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A Model of Contagious Currency Crises
with Application to Argentina

Prepared by Nada Choueiri

Authorized for distribution by Donald J. Mathieson

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Abstract

This paper proposes a model of contagious currency crises: crises transmit across countries by raising the risk premium on government bonds. Three types of equilibria can occur: a “no-collapse” equilibrium (crises never transmit from abroad); a “collapse” equilibrium (crises are inevitably contagious); or a “fundamentals” equilibrium (crises are contagious if domestic fundamentals are weak). A calibration exercise finds that the 1995 turmoil in Argentina coexisted with a combination of risk-averse investors and weak credibility in the currency board arrangement. This turmoil could only be attributed to a Tequila effect from the Mexican crisis alone if investors were excessively risk-averse.

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Author’s E-Mail Address: nchoueiri@imf.org

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I. INTRODUCTION

On December 20, 1994, Mexico was forced into a 15 percent devaluation followed by the adoption of a floating exchange rate, after a speculative attack reduced its stock of foreign exchange reserves to $6 billion, down from $29 billion in February of that year. Soon afterwards, Argentina, and to a lesser extent Brazil, suffered from financial problems as investors were reluctant to renew their loans to these countries. Argentina lost 40 percent of its reserves and 18 percent of its bank deposits in the first quarter of 1995, while Brazil's foreign exchange reserves declined by 20 percent. A similar scenario took place in the summer of 1997 after Thailand stopped defending the baht's fixed value against the dollar on July 2. A wave of speculative attacks rocked South-East Asian countries, leading to widespread depreciations that reduced the values of these countries' currencies by 30 to 50 percent as of December 1997, in comparison with the beginning of the year.²

The attacks in both the 1995 and 1997 episodes seemed often justified by fundamental problems in the targeted countries. For example, a large number of financial institutions in South East Asia were grappling with substantial amounts of unhedged foreign debt and non-performing assets. But there were cases in which these currency attacks seemed unwarranted by the underlying fundamentals, as was the case in the run on the Hong Kong dollar in October 1997.³ In both episodes of widespread financial turmoil, therefore, the temporal correlation of crises remains particularly striking. This phenomenon—that devaluation in a country can be the precursor and even one of the causes of financial troubles in other countries—has been dubbed contagion.

Prior to the 1990s, currency crises did not seem to spread across countries with the virulence and speed observed recently. It is therefore not surprising that in the 1980s the finance literature, while it focused on contagion in bank runs⁴, had little to say about contagious

²The Asian crises intensified in 1998, and the turmoil even reached Latin American markets. Brazil seems to have been mostly affected. First a period of financial distress in October 1997 (in which stock market prices fell by 15 percent) forced the central bank to double short-run borrowing rates to 46 percent in order to defend its currency, the Real. Although these rates declined later, the crisis further intensified in the summer of 1998, and culminated in the decision to float the currency on January 13, 1999.

³This run took place although the size of foreign exchange reserves in Hong Kong—around $88 billion at the time—was four times the size of its narrow money stock (M1). It forced the monetary authority to raise interest rates to 300 percent to fend off speculators, as reported in The Economist (November 1, 1997, p. 80).

⁴Theoretical models explaining bank runs can be found in Diamond and Dybvig (1983) and Chari and Jagannathan (1988), for example. Several empirical papers (such as Gorton, 1988)
speculative attacks on currencies. In the 1990s, however, the literature took a step in the latter direction when the interest of researchers focused on the co-movement of asset prices and capital flows across emerging financial markets. This co-movement was generally interpreted as supporting evidence for the presence of contagion in asset flows across countries. Calvo and Reinhart (1996), for example, argue that the significant correlation in bond and equity returns they found across emerging markets is indicative of such contagion. Valdes (1996), after accounting for fundamental factors, is still able to identify excess co-movement in country credit ratings and secondary market debt prices, hence ascertaining the contagion hypothesis.

The focus on contagion in the context of currency crises alone occurred mainly in the past few years, inspired by the collapse of the ERM in 1992 and the turmoil that swept financial markets in the aftermath of the Mexican crisis of December 1994. There are hence only a few empirical studies that investigate the reasons why crises seem to be temporally linked across countries. Eichengreen, Rose, and Wyplosz (1996) address this question using a sample of 20 industrialized countries over a thirty-year period. They find positive and significant correlation in the incidence of crises across countries, after accounting for domestic economic and political factors. Looking at emerging markets, Sachs, Tornell, and Velasco (1996) examine the countries that suffered from financial problems in the wake of the Mexican crisis. They find that lending booms, real appreciation and low reserve levels were common priors across the countries that had severe crises at the time, hence explaining their extreme susceptibility to the Tequila effect. Contagion in the aftermath of the Mexican crisis therefore seemed to have led to the worst turmoil in economies characterized by weak underlying fundamentals. Using a different approach, Schumaker and Frankel (1996) perform tests on country funds data and find significant adverse short-run spillover effects from the Mexican crisis to both Latin America and Asia. The long run effects across these two regions were different however. Fund managers are able to substitute between regions in the long run and hence chose to increase their exposure to Asia at the expense of Latin America.

Overall, the empirical literature clearly cannot refute the hypothesis of contagion effects in specific episodes of currency crises. The theoretical literature, however, offers limited analyses of such effects. The existing models generally relate contagion to either trade channels or liquidity considerations. Gerlach and Smets (1995), for example, model the role of trade channels as central to the transmission of crises. They show how a devaluation in a given country can lead to adverse monetary shocks and reserve losses in another country because of the inflationary effect of a rise in the latter's import prices. The devaluation can alternatively

\[4^{*}\text{...continued)}
examined historical episodes to identify the reasons behind bank panics.

\[5\text{See Calvo and Reinhart (1996) for a discussion of this branch of the literature.}

\[6\text{A rise in import prices leads to an increase in the general price level, which reduces money (continued...)}\]
cause a crisis in the second country by leading to a current account deficit as this country's exports are priced out of the first market. The trade channel, however, cannot account for the observation of contagion effects across countries that do not have strong trade linkages with one another. Valdes (1996) provides an alternative framework, in which investors' liquidity needs lead to contagion effects. According to his model, a crisis in a given country will drive investors to sell off assets of another country in order to raise funds, either because these funds are needed today or because cash needs are expected to materialize in the future. His model however only explains the trigger effect of a crisis, namely the sell-off of assets in other countries. There is no account on how this initial effect leads to a full-fledged crisis. Adopting a more general approach, Masson (1998) presents a balance-of-payments model in which a crisis could occur because of either adverse changes in domestic or external fundamentals or because of contagion effects. Masson's model, however, captures the latter factor by a positive shock to the exogenous probability of devaluation in a foreign country, and the transmission channel to the domestic economy operates through the bilateral real exchange rate. Therefore, as in Gerlach and Smets (1995), contagion in his model effectively operates through a fundamental economic link between the two countries.

