Abstract

This paper proposes a signaling model that offers a new perspective on why governments deviate from optimal tax smoothing and delay debt stabilization. In our model, dependable—but not fully credible—governments have an incentive to tighten the fiscal regime when the signaling effect on credit ratings is larger (that is, when a sufficiently large stock of debt has been accumulated). At this point, they may deviate from tax smoothing not to be mimicked by weak governments. The model predicts that primary balances and debt stocks are complementary inputs in the credit rating function as tests on Italian, Irish, Belgian, and Danish data show.

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I. INTRODUCTION

Since the early 1980s, several European countries have adopted fiscal consolidation programs aimed at stabilizing their public debt-to-GDP ratios. This policy was dictated by the need to reassure the markets that the fiscal regime was sustainable and avoid otherwise constantly increasing risk premia and debt financing costs. In turn, the success of the fiscal tightening and its cost depended critically on the speed at which credibility was regained. This interplay between fiscal variables and interest rates is the focus of the theoretical and empirical analysis of this paper. The theoretical part of the paper consists of a signaling model of fiscal policy, whereas the empirical part focuses on the experience of Italy, Ireland, Belgium, and Denmark, from the late 1970s to the 1990s.

The analytics of debt sustainability is well known. A country will have a sustainable fiscal regime if current and future primary balances, interest rates, and growth rates, are such that the government's intertemporal budget constraint is satisfied.\(^2\) Whereas the mathematics is unambiguous, policy prescriptions are not. The key difficulty is that all relevant variables are endogenous, so that the feedback effects of a fiscal package on growth-sensitive revenues and expenditures, as well as interest rates, are crucial to determine whether the measures taken are sufficient to stabilize the fiscal regime. At the same time, shocks to growth rates and interest rates affect the propensity of the government to initiate fiscal consolidation.

Whereas several recent papers have discussed the endogenous link between fiscal consolidation and growth,\(^3\) our paper focuses on that between fiscal consolidation and the credit rating component of interest rates. Modeling the endogenous link between fiscal variables and the credit standing provides an insight into which fiscal variables signal debt sustainability (the primary balance, as we will see).

Our study is also related to the literature on the determinants of large public debts and on the deviations from the “tax smoothing” theory of the government budget (Barro (1979)). We propose a different explanation of why fiscal stabilizations are often delayed, that could be considered complementary to those surveyed by Alesina and Perotti (1995b), which are

\(^2\)See, for example, Spaventa (1987).

\(^3\)Giavazzi and Pagano (1990 and 1995), for example, argue that there are instances in which a fiscal correction can be expansionary rather than contractionary. Their work is complemented by Alesina and Perotti (1995a and 1997), who point out that, to be expansionary, a fiscal tightening requires cuts in expenditure items such as transfer programs and public employment.
mainly based on the distributional consequences of fiscal adjustment\(^4\) and on the strategic use of government debt.\(^5\)

This paper links the timing of a fiscal correction to the credit standing and the debt level of a country. In our framework, a fiscal stabilization may be delayed if risk premia and debt levels are below a critical threshold so that no government has any incentive to tighten the fiscal regime: in this range, both dependable and weak policy makers choose to run primary deficits and build up the debt stock. Only when risk premia or debt levels exceed this critical threshold, interest payments become so large that dependable governments prefer to run primary surpluses, thus signaling the sustainability of the fiscal regime. Differently from other papers in this literature, we endogenously derive the threshold triggering fiscal stabilizations as a function of the reputation and preferences of the government.\(^6\) Moreover, when we allow for endogenous debt accumulation, we find a novel strategic role for government debt. In this case, the optimal policy of governments that are dependable—but not fully credible—is to strategically accumulate the critical amount of debt that allows them to signal. In this case, there is only one equilibrium, in which the dependable government first runs primary deficits and then deviates from optimal tax smoothing to signal its type.

An interesting case study is the Italian fiscal stabilization of the early 1990s. Figure 1 (top left panel) shows Italy's country rating against the time profile of primary balances. Although the rating improved somewhat during the 1980s when primary deficits were being reduced, it dropped considerably in the early 1990s when primary surpluses were about to be achieved and began to recover only in mid 1996. This evidence raises a number of questions: Is this seemingly non-monotonic relationship between ratings and primary balances an Italian peculiarity or is it common also in other instances of fiscal stabilization? And, can economic theory account for the observed behavior of credit ratings, primary balances, and debt stocks?

\(^4\)Fernandez and Rodrik (1991) show that the difficulty of identifying ex-ante individual gainers and losers from a reform could generate a bias toward the status quo. Similarly, Alesina and Drazen (1991) argue that a war of attrition between different social groups determines the timing of stabilization. In a related paper, Drazen and Grilli (1993) show that economic crises may have positive welfare effects when they prompt a fiscal stabilization.

\(^5\)Alesina and Tabellini (1990) and Persson and Svensson (1989) show that policy makers may accumulate strategically government debt to constrain the actions of their successors.

\(^6\)The existing literature often sets the trigger levels for the timing of stabilization exogenously. Bertola and Drazen (1993), for example, derive a nonlinear relationship between private consumption and government spending with government spending falling whenever it reaches exogenously given target points. An exogenous trigger level of government debt is also assumed in Sutherland (1995).
Figure 1. Rating and Primary Balance-to-GDP Ratio
Figure 1 shows that the same non-monotonic relationship between primary balances and credit ratings characterized the Irish, Danish, and Belgian, stabilization of the mid 1980s. In the late 1970s, these three countries enjoyed high ratings—the highest in the sample—despite sizeable primary deficits. As primary balances started improving in the early 1980s, ratings rapidly deteriorated. Only when primary surpluses were achieved in the mid 1980s, the ratings began to recover. This evidence not only confirms the non-monotonic relationship between credit ratings and primary balances noted for Italy, but also suggests that primary surpluses might have a signaling role.

Figure 2 sheds some light on the causes of the observed non-monotonic relationship by linking the evolution of credit ratings to the one of debt-to-GDP ratios. The small debt stock of all three countries in the late 1970s—the smallest in the sample—seems to account for the high ratings observed in those years, notwithstanding the primary deficits. When the debt stock rapidly increased in the early 1980s, possibly reflecting higher real world interest rates, as well as disinflationary monetary policies, the credit ratings deteriorated because investors did not know whether the primary balance would improve enough to make the fiscal regime sustainable. Figure 2 also shows that the debt-to-GDP ratio alone cannot fully account for the time-series behavior of ratings, which began to recover after the achievement of primary surpluses and in the presence of still-rising debt-to-GDP ratios. 7

In our view, the stylized facts of Figures 1 and 2 suggest that a bivariate analysis of the data might be misleading and that both debt stocks and primary balances concur to determine the credit rating of a country. This is also the first prediction of our theoretical model. The second prediction is that primary balances have a signaling role at high debt levels. This is indeed the original claim of our paper and, as discussed below, it distinguishes our model with endogenous uncertainty on the type of government in power from more conventional models with exogenous uncertainty. The testable implication is that debt stocks and primary balances not only concur to determine the credit rating of a country—as also predicted by models with exogenous uncertainty—but they are complementary inputs in the credit rating function until signaling is completed. We present econometric tests of this complementarity hypothesis below, but Figures 1 and 2 already provide a first indication of its validity. Primary balances seem to have a greater effect on credit ratings when the debt stock is large and when the primary balance is about to swing from a deficit into surplus. This greater importance of primary balances beyond a certain debt threshold can explain both the sudden deterioration of ratings at the beginning of the signaling phase, when there are still primary deficits, and their improvement after the achievement of a surplus, notwithstanding the large debt stock.

---

7In this respect, the sharp improvement of Italian ratings between 1983 and 1988 is somewhat puzzling because it took place in a context of rapidly growing debt-to-GDP ratios and almost unchanged primary balances. Neither our theoretical model nor our empirical estimates can account for this episode.
Figure 2. Rating and Debt-to-GDP Ratio
The paper is structured as follows. Section 2 presents the theoretical model, justifies its key assumptions and links it to the relevant economic literature. Section 3 discusses how to derive testable implications from our stylized signaling model and presents some econometric estimates, based on the data of Figures 1 and 2, that confirm the predictions of the model. Section 4 concludes. Appendices derive the main results.

II. THE THEORETICAL MODEL

In this section, we propose a model where credit ratings, primary surpluses, and debt stocks, are jointly determined. We show that, in a world where there is uncertainty about the type of government, primary surpluses can have a signaling role.

