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Money and Credit: Theory and Applications

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Abstract

We develop a theory of money and credit as competing payment instruments, then put it to work in applications. Buyers can use cash or credit, with the former (latter) subject to the inflation tax (transaction costs). Frictions that make the choice of payment method interesting also imply equilibrium price dispersion. We deliver closed-form solutions for money demand. We then show the model can simultaneously account for the price-change facts, cash-credit shares in micro payment data, and money-interest correlations in macro data. We analyze the effects of inflation on welfare, price dispersion and markups. We also describe nonstationary equilibria as self-fulfilling prophecies, which is standard, except here it entails dynamics in the price distribution.

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Table of Contents

1 Introduction

We develop a theory of money and credit as competing payment instruments, then put it to work in applications. This is a classic issue: as Lionel Robbins put it in his Introduction to von Mises (1953), "Of all branches of economic science, that part which relates to money and credit has probably the longest history and the most extensive literature." To bring this up to date we use a New Monetarist approach that involves taking the exchange/payment processes seriously (Section 2 reviews the literature). To get both cash and credit in the model, we adopt the venerable idea that the former is subject to the inflation tax while the latter involves transaction costs.[1] We consider both fixed and variable transaction costs, which turn out to work rather differently, and an unanticipated finding is that the variable cost specification outperforms the fixed cost in terms of theory and data.

An important ingredient is what Burdett and Judd (1983) call "noisy" search, which means sellers post prices, and each buyer sees a random number of them. This leads to a distribution of prices $F(p)$, wher any p in the nondegenerate support yields the same profit – intuitively, lower-price sellers earn less per unit but make it up on the volume. We integrate this into the model of money in Lagos and Wright (2005), with alternating centralized and decentralized markets, which is natural because at its core is an asynchronization of expenditures and receipts crucial for any analysis of money or credit. In the centralized market agents consume, work, adjust their cash balances and settle their accounts. In the decentralized market they trade different goods, as in Burdett-Judd, but with payment frictions: since buyers have no goods or services to offer by way of quid pro quo, they must use cash or credit. Consistent with conventional wisdom, they tend to use credit for large and cash for small expenditures.

[1] One needs some such device to get both money and credit into general equilibrium in a nontrivial way. Gu et al. (2016), e.g., prove the following: if credit conditions are loose money cannot be valued; if credit is tight money can be valued but then credit is not essential and changes in credit conditions are neutral. Transaction costs can get around this result.

Costly credit implies a simple demand for money and avoids an indeterminacy that plagues similar models (see below). The model also generates nominal stickiness. To see how, note that sellers post prices in dollars, since this is a monetary model. As the money supply M increases, $F(p)$ shifts so that the real distribution stays the same, but as long as the supports overlap some firms can keep the same p. So prices look sticky, even though sellers can always adjust at no cost. For a seller that sticks to p when M rises, the real price falls, but the probability of a sale increases, so changing p is simply not profitable. While Head et al. (2012) and others make a similar point, we avoid a technical problem in that approach. Also, while their model can match some features of price-change behavior quantitatively, we go beyond that by matching these features plus micro data on payment methods and macro data on money demand.[2]

In another application, we find small effects of inflation on welfare – e.g., eliminating $\pi = 10\%$ inflation is worth only 0.23% of consumption in the baseline setting where the welfare effects come mainly from impinging on the cash-credit margin. Even in an extension with endogenous participation, where π affects output directly, the impact of π on welfare is smaller than similar models, e.g. Lagos and Wright (2005). The reason is that we use posting instead of bargaining, and our agents can substitute between cash and credit. We also show the impact of π on markups and price dispersion is consistent with evidence. We also study different specifications for the process by which buyers sample prices. We also describe nonstationary equilibria where inflation and deflation arise as self-fulfilling prophecies, which is standard, except here it entails dynamics in the price distribution and not just the price level. Finally, we deliver closed-form solutions for money demand reminiscent of Baumol-Tobin, but in general equilibrium.

[2] As is standard, by money demand we mean the relationship between real balances and nominal interest rates. Head et al. (2012) have no credit, and hence cannot match the micro data, and do not match money demand at all well. Earlier related work like Caplin and Spulber (1987) or Eden (1994) do not go to the data. So, while we are not the first to capture sticky prices this way, one contribution here is quantitative.

Quantitatively, a fixed-cost specification can match the standard money demand observations but not these plus the money and credit shares in the payment data. A proportional-cost specification can match both. Either specification is consistent the salient price-change facts, including long durations, large average changes, many small changes, many negative changes, a decreasing hazard, and adjustment behavior that depends on inflation. Although we match these facts reasonably well, the fit is not perfect due to the discipline imposed by other observations; without this discipline – e.g., if we give up on money demand – the model can match price-change data virtually perfectly, but that is too easy. We think any theory trying to match the price-change facts should also confront the other facts, since they all pertain to monetary phenomena, and all have implications for monetary policy. Our objective is to match these simultaneously.

Section 2 reviews the literature. Section 3-4 describe the model and stationary equilibrium. Section 5 discusses calibration. Section 6-9 consider various extensions and applications. Section 10 concludes.

2 Literature

There is related work in several areas. New Monetarist papers are surveyed generally in Lagos et al. (2016), but particular models that use Burdett-Judd pricing are Head et al. (2012) and Wang (2014), who embed it in Lagos and Wright (2005), and Head and Kumar (2005) and Head et al. (2010), who embed it in Shi (1997). However, there is a technical problem with indivisible goods and price posting, as in Burdett-Judd, in monetary economies: it leads to an indeterminacy (i.e., a continuum) of stationary equilibria.[3] The papers get around this by assuming divisible goods, but then another problem arises – what should firms post? They

[3] This comes up in a series of papers spawned by Green and Zhou (1998). See Jean et al. (2010) for citations and more discussion, but here is a simple version of the problem: If all sellers post p then buyers' best response is to bring $m = p$ dollars to the market as long as p is not too high. If all buyers bring m then sellers' best response is $p = m$ as long as m is not too low. Hence, any $p = m$ in some range is an equilibria.

assume linear menus, where sellers set p and let buyers choose any q as long as they pay pq, but that is not generally a profit maximizing strategy, which seems like a serious issue. Here, with costly credit, the indeterminacy problem with indivisible goods goes away, so we can avoid the ad hoc assumption of linear menus.

Intuitively, holding more cash reduces the amount of costly credit buyers expect to use, which delivers a well-behaved money demand function and a unique equilibrium with money and credit. While we do not take a stand on whether divisible or indivisible goods are more realistic, indivisibility is an assumption on the physical environment, preferable to a restriction on pricing strategies. Also note the indeterminacy in question concerns stationary equilibria, not dynamic equilibria, which are discussed in Section 6.4. There we also make contact with theories of credit like Kiyotaki and Moore (1997), Gu et al. (2013) and references therein. Despite these technical differences, we share with Head et al. (2012) the goal of analyzing pricing without imposing menu costs (e.g., Mankiw 1985), letting sellers only change at exogenous points in time (e.g., Taylor 1980; Calvo 1983), or assuming inattention (e.g., Woodford 2002; Sims 2003). While those devices are interesting, we want to see how far we can go without them.[4] Caplin and Spulber (1987) and Eden (1994) take a similar approach, but do not use the microfoundations adopted here.

As regards empirical work on price adjustment, Campbell and Eden (2014) find in grocery-store data an average duration between price changes of 10 weeks, but we do not want to focus exclusively on groceries. Bils and Klenow (2004) find in BLS data at least half of prices last less than 4.3 months, or 5.5 months excluding sales. Klenow and Kryvtsov (2008) report durations from 6.8 to 10.4 months. Nakamura and Steinsson (2008) report 8 to 11 months, excluding substitutions and sales. These papers also find large fractions of small and negative price changes, plus evidence of a decreasing hazard. Eichenbaum et al. (2011) report a duration of

[4]Burdett and Menzio (2016) combine search as in our model with menu costs, making the analysis more difficult, even without money. Other nonmonetary search models with menu costs include Benabou (1988,1992a) and Diamond (1993).

11 months for reference prices (those most often quoted in a quarter). Cecchetti (1986) finds durations for magazine prices from 1.8 months to 14 years, while Carlton (1986) finds durations for wholesale prices from 5.9 to 19.2 months. Other empirical work is surveyed by Klenow and Malin (2010). We provide a summary of the findings in the Appendix C.

One issue in the menu cost literature is that average price changes are fairly big, suggesting high menu costs. However, there are also many small changes, suggesting low menu costs. Midrigan (2011) explains this with firms selling multiple goods, where paying a cost to change one price lets them change the rest for free (see also Vavra 2014). We account for realistic durations, large average changes, many small and negative changes, and repricing behavior that depends on inflation with that device. We can also get a decreasing hazard, as is problematic for other approaches (Nakamura and Steinsson 2008), and get price dispersion at low or zero inflation, consistent with evidence but not some other models (Campbell and Eden 2014). This suggests that search-based theories should be part of the conversation on price stickiness.[5]

A representative studies, Lucas (2000) and Cooley (1995) discuss the cost of inflation using money-in-the-utility-function or cash-in-advance models. They find eliminating an annual inflation of $\pi = 0.10$ is worth around 0.5% of consumption. Among much other work, we mention Dotsey and Ireland (1996) and Aiyagari et al. (1998) as related to our appraoch. In search-and-bargaining models Lagos et al. (2016) survey work that gets costs closer to 5.0%. Our findings are smaller, for reasons explained below. On inflation and price dispersion, empirical findings are mixed: Parsley (1996) and Debelle and Lamont (1997) find a positive relation;

[5] In discussions with people in the area, we found more or less agreement that these are the facts: (1) Prices change slowly, but exact durations vary across studies. (2) The frequency and size of changes vary across goods. (3) Two sellers changing at the same time do not typiclly pick the same $\hat{p}$. (4) Many changes are negative. (5) Hazards decline slightly with duration. (6) There are many small (beow 5%) and many big (above 20%) changes. (7) The frequency and size of changes, and fraction of negative changes, vary with inflation. (8) There is price dispersion even at low inflation. Our model is consistent with all these.

Reinsdorf (1994) finds a negative relation; Caglayana et al. (2008) find a U-shaped relation. On markups and inflation, a standard reference is Benabou (1992b), who reports a small but significant negative relationship. Benabou (1992a) and Head and Kumar (2005) explain this by inflation increasing dispersion and thus search effort. Here inflation decreases markups by directly affecting the cash-credit choice.

On money demand, we get exact solutions similar to Baumol (1952), Tobin (1956), Miller and Orr (1966) and Whalen (1966). The economic intuition is similar, involving a comparison between the opportunity cost of holding cash and the cost of tapping financial services. But those papers are partial-equilibrium analyses, or, more accurately, decision-theoretic analyses of how to manage one's money given that it is the only payment instrument. While such models are still being used to good effect (e.g., Alvarez and Lippi 2014), we like our setup because it is easy to integrate with standard macro, and allows us to investigate general equilibrium issues, like the emergence of inflation as a self-fulfilling prophecy.

On money and credit, one approach follows Lucas and Stokey (1987) by simply assuming some goods require cash and others allow credit. Papers that let individuals choose subject to a cost of credit include Prescott (1987), Freeman and Huffman (1991), Chatterjee and Corbae (1992), Lacker and Schreft (1996) and Freeman and Kydland (2000). See Nosal and Rocheteau (2011) for a general discussion; see Gomis-Porqueras and Sanches (2013), Li and Li (2013), and Lotz and Zhang (2015) for more recent work. There are various interpretations for these transaction costs, including resources used up in record keeping, screening, enforcement, etc. Other interpretations include saying that the cost of credit as a tax that can be avoided by using cash (e.g., Gomis-Porqueras et al. 2014), or that credit requires resources for monitoring (e.g., Wallace 2013; Araujo and Hu 2014).

Finally, the paper is related to an extensive nonmonetary literature on Burdett-Judd pricing, including the work in labor following Burdett and Mortensen (1998). Here, as in those models, if firms are homogeneous then theory does not pin down which one charges which p, only the distribution $F(p)$. With heterogeneity, lower-cost firms prefer lower p since they like high volume. Still, for any subset of sellers with the same marginal cost, theory does not pin down which one posts which p. This is relevant for retail, where the marginal cost is the wholesale price. Even if a few retailers get, say, quantity discounts, many others face the same wholesale price, making them homogeneous for our purposes. This bears on our discussion of sticky prices; it is unimportant for the other applications.

3 Environment

As in Lagos and Wright (2005), each $t = 1, 2...$ has two subperiods: first there is a decentralized market, called BJ for Burdett-Judd; then there is a frictionless centralized market, called AD for Arrow-Debreu. There is a set of firms (retailers) with measure 1, and a set of households with measure $\bar{b}$. Agents consume a divisible good x_t and supply labor ℓ_t in AD, while in BJ they consume an indivisible good y_t produced by the firms at unit cost $\gamma \geq 0$. Agents in the BJ market can use credit iff they access at a cost a technology to authenticate identity and record transactions. By incurring this cost, they can get BJ goods in exchange for commitments to deliver d_t dollars in the next AD; otherwise they need cash at the point of sale. We consider both a fixed cost δ and a proportional cost τ. Thus, the transaction cost is $C(d_t) = \delta \mathbf{1}(d_t) + \tau d_t$, where $\mathbf{1}(d_t)$ is an indicator function that is 1 iff $d_t > 0$. The cost is paid by buyers, but not much changes if it is paid by sellers.[6]

Household utility within a period is $U(x_t) + \mu \mathbf{1}(y_t) - \ell_t$, where $U'(x_t) > 0 > U''(x_t)$, $\mu > \gamma + \delta$ and $\mathbf{1}(y_t)$ is an indicator function. Let $\beta = 1/(1+r)$, $r > 0$, be a discount factor between AD today and BJ tomorrow; any discounting between BJ and AD is subsumed in the notation. Let x_t be AD numeraire, and assume it is produced one-for-one with ℓ_t, so the real wage is 1. All agents enter the BJ market for free (later we introduce a cost). Each firm in BJ maximizes profit by posting a

[6] This is similar to elementary tax-incidence theory, with a caveat: when the cost of credit is paid by sellers they may want to post different prices for cash and credit.

price, taking as given the CDF of other prices $F_t(p)$, with support $\mathcal{F}_t$. Every period a household in BJ randomly samples n firms – i.e., sees n independent draws from $F_t(p)$ – with probability α_n. As a benchmark we assunme α_0, α_1, $\alpha_2 > 0$ and $\alpha_n = 0\ \forall n \geq 3$, but this is generalized in Section 4.3.

