WP/99/141

INTERNATIONAL MONETARY FUND

Western Hemisphere Department

Nominal Exchange Rates and Nominal Interest Rate Differentials

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October 1999

Abstract

This paper reexamines some unsettled theoretical and empirical issues regarding the relationship between nominal exchange rates and interest rate differentials and provides a model for the behavior of exchange rates in the long run, where interest rates are determined in the bond market. The model predicts that an increase in the interest rate differential appreciates the home currency. We test the model for the U.S. dollar against the Deutsche mark, the British pound, the Japanese yen, and the Canadian dollar. The first two pairs of exchange rates—for which purchasing power parity seems to hold—display a strong relationship with interest rate differentials.

JEL Classification Numbers: F31, F41, C13

Keywords: Exchange rates, interest rate differential, bonds market, cointegration, common cycles

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\(^1\) This paper is part of a project on exchange rate behavior started while the author was at the Economics Department of the Reserve Bank of New Zealand. An earlier version of the paper was written in 1997 as a Reserve Bank of New Zealand working paper, and presented at the New Zealand Association of Economists Conference at Canterbury University. The views expressed in this version of the paper are those of the authors and do not necessarily represent those of the Reserve Bank of New Zealand. We are grateful to Sean Collins, Saul Lizondo, Nelson Mark, John McDermott, and participants at the Far Eastern Meeting of the Econometric Society in Singapore (July 1999) for their comments on an earlier draft of the paper. All remaining errors are our own responsibility.

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Contents

I. Introduction and Motivation ................................................................. 3

II. Exchange Rates and Interest Rates Differential in the Long Run ........... 5

III. Empirical Evidence ............................................................................. 10

A. Unit Roots and Cointegration .............................................................. 11
B. The Long-Run Elasticities of the Model ............................................ 13
C. The Performance of the Model and its Assumptions .......................... 15

IV. Summary ........................................................................................... 16

V. References .......................................................................................... 18

Tables

1. Tests for Unit Roots in Nominal Exchange Rates and Nominal Interest Rate Differentials ................................................................. 22
2. The Johansen-Juselius Maximum Likelihood Test for Cointegration .......... 23
4. The Vahid-Engle test for Common Cycles ......................................... 25

Figures

1. US & Germany 10-Year Government Bond Yield ................................ 26
2. US & Britain 10-Year Government Bond Yield .................................... 27
3. US & Japan 10-Year Government Bond Yield ..................................... 28
4. US & Canadian 10-Year Government Bond Yield ............................... 29
5. Exchange Rate & Interest Rate Differential ......................................... 30
6. Exchange Rate & Interest Rate Differential, GBP-USD ..................... 31
7. Exchange Rate & Interest Rate Differential, YEN-USD ..................... 32
8. Exchange Rate & Interest Rate Differential, CAD-USD .................... 33
9. Nominal Exchange Rate & a+b(i-i*), DM-USD .................................... 34
10. Nominal Exchange Rate & a+b(i-i*), GBP-USD ............................... 35
11. Nominal Exchange Rate & a+b(i-i*), YEN-USD ............................... 36
12. Nominal Exchange Rate & a+b(i-i*), CAD-USD ............................... 37
13. DM-USD Real Exchange Rate Trend ............................................... 38
14. GBP-USD Real Exchange Rate Trend ............................................. 39
15. YEN-USD Real Exchange Rate Trend ............................................. 40
16. CAD-USD Real Exchange Rate ....................................................... 41
I. INTRODUCTION AND MOTIVATION

The objective of this paper is not to review the literature on the relationship between nominal exchange rates and nominal interest rate differentials. However, we reexamine important and often disputed theoretical and empirical issues. We provide a model that links the level of the nominal exchange rate to nominal interest rate differentials under the assumptions of rational expectations, ex-ante purchasing power parity, and with the interest rate determined in the bond rather than in the money market. We emphasize issues such as the choice of the interest rate in the exchange rate determination model, correlation, unit roots, cointegration, common cycles and stability using monthly data from 1980–1997 for the exchange rates between the U.S. dollar and the following currencies: Deutsche mark (DM), British pound (GBP), Japanese yen (YEN), and the Canadian dollar (CAD).

Empirically, there seems to be no consensus on the appropriate interest rate variable—short versus long-term rates—to be used in exchange rate determination models. For example, Frankel (1979) uses long-term interest rates as a proxy for anticipated inflation, Kim and Mo (1995) use both short and long term rates, Bartolini and Bodnar (1995), McNown and Wallace (1994, 1989), Johansen and Juselius (1992) and Baillie and Pecchenino (1991) use short-term interest rates. The argument against the use of short-term interest rates is that central banks intervene in the money market to smooth out short-term interest rate movements affecting thereby their information content. On those grounds, empirical analysis of exchange rate using short-term interest rates requires an additional equation representing the behavior of the central bank (McCallum, 1994). On the other hand, long-term interest rates are the average of anticipated future short-term interest rates, which reflects anticipated returns to capital over the business cycle and anticipated inflation (Brunner and Meltzer, 1989). We use long-term interest rate differentials in this paper.

The issue of which interest rate is relevant to exchange rate determination models is not even theoretically settled. The question is “does the exchange rate move in the same direction with the interest rate differential or not?” In other words, does a rise in the domestic interest rate relative to the foreign interest rate appreciate or depreciate the home currency?3 Dornbusch (1976) and Frankel (1979) contend that a relative rise in domestic interest rates reflects a rise in the domestic real interest rate. A rise in domestic interest rates will attract foreign capital inflows and thereby bring on an appreciation of domestic currency, i.e., the exchange rate and the interest rate differential move in the same direction. This is also the conventional result in recent VAR analysis that tries and identifies monetary policy shocks (Christiano et al, 1998).

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3 We define the exchange rate as the foreign money price of a unit of a U.S. dollar, i.e., an increase in the exchange rate is an appreciation.
To the contrary, Mussa (1976, 1979), Frenkel (1976) and Bilson (1978, 1979) for example, argue that changes in interest rate differentials reflect changes in expected inflation differentials or the expected rate of currency depreciation. Thus, a rise in domestic interest rates indicates an increase in expected inflation. Asset holders will reduce their demand for the domestic currency, hence leading to its depreciation. Thus, the exchange rate and the interest rate differential move in the opposite direction. Empirical evidence on this issue is mixed and inconclusive.

