IMF Working Paper

WP/99/39

INTERNATIONAL MONETARY FUND

Monetary and Exchange Affairs Department

A Model of the Lender of Last Resort

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March 1999

Abstract

This paper develops a model of the lender of last resort. It provides an analytical basis for "too big too fail" and a rationale for "constructive ambiguity". Key results are that if contagion (moral hazard) is the main concern, the Central Bank (CB) will have an excessive (little) incentive to rescue banks and the resulting equilibrium risk level is high (low). When both contagion and moral hazard are jointly analyzed, the CB's incentives to rescue are only slightly weaker than with contagion alone. The CB's optimal policy may be non-monotonic in bank size.

JEL Classification Numbers: E50, E58, G21, G28

Keywords: Banking Crises, Financial Contagion, Moral Hazard, Lender of Last Resort

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[1] We would like to thank William E. Alexander, Peter Clark, Tito Cordella, Gianni De Nicolo, Xavier Freixas, Philipp Hartmann, Charles Siegman, Geoffrey Wood, and participants at an IMF seminar for helpful comments and conversations, and Satvinder Singh for research assistance.

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I. Introduction

There have been few formal models seeking to explain and to analyze how and why central banks have provided lender of last resort (LOLR) services to individual commercial banks, even though such acts have been a regular, albeit often contentious, part of a central bank's armory since Bagehot (1873). One reason why there have been few formal models of LOLR is that many, perhaps most, economists in this field believe that the provision of LOLR to individual banks is fundamentally misguided.

All economists accept the lessons of history that banking panics can occur, with depositors seeking to switch out of the deposits of (some, perceived as riskier) banks into currency, gold, foreign exchange or the deposits of those banks perceived as safer. But the argument of the above economists is that the central bank's role in such cases should be limited to open market operations (OMO), pumping extra cash into the system as a whole, in order to maintain the aggregate money stock at its desired level. As a generality the central banks should not lend to individual banks, e.g., through a discount window. In their view the market is as well or better informed, than is the central bank, about the relative solvency, or otherwise, of a bank short of liquidity. Given an aggregate sufficiency of high-powered money, illiquid (but solvent) banks will be able to borrow, e.g., in the interbank market, whereas potentially insolvent banks will be (appropriately) driven out of the system. Moreover, the monetary authorities will have certain (reputational) incentives to exercise forbearance (Kane, 1992) and rescue banks that should have been closed; and the pursuit of financial stability by direct intervention may, so it is argued, divert the central bank from achieving its primary goal of controlling the monetary aggregates so as to achieve price stability. See Bordo (1990), Humphrey (1989), Kaufman (1991), Schwartz (1988), amongst many others.[1]

Thus many, or most, "liberal" economists would prefer to leave the function of lending to individual banks to the market, rather than to central banks, restricting the latter to general open market operations. There are two ripostes to the "liberal" position. The first is to propose "market failure". Let us give a few practical examples of this.

When the Bank of New York computer malfunctioned in 1985, and would not accept incoming payments for bond market dealings, the resultant illiquidity position soon ballooned to a point where no one counterparty bank could take on the risk of making a sufficiently large loan. It would have required a coordinated syndicate, but such syndicates take time to organize, and time (as always) was scarce.

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[1] It was presumably on grounds such as these that some observers expect the European Central Bank not to assume any LOLR role, although the Maastricht Treaty (Articles 105.2, 105.5, 105.6) and the ESCB Status (Articles 22, 25.1) are rather ambiguous about this issue.

Next, in the aftermath of the BCCI failure, there were considerable deposit withdrawals from a string of small banks, run by Asians and serving the Asian community in the UK. They were (unjustly) tainted by association. They had relied, almost entirely, on deposits from the local community, and their names were not known in the wholesale banking market. Although illiquid, rather than insolvent, they were not getting help from the market, so the Bank of England assisted them.[2]

As a generality most central banks would argue that their supervisory role — or their ready access to supervisory information — should provide them with additional information, not available in the market. Moreover, when there is any large-scale need to redirect reserves, (as in the case of the Bank of New York, or in a potential market breakdown), there must be a coordination problem. No one commercial-counterparty can single-handedly assume the credit risk, and there is no incentive for a single commercial bank to take on the time, effort and cost of coordinating the exercise of sorting out the problem. The Bank of England would, we believe, tend to argue that most of its historical LOLR actions have primarily involved the provision of additional information combined with a coordinating role to encourage private sector financial institutions to resolve the problem, primarily by themselves,[3] as was also exemplified in the recent case of Long-Term Capital Management.

It would be an interesting historical exercise to go over a central bank's LOLR record to see how far such actions could be explained (justified?) as arising from particular examples of "market failure". There is a nice judgement to be made about the appropriate balance between "market failure" on the one hand and "official intervention failure" on the other hand.[4] But again this is not the purpose of this paper.

One of the rare recent examples of a formal model of LOLR is to be found in Freixas, Parigi and Rochet (1998). Using the framework of Diamond and Dybvig (1983), they analyze the moral hazard problem caused by bank managers' (stockholders') incentive to choose an inefficient technology that gives them some private benefit. This moral hazard problem, as in Holmstrom and Tirole (1998), sets an upper limit to the finance that would be provided at interim dates by outside investors. When liquidity shocks cannot be disentangled from solvency shocks, moral hazard on the commercial bank's investment creates a market failure. In the absence of central bank intervention there is excessive liquidation of banks: the optimal continuation threshold is above the (private) solvency threshold. Then "The role of the central

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[2] See, Bank of England (1996), "*Financial Stability review*, No. 1, Autumn; also see Bank of England, *Annual Report*, 1991-1992, and 1992-1993.

[3] The problem in the case of Barings was that there was insufficient information on the potential close-out cost of Leeson's derivative position. With the Bank of England (rightly) being unwilling to provide a guarantee to limit any such loss, no private institution was willing to buy Barings over the key weekend.

[4] See, for example, Bernanke (1983), Goodhart and Schoenmaker (1995), Gorton (1985), among others for related discussions.

banks is to mutualize the solvency shocks: lucky' banks will be taxed and unlucky' banks subsidized (first best contract)."

This is a stimulating and well-constructed model, but it does not address the macro-economic policy concerns of central banks, as the authors accept at the outset. They focus, instead, on the micro aspects of central banks' intervention. But their work did also provide yet another incentive, or goad, to attempt to model the main macro-policy consideration lying behind LOLR action; in our view LOLR has been primarily driven by macro, rather than micro, concerns. It is the purpose of this paper to model these.

So we wish to focus on our second reason for disputing the liberal position, of leaving it to the market to decide. Our main claim is that the liberal position is predicated on a certainty equivalent postulate, that is that the central bank is just as confident and knowledgeable about the optimal level of open market operations, high-powered money and aggregate money stock after the onset of bank failures and panic, as it would be if the panic was prevented. We find that, admittedly implicit rather than explicit, position difficult to accept. When failures occur, and people start to panic, their behavior is likely to become far less predictable. Policy mistakes become much more likely.

Let us take three examples. First, the bank failures in the USA in the 1930s shifted the high-powered money (H) to aggregate money (M) ratio. Although as Kaldor (1958) noted, the Fed's actions led to a much faster, than previous, growth in H during these years, M still fell. Second, after the 1987 stock market collapse, central banks lowered interest rates aggressively, scared of a replay of 1929, only to discover a couple of years later that they had overdone such ease. Third, in Japan now, some 90% of respondents to a survey in 1998 stated that they lacked confidence in their banks. Interest rates are rock bottom and H is growing very fast. What should the Monetary Policy Committee (MPC) of the Bank of Japan do? The published Minutes of the MPC reveal their uncertainties.

So the key, and original, feature in our model is that we formalize the loss, in the form of extra uncertainty that arises from allowing bank failures to occur. We assume that the central bank (CB) is trying to achieve an (exogenously given) desired (optimal) level of deposits in the system. When a failure occurs, of any size, the actual resultant change in deposits in the system as a whole, as depositors react, is an increasing function of the bank size, which function is known to the CB, and an additional stochastic component which also depends on the bank size.[5] The loss to the CB of getting macropolicy wrong is assumed to be quadratic. Since the CB knows the deterministic part of the change in deposit, it can immediately take open market operations to offset that known change, so the macro-policy loss from allowing failure to occur is simply due to the stochastic component, in fact to its variance.

[5] In addition to depositors' withdrawals, in a crisis commercial banks may also rationally withdraw from making new loans, as argued by Flannery (1996).

If a commercial bank comes to the CB seeking LOLR, and the CB turns it away, we assume that it will then certainly fail. If the bank receives such help, it may or may not turn out after the event to be insolvent. As has been argued by Goodhart (1988, 1995), the CB cannot discern exactly and immediately whether the commercial bank, coming to it for assistance, is illiquid **but** solvent, or illiquid **and** insolvent. Instead there is a probability (x), related to the overall riskiness of the banking system (h), that commercial bank will just be illiquid (S), or illiquid and insolvent (R). CB knows the value of x. If S happens, the CB faces no loss. If R happens and CB has rescued an insolvent bank, there is a cost to the CB. This cost may be reputational, or financial if the CB loses money when its LOLR loans cannot be fully repaid and/or if taxpayers' funds then have to be deployed.[6] We assume that there is a fixed (reputational) element and a proportional element to the cost (Z) to the CB of rescuing a failed bank, which also depends on bank size. When the CB is approached for LOLR assistance, it has (very quickly) to say no ($I_t = 1$), or yes ($I_t = 0$). The CB's problem is to choose I_t, so as to minimize the cost. The crucial aspect is that the cost of bank failure ($I_t = 1$) rises more rapidly with the size of the failing bank than the cost of bank rescue; we seek to justify this in more detail in Section 3.