The money supply per buyer evolves according to $M_{t+1} = (1+\pi) M_t$, with changes implemented in AD via lump-sum taxes if $\pi > 0$ or transfers if $\pi < 0$, but most results are the same if instead government uses seigniorage to buy AD goods. The AD price of money in numeraire is ϕ_t. In stationary equilibrium, π is the inflation rate, and the nominal interest rate is given by the Fisher equation $1+i = (1+\pi)(1+r)$. As is standard, the model growth rate satisfies $\pi > \beta - 1$, and in stationary equilibrium the Friedman rule corresponds to $\pi \to \beta - 1$. Note that it is easy to introduce bonds explicitly, but there is no need: $1+i$ is simply the dollars agents require in the next AD to give up a dollar in this AD market, and whether or not such trades occur in equilibrium they can be priced. Again, this is completely standard.

3.1 Firm Problem

Assuming α_1, $\alpha_2 > 0$ and $\alpha_n = 0\ \forall n \geq 3$ for now, profit for a firm posting p is

$$\Pi_t(p) = b_t \left[\alpha_1 + 2\alpha_2 \hat{F}_t(p_t)\right](p\phi_t - \gamma), \tag{1}$$

where $\hat{F}_t(p) \equiv 1 - F_t(p)$. We use b_t to denote the measure of participating households in the BJ market (also called tightness), where for now, because entry is free, $b_t = \bar{b}$. Thus, net revenue per unit is $p\phi_t - \gamma$, and the number of units is determined as follows: The probability a household contacts this firm and no other is α_1. Then the firm makes a sale for sure. The probability a household contacts this firm plus another is $2\alpha_2$, as it can happen in two ways, this one first and the other one second, or vice versa. Then the firm makes a sale iff it beats the other firm's price, which happens with probability $\hat{F}_t(p)$. This is all multiplied by tightness b_t to convert buyer probabilities into seller probabilities.

Profit maximization means every $p \in \mathcal{F}_t$ yields the same profit. As is standard in these models, $F_t(p)$ is continuous and $\mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$ is an interval.[7] Taking as given for now $\bar{p}_t$, and anticipating that $\bar{p}_t$ is not too high, so buyers do not reject it, $\forall p \in \mathcal{F}_t$ profit from p must equal profit from $\bar{p}_t$, which is

$$\Pi_t(\bar{p}_t) = b_t \alpha_1 \left(\bar{p}_t \phi_t - \gamma\right), \tag{2}$$

because the highest price firm never beats the competition. Equating (1) to (2) and rearranging immediately yields the equilibrium price distribution:

Lemma 1 $\forall p \in \mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$

$$F_t\left(p\right) = 1 - \frac{\alpha_1}{2\alpha_2} \frac{\phi_t \bar{p}_t - \phi_t p}{\phi_t p - \gamma}. \tag{3}$$

It is easy to check $F_t'\left(p\right) > 0$ and $F_t''\left(p\right) < 0$. Also, using $F(\underline{p}_t) = 0$ we get

$$\underline{p}_t = \frac{\alpha_1 \phi_t \bar{p}_t + 2\alpha_2 \gamma}{\phi_t\left(\alpha_1 + 2\alpha_2\right)}. \tag{4}$$

To translate from dollars to numeraire, let $q_t = \phi_t p_t$ and write the real price distribution as

$$G_t\left(q\right) = 1 - \frac{\alpha_1}{2\alpha_2} \frac{\bar{q}_t - q}{q - \gamma}. \tag{5}$$

We denote its support by $\mathcal{G}_t = [\underline{q}_t, \bar{q}_t]$, and let $\hat{G}_t\left(q_t\right) \equiv 1 - G_t\left(q_t\right)$.

3.2 Household Problem

Consider a stationary equilibrium, where real variables are constant and nominal variables grow at rate π. Framing the household problem in real terms, the state variable in AD is net worth, $A = \phi m - d - C(d) + I$, where ϕm and d are real money balances and real debt carried over from the previous BJ market, $C(d)$ is the transaction cost of using credit, and I is any other income. Generally, I includes

[7] There cannot be a mass of firms with the same p because any one of them would have a profitable deviation to $p - \varepsilon$, since they lose only ε per unit and make discretely more sales by undercutting others at p. Also, if there were a gap between p_1 and $p_2 > p_1$, a firm posting p_1 can deviate to $p_1 + \varepsilon$ and earn more per unit without losing sales.

transfers net of taxes, plus profit, since as in standard general equilibrium theory the firms are are owned by the households (this plays little role, except for making our welfare criterion unambiguous). All debt is settled in AD, so that households start BJ with a clean slate; they could roll over d from one AD market to the next at interest rate r, but since preferences are linear in ℓ, there is no point. Hence the state variable in BJ is simply real balances, z.

The AD and BJ value functions are $W(A)$ and $V(z)$. These satisfy

$$W(A) = \max_{x,\ell,z} \{U(x) - \ell + \beta V(z)\} \text{ st } x = A + \ell - (1+\pi) z, \tag{6}$$

where the cost of real balances z next period is $(1+\pi) z$ in terms of numeraire this period. Eliminating ℓ and letting x^* solve $U'(x^*) = 1$, after rearranging, we get

$$W(A) = A + U(x^*) - x^* + \beta \max_z O_i(z) \tag{7}$$

where the objective function for the choice of z is $O_i(z) \equiv V(z) - (1+i) z$, with i given by the Fisher equation. As is standard in Lagos-Wright models, we have (Appendix A contains all non-obvious proofs):

Lemma 2 $W'(A) = 1$ *and the choice of* z *does not depend on* A*.*

The BJ value function satisfies

$$V(z) = W(A) + (\alpha_1 + \alpha_2) \left[\mu - \mathbb{E}_H q - \delta \hat{H}(z) - \tau \mathbb{E}_H \max(0, q - z)\right]. \tag{8}$$

In (8), $\hat{H}(q) \equiv 1 - H(q)$, and $H(q)$ is the CDF of transaction prices,

$$H(q) = \frac{\alpha_1 G(q) + \alpha_2 \left[1 - \hat{G}(q)^2\right]}{\alpha_1 + \alpha_2}. \tag{9}$$

Notice $H(q)$ differs from the CDF of posted prices $G(q)$, because a buyer seeing multiple draws of q obviously picks the lowest. Also, notice the costs δ and $\tau(q - z)$ are paid iff $q > z$. Therefore, in terms of simple economics, the benefit of higher z is that it reduces the expected cost of having to tap credit.

4 Equilibrium

The above discussion characterizes behavior given $\bar{q}$, which will be determined presently. First we have these definitions:

Definition 1 *A stationary equilibrium is a list* $\langle G(q), z \rangle$ *such that: given* $G(q)$, z *solves the household's problem; and given* z, $G(q)$ *solves the firm's problem with* $\bar{q}$ *determined as in Lemma 3 below.*

Definition 2 *A nonmonetary equilibrium, or NME, has* $z = 0$*, so all BJ trades use credit. A mixed monetary equilibrium, or MME, has* $0 < z < \bar{q}$*, so BJ trades use cash for* $q \leq z$ *and credit for* $q > z$*. A pure monetary equilibrium, or PME, has* $z \geq \bar{q}$*, so all BJ trades use cash.*

Other variables, like x and ℓ, can be computed, but are not needed in what follows. Also, in NME prices must be described in numeraire q, while in MME or PME they can equivalently be described in numeraire or dollars.

The next step is to describe $\bar{q}$. To that end, we have the following useful results (again see Appendix A):

Lemma 3 *In NME,* $z = 0$ *and* $\bar{q} = (\mu - \delta)/(1+\tau)$*. In MME,* $z \in (0, \mu - \delta)$ *and* $\bar{q} = (\mu - \delta + \tau z)/(1+\tau)$*. In PME,* $\bar{q} = z \geq \mu - \delta$*.*

Lemma 4 *In MME,* $O_i(z)$ *is continuous. It is smooth and strictly concave* $\forall z \in (\underline{q}, \bar{q})$*, and linear* $\forall z \notin (\underline{q}, \bar{q})$*.*

In turn in what follows we study a fixed cost $\delta > 0$ with $\tau = 0$, and a variable cost $\tau > 0$ with $\delta = 0$.

4.1 Fixed Cost

Given $\delta > 0 = \tau$, as long as $\delta < \mu - \gamma$ there is a NME where all transactions use credit, which is similar to the original Burdett-Judd model (except for deferred settlement). We are more interested in outcomes where monetary is valued. Figure 1, based on Lemma 4, shows the objective function is linear $\forall z \notin (\underline{q}, \bar{q})$ with slope $O_i'(z) = -i < 0$. It is also easy to check $O_i''(z) < 0 \; \forall z \in (\underline{q}, \bar{q})$.

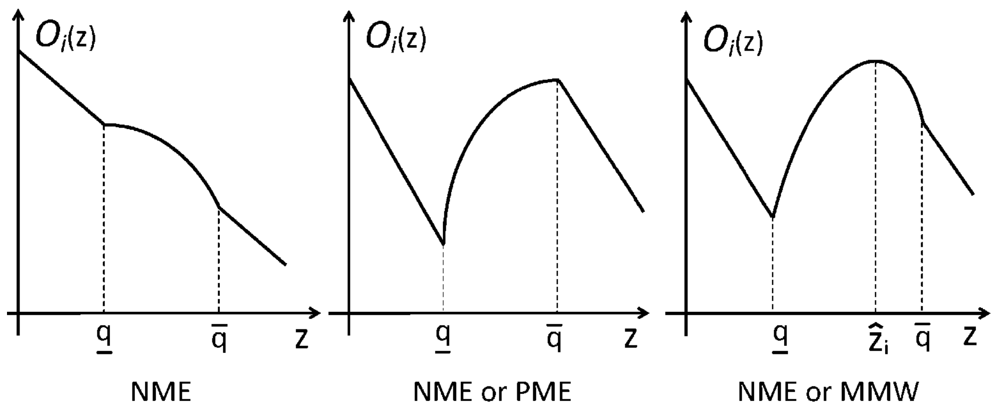


Figure 1: Possible Equilibria with a Fixed Cost

These results imply $\exists! \, z_i = \arg\max_{z \in [\underline{q}, \bar{q}]} O_i(z)$. If $z_i \in (\underline{q}, \bar{q})$, as required for MME, it satisfies the FOC

$$(\alpha_1 + \alpha_2) \, \delta H'(z_i) = i. \tag{10}$$

To check $z_i \in (\underline{q}, \bar{q})$, let $\hat{z}_i$ be the global maximizer of $O_i(z)$, and let $O_i^-(z)$ and $O_i^+(z)$ be the left and right derivatives. If $O_i^+(\underline{q}) \leq 0$ then $\hat{z}_i = 0$, as in the left panel of Figure 1. If $O_i^+(\underline{q}) > 0$ then we need to check $O_i^-(\bar{q})$. If $O_i^-(\bar{q}) \geq 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = \bar{q}$, as in the center panel. If $O_i^-(\bar{q}) < 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i \in (\underline{q}, \bar{q})$, as in the right panel. This leads to the following results:

Proposition 1 *In the fixed-cost model with $\alpha_n = 0 \; \forall n \geq 3$: (i) $\exists!$ NME; (ii) $\exists!$ MME iff $\delta < \bar{\delta}$ and $i \in (\underline{i}, \bar{\imath})$; (iii) $\exists$ PME iff either $\bar{\delta} < \delta < \mu - \gamma$ and $i < \hat{\imath}$, or $\delta < \bar{\delta}$ and $i < \underline{i}$; and the thresholds satisfy $\bar{\imath} \in (\underline{i}, \infty)$,*

$$\underline{i} = \frac{\delta \alpha_1^2}{2\alpha_2 (\mu - \delta - \gamma)} \text{ and } \bar{\delta} = \mu - \frac{\gamma (2\alpha_2^2 + 2\alpha_1\alpha_2)}{2\alpha_2^2 + 2\alpha_1\alpha_2 - \alpha_1^2}.$$

As Figure 2 shows, for money (credit) to be used the nominal rate i (transaction cost δ) cannot be too high. Also note that there is a continuum of PME when

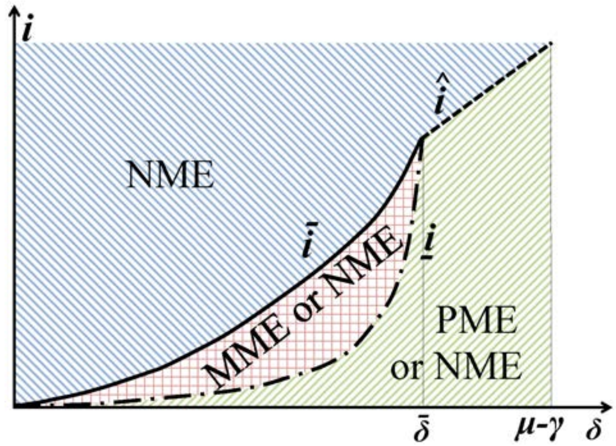


Figure 2: Equilibria with a Fixed Cost

they exist, for reasons discussed in fn. 3. One of the main benefits of costly credit is that we get uniqueness of the MME, which is our main object of interest. When MME exists, we can insert $G(q)$ into (10) and rearrange to get an explicit solution for money demand, i.e., for real balances as a function of i,

$$\hat{z}_i = \gamma + \left[\alpha_1^2 \delta \left(\mu - \delta - \gamma\right)^2 / 2\alpha_2\right]^{1/3} i^{-1/3}. \tag{11}$$

This is reminiscent of the famous square-root rule of Baumol (1952) or Tobin (1956), and even more like the cube-root rule of Miller and Orr (1966) or Whalen (1966). In those models, the usual story has an agent sequentially incurring expenses requiring currency, by assumption, with a fixed cost of rebalancing. The decision rule compares i, the opportunity cost of holding cash, with the benefit of reducing the number of financial transactions that are usually interpreted as trips to the bank. Our buyers make at most one transaction in before rebalancing z, but its size is random. Still, they compare the cost i with the benefit of reducing the use of financial services, again loosely interpretable as trips to the bank, although one might say they now go there to get a loan rather than to make a withdrawal.[8]

[8] While we do not model banking explicitly, we could do so following Berentsen et al. (2007), especially the version in Chiu and Meh (2012) with a fixed cost. Still, is can be useful to think about banks, heuristically, as in standard textbook discussions of Baumol-Tobin.

4.2 Variable Cost

Now consider $\tau > 0 = \delta$. This is in many respects easier, and avoids a technical issue with fixed costs that we waited until now to raise: In economies with non-convexities, like fixed costs, it can be desirable to let agents trade using lotteries.[9] One might try to argue that lotteries are infeasible, or unrealistic, but that seems awkward. Still, we do not analyze lotteries because the setup with a variable cost actually works better, and it has no role for lotteries. The main reason for covering a fixed cost at all is that it is used in many models discussed in Section 2 (in principle, those models should also consider lotteries).

