Sanctions and Civil Conflict

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Abstract

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Using a formal general equilibrium framework, this paper analyzes how sanctions imposed on the contestants in civil conflict affect the welfare of these contestants and the allocation of resources to conflict. It is shown that weak sanctions can hurt the contestant they are supposed to help, while strong sanctions augment the expected welfare of their intended beneficiaries. Moreover, sanctions are more likely to be successful if the contestant who is subject to sanctions can expect to derive a positive income in case of compliance. The likelihood of success rises as this income increases.

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I. INTRODUCTION

There is no denying, however, that economic pressures such as trade sanctions and the restriction of credit from international banks made both the South African government and the white business leaders of the 1980s calculate the price of apartheid and eventually come to the conclusion that it was too high. (Fredrickson 1999, p. 37).

Current [embargo] policy does not serve U.S. interests or further its objectives in Cuba. It neither advances the cause of human rights and a more open system nor reduces the possibility of another refugee outflow. (Smith 1996, p. 111).

Sanctions are actions by the outside parties that reduce the output in the sanctioned economy. These can be either economic sanctions (e.g. trade restrictions, disinvestment campaigns, etc.) or sanctions with economic consequences (e.g. military occupation of a part of the country).

In general, sanctions can be imposed against diverse countries and the objectives of the party imposing sanctions vary considerably from one episode to another. This paper analyzes a particular class of sanctions, namely, sanctions intended to influence the outcome of civil conflict, which is defined as a struggle between different groups for the control of economic rents. For example, international sanctions against South Africa were imposed because of the refusal by the white minority government to extend the franchise to the black majority. When the white minority abandoned its apartheid policies, the sanctions were lifted.

Besides sanctions there can be other factors that affect the output in the economy. Those factors will be referred to as system inefficiencies. For instance, in addition to sanctions, the South African economy under apartheid suffered due to decentralization of industry for political rather than economic purposes, overinvestment in capital, and bureaucratization (Lowenberg, 1997). Hence, international sanctions, the loss of output associated with the external pressure, must be considered in the context of, and as an increment to, system inefficiencies, the loss of output associated with the domestic economic policies of the regime.

Three questions can be raised regarding sanctions.

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2 Hufbauer, Schott, and Elliott (1990) list and analyze 116 cases of sanctions imposed by various countries in this century. The objectives of sanctions in those cases range from destabilizing a government or disrupting a major military adventure to changing the target country’s policy in some minor way.
(1) Do sanctions lead to the desired outcome of the conflict?

The cessation of hostilities in Rhodesia (now Zimbabwe) in 1980 followed the international sanctions against that country’s minority government. The South African apartheid was dismantled after more than forty years of sanctions. In both cases relatively weak international sanctions were imposed for a long time without significant effect, yet the intensity of sanctions has risen immediately preceding the end of conflict.³ Does this mean that strong sanctions accomplish their goals? Not necessarily, as the U.S. embargo against Cuba, which failed to destabilize the Castro regime, demonstrates.

Sanctions can help the “good” group win by weakening the “bad” group, or, ideally, by forcing the “bad” group to yield the control of economic rents outright. This observation brings forth the next question.

(2) If sanctions do lead to the desired outcome of conflict, what determines the scenario by which it is achieved?

Consider the example of the former Soviet Union. The most important economic deficiency of the Communist regime was associated not with the relatively insignificant international sanctions but with the pervasive government regulation of the economy, i.e. with system inefficiencies. However, when in the course of perestroika the moderate faction of the Communist Party hierarchy, the nomenklatura, challenged its conservative wing for the control of the state, the conservatives refused to let go. This was despite the fact that the inefficiencies associated with the centrally planned system were all too evident and that the moderates wanted to alleviate them. Nevertheless, instead of giving up, the conservatives put up a furious defense of the existing order (including the unsuccessful 1991 coup), and left the political scene only after their authority completely collapsed in the coup’s aftermath. To put it another way, the inefficiencies of the old system did not persuade the conservatives to give up, yet they could have helped the moderates to win the fight.⁴

³ In Rhodesia, the severity of sanctions was exacerbated by the Iranian revolution of 1979 after which the new rulers of Iran stopped the sale of oil to Rhodesia. The resulting fuel crisis was “...one of the most telling arguments being used by the Prime Minister Ian D. Smith to try to persuade his fellow whites to accept black majority rule, which, he hopes, would mean an end to economic sanctions” (West, 1979). The anti-apartheid international sanctions started after the World War II and then were gradually increasing in intensity with the strongest sanctions introduced in the mid-1980s (Lowenberg and Kaempfer, 1998, p. 164).

⁴ There is a significant body of literature devoted to the collapse of the Soviet regime, discussion of which is beyond the scope of this paper. For accounts of the role of the nomenklatura in collapse of the Soviet Union see the article by Kryshtanovskaya and White
In contrast, the white minority government of Rhodesia and the apartheid government of South Africa agreed to negotiate with the opposition and eventually yielded authority to their rivals while still well-positioned to prevent them from taking power. In other words, those governments surrendered.

(3) Do sanctions always augment the utility of their intended beneficiaries?

If the “good” group does not win, it bears the burden of sanctions together with the “bad” group. In other words, sanctions may hurt their intended beneficiaries ex post. Nevertheless, sanctions should at least increase the expected utility of their beneficiaries, i.e. help the “good guys” ex ante. Whether this happens in reality is not clear.

To answer these questions the paper develops a formal model of sanctions affecting the contestants in civil conflict. Exogenous sanctions introduce asymmetry in conflict by making victory more attractive for one side and less attractive for another side. The allocation of resources to conflict by the adversaries and, accordingly, their utility depends on the strength of these sanctions.

To properly account for the scenarios wherein the sanctioned party gives up the control over economic rents while still well-positioned to retain power, the paper formally introduces and analyzes surrender — a qualitatively new type of conflict equilibrium.

This paper complements the existing literature on sanctions by explicitly analyzing the impact of sanctions on the outcome of the conflict and the utility of the parties involved. While papers by Kaempfer and Lowenberg (1988) and Eaton and Engers (1999) focus on the origins of sanctions, this work treats sanctions as an exogenous factor and focus instead on the issue that the above-mentioned models do not address, namely, how exactly sanctions influence the sanctioned parties.6


5 Porter (1978) indicates that the black majority, the putative beneficiary, could suffer as a result of sanctions imposed against the white minority government of Rhodesia. Similarly, Hayes (1987) argues that sanctions led to higher unemployment among both whites and blacks in South Africa.