A. The Setup

We consider a three-date, two-period model. At time zero, the government issues a given stock of one-period debt $D_1$ and the public sets the interest factor $R_1$. At time one, the government pays back $D_1 R_1 (1 - \theta_1)$—where $\theta_1$ is the fraction of debt defaulted,\footnote{At this stage, we assume that the revenues from issuing $D_1$ are “put in the ground.” In Appendix 3, we relax this assumption by allowing the government to choose $D_1$, taxes, $\tau_0$, and expenditure, $g_0$ at time zero.} spends $g$, levies distortionary taxes $\tau_1$, and rolls over a stock of debt $D_2$ to the last period, while the public sets the interest factor $R_2$.\footnote{In this paper, we do not interpret default as inflating away the real debt value because it would not be consistent with our credit risk measure. In a framework that allowed for default through inflation, the maturity of the debt would play a role (see, for example, Calvo and Guidotti (1990) for a model with exogenous uncertainty and Drudi and Prati (1995) for a signaling model with endogenous uncertainty).} At time two, the government pays back $D_2 R_2 (1 - \theta_2)$ and levies distortionary taxes $\tau_2$. The government’s budget constraints are:

$$D_1 R_1 (1 - \theta_1) + g = \tau_1 + D_2$$

(1)

$$D_2 R_2 (1 - \theta_2) = \tau_2$$

(2)

\footnote{As noted below, the government can issue a positive stock of debt at time one only when default does not occur. Therefore, in equilibrium, the two actions of issuing new debt and defaulting on the outstanding stock of debt are incompatible.}
We assume that within each period the government moves first so that the timing of the game is:\textsuperscript{11}

<table>
<thead>
<tr>
<th>Time 0</th>
<th>Time 1</th>
<th>Time 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>$R_1$</td>
<td>$g, D_2, \theta_1, \tau_1$</td>
</tr>
</tbody>
</table>

We consider $D_1$ a parameter of the model so that the government chooses $g, D_2, \theta_1, \theta_2$ to minimize the cost function:

$$\frac{h}{2} \tau_1^2 + \frac{h}{2} \tau_2^2 - \alpha g$$

where $\frac{h}{2} \tau_i^2$ are distortionary costs of taxation and $\alpha g$ are the benefits of government expenditure.\textsuperscript{12}

After plugging the budget constraints into the objective function, the optimal policy of the government is the solution of the problem:

$$\text{Min}_{g, D_2, \theta_1, \theta_2} \frac{1}{2} \left[ D_1 R_1 (1 - \theta_1) + g - D_2 \right]^2 + \frac{1}{2} \left[ D_2 R_2 (1 - \theta_2) \right]^2 - \hat{\alpha} g$$

where $\hat{\alpha}$ is the ratio of the marginal benefit of government expenditure $\alpha$ to the parameter $h$ of the quadratic function of distortionary costs of taxation.

Finally, we assume that two types of government might be in charge. Type D ("dependable") can precommit not to default and always chooses $\theta_D^D = 0$, whereas type W ("weak") cannot precommit and chooses either $\theta_W^W = 1$ or $\theta_W^W = 1$, depending on his incentives to mimic type D policies in period one.\textsuperscript{13}

At time zero, the public does not know with certainty which government is in charge, but it believes with probability $1 - p_0$ that the government is type D. In the rest of the paper, we will loosely refer to the prior probability $1 - p_0$ as "initial reputation." At time one, the

\textsuperscript{11}The results of the paper would be analogous if the government's and the public's actions were simultaneous within each period. By contrast, an opposite timing with the public moving first would cause a multiplicity of equilibria.

\textsuperscript{12}In Appendix 1 of Drudi and Prati (1998), we show that minimizing (3) is equivalent to maximize the welfare of an economy populated by risk-neutral agents.

\textsuperscript{13}As shown in Appendix 1, it is never optimal for a weak government to choose a $\theta$ different from zero or one.
public will observe the policies implemented by the government in charge and revise the probability \( p_0 \) using Bayes' rule:

\[
p_1 = \frac{p_0 \cdot \text{Prob}(\theta_1 = \hat{\theta}_1, \tau_1 = \hat{\tau}_1, g = \hat{g} : \text{type W})}{p_0 \cdot \text{Prob}(\theta_1 = \hat{\theta}_1, \tau_1 = \hat{\tau}_1, g = \hat{g} : \text{type W}) + (1 - p_0) \cdot \text{Prob}(\theta_1 = \hat{\theta}_1, \tau_1 = \hat{\tau}_1, g = \hat{g} : \text{type D})}
\]

(5)

where all probabilities are conditioned on the type of government and \( \hat{\theta}_1 \), \( \hat{\tau}_1 \) and \( \hat{g} \) are the observed policies.

Under the additional assumptions of risk neutrality and zero risk-free interest rate, investors will set interest factors according to the no-arbitrage condition:

\[
R_1 (1 - \theta^e_1) = 1
\]

(6)

where \( \theta^e_1 \) is the default rate expected by investors.\(^{14}\)

**B. The Equilibria**

We investigate perfect Bayesian equilibria in pure strategies. Two types of equilibria may prevail: *separating equilibria* or *pooling equilibria*. In the first, the government W chooses \( \theta^w_1 = 1 \) and reveals itself at time one. In the second, type W mimics type D policies at time one and reveals its type in the last period by choosing \( \theta^w_2 = 1 \). Note that we use subscripts to refer to time and, when needed, superscripts to indicate equilibria (S for separating and P for pooling) and types (D for dependable and W for weak).

In the absence of uncertainty, type W would be unable to issue any debt at time zero because the public would anticipate its incentive to default at time one. By contrast, in the game of incomplete information considered in this paper, the uncertainty about the government in power allows type W to issue debt until uncertainty is resolved. Type W will reveal its type in period one (separating equilibria) when the cost of imitating type D policies is larger than the benefit of issuing debt between period one and period two. Type W will reveal its type in period two (pooling equilibria) when the opposite is true.

As we will see, this tradeoff is crucially affected by: the marginal benefits of expenditure (the higher is the parameter \( \alpha \), the greater is the expenditure that type W would like to finance in period one and the greater are the benefits of issuing debt between period

---

\(^{14}\)The game studied in this paper is part of a general class of dynamic games with a large player (the government) and a large number of small players. The play of the large players is observed and is therefore part of the public history of the game, while the individual plays of the small players are not observed, so that only their aggregate play is part of the public history of the game. Similar games in an infinite horizon context are in Chari and Kehoe (1990 and 1993) and Stokey (1991).
one and two), the marginal cost of taxation (the higher is the parameter \( h \), the lower are the taxes that type W is willing to levy in period one and the higher the cost of imitating type D policy), the initial stock of debt (the higher is \( D_1 \), the higher are the taxes that type D levies in period one, and the higher the cost of imitating its policy), and the initial reputation (the higher is \( 1 - p_0 \), the smaller are the risk premia, and the higher are the incentives to mimic type D policies).

In Appendix 1, we formally derive the equilibria. In this section, we state the main results in two propositions and provide the intuition behind them. First, consider separating equilibria. In this case, at time one, type W reveals its type by defaulting on the outstanding stock of debt. There are two possible cases. In the first, type D chooses the optimal unconstrained expenditure and taxation levels in a separating equilibrium and type W does not mimic them. In the second, the parameter values are such that type W would mimic type D’s unconstrained optimal policy; as a result, type D cuts expenditure below its optimal unconstrained level not to be confused with type W. Type D will choose this second signaling strategy only if the benefits of paying lower interest payments after signaling outweigh the cost of cutting expenditure. Evidently, the poorer is the initial reputation of the government, the higher are interest rate premia, and the stronger are the incentives to signal. The following proposition specifies the range of parameters in which each case of separating equilibrium exists.

**Proposition 1**: **Separating equilibria** hold for:

\[
\frac{1}{2(1-p_0)} < \frac{\alpha}{D_1} < \frac{1 + \sqrt{1-(1-p_0)^2}}{(1-p_0)^3} \quad \Rightarrow \quad \frac{D_1 R_1^S}{2} < \alpha < D_1 (R_1^S)^2 \sqrt{(R_1^S)^2 - 1}.
\]

**Case I**: type D runs a **primary budget surplus** in period one for (Table 1 shows equilibrium strategies and interest rates):

\[
\frac{1}{2(1-p_0)} < \frac{\alpha}{D_1} < \frac{1}{(1-p_0)} \quad \Rightarrow \quad \frac{D_1 R_1^S}{2} < \alpha < D_1 R_1^S.
\]

**Case II**: type D runs a **balanced primary budget** in period one for (Table 2 shows equilibrium strategies and interest rates):

\[
\frac{1}{(1-p_0)} < \frac{\alpha}{D_1} < \frac{1 + \sqrt{1-(1-p_0)^2}}{(1-p_0)^3} \quad \Rightarrow \quad D_1 R_1^S < \alpha < D_1 (R_1^S)^2 \sqrt{(R_1^S)^2 - 1}.
\]

**Proof**: see Appendix 1 for the proof.

The amount of debt maturing in period one determines the relevant case. In **Case I**, the debt plus interest maturing in period one is **large enough** \( (D_1 R_1^S > \alpha) \) to make type D run a **primary surplus** by choosing an expenditure level \( (g'^{SD} < \alpha) \) **smaller** than the smoothed level of taxation \( (\tau_1^s = \tau_2^s = \alpha) \). This policy allows the public to distinguish type D from type W, which prefers a higher expenditure level \( (g'^{SW} < g'^{SW} = \alpha) \). In **Case II**, the debt plus interest maturing in
Table 1. Separating Equilibrium (Case I)

Equilibrium range:

\[
\frac{D_1 R_1^S}{2} < \hat{\alpha} < D_1 R_1^S \Rightarrow \frac{1}{2(1-p_0)} < \frac{\hat{\alpha}}{D_1} < \frac{1}{(1-p_0)}
\]