In this paper, an alternative form of contagion is proposed and its implications on the incidence of currency crises is analyzed. Contagion is introduced in the model through a risk premium channel: a currency crisis in a given country affects another country by causing a rise in the risk premium that investors require to hold bonds issued by the latter country. This transmission mechanism can be due to the herding behavior of investors, rather than to fundamental factors such as trade channels or liquidity needs. Herding behavior is not necessarily an ad-hoc assumption: it can be justified by the globalization of capital markets in the 1990s, coupled with the presence of imperfect information. Indeed, as shown by Calvo and Mendoza (1997), by raising the number of investment opportunities and/or exerting pressures on managerial performances, globalization reduces the incentive to incur the cost of gathering country-specific information and induces conformity in investment decisions across fund managers. Both factors lead to herding behavior.7

Contagion here operates as follows. Consider an international investor who naturally diversifies risks by holding assets in several emerging markets, choosing the composition of her portfolio in accordance with the maximization of a mean-variance expected utility function. A crisis in a given country inevitably motivates the investor to reduce her exposure to the country in trouble. In a world of uncertainty, this will trigger a portfolio reallocation whereby the investor will withdraw from other markets as well. In particular, assets denominated in currencies that are expected to covary positively with the currency in trouble

6(...continued)
demand and leads to reserve losses in a fixed exchange rate system.

7In this paper, herding refers to “... a situation in which rational investors choose to react to a rumor, regarding (...) a country's asset return characteristics...,” as defined by Calvo and Mendoza (1997).
will become undesirable, and this is translated by a rise in the risk premium component of returns on these assets. A contagion effect is hence activated. Unless the return on such assets rises to compensate for the added risk, a decrease in the demand for these assets occurs, and hence the possibility of a currency crisis in their country of origin arises.

The present paper embeds the contagion mechanism above in a model of speculative attacks à la Krugman, developed by Flood and Marion (1998). An exchange rate crisis is assumed to have recently occurred in a foreign country and the contagious effects on the domestic economy are felt through an increase in the risk premium on domestic bonds. The model can theoretically yield three types of equilibria that are distinct primarily because of the underlying degree of uncertainty. In a world of extremely low uncertainty, the economy settles at a "no-collapse" equilibrium where a crisis in a foreign country does not lead to a crisis at home. Alternatively, the economy could be driven to a "collapse" equilibrium—characterized by an inevitable speculative attack on the fixed exchange rate system—in case investors face a large degree of uncertainty and are risk averse. For an intermediate range of uncertainty and risk aversion, the economy would settle at a "fundamentals" equilibrium, in which a crisis abroad causes a speculative attack at home only when domestic fundamentals are precarious.8

When calibrated to the Argentine and Mexican economies in 1995, the solution to the model is characterized by multiple equilibria of the type described above. Given the actual observations on the variances and covariances of the two countries' exchange rates over the 1970–95 period, it turns out that the turmoil in 1995 in Argentina could be attributed to contagion effects resulting from the Mexican collapse of December 1994 if investors' degree of risk aversion is sufficiently high.

The next section presents the model and sketches the analytical solution. Section III applies the model to Argentina. Section IV concludes.

II. THE MODEL

This section presents an economy that follows a fixed exchange rate system.9 For ease of exposition, the domestic economy will be referred to as Argentina, and domestic country variables will be indicated by the superscript $a$. The model extends the framework developed

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8The model therefore implies that preserving an exchange rate peg through tight economic policy is possible when there are foreign crises, but only if uncertainty is not too high and investors are not extremely risk averse.

9Argentina has had a currency board since the implementation of the Convertibility Plan in April 1991.
by Flood and Marion (1998) to a two-country set-up. The key modification is in allowing the risk premium on domestic bonds to account for contagion effects coming from a crisis that just occurred in a foreign country, which will be referred to as Mexico (foreign country variables will be indicated by the superscript $m$). Equilibrium in the foreign economy is not endogenously determined, however. In particular, the crisis in Mexico is exogenous to the model and is assumed to lead to a floating exchange rate system there. Hence a stochastic process for the Mexican exchange rate is an inherent part of the model. The latter also figures a process for the GDP ratio of Mexican debt in circulation as the size of such debt affects the portfolio allocation of international investors and hence has a bearing on the risk premium component of the rate of return paid by Argentina's government debt.

A. Model Specification

The basis of the model is a simplified stochastic version of the Argentine money market equilibrium condition:

$$m_t - p_t = -\alpha i_t + \delta e_t^a, \quad \alpha > 0, \delta < 0; \quad (1)$$

where $m_t$ is the logarithm of the money base normalized by domestic output, $p_t$ the logarithm of the price level, and $i_t$ is the interest rate. The Argentine economy is subject to real productivity shocks $e_t^a$ that affect money demand. The central bank does not hold government bonds; all public debt is in the hands of the private sector. This implies that monetization of fiscal deficits cannot take place, which eliminates a traditional channel by which misaligned fundamentals cause balance-of-payments crises à la Krugman. Large deficits leading to the accumulation of public debt would still create fundamental problems to the economy as they would translate into an increase in interest rates that jeopardize the

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10Flood and Marion (1998) present a Krugman-type speculative attack model in which a country's fixed exchange rate is abandoned either because of problems in economic fundamentals or because of a large shift in private sector expectations.

11In other words, $m_t = \ln (M_t/Y)$. It is therefore assumed that the income elasticity of money demand is unity. This assumption is consistent with rudimentary money demand regressions on monthly data for Argentina for the period 1991-1995, which yielded an estimate of 1.09 for that elasticity.

12A negative productivity shock is captured by $e_t^a > 0$ and causes a decline in money demand.

13This assumption is innocuous as the present set-up is primarily designed for fixed exchange-rate systems with arrangements similar to a currency board.

14See Krugman (1979).
exchange rate system. This is made clear in the next key equation of the model, the interest parity condition:

\[ i_t = i^* + E_t(s_{t+1}^a - s_t^a) + z[b_t^a Var_t(s_{t+1}^a) + b_t^m Cov_t(s_{t+1}^a, s_{t+1}^m)]. \] (2)

According to Equation (2) above, the domestic interest rate \( i_t \) has to exceed the constant safe rate of return on U.S. bonds \( i^* \) by an amount that compensates for two factors: the expected rate of depreciation of the domestic currency and the risk incurred by holding Argentine bonds.\(^{15}\) The expected rate of depreciation is measured by \( E_t(s_{t+1}^a - s_t^a) \), where \( s_t^a \) is the logarithm of Argentina's exchange rate expressed in pesos per dollar. The risk premium itself has two components and can be derived from a two-period utility-maximization problem of an investor allocating her wealth across holdings of a safe bond such as US treasury bills and bonds of two emerging markets, say Argentina and Mexico.\(^{16}\) The first component accounts for the adverse effect of an increase in the GDP ratio of government debt \( b_t^a \). The higher the expected volatility of the future domestic exchange rate, \( Var_t(s_{t+1}^a) \), the greater the degree of uncertainty facing investors and therefore the greater the risk resulting from an expansion in public debt.\(^{17}\) The second component of the risk premium accounts for the possibility of contagion effects resulting from a crisis in Mexico. Recall that, as explained in the introduction, imperfect information leads agents to adopt investment decisions based on rumors whose veracity would be too costly to ascertain. In this model, the existence of negative rumors circulating about Argentina’s peg (after the Mexican crisis takes place) is captured by investors’ ex-ante belief that Argentina and Mexican exchange rates covary positively. Given this positive covariance of the two rates, once a crisis in Mexico happens, it will raise expectations about an increase in the value of the exchange rate—in other words an abandonment of the peg—in Argentina as well. Formally, herding behavior in the wake of the Mexican crisis is captured by a positive value for \( Cov_t(s_{t+1}^a, s_{t+1}^m) \) in equation (2), and translates into an increase in the risk premium on domestic assets.\(^{18}\) The higher the GDP ratio of Mexican debt in circulation \( b_t^m \), the larger the premium required on holdings of Argentine bonds. Indeed, an increase in \( b_t^m \) is presumably associated with a greater risk of collapse of the peg in Mexico, or also with a larger depreciation of the short run equilibrium value of the

\( b_t^a \) is the ratio of total public debt to GDP in Argentina; \( b_t^m \) is the Mexican counterpart.