We estimate this cost both in a simple, single period setting and in a dynamic setting where both the probability of a failure (of size j) in any period, p_t, and the likelihood of a bank requiring LOLR being insolvent (x_t), are a function of CB's prior actions, which then influence the actions of banks and depositors. First we examine "contagion", which we model by assuming that p_{t+1} is a positive function of I_t and j_t.[7] Then we examine moral hazard, assuming p to be fixed, but allowing x_{t+1}, the probability of insolvency among LOLR candidate, to be a positive function of $(1 - I_t)$ and a negative function of I_t, that is banks reduce their preferred riskiness (h) after seeing failures and raise h after seeing a rescue $(1 - I_t = 1)$. Finally we put together a model simultaneously incorporating contagion and

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6 A further tenet of the "liberal" position on the use of LOLR is that CB losses can, and should, always be avoided by an appropriate requirement for collateral; indeed that the availability of appropriate "good" collateral should be the touchstone determining whether LOLR is made available at all. But there is yet another "liberal" principle constraining LOLR which is that it should only be made available at penal rates. The two principles are inconsistent. If a commercial bank seeking help from a CB will be charged a penal rate, and also potentially suffer reputational damage, it will seek first to use its "good" collateral to borrow in the open market. Only after it has exhausted its available market opportunities will it then seek help from the CB on less advantageous terms.

Even if a commercial bank seeking help from a CB will usually have used up its best collateral already, (to borrow on finer terms from the market), the CB may be able to extract such tough terms for its LOLR lending that its own resources are largely protected in the case of an insolvency. But some (junior) creditors would then be hit all the harder, and there would still be a reputational loss to the CB, perhaps the more severe if it was perceived as refusing to take its "share" of the losses – especially if it was also responsible for bank supervision.

7 The existence of such "contagion effects" is a contentious issue. See, e.g., Rochet and Tirole (1996) for theoretical discussions. Kaufman (1998) notes for the USA that "the variance in the annual bank failure rate was greater [than for non-banks]; bank failures were clustered in a small number of years. Such clustering is consistent with the presence of bank contagion and systemic risk and contributes to the widespread public fear of bank failure."

We offer no empirical evidence here either of the actual likelihood of contagion or, what is just as important, of CBs' perceptions of such likelihood. Readers can make their own subjective judgement of this.

moral hazard (acting now on both p, the probability of a bank seeking LOLR, **and** x, the probability that such a bank would be insolvent).

That is, broadly, where we are going. But, first, we repeat that the key feature in our model is that bank failures cause behavioral instability that, in turn, make macropolicy direction much more uncertain, and hence with a higher expected cost. To many that will be a glimpse of the obvious. But others will need more empirical evidence to support that claim. So in Section 2, we undertake some empirical studies. Our hypothesis is that a key monetary ratio, i.e., H/M becomes less predictable in the face of bank failures. We regress the H/M ratio against its own prior values and the concurrent interest rate level, an examine whether such an equation "breaks down" in periods of large bank failures.

Then in Section 3, we set out the model in detail, and solve it in a single period context, with bank riskiness (h), the probability of needing LOLR (p), and of then being found to be insolvent (x) all given. As is intuitively fairly obvious, the optimal policy in such cases is for the CB to save all banks bigger than a cut-off point, $\overline{j}$, and to allow all banks smaller than $\overline{j}$ to fail. In short we model "too big to fail", and show how and why that is the optimum policy for a CB to follow.

Of course, the CB's policy in one period affects private sector agents' behavior in subsequent periods. First, if the public sees a failure in period 1, it will get more twitchy, and the probability of a run in period 2 rises (p increases). We model this in Section 4.1. Naturally the effect of this is to lower $\overline{j}$, i.e., the CB will intervene to save smaller banks. Second, the CB's choice of I_t will influence the commercial banks' preferred riskiness in period $t+1$. In Section 4.2 we model such moral hazard by making h_{t+1} and hence x_{t+1} a function of I_t. The results of this show some interesting dynamics, involving cycling, whereby the system fluctuates between the commercial banks' choosing a safer risk profile and the CB rescues often, and the reverse (i.e., riskier profile and more failures), combined with a trend towards an equilibrium level dependent on the values of the main coefficients. Finally in Section 4.3 we combine contagion and moral hazard, with both p and x time-varying. Although this is mathematically complex, by using the Lagrange approach and linearizing the first order conditions around the steady states, we obtain a closed form solution. Our results extend our analyses for the case of contagion or moral hazard alone, and provide interesting comparisons amongst them.

Unlike the single period setting wherein the CB only rescues banks above a single threshold size, in a dynamic setting CB's optimal policy may be non-monotonic in bank size, and is time varying and contingent on the probability of a failure and the likelihood of a bank requires LOLR being insolvent. We find that, if contagion is the main concern, then the CB in general would have an excessive incentive to rescue banks. Its incentives to rescue big (small) banks are strong (weak) and thus the equilibrium risk level is high (low). If moral hazard is the main concern, then the CB in general would have little incentive to rescue banks; its incentives to rescue do not critically depend on bank size. When both contagion and moral hazard are included as major concerns, then the CB's incentives to rescue though LOLR is stronger than in the single period setting but weaker than in the dynamic setting with contagion alone.

Finally we conclude and indicate some directions for further research in Section 5.

II. Do Bank Failures Cause Uncertainty?

The Shorter Oxford English Dictionary (3rd Edition, Reprinted with Corrections, 1959) gives a number of definitions of "panic"; two of these are as follows:

> "A sudden and excessive feeling of alarm or fear, usually affecting a body of persons, and leading to extravagant or injudicious efforts to secure safety."
> "A condition of widespread apprehension in relation to financial and commercial matters, leading to hasty and violent measures, the tendency of which is to cause financial disaster."

Almost by definition, then, a panic is a situation in which behavior becomes less predictable, and sensible decision-making more difficult.

For the purpose of this exercise we have assumed that the monetary authorities know exactly what is the socially optimal level of bank deposit, D^*. In panic conditions this is less likely to be true. In a panic, such as in Russia and Indonesia in 1998, the external and internal value of the currency is likely to be under threat, so should one keep D^* down? But at the same time there is likely to be economic dislocation and a credit crunch, which suggests a case for a higher D^*. Dubinin or Geraschenko?

Such examples can be multiplied. But we shall stick to our strong assumption that D^* is known and given. So the problem in our model for a CB is to vary H, the high powered money base, by OMO, so as to hit D^*. What we shall assume here is that the CB tries to estimate the appropriate level of OMO by predicting the H/M ratio on the basis of a simplified (ARIMA) model, as follows:

$$\left(\frac{H}{M}\right)_t = d_0 + d_1\left(\frac{H}{M}\right)_{t-1} + d_2\left(\frac{H}{M}\right)_{t-2} + d_3 i_t + d_4 i_{t-1} + e_t. \tag{1}$$

Our hypothesis is that the residuals from this equation will be higher during, and in the immediate aftermath of, a panic. If so, we take this as evidence that it will be much harder for the CB to adjust OMO and H, so that $D = D^*$.

In three of the five cases that we examined, this hypothesis seemed to hold. These three cases were:

(1) USA: 1872-1914 and 1921-1940, annual data;[8]
(2) Australia: 1861-1913, annual data;[9]
(3) Mexico: May 1980 to November 1997, monthly data.[10]

In Chart 1 (USA), particular periods of instability in the H/M rates are 1873-1875, 1884-1886, 1893-1895 and 1907-1909. 1873 **was** a financial and economic panic even though the number of bank failures was not particularly high, and 1893 and 1907 were amongst the most severe banking panics of the National Banking era (see Sprague, 1910). The H/M ratio would rise (econometrically significant) when bank suspensions increased, and fall when suspensions fell back again.

The inter-war experience is even better known, largely from the work of Friedman and Schwartz (1963). The H/M ratio shot up amidst the bank closures in the early 1930s, so that even though the Fed expanded H at an unusually rapid rate (see Kaldor, 1958), M still fell. As closures declined in the mid 1930s, the H/M ratio became more predictable again, only for the residuals from our basic equation (1) to rise, alongside further bank suspensions at the end of the period.

For Australia, we do not have accurate data on bank failures year by year. What we do know, however, is that 1893 was a year of massive bank failures, so under our hypothesis that residual from our simple, predictive equation (1) should be larger in 1893. Chart 3 below shows that this was indeed so.

For Mexico, under our hypothesis the residuals from our simple predictive equation (1) should increase during and immediately after the crisis at end 1994/start 1995. The data shown in Chart 4 are consistent with that hypothesis.

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8 For the USA, the monetary data are taken from Friedman and Schwartz (1982). Data on bank failures (deposits, post 1918, and number of suspended banks), are taken from *Historical Statistics of the United States – Colonial Times to 1970*. The monetary data for H and M are annual averages; call money rates are annual averages of monthly data.

9 For Australia, the monetary data were mainly taken from Butlin, Hall and White (1971) and the interest rate data from Mitchell (1983). For deposits, the data are for the final quarter (average) of each year. For reserves, data are reported as of December for each year until 1900. For 1900-1912, data are as of June for each year, and so were averaged over two years in order to centre on December. Data on currency in circulation (end year) are taken from Mitchell (1983). Both H and M are thus approximately end-year.

10 The monthly data for Mexico are taken from IFS over the period May 1990 till November 1997. The interest rate used is the Treasury bill rate, reported as a monthly average.

Our hypothesis did not, however, receive support in every case. We had monthly data from IFS for Japan and for South Korea from January 1988 till January 1998, but there was no evidence of the H/M ratio becoming more unstable at the end of this period in either of these countries, though there have been anecdotal reports of a sharp (bank-failure-induced) rise in the H/M ratio in Japan since November 1997.

Perhaps the population in Japan and South Korea have been kept from panic, and greater flight into currency, by confidence in deposit insurance; i.e., that the State would guarantee the value of their deposit whatever happened to the solvency of their bank. The presence of 100% deposit insurance should thus serve to reduce the variance of the outcomes following the failure (i.e. in our terminology introduced in Section 3, to reduce k). This should, as will be further described in Section 3, raise the size threshold at which the authorities should go to the support of banks in difficulties.

