Can a Shorter Workweek Induce Higher Employment?  
Mandatory Reductions in the Workweek and Employment Subsidies

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Abstract

A reduction in the legal workweek may induce a degree of downward wage flexibility, while an employment subsidy to firms accommodates downward wage rigidity. It may be possible, therefore, to increase employment with a policy that combines a reduction in the workweek with an employment subsidy. In general, however, the long-run employment outcome is ambiguous, and a decline in output cannot be ruled out. More direct policy measures whose impact can be assessed with greater certainty—in particular, removing structural rigidities in the labor market—should be given priority to decrease long-term unemployment.

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I. INTRODUCTION

Mandatory reductions in the legal workweek to stimulate job creation have recently come back into focus in some European countries. This policy aims at lowering the high unemployment rate by trading-off fewer work hours per worker for a greater number of workers employed.\(^2\)

This paper examines the impact on employment of the combination policy of a mandatory reduction in the workweek accompanied with an employment subsidy. The analytical framework is a simple comparative static model of demand for workers and demand for work hours, including overtime. The analysis of the representative worker's optimal income and leisure choice and wage determination under the monopoly union model sheds some light on the possible impact of this policy on wage, supply of hours, and labor market equilibrium.


A common feature of these studies is that, in general, the impact of reductions in the standard work hours on labor demand is ambiguous and conditional on worker (union) responses.\(^3\) The broad fundamental lesson that emerges from the literature is that the first-order effect of a cut in the standard work hours is a decline in employment, as measured by the number of employed persons. This result obtains because a reduction in the standard hours

\(^2\)For example, France has already embarked on a legislative process to shorten the legal workweek from 39 to 35 hours. The French initiative provides for a subsidy for the firms that negotiate a reduction in work time and create jobs (or preserve jobs at risk); see Zanello (1998). Similar policies have been effected in Belgium and the legal workweek has been shortened in Germany through union agreements; see Rubin and Richardson (1977, Chapter 6) and Hunt (1999). Such proposals are under discussion in Italy.

\(^3\)Hoel (1986) shows that, although there may be some scope for increasing employment through a reduction in standard hours in a two-sector economy with fixed wages, such an outcome is by no means certain. In a monopoly union model of wage determination, Calmfors (1985) finds that wage and employment effects are broadly unclear. These findings are supported by Drèze and Modigliani (1981) and Drèze (1991).
increases the marginal cost of hiring an additional worker relative to the marginal cost of employing an additional hour of overtime work. An increase in employment is possible through second-order effects induced by a reduction in the standard work hours; for example, productivity gains from a reorganization of work process may help mitigate the negative impact of a reduction in the standard hours on employment. However, a generic case for such an outcome cannot be made without restrictions on the production function.  

The broad consensus among different studies is that, if the reduction in hours is accompanied with policies that serve to counteract the negative impact of this policy on employment, then employment may be increased. As elaborated by Hart (1987) and Drèze (1991), among others, incentives provided by employment subsidies to firms, intended to lower the quasi-fixed costs of additional hiring, may more than offset the increase in the cost of hiring due to the reduction in the standard work hours and, thus, have a favorable impact on employment. These basic results are robust to both comparative static and dynamic analysis, and, incorporating hours supply and wage responses to a cut in the standard hours does not appreciably alter the basic results.  

The thrust of the analysis of hours supply responses is the impact of a cut in the standard hours on wages, or, more broadly, on worker income. Maintaining the worker income level, in particular, maintaining at least the income level of the minimum wage earners, is an important consideration for the workers and unions, as well as the policy maker. This paper argues that it may be feasible to induce an increase in employment without a decline in worker income through the combination policy of lowering the standard work hours along with providing employment subsidies. However, for a given generalized subsidy that applies across-the-board, this result may not be applicable to the economy as a whole.

Maintaining worker income level is an overly restrictive policy objective, at least in the case of the workers who earn more than the minimum wage. As implied by the union models of wage determination, the focus may be on maintaining worker utility and not strictly the worker income level. The basic model of income-leisure choice shows that the utility level of the representative worker can be maintained at a lower income level than before the reduction in work hours, provided that the representative worker values leisure, and leisure and income are substitutes. As a result, wage as well as the total hours supplied by the representative

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4Hart (1987) provides some examples under different production function specifications. In a model with technical progress, FitzRoy (1988) shows that employment is likely to decline when the standard hours are lowered, unless there are countervailing employment gains from technical progress.

5See, for example, Booth and Schiantarelli (1987) for a dynamic model. A dynamic monopoly union model of wage setting and an efficiency wage model by the same authors (1988) confirms similar results. In contrast, Houpis (1993) argues that wages are likely to decline when the standard hours are lowered, resulting in higher employment.
worker may decline when the standard workweek is shortened. This result underscores that a cut in the standard workweek may induce a degree of downward wage flexibility, as workers trade-off more leisure for less income. However, under the monopoly union model of wage determination, the direction of change in wage in response to a cut in the standard hours is indeterminate. Nevertheless, with some downward wage flexibility, it may be possible for some firms and unions to renegotiate contracts with employment enhancing outcomes.

This paper argues that the degree of downward wage flexibility necessary to induce an increase in employment is less with an employment subsidy than without one. Thus, to some extent, an employment subsidy can accommodate for downward wage rigidity and improve the chances of increasing employment through a cut in the standard hours. However, this result is firm or industry specific because a generalized employment subsidy is not tailored specifically for a firm or industry to counteract the increase in labor costs resulting from the cut in the standard hours. For an exogenously given reduction in the standard hours and a generalized subsidy, the possible consequent decline in wages may not be adequate to induce an increase in employment in all firms or industries. Therefore, an increase in employment may not materialize for a significant number of firms or industries; in fact, overall employment may even decline. Finally, the paper also argues that the impact of this policy on capital and output levels remains ambiguous in general. Thus, the policy under consideration may have a detrimental effect on aggregate output, even if it induces an increase in overall employment. Evidence from industrial countries do not remove the ambiguities suggested by the theory.

In conclusion, a general case for lowering structural unemployment through a mandatory reduction in the legal workweek accompanied with an employment subsidy is uncorroborated by our model. This result is broadly supported by the previous literature. The policy implications are straightforward. To lower structural unemployment, first-order policies should involve measures whose impact can be assessed with greater certainty, such as those directly aimed at removing entrenched rigidities in the labor market.

Following this introduction, the impact of the above combination policies on employment is analyzed in the context of simple models of labor demand and hours supply in

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6These observations are supported by the empirical findings of Dagsvik, et al. (1988). These authors' simulations indicate that a reduction in the standard hours is likely to have a negative impact on the total supply of hours (especially among male employees). However, as also noted by FitzRoy (1988), if labor market becomes tighter and wages rise, some firms at the margin may exit markets and employment may be adversely affected.

7Booth and Schiantarelli (1987, 1988) and Houpis (1993) discuss some interesting cases of wage renegotiation in response to a cut in the standard hours (monopoly union models, efficiency wage models, cooperative Nash solution for wages). In an empirical investigation of union responses to hours of work, Earle and Pencavel (1990) observe that unionism tends to reduce full-time work hours.
Section II. Section III concludes. Some relevant background information is provided on the G7 labor markets in Appendix I. Selected technical issues relating to the analysis are discussed in Appendix II.

II. MAIN ANALYTICAL RESULTS

A. General Considerations

In this section, we present a simple comparative static model of labor demand in which total hours of work is determined endogenously, given the wage rate and other relevant parameters. The model incorporates capital but the capital accumulation process is not explicitly modeled, and the model is not a dynamic one. The analysis of the impact of a cut in the standard hours on capital stock sheds adequate light on the possible long term investment responses to this policy. As corroborated by the previous literature, the results that follow from comparative static models are robust to dynamic analysis. We choose a sufficiently general specification for the production function in order to highlight that firms subject to different technological constraints might not react uniformly to a given cut in the standard hours and a given generalized lump-sum subsidy. Thus, although it is possible that this policy may enhance employment in some sectors, its impact on overall employment is more difficult to assess.

