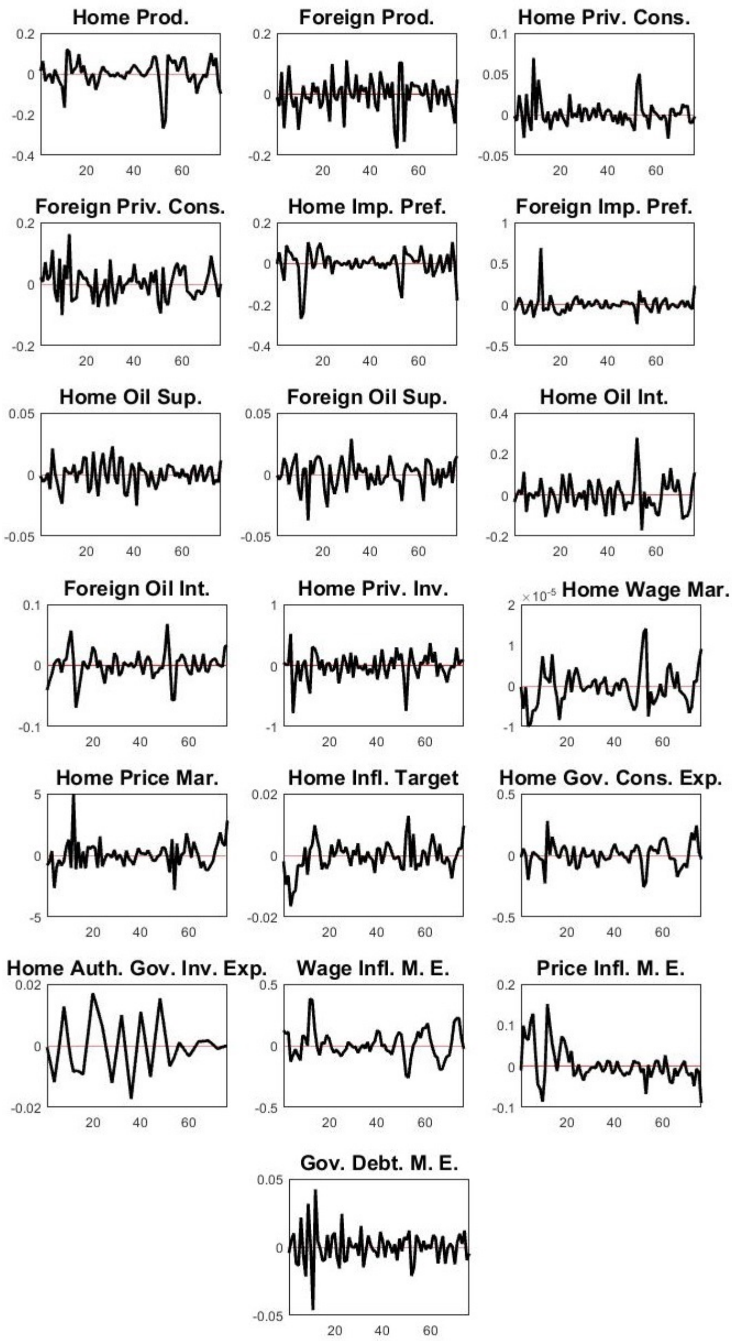
6.4 Smoothed Shocks

In the graphs below, the black line represents the estimate of the smoothed structural shocks derived from the Kalman smoother.