\(^{15}\) The derivation is explained in the Appendix.

\(^{16}\) The role of the volatility of financial variables in the occurrence of crises is not implausible. Indeed, using simple calculations, Calvo (1995b) shows that the probability of a crisis rises with the volatility of real M2.

\(^{18}\) In the 1990s, as most Argentine sovereign bonds have been issued in foreign currency, the risk associated with an increase in such debt is primarily a default risk. Hence in the application to Argentina in the next section, this covariance can be interpreted as a proxy for default risk.
post-crisis floating exchange rate there; this in turn increases the risk of a currency collapse in Argentina, given the positive covariance of the two countries’ exchange rates. $z > 0$ captures the degree of risk aversion of investors, with a value of zero indicating risk neutrality.

Domestic prices are set one period ahead. This rule allows deviation from PPP in order to capture the real appreciation usually observed in a fixed exchange rate system when pressures of a crisis arise:

$$p_t = E_{t-1} s_{t}^{a}. \quad (3)$$

Argentine and Mexican public debt$^{19}$ and the post-collapse Mexican exchange rate follow the stochastic processes specified below:

$$
\begin{align*}
    b_t^a &= \mu + \rho b_{t-1}^a + \gamma e_t^a, \quad \gamma > 0 \\
    b_t^m &= k + b_{t-1}^m + h e_t^m, \quad h > 0 \\
    s_t^m &= s_{t-1}^m + c e_t^m + d e_t^a, \quad c, d > 0 
\end{align*}
\quad (4)
$$

The model allows for two real disturbances, one affecting the Argentine economy ($e_t^a$) and the other affecting the Mexican economy ($e_t^m$). These two shocks are assumed to be white noise and independently distributed. In the last equation in (4), the post-collapse floating exchange rate that results in Mexico depends not only on the country's own shock but on Argentina's as well, because of the herding behavior of investors. As indicated in (1) and the first equation in (4), the real shock in Argentina affects both money demand and fiscal accounts. A negative productivity shock, for example (captured by a positive value of $e_t^m$), would both reduce money demand as well as raise the amount of domestic public debt issued (in order to cover the budget deficit increase). An alternative interpretation for the disturbance $e_t^m$ can be based on policy factors. A positive value of $e_t^m$ could capture an expansionary shift in fiscal policy, for example. Such a policy change would first increase the budget deficit finance and hence raise $b_t^a$. In fragile economies prone to money finance (a common characteristics of countries that adopt a fixed exchange rate system to control inflation) it would also cause the public to reduce its money holdings, expecting that the exchange rate peg cannot be sustained for long.

An explicit policy rule implemented by the Argentine government is needed to close the model. It will be assumed that the monetary authority defends the value of the peg as long as its reserves exceed a strictly positive threshold $\bar{r}$. In other words, as long as reserves are greater than $\bar{r}$, monetary policy is completely passive and accommodates any shifts in money demand so as to preserve the fixed exchange rate regime. If reserves fall below that threshold,

$^{19}$As in Flood and Marion (1998), it is assumed that there is a bond-financed fiscal deficit, but that government spending is also partly covered by tax levies which increase as government bond issues increase so as to satisfy transversality conditions.
however, the government would be ready to relinquish the peg as interest rates would have reached extremely high levels by then, hence choking economic activity.\textsuperscript{20,21} Furthermore, it is assumed that subsequent to a successful attack, the monetary authority will float the currency.

B. Origin of a Crisis

In case there is no crisis in Mexico and therefore no contagion effect at work, the covariance term in the risk premium is zero. The model becomes similar to the framework analyzed by Flood and Marion (1998), in which crises can result because of a shock to the domestic economy or because of weak domestic fundamentals; alternatively, crises could also be due to a shift in investors expectations regarding the future variance of the exchange rate.\textsuperscript{22}

The case that will be examined in this paper relates to the occurrence of a contagion-driven crisis in Argentina after a collapse of the peg in Mexico takes place. To fix ideas, the channel leading to a crisis in such a case is now explained in the light of the model of the previous section. As mentioned earlier, contagion works itself into the domestic economy through an increase in the risk premium on public bond holdings.\textsuperscript{23} The collapse of the peg in Mexico has

\[ \text{The precise magnitude of the threshold } \bar{f} \text{ depends on government preferences regarding the trade-off between recession or financial sector problems on the one hand and the value of preserving the fixed exchange rate system on the other. Note that if access to foreign borrowing is available, } \bar{f} \text{ could even be negative.} \]

\[ \text{This policy rule seems plausible in the Argentine case: in 1995 the government increased Central Bank credit to banks to contain the financial crisis. Although that did not violate the convertibility rule, it seemed to have been interpreted by the public as a sign that the government would be willing to introduce some flexibility in the currency board system in order to relieve economic pressures. See Uribe(1996) and D’Amato et al (1997) for an account of the events surrounding that episode.} \]

\[ \text{Flood and Marion (1998) find that in the absence of real economic shocks a crisis could occur either because of a large shift in investors expectations (triggered by a domestic economic shock not explicitly accounted for) or because of weak fundamentals (which they capture by a large value of domestic debt holdings relative to the worldwide stock of bonds). They distinguish a range of values of the fundamentals for which there is a possibility for a self-fulfilling crisis to succeed: a shift in expectations can make these fundamentals look precarious in an otherwise viable environment.} \]

\[ \text{Note that the risk premium could increase as a result of fundamental problems within the domestic economy. The lack of fiscal discipline, for example, could lead to an increase in the stock of public bonds in circulation which raises that premium. Hence both contagion effects and effects of domestic fiscal expansion work through the same mechanism in this model. Note also that another type of domestic problems leading to a crisis could arise from the real (continued...)} \]
yielded a floating exchange rate with a stochastic process described by the last equation in (4). Herding behavior leads investors to expect a collapse in Argentina with positive probability, forming expectations about both (i) the variance of the resulting exchange rate in Argentina \( \text{Var}_t(s_{t+1}^a) \) and (ii) its covariance with the Mexican exchange rate \( \text{Cov}_t(s_{t+1}^a, s_{t+1}^m) \). The greater the investors' anticipated value of the covariance of the two exchange rates (in other words the stronger the effect of herding behavior of investors), the greater the premium they require on their holdings of Argentine bonds and hence the greater the risk premium these bonds have to pay to encourage investors to hold them. This feeds into the money market through the interest rate and will lead to reserve losses, causing a crisis when the lower bound on reserves \( \mathfrak{r} \) is attained. Note that with this set-up the probability of a crisis occurring in Argentina at some undefined point in the future is equal to 1 when the shocks in the model are not bounded. Hence the question this paper addresses is not whether Argentina will ever experience a crisis. It is rather whether Argentina will experience a crisis in the next period given the occurrence of a crisis in Mexico today and assuming no further changes in domestic economic conditions.\(^{23}\) Furthermore, the issue is whether there is a role for fundamentals in either contributing to the occurrence of the crisis in Argentina or preventing the collapse of the peg.