Demand for labor input consists of two components: demand for workers, $N^D$, and demand for hours of work, $H^D$. With homogeneous labor, demand for labor input can be formulated as

$$L = L(N^D, H^D) ; L_N, L_H > 0.$$  \hspace{1cm} (1)

The relationship in (1) is not a simple one such as $L = N \cdot H$. The contribution of increases in $N$ and $H$ to total labor input (and, hence to output) is positive within the normal ranges of values for $N$ and $H$. Diminishing returns to $N$ and $H$ sets in outside the normal range of values. Given $H$, increasing $N$ results in a decline in the amount of capital per worker. Given $N$, increasing $H$ beyond a certain level results in fatigue. As the restriction on $L_N, L_H$ in (1) implies, the analysis will be conducted within the normal ranges of values for $N^D, H^D$.

\small

\footnote{For an explicit modeling of investment responses to a cut in standard hours in a dynamic context, see Bonatti (1998). While Bonatti's specifications for the production and utility functions are restricted to the Cobb-Douglas function, his results closely track ours.}

\footnote{For example, see Feldstein (1967), Rosen (1968), Ehrenberg (1971), Ehrenberg and Schuman (1982). For an introductory review of the related literature and analysis, see Ehrenberg and Smith (1988, 1996).}
In view of (1), the production function of a competitive firm can be expressed as

\[ Q = Q(L(N^D, H^D), K), \]

which, in general, can be rewritten as

\[ Q = Q(N^D, H^D, K), \] (2)

where \( Q_N = \frac{\partial Q}{\partial N} \), \( Q_H = \frac{\partial Q}{\partial H} \), \( Q_K = \frac{\partial Q}{\partial K} > 0 \) within the normal range of values for \( N^D \) and \( H^D \); \( H^D \) is equal to the sum of the exogenously determined (by law) regular or standard work hours, \( H_0 \), and the endogenously determined demand for overtime work hours (hours worked in excess of \( H_0 \)); and, \( K \) is the capital stock.\(^{10}\)

Typically, the wage paid for overtime hours is a legally determined multiple of the wage paid for regular hours. Thus, the cost of labor to the firm which employs \( N \) workers, each working \( H \) hours, can be expressed (in real terms) as

\[ C = N^D[wH_0 + w(H^D - H_0) + z]; m > 1, \] (3)

where \( w \) is the market wage, \( m \) is the exogenously determined overtime premium applied to the hours worked in excess of \( H_0 \), and, \( z \) is the quasi-fixed cost of employing an additional worker.

It is important to note that \( z \) is interpreted as a flow in (3). However, some items that qualify as fixed costs of hiring and firing (for example, search and training costs, severance pay, risks surrounding the quality of new hires) do not fit the description of a flow because such costs are incurred only once at the time of hiring and firing. As elaborated by Phelps (1994) in a dynamic model, such fixed costs need to be treated as stocks (the present value of flows of such costs over a time horizon). But some costs associated with hiring can be treated

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\(^{10}\)See, for example, Feldstein (1967), Ehrenberg and Smith (1988). For the specific assumptions concerning the production function, see Appendix II. In particular, the underlying assumption in (2) is that all three factors of production are substitutable in the long run. Alternative substitutability assumptions have been used in the literature. For example, in \( Q = Q(L(N^D, H^D), K) \), it may be assumed that the substitution between \( N^D \) and \( H^D \) is separable from the substitution between \( L \) and \( K \); see Hamermesh (1988); this alternative specification does not affect the results appreciably. Similar specifications have been considered by Hart (1987). If \( K \) were assumed constant, the results are not altered significantly.
as flows as in (3); for example, the firm’s regular contributions to unemployment insurance and social security for each period wages are paid.\footnote{We can express \( z \) as \( z = z_0 + z_w \), where \( z_0 \) represents fixed costs that are not wage-related (for example, work space for the worker) and \( z_w \) represents the wage-related costs (social security and unemployment insurance contributions by the employer). Parsing \( z \) into its components does not add appreciably to the results. The presence of \( z \) in the cost function ensures that the firm may prefer paying for the costlier overtime to hiring new workers; for more extensive arguments, see Ehrenberg and Smith (1988, Chapter 5). The employment subsidy in focus here is a marginal subsidy; for a comparative review of other types of employment subsidies, see Hart (1987, Chapter 9).}

As in the previous literature, the starting point of our analysis is an under-employment equilibrium. The monopoly union model indicates that an initial under-employment equilibrium is possible (see Section II.C.2 below). Along those lines, we assume that there are \( M \) identical workers in the labor force, and \( M \) is the sum of \( N^D \) employed workers and \((M-N^D)\) unemployed workers. For model consistency and simplicity, we further assume that a lump-sum tax, \( V \), is levied on firms’ profits to pay a lump-sum transfer, \( e \), to every employed and unemployed worker, and, to finance government consumption, \( G \); that is, \( V = G + eM \). The subsidy to firms, \( \sigma \), corresponds directly to a decrease in \( z \), and it is financed by a cut in government consumption, that is, \( \sigma = dz/dH_0 = dG/dH_0 > 0 \), and, changes in the quasi-fixed cost, \( z \), do not affect \( e \), that is, \( de/dH_0 = 0 \).\footnote{Even though a part of \( e \) is the employer’s share of social security contributions proportional to wage, we assume that the lump-sum subsidy paid to employers does not affect \( e \). Parsing \( e \) into its components (a portion proportional to wage and a portion independent of wage) does not significantly alter the basic results, either.} These simplifying assumptions indicate that the model abstracts from the effects of an increase or a decrease in distortionary taxes or a cut in other expenditures that may have an impact on employment and output.

Given \( e \), it follows that income per worker is

\[
Y = wH_0 + mw(H-H_0) + e .
\]

(4)

Throughout the following analysis, we will assume that \( m, H_0, \sigma, e \) are exogenously determined, while wage may be variable reflecting labor demand and supply responses to the reduction in \( H_0 \). We will refer to the policy that endeavors to increase employment through simultaneously lowering the standard hours and providing an employment subsidy as the \((H_0, \sigma)\) policy.
B. Effects on Labor Demand

Based on the standard arguments in (1)–(3), we can analyze the impact of a simultaneous reduction in $N_0$ and $z$ on demand for inputs $N^D, H^D, K$, and, on output, $Q$, of a competitive firm. Then, the possible impact of this policy on worker income, $Y$, can also be evaluated using (4). Thus, if the supply of both $N$ and $H$ is assumed to be perfectly elastic, then, given $w, m, H_0$, the equilibrium levels of $N, H$ are demand determined. Using (2) and (3), the firm's profit maximization problem can be expressed as

$$
\max_{N^D, H^D, K} \Pi = Q(N^D, H^D, K) - N[wH_0 + mw(H^D - H_0) + z] - rK - \nu ,
$$

where $\Pi$ is profit, $r$ is the cost of capital (assumed to be constant in the long run), and, $\nu$ is the firm's share of the lump-sum tax ($V$ divided by the number of firms). From the first-order conditions (f.o.c.) and second-order conditions (s.o.c.) that apply to (5), we can show that

$$
\frac{dN^D}{dH_0} = n_0 \left( \frac{dw}{dH_0} \right) + n_1 [\sigma - (m-1)w] ; n_1 < 0 ;
$$

$$
\frac{dH^D}{dH_0} = h_0 \left( \frac{dw}{dH_0} \right) + h_1 [\sigma - (m-1)w] ;
$$

$$
\frac{dK}{dH_0} = k_0 \left( \frac{dw}{dH_0} \right) + k_1 [\sigma - (m-1)w] ;
$$

$$
\sigma = \frac{dz}{dH_0} > 0 ; \quad n_1 = \frac{\partial N^D}{\partial z} < 0 ; \quad \frac{\partial N^D}{\partial H_0} = -n_1 (m-1)w > 0 ;
$$

$$
n_0 = \frac{\partial N^D}{\partial w} ; \quad h_0 = \frac{\partial H^D}{\partial w} ; \quad h_1 = \frac{\partial H^D}{\partial z} ; \quad k_0 = \frac{\partial K}{\partial w} ; \quad k_1 = \frac{\partial K}{\partial z} .
$$
The signs of all the coefficients, except \( n_1 < 0 \), are ambiguous.\(^{13}\) Therefore, for a given subsidy and a change in wage, it is not possible in general to ascertain the change in \( N^D \), \( H^D \), and \( K \) when the standard hours are lowered. For simplicity, we will treat all the coefficients as constants. We further make the plausible long run assumption that \( n_0 = \frac{\partial N^D}{\partial w} < 0 \), that is, *ceteris paribus*, at a higher wage, the firm employs fewer workers, and, conversely.\(^{14}\) Table 1 summarizes the effects of changes in the main parameters of the model on the optimal choice of \( N^D \), \( H^D \), \( K \), on output level, \( Q \), and, on worker income, \( Y \). For illustrative purposes, the signs of the relevant partial derivatives in the case of the Cobb-Douglas production function are also presented.