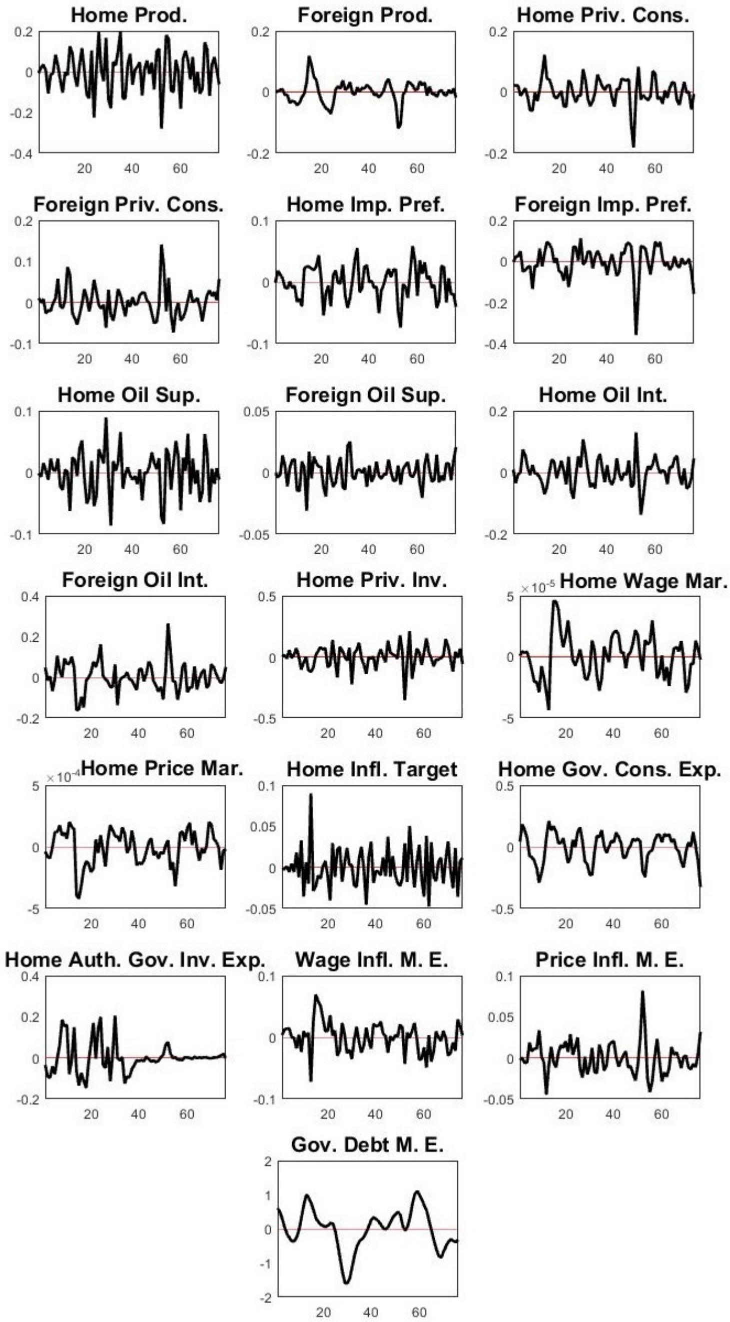
6.4.1 UK Model



6.4.2 Russia Model



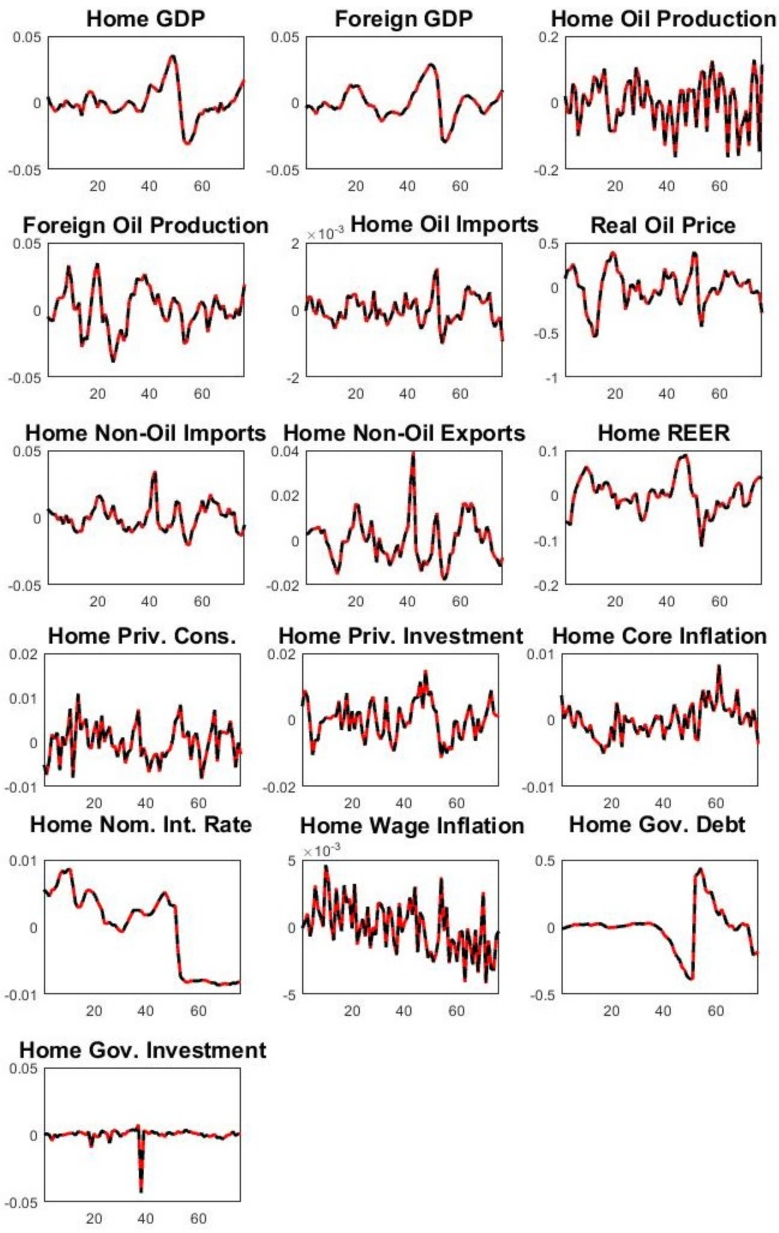
6.4.3 Saudi Arabia Model



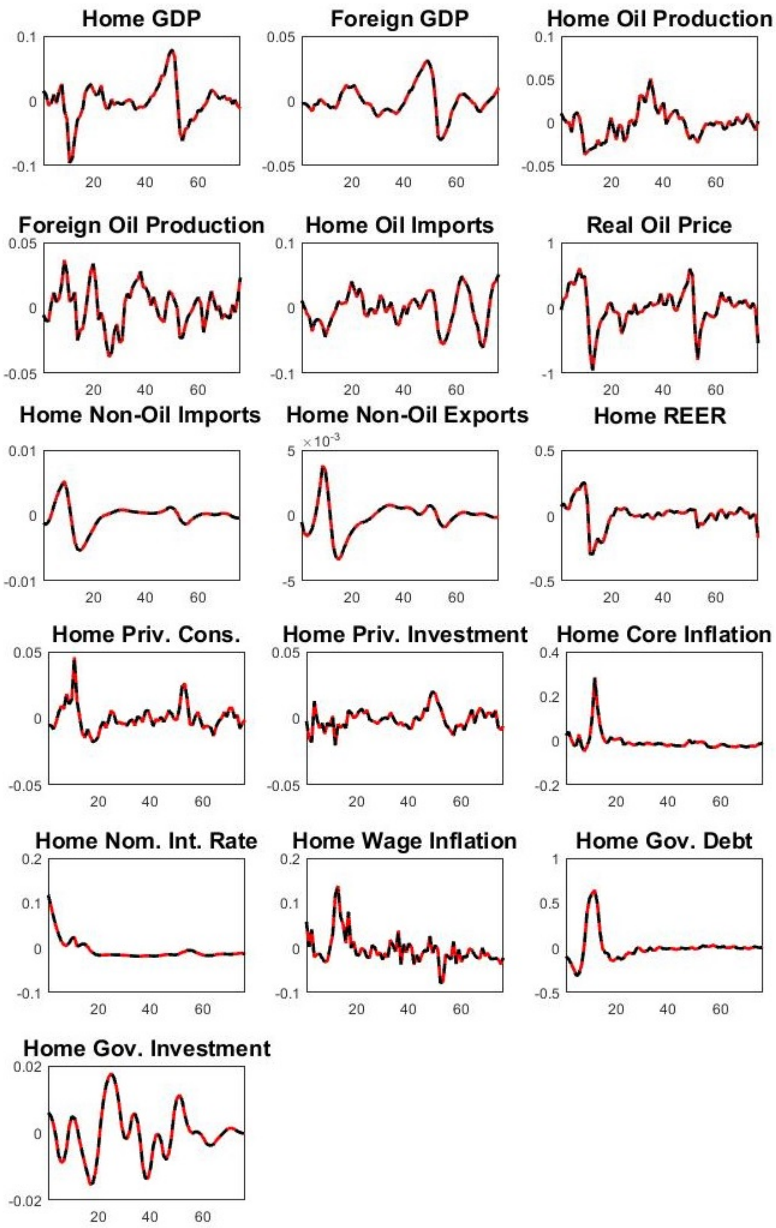
6.5 Historical and Smoothed Variables

In the graphs below, the dotted black line indicates the observed data whereas the red line indicates the estimate of the smoothed variables derived from the Kalman smoother.