Equilibrium policies:

\[
R_1^S = R_{1,W}^S = R_{1,D}^S = \frac{1}{1-p_0}
\]

\[
g_{1,W}^S = \hat{\alpha} \quad g_{1,D}^S = 2\hat{\alpha} - D_1 R_1^S < \hat{\alpha}
\]

\[
\theta_{1,W}^S = 1 \quad \theta_{1,D}^S = 0
\]

\[
\tau_{1,W}^S = \tau_{1,D}^S = \hat{\alpha}
\]

\[
D_{2,W}^S = 0 \quad D_{2,D}^S = \hat{\alpha}
\]

\[
R_2^S = \infty \quad R_{2,D}^S = 1
\]

\[
\theta_{2,W}^S = 1 \quad \theta_{2,D}^S = 0
\]

\[
\tau_{2,W}^S = 0 \quad \tau_{2,D}^S = \hat{\alpha}
\]

Primary balance of type D in period 1:

\[
\tau_{1,D}^S - g_{1,D}^S = \hat{\alpha} - (2\hat{\alpha} - D_1 R_1^S) > 0
\]
Table 2. Separating Equilibrium (Case II)

Equilibrium range:

\[ D_1 R_1^s \leq \hat{\alpha} < D_1 (R_1^s)^2 (R_1^s \cdot \sqrt{(R_1^s)^2 - 1}) \Rightarrow \frac{1}{1-p_0} < \frac{\hat{\alpha}}{D_1} < \frac{1 + \sqrt{1 - (1-p_0)^2}}{(1-p_0)^3} \]

Equilibrium policies:

\[ R_1^s = R_1^{s,w} = R_1^{s,d} = \frac{1}{1-p_0} \]
\[ g^{s,w} = g^{s,d} = \hat{\alpha} \]
\[ \theta_1^{s,w} = 1 \quad \theta_1^{s,d} = 0 \]
\[ \tau_1^{s,w} = \tau_1^{s,d} = \hat{\alpha} \]
\[ D_2^{s,w} = 0 \quad D_2^{s,d} = D_1 R_1^s < \hat{\alpha} \]
\[ R_2^{s,w} = \infty \quad R_2^{s,d} = 1 \]
\[ \theta_2^{s,w} = 1 \quad \theta_2^{s,d} = 0 \]
\[ \tau_2^{s,w} = 0 \quad \tau_2^{s,d} = D_1 R_1^s < \hat{\alpha} \]

Primary balance of type D in period 1:

\[ \tau_1^{s,d} - g^{s,d} = \hat{\alpha} - \hat{\alpha} = 0 \]
period one is so small \((D_1 < \alpha)\), that, if type D followed the optimal separating equilibrium tax and expenditure policies of Case I, it would run a primary deficit by choosing an expenditure level \((g^{SD} > \alpha)\) larger than the smoothed level of taxation \((\tau_1^{SD} \alpha^{SD} = \alpha)\). However, this tax and expenditure levels would make the separating equilibrium unsustainable because type W would mimic such large expenditure. To avoid a breakdown of the separating equilibrium, as long as the debt plus interest maturing in period one is small but not too small \((\alpha < D_1 < \alpha)\), type D deviates from the optimal—but unsustainable—Case I policies. To prevent mimicking, type D runs a balanced period-one primary budget rather than the primary deficit implied by Case I policies: expenditure is lower than it would be under Case I \((g^{SD} = \alpha)\) and period one taxes are higher than period two taxes \((\tau_1^{SD} = \alpha^{SD})\), implying a deviation from perfect tax smoothing.

In separating equilibria, type W repudiates the debt in period one \((\theta_1^{SW} = 1)\) so that the interest paid between period zero and period one includes a risk premium \((R_1 \alpha > 1)\), which is a function of the government's initial reputation \((1 - p_0)\). After separation, type W faces an infinite interest rate and does not issue any debt, whereas type D is able to issue debt at the risk-free interest rate.

**Proposition 2**: Pooling equilibria exist in the range:

\[
\frac{1 + \sqrt{1 - (1 - p_0)^2}}{(1 - p_0)^2} \leq \frac{\alpha}{D_1} \iff D_1 R_2^{P} (R_2^{P} + \sqrt{(R_2^{P})^2 - 1}) < \alpha.
\]

In pooling equilibria all government types run a primary budget deficit in period one (Table 3 shows equilibrium strategies and interest rates).

**Proof**: See Appendix 1 for the proof.

In pooling equilibria, the debt stock issued in period one is small enough \((D_1 < \alpha)\) to make type D willing to be mimicked in period one, as type D considers the marginal benefits of expenditure larger than the interest cost of being confused with type W. In pooling equilibria, type D runs a primary deficit in period one and chooses an expenditure large enough \((g^{P,D} > \alpha)\) to be imitated by type W in period one. As a consequence, given that type W defaults only in period two \((\theta_1^{PW} = \theta_2^{PW} = 0\) and \(\theta_2^{PW} = 1)\), the government pays the risk-free rate on the debt maturing in period one \((R_1 = 1)\).

Figure 3 shows the parameter ranges in which each type of equilibrium exists. Initial reputation \((1 - p_0)\) is on the horizontal axis, whereas the ratio of the marginal benefits of expenditure to the initial debt stock \((\frac{\alpha}{\alpha})\) is on the vertical axis. Pooling equilibria exist in an area in the top right-hand corner where a small initial stock of debt (a high \(\alpha\)) is associated with a small risk premium (a high \(1 - p_0\)). As we move toward the center of the box, the initial
Table 3. Pooling Equilibrium

Equilibrium range:

\[
D_1 R_2^p \left( R_2^p + \sqrt{(R_2^p)^2 - 1} \right) \leq \hat{\alpha} \iff \frac{1 + \sqrt{1 - (1 - p_0)^2}}{(1 - p_0)^2} \leq \frac{\hat{\alpha}}{D_1}
\]

Equilibrium policies:

\[
R_1^{p,w} = R_1^{p,d} = 1
\]

\[
R_2^p = R_2^{p,w} = R_2^{p,d} = \frac{1}{(1 - p_0)}
\]

\[
g^{p,w} = g^{p,d} = \frac{(1 + (R_2^p)^2)}{(R_2^p)^2} \hat{\alpha} - D_1 > \hat{\alpha}
\]

\[
\theta_1^{p,w} = \theta_1^{p,d} = 0
\]

\[
\tau_1^{p,w} = \tau_1^{p,d} = \hat{\alpha}
\]

\[
D_2^{p,w} = D_2^{p,d} = \frac{\hat{\alpha}}{(R_2^p)^2}
\]

\[
\theta_2^{p,w} = 1 \quad \theta_2^{p,d} = 0
\]

\[
\tau_2^{p,w} = 0 \quad \tau_2^{p,d} = \frac{\hat{\alpha}}{R_2^p}
\]

Primary balance of type D in period 1:

\[
\tau_1^{p,d} - g^{p,d} = \hat{\alpha} - \left[ \frac{(1 + (R_2^p)^2)}{(R_2^p)^2} \hat{\alpha} - D_1 \right] < 0
\]
Figure 3. Equilibrium Existence Ranges

- Upper limit of separating (case I)
- Separating equilibria II
- Pooling equilibria
- Lower limit of separating (case I)
- Lower limit of pooling
- \( g < 0 \)

\[ \frac{\alpha}{D_1} \]

\[ 1 - p_0 \]
stock of debt and the risk premia increase, and separating equilibria (Case I and Case II) prevail.

Multiple equilibria are possible in an intermediate range where there is an overlapping of the ranges in which pooling and Case II separating equilibria exist. In this area, if investors choose the risk-free rate in period zero \( (R_i^P = 1) \), the best response of both governments is to play their pooling equilibrium strategies; if investors, instead, choose the interest rate associated with separating equilibrium \( (R_i^S > 1) \), then the best response of both governments is to play their Case II separating equilibrium strategies. The fact that in the multiplicity area both types of policy makers prefer pooling equilibria (see Appendix 2) suggests that, if investors could coordinate their expectations, they would demand the risk-free rate in period zero and make pooling equilibria prevail. This would, indeed, be their optimal strategy given that both governments are ultimately trying to maximize the utility of the investors. However, as investors are atomistic, they may not coordinate their expectations on the risk-free rate and may well demand the higher rates associated with separating equilibria.\(^{15}\) As a result, in the range of parameters where multiple equilibria exist, investors could force an early—but suboptimal—resolution of uncertainty by failing to coordinate their expectations on the risk-free rate. In the three-period extension of Appendix 3, the multiplicity of equilibria disappears because type D would choose in period zero a level of \( D \) not in the multiplicity range.

C. Key Assumptions and Extensions

A first issue is whether our results depend critically on the two-period structure of the model. This question is taken up in Appendix 3 where we add one period to the model and allow the government to choose the optimal level of debt, \( D_0 \), taxes, \( \tau_0 \), and expenditure, \( g_0 \), at time zero. This extension has the advantage of making endogenous the evolution of the debt-to-GDP ratio and with it the timing of the switch from pooling to separating equilibria. In addition, it drastically simplifies the results of the model. In the three-period model, there is only one possible equilibrium in which the economy switches from a pooling equilibrium in period zero to a Case II separating equilibrium in period one. The main results of the two-period version are confirmed with the pooling in period zero being associated with a primary deficit and the separating in period one with a balanced primary budget. The key difference is that, in the three-period model, type D is allowed to choose \( D_1 \) optimally and eliminate welfare-inferior equilibria.