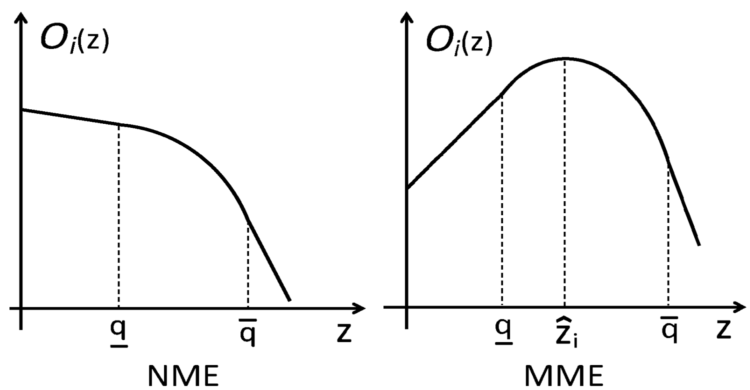


Figure 3: Possible Equilibria with a Variable Cost

The price distribution emerging from the firm's problem is similar Section 4.1, and in particular,

$$\bar{q} = \frac{\mu + z\tau}{1+\tau} \text{ and } \underline{q} = \frac{\alpha_1 (\mu + z\tau) + 2\alpha_2 \gamma (1+\tau)}{(\alpha_1 + 2\alpha_2)(1+\tau)}.$$

What is nice is that, as is easy to check, $O_i(z)$ is differentiable everywhere, including $q = \bar{q}$ and $q = \underline{q}$. Hence, as Figure 3 shows, there are only two possible outcomes: if $i > (\alpha_1 + \alpha_2)\tau$ there is a unique NME; and if $i < (\alpha_1 + \alpha_2)\tau$ there is a unique

[9] See Berentsen et al. (2002) for an analysis in related monetary models. The idea would be for a seller to post: "you get my good for sure if you pay p; if you pay $\tilde{p} < p$ then you get my good with probability $P = P(\tilde{p})$." In Section 4.1, when a buyer with $m = p - \varepsilon$ meets a seller posting p, he pays $p - \varepsilon$ in cash, ε in credit and δ in fixed costs; if ε is small, both parties would prefer to trade using cash only, to avoid δ, and have the good delivered with probability $P < 1$.

NME and a unique MME. A PME cannot exist because buyers are always willing to use credit with some probability. It turns out this is helpful quantitatively: it is easier to get a MME for reasonable parameters than in the fixed-cost model, because in that version, when δ is moderately high agents abandon credit, and we switch to PME, something that never happens with a variable cost.

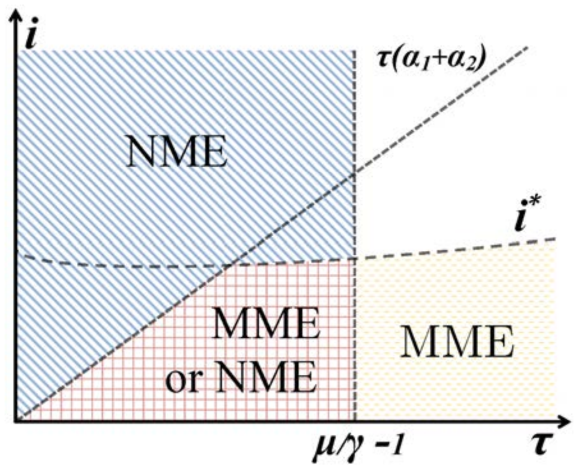


Figure 4: Equilibria with a Variable Cost

Proposition 2 *In the variable-cost model with $\alpha_n = 0$ $\forall n \geq 3$: (i) $\exists!$ NME iff $\tau \leq \mu/\gamma - 1$; (ii) $\exists!$ MME iff $i < \min\{\tau(\alpha_1 + \alpha_2), i^*\}$; (iii) $\nexists$ PME for $i > 0$; where $i^* = i^*(\tau)$ is the nominal rate that drives buyers' payoff to 0.*

As Figure 4 illustrates, MME exists for any value of $\tau > 0$ as long as i is not too big. Also, from (10) we again get a closed-form money demand function,

$$\hat{z}_i = \gamma + \frac{(\mu - \gamma)\left[\tau + (1+\tau)\sqrt{1 + 4\alpha_2 i/\alpha_1^2\tau}\right]}{1 + 2\tau + 4\alpha_2(1+\tau)^2 i/\alpha_1^2\tau}, \tag{12}$$

which is different from the fixed-cost version, but still very tractable, and still has a similar heuristic interpretation bank. As shown below, both specifications fit the money demand data quite well, although the variable-cost model does better at matching the micro payment data.

4.3 Generalized Sampling Distributions

Here we consider alternative specifications for the probability that a household randomly samples n prices.[10] To begin, let N be the maximum number of prices that can be sampled, which could be $N = \infty$. For a firm posting any p profit is

$$\Pi_t(p) = b_t(p\phi_t - \gamma)\sum_{n=1}^{N}\alpha_n n \hat{F}_t(p_t)^{n-1}, \tag{13}$$

while for one posting $\bar{p}_t$ profit is again given by (2). Using $F(\underline{p}_t) = 0$, we get

$$\underline{p}_t = \frac{\gamma}{\phi_t} + \frac{\alpha_1(\phi_t\bar{p}_t - \gamma)}{\phi_t\sum_{n=1}^{N}\alpha_n n}.$$

By virtue of equal profits, $\forall p \in [\underline{p}_t, \bar{p}_t]$,

$$(p\phi_t - \gamma)\sum_{n=1}^{N}\alpha_n n\left[1 - F_t(p_t)\right]^{n-1} = \alpha_1(\bar{p}_t\phi_t - \gamma), \tag{14}$$

from which we get $G_t(q)$ and $H_t(q)$. For households, in the fixed- and variable-cost models, the FOC's required for MME are respectively

$$\sum_{n=1}^{N}\alpha_n\delta H'(z_i) = i \text{ and } \sum_{n=1}^{N}\alpha_n\tau\left[1 - H(z_i)\right] = i. \tag{15}$$

When $N = 2$, (14) is linear in $F_t(p_t)$ and hence can be solved easily to get (3). However, we also get closed-form solutions with some parametric specifications for α_n. First, related to Mortensen (2005), consider a Poisson distribution for n, $\alpha_n = e^{-\eta}\eta^n/n!$, where $\eta = \mathbb{E}n$. Then (13) reduces to

$$\Pi_t(p) = b_t(p\phi_t - \gamma)\eta e^{-\eta F_t(p)}.$$

From this, plus the fact $e^x = \sum_{n=0}^{\infty} x^n/n$, we get

$$F_t(p) = 1 - \frac{1}{\eta}\left[\log(\phi_t\bar{p}_t - \gamma) - \log(\phi_t p - \gamma)\right].$$

From this we get analogs to Propositions 1 and 2:

[10] The results are sketched briefly here, but details are in Appendix A. Also, while these examples are useful for demonstrating the tractability and flexibility of the approach, one can skip to the applications below without loss of continuity.

Proposition 3 *In the fixed-cost model with a Poisson distribution for n: (i) $\exists!$ NME; (ii) $\exists!$ MME iff $\delta < \bar{\delta}$ and $i \in (\underline{i}, \bar{\imath})$; (iii) $\exists$ PME iff either $\bar{\delta} < \delta < \mu - \gamma$ and $i < \hat{\imath}$, or $\delta < \bar{\delta}$ and $i < \underline{i}$; and the thresholds satisfy $\bar{\imath} \in (\underline{i}, \infty)$,*

$$\underline{i} = \frac{e^{-\eta}\delta}{\mu - \delta - \gamma} \text{ and } \bar{\delta} = \mu - \frac{(1 - e^{-\eta})\gamma}{1 - 2e^{-\eta}}.$$

Proposition 4 *In the variable-cost model with a Poisson distribution for n: (i) $\exists!$ NME iff $\tau \leq \mu/\gamma - 1$; (ii) $\exists!$ MME iff $i < \min\{\tau(1 - e^{-\eta}), i^*\}$; (iii) $\nexists$ PME for $i > 0$; and $i^* = i^*(\tau)$ is the nominal rate that drives buyers' payoff to 0.*

As another example, related to Burdett et al. (2016), consider a Logarithmic distribution for n, $\alpha_n = -\omega^n/n\log(1-\omega)$, where $\omega \in (0,1)$. From the usual procedure, it is easy to derive

$$F_t(p) = 1 - \frac{\phi_t(\bar{p}_t - p)}{\omega(\phi_t\bar{p}_t - \gamma)}.$$

Notice that $F_t(p)$ is linear – i.e., p is uniformly distributed. Moreover, we have these analogs to Propositions 1 and 2:

Proposition 5 *In the fixed-cost model with a Logarithmic distribution for n: (i) $\exists!$ NME; (ii) $\exists!$ MME iff $\delta < \bar{\delta}$ and $i \in (\underline{i}, \bar{\imath})$; (iii) $\exists$ PME iff either $\bar{\delta} < \delta < \mu - \gamma$ and $i < \hat{\imath}$, or $\delta < \bar{\delta}$ and $i < \underline{i}$; and the thresholds satisfy $\bar{\imath} \in (\underline{i}, \infty)$,*

$$\underline{i} = -\frac{\delta}{\mu - \delta - \gamma\log(1-\omega)} \text{ and } \bar{\delta} = \mu - \frac{\gamma\log(1-\omega)}{1 + \log(1-\omega)}.$$

Proposition 6 *In the variable-cost model with a Logarithmic distribution for n: (i) $\exists!$ NME iff $\tau \leq \mu/\gamma - 1$; (ii) $\exists!$ MME iff $i < \min\{\tau, i^*\}$; (iii) $\nexists$ PME for $i > 0$; and $i^* = i^*(\tau)$ is the nominal rate that drives buyers' payoff to 0.*

In both the Poisson and Logarithmic cases, the results are similar to the baseline model, and again a variable cost of credit rules out PME and leads to a unique MME. In MME, with a Poisson distribution, the fixed- and variable-cost models respectively also deliver nice money demand functions,

$$\hat{z}_i = \gamma + \left[e^{-\eta}\delta(\mu - \delta - \gamma)\right]^{\frac{1}{2}} i^{-\frac{1}{2}} \text{ and } \hat{z}_i = \gamma + \frac{(\mu - \gamma)\tau e^{-\eta}}{(1+\tau)i + \tau e^{-\eta}}, \tag{16}$$

as they do with a Logarithmic distribution,

$$\hat{z}_i = \gamma - \frac{\delta}{i \log(1-\omega)} \text{ and } \hat{z}_i = \gamma + \frac{(\mu - \gamma)(1-\omega)^{i/\tau}}{1 + \tau - \tau(1-\omega)^{i/\tau}}. \tag{17}$$

All these specifications entail tight characterizations of the equilibrium set, as well as closed-form solutions for the price distribution and money demand. And they have intuitive economic interpretations about substituting between the use of money and credit given inflation and transaction costs. However, to ease the presentation, in the applications below we revert to $N = 2$.

4.4 Repricing Behavior

While this is not the only paper to make the point, and this is not the only point of the paper, let us sketch the search-based explanation of sticky prices. In the models presented above, $F_t(p)$ is uniquely determined, but an individual firm's price is not. Consider Figure 5, drawn for the calibrated parameters in Section 5.2. With $\pi > 0$, the density F'_{t+1} lies to the right of F'_t. Firms with $p < \underline{p}_{t+1}$ at t (Region A) must reprice at $t+1$, because while p maximized profit at t, it no longer does so at $t+1$. But as long as the supports $\mathcal{F}_t$ and at $\mathcal{F}_{t+1}$ overlap, there are firms with $p > \underline{p}_{t+1}$ at t (Region B) that can keep the same p at $t+1$ without reducing their profit. They are allowed to change, at no cost, but they have no incentive to do so.

Given this, consider the repricing strategy used in Head et al. (2012). If $p_t \notin \mathcal{F}_{t+1}$ then $p_{t+1}(p_t) = \hat{p}$ where $\hat{p}$ is a new price; and if $p_t \in \mathcal{F}_{t+1}$ then:

$$p_{t+1}(p_t) = \begin{cases} p_t & \text{with prob. } \sigma \\ \hat{p} & \text{with prob. } 1-\sigma \end{cases} \tag{18}$$

This defines a *payoff-irrelevant tie-breaking rule.* Different from Calvo pricing, where firms can be desperate to change p but are simply not allowed, our rule only applies to firms that are indifferent. Also, in the calibration below, $\sigma = 0.90$, so that only 10% of indifferent firms change p. Moreover, once we set σ there is a

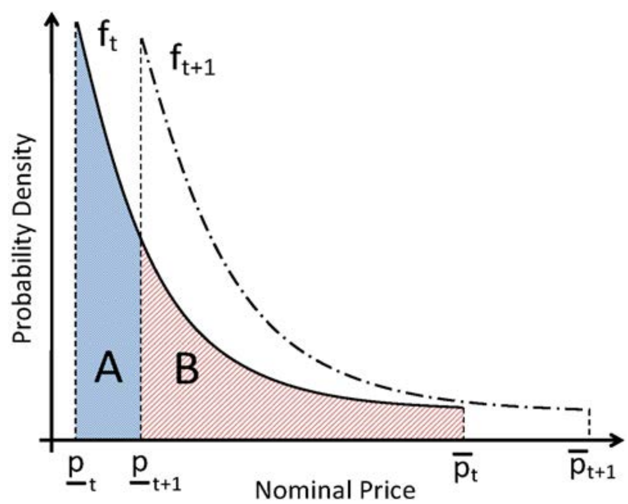


Figure 5: Nominal Price Densities and Inflation

unique symmetric equilibrium, where all sellers that pick a new $\hat{p}$ draw it from the same repricing distribution – i.e., there is the only one way to generates the equilibrium Burdett-Judd distribution. It is given by:

$$R_{t+1}(p) = \begin{cases} \dfrac{F_t\left(\frac{p}{1+\pi}\right) - \sigma[F_t(p) - F_t(\underline{p}_{t+1})]}{1 - \sigma + \sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\underline{p}_{t+1}, \bar{p}_t) \\ \dfrac{F_t\left(\frac{p}{1+\pi}\right) - \sigma[1 - F_t(\underline{p}_{t+1})]}{1 - \sigma + \sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\bar{p}_t, \bar{p}_{t+1}] \end{cases} \tag{19}$$

Using (19) we can compute repricing statistics from the model and compare them to the facts deemed interesting in the literature. While different values of σ generate different behavior, it is not the case that 'anything goes' (e.g., at high inflation most firms must adjust each period). Also, once we pin down σ using data, there are very precise predictions for observables. Hence, while the theory does not impose tight restrictions on any individual seller's behavior, it seems nonetheless interesting to ask how well it can account for average repricing behavior. At the very least, to the extent that the model is consistent with the price-change facts, we submit that the exercise provides a voice of caution about using the data to make inferences about Mankiw-style menu costs or Calvo-style arrival rates, since here we abstract from both.