Since deposit insurance will also have the effect of making banks choose a riskier strategy, (so that h and x also raise), there are two grounds for arguing, on the basis of our model, that the introduction of (100%) deposit insurance **should** have been associated with a much tougher, and less sympathetic, CB attitude towards banks coming to the authorities for support at times of difficulty and liquidity pressures. Instead, no doubt influenced by the searing example of the inter-war crisis, the introduction of deposit insurance was broadly accompanied by a much more sympathetic policy towards banks in trouble in the immediate post-war decades. In terms of our model, the actual cut-off point, above which any larger bank would be rescued, tended then to fall below the socially optimal level.

III. The Single Period Model

A. The Basic Setup

We assume a commercial banking system with many banks of varying sizes, but all of which choose a similar risk profile (h). This latter assumption is mathematically convenient, of course, but we shall also try to justify this assumption as a reasonable approximation to reality shortly. Given this preferred risk profile there is a probability (p) of a bank (or banks) holding j of the system's deposits coming to the CB asking for LOLR assistance in any period. If we set the initial volume of deposits at D^*, the desired level (D^* is exogenous), then $0 < j < D^*$.

If no bank seeks LOLR, as occurs with probability $(1 - p)$, CB takes no action, D remains equal to D^*, and the CB suffers no loss. If p occurs, with $j > 0$, the CB has to decide whether to say no to the request for LOLR ($I_t = 1$), or yes ($1 - I_t = 1$). There is a probability, x, where $x = f(h)$, that the bank (or banks) coming to the CB will also be insolvent. This is not revealed to the CB at this stage, and the CB also at this stage has to decide on its OMO.[11] By OMO the CB is assumed to be able to change D by any desired amount, OMO being unlimited in size and direction, so the CB can always achieve any desired expected value of D that it wants, i.e., it can make $\mathrm{E}D = D^*$.

If the central bank does not provide LOLR, the illiquid bank(s) will shut. This will cause the public to move out of deposits into cash, via the linear relationship, $\Delta D = B_1 j + j\epsilon$, where B_1 is a positive coefficient, known to the central bank, and ϵ is a stochastic variable, with $\mathrm{E}\epsilon = 0$ and $\mathrm{Var}(\epsilon) = k$, where k is also known to the CB. The loss from getting macropolicy wrong is assumed to be quadratic, and for simplicity as $(D - D^*)^2$.

The identity of the illiquid bank, which has been supported by LOLR, is then made known, i.e., whether it is still solvent, or not. If it is insolvent, with probability x, the central bank will face a cost Z, where $Z = n + B_2 j$ $(n > 0,\ B_2 > 0)$.

Thus, in the one period game, with h, f and x all given and constant, the CB wishes to minimize the loss functions,

$$\min\left[E(D - D^*)^2,\ EZ\right] \tag{2}$$

Of course if nature plays $1 - p$, nobody knocks at CB's door and there is no loss. We are only interested in the cases when p occurs.

The sequence of events is as in Figure 1.

B. The Solution of the Single Period Model

The solution to this one period model is as follows. Notice first that

$$E(D - D^*)^2 = (B_1 j - D^*)^2 + kj^2,$$

thus

$$\min[E(D - D^*)^2] = kj^2,$$

which is achieved at $B_1 j = D^*$ through OMO. Because the minimum cost of OMO is kj^2, a constant, while the expected cost of LOLR is,

$$EZ = (n + B_2 j)x.$$

LOLR is preferable if and only if:

$$EZ = (n + B_2 j)x \le kj^2 = \min[E(D - D^*)^2].$$

[11] All practical descriptions of LOLR activities reveal that the CB is under tremendous time-pressure to take decisions (e.g. before the market reopens) and has to do so when in possession of only sketchy details of the "true" financial position of the commercial bank needing help.

That is[12]

$$j \geq \bar{j} \equiv \frac{B_2 x + \sqrt{B_2^2 x^2 + 4knx}}{2k}. \tag{3}$$

Thus we reach our first result regarding the comparison between LOLR and OMO in a static setting.

Proposition 1 *In a static setting, LOLR is preferable to OMO if and only if the size of the bank seeking assistance is above a threshold level $\bar{j}$, defined in equation (3); while OMO is preferable to LOLR if and only if otherwise.*

Clearly the crucial feature of this model is that the costs of allowing a bank to fail rise at a faster rate with respect to the size of bank, i.e., j, here assumed to be quadratic, than the costs of rescuing a bank that **may** turn out to be insolvent, here taken to be linearly proportional to size j. But so long as the costs of failure rise consistently faster than the costs of rescue with respect to size, the same results will occur irrespective of the precise mathematical formulation.

How can we defend, and support, this asymmetry? Let us start with the quadratic loss function from policy error, kj^2. In virtually all the Central Bank Independence literature, policy errors, e.g. deviations of inflation from target, are taken to be quadratic. But the justification for quadratic loss functions is rarely profound, and often just based on mathematical convenience. More convincingly (to us), Allen and Gale (1998a) in their working paper on "Financial Contagion" analyze this as arising from a combination of incomplete markets and interconnectedness between a failing institution and other (similar) institutions. Incomplete markets are important because diversification is limited, e.g., banks located in country X will be particularly sensitive to country X risk, e.g. credit risk, interest rate risk, etc. Positive interconnectedness whereby the failure of one institution may worsen the position of another is a particularly prominent feature of banking through a variety of channels. Such channels include payments/settlements systems, e.g. Bankhaus Herstatt, interbank deposit and correspondent systems, e.g. Continental Illinois, and effects via asset prices. Thus in a further paper, on "Optimal Financial Crises" (1998b), Allen and Gale, write as follows, p. 1251,

> "One of the special features of the models described above is that the risky asset is completely illiquid. Since it is impossible to liquidate this risky asset, it is available to pay the late consumers who do not choose to early withdrawal. We next analyze what happens if there is an asset market in which the risky asset can be traded. It is shown that this case is very different. Now the banks may be forced to liquidate their illiquid assets in order to meet their deposit liabilities. However, by selling assets during a run, they force down the price and make the crisis worse. Liquidation is self-defeating, in

[12] The other root is $\underline{j} < 0$, which does not have economic meaning. Technically we also need parameter restriction such that $\bar{j} < D^*$, which we assume is not violated.

the sense that it transfers value to speculators with negative insurance. In this case, there is an incentive for the central bank to intervene to prevent a collapse of asset prices, but again the problem is not runs *per se* but the unnecessary liquidation they promote."

What we claim is that the risks arising from such interconnectedness are a strongly rising function of size, if not necessarily exactly quadratic. Would there have been such concern about Long-Term Capital Management if its position had not been so huge?

The next question is whether the costs of rescue rise less fast. Here there are two costs, reputational and financial costs, when the rescued bank is insolvent, with the latter falling on some combination of surviving banks, Central Bank and taxpayers. There is, we would argue, a significant fixed element in reputational costs. You either make an obvious, publicly observable, error of judgement, or you do not. Again, there is a fixed cost in making the taxpayer, or other surviving banks, face the reality of having to contribute to an ex post bailout at all. That fixed cost may be large, perhaps even so large that no banks, even the largest, will actually be supported, i.e., n is very large indeed, (e.g. see the problem in Japan of sharing out the costs of rescue). But, once the taxpayer has come to accept that she must bear the costs of rescue, then our assertion is that the disutility is just proportional to the cash burden. Although we cannot justify that claim theoretically, our experience with studying cases of bank failures, and their resolution, leads us to this view.

According to the above proposition, $\overline{j}$ only depends on x, k, n and B_2, but not on D^* or B_1. Some comparative statics on $\overline{j}$ further reveal that $\overline{j}$ increases as x increases, k decreases, n increases, or B_2 increases. These results have the following intuition: The CB should raise the threshold level of bank size (above which it wants to rescue), and thus only rescues bigger size banks, when the probability of insolvency x increases, when the risk of deposits moving out of the banking system k decreases, or when either the fixed cost (n) or variable cost (B_2) of rescuing banks that turn out to be insolvent increases.

Moreover, if the risk of deposits moving out of banking system k is very small, then $\overline{j}$ can be very large; thus OMO becomes optimal even for the largest bank; if the probability of insolvency x is very small, then $\overline{j}$ can be very small; thus LOLR becomes optimal even for very small banks.

We have now formally explained how a policy of "too big to fail" minimizes the CB's single period loss function, but if decisions whether to provide LOLR, or not, are a function of the size of j, of the commercial bank in difficulties, will that not then make their individual choice of risk profile (h) a function of size also, thereby contradicting our initial simplifying assumption? Not necessarily so. It is reasonable to assume that the benefits that a manager obtains, both pecuniary and non-pecuniary, from his (continued) position are a positive function of the size of his bank, as well as a function of its profitability. If so, the larger the bank, the less that the manager would want to put his status at risk (assuming normal risk aversion). So greater size may on the one hand be predicted to lead to more probable CB intervention, but on the other hand to make managers more unwilling to put their own position at risk.[13] Moreover, large size banks may be subject to a tighter monitoring and regulation by CB.

Of course, if the value of $\bar{j}$ was public knowledge, there would be a discontinuity. Banks just larger than the cut-off point would have an incentive to increase risk, whereas banks just below the cut-off point would become very risk averse. But, of course, $\bar{j}$ is not generally publicly observable.[14] This is partly because individual banks have idiosyncratic features making the CB more, or less likely, to intervene on their behalf,[15] partly because, as we will show in subsequent sections, $\bar{j}$ varies over time (and in a somewhat unpredictable fashion, depending on stochastic shocks), and partly because the monetary authorities maintain a policy in this respect of "constructive ambiguity". Indeed our model enables us to interpret this latter as a rational response by the CB precisely to prevent commercial banks' chosen risk profiles becoming a function of size.[16]

So we feel comfortable with our position that, even though the CB response to a commercial bank in difficulties will be a function of the size of that bank; a bank's chosen risk profile need not also be a function of size. Again we would also appeal to empirical findings. "Too big to fail" is widely accepted to be an almost universal phenomenon; yet we are not aware of any empirical finding that risk preference among bankers is a positive function of size.