To model a shift of equilibria within the two-period model, it would be necessary, instead, to invoke some kind of exogenous shock. Higher risk-free real interest rates, for example, would lead ceteris paribus to a faster dynamics of the debt-to-GDP ratio and could make separating equilibria prevail in parameter ranges that would otherwise be associated

\(^{15}\)The equilibrium refinement of Pareto Dominance (Fudenberg and Tirole, 1992, pp. 18-23) could rule out separating equilibria but its application requires players to be able to coordinate their actions.
with pooling equilibria. In the experience of Ireland, Belgium, and Denmark, of the early 1980s, an empirical counterpart for such shocks might be the increase in worldwide real interest rates and, perhaps, the tightening of the monetary policy regime associated with the creation of the EMS.

A second issue is whether our assumption that type D can precommit its policies is justified. This assumption was made in a number of papers that used game theoretical models to study monetary policy signals. As Cukierman and Liviatan (1991) pointed out, an alternative to this approach is to assume, as in Vickers (1986), that there are two types of policy makers with different preferences and that the public is initially uncertain about which of the two is in power. Following this second modeling strategy yields results very similar to those of this paper.

A third issue is whether a model with a continuum of types would yield different results. As it is difficult to imagine a continuum of types with varying degrees of precommitment ability, this extension is meaningful only in relation to a model with policy makers with different preferences. In this case, as policies are implemented, investors would update their prior probability that certain types are in power. As a consequence, signaling would no longer be instantaneous and credit ratings would change continuously once a signaling phase has begun.

Another feature of our model is that the weak government always defaults on the outstanding stock of debt. This does not prevent it from issuing debt, as long as investors do not know—because of incomplete information—that it is in power. Only when uncertainty is resolved, the weak government faces the traditional time-inconsistency problem and is unable

\footnote{See Barro (1986) and Persson and Tabellini (1990, Ch. 3-4) for a review of the literature.}

\footnote{Drudi and Prati (1993) assume that the two types of policy makers are characterized by different redistributive preferences. Only two insights of that model are lost in the simplified version of this paper. First, crisis equilibria in which no debt can be issued are no longer possible because type D is always willing to repay the debt no matter how large interest payments are. By contrast, crisis equilibria exist in the earlier model whenever the probability of a "left-wing" government being in power is high enough to generate risk premia so high that even a "right-wing" government would repudiate. Second, in this paper type W optimal repudiation rate is always \( \theta_{t-1} \), whereas a partial repudiation with \( 0 < \theta_{t} < 1 \) is possible in the previous version whenever the redistributive preferences of the "left-wing" government are not too extreme.}

\footnote{In this respect, this paper is different from Drudi and Giordano (1995), where the government defaults because of an exogenous shock to real interest rates, or Alesina, Prati and Tabellini (1990), where it defaults because of a self-fulfilling confidence crisis, whose likelihood depends on the maturity structure of the debt (see also Cole and Kehoe, 1996b).}
to issue any debt. The advantage of this setup is that we do not need to assume exogenous costs of default, as is often done in the literature, to have equilibria with a positive debt stock. Nevertheless, default is costly and type W takes very much into account the fact that it will be excluded from borrowing after a default to determine when it is optimal to stop mimicking the policies of a dependable government.  

Signaling models with features that resemble those of this paper are also common in the literature on sovereign lending. These papers, however, do not fully model the fiscal decision of the government and, as a consequence, do not characterize the equilibrium path of the debt stock, the primary balances, and the interest rates, in the same way.  

III. ECONOMETRIC TESTS OF THE SIGNALING MODEL

A. Testable Implications of the Signaling Model

Figure 3 shows that, in our model, when the debt stock is small and initial reputation is high (top right-hand corner), primary deficits—associated with pooling equilibria—prevail and ratings remain high because no government is expected to default. As the debt stock increases, primary deficits decline and, when dependable governments are about to switch to primary surpluses and signal, credit ratings drop (intermediate area of the box). Only when primary surpluses are consolidated, credit ratings improve. This implies that a testable prediction of our model is that ratings (RATE) are negatively related with the debt-to-GDP ratio (DY) and positively related with the primary balance-to-GDP ratio (PY). If our model holds, we should then be able to estimate a rating function:

\[
RATE = f\left(DY, PY \right)
\]

with \(f_{DY}<0\) and \(f_{PY}>0\), where \(f\) indicates the derivative with respect to the ith argument.

---

19It is important to note that this is not a "punishment" or a trigger strategy necessary to support equilibria with a positive debt stock, as in Grossman-van Huyck (1989) and Eaton and Gersowitz (1981), but rather the sequentially rational strategy of all agents once default has occurred, as in Chari and Kehoe (1990, 1993).


21As discussed in Section 2.3, the debt stock can increase either because we assume that in the two-period model of Section 2 the initial debt stock \(D_1\) is subject to an exogenous shock or because it evolves endogenously as predicted by the three-period extension of Appendix 3.
However, our model with *endogenous uncertainty* on the type of the policy maker is not the only one to predict a rating function of this type. Also a model with *exogenous uncertainty* on real interest rates or public expenditure could generate very similar predictions. With exogenous uncertainty, investors fear a default not because a weak policy maker might be in power (as in our model), but because a large enough shock to interest rates or public expenditure might hit the economy and force even a dependable government to default. In this class of models, for a given distribution of shocks, a default would be more likely the higher is the debt stock and the bigger is the primary deficit. As a consequence, also a model with exogenous uncertainty would predict that ratings be a negative function of the debt stock and a positive function of primary balances.

An exogenous uncertainty model would, however, be unable to explain why \( f_{PY} \) might vary over time as a function of the debt stock. This evidence would, instead, be consistent with our model, according to which the signaling power of primary balances (\( f_{PY} \)) varies with the stock of debt. *When the debt stock is low, pooling equilibria prevail and \( f_{PY} \) should be small* because primary balances do not signal the type of government in power. *When the debt stock is high, separating equilibria prevail and \( f_{PY} \) should be large* because primary balances have a signaling role. In sum, a unique prediction of our model is that PY and DY are *complements* in the rating function \( f(PY, DY) \), i.e., \( f_{PY, DY} > 0 \).23

There are several ways in which the predicted complementarity of PY and DY can manifest in the data. A literal interpretation of the model would suggest an instantaneous signaling, with \( f_{PY} \) increasing only temporarily when the debt stock reaches the critical threshold that divides the pooling and separating equilibrium areas. In practice, governments change and fiscal measures are often of a one-off nature, so that signaling may take place over several periods. In the limit, signaling may never be a once-and-for-all job and the complementarity between PY and DY might be detectable many years after the beginning of fiscal stabilization.

To allow for these different empirical counterparts of the theoretical model, we estimate two specifications of the rating function (7). The first one is associated with the idea that signaling is gradual but still confined to a specific time-interval with \( f_{PY} \) going back to its normal (presignaling) level once signaling is completed, no matter whether the debt-to-GDP ratio is still at relatively high levels. For each country, we identify a *signaling phase* associated with the period in which primary balances swing into surplus, and we test whether \( f_{PY} \) is significantly larger in it than in the previous and following periods. The second specification is

---

22There are several theoretical models that allow for exogenous uncertainty. See, for example, Missale, Giavazzi, and Benigno (1997), for the case of interest rate shocks, and Calvo and Guidotti (1990), for the case of public expenditure shocks.

23We are indebted to a referee for suggesting this interpretation of our model.
associated with the idea that signaling is not a once-and-for-all job and that $f_{py}$ is a positive function of the debt-to-GDP ratio.

The complementarity of PY and DY in the rating function $f(PY, DY)$ also implies that, if the government is dependable, the primary balance will always increase when the debt increases. The intuition is that the larger effects of the fiscal tightening on the credit rating make it always more profitable for the dependable government to tighten fiscal policy when the debt stock is large. To test this prediction, we check whether, in a regression of PY on DY, the estimated coefficient is positive and significantly greater than zero.

B. Results

For our estimates, we use the time-series data for Belgium, Denmark, Ireland, and Italy, shown in Figures 1 and 2. Data are semiannual with the primary balance-to-GDP ratio (PY) obtained as a linear interpolation of annual data. The credit rating (RATE) is published by Institutional Investor in March and September of each year and is available since 1979:02. The debt-to-GDP ratio (DY) is the end-of-period figure of June and December of each year. Preliminary empirical investigation based on cointegration tests suggests that the three variables RATE, PY, and DY, can be modeled as stationary processes. The samples are 1979:02-1995:01 for Ireland and Belgium, 1979:02-1992:01 for Denmark, and 1989:02-1998:02 for Italy.

\[\text{References}\]

\[\text{Footnotes}\]

24 Infra-annual data on interest payments, needed to derive semi-annual primary balances from the overall balances, were not available. Using non-interpolated series, or interpolated series instrumented with the non-interpolated ones to correct for a possible error-in-variables bias, we obtained very similar results.

