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Reading the Stars

by Peter Williams, Yasser Abdih, and Emanuel Kopp

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Western Hemisphere Department

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Abstract

Following the global financial crisis, significant uncertainty has existed around the U.S. economy's steady state equilibrium. This paper uses a factor model to provide a new approach to estimating "the stars" (i.e. the neutral interest rate, maximum employment, and the level and growth rate of potential output) that are most consistent with a medium-term equilibrium where inflation converges to the FOMC's two percent target. It is applicable to any country with an inflation targeting central bank. It also explicitly incorporates estimates of the extensive margin of slack in the labor market, which has proven to be an important factor in describing the post-financial crisis landscape.[1]

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I. INTRODUCTION

In standard macroeconomic models, key variables are viewed as fluctuating around their "stars" in response to short-term shocks and the business cycle. The stars define the economy's longer-term steady state once cyclical shocks have worked their way through the system and short-term price and adjustment rigidities have been overcome. For example, in a simple macro model, the steady state can be fully described by a combination of the natural rate of unemployment ($u*$), the neutral interest rate ($r*$), the level of potential output ($y*$), the growth rate of potential output ($g*$), and the central bank's inflation target ($\pi*$). The deviation of contemporaneous values from this longer-run steady state define the cyclical position of the economy and the difference between the policy rate and the neutral interest rate provides a measure of the stance of monetary policy. Policymakers take their bearings relative to these "stars" in order to conduct effective monetary and fiscal policy.

Navigating by the stars sounds simple but is operationally challenging.[2] The long-run steady state of the economy is not directly observable and it changes over time, albeit relatively slowly. As a result, the exact cyclical position of the economy relative to this steady state is subject to significant model and definitional uncertainty. These challenges are particularly true around cyclical turning points, following data revisions, or during periods of structural change. Of course, these conditions almost always describe the economy to some extent – making the challenges and pitfalls of using the stars rather apparent. Figure 1 shows that there has been a tension between the inflation gap's ($\pi - \pi*$) persistent shortfall over the last 30 years and existing estimates of the output gap. These estimates, while having considerable variation, suggest, on average, lower shortfalls of output and employment over the period than does the inflation gap.

A couple examples of these challenges for models, and model users, are useful. The Federal Reserve Board of Governor's FRB/US model derives very different estimates of the output gap depending on the estimation window. This suggests that parameter stability and changes in the behavior of inflation and the FOMC can have notable effects on estimates of the output gap. Estimates of the output gap from the benchmark neutral rate model of the Laubach and Williams (2003) model are rarely statistically different from zero in the last three decades (Figure 2), which suggests that largely stable inflation may pose a challenge for some estimation approaches as well (these issues are discussed in more depth below).[3] Based on these two examples, it seems that at least some of the challenges of the stars stem from

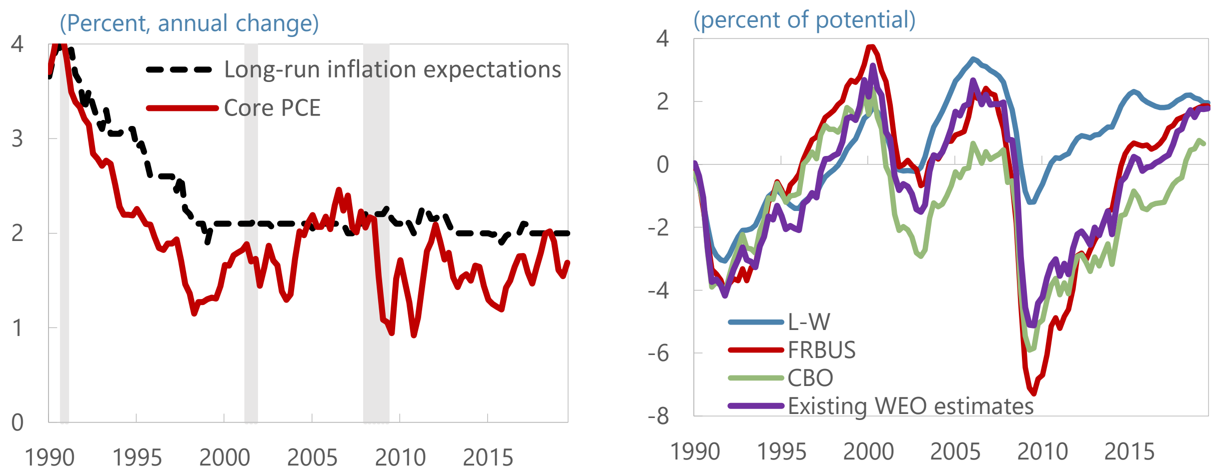
[2] See Powell (2018) for a policy maker's, somewhat skeptical, perspective on the challenges of modeling the stars and macroeconomy in real-time.

[3] See Meyer (2003), Orphanides and van Norden (2002), and Yellen (2015). As a data-driven example, see the August 2018 revisions to the Laubach-Williams model estimates. In response to the benchmark historical revisions to the U.S. GDP data, the model's estimate of potential GDP growth was revised downwards by around 0.4 percentage points and the neutral rate of interest was revised up by around 0.6 percentage points (Federal Reserve Bank of New York, 2018).

before the Crisis and may reflect the differences between the "Great Moderation" and prior periods. Policy makers have often interpreted these challenges as suggesting that policy should be relatively inertial (Yellen, 2017) and allow the economy to probe for the stars instead of reacting to preexisting notions about them (Meyer, 2000).