This is not to suggest that commercial banks' risk profile choices do not respond to the actions and signals of the CB. Indeed the next Section focuses directly on that. Rather we claim that our model simplification, whereby all banks react similarly to such CB signals, despite being of differing initial sizes, is sufficiently close to reality to make our model results interesting.

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13 It would, of course, be possible to formalize the objective function of commercial bank management, and construct a more complete game of CB/commercial bank intervention. We shall pursue this in related research concerned with the issue of the imposition of sanctions in response to excessive risk taking.

14 There are some exceptions to this dictum. The Comptroller of the Currency, at the time of the Continental Illinois crisis, stated that all larger banks would also be automatically rescued. The Japanese monetary authorities in recent years have made it publicly known that the large City banks are ring-fenced against failure.

15 For example Johnson Matthey's involvement in the gold market in London. Again, BCCI and Drexel Lambert could be let go, because their interconnectedness was low, and hence k was also low. Per contra, the interconnectedness of LTCM was high.

16 See, for example, Enoch, Stella and Khamis (1997) for empirical evidence.

IV. The Dynamic Model

In the single period case above, the probability of a commercial bank needing LOLR (p), the probability of it then also being insolvent (x) and its risk profile (h) were all taken as given. In this Section we consider the multi-period equilibrium in which, p, x, and h will become time-varying in response of the CB's actions and signals.

There are two main channels of inter-temporal interactions that we identify here. The first is contagion, whereby failure now, when the CB play $I_t = 1$, is likely to lead to more failures subsequently, i.e., $\partial p / \partial I_t > 0$. The second is moral hazard, whereby a rescue, $(1 - I_t = 1)$, is likely to cause commercial banks' to increase their risk profiles, thereby raising both p and x. We assume that $\partial p / \partial I_t > \partial p / \partial (1 - I_t)$. In reality, of course, moral hazard will be much worse if the CB provides LOLR assistance to an insolvent bank than to a solvent, but illiquid, bank, (when there might be no moral hazard). We have justified the above simplified assumption with the observation that generally commercial banks, and the general public, will not be able to observe for some time whether a LOLR support exercise involves an insolvent bank, or not, and so will have to condition on the LOLR action itself, rather than on the full characteristics of the banks supported.

In the one period game, the CB's loss function was: $\min \left[E(D - D^*)^2,\ EZ \right]$. In the multi-period game, this objective function generalizes to:

$$\min E_0 \left\{ \sum_{t=0}^{\infty} \delta^t p_t \left[j^2 k I_t + (n + B_2 j) x_t (1 - I_t) \right] \right\}, \tag{4}$$

where $0 < \delta < 1$ is discount factor. This is subject to the equation of motion of p_t or/and x_t.

In order to derive basic economic intuitions out of clear-cut closed form solutions, we start our analysis by allowing one of the two stochastic variables, p or x, to vary over time, holding the other fixed. We treat the analysis with p_t time-varying (x constant) as primarily about contagion and present it in Section 4.1; in Section 4.2 we present the analysis with x_t time-varying (p_t constant) as focussing on moral hazard. Then in Section 4.3, we allow both contagion and moral hazard to operate, i.e., with both p_t and x_t stochastically time-varying. With linearization of the first order conditions around the steady states, we obtain a closed form solution in this case also.

A. Contagion in Dynamic Model

To focus on the contagion problem, we can set $h_t + \varepsilon_t = x$ as constant. Thus, the objective function is:

$$\min_{I_t} E_0 \left\{ \sum_{t=0}^{\infty} \delta^t p_t \left[j^2 k I_t + (n + B_2 j) x (1 - I_t) \right] \right\}. \tag{5}$$

This is subject to the equation of motion for p_t, which will depend on the CB's actions, that is to refuse LOLR ($I_t = 1$) or grant LOLR ($1 - I_t = 1$), so that

$$p_{t+1} = \alpha_0 + (\alpha_1 + \beta_1 j)I_t + (\alpha_2 + \beta_2 j)(1 - I_t) + \alpha_3 p_t + \varepsilon_{t+1}, \qquad (6)$$

where ε_{t+1} is a stochastic term with mean zero and a constant variance. Note that allowing a bank to fail will raise the probability of future failures, i.e., contagion, so that $\alpha_1 > 0$ and $\beta_1 > 0$, but also providing LOLR assistance ($1 - I_t = 1$) will also raise the probability of failure by signaling the CB's greater willingness to rescue, so that $\alpha_2 > 0$ and $\beta_2 > 0$ also. We tend to believe that contagion will be stronger in this case than moral hazard, i.e., $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$, but that view does not affect the solution. With any CB action, in response to a call for assistance from a commercial bank lending to lead to further difficulties (i.e., p_t rising), there is a question whether the banking system is globally stable, (and past history suggests that such worries could have some foundation!). We show the necessary stability conditions and discuss their economic intuition below.

We can solve this dynamic programming problem by using the Lagrange method.[17] The Lagrange function of this question is[18]

$$\begin{aligned} \mathfrak{L} \quad = \quad & E_0\{\sum_{t=0}^{\infty} \delta^t p_t \left[j^2 k I_t + (n + B_2 j)x(1 - I_t)\right] \\ & -\delta^{t+1}\lambda_{t+1}\left[p_{t+1} - \alpha_0 - (\alpha_1 + \beta_1 j)I_t - (\alpha_2 + \beta_2 j)(1 - I_t) - \alpha_3 p_t - \varepsilon_{t+1}\right]\}. \end{aligned}$$

The first order conditions (FOC) of this Lagrange with respect to I_t and p_t yield:

$$p_t\left[j^2 k - (n + B_2 j)x\right] + \delta\left[\alpha_1 + \beta_1 j - (\alpha_2 + \beta_2 j)\right] E_t\lambda_{t+1} = 0, \qquad (7)$$

$$j^2 k I_t + (n + B_2 j)x(1 - I_t) + \delta\alpha_3 E_t\lambda_{t+1} = \lambda_t. \qquad (8)$$

For a quadratic objective function like ours, we can conjecture λ_t as a linear function:

$$\lambda_t = \rho_0 + \rho_1 p_t, \qquad (9)$$

thus,

$$E_t\lambda_{t+1} = \rho_0 + \rho_1 E_t p_{t+1} = \rho_0 + \rho_1\left[\alpha_0 + (\alpha_1 + \beta_1 j)I_t + (\alpha_2 + \beta_2 j)(1 - I_t) + \alpha_3 p_t\right].$$

Substituting this $E_t\lambda_{t+1}$ in (7) and (8) respectively, solving for ρ_0 and ρ_1, we finally arrive at the following proposition regarding the solution for I_t in the case of contagion, denoted as I_t^c.[19]

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17 Our problem in the case of contagion, or of moral hazard alone, is a standard dynamic programming problem and can be solved by using the standard dynamic programming approach. We decided to choose the Lagrange approach because it appears more efficient and useful in dealing with non-quadratic objective functions (the joint case of contagion and moral hazard in Subsection 4.3). For consistency in approach and easy exposition, we use the Lagrange approach for the whole paper. See Chow (1997) for more discussions of this method.

18 Technically, we should have a further Lagrange multiplier, τ_t, and include $\tau_t(1 - p_t)$ in the Lagrange function to take fully into consideration the case of $0 \leq p_t \leq 1$. For simplicity, we assume $0 < p_t < 1$ (and $0 < h_t < 1$) thus $\tau_t = 0$, and thus ignore it in the Lagrange function here (and below).

19 It is easy to check that the sufficient conditions for global optimization in this case of contagion, and moral hazard below, are satisfied.

Proposition 2 *In dynamic setting, the optimal monetary policy dealing with contagion case is:*

$$I_t^c = \gamma_0 + \gamma_1 p_t, \qquad (10)$$

whereby:

$$\gamma_0 = \frac{\delta\alpha_3 - \sqrt{\delta^2\alpha_3^2 - \delta}}{2 - \delta\alpha_3}\frac{(n + B_2 j)x}{j^2k - (n + B_2 j)x} - \frac{1}{2 - \delta\alpha_3}\frac{\alpha_0 + \alpha_2 + \beta_2 j}{\alpha_1 + \beta_1 j - (\alpha_2 + \beta_2 j)} > 0,$$

$$\gamma_1 = -\frac{\sqrt{\delta^2\alpha_3^2 - \delta}}{\delta[\alpha_1 + \beta_1 j - (\alpha_2 + \beta_2 j)]} < 0.$$

This result has the following implications. Notice first that $\gamma_1 < 0$ holds for all the given parameter settings, as long as $\delta\alpha_3^2 > 1$, i.e. $\alpha_3 > 1/\sqrt{\delta} > 1$, which we will further discuss in connection with the stability condition below. Therefore, if p_t rises, i.e., there is a structural shift making the banking system less stable, then the CB always wants to accommodate banks' calls for assistance more frequently, i.e. $\partial I_t^c/\partial p_t < 0$, regardless whether their sizes are above the threshold level or not. Thus it will accommodate banks' calls for assistance more frequently than in the one-period case and rescue "smaller" as well as all big banks. This should not be surprising when contagion is the main concern of the CB.