6 Kaempfer and Lowenberg (1988) view sanctions as a product of the interest group pressures in the sanctioning countries while Eaton and Engers (1999) explain the occurrence of sanctions as a result of incomplete information. The sanctioning party in these models determines the severity of sanctions or whether or not to impose sanctions at all according to its own preferences.
This paper also adds to the literature devoted to the economic analysis of conflict. The conflict literature concentrates on the endogenous decisions to allocate resources to appropriative activities and in general does not deal with the asymmetries brought about by sanctions. Exceptions are the works by McDermott (1997) and Grossman (1992). Both papers discuss the impact of outside actions on exploitation of one group by another (McDermott, 1997) and insurrections (Grossman, 1992). In these works, however, the outside actions have a direct effect on only one of the two contestants. The present model uses a more realistic assumption that as long as sanctions are imposed, they hurt all parties directly.\(^7\)

To sum it up, this paper contributes to our understanding of the impact of sanctions by combining the examination of sanctions with the analysis of conflict within a formal general equilibrium framework. The analysis indicates that sanctions are more likely to induce the desired regime change if defeat in conflict is not too unattractive for the contestant that is the target of sanctions. The paper also demonstrates that while strong sanctions do achieve their objective, weak sanctions are counterproductive.

The paper starts with the general discussion of conflict and description of the economy (sections II, III, and IV). Sections V and VI introduce sanctions, solve for the equilibrium allocations of resources to conflict, and explain how these equilibria depend on the severity of sanctions. After that the impact of sanctions on the utility of the parties involved in conflict is analyzed. The discussion of the fate of several political regimes illustrates the results (section VII). Section VIII concludes.

II. CIVIL CONFLICT

Civil conflict is modeled as a struggle between different groups for the control of economic rents. These rents can accrue from land or other natural resources or from physical and human capital. These rents can be extracted either as property income, or wages, or as tax revenue.

In this model the participants in civil conflict are an incumbent dominant group that initially controls the rents and a challenger that seeks to wrest control of the rents from the incumbent dominant group.\(^8\)

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\(^7\) Another distinction between this model and McDermott (1997) is that his model deals with exploitation and not with conflict. In McDermott (1997) any group can exploit its opponent, yet exploitation is costly. In the present model, only the winner can benefit at the expense of the loser, and groups have to invest in conflict to win.

\(^8\) This framework (the initial claimant vs. the challenger) is employed to analyze the appropriative conflict by Grossman and Kim (1995, 1996a, 1996b), Grossman (1999), McDermott (1997), and Rosendorff (1998). An alternative framework, which is not discussed
Conflict brings about either victory for the incumbent dominant group or victory for its adversary, the challenger. From the standpoint of a particular group, victory is an outcome that enables that group to collect the rents that were subject of the conflict. Although it is possible that the conflict leads to an arrangement according to which rental income is split between the contestants, this eventuality is not considered in the paper.

The focus of this study, however, is not the outcome of conflict (which may be a product of chance events) but rather the underlying equilibrium allocation of resources to conflict activities. Most models devoted to conflict and its applications (such as elections or litigation) rely on the Cournot-Nash equilibrium framework. The Cournot-Nash framework is appropriate for the models that consider engagement (positive investment in conflict by both sides) as the only type of conflict equilibrium. In contrast, one of the innovations of this paper is a comprehensive analysis of engagement together with two other qualitatively distinct types of conflict equilibria: deterrence and surrender. This wider scope of the analysis renders the Cournot-Nash framework inappropriate. Neither surrender, a decision to give up the contested rents, nor deterrence, an investment in defense that precludes offensive actions by one’s opponents, can realize as a Cournot-Nash equilibrium.

For example, in the Cournot-Nash framework if one group surrenders, the competing group invests nothing in conflict. Yet, as the latter group invests nothing, the group that has originally chosen to surrender would find it beneficial to change its strategy and invest in conflict; thus, surrender cannot be sustained in equilibrium. Similarly, deterrence cannot be sustained as a Cournot-Nash equilibrium: once the deterred group abstains from investing in offense, the opposing group would reduce its defense expenditure, which in turn would invite the previously-deterred group to invest in offense.

in this paper, implies no initial claimants. Hirschleifer (1995) utilizes such a setup wherein the resource that is subject to appropriation is initially in a common pool from which all agents attempt to appropriate.

For example, the incumbent can offer to transfer resources to the challenger in exchange for neither side investing in conflict. Such an outcome can be called partial victory/defeat or negotiated settlement. Throughout this paper it is assumed that incomes each group receives in case of victory or defeat are exogenous. In that case negotiated settlement is not possible because neither side can credibly commit to any proposed division of rents not backed by the corresponding investment in conflict. Allowing for incomes to be determined endogenously effectively introduces negotiated settlement as an equilibrium.

If the contested income is small relative to investment in conflict then it is possible that neither group would be willing to spend scarce resources on offense or defense. Such a possibility is not considered in this paper.
To properly account for all three equilibria it is assumed, following Grossman (1999) and Grossman and Kim (1995, 1996a,b), that the incumbent’s investment in defense against the challenger represents a commitment on the incumbent’s part. To put it more formally, the incumbent dominant group is a Stackelberg leader.\textsuperscript{11}

The incumbent’s investment in defense is a commitment because the institutional framework associated with the defense of the incumbent regime (standing armed forces, secret police, courts, prisons) is as a rule more rigid and hence can be changed or abolished at a higher cost compared to the investment in offensive actions made by the challenger.\textsuperscript{12}

Deterrence is a commitment of the large enough amount of resources to defensive activities by the incumbent dominant group to render allocation of resources to offensive actions by the challenger unprofitable in expectation. Deterrence results in victory for the incumbent dominant group. A good example of such an equilibrium is North Korea, where rulers have managed to deter the internal opposition over the course of the last 40 years.

Engagement means that both groups invest in conflict. The incumbent dominant group defends its status, while the challenger threatens this status by investing in offensive activities. Engagement can result in either victory (e.g. Castro’s successful struggle with the Cuban exiles) or defeat (e.g. overthrow of the incumbent Mobutu regime in Zaire in 1997) for the dominant group. Probability of victory for the incumbent depends positively on the investment in defense and depends negatively on the allocation of resources to offensive actions by the challenger.

Surrender is defined as a choice by the incumbent to give up what is contested even though conflict, or continued conflict, is a feasible alternative choice that would result in a nonnegligible probability of victory for the incumbent.\textsuperscript{13} By choosing surrender the incumbent

\textsuperscript{11} If the challenger wins, it becomes the incumbent (and a Stackelberg leader) in the next period, while the defeated incumbent becomes the challenger (and a follower). This analysis, which is developed in Gershenson and Grossman (2000), is not taken up here.