25 The country ratings of Institutional Investor are based on information provided by leading international banks. Bankers are asked to grade each of the countries on a scale of zero to 100, with 100 representing those with the least chance of default. The sample ranges from 75 to 100 banks. Banks are not permitted to rate their home countries. Individual responses are weighted by Institutional Investor using a formula that gives more importance to responses from banks "with greater worldwide exposure and more-sophisticated country-analysis systems."

26 The Danish sample is shorter because of a break in the Danish fiscal series.

27 The Italian sample is limited to the more recent period because all standard tests point to a structural break in the rating equation at the end of the 1980s. Given that every single estimated coefficient is significantly different in the pre-1989 and post-1989 sub-samples, the only solution is to estimate separate equations for the two sub-samples. In the estimates for the pre-1989 period (available upon request), all slope coefficients are non-significant.

(continued...)
First, we test whether PY and DY are complements in the rating function \( f(\text{PY}, \text{DY}) \) by checking whether the positive effect of PY on RATE is stronger during signaling phases. The estimated equation is:

\[
\text{RATE}_t = \beta_0 + \beta_1 \text{DY}_{t-1} + \beta_2 \text{PY}_{t-1} + \beta_3 (\text{DSIG} \ast \text{PY}_{t-1}) + \epsilon_t
\]  

(8)

DY and PY are lagged one period to avoid simultaneity bias\(^{28}\) and Newey-West standard errors robust to heteroskedasticity and serial correlation are reported for all estimates.\(^{29}\) DSIG is a dummy variable equal to one during signaling phases and zero otherwise. Signaling phases correspond to the periods in which primary balances swing from deficit to surplus in each country. We define them as the periods 1982:01-1988:01 in Ireland, 1982:02-1985:01 in Belgium, and 1982:02-1986:01 in Denmark. For Italy, we simply regress the rating on the debt stock and the primary balance because the 1989:02-1998:02 sample (the only one not affected by structural breaks, see footnote 26) is not long enough to be meaningfully split into signaling and post-signaling phases.

Estimation results are summarized in Table 4 (Model 1). The coefficients \( \beta_1 \), \( \beta_2 \) and \( \beta_3 \) are strongly significant and have the expected signs. An increase in the debt-to-GDP ratio of 10 percent is estimated to reduce the rating by 2.7 points in Ireland, 2.8 points in Belgium, 1 point in Denmark, and 3.6 points in Italy. A 1 percent improvement in the primary balance during a normal (nonsignaling) phase is estimated to raise the rating by 0.4 points in Ireland, 1.6 points in Belgium, 0.2 points in Denmark, and 0.9 points in Italy. A similar 1 percent improvement in the primary balance during a signaling phase is estimated to have an

\(^{27}\)(..continued)

This reflects the fact that our empirical specifications of the rating function cannot possibly account for the substantial improvement of Italian ratings from 1983 to 1989, which took place against the background of persistent primary deficits and constantly growing debt-to-GDP ratios (Figures 1 and 2). Indeed, our model has very little to say on the behavior of ratings in a pre-signaling phase, as most of the pre-1989 period should be considered. This suggests that richer models should be developed to account for the dynamics of ratings within periods that, in our model, would be associated with no release of information.

\(^{28}\)We also estimated instrumental variable regressions with contemporaneous DY and PY using lagged variables as instruments and obtained very similar results (available upon request).

\(^{29}\)We report estimates without lagged endogenous terms because in our model there is no economic reason to introduce them. Nevertheless, if we included one lag of the dependent variable to eliminate serial correlation of the residuals (in equation (8) or in any of the other specifications presented in this paper), the results would be similar, as shown in Drudi and Prati (1998).
Table 4. Rating Function Estimates

<table>
<thead>
<tr>
<th>Country</th>
<th>Parameter Estimates (Newey-West standard errors in parenthesis)</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\beta_0$</td>
<td>$\beta_1$</td>
</tr>
<tr>
<td>Ireland (1980:01 - 1995:01)</td>
<td></td>
<td>90.4 ** (3.9)</td>
<td>-0.266 ** (0.047)</td>
</tr>
<tr>
<td>Belgium (1980:01 - 1995:01)</td>
<td></td>
<td>103.9 ** (5.3)</td>
<td>-0.280 ** (0.061)</td>
</tr>
<tr>
<td>Denmark (1980:01 - 1992:01)</td>
<td></td>
<td>78.1 ** (0.6)</td>
<td>-0.099 ** (0.013)</td>
</tr>
<tr>
<td>Italy (1989:02 - 1998:02)</td>
<td></td>
<td>114.3 ** (5.6)</td>
<td>-0.358 ** (0.052)</td>
</tr>
</tbody>
</table>


1/ Data are semi-annual. Two (**) and one (*) stars mark statistical significance respectively at the one and five percent levels.
additional long-run effect of 1 point in Ireland, 1.5 points in Belgium, and 0.3 points in Denmark.

A second way to test whether PY and DY are complements in the rating function (7) is to verify whether the estimated coefficient of an additional term interacting PY and DY is positive. If signaling is not a once-and-for-all job, $f_{PY}$ may not only temporarily increase when stabilization begins, but it may remain high as long as the debt-to-GDP remains high. One way to capture this idea is to assume that $f_{PY}$ is equal to a constant plus a positive linear function of the time profile of the debt stock:

$$RATE_t = \beta_0' + \beta_1' DY_{t-1} + \beta_2' PY_{t-1} + \beta_3' * (DY_{t-1} - DY) + \epsilon_t' \quad (9)$$

where $DY$ is the sample mean of $DY^{30}$ and complementarity requires $\beta_3' > 0$.

Estimation results are summarized in Table 4 (Model 2). The coefficients $\beta_1'$ and $\beta_2'$ are strongly significant and similar to $\beta_1$ and $\beta_2$. The coefficient $\beta_3'$ is always positive, as expected, but it is significantly different from zero only in Belgium and Italy. For these two countries, specification (9) fits the data better, in terms of $R^2$, than specification (8), suggesting that, for some countries, it is better to assume that signaling is not a once-and-for-all job. This evidence also suggests that specification (9) might be more appropriate for gradual stabilizations, like those of Belgium and Italy.

The complementarity between PY and DY also implies that primary balances ought to increase with the stock of debt. This means that we should be able to estimate $\gamma_1 > 0$ in:

$$PY_t = \gamma_0 + \gamma_1 DY_{t-1} + \eta_t \quad (10)$$

where $DY$ is again lagged one period to avoid simultaneity bias.\textsuperscript{31} Estimation results are reported in Table 5. The estimated coefficient $\gamma_1$ is strongly significant and has the expected positive sign in all countries. A 10 percent increase in the debt-to-GDP ratio is estimated to

\textsuperscript{30}Expressing the debt-to-GDP ratio in deviation from its mean does not affect the fit of the regression, which remains the same, in terms of $R^2$, of a regression without such normalization. Normalizing the debt-to-GDP ratio is, however, necessary to be able to interpret $\beta_2'$ like $\beta_2$ in equation (8). Without normalization, $\beta_2'$ would be affected by the sample average of the debt-to-GDP ratio and could no longer be expected to be positive.

\textsuperscript{31}In practice, by using lagged regressors in all our specifications, we are estimating what would be the reduced form of a simultaneous model including both equation (9) (or equation (8)) and equation (10). Estimating jointly the two reduced form equations (to take into account possible cross-correlation of the residuals), or estimating a structural model (by substituting $PY_t$ to $PY_{t-1}$ in equations (8) or (9) and leaving equation (10) unchanged to have recursive identification), would yield results very similar to those presented in the paper.
<table>
<thead>
<tr>
<th>Model:</th>
<th>( PY_t = \gamma_0 + \gamma_1 DY_{t-1} + \eta_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Country</strong> (sample in parenthesis)</td>
<td><strong>Parameter Estimates</strong> (Newey-West standard errors in parenthesis) 1/</td>
</tr>
<tr>
<td></td>
<td>( \gamma_0 )</td>
</tr>
<tr>
<td>Ireland (1980: 01 - 1995: 01)</td>
<td>-16.6 ** (4.8)</td>
</tr>
<tr>
<td>Belgium (1980: 01 - 1995: 01)</td>
<td>-11.9 ** (1.4)</td>
</tr>
<tr>
<td>Denmark (1980: 01 - 1992: 01)</td>
<td>-14.2 ** (3.6)</td>
</tr>
<tr>
<td>Italy (1989: 02 - 1998: 02)</td>
<td>-18.0 ** (3.4)</td>
</tr>
</tbody>
</table>

**Legend:** \( PY \) = primary balance-to-GDP ratio (in percent) \( DY \) = debt-to-GDP ratio (in percent).

1/ Data are semi-annual. Two (**) and one (*) stars mark statistical significance respectively at the one and five percent levels.
increase the primary balance by 2.0 percent in Ireland, 1.4 percent in Belgium, 2.4 percent in Denmark, and 1.8 percent in Italy. Interestingly, primary balances react more slowly to the debt stock in those countries (Belgium and Italy) for which specification (9) performed better.