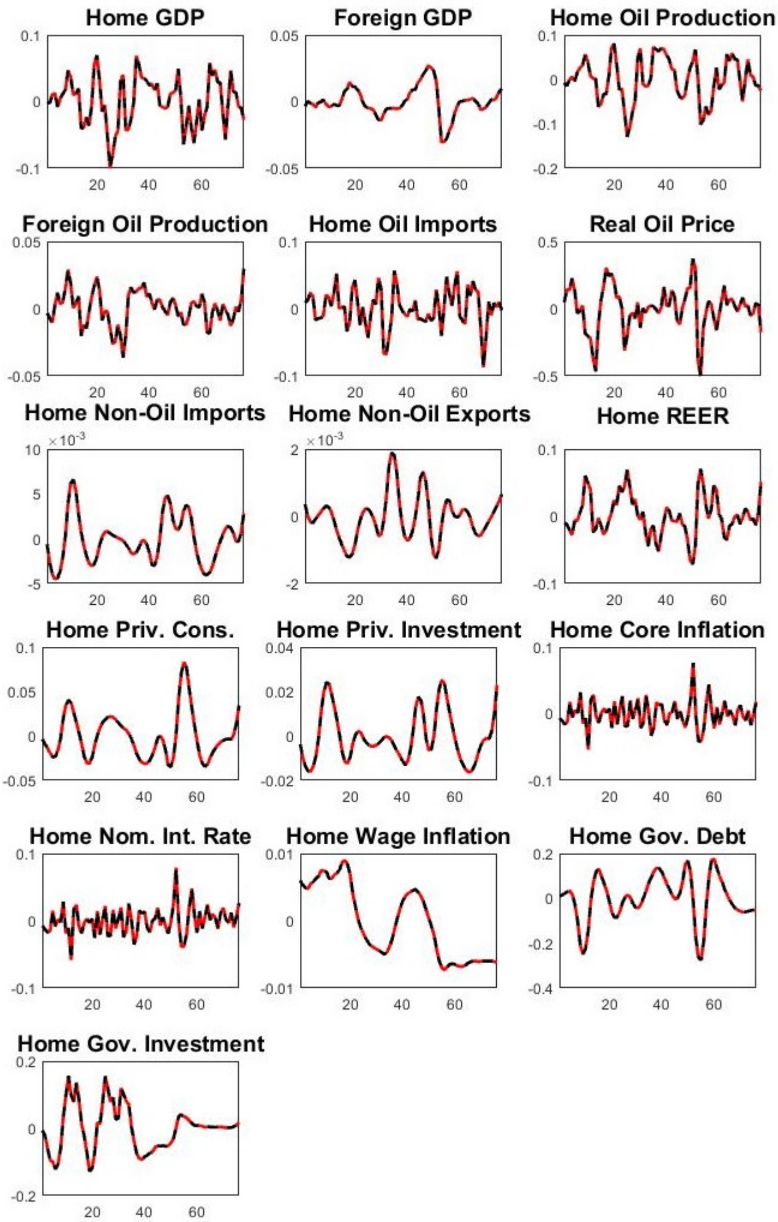
6.5.1 UK Model



6.5.2 Russia Model



6.5.3 Saudi Arabia Model

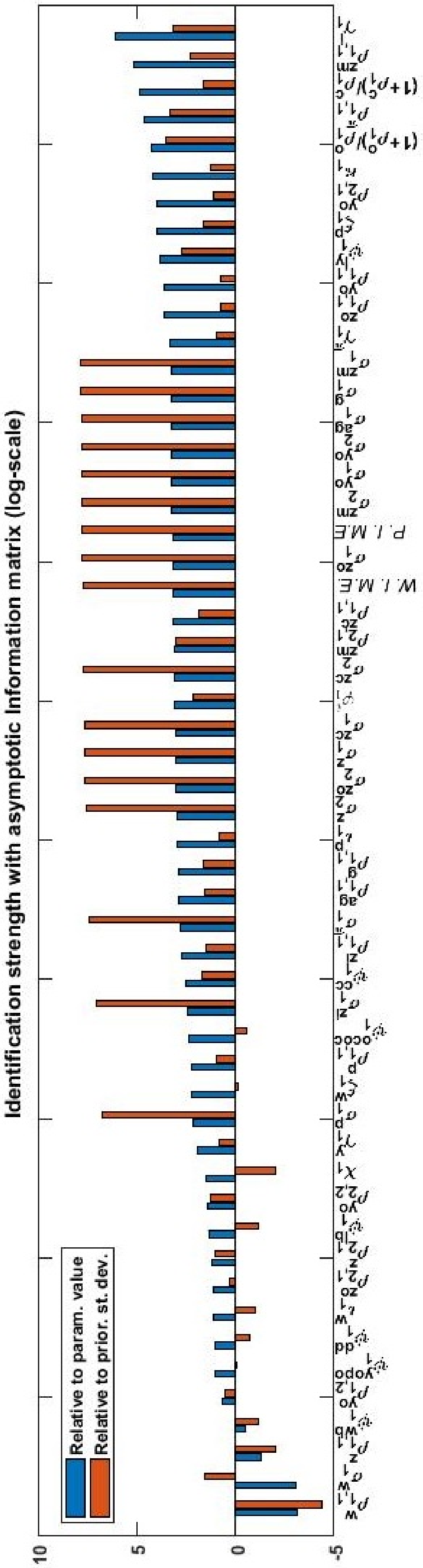


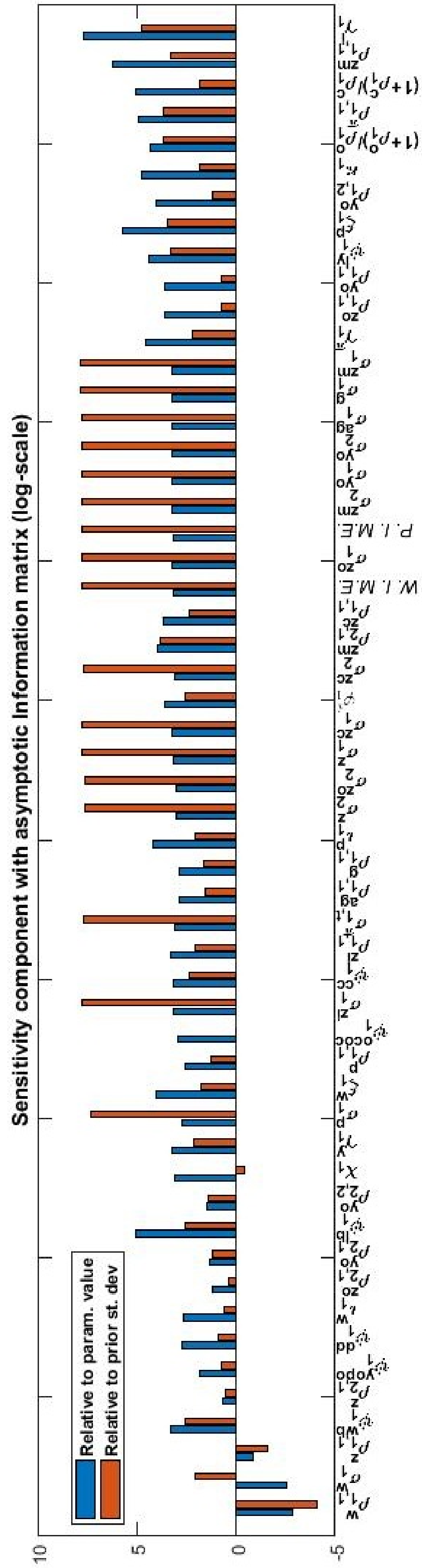
7 Appendix G: Identification Tests

7.1 UK Model

In the top panel of the graph below, the bar charts represent the identification strength of the parameters based on the Fischer information matrix normalised by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (orange bars).

In the bottom panel of the graph below, we show the sensitivity component of the parameters based on the moments information matrix normalised by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (orange bars).