C. Analytical Solution

The model is now solved analytically but as no closed form solution is possible, it will be simulated numerically, based on parameter estimates that reproduce Argentina's economy in the 1990s. The purpose will be to shed some light on the financial turmoil that took place in Argentina in March 1995 and its linkages with the Mexican 1994 crisis.

The solution procedure is analogous to the one formulated by Flood and Marion (1998) and previously used in the context of monetary models of the type developed in this paper.\(^{24}\) The core of this procedure is to solve for the shadow value of the Argentine exchange rate, \( s_n \), defined as the floating rate that would prevail if a successful attack takes place.\(^{25}\) Naturally an

\(^{23}\)(...continued)

sector disturbance \( e^a \).

\(^{24}\)Besides, to consider the occurrence of a crisis further in the future one has to formulate alternative policy rules to account for potential changes in domestic policies as the government anticipates contagion effects to result from the Mexican crisis. For example, the option of reducing debt (in other words tightening fiscal policy) in response to a foreign crisis can be used in order to avoid a full-fledged domestic crisis, when the economy settles in a "fundamentals" equilibrium.

\(^{25}\)See, for example, Flood and Garber (1984), or Blanco and Garber (1986).

\(^{26}\)For convenience, the superscript \( a \) will be dropped from the symbol of Argentina's exchange (continued...
attack takes place whenever it is worthwhile for investors to make it happen. In other words, a necessary and sufficient condition for an attack to occur is that \( \bar{s}_t > \bar{s}_t \) where \( \bar{s}_t \) is the fixed Argentine exchange rate. This proposition allows to solve for \( \bar{s}_t \) as a function of the other variables in the model. Once this is done, one can determine the range of values of the parameters and fundamentals for which an attack will take place.

By definition, \( \bar{s}_t \) equilibrates the money market when reserves have reached their lower bound \( \bar{r} \) and the peg is abandoned. Therefore, substituting equations (2) and (3) in the equilibrium condition (1) gives the following equation that determines the value of \( \bar{s}_t \):

\[
\bar{r} - E_{t-1}\bar{s}_t = -\alpha i^* - \alpha E_t(\bar{s}_{t+1} - \bar{s}_t) - \alpha z [b_t^a V\text{ar}(\bar{s}_{t+1}) + b_t^m Cov(\bar{s}_{t+1}, \bar{s}_t^m)] + \delta e_t^a. \tag{5}
\]

A couple of steps are needed in order to derive the value of \( \bar{s}_t \) from (5). First, note that

\[
E_{t-1}\bar{s}_t = (1 - \pi_{t-1})\bar{s} + \pi_{t-1}E_{t-1}(\bar{s}_t/\bar{s}_t > \bar{s}), \tag{6}
\]

where \( \pi_{t-1} \) is the period \( t-1 \) probability that a collapse will occur in period \( t \). Next, a linearization of the second term in equation (6) yields:

\[
\pi_{t-1}E_{t-1}(\bar{s}_t/\bar{s}_t > \bar{s}) = -\bar{s}(\bar{s} - \bar{s}_t) + \bar{s} \pi_{t-1} \tag{7}
\]

where \( \bar{s} \) is the average probability of an attack occurring in the next period and \( \bar{s}_t \) is the average value of the expected exchange rate next period conditional on an attack. To further aid in the solution, it is assumed that the post-attack variance of Argentina's exchange rate and its covariance with the Mexican rate are constant. Equations (6) and (7) can then be substituted in (5) to obtain an equation with one unknown, the shadow value of the exchange rate \( \bar{s}_t \). A reasonable guess for the solution form of the shadow exchange rate is

\[
\bar{s}_t = \lambda_0 + \lambda_1 b_{t-1}^a + \lambda_2 b_t^m + \lambda_3 e_t^m + \lambda_4 e_t^a. \tag{8}
\]

The Appendix shows the details of the steps followed in order to solve for the \( \lambda \)'s in (8), given the processes of \( b_t^a, b_t^m \) and \( s_t^m \) in (4). To simplify the analytical solution, the error terms

\[26(\text{continued})\]

rate in the rest of the paper. The Mexican rate will still be indicated by the superscript \( m \) to distinguish between the two rates.
" and $e_i$ are assumed to be identically distributed according to a uniform density with support $[-w;w]$, with $E(e_i) = E(e_i) = 0$. The values obtained for the $\lambda_i$ parameters are the following:

$$
\lambda_0 = \frac{1}{\pi/2 + 1/4 + \alpha} [(1 + \alpha)(\pi + \alpha i) + \alpha \beta_1 \mu + \alpha \beta_2 k + \alpha z \mu w^2 (\beta_3^2 + n + 1 \beta_2^2)/3
$$
$$
+ \alpha z kw^2 (\beta_3 c + \beta_4 d n + 1)/(n - 1) - 3\bar{s}/4 + \pi s/2 - \lambda_3 w/4].
$$

$$
\lambda_1 = \frac{\alpha p (z (\beta_3^2 + \beta_4^2)(n + 1)/(n - 1)) w^2/3 + \beta_1)}{\pi/2 + 1/4 + \alpha}
$$

$$
\lambda_2 = \frac{\alpha d (z (\beta_3 c + \beta_4 d(n + 1)/(n - 1)) w^2/3 + \beta_2)}{\pi/2 + 1/4 + \alpha}
$$

$$
\lambda_3 = h[z (\beta_3 c + \beta_4 d(n + 1)/(n - 1)) w^2/3 + \beta_2].
$$

$$
\lambda_4 = \gamma [z (\beta_3^2 + \beta_4^2(n + 1)/(n - 1)) w^2/3 + \beta_1] - \delta/\alpha.
$$

Section B of the Appendix shows that there can be at most two different solutions for equations (9) above, yielding two possible equilibria for $\bar{s}$, given a specific parameter configuration. Since it was impossible to derive a closed form solution analytically, the model was calibrated to reproduce the Argentine and Mexican economies, as specified in Section C of the Appendix. Before turning to the corresponding results, it is useful to note that there are three different types of equilibria that could occur; these will be referred to as “collapse,” “no-collapse” and “fundamentals” equilibria. A “collapse” equilibrium is one in which the economy inevitably experiences a speculative attack as a result of contagion. This is the case when the parameters of the model are such that $\lambda_0 > \bar{s}$. A “no-collapse” equilibrium is the opposite case where contagion can never affect the domestic economy and hence no attack occurs, and this corresponds to a parameter configuration yielding $\lambda_0 < \bar{s}$ and $\lambda_1, \lambda_2 = 0$. A “fundamentals” equilibrium corresponds to a case where the probability of collapse is between 0 and 1, and is a positive function of the level of domestic debt and real economic shocks. In such a case, $\lambda_0 < \bar{s}$ but $\lambda_1, \lambda_2$ are significantly different from 0. In a “fundamentals” equilibrium, the lower the level of initial debt, the lower the probability of a speculative attack triggered by contagion. In this case, fundamental factors play an important role in deterring such attacks should they threaten to propagate across countries.