C. Alternative Explanations of the Empirical Evidence

Are there alternative explanations that could account equally well for the empirical evidence? As noted in Section 3.1, a signaling model is not the only model predicting that credit ratings decline with the debt stock and rise with the primary balance. A model with exogenous shocks to public expenditure or interest rates would have similar implications. We have argued, however, that our model makes the additional prediction that, until signaling is completed, the debt stock and the primary balance should be complements in the rating function. This implies that signaling should be associated with high debt-to-GDP ratios and with primary balances swinging into surplus. Empirical tests broadly confirm the existence of these signaling effects.

Another explanation of the deterioration of the credit ratings of Ireland, Belgium, and Denmark in the early 1980s could be the contagion effects of the Mexican debt crisis, that might have induced holders of government debt to update their default priors. This explanation can be considered alternative but also complementary to our signaling hypothesis. In fact, a Mexico-induced revision of default priors may have accelerated the transition to a signaling equilibrium and caused a greater revision of ratings than the one that the Mexican debt crisis alone could have caused. (In terms of Figure 3, if 1-p₀ falls enough, the economy will move from a pooling to a separating equilibrium area.)

Another alternative explanation of the empirical evidence could be based on the assumption that investors are uncertain about the state of the public finances rather than the type of government. In this context, the beginning of fiscal consolidation may indicate that things are actually worse than previously thought and credit ratings may drop if consolidation is uncertain to succeed. Credit ratings would then recover when uncertainty is eliminated. This story is consistent with the observed correlations. However, uncertainty about the state of public finances is just a different modeling device that would require the government in power to solve a problem very similar to the one faced by the dependable government in our model.

IV. Conclusions

In this paper, we develop a signaling model to explain some stylized features of fiscal stabilization in Ireland, Belgium, and Denmark during the 1980s and in Italy during the early 1990s. The first prediction of our model is that credit ratings should be positively related with the primary balance and negatively related with the debt-to-GDP ratio. This explains why even a country with large primary deficits will have a high credit rating when the debt stock is small, and why that country will need a primary surplus to obtain the same rating when the debt stock is large. The intuition is that, when the debt stock is small, all governments would run primary deficits and no government would have any incentive to default, whereas, when
the debt stock is large, dependable governments would run primary surpluses to show that they are not of the weak (defaulting) type.

However, other types of models could predict a rating function positively related with primary balances and negatively related with the debt stock. For example, models with exogenous shocks to interest rates or government expenditure would predict a similar rating function without any need to introduce uncertainty on the type of government in power. But these models could not explain why the primary balance seems to have a greater-than-usual effect on credit ratings when the debt stock is large (Figures 1-2 and Table 4). This is, instead, the novel prediction of our model, which implies that primary balances and debt-to-GDP ratios are complementary inputs in the function determining the credit rating. The positive effect of primary balances on credit ratings increases with the debt-to-GDP ratio because it is at high debt-to-GDP ratios that the economy moves from a pooling to a separating equilibrium with primary balances having a strong signaling power. We successfully test this prediction on Italian, Irish, Belgian, and Danish data. Further research is certainly needed to verify the existence of signaling effects on a larger sample of countries.

The results of this paper shed a new light on the determinants of large public debts and on the deviations from the “tax smoothing” theory of the government budget. Our model implies that even dependable governments find optimal to delay the stabilization of the debt-to-GDP ratio. This happens because they have an incentive to tighten the fiscal regime when the signaling effect on credit ratings is larger (that is, when a sufficiently large stock of debt has been accumulated). Indeed, in Appendix 3 we show that when a dependable—but not fully credible—government is allowed to choose strategically the initial stock of debt, its optimal policy is to accumulate a debt stock large enough to determine a switch from pooling to separating equilibria and signal its type by deviating from optimal tax smoothing. In other words, to obtain an early resolution of uncertainty, dependable governments accelerate the dynamics of the debt stock and seemingly delay stabilization to optimally determine the signaling time. Although further analytical research is certainly needed to check the robustness of this result in an infinite horizon setting—and possibly with a continuum of types, we believe that our novel explanation of delayed stabilization may complement those based on distributional conflicts and on strategic debt accumulation aimed at constraining the actions of successive governments.

In regard to fiscal sustainability criteria, our paper suggests that standard measures should be interpreted with caution. On the one hand, as long as the debt stock is relatively small, explosive paths of the debt-to-GDP ratio with persistent primary deficits need not be a source of concern because both dependable and weak governments would run the same seemingly unsustainable policy. On the other hand, when the debt stock is large, explosive debt paths with persistent primary deficits are much more worrying because they signal that the government in power is not dependable and likely to default.
Propositions 1 and 2

**Proposition 1:** Separating equilibria hold for:

\[
\frac{1}{2(1-p_D)} < \frac{\hat{q}}{D_1} < \frac{1 + \sqrt{1-(1-p_D)^2}}{(1-p_D)^3} \quad \Rightarrow \quad \frac{D_1 R^S_1}{2} < \hat{q} < D_1 (R^S_1)^3 (R^S_1 + \sqrt{(R^S_1)^2 - 1}).
\]

**Case I:** type D runs a primary budget surplus in period one (**Table 1** shows equilibrium strategies and interest rates) for:

\[
\frac{1}{2(1-p_D)} < \frac{\hat{q}}{D_1} < \frac{1}{(1-p_D)} \quad \Rightarrow \quad \frac{D_1 R^S_1}{2} < \hat{q} < D_1 R^S_1.
\]

**Proof.** We derive the optimal strategy of type W in a separating equilibrium by solving problem (4) without precommitment and under the assumption that at the end of time one investors know that type W is in power:

\[
\text{Min}_{s^w, s^{w^p}} \frac{1}{2} \left[ D_1 R^S_1 (1-\theta^S_1) + g - D_2 \right] + \frac{1}{2} \left[ D_2 R^S_2 (1-\theta^S_2) \right]^2 - \hat{q} + g \quad (4.1)
\]

Solving backwards, the last period optimal strategy of type W at time two is clearly to default on the entire stock of debt outstanding by choosing \(\theta^S_2 = 1\) and \(\tau^S_2 = 1\). However, given that, in separating equilibrium, investors perfectly anticipate the last period policy of type W, they will not buy any debt at the end of time one, i.e. \(D^S_2 = 0\) and \(R^S_2 = \infty\). This implies that, at the beginning of time one, type W faces the problem:

\[
\text{Min}_{s^w, g^S_2} \frac{1}{2} \left[ D_1 R^S_1 (1-\theta^S_1) + g \right]^2 - \hat{q} \quad (4.2)
\]

whose only solution is \(\theta^S_1 = 1\) and \(g^S_2 = \tau^S_2 = \hat{q}\).

We derive the optimal strategy of type D in a separating equilibrium by solving problem (4) with precommitment to zero default in both periods and under the assumption that at the end of time one investors know that type D is in power:

\[
\text{Min}_{s^D, s^{D^p}} \frac{1}{2} \left[ D_1 R^S_1 + g^S - D_2 s^{D^p} \right]^2 + \frac{1}{2} \left[ D_2 s^{D^p} \right]^2 - \hat{g} \quad (4.3)
\]

Table 1 shows the optimal policy that solves this problem. Note that, to have a positive \(g^{S,D}\), we need to impose a limit on the range of parameters in which this equilibrium exists \((\frac{D_1 R^S_1}{2} < \hat{q})\).

Given that the prior probability at time zero that type W is in power is \(p_0\), the equilibrium strategy of risk-neutral investors in separating equilibrium is to set the interest factor on \(D_1\) at:
\[ R_1^{SW} = R_1^{SD} = R_1^S = \frac{1}{1-p_0} \]

and to set the one on \( D_2 \) using Bayes’ law:

\[ R_2^{SD} = 1, \quad \text{if } \theta_1^{SD} = 0 \text{ is observed} \quad R_2^{SW} = \infty, \quad \text{if } \theta_1^{SW} = 1 \text{ is observed} \]

Separating equilibria exist only if both type D and type W have no incentive to deviate from the separating equilibrium policies derived above. As long as type W does not mimic type D policies, type D would never deviate from his separating equilibrium strategy because this is optimal by construction under the hypothesis of separation and because any other strategy that confused him with type W would make him worse off (by not allowing him to issue any debt in period one and preventing tax smoothing). By contrast, to finance a larger expenditure without levying more taxes, type W may deviate from his separating equilibrium policies and mimic type D policies at time one, causing a breakdown of the separating equilibrium. In the deviation strategy, type W mimics type D strategy at time one and defaults on the entire stock of debt at time two.\(^{32}\) To rule out such deviation, we need to check the range of parameters for which \( C_{w}^{SD} > C_{w}^{S} \), where \( C_i^j \) is the cost for type i (i = W or D) either in equilibrium (j = S for separating and j = P for pooling) or in the deviation from equilibrium (j = SDev for the deviation from separating and j = PDev for the deviation from pooling). It is easy to verify that the above inequality is satisfied whenever \( g^{SD} < \bar{\alpha} \) or \( D_1 R_1^S > \bar{\alpha} \). The latter inequality provides the upper limit for the range of parameters in which Case I separating equilibria exist and implies that these equilibria are associated with primary surpluses.