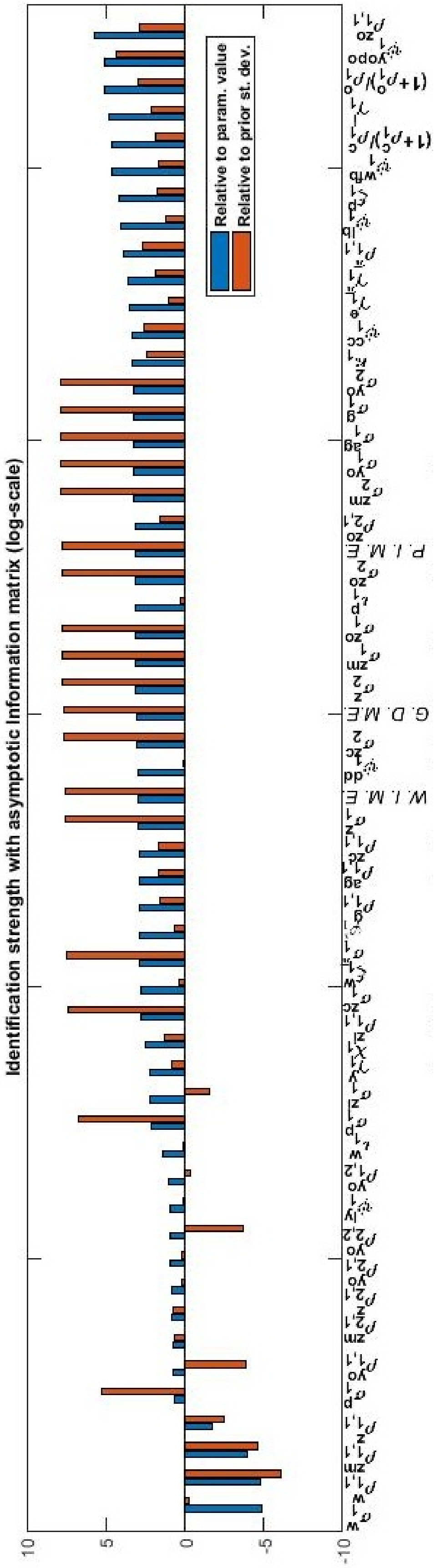

Identification strength with asymptotic Information matrix (log-scale)
Relative to param. value
Relative to prior. st. dev.
10
5
0
-5

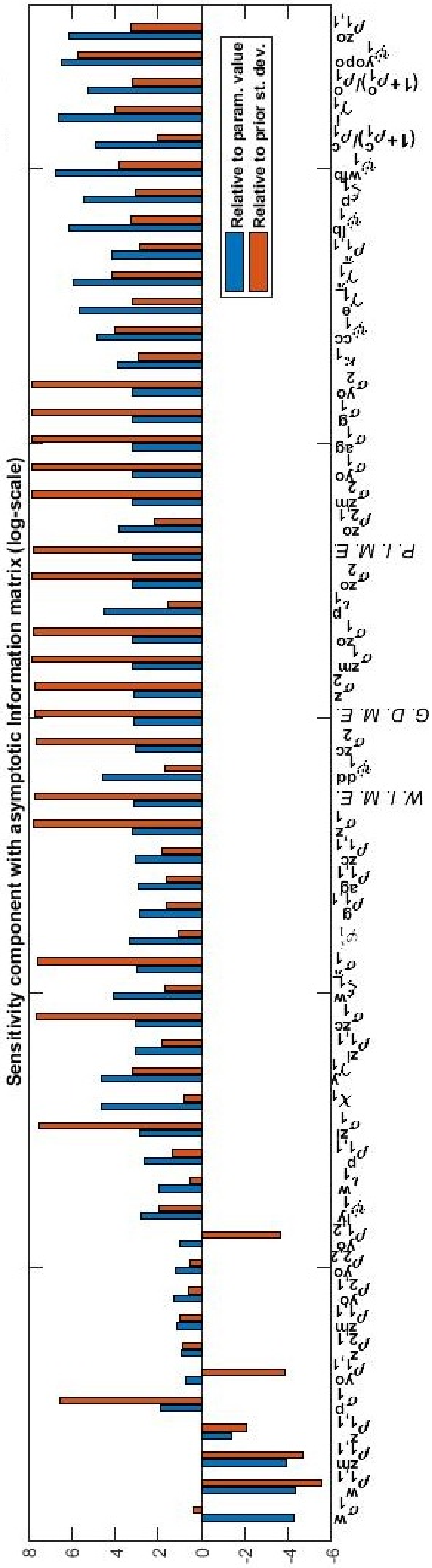

Sensitivity component with asymptotic Information matrix (log-scale)
Relative to param. value
Relative to prior st. dev
10
5
0
-5

7.2 Russia Model

In the top panel of the graph below, the bar charts represent the identification strength of the parameters based on the Fischer information matrix normalised by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (orange bars).

In the bottom panel of the graph below, we show the sensitivity component of the parameters based on the moments information matrix normalised by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (orange bars).

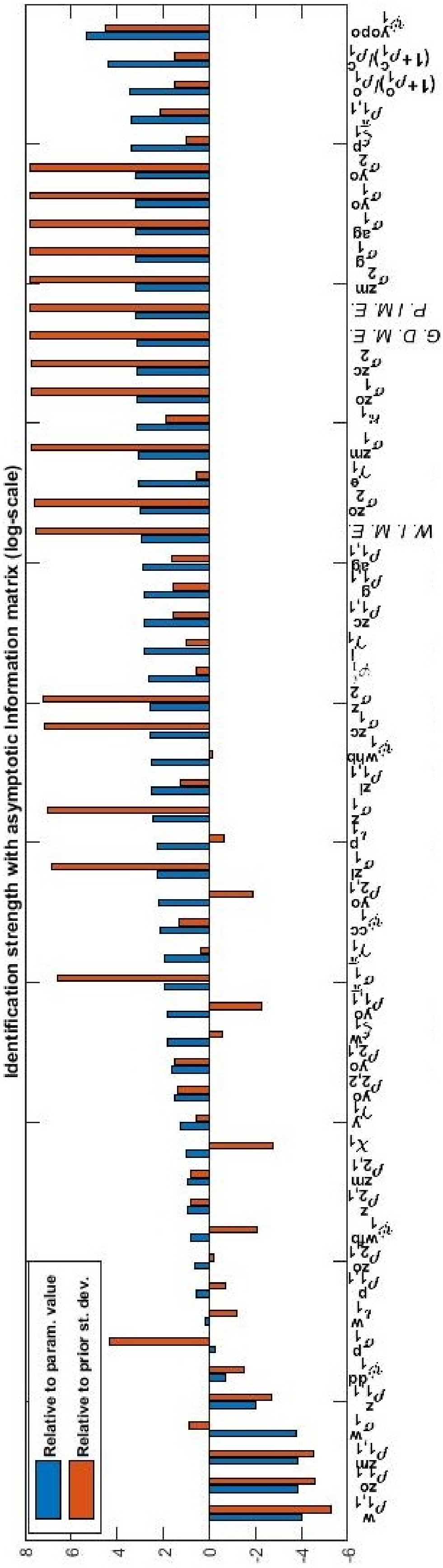

Identification strength with asymptotic Information matrix (log-scale)
Relative to param. value
Relative to prior st. dev.