5 Quantitative Results

In addition to confronting the price-change facts, we try to fit the money-credit shares in the payment data and a standard empirical notion of money demand. As in Lucas (2000), e.g., that notion is $L_i = \hat{z}_i / Y$, where $Y = x^* + (\alpha_1 + \alpha_2) \mathbb{E}_H q$ is output aggregated over AD and BJ. We use $U(x) = \log(x)$, so $x^* = 1$.[11] Formulae for L_i and its elasticity η_i are given in Appendix B, and we target these in the data. Other key statistics are the average BJ markup $\mathbb{E}_G q/\gamma$ and the aggregate markup across both AD and BJ. These are natural targets since BJ equilibrium can deliver anything from monopoly to marginal-cost pricing as α_1/α_2 varies, so the markup contains information about this ratio, while the aggregate markup contains information about the importance of AD and BJ. As mentioned above, the average duration between p changes pins down σ in the tie-breaking rule.

5.1 Data

We focus on 1988-2004, because the price-change observations are from that period, although in principle information from other periods can also be used to calibrate parameters. For money, the best available data is the M1J series in Lucas and Nicolini (2012) that adjusts M1 for money-market deposit accounts, similar to the way M1S adjusts for sweeps as discussed in Cynamon et al. (2006). Lucas-Nicolini have an annual series from 1915-2008 and a quarterly series from 1984-2013, and make the case that there is a stable relationship between these and (3-month T-Bill) nominal interest rates. We use their quarterly series, because the years correspond better to the price-change sample. In these data the average annualized nominal rate is $\mathbb{E}i = 0.041$, which implies $L_{\mathbb{E}i} = 0.277$ and $\eta_{\mathbb{E}i} = -0.116$.[12]

[11] Obviously this is a normalization. Generally, we can write utility as $\log(x) + \mu \mathbf{1}(y) - \psi \ell$, with μ capturing the importance of BJ vs AD goods, and ψ the importance of leisure. As is standard, ψ can be set to match average hours, but the results below do not depend on hours.

[12] The longer annual sample has $\mathbb{E}i = 0.038$, $L_{\mathbb{E}i} = 0.279$ and $\eta_{\mathbb{E}i} = -0.149$; using these gives similar results. We also tried truncating the data in 2004, to better match the pricing sample, and to avoid the financial crisis; that gave similar results, too.

Markup information comes from the U.S. Census Bureau Annual Retail Trade Report 1992-2008. At the low end, in Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, gross margins over sales range between 1.17 and 1.21; at the high end, in Specialty Foods, Clothing, Footwear and Furniture, they range between 1.42 and 1.44. Our target for the gross margin is 1.3, in the middle of these numbers. This implies a markup of 1.39, as discussed in Bethune et al. (2014). While this number is above what macro people often use, it is consistent with the micro data analyzed by Stroebel and Vavra (2015). Moreover, the exact value does not matter a lot over a reasonable range, similar to the findings in Aruoba et al. (2011). We choose the target for the aggregate markup to be 1.1, based on Basu and Fernald (1997). Since the BJ markup is 1.39 and the AD markup is 1.0, this implies the BJ market accounts for about 25% of total output.

On the shares of money and credit there are various sources. In terms of concept, we interpret monetary transactions broadly to include cash, check and debit card purchases. Here is the rationale: (1) Checks and debit cards use demand deposits that, like currency, are quite liquid and pay basically no interest. (2) As discussed in various papers on modern monetary economics, for some issues, it does not matter whether your liquid assets are in you pocket or your banker's. (3) For our purposes, the most interesting feature of credit is that it allows you to pay for BJ goods by working in the *next* AD market, while cash, check and debit purchases all require working in the *previous* AD market, and this matters a lot especially because BJ transactions are random, so you might have to carry liquid balances a long time before spending them. (4) This notion of money in the micro data is consistent with the use of M1J in the macro data. So, here monetary exchange includes cash, check and debit but not credit cards.

Earlier calibrations of monetary models proceeding in the same spirit (see Cooley 1995) target 16% for credit purchases, but more information is now available. In detailed grocery-store data from 2001, Klee (2008) finds credit cards account for 12% of purchases, although we do not want to focus on just groceries. In 2012 Boston Fed data, as discussed by Bennett et al. (2014) and Schuh and Stavins (2014), credit cards account for 22% of purchases in their survey and 17% in their diary sample. In Bank of Canada data, as discussed by Arango and Welte (2012), the number is 19%. While not literally identical, the Boston Fed and Bank of Canada data are close, and suggest a target of 20%. Also, this number is fairly stable over the relevant period, where the bigger changes have been into debit, out of checks and, to some extent, out of currency (Jiang and Shao 2014*a*,*b*).[13]

For price-change data we mainly use Klenow and Kryvtsov (2008), and benchmark their average duration of 8.6, but alternatives are also considered since there are differences across and within studies depending on details. Their average absolute price change is 11.3%, well above average inflation, because there are many negative changes. Since the Klenow-Kryvtsov data are monthly, the model period is a month, and model-generated money demand is aggregated to quarterly to line up with Lucas-Nicolini. A month also seems natural since it corresponds to credit card billing period. However, this does not matter much: as usual, a convenient feature of search models is that they can be fit to different frequencies simply by scaling parameters like arrival, discount and interest rates.

5.2 Basic Findings

Generally, while we cannot hit all the targets exactly, we get very close except where indicated. The results are in Table 1. Consider first the fixed-cost model, which hits all targets except the fraction of credit transactions, because our parameter search is constrained to stay within the region where MME exists. Trying to get 20% BJ credit transactions forces δ into a region where MME does not exist for some values of i in the sample. Hence, for this model we use the smallest δ

[13] These numbers are shares of credit transactions by volume. In Canadian data the fraction by value is double, 40%, since as theory predicts credit is used for larger purchase. However, in Boston Fed data, the fractions by value and volume are about the same. There seems to be no consensus why American and Canadian data differ on value, but in any case, we use volume.

consistent with MME at the maximum observed $i = 0.103$, which yields 11.9% credit transactions. This δ is about 4.7% of the BJ utility parameter μ, which comes primarily from matching average real balances. The value of γ, about one-third of μ, comes primarily from the BJ markup. The probability of sampling one price (two prices) in BJ is $\alpha_1 = 0.013$ ($\alpha_2 = 0.081$).

	BJ utility μ	BJ cost γ	credit cost δ or τ	$pr(n=1)$ α_1	$pr(n=2)$ α_2	tie breaker σ
Fix	8.62	2.91	0.404	0.013	0.081	0.90
Var	5.93	3.14	0.202	0.034	0.048	0.90

Table 1: Baseline Calibration

For the variable-cost model, in contrast, we approximate all targets very well, including 20% for BJ credit. Note the trade surplus, $\mu - \gamma$, is lower than the fixed-cost case, so BJ goods are now less important relative to AD goods. With $\mathbb{E}_H q$ around 4.21, the average transaction cost $\tau \mathbb{E}_H \max(0, q - z)$ is about 0.017. Scaled by BJ utility, $\tau \mathbb{E}_H \max(0, q - z) / \mu = 0.0029$, which is less than average credit cards fees, which are 1.5-2% without counting small fixed costs per transaction. The point is that we do not need big transactions costs to get money and credit both used, which makes sense giving relatively low inflation during the period. Also notice that α_1 (α_2) is higher (lower) than in the fixed-cost model. A constant across specifications is the tie-breaking parameter $\sigma = 0.90$, implying that indifferent sellers change prices only 10% of the time.

Figure 6 shows money demand, with the solid curve from the fixed-cost model and the dashed curve from the variable-cost model. The fit is good in both cases, although the curves are somewhat different at low values of i. While this difference can be important for other issues, it does not matter a lot for our applications. In general, we conclude that the variable-cost specification can easily match both money demand and micro payment data well, but the fixed-cost model has trouble with the latter, given our calibration method.

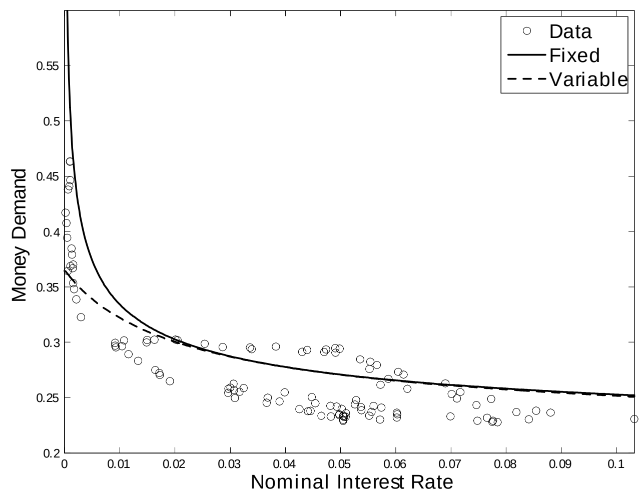


Figure 6: Money Demand for Different Specifications

6 Applications

6.1 Sticky Prices

It was discussed in Section 4.4, and in several earlier papers, how models can in principle generate the appearance of sticky prices without exogenous restrictions or costs on sellers' behavior. How well can they do quantitatively? Figure 7 shows the Klenow-Kryvtsov data plus model predictions of the price-change distribution. Both the fixed- and variable-cost versions capture the overall shape of the empirical histogram, although the fit is not perfect. We now argue, however, that the theory is broadly consistent with several facts considered important in the literature.

The average absolute change is 11.3% in the data, 20.3% in the model with a fixed cost of credit, and 12.3% with a variable cost. So at least the variable-cost model is close to the data. The fraction of small changes (below 5% in absolute value) is 44% in the data, 28% with a fixed cost, and 31% with a variable cost. So on this we are off but not dramatically so.[14] The fraction of big changes (above 20% in absolute value) is 16% in the data, 34% with a fixed cost, and 21% with

[14] Eichenbaum et al. (2015) find a fraction of small prices changes lower than other studies, and suggest this is because one needs correct for measurement error. While their point is valid, for this exercise we take the Klenow-Kryvtsov numbers at face value.

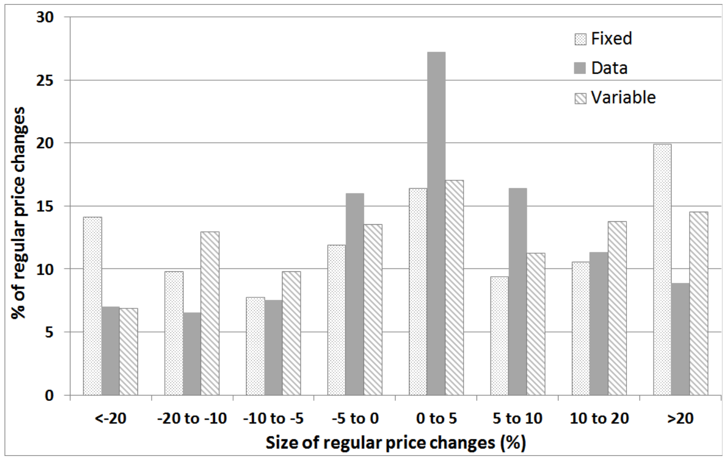
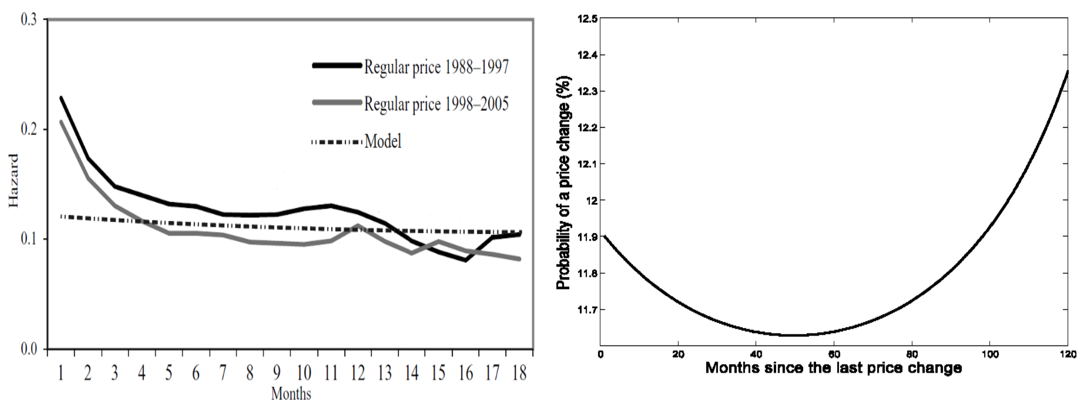


Figure 7: Distribution of Price Changes

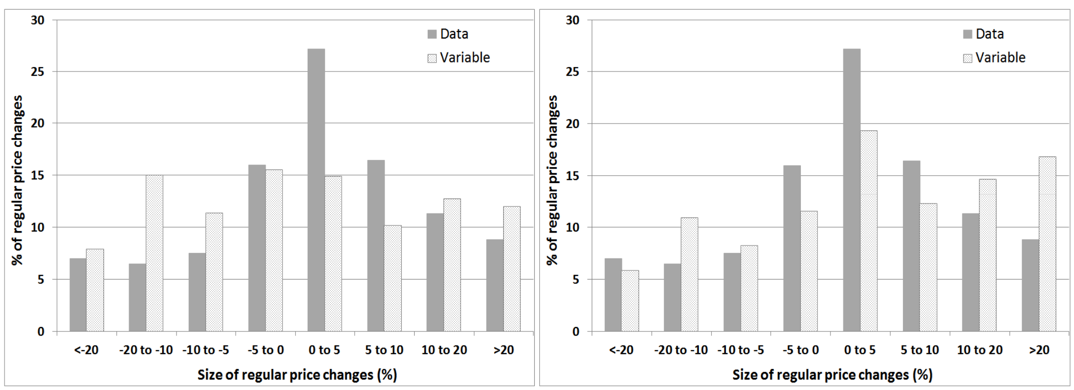
a variable cost, while the fraction of negative changes is 37% in the data, and 43% in both models. So on these we are slightly off. Given the literature says it is not easy to generate large average, many small, many big and many negative adjustments, this performance is reasonably good, but not perfect. To be clear, we do *not* calibrate to match these price-change statistics, but to match money demand, payment methods, and markups, although we did set σ to match average duration (robustness on this dimension is discussed below).