**Case II:** type D runs a balanced primary budget in period one (Table 2 shows equilibrium strategies and interest rates for):

\[ \frac{1}{(1-p_0)} \leq \frac{\alpha}{D_1} < \frac{1 + \sqrt{1-(1-p_0)^2}}{(1-p_0)^3} \quad \Rightarrow \quad D_1 R_1^S \leq \alpha < D_1 (R_1^S)^2 (R_1^S \sqrt{(R_1^S)^2 - 1}). \]

**Proof.** Case II separating equilibria may emerge in the range of parameters in which the stock of debt maturing in period one is so small, \( D_1 R_1^S \leq \bar{\alpha} \), that type W deviates from his Case I separating equilibrium strategy making Case I equilibria not viable. Case II separating equilibria will be possible in the range \( D_1 R_1^S \leq \bar{\alpha} \) only if type D finds optimal to restore the viability of a separating equilibrium by deviating from its Case I strategy and implementing policies that type W does not mimic.

The optimal strategy of type D solves the problem (4.3) subject to the constraint that type W must be unwilling to mimic, i.e. \( C_{w}^{SD} \geq C_{w}^{S} \), where \( C_{w}^{SDw} \) is now the cost for type W of mimicking the optimal Case II strategy of type D. This constraint amounts to a maximum level of expenditure for each level of taxes chosen by type D in period one:

\[^{32}\text{As off-equilibrium beliefs, we assume that investors believe that there is a zero probability of type W being in charge if they observe } \theta_1 = 0.\]
\[ g^{SD} \leq \frac{(\tau_1^{SD})^2}{2\hat{\alpha}} + \frac{\hat{\alpha}}{2} \]

Type D problem in Case II can then be rewritten as the following minimization problem in \( \tau_1^{SD} \):

\[ \text{Min}_{\tau_1^{SD}} \quad \frac{1}{2} \left[ \tau_1^{SD} \right]^2 + \frac{1}{2} \left[ D_1R_1^S + \frac{(\tau_1^{SD})^2}{2\hat{\alpha}} - \tau_1^{SD} \right] - \hat{\alpha} \left[ \frac{(\tau_1^{SD})^2}{2\hat{\alpha}} + \frac{\hat{\alpha}}{2} \right] \quad (4.4) \]

The unique real solution of this problem is \( \tau_1^{SD} = \hat{\alpha} \), which implies a balanced primary budget in period one. Given taxation, the other equilibrium values follow from the constraint on \( g^{SD} \) and the government budget constraints. Type W equilibrium strategies are the same as in Case I.

Case II separating equilibria exist only if both type D and type W have no incentive to deviate from the policies derived above. Type W cannot deviate by construction. Conversely, type D could deviate by reoptimizing at time one. In the deviation, type D chooses a set of policies in period one that do not allow the public to distinguish him from type W because he reckons that the benefit of a higher expenditure in period one more than compensates the cost of paying an interest premium between period one and two.\(^{33}\) Type D problem in the deviation is problem (4.3) modified to allow for \( R_2^{SD_{Dev}} = \frac{1}{1-p_0} > 1 \):

\[ \text{Min}_{g^{SD_{Dev}}, D_2^{SD_{Dev}}} \quad \frac{1}{2} \left[ D_1R_1^{SD_{Dev}} + g^{SD_{Dev}}D_2^{SD_{Dev}} \right] + \frac{1}{2} \left[ D_2^{SD_{Dev}}D_2^{SD_{Dev}} \right] - \hat{\alpha} \left[ \frac{(\tau_1^{SD})^2}{2\hat{\alpha}} + \frac{\hat{\alpha}}{2} \right] \quad (4.5) \]

with solution:

\[ D_2^{SD_{Dev}} = \frac{\hat{\alpha}}{\left[ R_2^{SD_{Dev}} \right]^2} \quad g_2^{SD_{Dev}} = \left[ 1 + \frac{\left( R_2^{SD_{Dev}} \right)^2}{\left[ R_2^{SD_{Dev}} \right]^2} \right] \hat{\alpha} - D_1R_1^S \]

To rule out the deviation of type D from Case II separating, we need to find the range of parameters in which

\[ C_D^{SD_{Dev}} > C_D^{S} \quad \text{or} \quad \frac{1}{2} \hat{\alpha}^2 + \frac{1}{2} \left[ \frac{\hat{\alpha}}{R_2^{SD_{Dev}}} \right]^2 - \hat{\alpha} \left[ 1 + \left[ R_2^{SD_{Dev}} \right]^2 \right] \hat{\alpha} - D_1R_1^S \]

\[ > \quad \frac{1}{2} \hat{\alpha}^2 + \frac{1}{2} (D_1R_1^S)^2 - \hat{\alpha}^2 \]

This inequality holds in a range whose limits are the two values of \( \hat{\alpha} \)s that solve the associated second order equation. The lower limit can, however, be neglected because it is outside the range of \( \hat{\alpha} \)s relevant for case II (\( D_1R_1^S \leq \hat{\alpha} \)). Therefore, after setting \( R_2^{SD_{Dev}} = R_1^S = \frac{1}{1-p_0} \), we conclude that Case II separating equilibria exist in the range:

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\(^{33}\)The off-equilibrium beliefs are the following: whenever investors observe the deviation strategy of type D in the separating equilibrium, they revert to the pooling pricing of bonds. This is justified from the observation that it would be in the interest of type W to mimic that policy in the deviation from separating equilibria.
\[ D_1 R_1^S \leq \hat{\alpha} < D_1 (R_1^S)^2 (R_1^S + \sqrt{(R_1^S)^2 - 1}) \]

**Proposition 2:** Pooling equilibria exist in the range:

\[ \frac{1 + \sqrt{1 - (1-p_0)^2}}{(1-p_0)^2} \leq \frac{\hat{\alpha}}{D_1} \iff D_1 R_2^P (R_2^P + \sqrt{(R_2^P)^2 - 1}) < \hat{\alpha}. \]

In pooling equilibria all government types run a primary budget deficit in period one (Table 3 shows equilibrium strategies and interest rates).

**Proof:** The optimal strategy of type \( W \) in pooling equilibrium is to choose \( \theta_2^{P,W} = 1 \) and \( \tau_2^{P,W} = 0 \) and to mimic type D optimal strategy in period one. Given that in pooling equilibria type W defaults only at time two, no revision of beliefs is possible at time one and the equilibrium strategy of risk-neutral investors is to set interest factors at:

\[ R_1^{P,W} = R_1^{P,D} = 1 \quad R_2^{P,W} = R_2^{P,D} = R_2^P = \frac{1}{1-p_0} \]

We derive the optimal strategy of type D in a pooling equilibrium by solving problem (4) with precommitment to zero default in both periods and under the assumption that investors set interest factors as indicated above:

\[ \text{Min}_{\xi^{P,D}, \xi^{P,D}} \frac{1}{2} \left[D_1 + g^{P,D} - D_2^{P,D}\right]^2 + \frac{1}{2} \left[D_2^{P,D} R_2^{P,D}_2\right]^2 - \hat{\alpha} g^{P,D} \]  \hspace{1cm} (4.6)

Table 3 shows the optimal policy that solves this problem.

Pooling equilibria exist in a range of parameters in which neither type W nor type D deviate from the above equilibrium strategies. Type W does not deviate from the pooling equilibrium strategy as long as type D runs a primary deficit at time one:

\[ g^{P,D} = g^{P,W} > \tau_1^{P,D} = \tau_1^{P,W} = \hat{\alpha} \]

The intuition is that, if type D chose to run a primary surplus by setting \( g^{P,D} < \tau_1^{P,D} = \hat{\alpha} \), type W would be better off defaulting on the outstanding stock of debt and running a balanced budget by setting \( g^{P,Den,W} = \tau_1^{P,Den,W} = \hat{\alpha} \). The primary deficit requirement can be obtained formally by checking in which range of parameters \( C_2^{P} > C_2^{W} \) or:

\[ \frac{1}{2} \hat{\alpha}^2 - \hat{\alpha}^2 > \frac{1}{2} (\tau_1^{P,D})^2 - \hat{\alpha} g^{P,D} \]

This inequality is satisfied for:

\[ g^{P,D} > \tau_1^{P,D} \quad \hat{\alpha} > D_1 R_1^{P} \quad D_2^{P,D} > D_1 \]
Note that the inequality $\hat{\alpha} > D_1(R_2^P)^2$ is always satisfied in the range of Proposition 2, which reflects the condition for no-deviation of type D that we are about to derive.