The references to the literature suggest that there are four key assumptions of a model intended to explain the relationship between exchange rates and interest rate differentials. First, there is the degree of substitutability of domestic assets and foreign assets, i.e., whether they are perfect or imperfect substitutes. Second, there is the relative speed of adjustment of prices in good and asset markets. Third, there is the process of interest rate determination. Finally, there is the process assumed for the formation of expectations, i.e., whether expectations are rational or take some other form. The important point is that the choice of the interest rate to use in empirical work is part and parcel of the particular combination of assumptions made in each case. For instance, if domestic and foreign assets are perfect substitutes, asset markets adjust instantaneously, and expectations are formed rationally, the domestic interest rate cannot differ from the foreign interest rate but by the expected change in the exchange rate.

Similarly, there is no consensus among economists on whether or not exchange rates and interest rate differentials share common cycles. Baxter (1994) applies approximate band-pass filters and finds a positive relationship between real exchange rates and real interest rate differentials with the strongest link at the trend and business cycle frequencies. There is no relationship between them at high frequencies (cycles of 2–5 quarters). One question is whether or not the nominal magnitudes have a similar relationship.

While there is agreement that exchange rates and perhaps nominal interest rates have unit roots just like other asset prices, there is a little agreement that interest rate differentials contain unit roots. This of course has implications for hypothesis testing. This issue is also complicated by the fact that there are many methods of testing for unit roots and all tests, perhaps, suffer from problems such as small sample bias and low power against stationary alternatives.

Again, there is no strong agreement among economists about cointegration. For example, see Messe and Rogoff (1983) and Messe and Rogoff (1988) research on real exchange rates and real interest rate differentials. Problems regarding the power of the tests are also present.

Faced with all those disagreements, Obstfeld and Rogoff (1996, p. 623) point at the strong visual correlation observed between exchange rates and interest rate differentials (real and nominal) that could not be uncovered by regressions. This puzzle is the motivation of this research.
In section two we discuss the long-run relationship between exchange rates and interest rate differentials. It is shown that the level of the exchange rate can be linked directly to the interest rate differential. We provide a simple model that explains such links under the assumptions of rational expectations, ex-ante purchasing power parity (PPP) and that interest rates are determined in the bond rather than in the money market. In section three we discuss unit roots, cointegration and common cycles. Monthly data from 1980 to 1997 for the DM-USD, GBP-USD, YEN-USD and the CAD-USD are used for formal testing. Different testing procedures are used. There is evidence that the exchange rates are approximately unit-root processes. However, the evidence regarding the order of integration of the interest rates differentials is mixed. When both the exchange rate and the interest rate differential are I(1), the evidence of common trends and cycles is also strong. The evidence becomes weak when the order of integration is ambiguous (e.g., CAD-USD and YEN-USD). Our model finds a significant relationship in the DM-USD and GBP-USD cases. It fails to find a significant relationship in the CAD-USD and YEN-USD cases. We speculate on the possible reasons for the failure of the model in the CAD-USD and YEN-USD pairs by making reference to the assumptions of our model. Because PPP does not seem to hold in the CAD-USD and YEN-USD pairs, the assumption of PPP may be important in explaining the failure of uncovered interest rate parity. Section four concludes the paper.

II. EXCHANGE RATES AND INTEREST RATES DIFFERENTIAL IN THE LONG RUN

We concentrate on the long-run relationship (low frequency) between nominal exchange rates and nominal interest rate differentials. This brings to the front issues of long-run common trends in exchange rates and interest rate differentials and the kind of assumptions likely to be consistent with the data at that frequency.

We assume that PPP holds in the long run when "real factors" are likely to become more important in the adjustment process. Baxter (1994, p.18) argues that competing theories of exchange rate determination agree that ex ante PPP holds in the long run (when price level adjustments are complete). In an exchange rate determination model with rational agents, deviations from PPP motivated by sticky prices are difficult to rationalize in the long run when all shocks become fully anticipated.

\[
\bar{e} = \bar{p}^* - \bar{p},
\]

(1)

where \(\bar{e}\) is the log of the long-run nominal exchange rate and \(\bar{p}^*\) and \(\bar{p}\) are the logs of the long-run equilibrium price levels abroad and in the home country, respectively.

Assume the Fisher relation holds at home and abroad:

\[
\bar{i} = \bar{r} + \bar{\pi}
\]

(2)
and

\[ \tilde{i}^* = \bar{r}^* + \bar{\pi}^* , \]

(3)

where \( \tilde{i} \) and \( i^* \) are the long-run nominal interest rates at home and abroad, \( \bar{r} \) and \( \bar{r}^* \) are the long-run real interest rates at home and abroad, and \( \bar{\pi} \) and \( \bar{\pi}^* \) are long-run expected inflation rates at home and abroad, respectively.

The assumption of long-run ex ante PPP implies that \( \bar{r} = \bar{r}^* \) in the long run to the extent that the real interest rate differential reflects an expected long-run profitability differential.\(^4\)

In the long run, (2) and (3) give:

\[ \bar{\pi}^* - \bar{\pi} = i^* - \tilde{i} , \]

(4)

which is consistent with the assumptions of financial markets efficiency and a high degree of substitutability among securities of different countries.

Equation (1) and (4) can be combined to obtain the change in the long-run exchange rate as a function of the differential in long-run nominal interest rates:

\[ \Delta \tilde{e} = i^* - \tilde{i} . \]

(5)

To allow the possibility of stochastic shocks, and using equation (4), equation (5) can be rewritten as follows:

\[ e_t = E_t e_{t+1} + (\pi_t - \pi_t^*) . \]

(6)

Alternatively, equation (6) can be written as:

\[ e_t = E_t e_{t+1} + (i_t - i_t^*) , \]

(6')

where \( E_t \) is the mathematical expectation operator conditional on the information set at time \( t \) and the variables \( \pi_t, \pi_t^*, e_t, i_t, i_t^* \) are defined as deviations from their long-run

\(^4\) The study of the implications of this assumption for growth convergence is beyond the scope of this paper.
values $\bar{r}, \bar{r}^*, \bar{e}, \bar{I}, \bar{I}^*$. Similarly, the variables $r_r, r_r^*$ are defined as deviations from the long-run values $\bar{r}, \bar{r}^*$, which can be viewed as the natural interest rates or rates of time preference.\(^5\)