Figure 1. Inflation and Output Gap Estimates

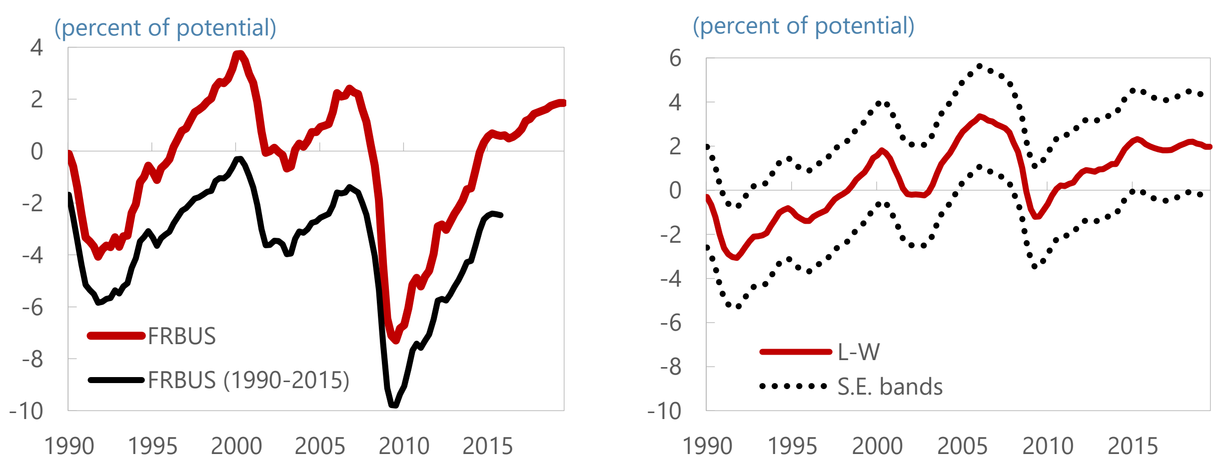
Inflation versus expectations | **Existing Output Gap Estimates**



.Note: *Existing WEO estimates* refers to the projections in the IMF's World Economic Outlook in October 2019, before this model started being used on the U.S. desk. *Sources: BEA, Federal Reserve Bank of New York, Federal Reserve Board of Governors, and authors estimates*

Figure 2. Output Gap Uncertainty

FRB/US output gap estimates | **Laubach-Williams Output Gap Estimates**



Note: FRB/US is the Federal Resreve Borad of Governors benchmark econometric model. It's supply side variables are estiamted in a state space model outlined in Fleischman and Roberts (2011) and Roberts (2014). The data for recent estimation stops in 2015 as that is the last date for which the supply-side model's separate database is available publicly. *Sources: Federal Reserve Bank of New York, Federal Reserve Board of Governors, and authors estimates.*

The policymakers' task not only includes setting policies relative to the stars but also to achieve an optimal policy path over the forecast horizon. This requires estimates not only about the contemporaneous cyclical position but also about the future path of the cycle, the steady state variables, and the distribution of possible outcomes around those paths. Incorrect measurement of the output gap will evidently lead to suboptimal policies[4] and can be particularly problematic for countries aiming to follow a cyclically adjusted fiscal rule[5] or have in place a counter-cyclical capital buffer. Finally, assessments of a country's external position can be sensitive to the measurement of the output gap.[6]

Unlike the other stars, policy makers do have some direct control over the inflation target, π^*. Over the last 20 years, many central banks have chosen low-but positive values for π^* (usually close to 2 percent in advanced economies). The relationship between realized inflation and the inflation target (which we assume is equivalent to equilibrium inflation) is one of the key determinants in estimating the location of the other stars.

To arrive at estimates of the various stars our modeling approach has four key ingredients (in rough order of importance to the results):

1. ***Identifying restrictions.*** Equilibrium is defined as inflation at the FOMC's inflation target (2 percent for most of the sample), wage growth equal to trend productivity growth plus long-run inflation expectations, the federal funds rate equal to the neutral rate, unemployment at the NAIRU, and output equal to potential.

2. ***A broader set of data*** is used to triangulate estimates of the various unobserved parameters including data on inflation, labor market outcomes, wages, and the non-oil current account.[7] The additional data narrows the statistical uncertainty around the various estimates but, because it introduces more parameters to the model, does require additional identifying restrictions and careful modeling.

3. ***The estimation window*** is from 1990Q1-2019Q4. This estimation window is shorter than common in the literature, but it ensures that the Phillips Curve and reaction function are relatively constant over the sample. This would not be true if we started extending the data back much further. This is one example where parameter stability is enhanced by using less data.

[4] Orphanides (2001).

[5] See Tooze, (2019) or Brooks and Fortun, (2019).

[6] Zsolt and Simon (2015).

[7] Stock and Watson (1998) and Fleischman and Roberts (2011) emphasize that multivariate approaches tend to yield lower errors in estimating the cycle.

4. ***The central bank's policy reaction function*** is explicitly modeled (as an inertial variant of a Taylor rule).

The results of this modeling exercise point to a different constellation of "stars" than is commonly assumed in most macro models. In particular:

- The U.S. had a relatively modest output gap at end-2019 of 0.7 percent.
- As a corollary, the estimated NAIRU is lower than is commonly assumed (at around, or slightly below, 4 percent). This appears consistent with the economy maintaining historically low unemployment without apparent signs of wage or price inflation.
- The potential growth rate is around 1.6 percent. It fell for nearly 15 years, troughing in 2013, and then rebounded slightly from these post-Crisis lows.
- The real neutral policy rate that is consistent with 2 percent inflation is modestly negative. This implies that the neutral nominal fed funds rate is between 1.75 and 2 percent.

More broadly, our approach seems to help resolve many of the issues that have puzzled macroeconomists since the financial crisis. Our estimates of slack seem more consistent with inflation's behavior in recent years and match measures of consumer (that is, citizen) sentiment quite well. In addition, the FOMC's 2019 rate cuts can be explained as a reaction to policy becoming slightly tight during 2018 given the lower-than-commonly appreciated level of the neutral rate.

The model does not suggest there was a sharply over-heating economy in the mid-2000s, before the housing bubble popped, a result some may question. Instead, the housing bubble largely shows up as declining productivity growth.[8] By linking slack, the neutral rate, and inflation together, the model leaves financial excesses to be dealt with by regulatory, and not monetary policy. This is in line with the views of most central bankers who see monetary policy (primarily concerned with inflation and employment outcomes) and regulatory policy (focused on the robustness of the financial system) as separate fields.

The paper is organized as follows. Sections II and III examine model formulation and estimation, respectively. Section IV discusses role of inflation and the Phillips Curve in measuring slack. Section V presents the estimation results. The last section offers the paper's implications.

[8] One could see increases in household and financial borrowing, funded by overseas inflows, as partially offsetting this effect on overall growth (which fell relatively less than did potential). Eventually the housing bubble popped, revealing the weakness in the supply-side.