a. Price Change Hazard over 18 Months b. Price Change Hazard over 10 Years

Figure 8: Price Change Hazards

Another observation to consider is the hazard rate, the probability of changing p as a function of the time since the last change. The left panel Figure 8 plots the data from Nakamura and Steinsson (2008) and the prediction from the variable-cost model. We do not generate enough action at low durations, but at least the hazard slopes downward, something Nakamura and Steinsson say is hard to get in theory. Now one should not expect to explain every nuance, and there is undoubtedly a lot missing in the model related to the hazard, including experimentation or learning (e.g., Bachmann and Moscarini 2014). Still, our hazard decreases for a while, before turning up at around 4 years, as shown in the right panel. It is U-shaped over a longer horizon because continuing inflation means any p eventually falls out of the equilibrium support.[15]



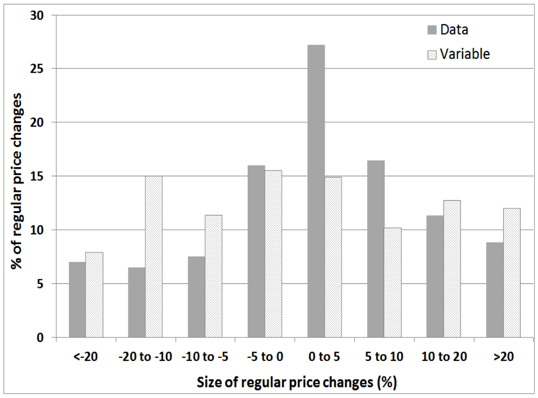
a. Average Duration 1 Month, $\sigma = 1.3\times e^{-10}$

b. Average Duration 16 Months, σ=0.95

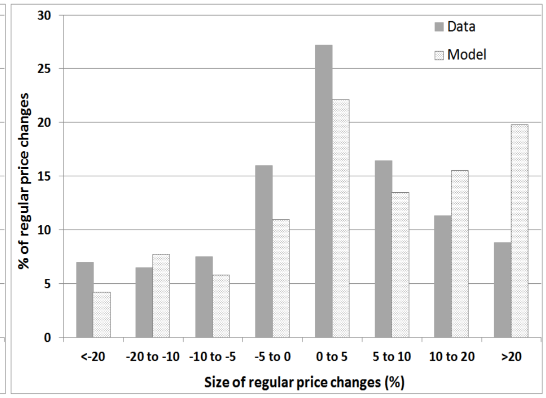
Figure 9: The Effect of Varying Duration

Figures 9 and 10 show the impact of counterfactually changing duration and inflation in the variable-cost model. The left panel of Figure 9 is for $\sigma \simeq 0$ and an expected duration of 1 month; the right is for $\sigma = 0.95$ and an expected duration of 16 months. Given there considerable variability in estimates of average price durations (see Appendix C), it is worth considering robustness with respect to σ. Evidently the right panel fits better than the benchmark duration of 8.6 months. However, with too much stickiness the fit gets quite bad: at $\sigma = 0.9999$, e.g., the fraction of negative changes drops to 1.5%. The left panel of Figure 10 sets π to

[15] Yet even at 10 years, our hazard is only up to 12.35%. Therefore some sellers can stick to prices for a very long time, as long as Cecchetti's (1986) magazines mentioned in Section 2.



a. 0% Annual Inflation



b. 20% Annual Inflation

Figure 10: The Effect of Varying Inflation

0, and the right to 20%. This is not a robustness check; rather, we want to know how repricing behavior depends on π, since this can be checked in data. As π increases, the fraction of negative adjustments falls, while both the frequency and size increase. This is not surprising, but still relevant, because it is consistent with the evidence in Klenow and Kryvtsov (2008), and difficult to explain with some models (e.g., the simplest Calvo model).

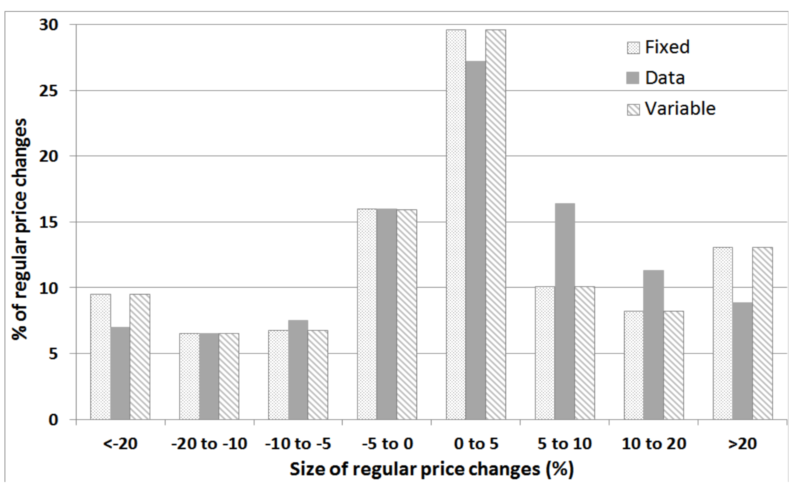


Figure 11: Distribution of Changes Ignoring Money Demand

To summarize the findings, while the fit is not perfect, overall it seems hard to argue that there is anything especially puzzling in the price-change data – it is pretty much what rudimentary search theory predicts. Moreover, this is true even with the discipline imposed by macro and micro observations on money and credit. If we ignore those observations we of course do better. Figure 11 shows a calibration that gives up on matching money demand. The fit obviously is very good. We conclude that it is easy for search models to capture the appearance of sluggish nominal prices quantitatively if we do not impose the discipline of other data, and even if we do the models capture broadly the facts. This is not to say there are no other models consistent with the facts; we only suggest that theories with search-type frictions constitute a viable candidate explanation.

6.2 Welfare

A genuinely classic economic question asks, what is the welfare cost of inflation? As is standard, we compute the percent change in consumption that is equivalent to changing π from a given level to some alternative, which we take to be 0. Given π, welfare is measured by

$$\Omega \equiv Y - (\alpha_1 + \alpha_2)\left\{\delta\left[1 - H_\pi(z_\pi)\right] + \tau \int_{z_\pi}^{\bar{q}} (q - z_\pi)\, dH_\pi\right\}, \tag{20}$$

where $Y = U(x^*) - x^* + (\alpha_1 + \alpha_2)(\mu - \gamma)$ adds the AD and BJ surpluses, while the remaining terms subtract the resource costs of credit.

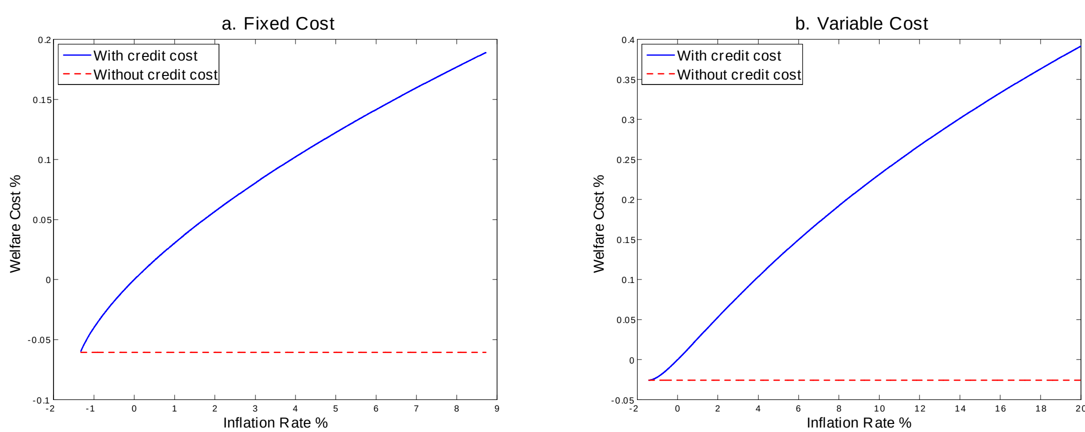


Figure 12: The Welfare Effects of Inflation

Figure 12 shows the welfare cost monotonically increases with inflation. The range of the horizontal axis is the range over which MME exists, going down to the

Friedman rule $\pi = \beta - 1$, and up to about 9% with a fixed cost (left panel) or 20% with a variable cost (right panel).[16] The welfare effects are small: with a variable cost, eliminating 10% annual inflation is worth .0.23% of consumption. This is less than estimates in Lucas (2000), and much less than Lagos and Wright (2005). Intuitively, changes in π here affect neither the intensive margin of trade, since the good is indivisible, nor the extensive margin, since the population of participants is fixed. Hence, the welfare effects is mainly due to inflation increasing the usage of credit, as shown in the right panel of Figure 13. Note that π can affect $G(q)$ as buyers economize on real balances, but this is a transfer between buyers and sellers, not a change in Ω.

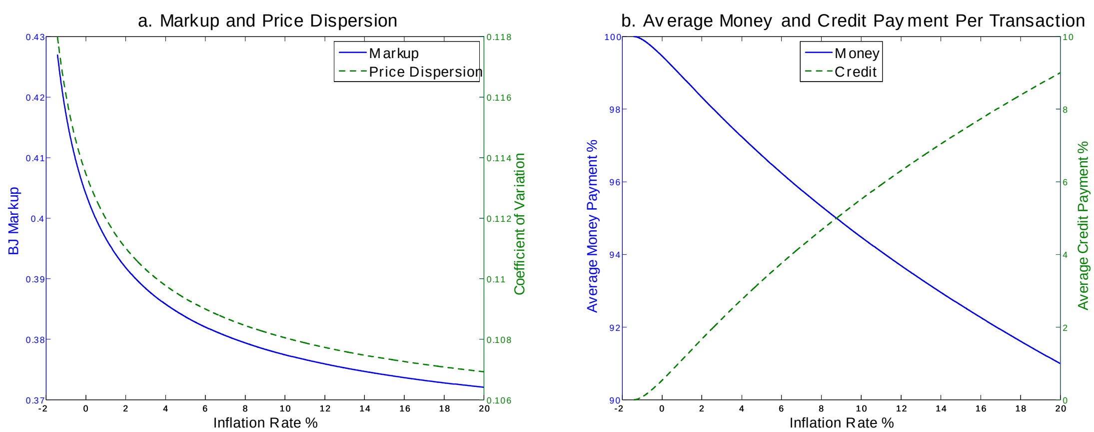


Figure 13: The Other Effects of Inflation

We revisit welfare in Section 6.3. First, let us briefly consider the relationship between inflation, markups and price dispersion. With a fixed cost of credit π does not affect $G(q)$, markups or dispersion. This is another reason to prefer a variable cost, where one can show implies $G(q)$ decreases with π in the sense of first-order stochastic dominance. Consequently, the average markup and dispersion (coefficient of variation) both decrease with π, as shown in Figure 13. In fact, both

[16] These thresholds are low, but this not surprising in a representative-agent context. Suppose we introduce heterogeneity across buyers, with some having zero or only very costly access to credit – e.g., the unbanked, who have to deal with loans sharks, pawnshops or payday advances. They would presumably continue to use cash up to higher thresholds. While this is worth pursuing, it must be reserved for future work.

$\bar{q}$ and $\underline{q}$ fall with π, but $\bar{q}$ falls faster. Benabou (1992) finds a small but significant negative relationship between markups and inflation, consistent with the model. On inflation and dispersion, Parsley (1996) and Debelle and Lamont (1997) find the relationship is positive, Reinsdorf (1994) finds it is negative, and Caglayana et al. (2008) find it is U-shaped. Hence the facts are not unequivocally established, but we can match the findings in Reinsdorf (1994). We also mention that matching the relationship between π and dispersion on markups does not necessarily mean that inflation has a big welfare cost.

6.3 Participation

We now let buyers choose whether to participate in the BJ market, at cost $k > 0$, to make output depend directly on inflation.[17] Let $W^1(A)$ and $W^0(A)$ be the AD value functions for households that enter and do not enter the next BJ market, respectively, so that $W(A) = \max\{W^1(A), W^0(A)\}$. In equilibrium where some but not all households enter, $W(A) = W^1(A) = W^0(A)$. This simplifies to $\beta\Psi = k$, where Ψ is the expected surplus from participation,

$$\Psi \equiv (\alpha_1 + \alpha_2)\left[\mu - \mathbb{E}_H q - \tau \mathbb{E}_H \max(0, q - z)\right] - i\hat{z}_i. \tag{21}$$

Buyers' arrival rates now depend on the buyer-seller ratio, or market tightness, $\alpha_n = \alpha_n(b_t)$. With entry, b adjusts to satisfy (21). An increase in π reduces b, and hence output, although a one-time unanticipated increase in M does not, as ϕ falls proportionately to leave ϕM and $G(q)$ the same (classical neutrality).

We need to parameterize the α's. Suppose that buyers attempt to solicit price quotes, and succeed with probability $s = s(b)$, with $s(0) = 1$, $s(\bar{b}) = 0$, $s'(b) < 0$, and $s''(b) > 0$. While this much is is standard, to generate price dispersion as in the baseline setup, any buyer who succeeds sees 1 price with probability $1 - \xi$ and sees 2 with probability ξ. Then $\alpha_1(b) = (1 - \xi)s(b)$ and $\alpha_2(b) = \xi s(b)$. As a special

[17] Similar monetary models with endogenous entry by buyers includes Liu et al. (2011), while those with entry by sellers include Rocheteau and Wright (2005); of course, prototypical search models with entry include Diamond (1982) and Pissarides (2000).

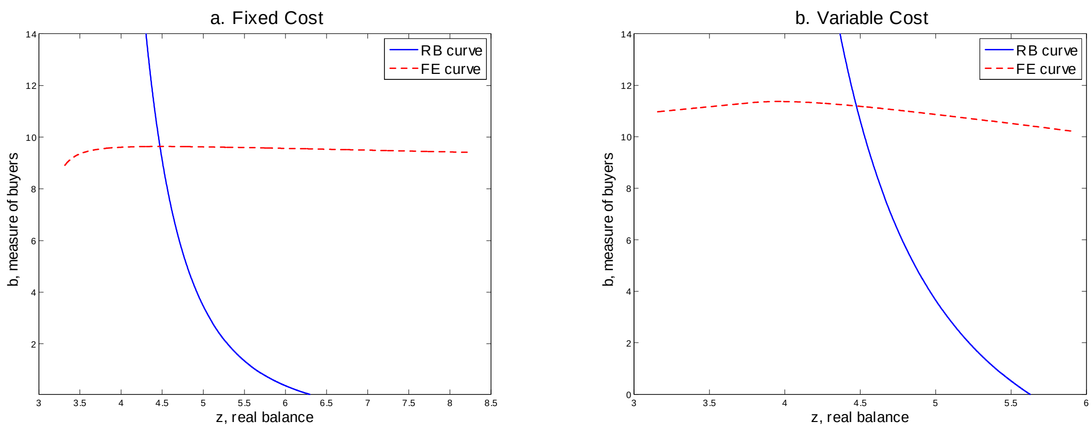


Figure 14: The Real Balance and Free Entry Curves

case of the money demand functions derived above, $\hat{z}_i$ now depends on b, as shown in Figure 14 by the RB (real balance) curve. Similarly, $\beta\Psi = k$ is shown as the FE (free entry) curve, and the curves intersect at MME. As Figure 14 shows, RB is decreasing and convex while FE is concave, implying a unique MME, from which $F(p)$, $G(q)$ and the rest of the endogenous variables are constructed as usual. It is easy to check that higher inflation shifts both curves toward the origin, reducing buyer entry and hence BJ output.