To elaborate on the links between equations (4), (5), (6) and (6'), we use a loanable funds model of interest rate determination in an open economy set-up. Deviations of the nominal interest rate from its long-run value are determined by a loanable funds equilibrium of the following form:

$$D_t + K_t = S_t + G_t.$$  

(7)

$$D_t^* + K_t^* = S_t^* + G_t^*,$$

(8)

where $D_t (D_t^*)$ represents the domestic (foreign) real private sector demand for long-term (zero-coupon, for simplicity) bonds; $K_t (K_t^*)$ represents real net capital inflows to the home country (abroad); $S_t (S_t^*)$ represents the real domestic (foreign) private sector supply of long-term bonds; and $G_t (G_t^*)$ is the real net borrowing by the domestic (foreign) government.\(^6\) Furthermore, it is necessary that net capital inflows to the home country match net capital outflows from abroad:

$$K_t = -K_t^*.$$  

(9)

Using the loanable funds framework, equation (7) and (8) are linearised for the home country as follows (a similar set of equations is assumed to hold abroad):

$$D_t = a_0 + a_1i_t - a_2E_t\pi_{t+1} + a_3E_ty_{t+1} - a_4E_tr_{t+1} + e_t^d$$

(10)

$$S_t = b_0 - b_1i_t + b_2E_t\pi_{t+1} + b_3E_ty_{t+1} + b_4G_t + e_t^s.$$  

(11)

---

\(^5\) As stated above, it is expected that $\bar{r} = \bar{r}^*$.

\(^6\) Cebula (1997) uses a similar framework to analyze the impact of capital inflows on long-term interest rates in France.
The demand for a long-term bond $D_t$ is positively related to its holding-period return, and the supply of a long-term bond $S_t$ is negatively related to its holding-period return. The holding-period return on a long-term (zero-coupon) bond equals its yield to maturity $i_t$ at the beginning of the period minus the expected change in that yield during the bond holding period. Assuming for simplicity that the bond holding period is equal to one, the expected change in the yield during the holding period can be decomposed into two components: (1) the expected deviation of $\pi_t$ from its long-run value $\bar{\pi}_t = E_t \pi_{t+1}$ and (2) the expected deviation of $r_t$ from its long-run value $\bar{r}_t = E_t r_{t+1}$. Therefore, an increase in expected inflation causes the demand for bonds to decline and the supply of bonds to increase.\(^7\) An expected increase in the real short-term interest rate $r_t$ reduces the demand for long-term bonds as well. To the extent that expected output proxies’ wealth on the demand-for-bonds side, and the expected profitability of investment opportunities on the supply-of-bonds side, expected output is positively correlated with both the demand for bonds and the supply of bonds. As a result, the effect of expected output on the long-term nominal interest rate is ambiguous and becomes an empirical matter. Finally, the net borrowing of the government increases bonds supply.\(^8\)

Without loss of generality, $r_t$, $\pi_t$, $y_t$ and $G_t$ are assumed to follow stochastic processes such as:

$$r_t = \Gamma_r r_{t-1} + \epsilon_t^r. \quad (12)$$

$$\pi_t = \Gamma_\pi \pi_{t-1} + \epsilon_t^\pi. \quad (13)$$

$$y_t = y_{t-1} + \delta + \epsilon_t^y. \quad (14)$$

$$G_t = \Gamma_o G_{t-1} + \epsilon_t^G, \quad (15)$$

\(^7\) The usual argument is that an increase in expected inflation increases the expected return on physical assets (higher nominal capital gains) which decreases the relative expected return on bonds and thus causes the demand for bonds to fall.

\(^8\) For simplicity, we abstract from risk or liquidity considerations in equation (10).
where $\Gamma$'s are auto-regressive coefficients such that $0 < \Gamma < 1$, and $\delta$ is a constant rate of growth. All $\varepsilon_i^*$ are white noise with zero mean and variance $\sigma_e^2 < \infty$. We assume identical processes for $r_i^*$, $y_i^*$, $\pi_i^*$, and $G_i$.

Using equation (7) - (14) for the home country and the corresponding equations for abroad yields the long-run interest rates:

$$i_t = \frac{1}{a_1 + b_1} \left[ -a_0 + b_0 + (a_2 + b_2)\Gamma_\pi \pi_t - (a_3 - b_3)(y_t + \delta) + a_4 \Gamma r_t + b_4 \Gamma G_t + w_t \right].$$

(16)

$$i_t^* = \frac{1}{a_1^* + b_1^*} \left[ -a_0^* + b_0^* + (a_2^* + b_2^*)\Gamma_\pi^* \pi_t^* - (a_3^* - b_3^*)(y_t^* + \delta^*) + a_4^* \Gamma r_t^* + b_4^* \Gamma G_t^* + w_t^* \right].$$

(17)

where $w_t = \varepsilon_t^* - \varepsilon_t^d$ and $w_t^* = \varepsilon_t^* - \varepsilon_t^d^*$.

Solving equation (6), the exchange rate becomes:

$$e_t = \frac{\Gamma_\pi \pi_t - \Gamma_\pi^* \pi_t^*}{1 - \Gamma_\pi}.$$  

(18)

Substituting equations (16) and (17) into (18), we obtain, after some algebra:

$$e_t = C + A(a_3 - b_3)(y_t + \delta) - B(a_3^* - b_3^*)(y_t^* + \delta^*) + A(a_1 + b_1)\Gamma r_t - B(a_1^* + b_1^*)\Gamma r_t^* - Aa_4 \Gamma r_t +$$

$$+ Ba_4 \Gamma G_t + Ab_4 \Gamma G_t^* - Bb_4 \Gamma G_t^* - Aw_t + Bw_t^*.$$

(19)

where:

$$A \equiv [(1 - \Gamma_\pi)(a_2 + b_2)]^{-1}, \ B \equiv [(1 - \Gamma_\pi^*)(a_2^* + b_2^*)]^{-1}, \text{ and } C \equiv A(a_0 - b_0) - B(a_0^* - b_0^*).$$

---

9 See Barr and Campbell (1997) for a similar assumption about the stochastic processes followed by the real interest rate and the inflation rate. Note that given the low power of available unit root tests, it may not be possible in practice to statistically distinguish between values of the $\Gamma$'s near 1 and values of the $\Gamma$'s equal to 1. We address this issue econometrically in the following section.
The long-run behavior of the nominal exchange rate can be obtained from equation (19) after recalling that in the long run, PPP holds\(^\text{10}\) and \(r = r^*_i\):

\[
e_i = C + A(a_i + b_i)y_i - B(a_i^* + b_i^*)y_i^* + \chi_i,
\]

(20)

where \(\chi_i = AG_i - BG_i^* - Aw_i + Bw_i^*\).