<table>
<thead>
<tr>
<th>( Q = Q(N, H, K) )</th>
<th>( \partial N^D )</th>
<th>( \partial H^D )</th>
<th>( \partial K )</th>
<th>( \partial Q )</th>
<th>( \partial Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \partial Q )</td>
<td>(+)</td>
<td>(-)</td>
<td>(3/)</td>
<td>(7)</td>
<td>(-)</td>
</tr>
<tr>
<td>( \partial w )</td>
<td>(-)</td>
<td>(-)</td>
<td>(7)</td>
<td>(7)</td>
<td>(-)</td>
</tr>
<tr>
<td>( \partial m )</td>
<td>(7)</td>
<td>(-)</td>
<td>(7)</td>
<td>(7)</td>
<td>(-)</td>
</tr>
<tr>
<td>( \partial z )</td>
<td>(7)</td>
<td>(7)</td>
<td>(7)</td>
<td>(7)</td>
<td>(7)</td>
</tr>
</tbody>
</table>

Source: Authors' calculations.

1/ (?/7) indicates that the sign of the relevant partial derivative is ambiguous.
2/ See Appendix II.
3/ Assumption.

\(^{13}\)The ambiguity of the signs of relevant partial derivatives (other than \( n_1 = \frac{\partial N^D}{\partial z} < 0 \)) is due to the *scale effect*. From (6), \( \frac{\partial N^D}{\partial H_0} = -(m-1)w_n > 0 \), that is, when \( H_0 \) declines \( N^D \) declines and this has a negative impact on output. The resulting scale effect may outweigh the increase in \( H^D \) and \( K \) due to the *substitution effect* arising from the increase in the cost of workers when \( H_0 \) is lowered (Appendix II). If \( Q, K \) are constant, then \( \frac{\partial H^D}{\partial H_0} < 0, \frac{\partial H^D}{\partial z} > 0 \) unambiguously. For discussions of this simpler short run case and of the scale effect, see Ehrenberg (1971), Ehrenberg and Schumann (1988), Hamermesh (1993), and Hart (1987).

\(^{14}\)With Cobb-Douglas technology, \( n_0 < 0 \) requires \( z > [(1-\beta)(m-1)wH_0]/\gamma \). When wage rises, \( N^D \) declines; but \( H^D \) also declines which has a positive impact on \( N^D \). Therefore, the firm may substitute more workers for fewer hours, even though wage is higher. But if the fixed cost of hiring, \( z \), is sufficiently large, then the negative impact of the increase in wage on \( N^D \) dominates the positive impact of the decline in \( H^D \), and \( N^D \) declines (Appendix II).
An important additional consideration when the standard hours are lowered is the level of worker income, in particular, income of the minimum wage earners. Indeed, the goal of income maintenance can be seen as the dual of the goal of net job creation, since it has a bearing on the evolution of labor costs and, therefore, on the likelihood of the success of the \((H_0, \sigma)\) policy. As also emphasized by Drèze (1991), if employment can be increased by this policy without a reduction in worker income, then upward pressures on wages, which may undermine employment creation, may be avoidable. But if worker income declines when this policy is implemented, upward pressures on wages may resume as unions attempt to maintain worker income at previous levels.\(^{15}\) In this case, the \((H_0, \sigma)\) policy is less likely to be effective in stimulating higher employment. Accordingly, we assume that, when the policy maker introduces the \((H_0, \sigma)\) policy with the objective of increasing employment, it also aims to ensure that worker income, \(Y\), remains at least unchanged \((dY/dH_0 \leq 0)\).

By (4), this additional policy objective requires that the following restriction hold:

\[
\frac{dY}{dH_0} = [H_0 + m(H^D - H_0)](\frac{dw}{dH_0}) + mw(\frac{dH^D}{dH_0}) - (m-1)w \leq 0 .
\] (7)

Then, making use of (7) and the solution for \(dH^D/dH_0\) in (6), we have the two policy objectives as below:

\[
\frac{dY}{dH_0} = [H_0 + m(H^D - H_0) + mwh_0](\frac{dw}{dH_0}) + mw(\sigma - (m-1)w) - (m-1)w \leq 0 ,
\]

\[
\frac{dN^D}{dH_0} = n_0(\frac{dw}{dH_0}) + n_1(\sigma - (m-1)w) \leq 0 .
\] (8)

We can use (8) to ascertain the magnitude of the subsidy and the degree of wage flexibility necessary to induce an increase in employment without adversely affecting the worker income level. We first discuss the simpler case of constant wage and then turn to the case of variable wage.

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\(^{15}\)The experience with union attempts to maintain take-home pay after cuts in the standard hours lends some support to this conjecture; see Rubin and Richardson (1997, Chapter 6). The empirical evidence presented by Hunt (1999) in the case of Germany further corroborate this conjecture.
1. Constant wage

An examination of the impact of the \( (H_0, \sigma) \) policy with constant wage is useful because it provides some insight into possible outcomes when wage is rigid, as it might be in the case of the legally predetermined minimum wage in the short run.

The two results in Table 1 that are immediately relevant for a cut in the standard hours are \( \partial N / \partial H_0 > 0 \) and \( \partial N / \partial \sigma < 0 \), which mean that a reduction in the legal workweek, \( H_0 \), has a negative impact, and, a reduction in the quasi-fixed cost of hiring, \( z \), has a positive impact on employment. Therefore, (6) indicates that, under the \( (H_0, \sigma) \) policy with a constant wage, labor demand rises when standard hours are lowered \( (dN^2/dH_0 < 0) \), if the subsidy is such that \( \sigma > (m-1)w \). This intuitively means that, employment rises, if the employment subsidy more than compensates for the increase in the relative cost of employing an additional worker due to the reduction in the legal workweek. Thus, a prima facie case can be made for a policy initiative to promote job creation through lowering the legal work week and providing employment subsidies, even when firms react at the margin by increasing their reliance on overtime hours.

However, the condition \( \sigma > (m-1)w \) is not sufficient to preserve worker income level. As (8) indicates, with a constant wage, it is feasible to induce an increase in employment \( (dN^2/dH_0 < 0) \) through the \( (H_0, \sigma) \) policy and maintain the worker income level \( (dY/dH_0 < 0) \), provided that the following two conditions hold:

\[
\sigma \geq (m-1)w ; \\
\frac{h_1}{h_1} + \frac{(m-1)w}{m} = \sigma .
\]

Recall that the sign of \( h \) is ambiguous. First suppose \( h > 0 \). Then, in order to satisfy (9), the value of \( \sigma \) should be chosen as

\[
(m-1)w \leq \sigma \leq (m-1)w + \frac{m-1}{m} \frac{1}{h} ; \quad h > 0 .
\]

On the other hand, if \( h < 0 \), then the appropriate value for \( \sigma \) to satisfy (9) is

\[
\sigma \geq (m-1)w \geq (m-1)w + \frac{m-1}{m} \frac{1}{h} ; \quad h < 0 .
\]
Therefore, it is feasible to increase employment without affecting worker income level through the \((H_0, \sigma)\) policy, provided that, for a given \(H_0\), the subsidy level is chosen such that (10) or (11) is satisfied.