\textsuperscript{12} Alternatively, one can justify the Stackelberg approach by assuming that one of the contestants prefers to lead while another prefers to follow. For general discussion on whether it is better to lead or to follow in a Stackelberg game with symmetric players see Gal-Or (1985). Dowrick (1986) studies under what conditions the Stackelberg players agree on the choice of roles. To require that contestants agree on the choice of roles in this model would severely limit the scope of possible equilibria (e.g. it would exclude deterrence). Therefore, it is assumed that the incumbent has the commitment power while the challenger does not.

\textsuperscript{13} This definition is not equivalent to the legal definition of surrender as an agreement to stop fighting imposed by one warring side on the other (Pillar, 1983, p. 15). The legal definition is inappropriate for the purposes of this paper since it refers only to the way hostilities end and
avoids (or, if the conflict has been ongoing, avoids the continuation of) the allocation of productive resources to conflict. Surrender implies defeat of the incumbent.

It is worthwhile to stress that surrender is not equivalent to being deterred. A group surrenders when it decides not to invest in conflict, even though, assuming there is no alternative to conflict, its optimal investment in conflict is nonnegative and results in a positive probability of victory. In contrast, a group is deterred when its optimal investment in conflict is zero.

According to the definition, surrender involves foregoing a nonnegligible probability of victory. In many cases the existence or non-existence of such a probability might not be clear. Nevertheless, this definition captures the traits of an equilibrium (whether or not scarce resources are allocated to conflict-related activities) that are of interest to economists. Abolition of the white minority rule in Rhodesia in 1980 and abolition of apartheid in South Africa in 1994 constitute modern examples of surrender.\textsuperscript{14}

The incumbent’s defeat, which follows either engagement or surrender, changes the identity of the group that collects the rents. That change affects the output of the economy if sanctions were imposed against the incumbent’s rule. The U.N. sanctions against the white minority governments in Rhodesia and in South Africa, and the U.S. embargo of Cuba may serve as examples of such sanctions.\textsuperscript{15}

In its treatment of conflict this paper relies mostly on the work by Grossman (1999) and Rosendorff (1998). Grossman (1999) develops a model of kleptocratic rivalry wherein the incumbent kleptocrat is a Stackelberg leader investing in the regime’s defense while the revolutionary leader is a follower investing in offense. Accordingly, the possible equilibrium allocations of resources to conflict are deterrence and engagement. Rosendorff (1998), while analyzing the conflict between white minority and black majority in South Africa during apartheid, introduces, although not explicitly defines, the idea of surrender as a choice to give up the fight.

Even though this paper focuses on civil conflict, the modeling framework is sufficiently general does not reveal much about the nature of conflict.

\textsuperscript{14} In contrast, the departure of Mobutu from Zaire in 1997 was not a surrender. By the time Mobutu agreed to leave, the old regime had disintegrated such that the probability of his eventual victory was negligible.

\textsuperscript{15} Defeat of the incumbent affects the total income in the economy in case of system inefficiencies as well. It happens if, for instance, the victorious challenger is less (or more) corrupt than the outgoing incumbent, or the challenger implements reforms that make economy more (or less) efficient.
and can be applied to other instances of contest as well, such as oligopolistic entry deterrence or gang warfare. For the model to be applicable the investment in conflict by one of the parties must constitute a commitment. For example, in case of oligopoly, the incumbent firm may try to deter its competition from entering the market by expanding its productive capacity. The expansion of productive capacity constitutes a commitment since such investment cannot be easily reversed. In contrast, limiting price to deter entry does not constitute a commitment, since the incumbent cannot commit to maintaining low prices after entry has occurred.  

III. THE ECONOMY

Consider groups $A$ and $B$, each populated by a given number of people. The groups divide available resources between consumption and competition for dominance. It is assumed that one of the groups (the incumbent) is initially dominant while the other group is the potential challenger.

The income of a group is contingent on whether that group is dominant. Income of the dominant group is called victory income. Income of the group which is not dominant is called defeat income. The determination of victory and defeat incomes depends on the nature of interaction between the groups.

If the participating groups exhaust the entire population of the economy, then the victory income includes wages as well as rent generated by the stock of physical capital, mineral deposits, arable land, etc. The defeat income can be wages (if the losing group remains in the economy and sells its labor, as in South Africa), or income earned abroad (if the losing group leaves). The latter scenario is applicable again to South Africa as well as to Rhodesia, and to the conflicts between the Hutus and the Tutsis in Rwanda and between the Bosnians and the Serbs in the Balkans.

If groups $A$ and $B$ are small and compete for the tax revenues collected from the general population, then the victory income consists of those tax revenues while the defeat income consists of either wages received at home or income that can be earned abroad. This is the case of competing elites or kleptocratic rivalry as in the Soviet Union (moderate vs. conservative

\footnote{Without commitment power the decision to deter entry by limiting price can be explained only if information is incomplete, as in the price-limit model developed by Milgrom and Roberts (1982).}

\footnote{Although South Africa had not experienced the mass exodus of whites after the transition to the majority rule, thousands of whites have left the country since 1994. According to the official South African data, nearly 39,000 South Africans, mostly white, emigrated between 1994 and 1997 ("White South Africa on the Wing," Economist June 6, 1998, pp. 43-4.)}
factions of the Communist Party hierarchy) or Zaire (Mobutu vs. Kabila).

The attractiveness of victory for each group is measured by the difference between its victory and defeat incomes. Total defeat and victory incomes of group A are denoted as \( y_A \) and \( Y_A \) respectively; total defeat and victory incomes of group B as \( y_B \) and \( Y_B \) respectively. The relationship between these four nonnegative exogenous numbers — \( y_A, Y_A, y_B, \) and \( Y_B \) — will be crucial in determining the equilibria of our model.

If groups are dividing a fixed income, then \( Y_A + y_B = y_A + Y_B \), or, equivalently, \( Y_A - y_A = Y_B - y_B \). In this case victory is equally attractive to either group. In general, however, victory needs not be equally attractive to all parties. For example, if there are sanctions associated with the dominance of group A, then victory is less attractive to group A than it is to group B. In fact, if sanctions are strong enough, then victory income of group A may be less than its defeat income. Section VI discusses how the difference between victory and defeat incomes of each group affects the possible equilibria of the model and analyzes what happens if victory income of one of the groups is less than that group's defeat income.