II. MEASURING THE BUSINESS CYCLE

We use trend-cycle decomposition in a state space framework to estimate the stars and the cyclical movements around these long-run values. Each observed series is viewed as a function of the cycle (cyc_t) and its lags, a time-varying trend (denoted by *), and, in some cases, exogenous driving forces, τ_t:

$$X_t - X_t^* = \lambda(L)cyc_t + \beta\tau_t + \varepsilon_t^X$$

Model dynamics are governed by a standard three equation New Keynesian model that connects output and inflation gaps and the stance of monetary policy.

Equation 1: The business cycle is modeled an autoregressive process that also incorporates the effect of deviations of the real policy rate from the estimated real neutral rate. The cycle is estimated both around GDP and Gross Domestic Income (which some have argued better reflects the cyclical behavior of the economy). [9]

$$(1)\ \ cyc_t = \rho_1 cyc_{t-1} + \rho_2 cyc_{t-2} + \vartheta_1 \frac{1}{2}\sum_{i=1}^{2}(r_{t-i} - r_{t-i}^*) + \varepsilon_t^{cyc}$$

Where $|\rho_1 + \rho_2| < 1$ ensures the cycle is stationary.[10]

Equation 2: Inflation deviates from long-run expectations as a result of past values of the business cycle, the change in relative import prices, and medium-term changes in productivity growth. Our data for long-run inflation expectations is based on the SPF survey's measure of long-run inflation expectations, adjusted into PCE equivalent terms by the Board of Governor's staff; after 1995 this it is equivalent to the FOMC's target.[11] The changes in productivity growth help account for relatively low levels of inflation during the late 1990s (see Gordon, 2011). Abdih, Balakrishnan and Shang (2016) show that such a Philips Curve specification is robust and fits the U.S. data well. Both core PCE inflation and the Dallas Fed's trimmed mean PCE inflation are modeled separately. This functional form ensures that notions of slack are consistent with inflation's deviation from its target.

$$2)\ \ \pi_t - \pi_t^* = \lambda_1 \frac{1}{4}\sum_{i=1}^{4} cyc_{t-i} + \beta_{i1} rpi_{t-1} + \beta_{i2}\Delta^{2yr}(lpg_t^*) + \varepsilon_t^{\pi}$$

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[9] See Nalewaik (2010).

[10] See Laubach-Williams (2003) and FRB/US.

[11] We assume that π_t^* is observed without measurement error. Our results are little changed if we allow for it to have an estimated measurement error or impose one of 25 basis points.

Equation 3: The policy rate is assumed to be captured by an inertial Taylor Rule where the deviation between the real policy rate and the real neutral rate reacts to lags of the output and inflation gaps.[12] The assumed lag structure allows for non-monotonic paths of policy rates in reaction to output or inflation shocks (see Hamilton et al (2016) and Orphanides and Williams (2002)). The neutral rate is assumed to be a unit root. Unlike many other models we do not explicitly link potential growth and the neutral rate; we see this flexibility as allowing the data to suggest a link if it is present.[13]

$$(3)\ r_t - r_t^* = \rho_3(r_{t-1} - r_{t-1}^*) + \rho_4(r_{t-2} - r_{t-2}^*) + \vartheta_2 cyc_{t-1} + \vartheta_3 \tilde{\pi}_{t-1} + \varepsilon_t^r$$

$$Where\ \widetilde{\pi_t} = \frac{1}{4}\sum_{i=1}^{4} cyc_{t-i} + \varepsilon_t^{\widetilde{\pi_t}}$$

$$r_t^* = r_{t-1}^* + \varepsilon_t^r$$

To incorporate the effects of unconventional monetary policies during the period where the federal funds rate was at the effective lower bound the model uses the Wu-Xia (2015) measure of the shadow fed funds rate.[14] The real rates are defined as the nominal rate deflated by the y/y change in core PCE.

These core equations are supplemented by further identities and identifying restrictions:

GDP and GDI are can be decomposed into a (common) cycle and trend component. The variance of the measurement error of GDP is assumed to be greater than that for GDI (see C.E.A. (2015) and Nalewaik (2010)).

$$(4)\ gdp_t = cyc_t + gdo_t^* + \varepsilon_t^{gdp}$$

$$(5)\ gdi_t = cyc_t + gdo_t^* + \varepsilon_t^{gdi}$$

$$\sigma^{u^{gdp}} > \sigma^{u^{gdi}}$$

Trend output (gdo_t^*) has a unit root and is driven by time-varying potential growth (g_{t-1}^*).

---

[12] The inflation gap term is set up in that way so the central bank see's through supply shocks.

[13] The modeling approach determines the neutral rate separately from potential GDP growth subject to the restrictions explicitly incorporated into the factor model (see Hamilton et al (2017), Schmelzing (2020), Vlieghe (2017) and Lunsford and West (2019) for a discussion of the relationship between long-run growth and the neutral rate).

[14] Our own initial explorations, as well as a review of Pescatori and Turunen (2015) and Wu and Zhang (2019), suggests that ignoring the shadow rate may bias estimates of the neutral rate. However, we share some of the cautions of Krippner (2019) on the use of shadow rates in assessing overall policy stimulus.

$$(6)\ \ gdo_t^* = gdo_{t-1}^* + g_{t-1}^*$$

This potential growth rate, in turn, is determined by trend labor force growth (lfg_t^*, which is approximated, with allowance for noise, by the CBO series for potential non-farm business hours worked) and trend labor productivity (lpg_t^*). Both of these series are also assumed to have a unit root.

$$(7)\ \ g_t^* = lfg_t^* + lpg_t^*$$
$$(8)\ \ lfg_t^* = lfg_{t-1}^* + \varepsilon_t^{lfg}$$
$$(9)\ \ lpg_t^* = lpg_{t-1}^* + \varepsilon_t^{lpg}$$

The NAIRU is based on $U3$ and assumed to follow a random walk, as is commonly done in the literature (see Laubach (2001)).

$$(10)\ \ U3_t^* = U3_{t-1}^* + \varepsilon_t^{nairu}$$

The unemployment gap is related to the contemporaneous and lagged value of the cycle by an Okun's law relationship (see Gonzalez-Astudillo and Roberts (2016))

$$(11)\ \ U3_t - U3_t^* = \lambda_2 cyc_t + \lambda_3 cyc_{t-1} + \varepsilon_t^{u3}$$