The foregoing result cannot be generalized to all firms and industries for a given set of values for \(H_0\) and \(\sigma\). Given \(\sigma\), it is possible that (10) or (11) is satisfied for some firms or industries but it may not be possible for others. This is to say that a generalized subsidy is not calibrated for all firms and industries. Furthermore, even when (10) or (11) is satisfied, the hours demand, capital demand, and output responses of the firm remain ambiguous. If \(H^D\) or \(K\) declines, this may have a negative impact on \(Q\) which, in turn, may result in a smaller increase in employment for a given subsidy level.\(^{16}\) On the other hand, salutary effects of the \((H_0, \sigma)\) policy on capital stock and output are equally plausible.\(^{17}\)

The subsidy scheme under the \((H_0, \sigma)\) policy may require significant budgetary resources. Therefore, a legitimate question at this juncture is whether effecting employment subsidies alone may be preferable to the \((H_0, \sigma)\) combination policy for employment creation.\(^{18}\) The subsidy lowers the quasi-fixed cost of hiring, \(z\), and increases the demand for workers, and, lowering the standard hours decreases the demand for workers, \(N^D\). Therefore, the increase in \(N^D\) under the \((H_0, \sigma)\) policy is smaller than the increase in \(N^D\) when only an employment subsidy is effected. Recall that, if \(z\) is lowered through a subsidy without a cut in the standard hours, then \(N^D\) increases unambiguously but the impact on demand for hours (the sign of \(\partial H^D/\partial z\)) remains ambiguous (Table 1). So, in general, it may be possible to maintain the worker income level by effecting a subsidy without a cut in the standard hours. On these grounds, this simpler policy should be preferred to the \((H_0, \sigma)\) policy because, for the same desired increase in employment, its budgetary cost is lower. Hence it may possibly have smaller deleterious macroeconomic effects through the budget.\(^{19}\)

\(^{16}\)If \(N^D\) declines as a result of the decline in output (scale effect), then either the increase in employment for a given subsidy is lower, or, a greater subsidy is necessary to induce a desired increase in employment.

\(^{17}\)For example, with Cobb-Douglas technology, the subsidy can be chosen such that (10) is satisfied, and this results in a decline in hours worked, an increase in the demand for workers and capital, and an increase in output (Appendix II).

\(^{18}\)This point was noted by Hart (1997), who recommended that “marginal employment subsidies should be given without the additional constraint of mandatory workweek reductions” (p. 229).

\(^{19}\)However, this argument needs to be treated with caution because it abstracts from the distortionary effects of an increase in taxes or a decrease in expenditures necessary to finance the subsidy paid to firms. Such distortions would weaken the case for employment subsidies.
But, in some cases, in spite of its higher budgetary cost, the \((H_0, \sigma)\) policy may be preferable to giving an employment subsidy without a cut in the standard hours. For example, with Cobb-Douglas technology, if only the quasi-fixed cost is lowered keeping \(H_0\) unchanged, then demand for hours declines unambiguously \((\partial H^D/\partial z > 0)\); this implies that worker income declines.\(^{20}\) However, under the \((H_0, \sigma)\) policy, it is possible to maintain worker income at the same level as before, while increasing the employment level. In such a case, giving a subsidy along with a cut in the standard hours may be preferable to giving a subsidy without a cut in the standard hours. Because the former policy attains the additional objective of maintaining workers' take-home pay, and its higher budgetary cost may be justified for this reason.\(^{21}\)

2. Variable wage

Constant wage is not a satisfactory assumption because, in the long run, wage will change reflecting labor demand and supply responses to the cut in \(H_0\). From (6) and (8), it is possible to show that, with variable wage, increasing employment \((dN^d/dH_0 \leq 0)\) while maintaining worker income level \((dY/dH_0 \leq 0)\) requires that the following conditions hold simultaneously:

\[
\frac{dN}{dH_0} \leq 0 \quad \text{if} \quad \frac{dw}{dH_0} \geq - \left( \frac{n_1}{n_0} \right) [\sigma - (m-1)w] . \tag{12}
\]

\[
\frac{dY}{dH_0} \leq 0 \quad \text{if} \quad \frac{w m h_1 \sigma}{(m-1)w + (w m h_1)(m-1)w - [H_0 + m(H^D - H_0)] + m w h_0} \left( \frac{dw}{dH_0} \right) . \tag{13}
\]

\(^{20}\)Under Cobb-Douglas technology, with wage and \(H_0\) constant, \(dY/dz = mw(dH^D/dz) > 0\).

\(^{21}\)The legally determined level of the overtime premium, \(m\), sets a benchmark for the economy as a whole. Increasing \(m\) has also been proposed to increase employment (for example, in the United States). For discussions of the possible impact of increasing \(m\) on employment, see Ehrenberg (1971), Ehrenberg and Schumann (1982), and Trejo (1993). As the results in Table 1 indicate, increasing the overtime premium alone has an ambiguous impact on employment. If \(m\) is altered along with a cut in standard hours, it may or may not be possible to increase employment without reducing worker income.
As noted earlier, the level of a generalized subsidy is not calibrated for a specific firm or industry. Thus, given the subsidy, we need to ascertain the magnitude of wage adjustment necessary to increase employment and maintain the worker income level. The inequality in (12) shows the magnitude of wage adjustment necessary for employment to rise under the \((H_0, \sigma)\) policy. However, for this value of \(dw/dH_0\) the inequality in (13) is not necessarily satisfied. Alternatively, in view of possible union attempts to maintain take-home pay, the value of \(dw/dH_0\) may be such that worker income level is preserved and (13) is satisfied, but this value does not necessarily satisfy (12) to induce an increase in employment. Thus, in general, the two main objectives of the \((H_0, \sigma)\) policy cannot necessarily be achieved simultaneously.

These results highlight the critical role wage flexibility plays in attaining the primary goal of increasing employment under the \((H_0, \sigma)\) policy. Maintaining worker income level is an overly strict condition in assessing the \((H_0, \sigma)\) policy, because workers might be willing to forego some income by accepting a cut in wages in return for more leisure and higher employment. This trade-off is critical in determining the extent of necessary wage flexibility for the \((H_0, \sigma)\) policy to be effective in inducing higher employment. With these preliminary observations, we now turn to the analysis of hours supply responses to the \((H_0, \sigma)\) policy.

C. Effects on Hours Supply

The impact of lowering the standard hours on wage and hours supply is analyzed in this section. This permits relaxing the constraint which rules out a decline in worker income and replacing it with the broader constraint that the representative worker’s or the union’s utility level be maintained. In this perspective, a reduction in the standard hours may facilitate some downward flexibility in take-home pay, at least for those workers earning more than the minimum wage.

1. Representative worker’s responses

The analysis of the representative worker’s optimal supply of hours shows that a cut in the standard hours may induce downward wage flexibility and a decline in total hours supplied.

The representative worker’s income is \(Y = wH_0 + mw(H^S - H_0) + e\), where \(H^S\) is the supply of hours. Let the maximum hours of leisure per workweek be \(T\), a constant. The worker’s problem is to maximize utility from income and leisure, as in

\[
\max_{(H^S)} U(wH_0 + mw(H^S - H_0) + e, T - H^S);
\]

\(f.o.c. : mwU_1 - U_2 = 0 \quad s.o.c. : (mw)^2U_{11} - 2mwU_{12} + U_{22} < 0\) \quad (14)
Using the f.o.c. and s.o.c. that apply to (14), we can show that $\partial H^c/\partial H_0 > 0$, that is, total hours of work supplied declines when $H_0$ is lowered, when income and leisure are substitutes ($U_{12} > 0$). In the normal range of the representative worker’s labor supply function, the decline in $H_0$ has an impact on the optimal choice of $Y$ and $H^c$ similar to the impact of an increase in non-labor income. Within the normal range of the labor supply function, an increase in non-labor income results in a decline in optimal $H$ and an increase in optimal $Y$. Therefore, as conjectured in (7) above, $\partial Y/\partial H_0 < 0$ appears to be a plausible description of the representative worker’s response to the decline in $H_0$, that is, the worker does not choose to lower his supply of hours, $H^c$, so much so that his income is lower.