IV. TECHNOLOGY OF CONFLICT

Assuming that group A is the incumbent dominant group, the technology of conflict is given by equation (1), which is the conventional contest success function, where \( P \) is the probability that group A wins.

\[
P = \frac{G_A}{G_A + \theta G_B}
\]  

(1)

\( G_A \) and \( G_B \) represent foregone consumption and denote investment in conflict by groups A and B respectively. The sizes of \( G_A \) and \( G_B \) are determined by the respective group leaders. The objective of each leader is maximization of expected consumption of his/her own group.\(^{18}\) The parameter \( \theta > 0 \) measures efficiency of the offensive weapons against the defensive fortifications. Expression (1) states that if either defense or offense expenditure are positive then the incumbent dominant group wins with probability \( P \in [0, 1] \); the probability of victory is not defined for \( G_A = G_B = 0.\(^{19}\)

\(^{18}\) It is assumed that utility is linear in consumption. Then maximization of expected utility is equivalent to maximization of expected consumption. In addition, the analysis abstracts from the problem of making and enforcing a collective choice.

\(^{19}\) In contrast, Ncary (1997a,b) uses the contest success function that is defined for all values of conflict expenditures, including \((0,0)\). It means that if neither side invests in conflict then the
This functional form of $P$ is a special case of the contest success function first introduced by Tullock (1980). Tullock (1980) allows for the odds in the conflict game to depend nonlinearly on the ratio of resources devoted to offense and defense. In contrast, the analysis here is restricted to cases wherein odds of the game depend linearly on the resource ratio. The obvious advantage of such an approach is its algebraic simplicity. Clearly, there is a loss of generality associated with restricting the attention to the linear case, yet this is not crucial for the purposes of the analysis. Nonlinearity renders the marginal cost of spending on conflict more or less steep and, consequently, affects the intensity of conflict. However, the focus of this paper is not allocation of resources to conflict per se but rather the impact of sanctions on such allocation. As a result, the linearity assumption provides considerable advantage in simplicity and clarity of exposition without compromising the objectives of the research.\(^{20}\)

It is clear that the chosen functional form of $P$ possesses the properties necessary for a contest success function. First, as long as investment in conflict is positive, the probability of victory for the dominant group depends positively on the amount of resources devoted to defense relative to the offense efficiency parameter $\theta$ times the amount of resources the challenger devotes to offense. Second, $P$ is a nonnegative number less or equal to one for any nonnegative values of defense and offense expenditure (unless $G_A = G_B = 0$). Finally, this particular form of $P$ makes the analysis tractable.\(^{21}\)

How exactly the groups interact is beyond the scope of the present paper: appropriative conflict can assume many forms, from violent encounters (military conflicts, tribal warfare, violent crime, etc.) to non-violent interaction (lawsuits, peaceful resolution of international disputes under the threat of military action, etc.). This analysis neither explains nor assumes violence. Also, the model determines only the probabilities of defeat and victory in case of engagement. It abstracts from the idiosyncratic facts (bad or good luck, for example) that determine the actual outcome.

contestants can claim a predetermined share of the total income, for example 50 per cent each. Such a scenario, however, does not seem to be applicable to civil conflicts. Specifically, if neither side invests in conflict, the general equilibrium framework does not justify any particular share of the contested income a group can claim.

\(^{20}\) For a detailed discussion of the contest success functions see Skaperdas (1996).

\(^{21}\) The drawback of the Tullock contest success function is that marginal benefit from investment in defense is infinitely large when such investment is very small. It implies that, as long as utility function is continuous and victory is more attractive than defeat, the incumbent never surrenders. Gershenson (1999) introduces fixed costs of conflict and destruction, which give rise to discontinuity in the utility function. This discontinuity renders very small investment in defense unprofitable and hence allows for surrender even if victory is more attractive than defeat.
V. SANCTIONS

Assume that sanctions are imposed against the dominance of the incumbent group \( A \). As a result of these sanctions, a fraction of total output of the economy \( s \in [0,1] \) is lost whenever group \( A \) is dominant.\(^{22}\)

Then

\[
Y_A + Y_B = (1 - s)(y_A + Y_B)
\]  

(2)

In other words, the total income in the economy if group \( A \) is dominant (the left-hand side) is a fraction of the total income in the economy if group \( B \) is dominant \( (y_A + Y_B) \).

It is easy to see from (2) that in the presence of sanctions, gain from victory to group \( A \) is less than corresponding loss to group \( B \) and this difference increases with the severity of sanctions:

\[
Y_B - y_B = Y_A - y_A + s(y_A + Y_B)
\]  

(3)

According to (2) and (3), the total loss in the economy due to sanctions is measured by \( s(y_A + Y_B) \). If it is assumed that sanctions affect both group \( A \) and group \( B \) directly. Specifically,

\[
Y_A - y_A = \Omega - \lambda s(y_A + Y_B)
\]  

(4)

\[
Y_B - y_B = \Omega + (1 - \lambda) s(y_A + Y_B)
\]  

(5)

According to (4) and (5), if there are no sanctions, then the difference between victory and defeat income for both groups is the same and equals to \( \Omega \geq 0 \). Sanctions reduce the victory income and thereby make victory less attractive for the incumbent dominant group \( A \), as shown by (4). Sanctions reduce the defeat income and, accordingly, make victory more attractive for the challenger group \( B \), as shown by (5). The parameter \( \lambda \in (0,1) \) is the fraction of the loss associated with sanctions borne by group \( A \). Similarly, \( 1 - \lambda \) is the fraction of the loss borne by group \( B \). \( \lambda \) reflects the structure of the economy and is considered exogenous.

The impact of sanctions is twofold. First, sanctions reduce income of group \( A \) if it is victorious since fraction \( s \) of total output is lost. Second, sanctions make victory more attractive for the challenger. By ousting the dominant group the challenger not only acquires economic rents but also puts an end to sanctions and a corresponding loss of output and consumption.