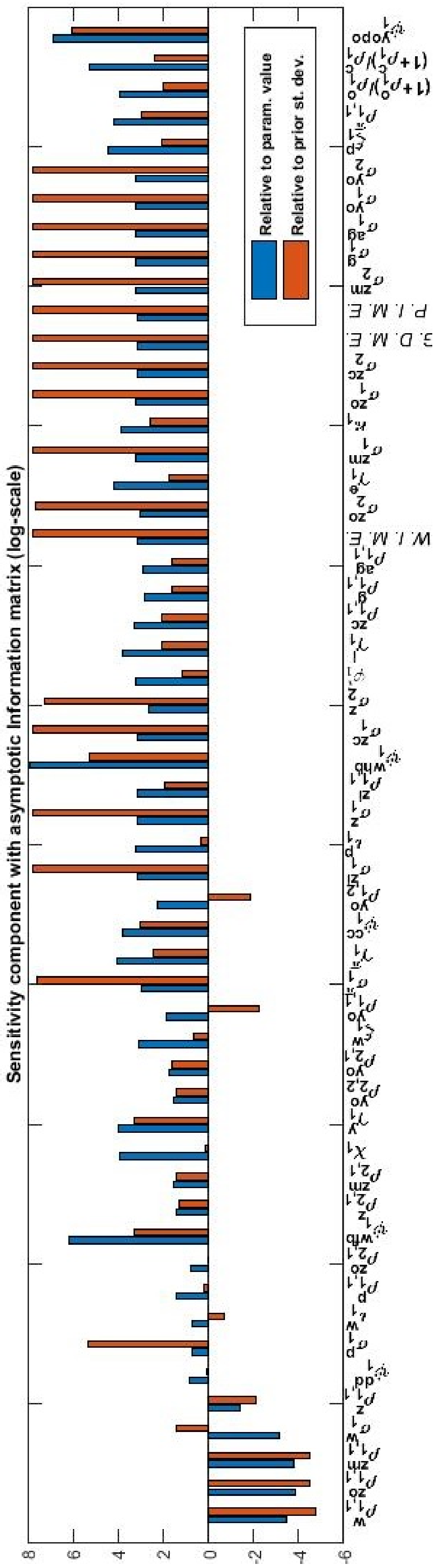

Sensitivity component with asymptotic Information matrix (log-scale)
Relative to param. value
Relative to prior st. dev.

7.3 Saudi Arabia Model

In the top panel of the graph below, the bar charts represent the identification strength of the parameters based on the Fischer information matrix normalised by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (orange bars).

In the bottom panel of the graph below, we show the sensitivity component of the parameters based on the moments information matrix normalised by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (orange bars).

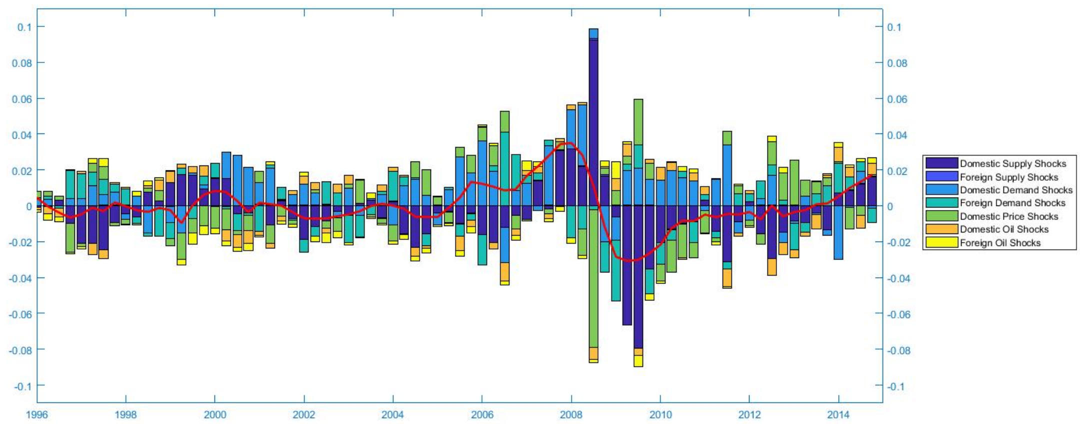

Identification strength with asymptotic Information matrix (log-scale)
Relative to param. value
Relative to prior st. dev.


Sensitivity component with asymptotic Information matrix (log-scale)
Relative to param. value
Relative to prior st. dev.

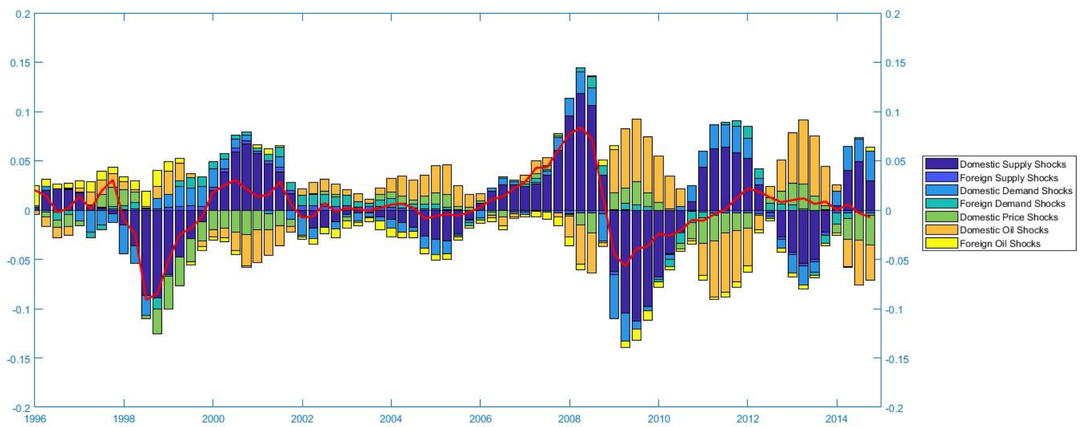
8 Appendix H: Historical Decompositions for the Full Sample

Below we present the GDP historical decompositions for UK, Russia and Saudi Arabia as obtained from our estimated models. The red lines show the real GDP.