However, the worker can be at least as well off after the reduction in $H_0$ at a lower income level. By differentiating the utility function in (14) with respect to $H_0$ with $m$ and $e$ constant, we can show that worker utility remains constant when wage declines such that

$$\frac{dU}{dH_0} \leq 0 \quad \text{if} \quad 0 < \frac{dw}{dH_0} \leq \frac{(m-1)w}{m(H-H_0) + H_0}, \quad (15)$$

which can be interpreted as a lower limit on wage flexibility. By differentiating the f.o.c. in (14) with respect to $H_0$, we can further show that the supply of hours declines ($\partial H^c/\partial H_0 > 0$) when standard hours are lowered and wage is decreased such that worker utility is constant.

2. Monopoly union responses

The union (as well as the representative worker) is not only concerned with wages and hours worked but also with the level of employment. The usual conjecture is that the monopoly union chooses wage to maximize

$$\max_{w} V = N^D U(wH_0 + mw(H^D - H_0) + e, T-H^D) + (M-N^D)\overline{U},$$

$$\overline{U} = U(e, T);$$

$$\text{f.o.c.: } V_w = 0; \text{ s.o.c.: } V_{ww} < 0,$$

---

$^{22}$Similarly, within the normal range of the labor supply function where the substitution effect dominates, it can be argued that the signs of $\partial H^c/\partial w$ and $\partial H^c/\partial m$ are positive, whereas the sign of $\partial H^c/\partial e$ is negative.

$^{23}$See, for example, Ehrenberg and Smith (1983, Chapter 6). As non-labor income is increased beyond the normal range, income effect dominates the substitution effect and $Y$ declines along with $H^c$. 
where $M$ is the constant number of workers (union members) in the labor force. Those who are left unemployed are chosen by a random draw. In view (6), the f.o.c. that applies to (16) is

$$\frac{dV}{dw} = (U-\bar{U})n_0 + N^D_U[H_0 + m(H^D - H_0)] + N^D_m(wU_1 - U_2)h_0 = 0. \quad (17)$$

The change in the equilibrium wage when the standard hours are lowered ($dw/dH_0$) can be derived by differentiating the f.o.c. in (17) with respect to $H_0$ and making use of the s.o.c. that applies to (16). It can be shown that the sign of $dw/dH_0$ is indeterminate. Therefore, given $\sigma$, the new wage level may be higher or lower than the level before the $(H_0, \sigma)$ policy is implemented.

An interior solution to (17) implies that, at the equilibrium wage, $N^D < M$. So, the monopoly union model indicates that there is a limit to downward wage flexibility, even when there is unemployment. Therefore, even if wage declines when the standard hours are cut, the decline in wage may not be sufficient to satisfy the condition for higher employment in (12).

3. Wage flexibility, total hours worked, and output

Wage flexibility is usually identified with downward flexibility ($\partial w/\partial H_0 > 0$). In the present context, downward wage flexibility means that workers are willing to take a cut in wages in return for more leisure and, possibly, for higher employment, when the standard hours are lowered. Furthermore, it has been argued that a decline in the standard hours results in a decline in total hours worked ($\partial H/\partial H_0 > 0$). Both conjectures are supported by the above analysis of the representative worker’s responses.24

In order to evaluate the implications of wage flexibility for the $(H_0, \sigma)$ policy, let us revisit the employment creation condition in (12). This condition indicates that, when the standard hours are lowered and no subsidy is given ($\sigma = 0$), then employment rises only if wage declines by more than $(n_i/n_0)(m-1)w$. However, if a subsidy is given along with the cut in the standard hours, even when $\sigma < (m-1)w$, the necessary decline in wage for increased employment is less than when no subsidy is given. Therefore, to some extent, the subsidy to

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24In the case of the union, under restrictive but plausible assumptions, Houpis (1993) showed that the union would be willing to accept a cut in wages. Along those lines, Trejo (1993) argued that, if the overtime premium were increased with the objective of increasing employment, employers would settle for lower wages in the long run so as to keep the number of workers more or less unchanged. Similarly, in the long run, wages may be lowered so as to at least maintain the number of workers at the initial level when the standard hours are lowered. Furthermore, union preferences have historically favored shorter work hours.
employers accommodates for downward wage rigidity. As noted earlier, if wage is completely rigid \( \partial w / \partial H_0 = 0 \), then \( \sigma > (m-1)w \) is necessary for employment to rise. If \( \sigma > (m-1)w \), employment may be increased when the standard hours are lowered, even if wage rises within the limits indicated by the condition in (12).

Nevertheless, as implied by the analysis of the representative worker's responses and emphasized by Houpios (1993) and Drèze and Modigliani (1981), a case for downward wage flexibility can be made with lower standard hours, particularly if the initial equilibrium is a suboptimal one where the total hours worked is above the optimum the representative worker prefers. Then, a cut in the standard hours is more likely to produce a decline in wage and in the total hours worked. Therefore, even when \( \sigma < (m-1)w \), the employment increasing condition in (12) may well hold.

In general, the impact of the \((H_0, \sigma)\) policy on output level is ambiguous, even if this policy serves to increase the level of employment. From (2), the output response to a cut in the standard hours can be derived as \( dQ/dH_0 = Q_q(dN/dH_0) + Q_q(dH/dH_0) + Q_q(dK/dH_0) \). The sign of \( dQ/dH_0 \) is ambiguous, even if we assume that employment rises \( (dN/dH_0 < 0) \) and total hours worked declines \( (dH/dH_0 > 0) \) when the standard hours are lowered.\(^{25}\)

III. CONCLUSIONS

The simple analysis of a cut in the legal workweek along with providing a generalized employment subsidy offers no compelling reason \( a \ priori \) that a higher long run equilibrium level of employment can be achieved through this policy. However, a cut in the standard workweek may induce a degree of downward wage flexibility, while an employment subsidy accommodates for downward wage rigidity to some extent. For a given subsidy level, micro level settlements between firms and workers may result in mutually acceptable levels of wages and work hours at a higher level of employment in some sectors. However, such an outcome cannot be generalized to the economy as a whole. This is because, for a given generalized employment subsidy, downward wage flexibility induced by the cut in the legal workweek may not be sufficient to result in an increase in employment in all sectors of the economy.

A decline in wages as well as a decline in total hours worked in response to a cut in the standard hours may be an analytically plausible outcome. Such a favorable outcome lends support to the view that employment may be increased when the standard hours are lowered. Indeed, achieving higher employment through a decline in total hours worked is the primary

\(^{25}\)More restrictive conditions on the degree of wage flexibility are necessary to ensure that output will not decline when the standard hours are lowered. The ambiguous impact of the \((H_0, \sigma)\) policy on output in the case of the Cobb-Douglas production function is discussed in Appendix II.
goal under the \((H_\alpha, \sigma)\) policy. However, even if wage and total hours worked decline, and aggregate employment is increased as a result of this policy, the outcome for aggregate output remains ambiguous. Consequently, the outcome may well be that more workers are producing fewer goods and services. Such a trade-off, when feasible and sustainable, is a legitimate if an inefficient policy choice, often observed in the form of over employment in state-owned enterprises and the bureaucracy in many countries.\(^{26}\)

In view of the theoretical uncertainties surrounding the impact of the \((H_\alpha, \sigma)\) policy on overall employment, such a policy would have to be evaluated on the basis of what is achieved in practice at the worker-firm level. Empirical results from industrial countries are mixed.\(^ {27}\) Evidence from industrial countries (1975–88) presented by Layard, Nickell, and Jackman (1991) has shown that unemployment rose the most in countries where the work hours have declined the most. A recent favorable verdict has been rendered by Rubin and Richardson (1997), who concluded that a shorter workweek resulted in higher employment in the United Kingdom, in particular, in the case of manual workers in manufacturing industries. In the case of France, Zanello (1998) reported that, within the two months after the adoption of the 35-hour initiative, 258 agreements to reduce the work hours of 30,000 workers had been signed, of which 170 requested subsidies. Net job creation amounted to about 2,500. Typically, the agreements involved full maintenance of the weekly wages. At the beginning of 1999, Kirman reported that the number of concluded contracts had reached a total of 523, with small to medium size firms dominating the total. Some firms reached agreements with their workers to add extra work shifts and to increase holidays and days-off. The policy may have induced some downward wage flexibility. Some firms agreed to a salary freeze over the next several years to cover the increased costs of the reduced work week. In many firms, new hires were to be paid less than those already employed on weekly basis. In contrast, in the case of Germany, longer term (1984–94) empirical results obtained by Hunt (1999) show that wage restraint (at least in the manufacturing sector) in the face of the decline in the standard hours did not materialize; the reduction in work hours was associated with an increase in wages, offsetting possible employment gains from the cut in the standard hours; in fact, the reduction in the work hours might have a detrimental impact on overall employment.