Type D could deviate from its pooling equilibrium strategy by reoptimizing at time one. In the deviation, type D chooses a set of policies in period one that distinguish him from type W because he reckons that the benefit of paying a risk-free interest rate between period one and two is larger than the cost of cutting expenditure to a level that type W would not mimic. The optimal strategy of type D in the deviation is the one that solves the problem (4.6) with $R_2^{P,D} = R_2^{P,Dev,D} = 1$ and subject to the constraint that type W must not mimic. This constraint (analogous to the one derived above in the proof of Proposition 1 for Case II separating equilibria) amounts to a maximum level of expenditure for each level of taxes that type D chooses in period one in the deviation from pooling:

$$g^{P,Dev,D} \leq \frac{\hat{\alpha}}{2} + \frac{(\tau_1^{P,Dev,D})^2}{2\hat{\alpha}}.$$

Type D problem in the deviation from pooling can then be rewritten as the following minimization problem in $\tau_1^{P,Dev,D}$:

$$\min_{\tau_1^{P,Dev,D}} \left[ \frac{1}{2} \left[ \tau_1^{P,Dev,D} \right]^2 + \frac{1}{2} D_1 + \frac{(\tau_1^{P,Dev,D})^2}{2\hat{\alpha}} - \frac{\hat{\alpha}^2}{2} - \tau_1^{P,Dev,D} \right] - \hat{\alpha} \left[ \frac{(\tau_1^{P,Dev,D})^2}{2\hat{\alpha}} + \frac{\hat{\alpha}}{2} \right].$$

The unique real solution of this problem is $\tau_1^{P,Dev,D} = -\hat{\alpha}$, which implies $g^{P,Dev,D} = -\hat{\alpha}$ and a balanced primary budget in period one.

To rule out the deviation of type D from pooling, we need to find the range of parameters in which $C^{P,Dev}_D > C^P_D$, or

$$\frac{1}{2} \hat{\alpha}^2 + \frac{1}{2} D_1^2 - \hat{\alpha}^2 > \frac{1}{2} \hat{\alpha}^2 + \frac{1}{2} \left( \frac{\hat{\alpha}}{R_2^P} \right)^2 - \hat{\alpha} \left[ \frac{1+R_2^{P,D}}{(R_2^{P,D})^2} - D_1 \right]$$

This inequality is satisfied in the following two separate ranges:

$$\hat{\alpha} < D_1 R_2^P \left( R_2^P - \sqrt{(R_2^P)^2 - 1} \right) \quad \hat{\alpha} > D_1 R_2^P \left( R_2^P + \sqrt{(R_2^P)^2 - 1} \right)$$

We conclude that pooling equilibria exist only in the upper range because only in this one the condition $\hat{\alpha} > D_1(R_2^P)^2$ for the no-deviation of type W is satisfied.

\*\*We assume that in the deviation off-equilibrium beliefs are such that, if the optimal deviation policy of type D (derived below) is observed, investors require the risk-free interest rate.
Pareto Dominance in Multiplicity Range

This appendix proves that in the multiplicity range both type W and type D prefer pooling equilibria to Case II separating equilibria. 

**Type W.** Type W will prefer pooling equilibria to Case II separating equilibria if \( C^p_w < C^s_w \) or

\[
\frac{1}{2} \hat{a}^2 - \hat{a} g^{p,w} < \frac{1}{2} \hat{a}^2 - \hat{a}^2.
\]

This inequality is always satisfied in the multiplicity range because in that range pooling equilibria exist and, to prevent a deviation of type W from pooling, the inequality \( g^{p,w} > \hat{a} \) must hold (i.e., pooling equilibria exist only if there is a primary deficit in period one, see proof of Proposition 2 in Appendix 1).

**Type D.** Type D will prefer pooling equilibria to Case II separating equilibria if \( C^p_D < C^s_D \) or

\[
\frac{1}{2} \hat{a}^2 + \frac{1}{2} \left( \frac{\hat{a}}{R'^{p,D}} \right)^2 - \hat{a} g^{p,D} < \frac{1}{2} \hat{a}^2 + \frac{1}{2} \left( D_i R_i s^D \right)^2 - \hat{a}^2.
\]

This inequality is always satisfied in the multiplicity range because in that range pooling equilibria exist and, to prevent a deviation of type D from its pooling strategy, the inequality \( C^{pDev}_D > C^p_D \) must hold (see proof of Proposition 2 in Appendix 1), but, given that \( C^s_D > C^{pDev}_D \), also the inequality \( C^s_D > C^p_D \) holds.
Three-Period Model

Proposition 3: In the three-period model, there is only one equilibrium with type W and type D choosing Case II separating equilibrium strategies in periods one and two (Table 3) and the following pooling equilibrium strategy in period zero:

\[
\tau_0 = \hat{\alpha} \quad , \quad g_0 = \hat{\alpha} \left( \frac{1 + (R_1^W)^2}{(R_1^W)^2} \right) \quad , \quad D_1 = \frac{\hat{\alpha}}{(R_1^W)^2}.
\]

In this equilibrium, the primary budget is in deficit in period zero (pooling) and is balanced in period one (separating).

Proof. The model needs to be solved backwards to insure sequential rationality. This implies that the results of Propositions 1 and 2 still characterize the possible equilibria of the game starting at time one for any given D_1 and that we need to study how these equilibrium strategies for periods one and two can be combined with the optimal strategy in period zero. There are four types of possible equilibria: (1) pooling in period one combined with pooling in period zero; (2) Case I separating in period one combined with pooling in period zero; (3) Case II separating in period one combined with pooling in period zero; (4) separating in period zero. Our solution strategy is the following. First, we derive the optimal policies and the conditions for the existence of each of the four possible equilibria. Second, we let type D choose the debt level, D_1, that minimizes overall costs.

(1) pooling in period one combined with pooling in period zero ("pooling-pooling").
Type D finds its optimal strategy at time zero by solving its loss minimization problem subject to the constraint on D_1 in Table 3 and by taking the pooling equilibrium strategies in period one and two of Table 3 as given. The solution is:

\[
\tau_0 = \hat{\alpha} \quad \quad \quad -\hat{\alpha} < D_1 < \frac{\hat{\alpha}}{R_2^W (R_2^W + \sqrt{(R_2^W)^2 - 1})}
\]

Type D can choose any D_1 in the specified range because, with R_1=1, g_0 and g_1 have the same marginal utility so that type D can shift expenditure between period zero and period one by varying D_1 without affecting welfare. The left-hand-side inequality on D_1 follows from the optimal policy \(\tau_0 = \hat{\alpha}\) combined with the non-negativity requirement for g_0, whereas the right-hand-side inequality follows from Proposition 2.

The pooling-pooling equilibrium strategy will be viable only if neither type W nor type D find it optimal to deviate from it. Type W will never deviate from it because the cost of not mimicking type D policy in period zero (and then adopt an "autarchy" strategy in which he runs balanced budgets in period zero and one by choosing \(\tau_0 = g_0 = \tau_1 = g_1 = \hat{\alpha}\)) is larger than the cost of mimicking it. Type D will also never deviate from the pooling-pooling strategy. To

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Appendix 4 of Drudi and Prati (1998) provides more details on this proof.
show it, we first need to find the primary surplus at time zero at which type W prefers the "autarchy" strategy and then show that type D always prefers the pooling-pooling strategy to running such a large surplus at time zero. It can be shown that: (a) type W's "autarchy" requires type D to choose a larger than optimal level of taxes in period zero, \( \tau_0 > \hat{\tau}(1 + \sqrt{2}) \), together with \( D_1 = -\tau_0 x - \hat{\alpha} \), \( g_0 = 0 \), \( \tau_1 = \tau_2 = \hat{\alpha} \), \( g_1 = 2 \hat{\alpha} - D_1 \); (b) for \( \tau_0 > \hat{\tau}(1 + \sqrt{2}) \), type D will never deviate from "pooling-pooling."

(2) Case I separating in period one combined with pooling in period zero
Type D finds its optimal strategy at time zero by solving its loss minimization problem subject to the constraint on \( D_1 \) in Table 1 and by taking Case I separating equilibrium strategies in period one and two of Table 1 as given. The solution is:

\[
\tau_0 = \hat{\alpha} \quad \text{lim inf} \left( D_1 > \frac{\hat{\alpha}}{R_1 \hat{\delta}} \right)
\]

Note that the solution for \( D_1 \) is a corner solution corresponding to the minimum \( D_1 \) at which Case I Separating equilibria exist. This happens because the derivative of the cost function with respect to \( D_1 \) is always positive implying that \( D_1 \) should be chosen as small as possible. At time zero, type W never deviates from this equilibrium because type D always runs a primary deficit. Also type D never deviates for \( \tau_0 > \hat{\tau}(1 + \sqrt{2}) \).

(3) Case II separating in period one combined with pooling in period zero
Type D finds its optimal strategy at time zero by solving its loss minimization problem subject to the constraint on \( D_1 \) in Table 2 and by taking the Case II separating equilibrium strategies in period one and two of Table 2 as given. The solution is:

\[
\tau_0 = \hat{\alpha} \quad D_1 = \frac{\hat{\alpha}}{R_1 \hat{\delta}^2} \quad g_0 = \hat{\alpha} \left( 1 + \frac{(R_1 \hat{\delta})^2}{(R_1 \hat{\delta})^2} \right)
\]

At time zero, type W never deviates from this equilibrium because type D always runs a primary deficit. Also type D never deviates for \( \tau_0 > \hat{\tau}(1 + \sqrt{2}) \).

(4) separating in period zero
The separating equilibrium in period zero never exists because type D will always prefer equilibria (1), (2), and (3) above. This follows from the fact that the conditions for no deviation of type D at time zero from such equilibria are always satisfied.

Equilibrium selection
Type D chooses the \( D_1 \) in period zero that corresponds to the equilibrium that minimizes the overall cost. This is equilibrium (3).
REFERENCES