The model predicts \(A\) and \(B\) to be < 1 in magnitude. Thus, an increase in the long-term nominal interest rate at home—ceteris paribus—will appreciate the domestic currency. This is consistent with the idea that an increase in the long-run interest rate differential induces capital inflows, which appreciate the domestic currency. In the model (equation 16), an increase in the long-term interest rate differential may be due to a persistent increase in the real interest rate in the home country (Dornbusch suggested the same relationship but involved short-term interest rates), or may be due to a persistent increase in inflation in the home country, as suggested by Frenkel. Whatever the reason, the home currency will appreciate in the long run, as suggested by Dornbusch, rather that depreciate, as suggested by Frenkel.

III. EMPIRICAL EVIDENCE

We use data from the International Monetary Fund’s *International Financial Statistics* (IFS). The exchange rates are the DM-USD, GBP-USD, YEN-USD, and the CAD-USD at the end of each month.\(^\text{11}\) We excluded the U.S. dollar-French Franc and the U.S. dollar-Italian Lire from the sample because France and Italy maintained some significant capital control measures until late 1980s, which may bias the tests given the long-run assumptions made in the model. Britain abolished exchange controls in 1979 (Artis and Taylor, 1989). Japan abolished its exchange control policies in 1980 (Fukao, 1990).

The long-term interest rate is the 10-year government bond yield rate (IFS, line 61, where e.g., six percent is 6.0 instead of 0.06). We use monthly data from January 1980 to July 1997. The period of the early float did not represent a clean-float. It included numerous episodes of government intervention in foreign exchange markets, and most importantly, 1980 is considered far enough in time from the initial conditions of the new exchange rate regime.

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\(^\text{10}\) Note that productivity growth in the home country and in the rest of the world are equal (i.e., \(\delta = \delta^*\)) if the change in the real exchange rate is zero in the long run (Harrod-Balassa-Samuelson hypothesis). See e.g., Obstfeld and Rogoff (1996).

\(^\text{11}\) Recall that exchange rates are measured in natural logarithms.
We plot the data in figures 1–8. Figures 1 to 4 plot the German, British, Japanese, Canadian interest rates and the U.S. interest rate. The U.S.-German and the U.S.-British interest rate differentials are larger in magnitude than the U.S.-Japanese and the U.S.-Canadian interest rate differentials. Figures 5 to 8 plot the exchange rates on the LHS axis and the interest rate differentials on the RHS axis. The interest rate differential is defined as the 10-year government bond yield of the US minus that of the foreign country (i.e., Germany, Britain, Japan, and Canada). Visually, there seems to be a strong relationship between the DM-USD and the GBP-USD exchange rates and the corresponding interest rate differentials rather than between the YEN-USD and the CAD-USD exchange rates and the corresponding interest rate differentials. It will be shown that the results of all different statistical tests we use in the rest of this paper are consistent with the visual patterns in figures 1–8.

A. Unit Roots and Cointegration

In testing for unit roots, cointegration and common cycles our strategy is to use different tests. A decision is then made based on whether the results of these various tests converge or not. Thus, we will be seeking consensus among different tests. For example, when different tests for unit roots move in one direction, e.g., indicating a unit root, we are a little more confident in the results than when the tests diverge.

Nominal exchange rates as well as nominal interest rates are asset prices. There seems to be agreement among researchers that these variables, during the post Bretton-Woods era, may contain unit roots. The literature on unit roots and cointegration is vast and it will not be reviewed here. Suffice it to say that there is a valid concern among economists about the appropriateness of the tests for unit roots and their power against stationary alternatives. The choice of a particular testing methodology is not straightforward. Ultimately, we may not be able to determine whether a particular time series contains a unit root or not. It seems inevitable, however, that we have to make a choice. We use two popular methods to test for unit roots: the ADF test (Dickey and Fuller, 1979, 1981, and Said and Dickey, 1984), and the Phillips-Perron test (Phillips (1987) and Perron (1988)). We test for unit roots from January 1980 to July 1997.

We report the results for unit root tests in table 1. The two tests we consider in this paper seem to agree that both time series may be approximately unit roots during the post Bretton-Woods era. The tests indicate that the exchange rates are unit roots. However, the ADF and the Phillips-Perron tests disagree twice regarding the interest rates differentials.

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12 The choice of the lag structure always has been an issue. The objective of the lags is to remove serial correlation. With this objective in mind, we look at different criteria to choose the lags. For example, the lag order is set as the highest significant lag order—using an approximate 95 percent confidence interval—from either the autocorrelation function or the partial autocorrelation function of the first-differenced series. The maximum lag order is the square root of the sample size. We also test backward using F tests, and look at Schwarz IC and AIC. Unnecessary lags are eliminated. Every time, we check for serial correlation using the Bartlett - Kolmogrov - Smirnov test for white noise.
The ADF test finds the YEN-USD 10-year interest rate differential to be stationary, while the Phillips-Perron test finds a unit root in it. Then the ADF test finds the CAD-USD 10-year interest rate differential to be a unit root process, while the Phillips-Perron test finds it to be stationary. These results seem to be consistent with our judgements on figures 1–8. The YEN-USD and the CAD-USD interest rate differentials are so small in magnitude they are likely to be stationary (fluctuate around zero). The DM-USD and the GBP-USD interest rate differentials on the other hand seem to diverge for prolonged periods of time; they are likely to have trends.

To test the null hypothesis that the exchange rate is not cointegrated with the interest rate differential, we first try the Engle-Granger (1987) and the Engle and Yoo (1987) procedures. We test the null hypothesis that the residuals from the OLS regressions of the nominal exchange rate on a constant, or on a constant and a trend, and the interest rate differential are unit roots. Typically, the ADF test is used as recommended by Engle and Granger’s original paper, but we also use the Phillips-Perron-Phillips-Ouliaris test to test the same hypothesis. Given our sample, both tests seem to indicate that there is no statistical evidence of cointegration in all pairs of data. We are not aware of any evidence of cointegration between these two variables in the literature based on the Engle-Granger test.