Moreover, the system will tend to a long-run equilibrium value for p, set as p^c, where p^c is given by the following equation

$$\begin{aligned} p^c = \; & \frac{\delta}{(2 - \delta\alpha_3)(\sqrt{\delta^2\alpha_3^2 - \delta} - \delta\alpha_3 - \delta)}\{(1 - \delta\alpha_3)(\alpha_0 + \alpha_2 + \beta_2 j) \\ & + (\delta\alpha_3 - \sqrt{\delta^2\alpha_3^2 - \delta})[\alpha_1 + \beta_1 j - (\alpha_2 + \beta_2 j)]\frac{(n + B_2 j)x}{j^2k - (n + B_2 j)x}\}. \end{aligned}$$

Since $\sqrt{\delta^2\alpha_3^2 - \delta} - \delta\alpha_3 - \delta > 0$ and $\delta\alpha_3 - \sqrt{\delta^2\alpha_3^2 - \delta} > 0$, then if $1 - \delta\alpha_3 > 0$ (that is $1/\sqrt{\delta} < \alpha_3 < 1/\delta$) and hence $2 - \delta\alpha_3 > 0$, the value of p^c critically depends on whether $j^2k - (n + B_2 j)x > 0$ or not. If $j^2k - (n + B_2 j)x > 0$, that is the cost of LOLR is smaller than that of OMO for a given j (i.e., the bank size is larger than the cutoff size in the one period model), then $\frac{(n+B_2 j)x}{j^2k-(n+B_2 j)x} > 0$. Comparative statics analysis on p^c in this case indicates that p^c goes up if α_0 (the constant level in p_{t+1}), or $(\alpha_1 + \beta_1 j)$ goes up, or $(n + B_2 j)x$ goes up, or j^2k goes down. These results are consistent with intuition, because when $j^2k - (n + B_2 j)x > 0$, the CB has more incentive to provide LOLR. A higher α_0 implies a higher constant risk level in the banking system; a higher $\alpha_1 + \beta_1 j$ implies a stronger effect of providing LOLR on the risk level of the banking system; a higher $(n + B_2 j)x$ implies higher costs of LOLR, and a lower j^2k implies lower costs of OMO. The effect of $\alpha_2 + \beta_2 j$ on p^c is more complicated, as the main instrument is LOLR. The system will fluctuate around p^c as ε_t varies stochastically.

If $j^2k - (n + B_2j)x < 0$, that is the cost of LOLR is bigger than that of OMO for a given j (i.e., the bank size is smaller than the cutoff size in the one period model), however, then $\frac{(n+B_2j)x}{j^2k-(n+B_2j)x} < 0$, and for $p^c > 0$ we need

$$(1 - \delta\alpha_3)(\alpha_0 + \alpha_2 + \beta_2 j) > (\delta\alpha_3 - \sqrt{\delta^2\alpha_3^2 - \delta})[\alpha_1 + \beta_1 j - (\alpha_2 + \beta_2 j)]\frac{(n + B_2j)x}{(n + B_2j)x - j^2k}.$$

Comparative statics analysis on p^c in this case indicates that p^c goes up if α_0 goes up, or $(\alpha_2 + \beta_2 j)$ goes up, or j^2k goes up, or $(n + B_2j)x$ goes down. These results are also consistent with intuition, because when $j^2k - (n + B_2j)x < 0$, the CB has more incentive to provide OMO rather than LOLR. A higher $\alpha_2 + \beta_2 j$ implies a stronger effect of providing OMO on the risk level of the banking system. The effect of $\alpha_1 + \beta_1 j$ on p^c is more complicated, as the main instrument in this case is OMO.

Notice further that, if failures have occurred in a series of large banks, i.e., $j^2k - (n + B_2j)x > 0$, then p^c may even be close to its upper bound, 1, for some parameter configurations. The intuition is that when the CB is only concerned with contagious risk, which is more severe for large banks, it would excessively rescue too many banks so that the banking system risk level would then become extremely high. Similarly, if the only failures to have occurred were among small banks, i.e., $j^2k - (n + B_2j)x < 0$, p^c may even be close to its lower bound, zero, for some parameter configurations. The intuition is that even when the CB is concerned with contagious risk, yet the troubled banks have been small, it would limit access to LOLR and rather use OMO, so that the banking system risk level would be extremely low.

Furthermore, from the equation of motion for p_t above and substituting I_t, we get:

$$p_{t+1} = \alpha_0 + (\alpha_1 + \beta_1 j)\gamma_0 + (\alpha_2 + \beta_2 j)(1 - \gamma_0) + \left[\alpha_3 - \frac{\sqrt{\delta^2\alpha_3^2 - \delta}}{\delta}\right] p_t + \varepsilon_{t+1}.$$

Thus the stability condition (on α_3 and δ) is:

$$\delta(\alpha_3 - 1) < \sqrt{\delta^2\alpha_3^2 - \delta}.$$

Combining this condition with $1/\sqrt{\delta} < \alpha_3 < 1/\delta$, and noticing that $\frac{1+\delta}{2\delta} > \frac{1}{\sqrt{\delta}}$ for $0 < \delta < 1$, we thus have:

$$\frac{1+\delta}{2\delta} < \alpha_3 < \frac{1}{\delta}. \tag{11}$$

It is easy to check that for $0 < \delta < 1$ there is a non-empty set for α_3.

From p^c, it is clear that another conditions for p^c to be stable is required:

$$j^2k \neq (n + B_2j)x.$$

Finally, notice that the result of the above proposition can also be interpreted in terms of bank size, i.e., j in our model. Setting $I_t^c = 0$ in (10), we get:

$$\gamma_0(j) + \gamma_1(j)p_t = 0. \tag{12}$$

This is a quadratic function of j, thus there exist two real-number roots. These roots are obviously functions of all the parameters and variables including p_t. Thus, when p_t changes, CB should adjust its optimal policy accordingly. Therefore, a same size bank may be rescued by the CB for a given p_t, but may not be rescued when p_{t+1} changes. This shifting of CB's policy reflects the time dynamics and is caused by the changes in time-varying variable, p_t.

More importantly, even for the same value of p_t, the CB may optimally rescue only these banks that have the "right sizes". For the above quadratic equation (12), there may be two real number roots which are non-negative yet smaller than D^*. These roots also clearly define useful bank size categories and the CB should rescue these banks ($I_t = 0$) within such categories, but not to do so for those banks which are not in the categories. We record this results as the following corollary:

Corollary 3 *Denoting* $0 < j_1^c < j_2^c < D^*$ *as these real number roots such that*

$$\gamma_0(j_k^c) + \gamma_1(j_k^c)p_t = 0,$$

then the CB should only rescue these banks whose sizes are equal to or closely around j_k^c, $1 \leq k \leq 2$.

For a variety of possible parameter settings, a small or a large bank may be rescued, while banks whose sizes are intermediate may not be. Consequently, the CB's optimal behavior, if non-monotonic in bank size, may appear ambiguous if viewed from outside, which will further enhance the constructive ambiguity which we described in Section 3.

Next we turn to the case of moral hazard.

B. Moral Hazard in Dynamic Model

We focus on moral hazard by switching off contagion (i.e., holding p constant), but allowing universal risk preference (h) to increase alongside with the probability of a bank requiring assistance also being insolvent (i.e., x rises).

We assume that x is a linear function of h, and we can, without loss of generality, set the coefficient equal to unity, so $x_t = h_t$. With p constant, we can drop that from the CB's objective function, which involves setting I_t so as to minimize:

$$\min_{I_t} E_0 \left\{ \sum_{t=0}^{\infty} \delta^t \left[j^2 k I_t + (n + B_2 j) h_t (1 - I_t) \right] \right\}, \tag{13}$$

subject to the equation of motion for h, whereby:

$$h_{t+1} = a_0 - (a_1 + b_1 j) I_t + (a_2 + b_2 j)(1 - I_t) + a_3 h_t + e_{t+1}. \tag{14}$$

Where e_{t+1} is a stochastic term with mean zero and a constant variance, and $a_1 > 0$, $b_1 > 0$,- $a_2 > 0$, $b_2 > 0$, with $a_1 < a_2$ and $b_1 < b_2$.

Again, the Lagrange function of this question is:

$$\begin{aligned} L &= E_0\{\sum_{t=0}^{\infty} \delta^t \left[j^2 k I_t + (n + B_2 j) h_t (1 - I_t)\right] \\ &\quad -\delta^{t+1} l_{t+1} \left[h_{t+1} - a_0 + (a_1 + b_1 j) I_t - (a_2 + b_2 j)(1 - I_t) - a_3 h_t - e_{t+1}\right]\}. \end{aligned}$$

The FOC of this Lagrange with respect to I_t and h_t yield:

$$j^2 k - (n + B_2 j) h_t - \delta(a_1 + b_1 j + a_2 + b_2 j) E_t l_{t+1} = 0, \tag{15}$$

$$(n + B_2 j)(1 - I_t) + \delta a_3 E_t l_{t+1} = l_t. \tag{16}$$

Again, we can conjecture l_t as a linear function:

$$l_t = r_0 + r_1 h_t, \tag{17}$$

thus,

$$E_t l_{t+1} = r_0 + r_1 E_t h_{t+1} = r_0 + r_1 \left[a_0 - (a_1 + b_1 j) I_t + (a_2 + b_2 j)(1 - I_t) + a_3 h_t\right].$$

Substituting this $E_t l_{t+1}$ in (15) and (16) respectively, solving for r_0 and r_1, we finally arrive at the following proposition regarding the solution for I_t in the case of moral hazard, denoted as I_t^m.

Proposition 4 *In dynamic setting, the optimal monetary policy dealing with moral hazard case is:*

$$I_t^m = g_0 + g_1 h_t, \tag{18}$$

whereby:

$$\begin{aligned} g_0 &= -\frac{\delta a_3 - \sqrt{\delta^2 a_3^2 - \delta}}{2 - \delta a_3} - \frac{1}{2 - \delta a_3} \frac{a_0 + a_2 + b_2 j}{a_1 + b_1 j + a_2 + b_2 j} < 0, \\ g_1 &= \frac{\sqrt{\delta^2 a_3^2 - \delta}}{\delta(a_1 + b_1 j + a_2 + b_2 j)} > 0. \end{aligned}$$

This result has the following implications. Notice first that $g_1 > 0$ holds for all given parameter settings. Therefore, if h_t rises, i.e., there is a structural shift making the risk level in the banking system higher, then the CB will accommodate less often, i.e. $\partial I_t^m / \partial h_t > 0$, regardless of the banks' sizes. This implies that when the risk level in the banking system is higher, the CB will respond to banks' call for LOLR less often; and vice versa, when the risk level in the banking system is lower, the CB will respond to banks' call for LOLR more often. This is obviously the opposite case to contagion, whereby $\partial I_t^c / \partial h_t < 0$, and thus the CB always wants to accommodate banks' calls for assistance more often.