Finally, employment subsidies may require substantial budgetary resources, increase the fiscal burden, and may have a permanent adverse effect on aggregate output and employment. This possibility compounds the uncertainties surrounding the macroeconomic

\(^{26}\) Whether the \((H_\alpha, \sigma)\) policy is a superior policy to directly subsidizing over employment in state-owned enterprises and the bureaucracy is an interesting question that remains outside the scope of this paper.

\(^{27}\) For a recent review, see OECD (1998).
impact of this policy.\textsuperscript{28} The distortionary effects of tax increases or expenditure cuts to finance employment subsidies need to be taken into consideration. The distortionary effects of the \((H_\theta, \sigma)\) policy are not captured in the present model because, for simplicity, we posited only a lump-sum tax and a lump-sum subsidy.\textsuperscript{29} However, the quasi-fixed costs of hiring, \(z\), contain distortionary tax elements in the form of mandatory employer contributions to employees' social security and unemployment benefits. So, a significant part of the high labor costs resulting in low employment is policy-induced to begin with. By adopting a \((H_\theta, \sigma)\)-type policy, the policy maker is attempting to lower labor costs through a subsidy scheme predicated on a mandatory reduction in work hours in order to lower policy-induced unemployment. Such a policy does not appear to be the first-best response to high unemployment.\textsuperscript{30} Thus, an important question is why the policy maker does not use the more direct policy of simply cutting social security-related taxes and lowering the quasi-fixed costs of hiring to generate higher employment. The reason again is the preponderance of the second main policy objective of maintaining the worker income level. Unless other (distortionary) taxes are levied to compensate for the revenue loss, a cut in social security-related taxes might necessitate a cut in worker benefits, resulting in a decline in worker welfare. Social implications and the political inexpedient of more direct policies to lower unemployment—arguably with less ambiguous outcomes than the \((H_\theta, \sigma)\)-type policies—also play an important role in the policy maker's choice between the first- and second-best policy instruments.

More direct policy measures whose impact can be assessed with greater certainty should be given priority over lowering the standard workweek to reduce structural unemployment. Measures to remove structural rigidities in the labor market and to promote wage flexibility more directly have more promising prospects toward achieving that goal.\textsuperscript{31}

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\textsuperscript{28}For an extensive discussion of macroeconomic effects, see Hart (1987, Chapter 9).

\textsuperscript{29}The presence of a distortionary tax and increasing it to finance the employment subsidy would serve to strengthen the basic results of our model. In particular, the case for giving an employment subsidy to enhance employment would weaken since such a tax (on profits or worker income) would have to be weighed against the ambiguous employment impact of a cut in the standard hours.

\textsuperscript{30}As Hoel (1986) also observes, "even if a shortening of the workday does reduce unemployment, it might well be an 'inefficient' policy in the sense that there exist other ways of achieving the same reduction in unemployment, ways in other respects preferred by all agents to shortening the workweek" (p. 84).

\textsuperscript{31}Blanchard, et al. (1986) lay out a comprehensive agenda for employment creation in Europe through improving wage flexibility, reinforced by both supply and demand side macro policies.
### Table 2. The G7 Countries: Selected Labor Market Indicators (1997)

<table>
<thead>
<tr>
<th></th>
<th>Work hours per week</th>
<th>Days per year</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Legal (all employees)</td>
<td>Normal (full-time)</td>
<td>Paid vacation</td>
<td>Official holidays</td>
</tr>
<tr>
<td>Canada</td>
<td>40-48</td>
<td>41</td>
<td>37.3</td>
<td>12</td>
</tr>
<tr>
<td>France</td>
<td>39</td>
<td>39.1</td>
<td>40.5</td>
<td>25</td>
</tr>
<tr>
<td>Germany</td>
<td>48</td>
<td>40</td>
<td>40.4</td>
<td>24</td>
</tr>
<tr>
<td>Italy</td>
<td>48</td>
<td>38.6</td>
<td>39.8</td>
<td>25</td>
</tr>
<tr>
<td>Japan</td>
<td>44</td>
<td>47.5</td>
<td>49</td>
<td>10</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>(not defined)</td>
<td>43.9</td>
<td>45.7</td>
<td>20-30</td>
</tr>
<tr>
<td>United States</td>
<td>40</td>
<td>43.2</td>
<td>44.5</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Duration of work</th>
<th>In percent of total employment</th>
<th>Legal max. weekly overtime hrs.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Self-employed</td>
<td>Part-time employed</td>
</tr>
<tr>
<td></td>
<td>effective 1/</td>
<td>Life-cycle 2/</td>
<td>employed</td>
</tr>
<tr>
<td>Canada</td>
<td>1,721</td>
<td>...</td>
<td>10.9</td>
</tr>
<tr>
<td>France 3/</td>
<td>1,539</td>
<td>60,635</td>
<td>11.3</td>
</tr>
<tr>
<td>Germany</td>
<td>1,519</td>
<td>64,578</td>
<td>9.2</td>
</tr>
<tr>
<td>Italy</td>
<td>1,682</td>
<td>61,825</td>
<td>34.8</td>
</tr>
<tr>
<td>Japan</td>
<td>1,990</td>
<td>71,123</td>
<td>11.8</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1,731</td>
<td>73,904</td>
<td>43.9</td>
</tr>
<tr>
<td>United States</td>
<td>1,967</td>
<td>61,343</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Sources: OECD (1998).

1/ Average effective duration per employee (hours per year).
3/ The yearly limit on overtime hours is 130 hours, corresponding to less than 3 hours per week for a 47-week work year.
Figure 1. G7 Countries: Unemployment Trends, 1970-98
(Canada, France, Germany)

Chart 1. G7 Countries: Unemployment Trends, 1970-98 (continued)
(Italy, Japan, United Kingdom, United States)

Source: IMF World Economic Outlook
Selected Technical Discussions

1. General profit maximization conditions with substitutability among $N$, $H$, $K$

The $f.o.c.$ and $s.o.c.$ that apply to the firm’s profit maximization problem in (5) are

\[
\begin{align*}
\Pi_N &= Q_N - [wH_0 + mw(H-H_0) + z] = 0; \\
\Pi_H &= Q_H - mwN = 0; \\
\Pi_K &= Q_K - r = 0;
\end{align*}
\]

\[
\begin{align*}
\text{s.o.c.} \\
\Pi_{NN}, \Pi_{HH}, \Pi_{KK} < 0; \\
\Pi_{NN}\Pi_{KK} - \Pi_{NK}^2 > 0; \quad \Pi_{NN}\Pi_{HH} - \Pi_{NH}^2 > 0; \quad \Pi_{HH}\Pi_{KK} - \Pi_{HK}^2 > 0; \quad \Delta = \Pi_{NN} (\Pi_{HH}\Pi_{KK} - \Pi_{HK}^2) - \Pi_{HH} (\Pi_{NH}\Pi_{KK} - \Pi_{NK}\Pi_{HK}) \\
&\quad + \Pi_{NK} (\Pi_{NH}\Pi_{HK} - \Pi_{HH}\Pi_{NK}) < 0; \quad \text{where}
\end{align*}
\]

\[
\begin{align*}
\Pi_{NH} &= (Q_{NH} - mw); \quad \Pi_{NK} = Q_{NK}; \quad \Pi_{HK} = Q_{HK}; \\
\Pi_{NN} &= Q_{NN}; \quad \Pi_{HH} = Q_{HH}; \quad \Pi_{KK} = Q_{KK}.
\end{align*}
\]