We also use the Engle-Granger procedure to test for no-cointegration between real exchange rates and the corresponding real interest rate differentials. We fail to reject the null of no-cointegration. Therefore, we confirm the findings of Meese and Rogoff (1988), Edison and Pauls (1993), and Kawai and Ohara (1997).

We also test for no-cointegration between nominal exchange rates and interest rate differentials using a second approach. Gonzalo (1994) compares five different residual-based tests for cointegration including the Engle-Granger test. Among them, he recommends using the Johansen-Juselius (1990) method. This test has been very popular in the literature, but just like any other unit root test, it is highly criticised for its lack of power in finite samples, and—among other problems—its sensitivity to the choice of the lag length. We use this test and report the results in table 2. We correct the critical values for small sample using the Cheung and Lai (1993) approach. We use two different ways to evaluate the lag length. We fit a general lag model. Then we eliminate the unnecessary lags by testing backward using F tests and the SC criterion. The residuals of

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13 The same approach to selecting the lag structure in the tests for unit roots in individual series is used here.

14 We do not report the results but they are available upon request.

15 We start with a lag structure similar to that we adopted in the Engle-Granger test. We make our choice based on the multivariate serial correlation tests recommended by Johansen and reported in CATS.
the models are carefully checked for whiteness each time using multivariate Ljung-Box, LM1 and LM4 tests. At the 95% level, we find evidence of cointegration between the nominal exchange rates and the nominal interest rate differentials based on the λ max statistics for all pairs of currencies and interest rate differentials. In the cases of the DM-USD and the GBP-USD the statistics are very significant and the λ trace is also highly significant. However, in the cases of the YEN-USD and the CAD-USD, the trace statistics can only reject the null hypothesis of zero cointegration vectors at the 90% level.\textsuperscript{16}

Strictly speaking, results for the pairs YEN-USD and CAD-USD are inconclusive. This is consistent with the unit root tests reported earlier, i.e., exchange rates clearly contain a unit root while the evidence on unit roots for the corresponding interest rate differentials is instead mixed. These results are also consistent with figures 1–4. The movements of the U.S. interest rate are close to the movements of the Japanese interest rates and very close to the Canadian interest rate. Thus, the differentials are small and may be stationary. If they are really stationary, we will not expect tests for cointegration between an I(1) variable and an I(0) variable to be meaningful.

B. The Long-Run Elasticities of the Model

We estimate the long-run elasticities of the model using a third method. We estimate the reduced-form equation (20) by the nonlinear dynamic least squares of Phillips and Loretan (1991). They show that this single-equation technique is asymptotically equivalent to a maximum likelihood on a full system of equations under Gaussian assumptions. This technique provides estimators that are statistically efficient, and whose t-ratios can be used for inference in the usual way. Most importantly, this method takes into account both the serial correlation of the errors and the endogeneity of the regressors that are present when there is a cointegration relationship. We estimate two regressions, the restricted and the unrestricted given by equation (21) and (22) respectively:

\[
e_t = a + b(i - i^*)_t + \sum_{i=-k}^{k} \delta_i \Delta(i - i^*)_{t-i} + \rho [e_{t-1} - a - b(i - i^*)_{t-1}] + \nu_t,
\]

(21)

\[
e_t = a + b_1 i_t + b_2 i^*_t + \sum_{i=-k}^{k} \varphi_i \Delta i_t + \sum_{i=-k}^{k} \theta_i \Delta i^*_t + \rho [e_{t-1} - a - b_1 i_{t-1} - b_2 i^*_{t-1}] + \eta_t,
\]

(22)

\textsuperscript{16} The \( \lambda_{\max} \) statistic has a sharper alternative hypothesis than the \( \lambda_{\max} \) statistic. In case of conflict, the former is to be preferred to the latter.
where $e_i$ is the nominal exchange rate, $i_i$ and $i_r$ are the nominal 10-year government bond yields for the U.S. and other countries respectively, $\nu_i$ and $\eta_i$ are Gaussian error terms with classic properties. We are mostly interested in $\hat{a}$, $\hat{b}$ in the restricted regression and $\hat{b}_1$ and $\hat{b}_2$ in the unrestricted regression, because they are the parameter estimates of the long-run relationship between the nominal exchange rate and the nominal 10-year bond yield differential.

Results of the nonlinear least square estimation for all pair of currencies are reported in table 3. The parameters of the restricted regression are significant except the slope parameter, $\hat{b}$, in the CAD-USD regression. The slope parameter, $\hat{b}$, in the YEN-USD regression may be significant at the 10% level only. We report the t-ratios and the p-values. These results confirm the Johansen-Juselius test results of table 2. Recall that the YEN-USD and the CAD-USD exchange rate-interest differentials were cointegrated only at the 90% level using the $\lambda$trace statistics. They are also consistent with figures 1–8. The various diagnostics of the residuals indicate that the residuals are white noise, serially uncorrelated, and homoscedastic. The magnitudes of the parameters are consistent with the model’s prediction. The magnitudes of $\hat{b}$ are almost identical for the DM-USD, GBP-USD, and CAD-USD cases (i.e., 0.12, 0.10 and 0.10 respectively). The magnitudes of the slope coefficients are less than unity and have the signs predicted by the model. In the long run, an increase in the nominal interest rate differential appreciates the home currency. As expected from the model, an upward deviation of inflation from its previous trend in the home country, or an increase of the real interest rate above its previous trend in the home country, increase the interest rate differential and appreciate the domestic currency.

The unrestricted regression produces similar results. However, the restrictions (7 restrictions on the levels and the dynamics) only hold in the DM-USD case as indicated by the F test. Also, we report a chi-square test of the restriction that the long-run coefficient on the US interest rate is equal in magnitude (with opposite sign) to the foreign interest rate coefficient. We cannot reject the hypothesis that the long-run coefficients are equal in magnitude in the cases of DM-USD and CAD-USD.

Finally, we split the sample in two periods, the 1980s and the 1990s. We re-estimate the same regressions. Evidence of instability is found in the sense that goodness of fit deteriorates. The relationship between the exchange rate and the interest rate differential breaks down during the 1990s. This is true in all pairs except the DM-USD.