The system will tend to a long-run equilibrium value for h, set as h^m, where h^m is given by the following equation

$$h^m = \frac{\delta[(3-\delta a_3)(a_0+a_2+b_2 j)+(\delta a_3-\sqrt{\delta^2 a_3^2-\delta})(a_1+b_1 j+a_2+b_2 j)]}{(2-\delta a_3)(\sqrt{\delta^2 a_3^2-\delta}-\delta a_3-\delta)}. \tag{19}$$

For $\delta a_3^2 < 1$ and $\delta a_3 < 2$, comparative statics analysis on h^m indicates that h^m goes down if a_0 (the constant level in h_{t+1}), or $a_1 + b_1 j$, or $a_2 + b_2 j$ goes down.

The equilibrium risk level in moral hazard does not so critically depend on the sizes of failing banks as it did in the previous pure contagion case. Although the equilibrium risk level does depend on the average size of failing banks, and the higher is that average size, the higher is the equilibrium risk level, the difference is much smaller than in contagion. These results are again consistent with intuition, because when moral hazard is the sole concern of the CB, then it should always have strong incentive to reject the request from the troubled banks for LOLR, and as a result of such an extreme policy, the equilibrium risk level in the banking system should be quite low, for small and large banks as well.

Furthermore, from the equation of motion for h_t above and substituting I_t, we get:

$$h_{t+1} = a_0 + a_2 + b_2 j - (a_1 + b_1 j + a_2 + b_2 j) g_0 + \left[a_3 - \frac{\sqrt{\delta^2 a_3^2 - \delta}}{\delta}\right] h_t + e_{t+1}.$$

Thus, similar to the case of contagion alone, the stability conditions are:

$$\delta(a_3 - 1) < \sqrt{\delta^2 a_3^2 - \delta}.$$

Again combining this condition with $1/\sqrt{\delta} < a_3 < 1/\delta$, and noticing that $\frac{1+\delta}{2\delta} > \frac{1}{\sqrt{\delta}}$ for $0 < \delta < 1$, we thus have:

$$\frac{1+\delta}{2\delta} < a_3 < \frac{1}{\delta}. \tag{20}$$

And it is easy to check that there is a non-empty set for a_3.

In the case of moral hazard, bank size does not play a key role. When contagion is the main concern of the CB, it is the big banks which worry the CB most, and require the CB's prompt LOLR action. If moral hazard is the main concern of the CB, CB's rescuing policy is more uniform across banks with different sizes; The banking system can be more easily stabilized (than that in the case of contagion alone); And the stability conditions do not depend on the size of the illiquid banks.

Once again, the threshold (j) for LOLR with moral hazard is time-varying. Setting $I_t^m = 0$, we get:

$$g_0(j) + g_1(j) h_t = 0. \tag{21}$$

Again, this is a quadratic function of j, thus there exist two real-number roots. These roots are obviously functions of all the parameters and variables including h_t. Thus, when h_t changes, CB should adjust its optimal policy accordingly. Therefore, a same size bank may be rescued by the CB for a given h_t, but may not be rescued when h_{t+1} changes. This changing of CB's policy reflects the time dynamics and is caused by the changes in time-varying variable, h_t.

More importantly, even for the same value of h_t, the CB may optimally rescue only these banks that have the "right sizes". For the above quadratic equation (21), there may be two real number roots which are non-negative yet smaller than D^*. These roots also clearly define useful bank size categories and the CB should rescue these banks ($I_t = 0$) within such categories, but not to do so for those banks which are not in the categories. We record this results as the following corollary:

Corollary 5 *Denoting* $0 < j_1^m < j_2^m < D^*$ *as these real number roots such that*

$$g_0(j_k^m) + g_1(j_k^m)p_t = 0,$$

then the CB should only rescue these banks whose sizes are equal to or closely around j_k^m, $1 \leq k \leq 2$.

C. Contagion and Moral Hazard in Dynamic Model

When both p and x are allowed to be time-varying, we assume (as above) that x is a linear function of h, and we set the coefficient equal to unity, so $x_t = h_t$. Thus the new problem is to set I_t optimally so as to minimize:

$$\min_{I_t} E_0 \left\{ \sum_{t=0}^{\infty} \delta^t p_t \left[j^2 k I_t + (n + B_2 j) h_t (1 - I_t) \right] \right\}, \tag{22}$$

subject to the equations of motion for h_t and p_t, whereby:

$$h_{t+1} = a_0 - (a_1 + b_1 j) I_t + (a_2 + b_2 j)(1 - I_t) + a_3 h_t + \varepsilon_{t+1}, \tag{23}$$

$$p_{t+1} = \alpha_0 + (\alpha_1 + \beta_1 j) I_t + (\alpha_2 + \beta_2 j)(1 - I_t) + \alpha_3 p_t + e_{t+1}, \tag{24}$$

where again ε_{t+1} and e_{t+1} are stochastic terms with mean zero and constant variances, and all other coefficients are positive.

Again, the Lagrange function of this question is:

$$\begin{aligned} \pounds \quad = \quad & E_0\{\sum_{t=0}^{\infty} \delta^t p_t \left[j^2 k I_t + (n + B_2 j) h_t (1 - I_t) \right] \\ & -\delta^{t+1} l_{t+1} \left[h_{t+1} - a_0 + (a_1 + b_1 j) I_t - (a_2 + b_2 j)(1 - I_t) - a_3 h_t - \varepsilon_{t+1} \right] \\ & -\delta^{t+1} \lambda_{t+1} \left[p_{t+1} - \alpha_0 - (\alpha_1 + \beta_1 j) I_t - (\alpha_2 + \beta_2 j)(1 - I_t) - \alpha_3 p_t - e_{t+1} \right] \}. \end{aligned}$$

The FOC of this Lagrange with respect to I_t, h_t and p_t yield, respectively:

$$p_t\left[j^2k-(n+B_2j)h_t\right]-\delta(a_1+b_1j+a_2+b_2j)E_tl_{t+1}+\delta\left[\alpha_1+\beta_1j-(\alpha_2+\beta_2j)\right]E_t\lambda_{t+1}=0, \tag{25}$$

$$p_t(n+B_2j)(1-I_t)+\delta a_3E_tl_{t+1}=l_t, \tag{26}$$

$$j^2kI_t+(n+B_2j)h_t(1-I_t)+\delta\alpha_3E_t\lambda_{t+1}=\lambda_t. \tag{27}$$

Intuitively, the joint case of contagion and moral hazard is some form of convex combination of each case alone, with both h_t and p_t changing over time. As discussed above, the contagion case is a special case of the joint case, with h_t fixed; and similarly, the moral hazard case is a special case of the joint case, with p_t fixed.

To solve a problem with a quadratic first order conditions like ours, we shall start with linearization of the FOC around the steady state.[20] Doing so leads to:

$$\begin{aligned} 0 \quad = \quad & (n+B_2j)\overline{h}\overline{p}-(n+B_2j)\overline{p}h_t+[j^2k-(n+B_2j)\overline{h}]p_t \\ & -\delta(a_1+b_1j+a_2+b_2j)E_tl_{t+1}+\delta\left[\alpha_1+\beta_1j-(\alpha_2+\beta_2j)\right]E_t\lambda_{t+1}, \end{aligned} \tag{28}$$

$$l_t=(n+B_2j)\overline{I}\overline{p}-(n+B_2j)(1-\overline{I})p_t-(n+B_2j)\overline{p}I_t+\delta a_3E_tl_{t+1}, \tag{29}$$

$$\lambda_t=(n+B_2j)\overline{h}\overline{I}+(n+B_2j)(1-\overline{I})h_t+[j^2kI_t-(n+B_2j)\overline{h}I_t+\delta\alpha_3E_t\lambda_{t+1}. \tag{30}$$

Where $\overline{h}$ and $\overline{p}$ denote the steady state for h_t and p_t respectively, and $\overline{I}$ denotes I_t evaluated at $\overline{h}$ and $\overline{p}$.

With a linear FOCs, we can conjecture

$$\begin{pmatrix} l_t \\ \lambda_t \end{pmatrix}=\begin{pmatrix} r_0 \\ \rho_0 \end{pmatrix}+\begin{pmatrix} r_1 & 0 \\ 0 & \rho_1 \end{pmatrix}\begin{pmatrix} h_t \\ p_t \end{pmatrix}. \tag{31}$$

Following the same procedure as in the case of contagion, or moral hazard, we finally arrive at the following proposition regarding the solution for I_t in the joint case of contagion and moral hazard, denoted as I_t^*.

Proposition 6 *In dynamic setting, the optimal monetary policy dealing with both contagion and moral hazard is:*

$$I_t^*=\mu_0+\mu_hh_t+\mu_pp_t, \tag{32}$$

whereby:

$$\begin{aligned} \mu_0 &= \overline{I}-\mu_h\overline{h}+\mu_p\overline{p}, \\ \mu_h &= \frac{2\overline{p}+\delta a_3\Delta_h}{\delta\Delta_h(a_1+b_1j+a_2+b_2j)}, \\ \mu_p &= -\frac{2[j^2k-(n+B_2j)\overline{h}]+\delta\alpha_3\Delta_p}{\delta\Delta_p[\alpha_1+\beta_1j-(\alpha_2+\beta_2j)]}, \\ \Delta_h &= (a_1+b_1j+a_2+b_2j)(1-\overline{I})-2a_3\overline{p}-\sqrt{(a_1+b_1j+a_2+b_2j)^2(1-\overline{I})^2-4/\delta}, \end{aligned}$$

[20] Because both the objective function and two constraints are convex in I_t, h_t and p_t, global optimality is warranted. With this in mind, we can quite comfortably linearize the first order conditions around the steady states to solve the problem analytically.

$$\Delta_p = (n+B_2j)(1-\overline{I}) - 2\alpha_3[j^2k-(n+B_2j)\overline{h}] - \sqrt{\{(n+B_2j)(1-\overline{I}) - 2\alpha_3[j^2k-(n+B_2j)\overline{h}]\}^2 - 4[j^2k-(n+B_2j)\overline{h}]^2/\delta}.$$

This result has the following implications. Notice first that $\mu_h > 0$ holds, except for the case in which $\Delta_h < 0$ and $2\overline{p} + \delta\alpha_3\Delta_h > 0$, which happens if $\delta\alpha_3\overline{p}(a_1 + b_1 j + a_2 + b_2 j) > 1$ and $2\overline{p} + \delta\alpha_3\Delta_h > 0$. In general, we shall expect $\mu_h > 0$, and thus similar to the case of moral hazard alone, if h_t rises, i.e., there is a structural shift making the risk level in the banking system higher, then the CB will accommodate less often, i.e. $\partial I_t^*/\partial h_t > 0$, regardless whether the banks' sizes are above or below some cutoff points. Again, this implies that when the risk level in the banking system is higher, the CB will respond to banks' call for LOLR less often; and vice versa, when the risk level in the banking system is lower, the CB will respond to banks' call for LOLR more often. Notice that, unlike in the case of moral hazard alone, μ_h becomes dependent on many parameters, including the equilibrium states, $\overline{h}$, $\overline{I}$, and $\overline{p}$ which were not in the moral hazard case alone.