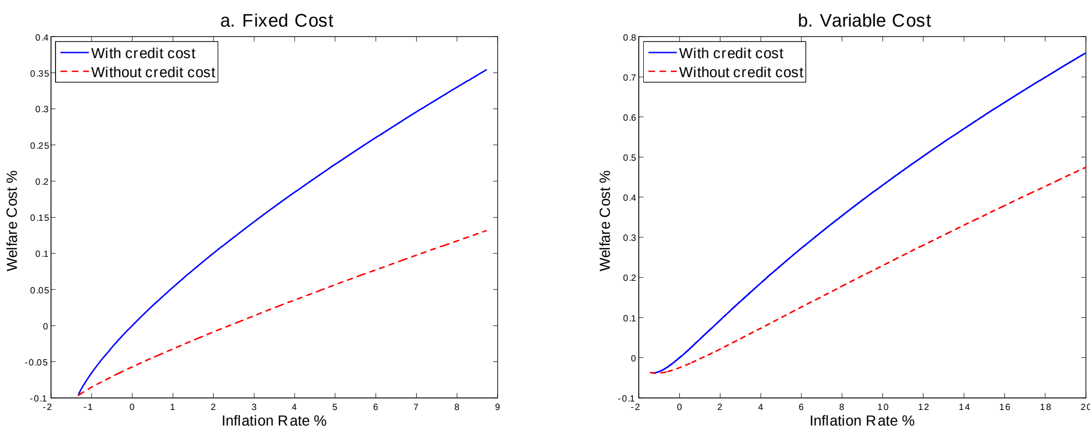


Figure 15: The Welfare Effects of Inflation with Entry

While out theory is consistent with the appearance of sticky prices, the implications are different from models with constraints on changing prices. In those

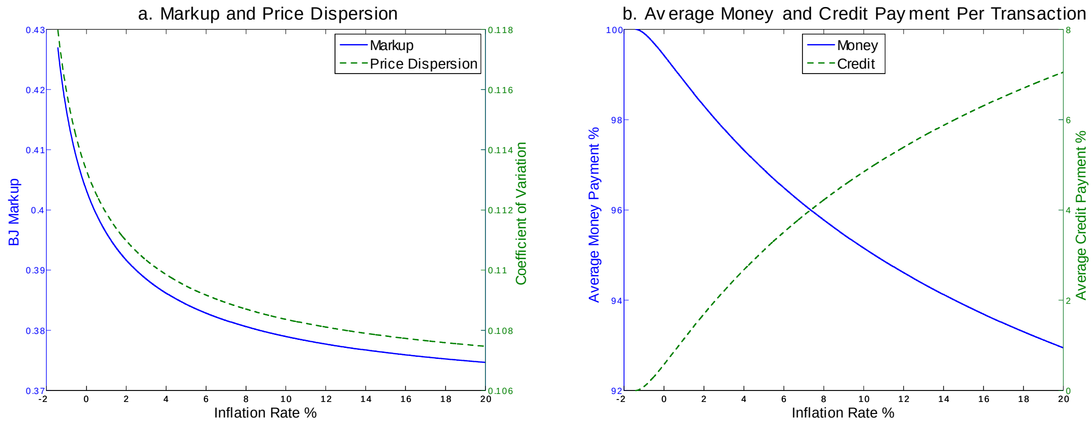


Figure 16: The Other Effects of Inflation with Entry

models, a one-time unanticipated jump in M has real effects. This is because at least some firms do not adjust p, even though they would like to, absent the assumed constraints, and hence the nominal distribution $F(p)$ does not change enough to keep the same real distribution $G(q)$. Hence, prices turn in favor of buyers, making b and output increase. In contrast, in our model economy, a surprise jump in M affects neither $G(q)$ nor b. A policy advisor seeing only a fraction of sellers adjusting p each period in our economy may conclude that a jump in M would have real effects; that would be wrong. Although not surprising, it is worth emphasizing that for policy prescriptions it is not actually enough to say prices are sticky in the data, it is important to know why.

As Figure 15 shows, compared to the benchmark model, the welfare cost of inflation approximately doubles, because it not only increases resources used to support more credit, an increase in π also decreases participation. Figure 16 demonstrates how π affects markups, price dispersion, and payment methods in the variable-cost model. Compared to Figure 13, endogenizing participation does not change the impact of inflation on the markup or price dispersion a lot. In particular, now fewer buyers enter the BJ market at higher π, but since that leads to higher arrival rates for those that enter, their reduction in real balances is attenuated.

6.4 Dynamics

General equilibrium monetary models can have nonstationary equilibria because money conveys liquidity. However, that is also true of other assets, and actually has little to do with the fiat nature of currency. It has more to do with liquidity being at least partially self-referential – how much you are willing to give for an asset depends on how much you think others will give in the future – an idea at the heart of search-based models at least since Kiyotaki and Wright (1989). To pursue this in our framework, assume m has a flow return ρ. If $\rho > 0$, m can be interpreted as a share in a technology or 'tree' bearing a dividend of 'fruit' in terms of numeraire, as in standard finance. If $\rho < 0$, it can be interpreted as a storage cost, as in some models of commodity money, which makes m a poor saving vehicle but potentially still valuable as a medium of exchange. And $\rho = 0$ means fiat currency. Also, here we revert to $k = 0$ so $b = \bar{b}$, focus on the variable-cost model, and keep the supply M fixed.[18]

The household's problem is now

$$W(A) = A + U(x^*) - x^* + \beta \max_z O_i(z)$$

where $A = \rho m + \phi m - d - C(d) + I$ includes dividend income ρm, and $O_r(z) = V(z) - (1 + r) z$. The Euler equation is

$$\phi_t = \frac{\phi_{t+1} + \rho}{1 + r} \left[1 + (\alpha_1 + \alpha_2) \tau \hat{H}(\hat{z}) \right], \tag{22}$$

where we do not impose stationarity of ϕ_t. If $\alpha_1 = \alpha_2 = 0$ (i.e., if we shut down the BJ market), (22) is a standard asset-pricing equation, it implies there is a unique equilibrium, and in equilibrium $\phi_t = \rho/r \; \forall t$. This is because any other solution to the difference equation is explosive and violates transversality.[19]

More generally, (22) is augmented on the RHS by a *liquidity premium* capturing the expected reduction in the credit costs, $(\alpha_1 + \alpha_2) \tau \hat{H}(\hat{z})$, and that dramatically

[18] It is not hard to let M change over time, but that is less interesting for real assets than for fiat currency, where one can think of π as a policy choice.

[19] See, e.g., Rocheteau and Wright (2013) for details in a class of related models.

changes the equilibrium set. Inserting $H(\hat{z})$, after some algebra we get

$$\phi_t = \frac{\phi_{t+1}+\rho}{1+r}\left\{1+\frac{\tau\alpha_1^2}{4\alpha_2}\frac{\left[\mu-\left(\rho+\phi_{t+1}\right)\right]\left[\mu+\left(\rho+\phi_{t+1}\right)(1+2\tau)-2\gamma(1+\tau)\right]}{(1+\tau)^2\left(\rho+\phi_{t+1}-\gamma\right)^2}\right\}$$

This dynamical system gives today's asset price in terms of tomorrow's, $\phi_t = \Phi\left(\phi_{t+1}\right)$. The left panel of Figure 17 shows $\phi_t = \Phi\left(\phi_{t+1}\right)$ and the inverse $\phi_{t+1} = \Phi^{-1}(\phi_t)$, for the calibrated parameters, including $\rho = 0$. This yields a unique steady state MME at $\phi \approx 4.4$. As is typical with fiat currency, the monetary (nonmonetary) steady state is unstable (stable), implying there are equilibria where $\phi \rightarrow 0$. This features inflation as a self-fulfilling prophecy.

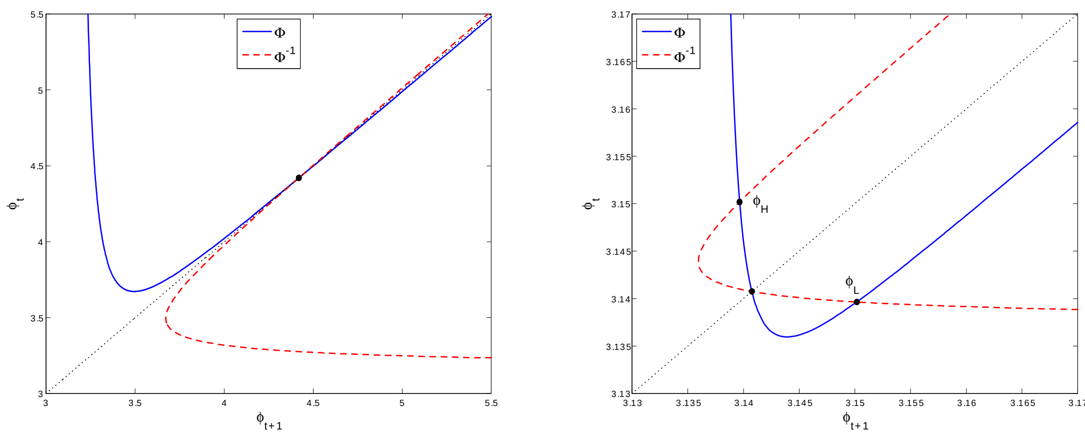


Figure 17: Phaseplane for Dynamic Equilibria

The right panel of Figure 17 makes one change in parameters, reducing α_1 from 0.034 to 0.0001. There is still a unique steady state MME, now with $\phi \approx 3.14$. However, textbook methods (e.g., Azariadis 1993) imply the following: because $\Phi' < -1$ at the monetary steady state, Φ and Φ^{-1} also cross off the 45^o line at (ϕ_L, ϕ_H) and (ϕ_H, ϕ_L). This is an equilibrium with a cycle of period 2, where ϕ oscillates between ϕ_L and ϕ_H as a self-fulfilling prophecy. Heuristically, if $\phi_{t+1} = \phi_L$ is low then liquidity will be scarce at $t+1$, making buyers want more of the asset at t, and thus making the price $\phi_t = \phi_H$ high. While it is not atypical for different monetary models to have cyclic equilibria, this intuition comes from the search literature (Rocheteau and Wright 2013), and is different from OLG models, say, where the results are described in terms of backward-bending labor supply or savings functions (Azariadis 1993). Moreover, a novelty here is that there are fluctuations in the price distribution $F(p)$, not just the price level, but the dynamics are still easy because one number $\bar{p}$ is sufficient to pin down $F(p)$ by virtue of (3). A second novelty is the role of liquidity: buyers can use credit at any time, but prefer to use assets at least some time, to reduce transaction costs.

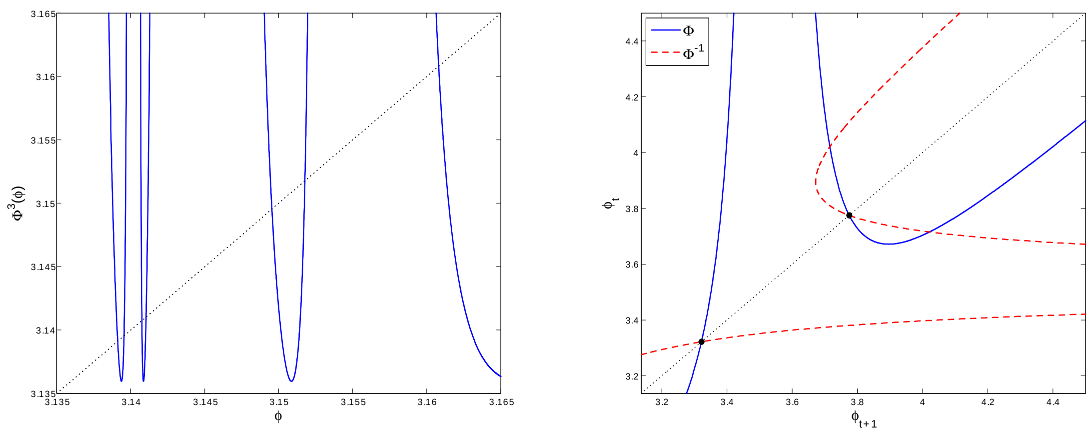


Figure 18: Examples with a 3-Cycle and with Two Steady States

For the same parameters that generate the 2-cycle, the left panel of Figure 18 shows the third iterate $\Phi^3(\phi)$. In addition to the steady state, $\Phi^3(\phi)$ has 6 intersections with the 45^o line. This means there exist a pair of 3-cycles. Standard results (again see Azariadis 1993) tell us that the existence of a 3-cycle implies the existence of N-cycles $\forall N$ by the Sarkovskii theorem, as well as chaotic dynamics by the Li-Yorke theorem. Thus we can generate a large set of perfect-foresight dynamics, if not for the calibrated parameter values, for values that are fairly close. There are also stochastic (sunspot) equilibria for these parameters, with random fluctuations in ϕ, $F(p)$ and other endogenous variables, illustrating how costly credit can generate excess volatility as a self-fulfilling prophecy.[20]

[20] A proof that sunspot equilibria exist, going back to Azariadis and Guesnerie (1986), is to suppose the outcome depends on an extrinsic two-state Markov process, $s \in \{s_1, s_2\}$, where $\varepsilon_s = prob(s_{t+1} \neq s | s_t = s)$. If $\varepsilon_1 = \varepsilon_2 = 1$ this reduces to a 2-cycle, the existence of which we just proved by example. By continuity there are equilibria for $\varepsilon_s < 1$.