---

17 We start with a lag-lead structure similar to that in Johansen. We find a three lag-lead structure to be sufficient to eliminate the serial correlation. However, because we are conscious about Phillips and Loretan warning of over-fitting, we start reducing the number of leads by one. Every time, we check the serial correlation, the parameter estimates, and their significance. We find that a structure of three lags and two leads gives the same results as a three lag-lead structure. A further reduction in the leads introduces some serial correlation.
We provide a graphical presentation of the long-run relationships. We compare the long-run correlation between the exchange rates and the corresponding interest rate differentials. For each pair, we compute $f_i = \hat{a} + \hat{b}(i - i^*)$. Then we plot the exchange rates and $f_i$. The plots are shown in figures 9–12. The correlation between the exchange rates and $f_i$ are visually clear in the DM-USD and the GBP-USD cases. The correlation values between the exchange rates and the corresponding $f_i$ are 0.80 ($R^2 = 0.64$) and 0.40 ($R^2 = 0.16$). Interestingly, the model captures quite well the appreciation undergone by the U.S. dollar until the mid-1980s. Figures 11 and 12 for YEN-USD and CAD-USD are much less impressive. The correlation values are low, 0.28 ($R^2 = 0.08$) and 0.22 ($R^2 = 0.05$) respectively. This is because the estimates of $\hat{b}$ were not statistically significant at the 95 percent confidence level in those cases.

We conclude that the DM-USD, GBP-USD, YEN-USD and CAD-USD contain unit roots. The interest rate differentials between the U.S. and Germany and the U.S. and Britain contain unit roots, but it is unclear whether the interest rate differentials between the U.S. and Japan and the U.S. and Canada contain unit roots. Consequently, we found the DM-USD and the GBP-USD and the corresponding interest rate differentials to be cointegrated. In contrast, the cointegration relationships between the YEN-USD, the CAD-USD and the corresponding interest rate differentials are not robust.

Finally, we use the Vahid-Engle (1993) method to test for common cycles for all pairs of currencies and interest rate differentials although technically the test is conditional on the presence of cointegration. Therefore, results for the pairs YEN-USD and CAD-USD are technically correct only if we accept the unit root tests that find a unit root in the corresponding interest rate differentials and the cointegration tests based on the $\lambda$ max statistics. Results are reported in table 4. In the DM-USD and the YEN-USD cases, we cannot reject the null hypothesis that there is at least one common cycle. In the pair CAD-USD the evidence is weaker. Finally, we clearly reject the null hypothesis in the pair GBP-USD.

C. The Performance of the Model and its Assumptions

Why do exchange rates and interest rate differentials for the pairs DM-USD and GBP-USD have a common trend while they most likely do not have a common trend for the pairs YEN-USD and CAD-USD? We suggest that a possible explanation may be related to two of the key assumptions of the model: (1) long-run PPP, and (2) a high degree of capital mobility as represented by the assumption of equality between domestic and foreign real interest rates. If these assumptions do not hold, then it is natural to look into the behavior of real exchange rates to explain the relatively poorer performance of the model in the pairs YEN-USD and CAD-USD.

Figures 13–16 show the stochastic trends of the real exchange rates of all pairs of countries analysed in this study. These trends have been calculated using the Approximate Band-Pass filter of Baxter and King (1995), and are defined as the
components of the real exchange rate series that have frequencies longer than 96 months (this definition follows Baxter's, 1994). We extended the sample back to January 1975 to gain further perspective and offset the three years of data points lost by using the filter.

Figures 13 and 14 seem to indicate that the assumption of "long-run PPP" is acceptable for the pairs DM-USD and GBP-USD during the sample period. Figures 15 and 16 seem to indicate the opposite for the pairs YEN-USD and CAD-USD. A full analysis of the reasons behind the different behavior of those real exchange rates is beyond the scope of this paper. However, one may speculate that as the U.S. and Canada are affected by persistent asymmetric shocks, it is possible that the real exchange rate meanders for far longer than the length of the sample used in this study. Persistent terms of trade shocks, for instance, are much more important for Canada than for the U.S. Amato and Van Norden (1995) show that the CAD-USD real exchange rate and the terms of trade are correlated and cointegrated. They also show that the "causality" runs from the terms of trade to the real exchange rate. Similarly, in the case of the pair YEN-USD, it could be that the assumption of perfect capital mobility is not strictly appropriate for Japan given the weight of tradition in the conduct of financial activities in the country, and that the domestic financial market liberalization occurred only recently (Viner, 1988). Alternatively, a positive differential in productivity growth between Japan and the U.S. may explain the downward slope in the real exchange rate between the two countries and why PPP does not hold.

IV. SUMMARY

In this paper, we provide a model of the long-run behavior of the nominal exchange rate based on the assumption that interest rates are determined in the bond market rather than in the money market. We test the model by using monthly data for the DM-USD, the GBP-USD, the YEN-USD, and the CAD-USD from January 1980 to July 1997.

We use two different tests to test the null hypothesis that the nominal exchange rates and the long-term nominal interest rate differentials are unit root processes. We confirm previous findings that the null hypothesis that the nominal exchange rate contains a unit root cannot be rejected. However, it is still unclear whether all interest rate differentials are unit root processes. The interest rate differentials between the U.S. and Germany and the U.S. and Britain contain a unit root, but it is unclear whether the interest rate differentials between the U.S. and Japan and the U.S. and Canada contain unit roots. Consequently, we find that the DM-USD and the GBP-USD exchange rates and the corresponding interest rate differentials are cointegrated, but the cointegration relationships between the YEN-USD, the CAD-USD exchange rates and the corresponding interest rate differentials are ambiguous. The finding of cointegration is sensitive to the methodology used. The Engle-Granger type test cannot reject the null hypothesis that the exchange rates and the interest rate differentials are not cointegrated for all pairs of countries. However, cointegration can be found using the Johansen-Juselius (1990) [and the Phillips-Loretan (1991)] method[s].
On the basis that nominal exchange rates and long-term nominal interest rate differentials are cointegrated, we investigate the presence of common cycles. Using the Vahid-Engle (1993) method, we find strong statistical evidence of common cycles between nominal exchange rates and interest rate differentials in the cases of DM-USD and YEN-USD, weaker evidence in the case of CAD-USD, and no evidence in the case of GBP-USD.