The prime age employment-to-population ratio (*PAEPOP*) follows a similar, if somewhat more inertial, Okun's relationship except *PAEPOP** (unlike $U3^*$) is assumed to be a constant.[15]

$$(12)\ \ PAEPOP_t - PAEPOP^* = \lambda_4 cyc_t + \lambda_5 \frac{1}{3}\sum_{i=1}^{3} cyc_{t-i} + \varepsilon_t^{PAEPOP}$$

Nominal wage growth (measured by the employment cost index and the Atlanta Fed wage tracker) is assumed, over the long-run, to be a function of trend inflation and trend labor productivity growth with a unity coefficient on each.[16] Cyclical deviations from trend output (averaged over the past year) cause wage growth to temporarily deviate from these long-term drivers:

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[15] Given our sample this assumption seems reasonable to us. If the model were to be extended backwards more dynamics would need to be added to the labor market trends.

[16] Abdih and Danninger (2018) measure trend inflation by expected long run (10-year) PCE inflation reported in the Survey of Professional Forecasters (SPF) and trend labor productivity growth by passing the actual data through the Baxter and King (1999) filter.

$$(13)\ wage\ growth_t - (\pi_t^* + lpg_t^*) = \lambda_6 \frac{1}{4}\sum_{i=0}^{3} cyc_{t-i} + \varepsilon_t^{wages}$$

Finally, the model incorporates a relationship between the U.S. business cycle and the non-oil current account position (relative to its trend position)[17]. Where domestic demand outstrips supply there will be downward pressure on the current account relative to its trend value (similarly, the stronger the cyclical position is of trading partners the more positive will be the U.S. current account). [18]

$$(14)\ \ cabxo_t - cabxo_t^* = \lambda_7 \sum_{i=0}^{1} cyc_{t-i} + \beta_3 cyc_t^{RoW} + \ \varepsilon_t^{gtb}$$

These different series can be seen in Figure 3. Clear cyclical patterns can be observed in all the data. The more traditional measures are important for inclusion as they are the benchmark series which form the basis of economist, policy maker, and market participant communications (activity relates to GDP, unemployment to the U3 rate, and inflation to core PCE). The more novel series are often more cyclical than the standard measures in each sector; this aids in estimation even though it necessitates some additional parameters. It should also be clear that the different series are never in full agreement; our approach can be seen as letting the model average across a range of correlated indicators.

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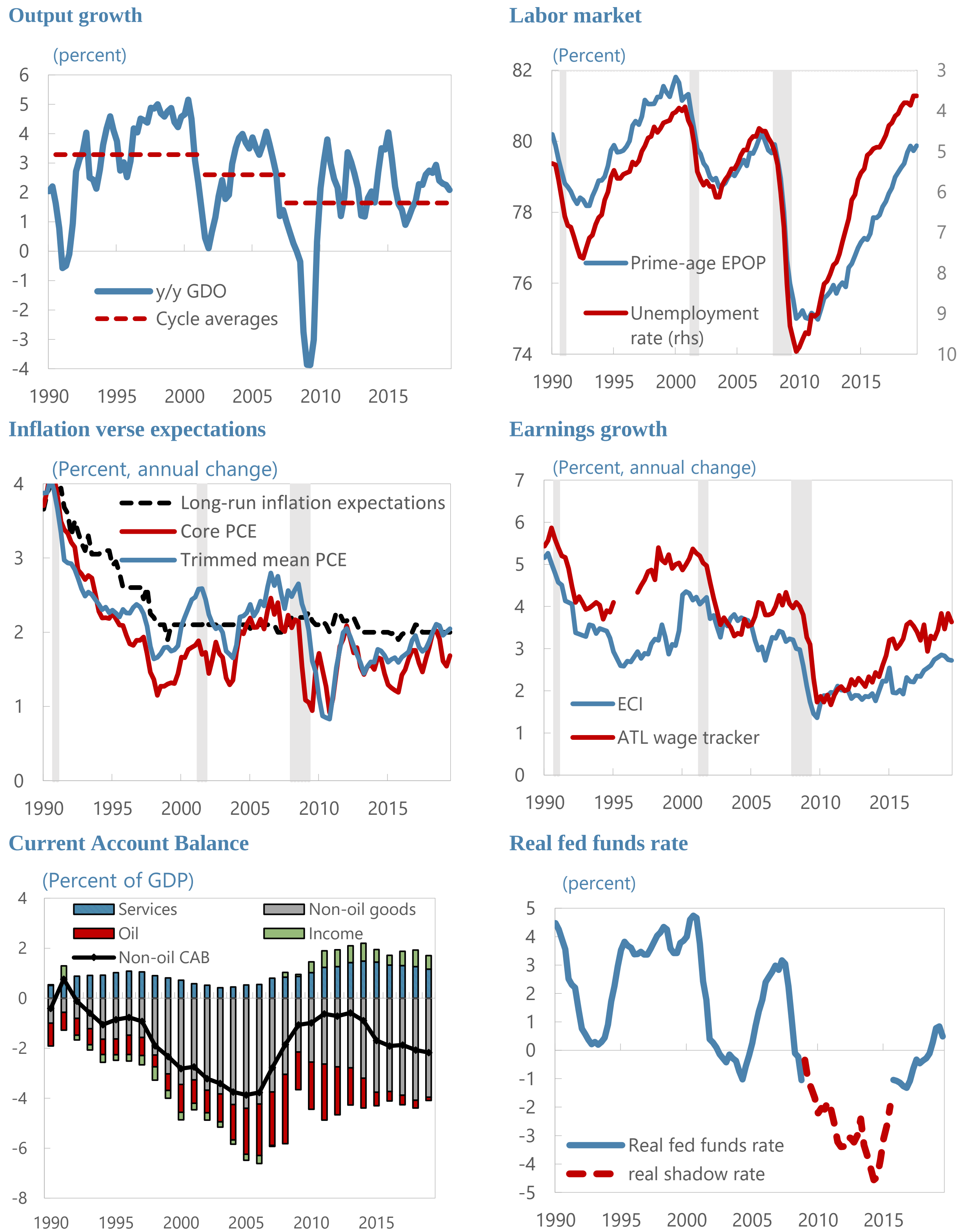
[17] See Cubeddu et al (2019).