By differentiating the $f.o.c.$ with respect to $H_0$ and making use of the $s.o.c.$, we can show that
\[
\frac{dN^D}{dH_0} = n_0 \frac{dw}{dH_0} + n_1 [\sigma - (m-1)w] ; \quad \sigma = \frac{dz}{dH_0} > 0 ; \quad n_0 = \frac{\partial N^D}{\partial w} ; \quad n_1 = \frac{\partial N^D}{\partial z} ;
\]

\[
n_0 = \frac{(Q_{22}Q_{23} - Q_{23}^2)[H_0 + m(H^D - H_0)] + mN^D[Q_{13}Q_{23} - Q_{33}(Q_{12} - mw)]}{\Delta} ;
\]

\[
n_1 = \frac{(Q_{22}Q_{23} - Q_{23}^2)}{\Delta} < 0 ; \quad \frac{\partial N^D}{\partial H_0} = - n_1 (m-1)w > 0 ;
\]

\[
\frac{dH^D}{dH_0} = h_0 \frac{dw}{dH_0} + h_1 [\sigma - (m-1)w] ; \quad h_0 = \frac{\partial H^D}{\partial w} ; \quad h_1 = \frac{\partial H^D}{\partial z} ;
\]

\[
h_0 = - \frac{[Q_{33}(Q_{12} - mw) - Q_{23}Q_{13}][H_0 + m(H^D - H_0)] - mN^D[Q_{11}Q_{33} - Q_{13}^2]}{\Delta} ;
\]

\[
h_1 = - \frac{Q_{33}(Q_{12} - mw) - Q_{23}Q_{13}}{\Delta} ; \quad \frac{\partial H^D}{\partial H_0} = - h_1 (m-1)w ;
\]

\[
\frac{dK}{dH_0} = k_0 \frac{dw}{dH_0} + k_1 [\sigma - (m-1)w] ; \quad k_0 = \frac{\partial K}{\partial w} ; \quad k_1 = \frac{\partial K}{\partial z} ;
\]

\[
k_0 = - \frac{[Q_{13}Q_{22} - Q_{23}(Q_{12} - mw)][H_0 + m(H^D - H_0)] + mN^D[Q_{11}Q_{23} - Q_{13}(Q_{12} - mw)]}{\Delta} ;
\]

\[
k_1 = - \frac{Q_{22}Q_{13} - Q_{23}(Q_{12} - mw)}{\Delta} ; \quad \frac{\partial K}{\partial H_0} = - k_1 (m-1)w .
\]

The ambiguity of the signs of the coefficients (except \(n_1 < 0\)) reflects the interaction of counteracting scale effect and substitution effect in response to changes \(w\) or \(H_0\). We assume \(n_0 = \partial N^D/\partial w < 0\).
2. The Cobb-Douglas case with constant wage

The following is an example with the Cobb-Douglas production function. With \( Q = A N^\alpha H^\gamma K^\beta \); \( 0 < \alpha, \gamma, \beta < 1 \), \( \alpha + \gamma + \beta = 1 \); \( \alpha > \gamma \); \( A > 0 \), and the additional constraint that \( z > (m-1)wH_0 \), we can show that

\[
H^* = \left( \frac{\gamma}{\alpha - \gamma} \right) \left( \frac{1}{mw} \right) \left[ z - (m-1)wH_0 \right];
\]

\[
N^* = \frac{N_0}{(H^*)^{1-\alpha-\beta}} , \quad N_0 = A \frac{1}{1-\alpha-\beta} \left( \frac{\gamma}{mw} \right)^{1-\beta} \left( \frac{\beta}{r} \right)^{1-\alpha-\beta} ;
\]

\[
K^* = \frac{K_0}{(H^*)^{1-\alpha-\beta}} , \quad K_0 = A \frac{1}{1-\alpha-\beta} \left( \frac{\alpha}{mw} \right)^{1-\beta} \left( \frac{\beta}{r} \right)^{1-\alpha-\beta} ;
\]

\[
Q^* = \frac{Q_0}{(H^*)^{1-\alpha-\beta}} = \frac{rK^*}{\beta} , \quad Q_0 = A \frac{1}{1-\alpha-\beta} \left( \frac{\gamma}{mw} \right)^{1-\beta} \left( \frac{\beta}{r} \right)^{1-\alpha-\beta} ;
\]

\[
\Pi^* = \left( \frac{1-\beta}{\beta} \right) rK^* - \nu ; \quad Y^* = \frac{\gamma z - \alpha(m-1)wH_0}{\alpha - \beta} + e ;
\]

where (*) denotes the equilibrium values.\(^{32}\) For workers to offer positive hours of work in equilibrium \((Y^*, H^* > 0)\), it is necessary that \( \gamma z - \alpha(m-1)wH_0 > 0 \). The firm’s output is exhausted since \( Q^* = \Pi^* + N^*C^* + rK^* + \nu \). From (20), with constant wage,

\[
\frac{dY^*}{dH_0} \leq 0 \quad \text{if} \quad \sigma \leq \frac{\alpha}{\gamma} (m-1)w \quad ; \quad \frac{dH^*}{dH_0} \geq 0 \quad \text{if} \quad \sigma \geq (m-1)w ;
\]

\[
\frac{dN^*}{dH_0} \leq 0 \quad \text{if} \quad \frac{dH^*}{dH_0} \geq 0 .
\]

\(^{32}\)Notice in (20) that, while \( N^*, K^* \) are scale dependent, \( H^* \) is scale independent, that is, \( H^* \) is not affected by the level of output, hence \( \partial H^*/\partial H_0 < 0 \) unambiguously. Keeping \( Q \) constant, when \( H_0 \) is lowered, \( N^* \) declines and \( H^* \) rises due to the substitution effect. But \( Q^* \) declines when \( N^* \) declines. If \( H^* \) were scale dependent, then the decline in \( Q^* \) would result in a decline in \( H^* \) due to the scale effect, hence the sign of \( \partial H^*/\partial H_0 \) is ambiguous when \( Q^* \) is variable.
Therefore, the policy objectives of increased employment \( (\partial N^*/\partial H_0 < 0) \) and increased worker income \( (\partial H^*/\partial H_0 < 0) \) can be achieved simultaneously under the \((H_0, \sigma)\) policy, if

\[
(m-1)w < \sigma < \left(\frac{\alpha}{\gamma}\right)(m-1)w,
\]

which can hold since \(\alpha/\gamma > 1\) for \(H^* > 0\); notice that (22) is the equivalent of (10). It is also clear from (20) that, if \(\sigma\) is chosen such that (22) is satisfied, then, \(\partial H^*/\partial H_0 > 0\), therefore, \(\partial K^*/\partial H_0 < 0\) and \(\partial Q^*/\partial H_0 < 0\), that is, total hours worked declines, and, capital and output levels rise when the standard hours are lowered.

3. The Cobb-Douglas case with variable wage

By differentiating \(H^*, N^*, K^*\) and \(Q^*\) in (20) with respect to \(H_0\), we can show that

\[
\frac{1}{H^*} \frac{dH^*}{dH_0} = \frac{1}{\Phi} \left[ (\sigma-(m-1)w) - \frac{z}{w} \frac{dw}{dH_0} \right]; \Phi = z-(m-1)wH_0 > 0;
\]

\[
\frac{1}{N^*} \frac{dN^*}{dH_0} = -\frac{1}{\Phi(1-\alpha-\beta)}[(\gamma z-(1-\beta)(m-1)wH_0)(\frac{1}{w} \frac{dw}{dH_0}) + (1-\gamma-\beta)(\sigma-(m-1)w)]
\]

\[
\frac{1}{K^*} \frac{dK^*}{dH_0} = -\frac{1}{\Phi(1-\alpha-\beta)}[(\gamma z-\alpha(m-1)wH_0)(\frac{1}{w} \frac{dw}{dH_0}) + (\alpha-\gamma)(\sigma-(m-1)w)];
\]

\[
\frac{1}{Q^*} \frac{dQ^*}{dH_0} = \frac{1}{K^*} \frac{dK^*}{dH_0}.
\]