There seems to be strong statistical evidence, at least for the pairs DM-USD and GBP-USD, that an increase in the interest rate in the home country induces capital inflows and appreciates the domestic currency. Consistent with the model that assumes that interest rates are determined in the bond market, an increase in the long-term nominal interest rate differential, either due to an increase in long-run inflation in the home country, or due to a persistent increase in the real interest rate in the home country, is consistent with an appreciation of the home currency. Referring to a long-standing debate, this result is in contrast to Frenkel (1976), Musa (1976, 1979), and Bilson (1978, 1979), and in agreement with Dornbusch (1976) and Frankel (1979).

We speculate on the possible reasons for the failure of the model in the CAD-USD and YEN-USD pairs by making reference to the assumptions of our model. Because PPP does not seem to hold in the CAD-USD and YEN-USD pairs, the assumption of PPP may be most important in explaining the failure of uncovered interest rate parity. To be able to better pin down the possible transmission mechanism, however, we are working on the estimation of the relationship between nominal exchange rates and short-term nominal interest rate differentials. Our preliminary results suggest that for the same pair of countries, an increase in the short-term interest rate differential, causes a capital outflow, and depreciates the domestic currency. This is in contrast to Dornbusch (1976) and Frankel (1979), and in agreement with Frenkel (1976), Musa (1976, 1979), and Bilson (1978, 1979). It may be, as suggested by McCallum (1994), that short-term interest rate smoothing by central banks has to be accounted for to obtain a correct specification of the co-movements of exchange rates and interest rate differentials along the whole term structure.
V. REFERENCES


Table 1
Tests for Unit Roots in Nominal Exchange Rates
and Nominal Interest Rate Differentials

\[ \Delta y_t = \text{constant} + \text{trend} + \rho y_{t-1} + \sum_{i=1}^{k} \delta_i \Delta y_{t-i} + \epsilon_t \]

1980:1 to 1997:7

<table>
<thead>
<tr>
<th></th>
<th>Lags</th>
<th>ADF</th>
<th>Phillips – Perron z</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM-USD</td>
<td>13</td>
<td>-2.5518</td>
<td>-10.299</td>
</tr>
<tr>
<td>GBP-USD</td>
<td>14</td>
<td>-2.9817</td>
<td>-8.1126</td>
</tr>
<tr>
<td>YEN-USD</td>
<td>0</td>
<td>-1.4052</td>
<td>-5.9848</td>
</tr>
<tr>
<td>US/Canadian</td>
<td>11</td>
<td>-2.2438</td>
<td>-4.8073</td>
</tr>
<tr>
<td>US-1-Germany</td>
<td>2</td>
<td>-1.8237</td>
<td>-10.141</td>
</tr>
<tr>
<td>US-1-Britain</td>
<td>2</td>
<td>-2.7944</td>
<td>-18.384</td>
</tr>
<tr>
<td>US-1-Japan</td>
<td>6</td>
<td>-3.7273</td>
<td>-11.929</td>
</tr>
<tr>
<td>US-1-Canada</td>
<td>7</td>
<td>-1.6701</td>
<td>-29.474*</td>
</tr>
</tbody>
</table>

The exchange rate is measured in natural logarithms. Lags are the same across all three tests. Interest rates are the 10-year bond rates.

ADF is the ADF statistic for \( H_0 \): unit root. The 5% critical value is -3.41.

Phillips-Perron is the Phillips-Perron statistic for \( H_0 \): unit root. The 5% critical value is -21.7. We do not report the statistics for joint hypothesis tests for \( \rho \) and the constant, \( \rho \) and trend, and \( \rho \), constant and trend.
Table 2
The Johansen-Juselius Maximum Likelihood Test for Cointegration
1980:1 to 1997:7

<table>
<thead>
<tr>
<th></th>
<th>Eigen Values</th>
<th>λ max</th>
<th>Trace</th>
<th>Ho: r</th>
<th>p-r</th>
<th>λ max 95%</th>
<th>Trace 95%</th>
<th>lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM-USD</td>
<td>0.0012</td>
<td>0.2033</td>
<td>0.2049</td>
<td>0</td>
<td>2</td>
<td>12.33</td>
<td>16.51</td>
<td>1</td>
</tr>
<tr>
<td>Residuals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L-B ~ $\chi^2_{126}$</td>
<td>115.656</td>
<td>(0.74)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM1 ~ $\chi^2_4$</td>
<td>2.209</td>
<td>(0.70)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM4 ~ $\chi^2_4$</td>
<td>0.343</td>
<td>(0.99)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GBP-USD</td>
<td>0.1542</td>
<td>21.77</td>
<td>24.87</td>
<td>0</td>
<td>2</td>
<td>12.45</td>
<td>16.67</td>
<td>2</td>
</tr>
<tr>
<td>Residuals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L-B ~ $\chi^2_{122}$</td>
<td>102.446</td>
<td>(0.90)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM1 ~ $\chi^2_4$</td>
<td>7.106</td>
<td>(0.13)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM4 ~ $\chi^2_4$</td>
<td>3.672</td>
<td>(0.45)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>YEN-USD</td>
<td>0.0779</td>
<td>16.46</td>
<td>17.28</td>
<td>0</td>
<td>2</td>
<td>13.21</td>
<td>17.69</td>
<td>8</td>
</tr>
<tr>
<td>Residuals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L-B ~ $\chi^2_{170}$</td>
<td>180</td>
<td>(0.29)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM1 ~ $\chi^2_4$</td>
<td>4.580</td>
<td>(0.33)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM4 ~ $\chi^2_4$</td>
<td>3.424</td>
<td>(0.49)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAD-USD</td>
<td>0.0700</td>
<td>15.17</td>
<td>16.42</td>
<td>0</td>
<td>2</td>
<td>12.45</td>
<td>16.67</td>
<td>2</td>
</tr>
<tr>
<td>Residuals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L-B ~ $\chi^2_{202}$</td>
<td>214.305</td>
<td>(0.26)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM1 ~ $\chi^2_4$</td>
<td>10.395</td>
<td>(0.03)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM4 ~ $\chi^2_4$</td>
<td>6.869</td>
<td>(0.14)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

r is the number of cointegrated vectors.
p is the number of variables.
The 95% critical values corrected for small samples using Cheung and Lai (1993) are also used to evaluate the results.
The models include drift terms. **L-B** is a multivariate Ljung-Box test based on estimated auto cross-correlation of the first T/4 lags. The **LM1** and **LM4** are the Langrange multiplier tests.
Table 3
The Phillips-Loretan Nonlinear Dynamic Least Square Estimator
1980:1 to 1997:7