\(^{22}\) Some costs of sanctions can be sunk, i.e. not recovered after the sanctions are lifted. For example, the South African economy still suffers from the consequences of the 1980s disinvestment campaign (Thurow, 2000). Sunk costs reduce the magnitude of \( s \).
Throughout this paper the strength of sanctions \( s \) and the offence efficiency \( \theta \) are treated as parameters independent of each other. More generally, however, \( \theta \) can depend on \( s \), i.e. \( \theta = \theta(s) \). Then, a negative effect of \( s \) on \( \theta \) signifies what Kaempfer and Lowenberg (1992, p. 156) call a “rally-around-the-flag” effect, i.e. greater resolve of the incumbent group to continue its policies in defiance of the pressure from abroad.\(^{23}\) Levy (1999) suggests that, at least at some point, such was one of the effects of the international sanctions on South Africa. At the same time, international sanctions, by showing the world’s solidarity with the challenger, can have positive impact on the challenger’s resolve. Lowenberg and Kaempfer (1998, p. 192) note that the anti-apartheid sanctions helped the black majority to “organize collective actions among its members”. Thus, by treating \( \theta \) as independent of \( s \) it is assumed that those two effects cancel out. The proposed analytical framework, however, can easily accommodate situations when \( \theta \) is a function of \( s \).

VI. EQUILIBRIUM

The expected consumption of the incumbent dominant group \( A \) is

\[
C_A = PY_A + (1 - P)y_A - G_A
\]

(6)

The expected consumption of the challenger group \( B \) is then

\[
C_B = Py_B + (1 - P)Y_B - G_B
\]

(7)

At the first stage of the game the incumbent dominant group selects \( G_A \) to maximize its own expected consumption, taking into account the challenger’s best response. In the second stage the challenger responds with the expected consumption maximizing choice of \( G_B \), taking the defense spending by the incumbent as given.

The equilibrium analysis yields the following results (derived in Appendix I).

*Proposition 1. There exists \( s^* \), the threshold value of the sanctions’ strength, \( 0 \leq s^* \leq 1 \),

\[
s^* = \frac{\Omega}{\lambda(y_A + Y_B)}
\]

\(^{23}\) There are other instances wherein sanctions, if viewed as unjustified or too severe, lead to the increased support for the pressed group. See, for example, Akerlof and Yellen (1994) who argue that the tough government actions against the criminal gangs can benefit the gangs by reducing the community members’ willingness cooperate with the law enforcement agencies.
such that the incumbent surrenders if and only if \( s \geq s^* \).

The value of \( s^* \) depends positively on the attractiveness of victory in the absence of sanctions, \( \Omega \), and depends negatively on the fraction of loss associated with sanctions borne by group A, \( \lambda \). In addition, \( s^* \) depends negatively on the total income in the economy if group B is dominant, \( y_A + Y_B \). For a high \( \Omega \) (victory is attractive), \( s^* \) approaches one and A surrenders if and only if sanctions are strong. In contrast, for a high \( \lambda(y_A + Y_B) \) (the burden associated with sanctions borne by group A is substantial) \( s^* \) approaches zero and even weak sanctions lead to the incumbent's surrender.

**Proposition 1A.** Low defeat income for the incumbent makes its surrender less likely.

This result follows directly from Proposition 1.

**Proposition 2.** For \( s < s^* \) there exists a convex decreasing function of \( s \),

\[
 f(s) = \frac{1}{2} \frac{Y_A - y_A}{Y_B - y_B} = \frac{1}{2} \frac{\Omega - \lambda s(y_A + Y_B)}{\Omega + (1 - \lambda)s(y_A + Y_B)}
\]

such that

(i) the incumbent deters the challenger if and only if \( \theta \leq f(s) \)

(ii) the equilibrium is engagement if and only if \( \theta > f(s) \)

To summarize the results, the incumbent dominant group surrenders for \( s \geq s^* \). The choice between surrender and the other two options (engagement, deterrence) does not depend on the offense efficiency parameter \( \theta \) as long as \( \theta > 0 \).\(^{24}\) If \( s < s^* \), then the equilibrium is engagement for high offense efficiency parameter, \( \theta > f(s) \), and deterrence for low offense efficiency parameter, \( \theta \leq f(s) \).

\(^{24}\) This follows from the fact that as long as it is attractive to remain dominant (\( Y_A > y_A \) which is equivalent to \( s < s^* \)) the benefit of infinitesimal investment in defense is infinitely large:

\[
 \lim_{G_A \to 0} \frac{dC_A}{dG_A} = +\infty
\]

if and only if \( \theta > 0 \) and \( Y_A > y_A \).
Chart 1 illustrates these findings. Deterrence is the equilibrium for sufficiently weak sanctions $s$ (which makes victory attractive for the incumbent) and low offense efficiency parameter $\theta$ (which makes deterrence less expensive). If $\theta$ is high, which makes deterrence expensive, and $s$ is low (to guarantee the attractiveness of victory for the incumbent), the equilibrium is engagement. The deterrence / engagement threshold value of $\theta$ is a decreasing function of $s$. High $s$ makes victory more desirable for the challenger and less attractive for the incumbent. If $s$ is high, the incumbent is less willing to select relatively more expensive deterrence to ensure its victory. In other words, the exogenous increase in the severity of sanctions $s$ induces shifts from deterrence (if $\theta$ was sufficiently low) to engagement and then to surrender.\textsuperscript{25}

Our results imply that presence of sanctions is a necessary condition for surrender; if $s = 0$ then the equilibria are either engagement or deterrence.\textsuperscript{26} The model also predicts that severe sanctions lead to surrender.

\textsuperscript{25} It is shown in Appendix I, equation (14), that as the strength of sanctions $s$ approaches its threshold value $s^*$ from below, the equilibrium probability of the incumbent's victory approaches zero in the limit. To put it another way, in ostensible contrast to the definition of surrender introduced in Section II, the switch from engagement to surrender does not involve giving up the nonnegligible probability of victory on the incumbent's part. To reconcile this observation with the proposed definition of surrender, note that, for the reasons of clarity and brevity, the model presented in this paper neither allows for engagement to be destructive nor considers the fixed costs of conflict. A more comprehensive model (analyzed in Gershenson, 1999) incorporates those possibilities. With either destruction or fixed costs the decision to surrender will indeed imply forfeiting a nonnegligible probability of victory.

\textsuperscript{26} If there are no sanctions, $s = 0$, then $Y_A - y_A = Y_B - y_B$, the threshold value for the offense efficiency parameter $\theta = \frac{1}{2}$, and the probability of the dominant group's victory in case of engagement $P = \frac{1}{2\theta}$. These results were obtained in the model of kleptocratic rivalry by Grossman (1999).
Chart 1. Equilibria in Civil Conflict

The resulting equilibrium depends on the offense efficiency parameter $\theta$ and the severity of sanctions $s$. 
A. Sanctions and Conflict Expenditure

The impact of sanctions on investment in conflict is derived in Appendix II. Not surprisingly, investment in conflict by the incumbent, $G_A$, depends negatively on $s$. An increase in $s$ reduces the attractiveness of victory for the incumbent. As a result the incumbent is less willing to expend scarce resources pursuing victory.