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27To aid in the analytical solution, the continuous uniform distribution of $e_i$ will be approximated by a discrete $n$-point uniform distribution over the same interval $[-w;w]$. See section B of the Appendix for details. In the numerical solution, however, $n$ is allowed to go to infinity.
III. CONTAGION: A NUMERICAL EXAMPLE

Table 3 in the Appendix provides most of the parameter values employed in the calibration, along with their respective sources. In order to complete the calibration, values are needed for the following parameters: \( z \) (the risk aversion parameter), \( c \) and \( d \) (the parameters indicating the sensitivity of Mexico's post-collapse floating exchange rate to real Mexican and Argentine shocks respectively), \( \delta \) (the parameter controlling the reaction of money demand to real economic shocks in Argentina) and \( \gamma \) and \( h \) (the parameters indicating the effect on public debt of real domestic shocks in Argentina and Mexico respectively). \( \delta, \gamma \) and \( h \) capture the sensitivity of the economy to domestic shocks, with higher values indicating greater effect of the latter shocks compared to foreign shocks. Obtaining precise values for the five parameters above is virtually impossible; hence the model was simulated for all possible combinations of parameter values drawn from the following sets: \( \delta \in [0.05, 0.10, 0.15, 0.20, ... , 0.95, 1] \); \( \gamma \in [0.1, 0.3, 0.5, ... , 4.7, 4.9] \); \( \delta \in [0.01, 0.04, 0.07, ... , 0.97, 1] \). Solutions were computed for each possible vector \( (\delta, \gamma, h, z, c, d) \) drawn from the above sets and a tally of the results is given in the first row of Table 1. \(^{28}\) The remaining rows of the table provide an idea of the sensitivity of the results to a change in the degree of risk aversion \( z \) or in the degree of responsiveness of the economy to domestic shocks, as captured by \( \gamma \) and \( h \). Solutions were computed by fixing one parameter value as indicated in the first column of the table and letting all other parameters vary across their respective ranges. The results are given in rows (2)–(7) of the table.

Table 1 shows that the solution to the model, when calibrated to the Argentine and Mexican economies in 1995, is generally characterized by two possible equilibria: a “no-collapse” equilibrium and a “collapse” equilibrium. Indeed, 50 percent of acceptable solutions are associated with multiple equilibria of these two types. 28 percent of the acceptable cases turn out to produce multiple equilibria of the “no-collapse” and “fundamentals” types, however. In these cases a speculative attack either never occurs or only takes place if domestic fundamentals are weak enough. \(^{29}\) Rows (2)–(4) indicate that an increase in the degree of risk

\(^{28}\)The table entries refer to the percentage of acceptable solutions with characteristics corresponding to the column headings. These characteristics are as follows: \( 2yc \) indicates solutions that correspond to two "collapse" equilibria; \( 2nc \) indicates solutions that correspond to two “no-collapse” equilibria; \( 1nc1yc \) indicates solutions that correspond to one “collapse” and one “no-collapse” equilibria; \( 1nc1fe \) indicates solutions that correspond to one “no-collapse” and one “fundamentals” equilibria; and \( 1yc1fe \) indicates solutions that correspond to one “collapse” and one “fundamentals” equilibria. The ratio of solutions corresponding to two “fundamentals” equilibria was always negligible if not zero, so it is not reported here.

\(^{29}\)Note however that the model does not allow for a banking sector, hence the fundamentals considered here are restricted to characteristics of the fiscal and monetary policies followed at the time. It would be interesting to find out whether the likelihood of a “fundamentals” (continued...)
aversion is associated with an increase in the risk of settling at a collapse equilibrium. It is also interesting to note that a greater effect of domestic disturbances on the economy is unquestionably associated with a shift to a “no-collapse” equilibrium, as implied by rows (5)–(7) of Table 1. Indeed, an increase in $\gamma$ and $h$ represents a greater relative effect of domestic shocks and therefore greater isolation of the economy from foreign disturbances, and hence less susceptibility to contagion.

<table>
<thead>
<tr>
<th>(1)</th>
<th>All parameter ranges</th>
<th>2yc</th>
<th>2nc</th>
<th>Inclyc</th>
<th>Inclfe</th>
<th>IycIfe</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2)</td>
<td>$z=1$</td>
<td>4</td>
<td>32</td>
<td>47</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>(3)</td>
<td>$z=3$</td>
<td>9</td>
<td>33</td>
<td>47</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>(4)</td>
<td>$z=5$</td>
<td>8</td>
<td>30</td>
<td>54</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>(5)</td>
<td>$\gamma=h=0.1$</td>
<td>4</td>
<td>1</td>
<td>85</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>(6)</td>
<td>$\gamma=h=0.5$</td>
<td>0</td>
<td>61</td>
<td>0</td>
<td>39</td>
<td>0</td>
</tr>
<tr>
<td>(7)</td>
<td>$\gamma=h=0.9$</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. Classification of Feasible Equilibria

Given the existence of multiple equilibria for the Argentine economy subsequent to the Mexican crisis, would it be possible to determine the one at which the economy is expected to settle? In other words, is there a factor that explains why the Argentine economy experienced a crisis in 1995 instead of moving to a “no-collapse” equilibrium?\(^{30}\) It turns out that the...

\(^{29}\)(...continued)
equilibrium would increase if the model is modified to account for the existence of a banking sector, given the simultaneous banking crisis that took place in Argentina at the time. This however is an extension beyond the scope of this paper.

\(^{30}\)Note that the financial turmoil in Argentina did not lead to an abandonment of the currency board system then, and the currency peg still holds. Although this seems not to fit the criteria for a crisis as specified in the model, it is important to recall that steps were implemented that succeeded at fending off speculators. In contrast, the model of the preceding section assumes that no policy intervention takes place to counteract speculative attacks. Hence, although the attack was not successful at breaking down the peg, the Argentine episode can still be

(continued...)
equilibria produced by the model can be distinguished by the associated variance of Argentina's exchange rate and its covariance with the Mexican rate. Indeed, all “no-collapse” equilibria corresponded to low variance and covariance values, in the order of 0.01, whereas the other two types of equilibria generally corresponded to variances and covariances at least ten times greater in magnitude. The interpretation of this observation is straightforward: in periods of low uncertainty, captured by relatively low values of the exchange rates’ variance and covariance, the risk premium is not greatly affected by the occurrence of a foreign crisis, and hence does not increase enough to lead to large pressures in domestic financial markets. In other words, “no-collapse” equilibria are the product of low uncertainty which minimizes herding behavior. Hence, since a crisis occurred in Argentina in 1995, then it must be the case that credibility in the currency board arrangement was weak, creating uncertainty about the sustainability of the exchange rate peg.