Moreover, because $\Delta_p > 0$, $\mu_p < 0$ if and only if

$$2[j^2k - (n+B_2j)\overline{h}] + \delta\alpha_3\Delta_p > 0.$$

A strong condition for this to be true is

$$j^2k > (n+B_2j)\overline{h},$$

that is the cost of LOLR is smaller than that of OMO when the risk level is in equilibrium. More precisely,

$$j \geq \overline{j}^* \equiv \frac{B_2\overline{h} + \sqrt{B_2^2\overline{h}^2 + 2k(n\overline{h} - \delta\alpha_3\Delta_p)}}{2k},$$

thus $\overline{j}^* < \overline{j}(x = \overline{h})$ and it is smaller than the cutoff size in the one period model.

We may compare this result with that for contagion alone. In the case of contagion alone, $\gamma_1 < 0$ holds regardless of bank size, and so the CB always had an incentive to rescue banks. In the joint case, the CB will only provide LOLR for large banks. The CB is still willing to provide LOLR even if moral hazard effects are considered, but its incentives to provide LOLR are not as strong as in the case where moral hazard is excluded.

Similarly, the system will tend to a long-run equilibrium value for both h and p, set as $\overline{h}$ and $\overline{p}$, which are more complicated than in the case of moral hazard or contagion alone. But it is quite easy to see that both the equilibrium risk levels, $\overline{h}$ and $\overline{p}$, critically depend on the average size of failing banks. $\overline{p}$ critically depends on bank size, because μ_p critically depends on bank size. To see that $\overline{h}$ also critically depends on bank size, we shall take notice that the equilibrium $\overline{p}$ ultimately affect $\overline{h}$.

The dynamics of the joint case of contagion and moral hazard are more interesting - and complicated. From our above analysis, we can see that the system's values will, as before, fluctuate with the stochastic shocks, but also both contagion and moral hazard provide an inbuilt cycling mechanism. When h_t is (temporarily) low, the CB will be induced to accommodate more (i.e., play $(1 - I_t = 1)$), but that will signal the commercial banks to raise h_t and raise the contagion risk p_t, which will cause the CB to refuse accommodation more often, and so on. For plausible values of the coefficients, however, we would expect this cycle to be damped. But the combination of stochastic shocks to riskiness together with this inbuilt damped cycling mechanism will always leave the choice of whether to accommodate, or not, fluctuating around its long-run equilibria.

Finally, there may be multiple values of j around which CB's policy should shift, and such values will always be changing over time, more often and perhaps more strongly than in the previous case of moral hazard case alone, as can be seen more clearly below.

If we set $I_t^* = 0$ in (32), we get:

$$\mu_0(j) + \mu_1(j)h_t + \mu_2(j)p_t = 0. \tag{33}$$

This is a high-order equation (higher-order than quadratic as in the case of contagion or moral hazard alone) in j, hence there may exist several real-number roots which are obviously functions of all the parameters and variables including h_t and p_t. Thus, when h_t and/or p_t change, CB should adjust its rescuing policy accordingly. Therefore, a bank of equal size may be rescued by the CB for a set of values for h_t and p_t, but may not be rescued when $h_{t+1}$and p_{t+1} change. Such a shift in CB's policy reflects the time dynamics and is caused by the changes in time-varying variables, h_t and p_t.

Moreover, even for the same value of h_t and p_t, the CB may optimally rescue only these banks that have the "right sizes". For the above high-order equation (33), there may be several real number roots which are non-negative yet smaller than D^*. These roots also clearly define bank size categories and the CB should rescue these banks ($I_t = 0$) within such categories, but not rescue banks outside these categories.

V. Conclusion

This paper has developed a model of the lender of last resort. In a simple one-period setting, the CB should only rescue banks above a threshold size. This result provides an analytical basis for the well known "too big too fail" syndrome. If that key threshold size were known to commercial banks, it would influence their risk preferences. To avoid this the regulatory authorities should, and do, use "constructive ambiguity" to make their decisions on which banks they are likely to rescue. In a dynamic setting, wherein both the probability of a failure and the likelihood of a bank requiring LOLR being insolvent in each period are a function of CB's prior actions, which then influence the actions of banks and depositors, we focus our analysis on the effects of contagion and/or moral hazard. We show that CB's optimal rescuing policy, whether to support or not, depends not only on bank size, but also on the time-varying variables, such as the probability of a failure and the likelihood of a bank requiring LOLR being insolvent.

Unlike the single period setting wherein the CB only rescues banks above a single threshold size, in dynamic setting CB's optimal rescuing policy may be non-monotonic in bank size, and its optimal policy is time-varying. More importantly, we have found that if contagion is the main concern, then the CB in general would have an excessive incentive to rescue banks through LOLR, though its incentives to rescue big (small) banks are very strong (weak) and thus the equilibrium risk level is high (low). If moral hazard is the main concern, then the CB in general would have little incentive to rescue banks through LOLR, its incentives to rescue do not critically depend on bank size. When both contagion and moral hazard are included as main concerns, then the CB's incentives to rescue though LOLR are stronger than in the single period setting but weaker than in the dynamic setting with contagion alone, and the CB's optimal policy in handling moral hazard is similar to the dynamic setting with moral hazard alone.

A key result coming out of our model is that contagion is the key factor affecting CB's incentive in providing LOLR, while moral hazard is not. When contagion is the main concern, the CB has a very strong incentive to provide LOLR. When moral hazard is also included, in the joint case, even though it weakens the CB's incentive to rescue in general, its effects are quite weak, and the qualitative features of CB's incentive remain the same as when contagion is the main concern. This is so because moral hazard is only an unpleasant by-product of contagion. If it was not for worries about contagion, then CB's incentive to provide LOLR would be very weak, and consequently there would be very little moral hazard.

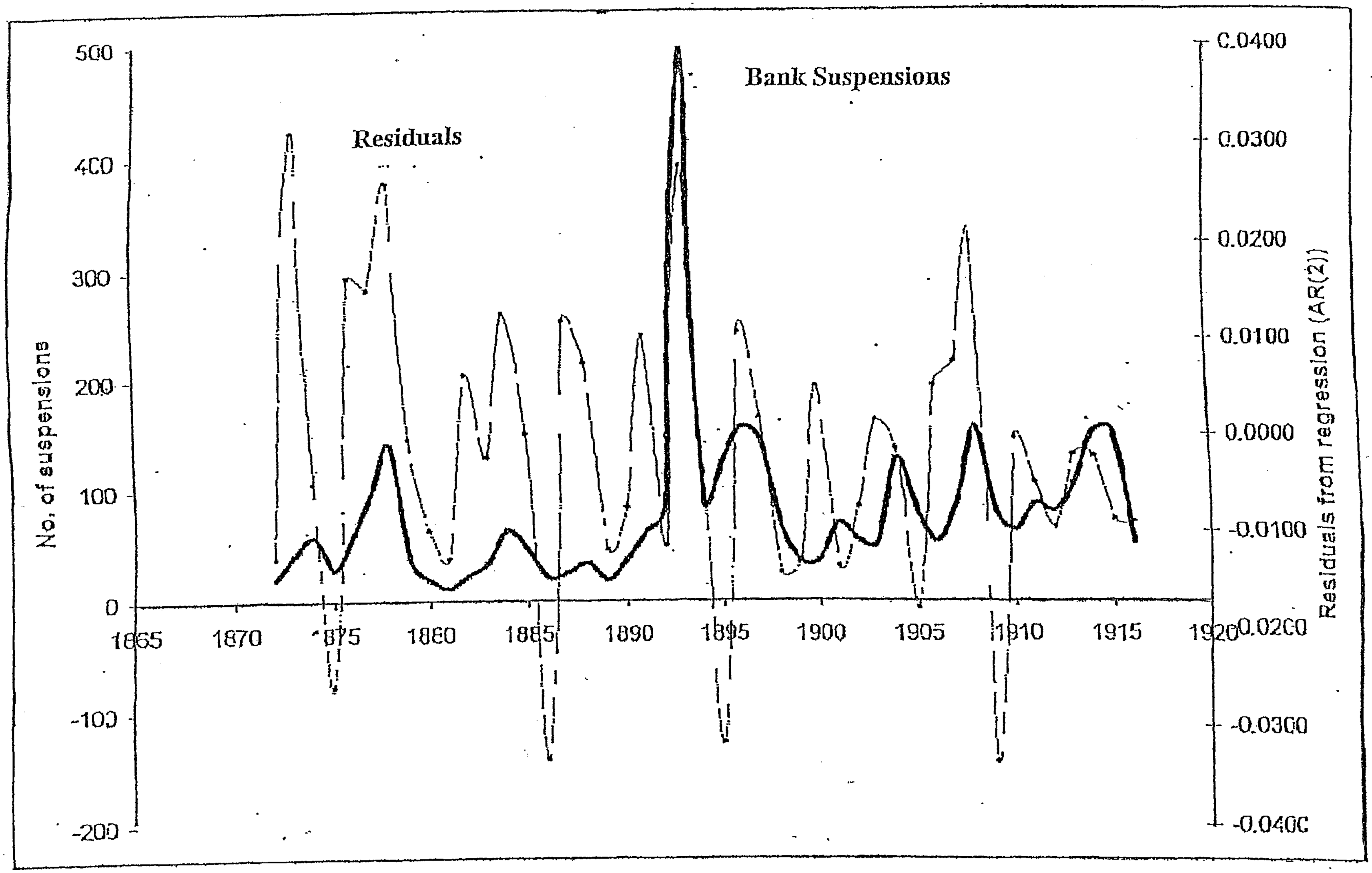
This conclusion has some implications for the ongoing debates over CB's, and in particular the IMF's, rescuing policy in financial crises. When contagion becomes a main concern, even if moral hazard is also present, LOLR becomes (perceived as) necessary and justified. Attacks on such LOLR policies, largely based on arguments from moral hazard, are insufficient and unsatisfactory unless they also address the possibility of contagion.[21]

Turning finally to further possible research, in our model we have assumed that commercial bank managers' appetite for risk remains constant as size varies. This was because we assumed that the incentive for risk-taking inherent in too-big-to-fail was (roughly) balanced by the greater risk aversion of managers in high status large banks. This assumption

[21] For international perspectives on the LOLR function, see Fischer (1999) and Giannini (1998).