In contrast, the relationship between the investment in conflict by the challenger, $G_B$, and $s$ is not monotonic. This is explained by the dual effect of $s$ on $G_B$. First, an increase in $s$ augments the attractiveness of victory for the challenger. Accordingly, the challenger is willing to spend more in order to win. Second, high $s$ means low defense investment by the incumbent, as discussed in the preceding paragraph. Then the challenger does not need to invest a lot in offense to overtake the incumbent.

Chart 2 presents the results. If the offense efficiency parameter $\theta$ is not too high (Panels A and B) then the first effect is stronger for low values of $s$. An initial increase in the strength of sanctions leads to more investment in conflict on the part of the incumbent. As $s$ increases, the second effect overtakes the first: an increase in $s$ eventually reduces the optimal investment in offense to zero. In other words, for the not too high values of $\theta$, there exists $s^*, 0 < s < s^*$, that maximizes the challenger's investment in offense.

If the offense efficiency parameter is sufficiently high, then the second effect dominates for all values of $s$. Strong sanctions unambiguously reduce the investment in offense in that case (Panel C).

B. Sanctions and Consumption

To understand the impact of sanctions on expected utility of the challenger group we need to evaluate the function $C_B(s, \theta)$ for $s \in [0, 1]$ and $\theta > 0$. Our findings, derived in Appendix III, are illustrated by the indifference map for the challenger group $B$ in Chart 3.\textsuperscript{27} The preferences are concave such that the thicker lines correspond to the higher utility levels. There is also a bliss region corresponding to the incumbent’s surrender since in that case the challenger attains the highest possible utility.

Proposition 3. Weak sanctions can hurt the challenger. The inefficiency of challenger in his struggle against the incumbent (low $\theta$) and the heavy burden of sanctions that falls on the challenger (low $\lambda$) exacerbate this effect.

\textsuperscript{27} Since utility is assumed to be linear in consumption, the indifference map is equivalent to the isoconsumption map.
Chart 2. Impact of Sanctions on the Offense Expenditure by the Challenger

Panel A. Low offense efficiency

Panel B. Moderate offense efficiency

Panel C. High offense efficiency
Chart 3. Indifference/Isoconsumption Map for the Challenger

Thicker lines correspond to higher utility/consumption.
To see the impact of an increase in sanctions on the challenger’s utility we need to pick a point on the graph (Chart 3) and then move horizontally to the right. Suppose we start with low offense efficiency parameter $\theta$ and no sanctions, $s = 0$. Then, moving rightward we originally descend to lower indifference curves. After a while, though, as $s$ gets closer to $s^*$, we start climbing to higher indifference curves and eventually reach our bliss plateau at $s = s^*$. In contrast, starting with high $\theta$ we immediately start climbing up until we reach the bliss plateau at $s = s^*$.

In addition, as shown in Appendix III, the lower the value of $\lambda$ is, the more elongated the indifference curves are. That means that if the challenger bears the heavy burden of sanctions (low $\lambda$), then weak sanctions are more likely to hurt the challenger.

**Proposition 4. Sufficiently strong sanctions benefit the challenger.**

These results arise because of the twofold impact of sanctions. On the one hand, sanctions make the challenger’s victory more likely. On the other hand sanctions reduce incomes in the economy if the challenger fails. For weak sanctions the latter effect outweighs the former, while for strong sanctions the reverse is true. This relatively stronger negative impact of weak sanctions on the challenger’s expected utility explains why the switch from deterrence to engagement due to an increase in sanctions’ strength reduces the expected utility of the challenger (move to a lower indifference curve in Chart 3).

**VII. RECENT SANCTIONS EXPERIENCE**

The analysis in this paper has clarified how sanctions affect the struggle for economic rents. This section is intended to illustrate the results derived above.

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28 The similar in spirit results were obtained by McDermott (1997) who analyzed how sanctions affect exploitation of one party by another and argued that strong sanctions put an end to exploitation while weak sanctions lead to the increase in the intensity of exploitation. Grossman (1992) analyzes the impact of foreign aid on insurrection, a particular form of conflict, and demonstrates that foreign aid to the ruler can benefit the general population. Aid prompts the ruler to increase the transfers to the people at large in order to mitigate their subversive activity. Then, according to this logic, sanctions against the ruler, which can be thought of as negative aid, harm the general population. To rephrase the Grossman’s (1992) result in the language of the present paper, sanctions can hurt the dominated group because the dominant group would lower transfers to the dominated group. The Proposition 3, however, is a stronger statement, for it argues that sanctions can hurt the dominated group even if there are no intergroup transfers.
The preceding two decades witnessed the demise of three odious political regimes: the cessation of minority rule in Rhodesia in 1980, the collapse of the Soviet Union in 1991 and the abolition of apartheid in South Africa in 1994. Civil conflict was the driving force behind the breakdown of each regime. Both sanctions and system inefficiencies affected each political system. However, the impact of sanctions, and the way in which the regime met its demise in the Soviet Union differed from those in Rhodesia and in South Africa.

As was mentioned in the beginning of the paper, the incumbent Soviet conservatives refused to let go in the face of mounting system inefficiencies associated with the old system. In the language of the model, they have chosen engagement and lost. In contrast, the incumbent rulers of Rhodesia and of South Africa surrendered, since they agreed to yield power while still firmly in control of the country. Why did the Rhodesian and the South African rulers surrender while the Soviet conservatives did not? The model suggests that high defeat income makes surrender more likely while low defeat income has the reverse effect.

The incumbent’s defeat income was high in Rhodesia and in South Africa because the whites, the incumbent group in both countries, were in general skilled and thus could apply themselves with reasonable success either in their country after the transition or abroad. Also, and this may explain the timing of transition in South Africa, the dissolution of the worldwide Communist bloc reduced the fear among the whites that the victorious African National Congress would attempt to establish a Communist regime. This additional increase in the defeat income made the South African whites even more receptive to the idea of surrender.

Conversely, the defeat income was low in the Soviet case. The conservative nomenklatura members could not imagine themselves as productive members of the society outside the...

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29 According to the pre-transition study by Guelke (1991, p. 160), “From within the dominant community inside South Africa, majority rule appears very far from: being inevitable in the foreseeable future.” This sentiment is shared by Lowenberg (1997, p. 71) in his post-transition analysis: “Certainly, the National Party government [of apartheid South Africa] could have clung to power virtually indefinitely, given the superior military and police technology of the white state relative to its opponents.” The whites of Rhodesia were engaged in brutal civil war with the rebels for several years before abandoning power. Although the war was turning into the rebels’ favor, there seemed to be a nonnegligible probability of a costly victory for the white minority government. For a discussion on Rhodesia see Matthews (1990) and Ginifer (1995).