When $\rho = 0$, one might think this dynamic multiplicity arises because there are two steady states, $\phi > 0$ and $\phi = 0$. However, we can eliminate the equilibrium with $\phi = 0$ by setting $\rho > 0$, and as long as ρ is not too big, by continuity the qualitative results are the same. Heuristically, the dynamic equilibria should not be interpreted as approximating fluctuations across two steady states, but around one steady state; this is an example where multiple steady states are not necessary for complicated dynamics. At the same time, setting $\rho < 0$ leads to two steady states, say ϕ_1 and ϕ_2, with $\phi_2 > \phi_1 > 0$, as shown in the right panel of Figure 18, drawn for the same parameters except $\rho = -0.4$. Since the lower steady state ϕ_1 is stable, in this configuration we can construct sunspot equilibria fluctuating around it.[21]

Summarizing, models with costly credit admit cyclic, chaotic and stochastic dynamics, with price distributions and the use of money and credit varying over time due to 'animal spirits.' This does not require fiat money, as the results hold for $\rho \neq 0$. It has to do with a trade-off between paying transaction costs on credit and accepting low returns on liquid assets. The return on fiat money is low, for obvious reasons, at least away from the Friedman rule. It may be less obvious for real assets but the point is similar: if an asset is useful in exchange its price is above the fundamental ρ/r, as seen in (22), and high asset prices mean low returns. This is well known, if perhaps less well known in a context where credit is always available but costly.[22]

[21]A method Azariadis (1981) uses in OLG models is this: We seek $(\phi_1, \phi_2, \varepsilon_1, \varepsilon_2)$ such that $\phi_1 = \varepsilon_1 \Phi(\phi_2) + (1 - \varepsilon_1) \Phi(\phi_1)$ and $\phi_2 = \varepsilon_2 \Phi(\phi_1) + (1 - \varepsilon_2) \Phi(\phi_2)$, where $\varepsilon_s \in (0, 1)$ and wlog $\phi_2 > \phi_1$. These equations are linear in, and hence easy to solve for, ε_1 and ε_2. Whenever $\Phi'(\phi_s) > 1$ at a steady state ϕ_s, for any ϕ_1 in some range to the left of ϕ_s and any ϕ_2 in some range to the right of ϕ_s, one can check $\varepsilon_1, \varepsilon_2 \in (0, 1)$, which is all we need to have a proper sunspot equilibrium.

[22]We close this part of the discussion by mentioning that in the above analysis assets are interpreted as facilitating trade as media of exchange – buyers hand them over by way of quid pro quo – but this story can be changed. In fact, the equations and conclusions are identical under the alternative interpretation that assets are used as collateral, and the results can be recast in terms of secured vs costly unsecured credit, rather money vs credit, making contact with the literature following Kiyotaki and Moore (1997). In the interest of space, for details we refer readers to the discussion in the survey by Lagos et al. (2016). However, the basic idea is

7 Conclusion

This paper has explored models of alternative payment methods, with money and costly credit as the leading example. For this we combined Burdett-Judd price posting with the Lagos-Wright monetary model. We are not the first to combine these ingredients, and this was not meant to be the main contribution; the contribution concerned the introduction of costly credit. This was useful, technically, because it resolved an indeterminacy problem in other models with money and price posting, and implied a unique stationary equilibrium where both money and credit are used. For both fixed and variable transaction costs, and for different assumptions about the way households sample prices, we derived exact money demand functions that resemble classic results in the literature, but we think with better microfoundations. These functions can match the macro data, and at least the variable-cost model can also match the money-credit shares in micro data.

In one application, we showed how the theory can account for the price-change data. It accounts for this very well if we do not impose the discipline of matching other observations, and fairly well if we do impose it. By accounting for the price-change data, we only mean there are equilibrium outcomes that are roughly consistent with the evidence. To be clear, the theory does not pin down which seller posts which price in the cross section, and hence does not pin down price-change behavior in the time series. However, once one sets the parameter σ in a pay-off irrelevant tie-breaking rule, there is a unique (symmetric, stationary, monetary) equilibrium with very precise predictions about price-change behavior. What we did is to calibre σ to the average duration of a price, and then compared these predictions to the facts.[23]

easy: purchases in frictional markets for goods, inputs, or anything else can have constraints that are relaxed by asset holdings, and this leads to very similar outcomes whether the assets are used to finalize spot trades, or forfeited after any (off-the-equilibrium-path) default.

[23] As reported in fn. 5, recall the stylized facts: (1) Empirical price durations vary across studies, but are typically fairly long. (2) The frequency and size of price changes vary across goods. (3) Two sellers changing at the same time do not generally pick the same new price. (4) Many changes are negative. (5) Hazards decline with duration. (6) There are many small but also

Another application revisited the welfare cost of inflation, from which we learned the following: while search-based models with bargaining generate large welfare costs, this is not the case in otherwise similar models with price posting. We found this in our baseline specification, where inflation impinged mainly on the costly use of credit, and in an extension where it also impinged on participation. This extension is also interesting in its own right, highlighting as it does an entry channel through which monetary policy affects frictional goods markets and hence aggregate economic activity. A related application considered the relationships between inflation, markups and price dispersion, where the model was shown to be consistent with some findings in the empirical literature. A final application discussed endogenous dynamics. While the mathematics in that discussion are not new, there are some novel economic ideas – e.g., fluctuations in a price distribution, not just a price level, and liquidity considerations emerging from assets reducing the resource cost of using credit. Many other extensions are possible, such as incorporating heterogeneity, or combining menu-cost and search-based monetary models; these are left for future work.

many big changes. (7) The frequency and size of price changes, as well as the fraction of negative changes, vary with inflation. (8) There is price dispersion even at low inflation. Our model is consistent with all these, although we did not play up (2); it seems clear, however, that different values for the preference and cost parameters μ and γ, or arrival rates rates α_n, as is reasonable for different goods, will affect price-change behavior.

Appendix A: Proofs of Non-obvious Results

Derivation of (8): The BJ value function can be written

$$V(z) = W(A) + \alpha_1 \int_{\underline{q}}^{z} (\mu - q)\, dG_1(q) + \alpha_1 \int_{z}^{\bar{q}} [\mu - q - \delta - \tau(q-z)]\, dG_1(q)$$
$$+ \alpha_2 \int_{\underline{q}}^{z} (\mu - q)\, dG_2(q) + \alpha_2 \int_{z}^{\bar{q}} [\mu - q - \delta - \tau(q-z)]\, dG_2(q),$$

where $G_n(q) = 1 - \hat{G}(q)^n$ is the CDF of the lowest of n draws from $G(q)$. The first term is the continuation value if a buyer does not trade. The second is the probability of meeting a seller with $q \le z$, so only cash is used, times the expected surplus, which is simple because $W'(A) = 1$. The third is the probability of meeting a seller with $q > z$, so credit must be used, which adds fixed cost δ and variable cost $\tau(q-z)$. The last two terms are similar except the buyer meets two sellers. The rest is algebra. ■

Proof of Lemma 3: For part (i), in NME, buyers' BJ surplus is $\Sigma = \mu - q - \delta - \tau q$. Note $\Sigma = 0$ at $q = (\mu - \delta)/(1+\tau)$, so no buyer pays more than this. If $\bar{q} < (\mu - \delta)/(1+\tau)$ then the highest price seller has profitable deviation toward $(\mu - \delta)/(1+\tau)$, which increases profit per unit without affecting sales. Hence $\bar{q} = (\mu - \delta)/(1+\tau)$.

For part (ii), in MME, for $q > z$, $\Sigma = \mu - q - \delta - \tau(q-z)$. Note $\Sigma = 0$ at $q = (\mu - \delta + \tau z)/(1+\tau)$, and repeat the argument for NME to show $\bar{q} = (\mu - \delta + \tau z)/(1+\tau)$. The definition of MME has $z < \bar{q} = (\mu - \delta + \tau z)/(1+\tau)$, which reduces to $z < \mu - \delta$

For (iii), in PME, given buyers bring z to BJ they would pay z. Hence $\bar{q} \ge z$, as $\bar{q} < z$ implies the highest price seller has profitable deviation. We also have to be sure there is no profitable deviation to $q > z$, which requires buyers using some credit. The highest such q a buyer would pay solves $\Sigma = \mu - q - \delta - \tau(q-z) = 0$, or $q = (\mu - \delta + \tau z)/(1+\tau)$. There is no profitable deviation iff $(\mu - \delta + \tau z)/(1+\tau) \le z$, which reduces to $z \ge \mu - \delta$. ■

Proof of Proposition 1: Part (i), for fiat currency $\phi = 0$ is always self-fulfilling, so we can set $G(q)$ according to (5), corresponding to equilibrium in the original BJ model.

For (ii), from Figure 1, MME exists iff three conditions hold: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > O_i(0)$. Now (a) is equivalent to $(\alpha_1 + \alpha_2)\,\delta H^-(\bar{q}) < i$, which holds iff $i > \underline{i}$. Then (b) is equivalent to $(\alpha_1 + \alpha_2)\,\delta H^+(\underline{q}) > i$, which holds iff $i < \tilde{\imath}$ where $\tilde{\imath} = \delta\,(\alpha_1 + 2\alpha_2)^3\,/2\alpha_1\alpha_2\,(\mu - \delta - \gamma) > \underline{i}$. Also, (c) is equivalent to $(\alpha_1 + \alpha_2)\,\delta H\,(z_i) - iz_i > (\alpha_1 + \alpha_2)\,\delta H\,(0)$, which holds iff $\Delta\,(i) > 0$ where

$$\Delta\,(i) = -i\gamma + \frac{\delta\,(\alpha_1 + 2\alpha_2)^2}{4\alpha_2} - i^{\frac{2}{3}}\delta^{\frac{1}{3}}\alpha_1^{\frac{2}{3}}\alpha_2^{-\frac{1}{3}}\,(\mu - \delta - \gamma)^{\frac{2}{3}}\,(2^{-\frac{1}{3}} + 2^{-\frac{4}{3}}).$$

Notice $\Delta\,(0) > 0 > \Delta\,(\tilde{\imath})$ and $\Delta'\,(i) < 0$. Hence $\exists!\ \bar{\imath}$ such that $\Delta\,(\bar{\imath}) = 0$, and $\Delta\,(i) > 0$ iff $i < \bar{\imath}$. It remains to verify that $\bar{\imath} > \underline{i}$, so that (a) and (c) are not mutually exclusive. It can be checked that this is true iff $\delta < \bar{\delta}$. Hence a MME exists under the stated conditions. It is unique because $\bar{q} = \mu - \delta$, which pins down $G\,(q)$, and then $\hat{z}_i = \arg\max_{z \in (\underline{q}, \bar{q})} O_i\,(z)$.

For (iii), from Figure 1, PME exists iff three conditions hold: (a) $O_i^-(\bar{q}) > 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(\bar{q}) > O_i(0)$. Now (a) holds iff $i < \underline{i}$ and (b) holds iff $i < \tilde{\imath}$. Condition (c) holds iff $i < \hat{\imath}$. For $\delta > \bar{\delta}$, it can be checked that $\hat{\imath} < \underline{i}$ and $\underline{i} < \tilde{\imath}$, so the binding condition is $i < \hat{\imath}$. For $\delta < \bar{\delta}$, it is easily checked that $\hat{\imath} > \underline{i}$, and $\underline{i} < \tilde{\imath}$, so the binding condition is $i < \underline{i}$. ■

Proof of Proposition 2: Part (i), with fiat currency $\phi = 0$ is always self-fulfilling, so there is a NME iff buyers' payoff from in the BJ market is nonnegative, $(\alpha_1 + \alpha_2)[\mu - (1+\tau)\mathbb{E}_H q] \geq 0$. Substituting $\mathbb{E}_H q$ into this, after some algebra we can show this holds iff $\tau \leq \mu/\gamma - 1$.

For (ii), from Figure 3, MME exists iff three conditions hold: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $\Psi_M > 0$ where

$$\Psi_M = (\alpha_1 + \alpha_2)\,[\mu - \mathbb{E}_H q - \tau\mathbb{E}_H \max\,(0, q - z_i)] - iz_i$$

is buyers' payoff from the BJ market. Now (a) holds automatically since $O_i^-(\bar{q}) = -i$. Then (b) is equivalent to $(\alpha_1 + \alpha_2)\,\tau\hat{H}^+(\underline{q}) > i$, which holds iff $i < (\alpha_1 + \alpha_2)\tau$. And (c) is equivalent to

$$\Psi_M = \alpha_2\,(\mu - \gamma) + \frac{\alpha_1\tau\,(\mu - z_i)}{1+\tau} - \frac{\alpha_1^2\tau(\mu - z_i)^2}{4\alpha_2(1+\tau)^2(z_i - \gamma)} - iz_i = \Psi\,(z_i) - iz_i > 0.$$

Notice Ψ_M is strictly concave and continuous in i, $\lim_{i\to 0}\Psi_M > 0$, and $\lim_{i\to\infty}\Psi_M < 0$. Hence there exists a unique solution to $\Psi_M = 0$, which defines i^*, so $\Psi_M > 0$ holds $\forall i < i^*$. Hence, a MME exists under the stated conditions. It is unique because $O_i''(z_i) = V''(z_i) < 0$, and then $\hat{z}_i = \arg\max_{z \in (\underline{q}, \bar{q})} O_i\,(z)$.

Finally, for part (iii), from Figure 3 it is clear that there is no PME in the variable-cost model. ■

Proof of Proposition 3: Substituting α_n into (13) we have

$$\Pi_t(p) = b_t\left(p\phi_t - \gamma\right)\eta e^{-\eta}\sum_{n=1}^{\infty}\frac{\left[\eta\hat{F}_t\left(p_t\right)\right]^{n-1}}{(n-1)!} = b_t\left(p\phi_t - \gamma\right)\eta e^{-\eta F_t(p)},$$

since $e^x = \sum_{i=0}^{\infty} x^i/i!$. As a special case, $\Pi_t(\bar{p}_t) = b_t\left(\bar{p}_t\phi_t - \gamma\right)\eta e^{-\eta}$. Equal profit implies

$$\begin{aligned} F_t\left(p\right) &= 1 - \frac{1}{\eta}\left[\log\left(\phi_t\bar{p}_t - \gamma\right) - \log\left(\phi_t p - \gamma\right)\right] \\ G_t\left(q\right) &= 1 - \frac{1}{\eta}\left[\log\left(\bar{q}_t - \gamma\right) - \log\left(q - \gamma\right)\right], \end{aligned}$$

with $\bar{q}$ as in the baseline model and $\underline{q}_t = e^{-\eta}\bar{q}_t + \left(1 - e^{-\eta}\right)\gamma$. Algebra the yields

$$H_t\left(q\right) = \frac{\sum_{n=1}^{\infty}\alpha_n\left[1 - \left[1 - G_t\left(q\right)\right]^n\right]}{\sum_{n=1}^{\infty}\alpha_n} = \frac{1 - e^{-\eta}\left(\bar{q}_t - \gamma\right)/\left(q - \gamma\right)}{1 - e^{-\eta}}.$$

In the fixed-cost model, (i) holds as in Proposition 1. For (ii), follow Proposition 1 and check : (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > O_i(0)$. Now (a) holds iff $\sum_{n=1}^{\infty}\alpha_n\delta H^-(\bar{q}) < i$ iff $i > \underline{i} = e^{-\eta}\delta/(\mu - \delta - \gamma)$. Then (b) holds iff $\sum_{n=1}^{\infty}\alpha_n\delta H^+(\underline{q}) > i$ iff $i < \tilde{\imath} = e^{\eta}\delta/(\mu - \delta - \gamma) > \underline{i}$. And (c) holds iff $\sum_{n=1}^{\infty}\alpha_n\delta H\left(z_i\right) - iz_i > \sum_{n=1}^{\infty}\alpha_n\delta H\left(0\right)$ iff $\Delta\left(i\right) > 0$, where

$$\Delta\left(i\right) = \delta - 2\left[e^{-\eta}\delta\left(\mu - \delta - \gamma\right)i\right]^{\frac{1}{2}} - i\gamma.$$