We have assumed that \(n_0 = \partial N/\partial w < 0\), which requires that \(\gamma z - (1-\beta)(m-1)wH_0 > 0\); also, by (20), \(\gamma z - \alpha(m-1)wH_0 > 0\), which, by (23), indicates that \(dQ^*/dH_0, dK^*/dH_0 < 0\), that is, if wage rises \(K\) and \(Q\) decline, and, conversely. Furthermore, we can show that \(\gamma z - \alpha(m-1)wH_0 > 0\). With these observations, it follows that

\[
-\frac{1-\gamma-\beta}{\gamma z-(1-\beta)(m-1)wH_0} < -\frac{\alpha-\gamma}{\gamma z-\alpha(m-1)wH_0} < 0.
\]

When \(\sigma - (m-1)w > 0\), using (23) and (24) we can show that
\[ \frac{1}{H^* \, dH_0} \frac{dH^*}{dN^*} < 0 , \quad \frac{1}{N^* \, dH_0} \frac{dN^*}{dQ^*} > 0 \]

\[ \text{if} \]

\[ \frac{(1-\gamma-\beta)(\sigma-(m-1)w)}{\gamma z-(1-\beta)(m-1)wH_0} \frac{1}{w \, dH_0} \frac{dw}{dH_0} \begin{array}{c} < 0 \end{array} \frac{(\alpha-\gamma)(\sigma-(m-1)w)}{\gamma z-\alpha(m-1)wH_0} \begin{array}{c} < 0 \end{array} \frac{w}{z} \]

This demonstrates that, when \( (\sigma-(m-1)w) > 0 \), there is room for even a wage increase \( (dw/dH_0 < 0) \) under the \((H_0, \sigma)\) policy. However, wage may increase such that output declines, even if employment is increased when the standard hours are cut. The decline in total hours in response to the cut in the standard hours may induce an increase in demand for workers even at a higher wage but output may decline. However, if wage declines when the standard hours are cut \( (dw/dH_0 > 0) \), then (25) indicates that the total hours worked declines, and both employment and output rise.

When \( (\sigma-(m-1)w) < 0 \), then

\[ \frac{1}{H^* \, dH_0} \frac{dH^*}{dN^*} > 0 , \quad \frac{1}{N^* \, dH_0} \frac{dN^*}{dQ^*} < 0 \]

\[ \text{if} \]

\[ \frac{1}{w \, dH_0} \frac{dw}{dH_0} > \frac{(1-\gamma-\beta)(\sigma-(m-1)wH_0)}{\gamma z-(1-\beta)(m-1)wH_0} > \frac{(\alpha-\gamma)(\sigma-(m-1)wH_0)}{\gamma z-\alpha(m-1)wH_0} > 0 . \]

This demonstrates that, for employment and output to rise when the standard hours are cut, a degree of downward wage flexibility \( (dw/dH_0 > 0) \) is necessary. However, with a subsidy, the required downward wage flexibility is less than it would be without a subsidy. When (26) holds, it can also be shown that worker income rises even though wage declines, because total hours worked increases. Whether such an overall outcome could be mustered depends on union responses to higher income with more work hours, which we have shown to be indeterminate by (16) and (17).

4. Legally maximum overtime hours

A possible increase in total hours worked and a decline in wage when the standard hours are cut—such as the outcome in (26)—requires that worker preferences are for more
hours and higher income. Such an outcome further requires that the increase in total hours worked does not exceed the legally maximum hours of work per period. In view of the documented union preferences for shorter hours, it has been argued that a cut in the standard hours may result in a decline in total hours worked, along with a decline in wage. The \((H_\phi, \sigma)\) policy envisions such an outcome, since achieving higher employment through a decline in total hours worked is its primary goal. But even with the envisaged outcome of a decline in total hours worked and a decline in wage, it can be argued that the impact on output of the \((H_\phi, \sigma)\) policy remains ambiguous. In this context, it can also be shown that manipulating the legal maximum for overtime hours while implementing the \((H_\phi, \sigma)\) policy has an ambiguous impact on employment and output.

Suppose that the firm operates at the legally maximum total hours, \(\hat{H}\). With Cobb-Douglas technology, the firm’s profit maximization problem and the solutions for \(N, K, Q\) are

\[
\begin{align*}
\max_{(N, K)} (AH^n)N^\gamma K^{\beta} - N[wH_0 + mw(H-H_\phi) + z] - rK - \nu ;

N^* &= A^{1-\alpha/\beta} (\alpha)^{1-\alpha/\beta} (\beta)^{\beta(1-\alpha/\beta)} \left( \frac{\hat{H}^\gamma}{C^{(1-\beta)}} \right)^{1/1-\alpha/\beta} ; \\
K^* &= A^{1-\alpha/\beta} (\alpha)^{\frac{\alpha}{1-\alpha/\beta}} (\beta)^{\frac{\beta}{1-\alpha/\beta}} \left( \frac{\hat{H}^\gamma}{C^{\alpha}} \right)^{1/1-\alpha/\beta} ; \\
Q^* &= \frac{rK^*}{\beta} ; \quad \Pi^* = \left( \frac{1-\alpha/\beta}{\beta} \right) rK^* - \nu .
\end{align*}
\]

Using (27), we can show that
\[
\frac{1}{N^*} \frac{dN^*}{dH_0} < 0, \quad \frac{1}{Q^*} \frac{dQ^*}{dH_0} > 0
\]

if

\[
- \frac{\sigma - (m-1)w}{H_0 + m(H-H_0)} + \frac{\gamma C - (1-\beta)mw\dot{H}}{(1-\beta)\dot{H} [H_0 + m(H-H_0)]} \left( \frac{d\dot{H}}{dH_0} \right) < \frac{1}{w} \frac{dw}{dH_0} < \\
- \frac{\sigma - (m-1)w}{H_0 + m(H-H_0)} + \frac{\gamma C - \alpha mw\dot{H}}{\alpha \dot{H} [H_0 + m(H-H_0)]} \left( \frac{d\dot{H}}{dH_0} \right),
\]

(28)

where

\[
\frac{\gamma C - (1-\beta)mw\dot{H}}{(1-\beta)\dot{H} [H_0 + m(H-H_0)]} < \frac{\gamma C - \alpha mw\dot{H}}{\alpha \dot{H} [H_0 + m(H-H_0)]}.
\]

The percent change in wage at the new labor market equilibrium may be such that (28) holds. This is sufficient to demonstrate that, if total hours worked and wage decline when the standard hours are cut, employment may rise but output may decline: more workers produce fewer goods and services.

If the legally maximum overtime hours are kept the same or lowered when the standard hours are lowered, then \(d\dot{H}/dH_0 \geq 1\); if the legally maximum overtime hours are increased when the standard hours are lowered, then \(d\dot{H}/dH_0 < 1\). The result in (28) does not depend on the magnitude of \(d\dot{H}/dH_0\) relative to unity but depends rather on the degree of wage flexibility.

Then, the question is, is decreasing or increasing the legal maximum for overtime hours more likely to facilitate greater downward wage flexibility? Examining the impact of a change in \(\dot{H}\) on worker income provides some insight in answer to this question. Rearranging (7) we can show that

\[
\frac{dY}{dH_0} = mw \left( \frac{d\dot{H}}{dH_0} - \frac{m-1}{m} \right) + [H_0 + m(H-H_0)] \frac{dw}{dH_0}.
\]

(29)

Therefore, if \(d\dot{H}/dH_0 \leq (m-1)/m < 1\), then the first term in (29) is negative and, for \(dY/dH_0 \leq 0\), wage may remain unchanged or decrease when the standard hours are lowered \((dw/dH_0 > 0\). This is because the increase in total hours worked due to raising the legal maximum for overtime hours more than compensates for the decline in wage. However, if \(d\dot{H}/dH_0 >

(m-1)/m, then the first term in (29) is positive and, for dY/dH₀ ≤ 0, wage rate needs to rise when the standard hours are lowered (dw/dH₀ < 0). In this case, the adjustment in H is not sufficient to maintain worker income without an increase in wage. These observations suggest that if the legal maximum for overtime hours is increased when the standard hours are cut, there may be greater room for downward wage flexibility.

However, with a general production function, it can be shown that the signs of ∂N/∂H, ∂Q/∂H are ambiguous. Therefore, it is not possible to make a general case for inducing an increase in employment through manipulating the legal maximum for total hours worked under the (H₀, q) policy.
BIBLIOGRAPHY