At this point an important question arises: Is weak credibility the main culprit for the Argentine crisis? Weren't there additional important factors at work that contributed to the turmoil? In order to examine that question, it was essential to assume a quantitative measure for the uncertainty prevalent at the time so as to focus uniquely on the equilibria that could arise in case of lack of sufficient credibility. It seems plausible to derive this value from the historical experience of Argentina with exchange rate volatility. Argentine data indicates that the black market rate had a sample variance of 0.15 over the period 1970:1-1995:5; furthermore, the covariance of that rate with the Mexican exchange rate over periods of relatively floating rates in the latter country ranges from about 0.46 to 0.53, depending on the definition of these periods. Accordingly, it was assumed that the solutions producing a variance of the Argentine exchange rate of 0.12 to 0.18 and a covariance of that rate with the Mexican rate of 0.42 to 0.58 would be the ones associated with significant uncertainty, and hence weak credibility. Table 2 presents a tally of this restricted set of solutions: these were computed for all combinations of (δ, γ, h, c, d) drawn from the sets specified above, and for values of z specified in the first column of the table.

\[\ldots \text{continued}\]

considered a crisis given the observed speculation against the currency and assuming that a collapse could have occurred had there been no government initiative to address the situation.
Table 2. Classification of Feasible Equilibria
Restricted Variance and Covariance

<table>
<thead>
<tr>
<th>Risk Aversion Parameter</th>
<th>% of solutions with &quot;collapse&quot; equilibrium</th>
<th>% of solutions with &quot;no-collapse&quot; equilibrium</th>
<th>% of solutions with &quot;fundamentals&quot; equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>z=1</td>
<td>0</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>z=2</td>
<td>0</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>z=3</td>
<td>0</td>
<td>33</td>
<td>67</td>
</tr>
<tr>
<td>z=4</td>
<td>86</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>z=5</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2 shows that weak credibility is not a sufficient condition for a crisis to result in Argentina as a result of a crisis in Mexico. The degree of risk aversion of investors, $z$, has to be high enough to produce a "collapse" equilibrium. Indeed, if $z$ is close to 5, the majority of solutions are of the "collapse" equilibrium type. If however it is around 1 or 2, then the economy will rather move to a "no-collapse" equilibrium.\(^{31}\) Accordingly, in addition to economic uncertainty, a relatively large degree of risk aversion of investors must have contributed to the Argentine crisis of 1995. Moreover, it is important to note that the role of fundamentals is not completely ruled out in this setting as for an intermediate range of risk aversion these turn out to be crucial in determining whether the crisis would take place or not. Hence, if investors are not sufficiently risk averse, then the 1995 crisis must have been driven by a combination of lack of credibility in the exchange-rate system, risk aversion of investors, and weak domestic economic fundamentals.\(^{32}\)

\(^{31}\) As a sensitivity check, another set of solutions was defined, based on lower bands for the variance and covariance. The results showed that the lower the band adopted, the greater the degree of risk aversion $z$ need to produce a collapse equilibrium. In other words, the lower the degree of uncertainty (and hence the greater the confidence attached to the currency peg) the higher the degree of risk aversion needed to produce a collapse equilibrium.

\(^{32}\) In this respect, it would be useful to derive a methodology that would allow to estimate the degree of risk aversion $z$. Given the linearizations and other assumptions involved in solving the background investor model to obtain the risk premium (see the Appendix) such an endeavor is not straightforward and is beyond the scope of this paper. Calvo and Mendoza (1997) performed calculations leading to the conclusion that the coefficient of risk aversion in their model is within the range $[0;1]$. It is hard to map this result into the present model to specify a plausible range of values for $z$ because of the differences between their (continued...)
IV. CONCLUSION

This paper presented a framework illustrating the transmission of currency crises across countries. Contagion was modeled as the consequence of rational herding behavior of investors who diversify risk by holding assets of several countries and form ex-ante expectations about the co-movement of exchange rates across these countries, possibly because of lack of accurate information. A crisis in one country leads to an increase in the likelihood of a crisis in another country when investors expect a positive covariance of these countries' exchange rates. This contagion phenomenon translates into an increase in the risk premium required to hold domestic bonds. The resulting pressures of a crisis are transmitted to the money market through the domestic interest rate. For a benchmark of parameter values, calibrated to reproduce the Argentine economy, the model yields multiple equilibria. The different equilibria that the economy settles at are associated with different levels of uncertainty prevailing in the economy, as captured by investors’ expectations regarding the co-movement of the two countries’ exchange rates. In particular, the “no-collapse” equilibrium corresponds to a relatively low degree of uncertainty. If therefore one believes that the Convertibility plan in Argentina was credible enough to reduce exchange-rate uncertainty to near-zero levels then the model implies that contagion was not the main cause for the Argentine financial turmoil of 1995. However, if one is to extrapolate from historical values of the exchange rate variance in Argentina and its covariance with the Mexican rate over the non-fixed periods in 1970–95, then the model provides a rationale for the argument that contagion contributed to the turmoil, but only if investors are believed to be sufficiently risk averse.

It is important to note that the economic fundamentals accounted for in the paper, namely the stock of public debt, do not seem to play a critical role in determining whether or not the Argentine economy is driven to a crisis equilibrium. This is not surprising because that aspect of the economy was not problematic at the time. Argentine’s financial sector fundamentals, however, were deteriorated in 1995. Hence it would be useful to extend the model to employ a broader spectrum of fundamentals that would account for the state of the private banking sector in addition to government fiscal accounts.

\[^{32}\text{(...continued)}\]

model and the one presented in the Appendix. However, back-of-the-envelope calculations seem to indicate that \( z \) cannot exceed the range of values covered in Table 2.
A. Derivation of the Risk Premium

A rationale for the risk premium adopted in equation (3) in the text can be derived as follows. Consider the two-period problem of an investor choosing to allocate a given stock of wealth \( W_0 \) across one-period holdings of Argentine bonds \( B^a \), Mexican bonds \( B^m \), and U.S. Treasury bills \( B^t \). All bonds are denominated in the currency of the respective country of origin. The predetermined rates of interest these bonds pay are \( i^a \), \( i^m \) and \( i^t \) respectively. \( X^a \) is the dollar value of the Argentine peso and \( X^m \) that of the Mexican peso. The investor maximizes the mean-variance utility function

\[
U_0 = E_0(W_1) - gV(W_1/W_0)
\]

subject to the wealth constraint

\[
W_0 = B^t + B^a X^a_0 + B^m X^m_0.
\]

Second period wealth is given by \( W_1 = B^t (1+i^t) + B^a (1+i^a) X^a_1 + B^m (1+i^m) X^m_1 \). The first order conditions with respect to \( B^a \) and \( B^t \) can be combined to yield the following form for the risk premium required on Argentine bonds:

\[
(1+i^a) \frac{X_1^a}{X_0^a} - (1+i^t) = \frac{2g(1+i^a)}{W_0^2} [B^a X_0^a (1+i^a) V \left( \frac{X_1^a}{X_0^a} \right) + B^m X_0^m (1+i^m) COV \left( \frac{X_1^a}{X_0^a}, \frac{X_1^m}{X_0^m} \right)]. \tag{12}
\]

The higher the expected variance of the future exchange rate in Argentina or the higher the stock of Argentine bonds in circulation, the higher the risk premium. This is the component of the premium compensating for "own-country" risk. The second component of the premium compensates for "contagion" risk. Allowing for contagion in this set-up is equivalent to assuming that the stochastic processes of the Mexican and Argentine exchange rates are interdependent such that \( COV \left( X_1^a / X_0^a, X_1^m / X_0^m \right) > 0 \). Hence contagion works its way into the domestic economy by raising the premium required on holding domestic bonds.