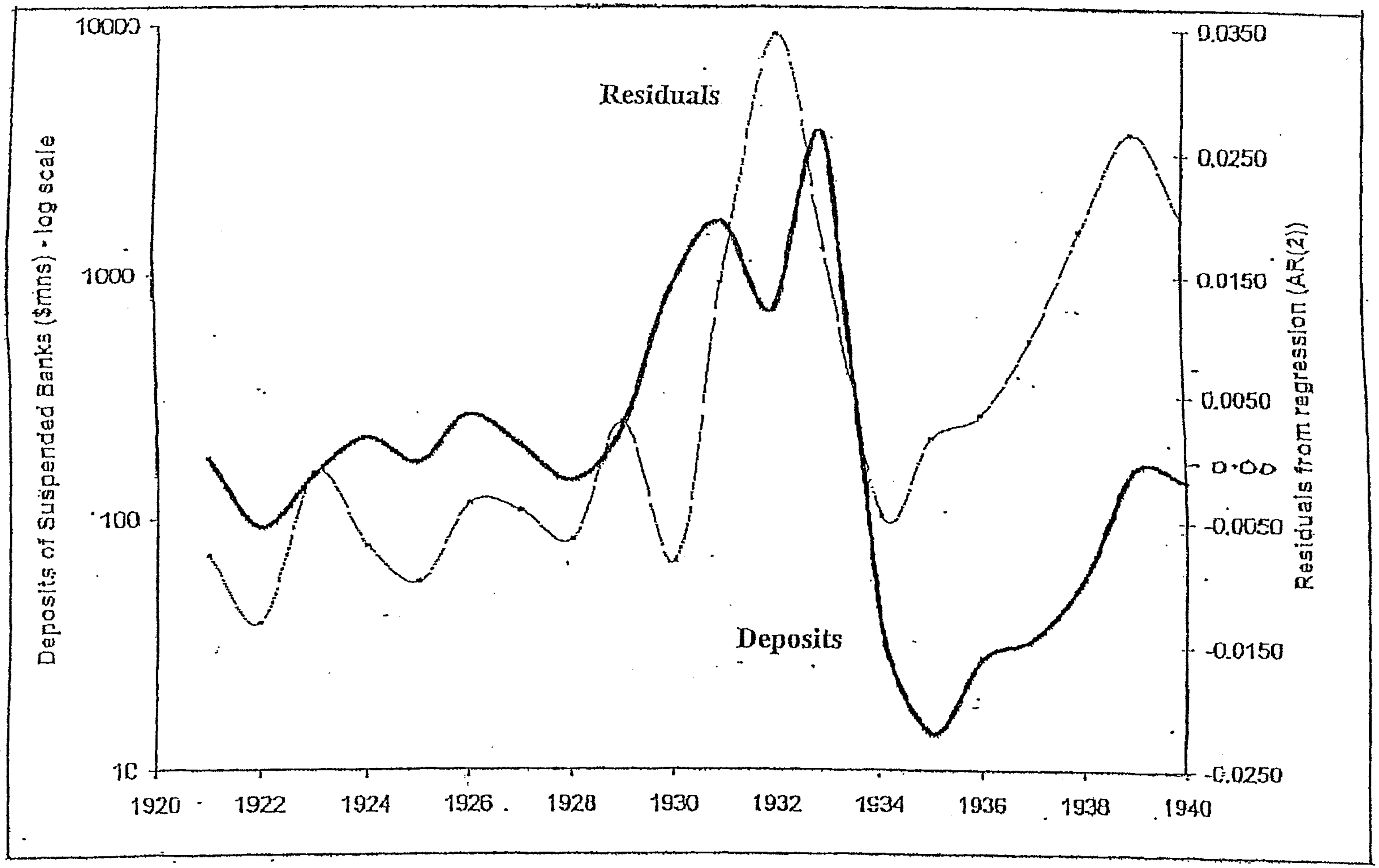
can be examined. In future work, we intend to model the incentives on commercial bank managers, in such a game, more rigorously.

Chart 1. United States: Bank Suspensions



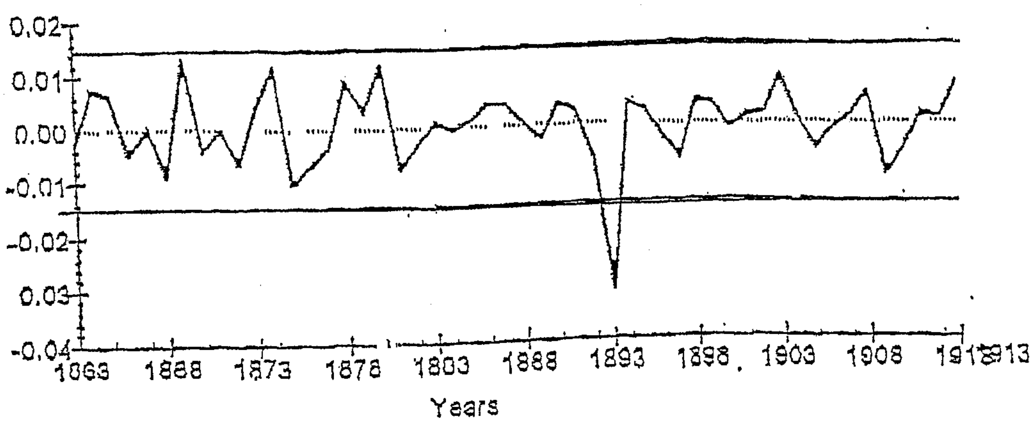
Sources: Friedman and Schwartz (1963, 1982), Historical Statistics of the United States - Colonial Times to 1970.

Chart 2. United States: Deposits of Suspended Banks



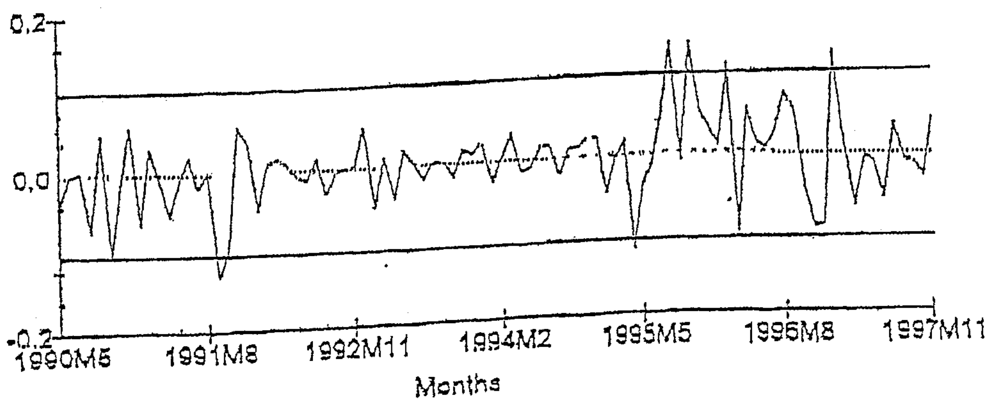
Sources: Friedman and Schwartz (1963, 1982), Historical Statistics of the United States - Colonial Times to 1970.

Chart 3. Australia: Plot of Residuals and Two Standard Error Bands



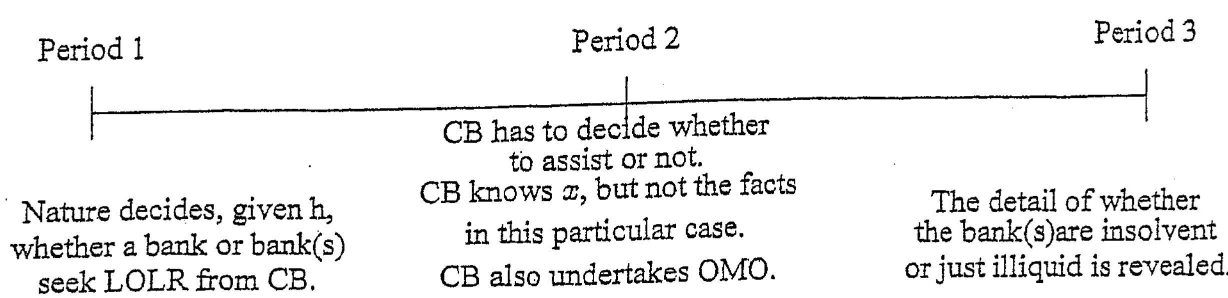
Sources: Butlin, Hall and White (1971) and Mitchell (1983).

Chart 4. Mexico: Plot of Residuals and Two Standard Error Bands



Sources: IFS, May 1990- Nov. 1997.

Figure 1: Time Line



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