[18] Darvas and Simon (2015) argue that the Phillips Curve should, instead, be augmented by the current account to account for the absorption of excess demand in tradeable goods and services.

Figure 3. Drivers of the Business Cycle



Sources: Bureau of Economic Analysis, Federal Reserve Banks of Atlanta and Philadelphia, Federal Reserve Board of Governors, and authors estimates

Note: GDO is the average of real GDP and GDI.

III. Model Estimation

The model is estimated using a Kalman Filter which allows it to handle missing observations in some series and impose the identifying restrictions described above. We estimate the model over for 1990Q1-2019Q4 (during which time we view the Phillips curve and central bank reaction function as more stable than over a longer time series).[19]

The model utilizes a broader set of data and clearer identifying assumptions about what slack is, making it less prone to some of the estimation problems documented in the literature. The "pile-up" problem (see Stock and Watson (1998) and Roberts (2001)) seems less problematic for trend growth rates of the labor force (lfg_t^*) and productivity growth (lpg_t^*) as a result of the larger number of cyclical variables in this model (relative to the number of underlying trends).[20] Wage data helps pin down the trend level of productivity growth (via the wage Phillips Curve) which gives a cleaner estimation of potential growth (rather than being solely derived from the estimated relationship between the output gap and inflation).

Nonetheless, additional identifying restrictions are imposed:

- The measurement error for the two inflation and wage measures are assumed to be equal. But they are allowed to have different coefficients on slack, reflecting the different degrees of cyclicality apparent in the series.
- The standard deviation of the measurement error of the Wu-Xia shadow rate is set to 25 basis points (substantially higher than the estimated s.e. for the real fed funds rate).
- The cycles in the model are more volatile than the trend (i.e. $\sigma(\varepsilon_t^{cyc}) > \sigma(\varepsilon_t^{lpg}) > \sigma(\varepsilon_t^{lfg})$, $\sigma(\varepsilon_t^{r-r^*}) > \sigma(\varepsilon_t^{r^*})$
- The variance of trend unemployment is equal to that of the CBO's long-run NAIRU estimate (i.e. $\sigma(\varepsilon_t^{u*}) = 0.15$). [21]

[19] Stock and Watson (2007) and Blanchard, Cerutti, and Summers (2015) show that, since the late-1980s, the volatility of trend inflation and the persistence of inflationary shocks have both fallen and the slope of the Phillips Curve has been flat but stable. Watson (2014) shows inflation to be much less volatile since the early 1990s with shocks becoming less persistent.

[20] We have nine independent states to estimate. For three of these series the pile up problem is a reasonable concern (the two components of potential output growth (lpg_t^*, lfg_t^*) and the neutral real fed funds rate (r_t^*). In simpler models this ratio of data-to-states tends be much closer to one-to-one. For example, in the benchmark models of Laubach-Williams and Holston-Laubach-Williams there are two series which enter the likelihood and four independent latent series.

[21] Unlike the median-unbiased estimator, which is used to increase the variance of the trends and keep them from being constant, our model performs best and most realistically when the variance of the key trend (the NAIRU) is slightly lower than the model would estimate if left unconstrained. Without this constraint the model's in-sample likelihood improves but estimates of the trend variables show larger fluctuations through time (which seems inconsistent with our view that these variables are time-varying but slow-moving) and are slightly less stable in quasi-real-time evaluation.

IV. A WORD ABOUT PHILLIPS CURVES

Before turning to our results, it is worth spending a moment to compare the Philips Curve used in the framework described above with that of other approaches (notably Laubach-Williams and the FRBUS model). These differences are quite important as the model's inflation process is a key determinate of the level and uncertainty of the estimated output gaps.[22]

This paper's Philips Curve is anchored around the long-run inflation expectations (π_t^*). As such, in equilibrium—where output is at potential across a four quarter average, relative import price effects wash out, and productivity growth is at its long-term value—then inflation (measured here as the four-quarter change in core PCE or as the Dallas Fed trimmed means inflation)—will be at expectations. Insofar as expectations are well anchored at the Fed's medium-term target, then inflation should gravitate over time to the Fed's target. The relatively tight link between inflation and the cyclical component allows for the output gap to be measured with relatively small standard errors.

$$\pi_t - \pi_t^* = \lambda_1 \frac{1}{4}\sum_{i=1}^{4} cyc_{t-i} + \beta_{i1} rpi_{t-1} + \beta_{i2}\Delta^{2yr}(lpg_t^*) + \varepsilon_t^{\pi}$$

Laubach-Williams (2003) framework uses an accelerationist Phillips Curve where inflation has a unit root and is not anchored to any long-run value (in the PCs below, cyc_t refers to the output gap while Z_t is shorthand for the model's supply shocks).[23] This approach results in the unexplained component of inflation being relatively noisy and having minimally apparent cyclical properties. Importantly, this approach generates an output gap estimate which is not necessarily consistent with the Fed's target or long-run inflation expectations.[24]

$$\pi_t = \rho_1 \pi_{t-1} + \rho_2 \sum_{i=2}^{4} \pi_{t-i} + (1 - \rho_2 - \rho_2) \sum_{i=5}^{8} \pi_{t-i} + \beta cyc_{t-1} + \gamma * Z_{t-1} + \varepsilon_t^{\pi}$$

Finally, the FRB/US model uses an anchored expectations New Keynesian Phillips Curve with a lagged deviation between inflation and the long-run expectations that allow short-term shocks to have persistence (see Roberts (2014)). The model has a more detailed supply structure that allows it to incorporate a range of supply shocks to inflation and real activity.

$$\pi_t - \pi_{t-1}^* = \rho(\pi_{t-1} - \pi_{t-1}^*) + \beta cyc_t^{avg} + \gamma * Z_{t-1} + \varepsilon_t^{\pi}$$

[22] Rearranging any of these three Phillips Curves so that the output gap is on the left-hand side makes the implied process that drives the output gap estimates clearer than the standard presentation, see Figure 4.

[23] Of course, policy makers act to stabilize inflation over the medium-term. In this case, while inflation would follow a unit root on the margin, its overall behavior would remain constrained.