In order to transform the risk premium given by the right-hand side of equation (12) into the form adopted in equation (2) of the text, the following set of notations and assumptions are used:
\[ \bar{\bar{\bar{i}}}^a = \bar{\bar{i}}^b = \bar{\bar{i}} = i^* \]
\[ z = 2g(1 + \bar{i})^2 / W_0 \]
\[ V_i(X_{t+1}^a) = V_i(s_{t+1}^a) \]
\[ \text{COV}_i(X_{t+1}^a, X_{t+1}^m) = \text{COV}_i(s_{t+1}^a, s_{t+1}^m) \]  

(13)

\[ \frac{B_t^a X_t^a}{W_t} = b_t^a \]
\[ \frac{B_t^m X_t^m}{W_t} = b_t^m \]

A bar above a variable indicates a steady state value. \( S^a = 1/X^a \) and \( S^m = 1/X^m \) are the respective exchange rates of Argentina and Mexico, and \( s^a \) and \( s^m \) their natural logarithms. In the last two equations in (13), \( b_t^a \) and \( b_t^m \) represent the investor’s exposure to the debt of the two countries. They will be proxied by the GDP ratios of Argentine and Mexican bonds respectively.

B. Derivation of the Shadow Value of the Exchange Rate

The solution for \( \bar{s}_t \) is found using equations (4)-(7). Assuming that the variance and covariance terms in the premium are constant after the attack, one can stipulate a linear solution to the model of the following form:

\[ \bar{s}_t = \lambda_0 + \lambda_1 b_{t-1}^a + \lambda_2 b_{t-1}^m + \lambda_3 e_t^m + \lambda_4 e_t^a. \]

In order to solve for \( \bar{s}_t \), it is necessary first to determine \( \pi_{s,t}, E_{s,t} (\bar{s}_t / \bar{s}_t > \bar{s}) \) and \( E_t (\bar{s}_{t+1} - \bar{s}_t) \). To aid in the determination of these three expressions, it is useful to approximate the continuous uniform distribution of the error term \( e_t^a \) by a discrete uniform distribution, while maintaining the assumption of a continuous uniform distribution for \( e_t^m \) over the interval \([-w;w] \). Hence the density functions of \( e_t^a \) and \( e_t^m \) are given by \( g(.) \) and \( f(.) \) defined as follows.\(^{33}\)

\(^{33}\)In the density \( g(.) \), \( n \) is restricted to the set of odd integers greater than 1.
\[ g(e^a) = \frac{1}{n} \quad \text{for } e^a \in \Omega^a = [0, \pm \frac{2}{n-1} w, \pm \frac{4}{n-1} w, \pm \frac{n-3}{n-1} w, \pm w] \]
\[ 0 \quad \text{otherwise.} \]
\[ f(e^m) = \frac{1}{2w} \quad \text{for } e^m \in [-w, w] \]
\[ 0 \quad \text{otherwise.} \]

Given \( g(.) \) and \( f(.) \), the time \( t-1 \) probability of a crisis occurring in period \( t \) can be solved for as follows:

\[ \pi_{t-1} = Pr(\bar{s}_i > s) \]
\[ = Pr(\lambda_3 e^m_t + \lambda_4 e^a_t > s - \lambda_0 - \lambda_1 b^a_{t-1} - \lambda_2 b^m_{t-1}) \]
\[ = \sum_{x \in \Omega^a} [Pr(e^m_t > (s - \lambda_0 - \lambda_1 b^a_{t-1} - \lambda_2 b^m_{t-1} - \lambda_4 x)/\lambda_3) Pr(e^a_t = x)] \]
\[ = \frac{1}{n} \sum_{x \in \Omega^a} Pr(e^m_t > J_{t-1} - (\lambda_4/\lambda_3) x) \]
\[ = \frac{1}{n} \sum_{x \in \Omega^a} (1 - F(J_{t-1} - (\lambda_4/\lambda_3) x)) \]

where \( J_{t-1} = (s - \lambda_0 - \lambda_1 b^a_{t-1} - \lambda_2 b^m_{t-1}) / \lambda_3 \) and \( F \) is the cumulative density function associated with the uniform density function \( f \) of \( e^m_t \). Solving the above sum yields the following:

\[ \pi_{t-1} = (w\lambda_3 - \bar{s} + \lambda_0 + \lambda_1 b^a_{t-1} + \lambda_2 b^m_{t-1}) / 2w\lambda_3. \]  \hspace{1cm} (17)

Second,

\[ E_{t-1}(\bar{s}_i / \bar{s}_i > \bar{s}) = \lambda_0 + \lambda_1 b^a_{t-1} + \lambda_2 b^m_{t-1} + E_{t-1}(\lambda_3 e^m_t + \lambda_4 e^a_t / \bar{s}_i > \bar{s}). \]  \hspace{1cm} (18)
But

$$E_{t-1}(\lambda_s e_t^m + \lambda_a e_t^a / \tilde{s} > \tilde{s}) = \sum_{x \in \Omega} E(\lambda_s e_t^m + \lambda_a x / \tilde{s} > \tilde{s} ; e_t^a = x) Pr(e_t^a = x)$$

$$= \frac{1}{n} \sum_{x \in \Omega} \int_{J_{t-1}}^{J_t} \frac{(\lambda_s e_t^m + \lambda_a x) f(e^m) de^m}{1 - F(J_{t-1} - (\lambda_a / \lambda_s)x)}$$

$$= (\tilde{s} + w\lambda_3 - \lambda_0 - \lambda_1 b_{t-1}^a - \lambda_2 b_{t-1}^m) / 2.$$

(19)

Hence

$$E_{t-1}(\tilde{s} / \tilde{s} > \tilde{s}) = (\lambda_0 + \lambda_1 b_{t-1}^a + \lambda_2 b_{t-1}^m + s + w\lambda_3) / 2.$$ (20)