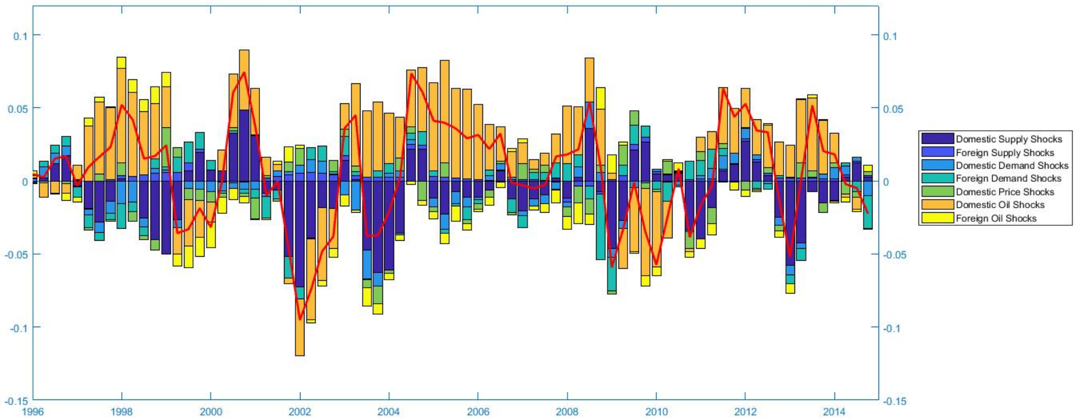
8.1 UK Model



8.2 Russia Model



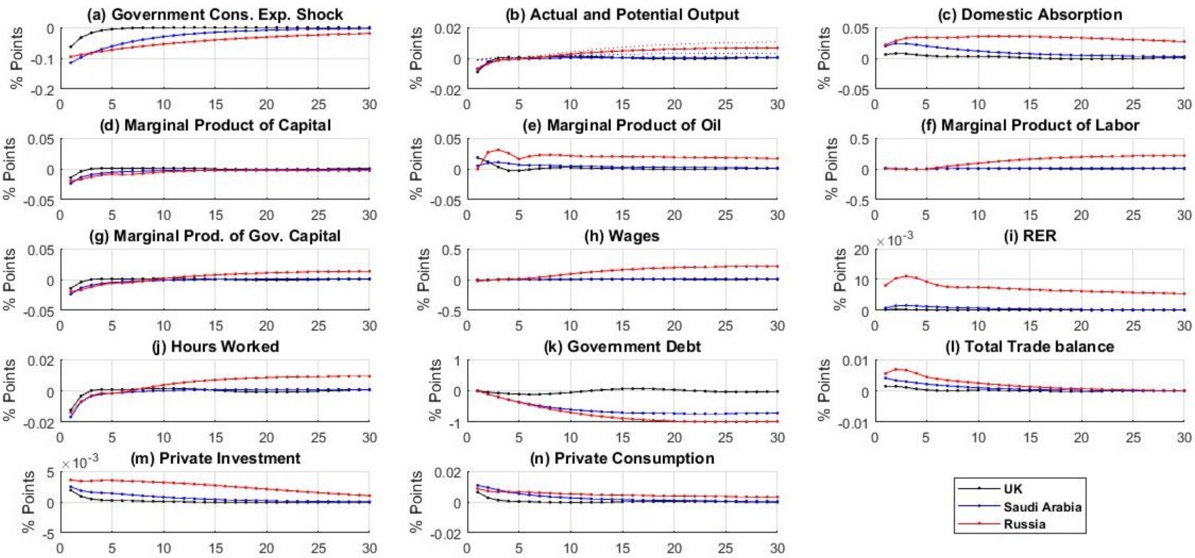
8.3 Saudi Arabia Model



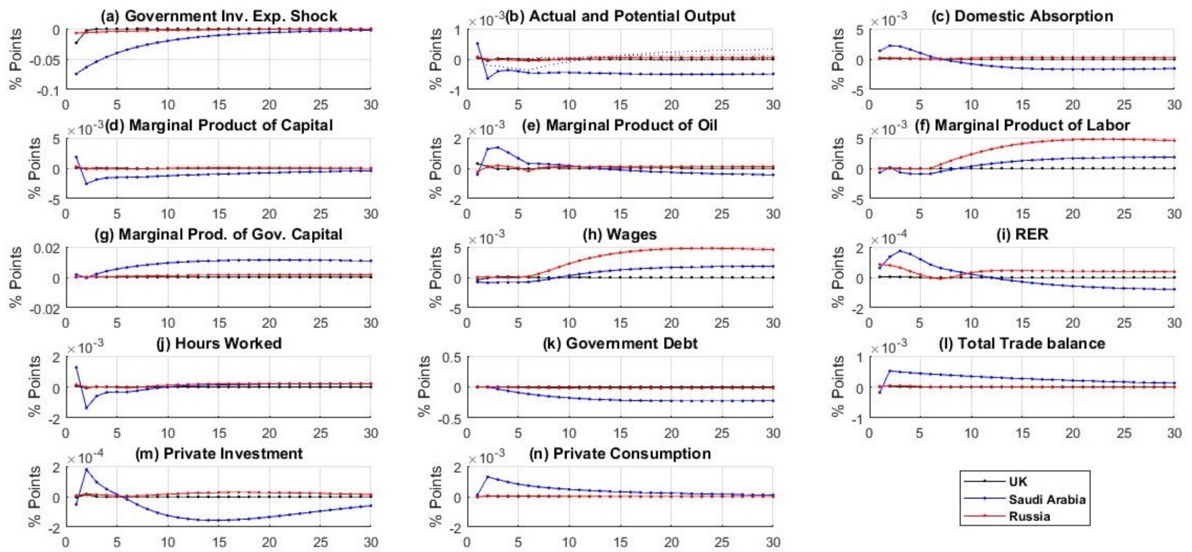
9 Appendix I: Estimated IRFs

Below, we present the impulse responses of several macroeconomic aggregates for one-standard deviation of the relative estimated shock in the three countries.

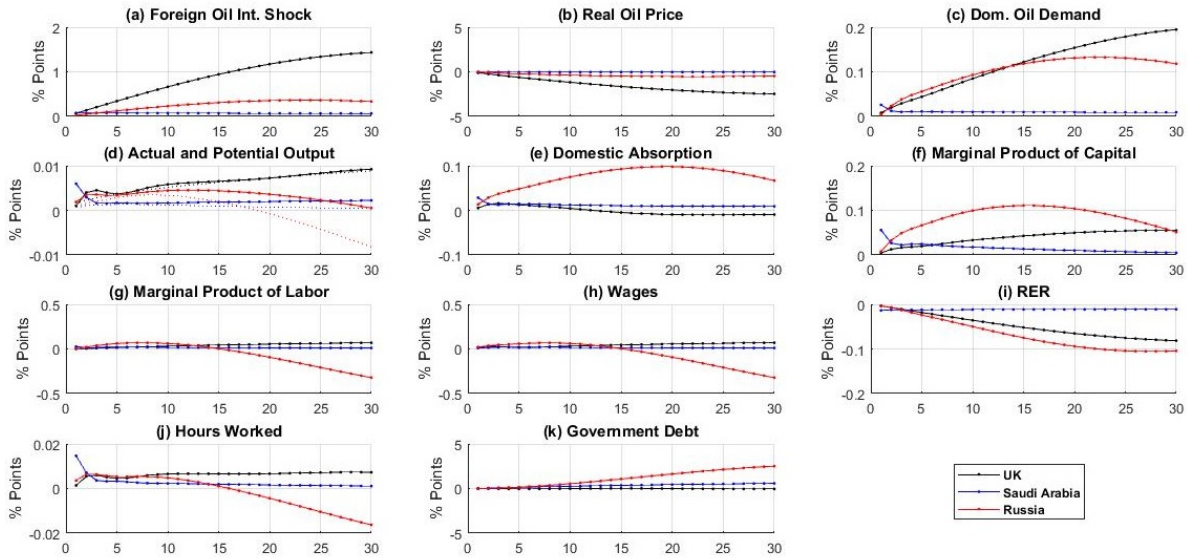
9.1 Government Consumption Spending Shock