[24] This is because medium-term changes in inflation imply the level of the output gap, so a level of inflation which is below (above) the FOMC's target could still imply a positive (negative) output gap depending on recent changes in inflation.

As Table I shows, the performance of all three models in fitting core PCE is similar (when adjusting for the different samples). However, basic model fit criteria are not the ideal way to compare them; the point of these models is to describe deviations in real activity and inflation from their equilibria. Anchoring inflation around long-run expectations allows for greater precision and analytic clarity in estimating the (unobserved) output gap.

Table 1. Phillips Curve Accuracy

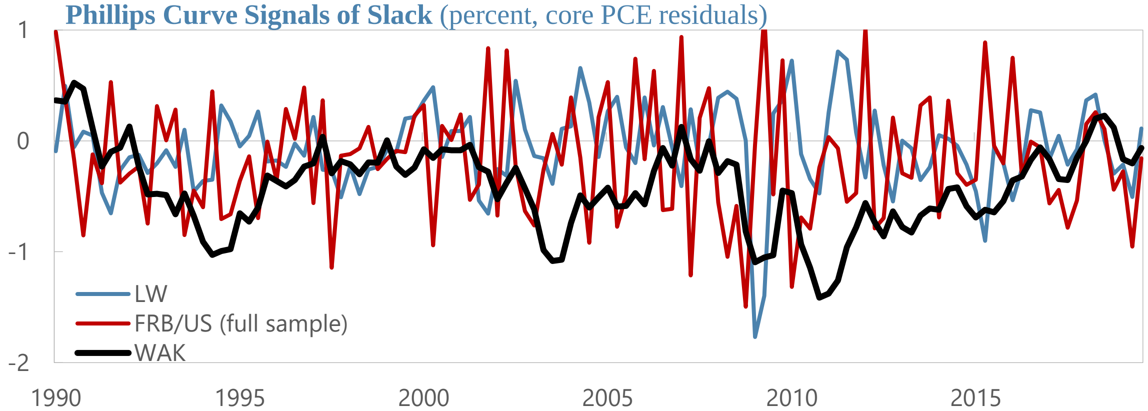
	P.C. standard error (core PCE, q/q saar)
Laubach-Williams - Full sample	0.76
FRB/US[1]	
Beginning in 1965	0.78
Beginning in 1990	0.5
Williams, Abdih, and Kopp[2]	0.52

1. For FRBUS the results end in 2015 as that is the last date publicly available in the supply-side model database.
2. Our baseline model is estimated on y/y core PCE inflation but here we use q/q saar inflation, without allowing for any additional shorter-term dynamics. This makes our estimates comparable with the other models. If we included moving average terms, the fit of our model would have improved without altering the interpretation (see Kim (2009)). *Source: Federal Reserve Bank of New York, Federal Reserve Board of Governors, and authors estimates.*

Figure 4 shows how the state space models use inflation to generate output gap estimates. For example, in our model the chart displays the estimate difference between realized inflation and its non-slack drivers $(\pi_t - (\pi_t^* + \beta_{i1} rpi_{t-1} + \beta_{i2} \Delta^{2yr}(lpg_t^*))$. This presentation makes the differences in the output gap standard errors more clear given inflation's importance in estimating the level of slack (see Table III below for more detail).

Figure 4. What the Phillips Curve Implies about Slack

Phillips Curve Signals of Slack (percent, core PCE residuals)



Note: Data shown is from rearranging the three PCs using their estimated parameters and data, so that the output gap term (βcyc_t) and noise are what remain. It is worth noting that a moving average of the FRB/US series would look similar to our results. *Source: Authors estimates based on the above models*

V. Results – More Slack and Lower Rates

Table II shows and Figure 5 show the model results. The estimated stars generally suggest a more pessimistic narrative for the economy over the last 20 years than most other models; however, the parameters that govern the model's dynamics and internal relationships are consistent with various other studies in the literature. Some observations are worth making:

- The dynamics of the cyclical component is very slow moving ($\rho_1 + \rho_2$ close to 1), the policy rate has only a small effect on the cycle, the impact of slack on wage and prices is small (consistent with well-anchored expectations), and the Okun's law coefficient is broadly in line with other estimates.

Table 2. Key Parameter Estimates

Dynamic Equations				
IS-Equation	ρ_1=1.67	ρ_2=-0.69	ϑ=-0.02	
Phillips Curve	λ_1 =0.15			
Taylor Rule	ρ_3=1.6	ρ_4=-0.7	ϑ_1=0.048	ϑ_2=0.036
Measurement Equations				
Okun's Law – U3	λ_2 = -0.1	λ_3= -0.52		
Okun's Law – PAEPOP	λ_4 = 0.06	λ_5 = 0.59		
Wage Phillips Curve slope	λ_6 = 0.19			
Sources: Authors Estimates				