Given $\Delta\left(0\right) > 0 > \Delta\left(\tilde{\imath}\right)$ and $\Delta'\left(i\right) < 0$, $\exists!\ \bar{\imath}$ such that $\Delta\left(\bar{\imath}\right) = 0$, and $\Delta\left(i\right) > 0$ iff $i < \bar{\imath}$. It remains to verify $\bar{\imath} > \underline{i}$, so that (a) and (c) are not mutually exclusive. This is true iff $\delta < \bar{\delta}$, where $\bar{\delta} = \mu - \left(1 - e^{-\eta}\right)\gamma/\left(1 - 2e^{-\eta}\right)$. Hence, MME exists under the stated conditions. The rest of the proof is the same as Proposition 1, except with $\hat{\imath} = \delta(1 - e^{-\eta})/(\mu - \delta)$. ■

Proof of Proposition 4: For (i) we again follow the proof of Proposition 2 and check

$$\Phi_N = \sum_{n=1}^{\infty}\alpha_n\left[\mu - \left(1 + \tau\right)\mathbb{E}_H q\right] \geq 0.$$

After substituting $\mathbb{E}_H q$, we get $\Phi_N = (1 - e^{-\eta} - \eta^{-\eta})[\mu - \gamma(1 + \tau)]$. Thus NME exists iff $\tau \leq \mu/\gamma - 1$. To prove (ii), we again check: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > 0$, where

$$O_i(z_i) = \sum_{n=1}^{\infty}\alpha_n\left[\mu - \mathbb{E}_H q - \tau\int_{\hat{z}_i}^{\bar{q}}\left(q - \hat{z}_i\right)dH\right] - i\hat{z}.$$

Now (a) always holds, and (b) holds iff $i < (1 - e^{-\eta})\tau$. For (c), substitute α_n and H and simplify to get

$$\begin{aligned} O_i\left(z_i\right) &= \left(1-e^{-\eta}\right)\mu - \left(1-e^{-\eta}-\eta e^{-\eta}\right)\gamma - \eta e^{-\eta}\frac{\mu + z_i\tau}{1+\tau} - \frac{\tau e^{-\eta}\gamma\left(\mu - z_i\right)}{\left(1+\tau\right)\left(z_i-\gamma\right)} \\ & \quad - \frac{\tau e^{-\eta}\left[\mu-\gamma+\tau\left(z_i-\gamma\right)\right]}{1+\tau}\log\frac{\mu-\gamma+\tau\left(z_i-\gamma\right)}{\left(1+\tau\right)\left(z_i-\gamma\right)}. \end{aligned}$$

One can show $O_i''(z_i) < 0$. Since z_i is strictly decreasing in i, $O_i''(z_i)$ is strictly convex in i on $[0,\infty)$. Moreover, $\lim_{i\to 0} O_i(z_i) > 0$ and $\lim_{i\to\infty} O_i(z_i) < 0$. There is a unique solution to $O_i(z_i) = 0$ and that defines i^*, so $O_i(z_i) > 0\ \forall i < i^*$. Hence, there exists a unique MME iff $i < \min\left\{\tau\left(1-e^{-\eta}\right), i^*\right\}$. Finally, as in the proof of Proposition 2, (iii) is true. ■

Proof of Proposition 5: Substituting α_n into (13) we have

$$\Pi_t(p) = b_t\left(p\phi_t - \gamma\right)\sum_{n=1}^{\infty}\left[-\frac{\omega^n}{\log\left(1-\omega\right)}\right]\left[\hat{F}_t\left(p_t\right)\right]^{n-1},$$

and $\Pi_t(\bar{p}_t) = -b_t\left(\bar{p}_t\phi_t - \gamma\right)\omega/\log\left(1-\omega\right)$. Now equal profit implies

$$F_t\left(p\right) = 1 - \frac{\phi_t\left(\bar{p}_t - p\right)}{\omega\left(\phi_t\bar{p}_t - \gamma\right)} \text{ and } G_t\left(q\right) = 1 - \frac{\bar{q}_t - q}{\omega\left(\bar{q}_t - \gamma\right)},$$

with $\bar{q}$ as in the baseline models and $\underline{q}_t = (1-\omega)\bar{q}_t + \omega\gamma$. Also,

$$H_t\left(q\right) = 1 - \frac{\log\left[1-\omega\left[1-G_t(q)\right]\right]}{\log\left(1-\omega\right)} = 1 - \frac{\log\left(q-\gamma\right) - \log\left(\bar{q}_t-\gamma\right)}{\log\left(1-\omega\right)},$$

where we used $\sum_{n=1}^{\infty} x^n/n = -\log(1-x)$.

In the fixed-cost model, (i) holds as in Proposition 1. To show (ii), we check: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > O_i(0)$. Now (a) holds iff $i > \underline{i} = -\delta/[(\mu-\delta-\gamma)\log(1-\omega)]$, and (b) holds iff $i < \tilde{\imath} = -\delta/[(1-\omega)(\mu-\delta-\gamma)\log(1-\omega)] > \underline{i}$. Then (c) holds iff $\Delta\left(i\right) > 0$, where

$$\Delta\left(i\right) = \delta - \frac{\delta\left[\log\left(\hat{z}_i-\gamma\right) - \log\left(\mu-\delta-\gamma\right)\right]}{\log\left(1-\omega\right)} - i\gamma + \frac{\delta}{\log\left(1-\omega\right)}.$$

It is easy to check $\Delta'\left(i\right) < 0$, $\lim_{i\to 0}\Delta(i) > 0$, and $\exists!\ \bar{\imath}$ such that $\Delta\left(\bar{\imath}\right) = 0$. Hence $\Delta\left(i\right) > 0$ iff $i < \bar{\imath}$. For (a) and (c) to not be mutually exclusive, we check $\bar{\imath} > \underline{i}$. This holds iff $\delta < \bar{\delta}$, where

$$\bar{\delta} = \mu - \frac{\gamma\log\left(1-\omega\right)}{1+\log\left(1-\omega\right)}.$$

Thus MME exists. Uniqueness follows Proposition 1, as does (iii), except now $\hat{\imath} = \delta / (\mu - \delta)$. ■

Proof of Proposition 6: For (i), we check

$$\Phi_N = \left[1 - \frac{\omega}{\log(1-\omega)}\right] [\mu - \gamma(1+\tau)] \geq 0,$$

which holds iff $\tau \leq \mu/\gamma - 1$. For (ii) we check: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > 0$. Now (a) always holds and (b) holds iff $i < \tau$. Then for (c), write

$$\begin{aligned} O_i(z_i) &= \mu - \gamma + \frac{(\omega+\tau)\mu - (1+\tau)\omega\gamma + (\omega-1)\tau\hat{z}_i}{(1+\tau)\log(1-\omega)} \\ &\quad + \frac{\gamma\tau}{\log(1-\omega)} \log\left[\frac{\mu - \gamma + \tau(\hat{z}_i - \gamma)}{(1+\tau)(\hat{z}_i - \gamma)}\right]. \end{aligned}$$

Since $\log(1-\omega) < 0$, we have $O_i'(z_i) > 0$. Given $\partial\hat{z}_i/\partial i < 0$, we also have $\partial O_i(z_i)/\partial i < 0$. Define i^* as the solution to $O_i(z_i) = 0$, so $O_i(z_i) > 0$ holds $\forall i < i^*$. Hence, there is a MME iff $i < \min\{\tau, i^*\}$, and it is unique since $O_i''(z_i) < 0$. Finally, (iii) follows Proposition 2. ■

Appendix B: Formulae for Calibration

Consider first the variable-cost model. Inserting $\bar{q}$ and $\underline{q}$, we get

$$\begin{aligned} G(q) &= 1 - \frac{\alpha_1}{2\alpha_2} \frac{\mu - q + \tau(\hat{z}_i - q)}{(1+\tau)(q-\gamma)}, \\ H(q) &= 1 - \frac{\alpha_1^2 [\mu - q + \tau(\hat{z}_i - q)][\mu + \tau\hat{z}_i + (q - 2\gamma)(1+\tau)]}{4\alpha_2(\alpha_1+\alpha_2)(1+\tau)^2(q-\gamma)^2}. \end{aligned}$$

The fraction of monetary transactions and the markup are therefore

$$\begin{aligned} H(\hat{z}_i) &= 1 - \frac{\alpha_1^2(\mu - \hat{z}_i)[\mu + \tau\hat{z}_i + (\hat{z}_i - 2\gamma)(1+\tau)]}{4\alpha_2(\alpha_1+\alpha_2)(1+\tau)^2(\hat{z}_i-\gamma)^2}, \\ \frac{\mathbb{E}_G q}{\gamma} &= 1 + \frac{\alpha_1(\mu + \tau\hat{z}_i - \gamma + \tau\gamma)\log(1 + 2\alpha_2/\alpha_1)}{2\alpha_2\gamma(1+\tau)}, \end{aligned}$$

where $\hat{z}_i$ is given in the text. From this we get

$$\begin{aligned} L_i &= \frac{(1+\tau)\hat{z}_i}{\alpha_1(\mu + \hat{z}_i\tau) + (1+\tau)(1+\alpha_2\gamma)}, \\ \eta_i &= \frac{\alpha_1\mu + (1+\tau)(1+\alpha_2\gamma)}{\alpha_1(\mu + \hat{z}_i\tau) + (1+\tau)(1+\alpha_2\gamma)} \frac{\partial\hat{z}_i/\partial i}{\hat{z}_i/i}. \end{aligned}$$

Consider next the fixed-cost model. Inserting $\bar{q}$ and $\underline{q}$, we get

$$\begin{aligned} G(q) &= 1 - \frac{\alpha_1}{2\alpha_2} \frac{\mu - \delta - q}{q - \gamma}, \\ H(q) &= 1 - \frac{\alpha_1^2(\mu - \delta - q)(\mu - \delta + q - 2\gamma)}{4\alpha_2(\alpha_1+\alpha_2)(q-\gamma)^2}. \end{aligned}$$

The the fraction of monetary transactions and the markup are

$$\begin{aligned} H(\hat{z}_i) &= \frac{[2\alpha_1\alpha_2(\mu-\delta-\gamma)/\delta]^{2/3} i^{2/3} - \alpha_1^2}{4\alpha_2(\alpha_1+\alpha_2)}, \\ \frac{\mathbb{E}_G q}{\gamma} &= 1 + \frac{\alpha_1(\mu - \delta - \gamma)\log(1 + 2\alpha_2/\alpha_1)}{2\alpha_2\gamma}. \end{aligned}$$

From this we get

$$\begin{aligned} L_i &= \frac{\gamma + \left[\alpha_1^2\delta(\mu-\delta-\gamma)^2/2\alpha_2\right]^{1/3} i^{-1/3}}{1 + \alpha_1(\mu-\delta) + \alpha_2\gamma} \\ \eta_i &= \frac{-1}{3 + 3\gamma\left[\alpha_1^2\delta(\mu-\delta-\gamma)^2/2\alpha_2\right]^{-1/3} i^{1/3}}. \end{aligned}$$

Appendix C: Summary of Empirical Findings on Price Changes Statistics

Studies	Data source	Sample period	Empirical evidences [1] Monthly change frequency (%)		Duration (month)		Absolute size of p changes (%)		Fraction of p changes below 5%	Hazard rates
			Mean	Median	Mean	Median	Mean	Median		
Using BLS CPI Research Database										
Bils and Klenow (2004)		1995M1-1997M12	26.1	20.9	...	4.3 (5.5)				
Klenow and Kryvtsov (2008) [2]			36.2 (29.9)	27.3 (13.9)	6.8 (8.6)	3.7 (7.2)	14.0 (11.3)	11.5 (9.7)	39.8 (44.3)	Declining pooled hazard; flat w/o survivor bias
Nakamura and steinsson (2008)		1988M2-2005M1	23.9-27.7 (19.7-23.1)	19.4-21.7 (10.6-13.0)	7.5-9.0 (9.0-10.4)	4.1-4.6 (7.2-8.9)		10.7 (8.5)		Declining hazard
Klenow and Malin (2010)		1988M1-2005M12 1988M1-2009M10	...	...	6.2 (8.0)	3.4 (6.9)	...	...	...	...
Eichenbaum et. al. (2014) [3]		1988M1-2011M7	22.0 (13.5)	...	4.5 (7.4)	...	...	...	40.6 (46)	...
Kehoe and Midrigan (2014) [4]		1988M1-2005M12	22.0 (6.9)	...	...	...	...	...	...	...
Using Scanner data set [5]										
Burstein and Hellwig (2007)	Dominick's	1989-1997	41 (26)	...	2.5 (4)	...	13 (11)	...	...	...
Nakamura (2008)	AC Nielsen ScanTrak	2004	43.9 (19.0)	42.7 (17.5)	...	...	...	...	...	...
Midrigan (2011) [6]	Dominick's	1989-1997	34 (2.9)	...	...	...	20 (11)	...	25	...
Eichenbaum et. al. (2011) [6]	A Large U.S. Retailer	2004-2006	43	...	0.6	...	...	...	...	...
Campbell and Eden (2014)	AC Nielsen ERIM	1985-1988	29	...	2.6	...	...	...	...	Declining hazard
Eichenbaum et. al. (2014) [7]	A Large U.S. Retailer	2004	...	...	...	...	...	30	5.2	...

(1) For studies that report statistics of both regular (i.e., excl. sales) and posted prices, the figures in parenthese correspond to those of regular prices.
(2) The paper also reported other estimates of durations, with the highest at 13.4 (mean) and 10.6 (median) duration when restricting the sample to 1998-2004 only.
(3) The paper also claimed that many small price changes are due to measurement errors and quality adjustment. Once removing these, the fraction drops to 32.2% for regular price changes and 24.4% for posted prices.
(4) This paper used the same data as Nakamura and Steinsson (2008), but different algorithm to calcualte price changes.
(5) Most scanner data are available on weekly basis, but here only monthly frequencies of price changes are reported, except Midrigan (2011) and Eichenbaum et. al. (2011) which reports a weekly frequency.
(6) Numbers of price change frequency are weekly frequencies.
(7) The paper also reported statistics calculated using the unit value index (UVI)-based approach; the median change in UVI-based prices is 10 percent and 31.5 percent of the changes are smaller than 5 percent in absolute terms.

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