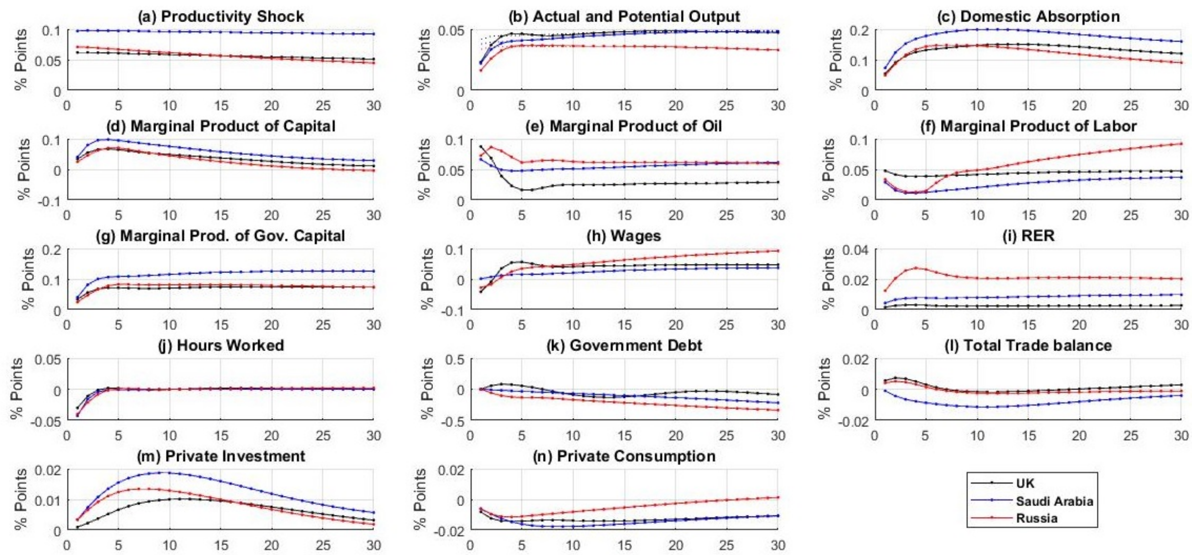
9.2 Government Investment Spending Shock