Next, let $\bar{s}_{t+1} = \beta_0 + \beta_1 b_t^a + \beta_2 b_t^m + \beta_3 e_t^a + \beta_4 e_t^m$ be the shadow exchange rate that equilibrates the money market at $t+1$ after an attack occurs in period $t$. Then $E_{t}(\bar{s}_{t+1} - \tilde{s}) = \beta_0 + \beta_1 (= \frac{\mu + \rho b^a + e^a}{\lambda_0} + \beta_2 (k + lb^m + e^m) - \lambda_0 - \lambda_1 b^a - \lambda_2 b^m - \lambda_3 e^a - \lambda_4 e^m$. The $\beta$ coefficients can be solved for by applying the method of undetermined coefficients to the money market equilibrium condition in period $t+1$. This yields the following values:

$$\beta_0 = \bar{s} + \alpha (i^* + \beta_1 \mu + \beta_2 k + zw^2/3 \frac{[\mu(\beta_3^2 + \beta_4^{n+1}) + k(\beta_3 c + \beta_4^{n+1})]}{n-1})$$

$$\beta_1 = \frac{\alpha \rho z \left[\beta_3^2 + \beta_4^{n+1} / (n-1)\right] w^2/3}{1 + \alpha(1 - \rho)}$$

$$\beta_2 = \frac{\alpha \rho z \left[\beta_3 c + \beta_4^{n+1} / (n-1)\right] w^2/3}{1 + \alpha(1 - \rho)}$$

$$\beta_3 = h[\beta_2 z + \beta_3 c + \beta_4^{n+1} / (n-1)] w^2/3]$$

$$\beta_4 = h[\beta_2 z + \beta_3 c + \beta_4^{n+1} / (n-1)] w^2/3] - \delta / \alpha$$

(21)

Note that

$$\text{Var}(\bar{s}_{t+1}) = \beta_3^2 \text{Var}(e_t^m) + \beta_4^2 \text{Var}(e_t^a) = \beta_3^2 \frac{w^2}{3} + \beta_4^2 \frac{n+1}{n-1} \frac{w^2}{3}$$

$$\text{Cov}(\bar{s}_{t+1}, e_t^m) = E[(\beta_3 e_t^m + \beta_4 e_t^a)(ce_t^m + de_t^a)]$$

$$= \beta_3 c^2 \frac{w^2}{3} + \beta_4 d^{n+1} \frac{w^2}{n-1}$$

(22)

Substituting (17), (20), (21), (4) and (6)–(7) in equation (5) and using the method of undetermined coefficients again gives the solution for the shadow rate value as described in equations (8)–(9) of the text.
Note that the last four equations in (21) imply that $\beta_3$ solves the following second-order equation:

$$
0 = \beta_3^3 \frac{w^2}{3} \frac{1 + \alpha}{(1 + \alpha - \alpha \rho)} \frac{d(n + 1)}{n - 1} + \frac{1}{d} \frac{3(1 + \alpha - \alpha \lambda)}{h(1 + \alpha)zw^2} \left[ -c \right]^2 ] - \beta_3 \frac{3(1 + \alpha - \alpha \lambda)}{h(1 + \alpha)zw^2} - c] - \frac{\delta}{\alpha}
$$

(23)

Finding a closed-form solution to the model entails finding first the roots of the above expression. Each acceptable root will determine a solution for $s$ that can be found by solving equations (21) and (9) for the $\beta$ and $\lambda$ coefficients using this value of $\beta_3$. Both roots are acceptable when the discriminant $\Delta$ of (23) is strictly positive. Assuming $n \to \infty$, this discriminant is as follows:

$$
\Delta = \left[ \frac{3(1 + \alpha - \alpha \lambda)}{h(1 + \alpha)zw^2} - c \right]^2 + \frac{4 \delta}{\alpha} \frac{w^2}{3} \frac{1 + \alpha}{(1 + \alpha - \alpha \rho)} \frac{d(n + 1)}{n - 1} \left[ \frac{1}{d} \frac{3(1 + \alpha - \alpha \lambda)}{h(1 + \alpha)zw^2} - c \right]^2.
$$

Since $\delta < 0$, the sign of $\Delta$ cannot be determined analytically. Hence the model is calibrated to the Argentine economy (see the following section) to find a numerical solution.

C. Parameter Values for the Calibration to Argentina’s Economy

Some Relevant Statistics, used in setting parameter values in Table 3.\textsuperscript{35}

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>$281$ billion</td>
<td>$31.9$ percent</td>
<td>$8.5$ billion</td>
<td>$1$ peso = $1$</td>
<td>$5.8$ percent</td>
<td>$31$ percent</td>
</tr>
<tr>
<td>Mexico</td>
<td>Total Debt/GDP (1994)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>USA</td>
<td>3-months TB interest rate (1994:12)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$5.64$ percent</td>
</tr>
</tbody>
</table>

\textsuperscript{34}The roots have to be real and positive to be acceptable from an economic point of view and be consistent with the model's assumptions.

\textsuperscript{35}Sources: World Development Indicators (World Bank, February 1997) and International Financial Statistics (IMF, 1998).
Table 3. Baseline Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value$^{36}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>GDP</td>
<td>$280$ bn</td>
</tr>
<tr>
<td>$\hat{R}$</td>
<td>Foreign exchange reserves threshold</td>
<td>$6$ bn</td>
</tr>
<tr>
<td>$r$</td>
<td>Log ($\hat{R}/Y$)</td>
<td>$-3.8$</td>
</tr>
<tr>
<td>$I^*$</td>
<td>US interest rate</td>
<td>$0.0564$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Average probability of collapse</td>
<td>$0.5\Phi$</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>Log of the fixed Argentine exchange rate</td>
<td>$0$</td>
</tr>
<tr>
<td>$w$</td>
<td>Width of the uniform distribution of the shocks</td>
<td>$2\Phi$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Semi-interest elasticity of money demand</td>
<td>$3.2^{37}$</td>
</tr>
<tr>
<td>$b^a_0$</td>
<td>Period-zero debt to GDP ratio in Argentina</td>
<td>$0.3$</td>
</tr>
<tr>
<td>$b^m_0$</td>
<td>Period-zero debt to GDP ratio in Mexico</td>
<td>$0.4$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Drift parameter of the AR(1) process for $b^a_t$</td>
<td>$0.42^{38}$</td>
</tr>
<tr>
<td>$k$</td>
<td>Drift parameter of the AR(1) process for $b^m_t$</td>
<td>$0.07$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>AR parameter of the process for $b^a_t$</td>
<td>$0.68$</td>
</tr>
<tr>
<td>$l$</td>
<td>AR parameter of the process for $b^m_t$</td>
<td>$0.83$</td>
</tr>
</tbody>
</table>

$^{36}$A '$\Phi$' marks a parameter value identical to the one adopted in Flood and Marion (1998).

$^{37}$Several authors estimated different forms of money demand functions for Argentina. Calvo (1996) and Calvo and Leiderman (1992) find the following values for $\alpha$: $3.18$, $3.2$ and $4.13$, and these compare to several other findings in the literature.

$^{38}\mu$, $\rho$, $k$ and $l$ were estimated by fitting an AR(1) process to quarterly data on the ratio of public debt to GDP in Argentina and Mexico respectively, for the period $1970:4$-$1994:4$. 

References