30 As Levy (1999) puts it, “The final key ingredient in the change was the fall of communism in Eastern Europe and Mikhail Gorbachev’s forswearing of regional proxy wars. The Afrikaner government of South Africa saw the ANC as a party of godless communists, ready to take their country into the communist bloc. Once the communist bloc fell apart and a withdrawal of Cuban troops from Angola was negotiated, a political deal with the ANC became conceivable.”
communist system. Hence, they were not compelled to surrender even in the face of enormous system inefficiencies. Similarly, there is no indication that the current rulers of Cuba, the subject of U.S. embargo, or North Korea, ravaged by the economic mismanagement, are going to surrender (allow free elections, liberalize the economy) in spite of the enormous inefficiencies associated with their regimes. As in the Soviet case, the ruling elites in Cuba and North Korea are not productive outside the system. The ruling party functionaries are not likely to possess the skills necessary for success in a free market environment: the defeat income and the willingness to surrender is low.

**VIII. SUMMARY**

This paper analyzes how sanctions, which are imposed on adversaries in civil conflict, affect the utility of the participants and the allocation of resources to conflict. This work does not attempt to explain why sanctions are imposed; instead it concentrates on how sanctions work.

Civil conflict is a struggle for rents between the incumbent (group that initially controls the resources in the economy) and the challenger (group that wants to wrestle the control over the economy from the incumbent). The commonly considered equilibria in civil conflict are engagement and deterrence. To properly account for all the possible types of resource allocation, the paper extends the formal analysis of conflict to include a qualitatively new type of equilibrium — surrender.

Without sanctions, the equilibrium is deterrence, if challenger is not effective in its struggle against the incumbent, and engagement, if the challenger is effective. Strong enough sanctions lead to surrender of the incumbent regardless of the effectiveness of the challenger.

31 The dissolution of the Soviet Union put an end to the Soviet assistance to Cuba and led many observers to believe that, with the U.S. embargo becoming much more influential, the collapse of the Castro regime is imminent (see, for instance, Purcell, 1992). Nevertheless, the Castro’s government continues to dominate Cuba and is not close to its political demise, as described by Dominguez (1993) and Smith (1996). On recent North Korean trends see Eberstadt (1997) and Noland (1997).

32 Moreover, the unwillingness of the Cuban elite to give up might be exacerbated by the expectation that liberalization will bring in former citizens (Cuban exiles from Florida, for example). The fact that the newcomers are likely to impose additional restrictions on opportunities for the former government officials reduces the defeat income even further. As Dominguez (1993, p. 100) puts it, referring to the Cuban military corps, “...the Nuremberg-style trials that Miami radio stations promise await them ... leave Cuba’s army officers loyal to the regime, grateful for their perks, and unmotivated to revolt.”
If the equilibrium is deterrence, then the imposition of sanctions makes deterrence more expensive for the incumbent to sustain, and eventually forces the incumbent to switch to engagement. However, the switch from deterrence to engagement reduces the expected utility of the challenger. This happens because the negative effect on the challenger’s utility due to sanctions (lost output) outweighs the positive effect (positive probability of victory for the challenger).

In general, it is shown that weak sanctions imposed against the incumbent dominant group’s rule can actually hurt the challenger while strong sanctions benefit the challenger. The challenger in turn prefers sufficiently severe sanctions to no sanctions at all, even if the sanctions do not force the incumbent dominant group to surrender.

The model suggests that sanctions are more likely to work when the income that the incumbent collects in case of defeat is large. This finding helps to explain the significance of sanctions associated with the apartheid regime in South Africa and in Rhodesia under white minority government, as well as inability of sanctions to induce surrender of the incumbent Castro government in Cuba.
EQUILIBRIUM

Since the challenger is a follower in a Stackelberg game, we start with solving for the challenger’s optimal investment in offensive actions taking investment in defense by the incumbent as given. The challenger’s problem is

$$\max_{G_B} C_B$$

subject to

$$G_B \geq 0$$

If the dominant group decides to surrender, it selects $G_A = 0$. Since the probability of the dominant group’s victory given by (1) is not defined for $G_A = G_B = 0$, the challenger responds with the smallest possible positive value of offense spending, $G_B = \epsilon > 0$ (e.g. if the challenger cannot buy less than 1 gun, then $\epsilon$ is the value of a gun). Then $P = 0$, i.e. the challenger wins with probability 1. The value of $\epsilon$ is assumed to be small, so its impact on the challenger’s consumption is negligible.

The solution to the challenger’s problem is (using Kuhn-Tucker conditions)

$$G_B^*(G_A) = \begin{cases} 
\epsilon & \text{if } G_A = 0 \\
\sqrt{G_A \left[ \Omega + (1 - \lambda)s(y_A + Y_B) \right]} - \frac{G_A}{\theta} & \text{if } 0 < G_A < G_A^D \\
0 & \text{if } G_A \geq G_A^D 
\end{cases}$$  \hspace{1cm} (8)

where

$$G_A^D = \theta \left[ \Omega + (1 - \lambda)s(y_A + Y_B) \right]$$  \hspace{1cm} (9)

The expression $\Omega + (1 - \lambda)s(y_A + Y_B)$ equals $Y_B - y_B$ and measures the attractiveness of victory for group $B$. The larger $s$ is, i.e. the stronger sanctions are, the more attractive victory for group $B$ is. Similarly, the smaller $\lambda$ is, i.e. the larger the burden of sanctions that group $B$ bears is, the more attractive victory becomes for that group. As a result, the optimal investment in offensive activities $G_B$ depends positively on $s$ and negatively on $\lambda$ for a given $G_A < G_A^D$.

$G_A^D$ is the deterrent value of defense spending. Since the sanctions make victory more valuable to the challenger, the larger $s$ is, the more expensive it is for the dominant group to deter an attack. The deterrent value of $G_A$ depends positively on $s$ and negatively on $\lambda$. 

The problem of maximizing expected consumption for the incumbent dominant group can be stated as follows

\[
\max_{G_A} \left\{ y_A + \frac{G_A}{G_A + \theta G_B} (Y_A - y_A) - G_A \right\}
\]

subject to

\[
G_A \geq 0 \quad G_B = G_B^*(G_A)
\]

where \(G_B^*(G_A)\) is given by (8).