9.3 Foreign Oil Intensity Shock

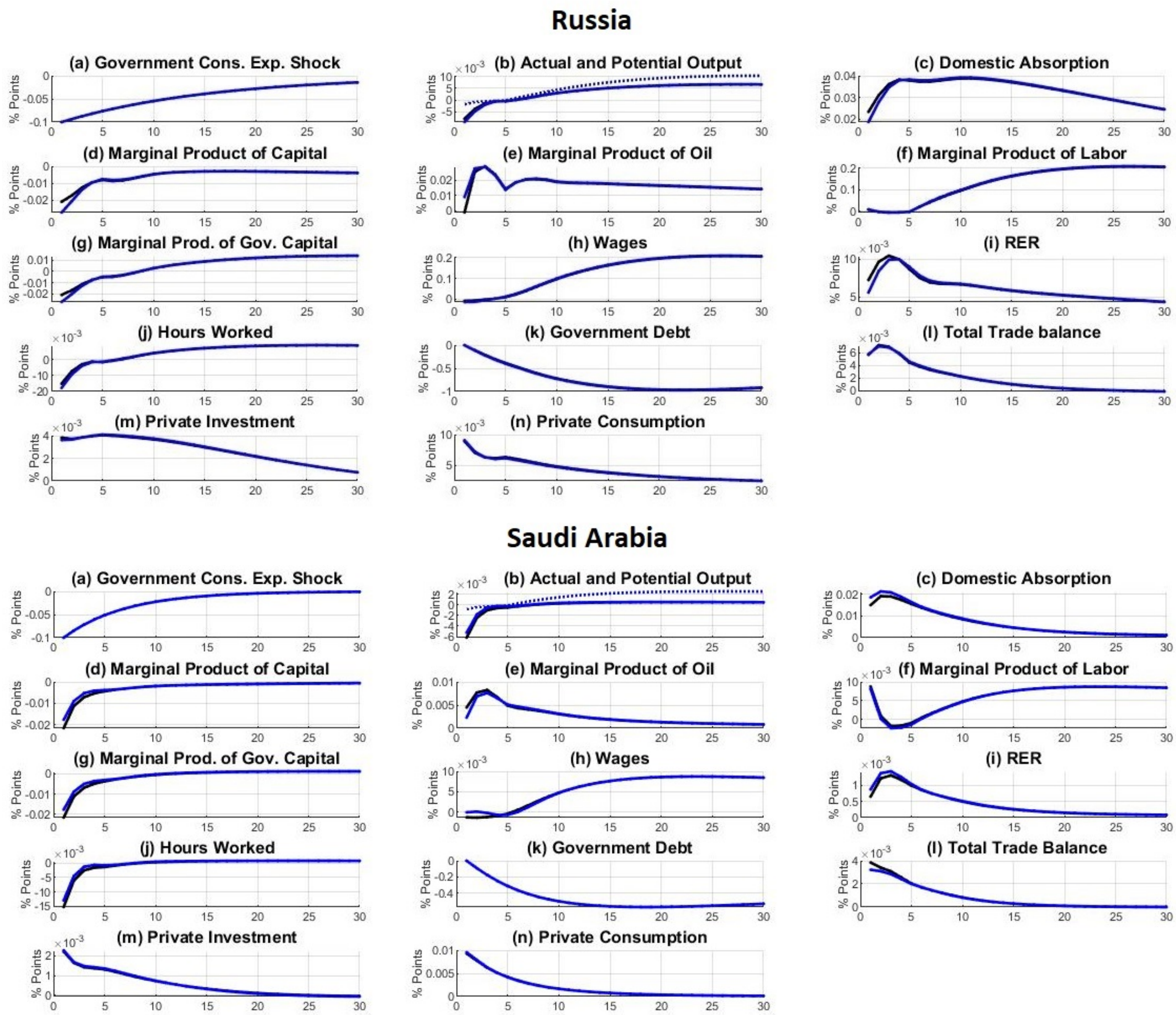


9.4 Domestic Productivity Shock



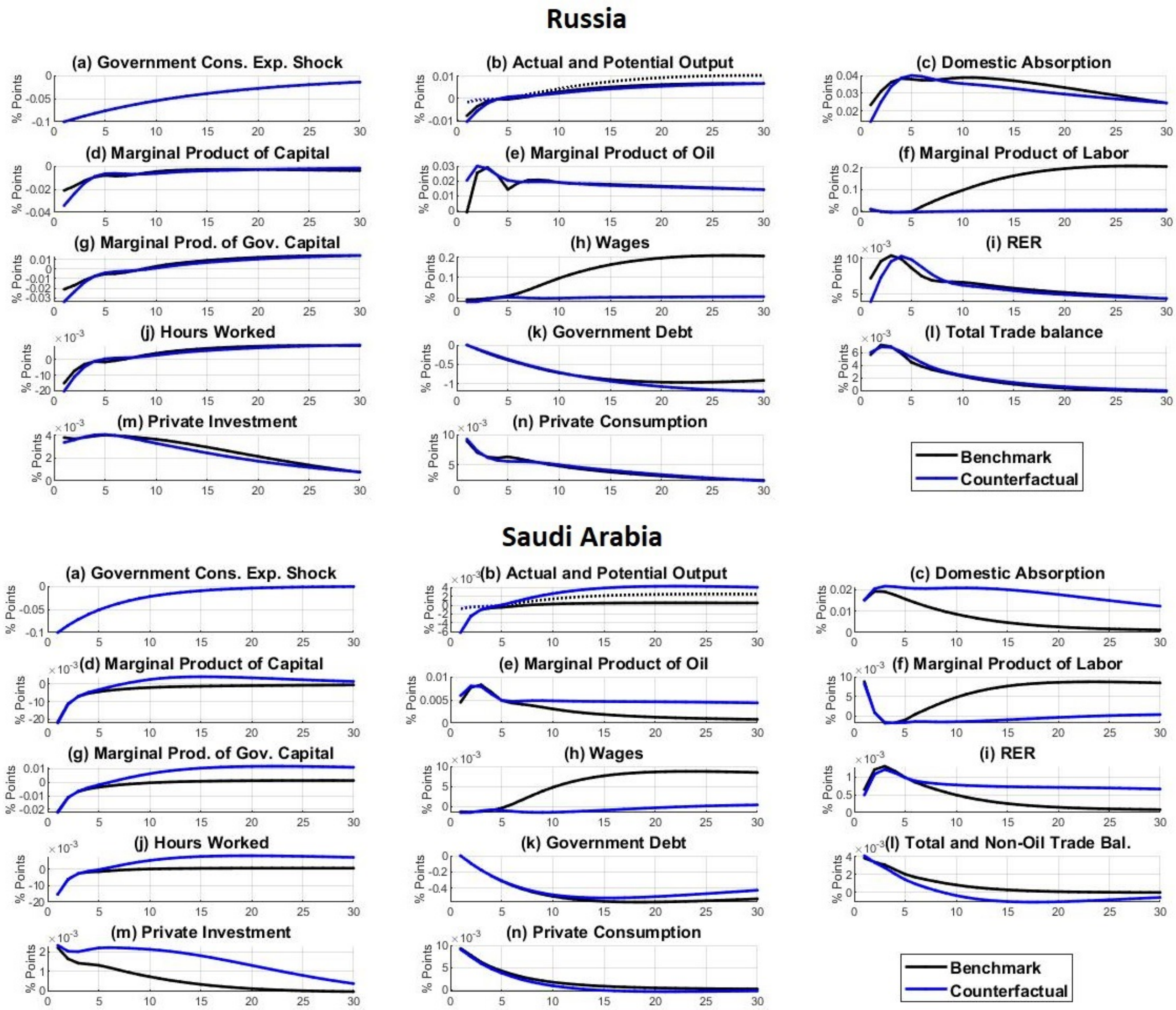
10 Appendix J: Counterfactual Analysis

10.1 Monetary reforms with spending led fiscal consolidation



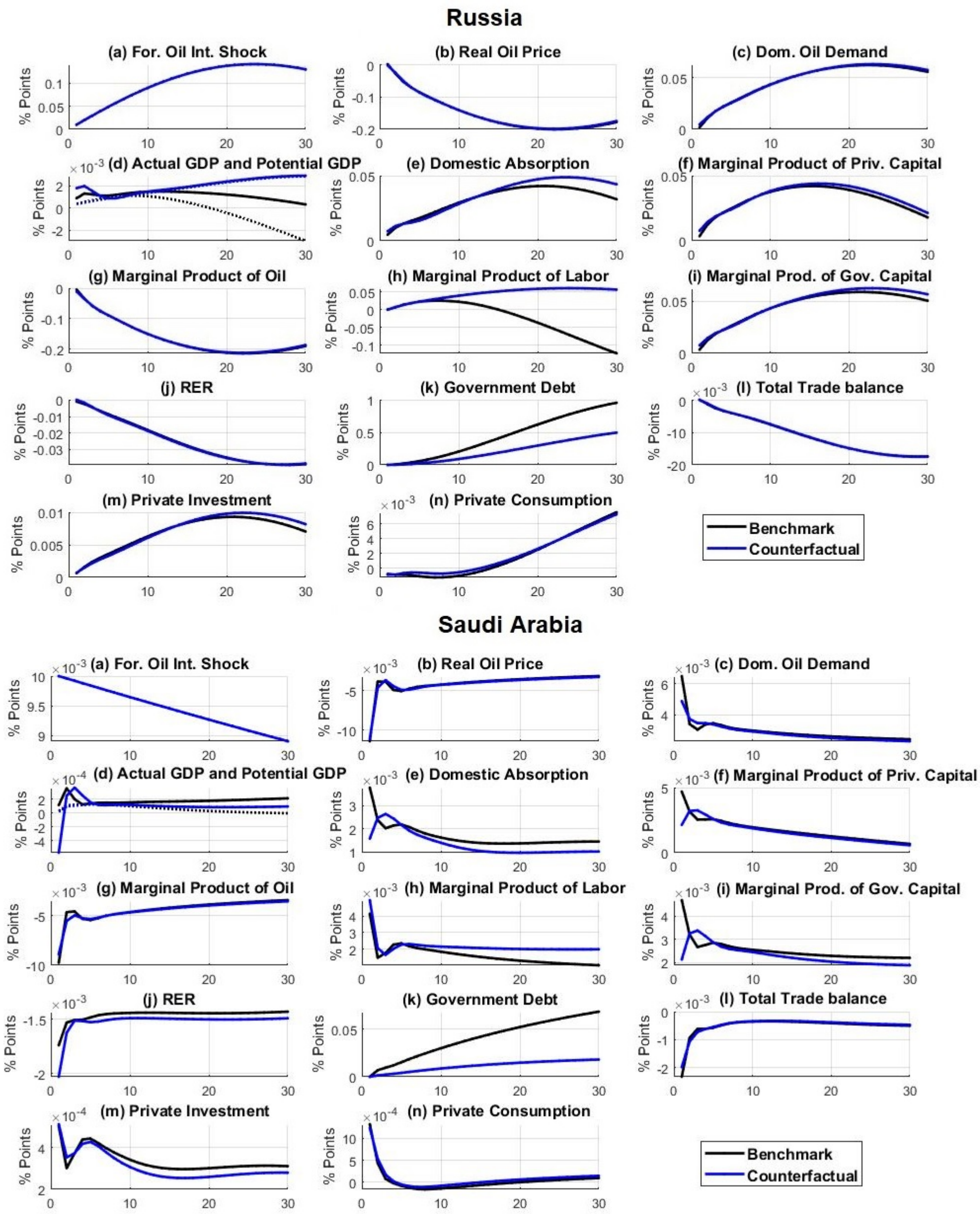
Notes: Simulated 10% reduction in government consumption spending.

10.2 Fiscal reforms with spending led fiscal consolidation



Notes: Simulated 10% reduction in government consumption spending.

10.3 Monetary and fiscal reforms with a global shift away from fossil fuels



Notes: Simulated 1% increase in foreign oil intensity.