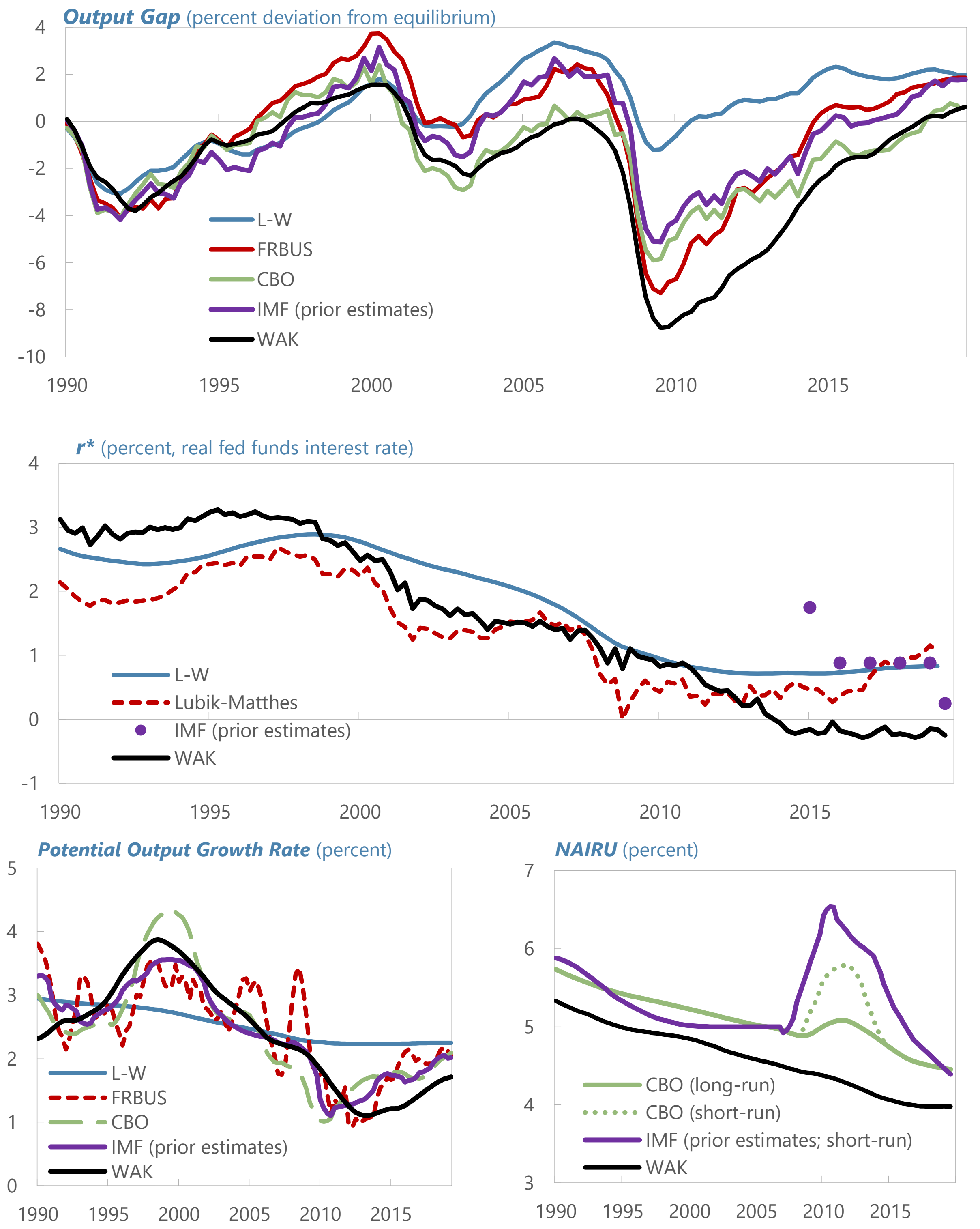
- The output gap is estimated as being more negative following the financial crisis than in other work. The broader dataset used in our modeling exercise allows for information on the extensive margins of labor supply (i.e., as workers are brought off of the sidelines into the active labor market) to play an important role in determining the split between trend and cycle. In addition, output remains below potential for an extended part of the chosen sample period (consistent with below target inflation).

- Following a secular decline in the potential growth rate between the late 1990s and 2010, there seems to be a modest improvement in potential growth over the past 5 or 6 years. This increase in trend productivity could be part of the explanation for why inflation has remained well below target for a prolonged period.

- The neutral rate, like the potential growth rate, has seen a secular decline since the mid-1990s. The contemporaneous estimate of the neutral real federal funds rate is modestly negative (which is well below that found in most other studies). This long-run decline is consistent with market-based results from most term premium models (see Adrian et al, 2013; Kim and Wright, 2005; and Kopp and Williams, 2018).

- The labor market is key to defining the cycle. This reflects the stationary nature of the labor market indicators, which makes building a mean-reverting output gap around them more natural, and their lower levels of high-frequency noise when compared to the national accounts. The NAIRU is time varying but moves slowly and consistently declines. Changes in the unemployment rate map closely into the model's measure of the cyclical position of the economy (like Fleischman and Roberts (2011)).

Figure 5. A Comparison of Stars



Sources: Congressional Budget Office, Federal Reserve Banks of New York and Richmond, Federal Reserve Board of Governors, and authors estimates.

- Hysteresis (where cyclical effects have direct effects on the stars) is suggested as possible but does not appear to be a strong feature of the results. The trends in the stars appear longer than traditional business cycles, which makes estimating direct links to the output gap difficult.[25] The timing of the movements in the r^* and g^* point to possible links between the boom-and-bust cycle in housing prices and household debt or persistent effects from the end of the IT investment boom in the 1990s.

- The standard errors of the full sample estimates indicate greater precision in the estimates of the various unobserved parameters and states (Table III compares the output gap estimates with Laubach-Williams and FRB/US[26]). These standard errors are closely linked to the noisiness of the Phillips Curves' signals in Figure 4.

- The most basic version of the model (that just utilizes data only on core PCE, GDP, the ECI, and unemployment rate) has smaller standard errors than alternative approaches. This is attributable to the structure of the model and the identifying restrictions. Inclusion of the unemployment rate (and, to a lesser extent, other labor market variables) in the model is particularly important since it results in a more precise estimation of the cycle than using only activity data. In addition, estimating any model (with fixed parameters) over the post-1990 sample period reduces the standard errors. Finally, the additional data contributes to an incremental improvement in the precision of the estimates.

- The simpler version of the model has better quasi-real-time performance than the full model. This shows the trade-off between deploying more data in the estimation versus the additional parameters that need to be estimated. Nonetheless, the full model's quasi-real-time performance compares favorably with other methodologies.

- Our output gap has a strong degree of inertia. This is helpful in describing the data, but it can lead to revisions around cyclical turning points (a nearly ubiquitous problem in statistical filters that was mentioned in the introduction, see Figure 6).

- When evaluating different approaches to estimating the stars, it is important to consider not just model fit exercises (an area where we think our constraints and factor approach yield good results) but also that the results are consistent with the country's policy framework. In addition, models such as ours, where the level of the output gap is closely related to inflation's level versus its target, seem to yield estimates of slack which are more consistent with the lived experience in those countries than using accelerationist or strongly inertial Phillips Curve-based models.

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[25] Convergence or significance were difficult to achieve in early attempts to include hysteresis effects in either the NAIRU or productivity growth (our use of single-stage maximum likelihood estimation likely did not help in this regard).