<table>
<thead>
<tr>
<th>Restricted Model</th>
<th>Unrestricted Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_t = a + b(i-f_t) + \sum_{i-t}^1 \delta_i (i-f)<em>{t-i} + \rho { e</em>{t-i} - a - b(i-f)_{t-i} } + \nu_t )</td>
<td>( e_t = a + b_1 (i-f_t) + b_2 (i-f_t)^2 + \sum_{i-t}^1 \delta_i (i-f)<em>{t-i} + \sum</em>{i-t}^1 \theta_i (i-f)<em>{t-i} + \rho { e</em>{t-i} - a - b_1 (i-f_t) - b_2 (i-f_t)^2 } + \nu_t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>DM-USD</th>
<th>GBP-USD</th>
<th>YEN-USD</th>
<th>CAD-USD</th>
<th>DM-USD</th>
<th>GBP-USD</th>
<th>YEN-USD</th>
<th>CAD-USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>0.34(8.35)</td>
<td>-0.36(-7.07)</td>
<td>3.67(4.78)</td>
<td>0.42(3.65)</td>
<td>0.09(0.49)</td>
<td>-0.12(-1.10)</td>
<td>3.77(21.5)</td>
<td>0.55(2.75)</td>
</tr>
<tr>
<td></td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.0002]</td>
<td>[0.6174]</td>
<td>[0.2690]</td>
<td>[0.0001]</td>
<td>[0.0058]</td>
</tr>
<tr>
<td>( b )</td>
<td>0.12(5.60)</td>
<td>0.10(3.04)</td>
<td>0.31(1.76)</td>
<td>0.10(1.50)</td>
<td>0.93(51.8)</td>
<td>0.90(33.6)</td>
<td>0.95(61.2)</td>
<td>0.97(68.8)</td>
</tr>
<tr>
<td></td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.0776]</td>
<td>[0.1350]</td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.0001]</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.94(51.3)</td>
<td>0.93(50.1)</td>
<td>0.98(132)</td>
<td>0.97(79.8)</td>
<td>0.11(5.80)</td>
<td>0.09(3.38)</td>
<td>0.15(4.43)</td>
<td>0.09(1.50)</td>
</tr>
<tr>
<td></td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.0001]</td>
<td>[0.1389]</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.06(-1.83)</td>
<td>-0.11(-4.88)</td>
<td>-0.03(-0.66)</td>
<td>-0.10(-1.62)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.0665]</td>
<td>[0.0001]</td>
<td>[0.5087]</td>
<td>[0.1052]</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( Se )</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>DW</td>
<td>1.85</td>
<td>1.77</td>
<td>1.83</td>
<td>2.01</td>
<td>1.90</td>
<td>1.80</td>
<td>1.87</td>
<td>2.11</td>
</tr>
<tr>
<td>Breusch-Pagan</td>
<td>6.10</td>
<td>9.53</td>
<td>7.75</td>
<td>7.16</td>
<td>17.88</td>
<td>13.41</td>
<td>7.69</td>
<td>11.79</td>
</tr>
<tr>
<td>ARCH (12)</td>
<td>10.35</td>
<td>15.52</td>
<td>12.78</td>
<td>8.68</td>
<td>12.00</td>
<td>21.86</td>
<td>13.91</td>
<td>13.31</td>
</tr>
<tr>
<td>Bartlett's-Kolmogrov-Smirnov</td>
<td>0.0611</td>
<td>0.0982</td>
<td>0.0692</td>
<td>0.0659</td>
<td>0.0583</td>
<td>0.0810</td>
<td>0.0699</td>
<td>0.0714</td>
</tr>
<tr>
<td>F</td>
<td>2.048</td>
<td>4.68*</td>
<td>6.79*</td>
<td>10.25*</td>
<td>0.0456</td>
<td>0.0390</td>
<td>0.3062</td>
<td>0.3469</td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>3.77[0.0518]</td>
<td>5.611[0.0178]</td>
<td>35.32[0.0001]</td>
<td>0.77[0.3772]</td>
<td>10.78[0.001]</td>
<td>3.05[0.081]</td>
<td>4.27[0.039]</td>
<td>0.03[0.856]</td>
</tr>
</tbody>
</table>

Se: Standard error of the estimates. The t-ratios are in parentheses and P-values are in square brackets. Bartlett's-Kolmogrov-Smirnov 5% critical value is 0.1587. F tests the restriction on the long-run parameters and the dynamics. \( \chi^2 \) is to test the restrictions on the long-run parameters only.
Table 4
The Vahid-Engle Test for Common Cycles
1980:1 to 1997:9

<table>
<thead>
<tr>
<th></th>
<th>Squared Canonical Correlation</th>
<th>k</th>
<th>$C(k,s) \sim \chi^2$</th>
<th>H₀: s</th>
<th>Degrees of Freedom</th>
<th>P-value</th>
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</thead>
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<tr>
<td>DM-USD</td>
<td>0.012417</td>
<td>1</td>
<td>2.63</td>
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<td>0.2685</td>
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<td></td>
<td>0.078758</td>
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<td>19.93*</td>
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<td>0.0188</td>
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<td>22.98*</td>
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$C(k,s) = -(T - k - 1) \sum_{i=1}^{s} \ln (1 - \lambda_i^2)$, where $T$ is the sample size, $k$ is the lag length, and $\lambda$ is the canonical correlation. $s$ is the smallest squared canonical correlation between the exchange rate and the interest rate differential. The degrees of freedom are $s^2 - spk + sr - sp$, where $r$ is the number of cointegrated vectors (i.e., 1), and $p$ is the dimension of the system (i.e., 2). An asterisk means significant at the 5% level.
Figure 3: US & Japan 10-Year Government Bond Yield
Figure 5: Exchange Rate & Interest Rate Differential
DM-USD
Figure 9: Nominal Exchange Rate & a+b(i-i*)
DM-USD
Figure 12: Nominal Exchange Rate & $a+b(i-i^*)$
CAD-USD