[26] Estimates of g* and r* are less precise than the output gap because they are less directly observed (more latent). In g*'s case this is because fewer series give us direct information about it than the output gap (i.e. it only directly enters into 4 measurement equations). The same issue applies to r* and it is accentuated by the relatively flat slope of the IS-equation.

Table 3. Output Gap Estimation Certainty

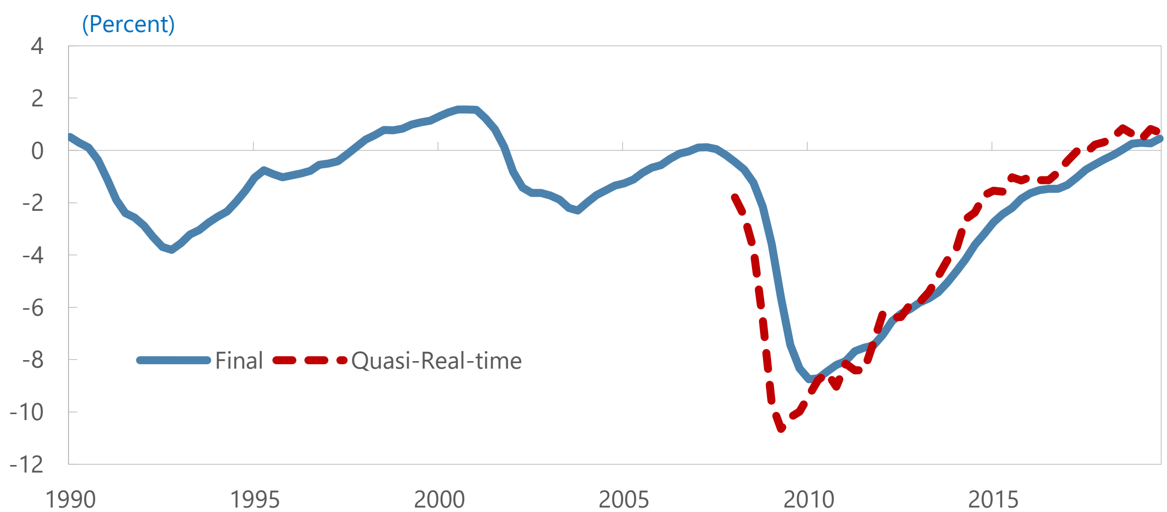
(expanding window, end points from 2008Q1-2019Q3)

	Full sample estimates[1]		Quasi-real time performance	
	Avg. s.e.	Final s.e.	RMSE[2]	Revision Ratio[3]
Laubach-Williams				
Beginning in 1965[4]	2.28	3.10	1.43	1.34
FRB/US[5]				
Beginning in 1965	0.85	0.98	0.85	0.30
Beginning in 1990	0.62	0.8	0.66	0.24
Williams, Abdih, and Kopp				
GDP, core PCE, ECI, U3 and interest rates (most basic model)	0.16	0.3	0.48	0.15
Full model	0.13	0.22	0.7	0.22

1. These results are for the standard error (s.e.) of the output gap.
2.. RMSE is the root mean squared error between the final smoothed estimate and the end points of the expanding window estimates.
3. Revision ratio is the ratio between of the RMSE and the standard deviation of the output gap over 2008-2019; it adjusts for the relative volatility of each models output gap. It is the preferred one as the first is not scale invariant.
4. We struggled to get convergence when estimating LW with sample beginning after the mid-1980s and so these results are excluded from this table. In our view this suggests that since inflation was stabilized by the Fed in the mid-1980s, the accelerationist model of inflation is not particularly useful for describing inflation or inferring the output gap, a result consistent with the LW output gap rarely being statistically significantly different from zero in our sample.
5. For FRBUS the results end in 2015 as that is the last date available in the supply-side model database.

Sources: Federal Reserve Bank of New York, Federal Reserve Board of Governors, and authors estimates.

Figure 6. Quasi-Real Time Estimate of the Output Gap



Source: Authors estimates

VI. CONCLUSIONS

As is common in other studies of this type, estimates of the equilibrium real interest rate have fallen significantly over time and r* is now modestly negative (see Kiley (2019) for similar findings). A lower r^* implies that the federal funds rate was close to its neutral level over the past few years and monetary policy was less accommodative than generally thought.

The decline in the neutral rate has wide-ranging ramifications. It will necessarily have implications for the appropriate tools for monetary policy, the size of the term premium, and the speed of recovery in risk assets after recessions. Low levels of r* may hamper economic stabilization attempts as the effective lower bound becomes more binding.[27] Persistently low rates could create financial fragility by incentivizing risk-taking and higher leverage in attempts to generate constant nominal returns.[28] With less room for monetary policy to respond, counter-cyclical fiscal policy will become more important and judgments on the future path of government debt will need to account for much lower interest rates.[29]

The secular decline in potential growth suggests that there were negative effects from the housing boom and the global financial crisis which were starting to be reversed in the past few years. These effects complicate the ability to extract estimates of the stars from the data and accurately triangulate changes in potential growth and in the NAIRU. Policy makers need to incorporate the possibility of such effects (in both directions) in their broader assessment of the economy and in arriving at the appropriate policy setting.

Persistently low inflation over the last decade suggests that the Philips Curve is flat, the NAIRU has fallen, and the output gap at the end of 2019 was relatively modest (despite a decade of expansion and very low unemployment rates). There was some expectation that "running the economy hot" would push inflation above target but labor market slack appears to have been larger than was thought even a few years ago (see Reifschneider and others, 2015). Inflation, wages, and alternative measures of labor market slack were consistent with each other, a large output gap, and a low NAIRU. This suggests that extensive margins of slack should be directly integrated into modeling of the business cycle.

The framework proposed here can help policy makers learn about the changing structure of the economy in a way that is fully consistent with their inflation targets and easily incorporates a wide variety of measures of economic activity.

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[27] Bernanke, Kiley and Roberts (2019), Gali and others (2019), Kiley (2019).

[28] Borio and Disyatat (2014); Claessens and others (2016).

[29] Blanchard (2019) and Blanchard and Ubide (2019). Rachel and Summers (2019) analyze the potential for fiscal policy to raise r* and Furman (2016) discusses the appropriate stance of fiscal policy in the presence of constraints to monetary policy (like the zero-lower bound).

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