\(Y_A - y_A\) measures the attractiveness of continued dominance for group A in terms of the additional income that group appropriates. From (4), the severity of sanctions \(s\) and the fraction of loss borne by group \(A, \lambda\), reduce the attractiveness of dominance for \(A\).

The possible configurations are represented in Chart 4. If victory income exceeds defeat income for the incumbent (Case 1), then the expected consumption function either does or does not have an interior maximum in \([0, G_A^D]\).33 If there is no interior maximum, then \(G_A = G_A^D\) maximizes the expected consumption of the incumbent; the equilibrium is deterrence. If there is an interior maximum, \(G_A^*\), then it is the incumbent's choice; the equilibrium is engagement. If victory income is less or equal to the defeat income for the incumbent due to strong sanctions (Case 2), then investment in defense in counterproductive. The incumbent chooses \(G_A = 0\) to maximize its expected consumption; the equilibrium is surrender.

Formally, the solution is

\[
G_A = \begin{cases} 
0 & \text{if } s \geq s^*; \text{ sur} \\
G_A^* = \frac{1}{4} \frac{(Y_A - y_A)^2}{\theta (Y_B - y_B)} & \text{if } s < s^* \text{ and } \theta > \frac{1}{2} \frac{Y_A - y_A}{Y_B - y_B}; \text{ eng} \\
G_A^D = \theta (Y_B - y_B) & \text{if } s < s^* \text{ and } \theta \leq \frac{1}{2} \frac{Y_A - y_A}{Y_B - y_B}; \text{ det}
\end{cases}
\]

where

\[
s^* = \frac{\Omega}{\lambda (y_A + Y_B)}
\]

33 Expected consumption of the incumbent, \(C_A\), is defined for any positive investment in defense \(G_A\). However, investment in defense in excess of the deterrent value \(G_A^D\) only reduces the expected consumption.
Chart 4. Expected Consumption of the Incumbent as a Function of Defense Expenditure

Case 1. Victory income is larger than defeat income

Case 2. Victory income is not larger than defeat income: surrender
The threshold value for the severity of sanctions, \( s^* \), is defined as the value of \( s \) such that the victory income of group \( A \) equals its defeat income, \( Y_A = y_A \). It is derived by substituting \( Y_A = y_A \) into (4). Thus \( s \geq s^* \) is equivalent to \( Y_A \leq y_A \). In other words, if \( s \geq s^* \) then the marginal benefit from defense spending is nonpositive for any positive \( C_A \).

To show that \( s^* \in [0,1] \) we need to use our assumption that both defeat and victory incomes are nonnegative. If \( s = 1 \) then, according to (2), total income in the economy if group \( A \) is dominant equals to zero. Thus, nonnegativity of income implies that if \( s = 1 \) then \( Y_A = y_B = 0 \) and the expression (4) can be rewritten as

\[
-y_A = \Omega - \lambda(y_A + Y_B)
\]

In other words,

\[
\Omega - \lambda(y_A + Y_B) \leq 0
\]

and

\[
\frac{\Omega}{\lambda} \leq y_A + Y_B
\]  

From (12) and (13) it follows that

\[
s^* \in [0,1]
\]

This concludes the proof of Proposition 1.

To substantiate Proposition 2 we examine the deterrence / engagement threshold value of \( \theta \). By assumption, \( Y_A - y_A \) and \( Y_B - y_B \) depend on \( s \). Then this threshold value is a function of \( s \) as well. Denote this threshold value as \( f(s) \). From (11)

\[
f(s) = \frac{1}{2} \frac{Y_A - y_A}{Y_B - y_B} = \frac{1}{2} \frac{\Omega - \lambda y_A + Y_B}{\Omega + (1 - \lambda) y_A + Y_B}
\]

Simple substitution shows that that \( f(0) = 1/2 \) and \( f(s^*) = 0 \). It can easily be demonstrated that \( f'(s) < 0 \) and \( f''(s) > 0 \).

For the analysis in Appendix III we will need to know the equilibrium probability of victory for the incumbent dominant group \( A \), which is obtained by substituting (8) and (11) into (1).

\[
P = \begin{cases} 
0 & \text{if } s \geq s^* \\
\frac{1}{2} \frac{Y_A - y_A}{Y_B - y_B} & \text{if } s < s^* \text{ and } \theta > \frac{1}{2} \frac{Y_A - y_A}{Y_B - y_B} \\
1 & \text{if } s < s^* \text{ and } \theta \leq \frac{1}{2} \frac{Y_A - y_A}{Y_B - y_B}
\end{cases}
\]  

(14)
IMPACT OF SANCTIONS ON INVESTMENT IN CONFLICT

Consider first the investment in conflict by the incumbent, $G_A$. The equilibrium value of $G_A$ is given by (11). Since $Y_A - y_A$ depends negatively, while $Y_B - y_B$ depends positively on $s$, it follows that $G_A$ depends negatively on $s$ both in case of deterrence and in case of engagement.

Now let’s analyze the impact of sanctions on the investment in conflict by the challenger, $G_B$. In cases of deterrence and surrender the challenger invests in conflict either zero, or a negligible amount close to zero respectively, as follows from the discussion in Appendix I. The only non-trivial case is therefore engagement.

In case of engagement the equilibrium value of $G_B$ can be derived by substituting (11) into (8), which yields

$$G_B = \frac{1}{4} \frac{Y_A - y_A}{\theta} \left[ 2 - \frac{Y_A - y_A}{\theta(Y_B - y_B)} \right]$$

Taking the partial derivative of $G_B$ with respect to $s$ and remembering that

$$Y_A - y_A = \Omega - \lambda s(y_A + Y_B) \quad (15)$$
$$Y_B - y_B = \Omega + (1 - \lambda) s(y_A + Y_B) \quad (16)$$

produces the following result:

$$\frac{\partial G_B}{\partial s} > 0 \quad \text{if and only if}$$

$$(1 - \lambda) \frac{1}{2\theta} \left( \frac{Y_A - y_A}{Y_B - y_B} \right)^2 > \lambda \left[ 1 - \frac{Y_A - y_A}{\theta(Y_B - y_B)} \right] \quad (17)$$

Observing that

$$\lim_{s \to s^*} (Y_A - y_A) = 0$$

$$\lim_{s \to f^{-1}(\theta)} (Y_A - y_A) = 2\theta(Y_B - y_B)$$
and

\[ \lim_{s \to 0} (Y_A - y_A) = \lim_{s \to 0} (Y_B - y_B) = \Omega \]

the following results emerge:

1. If \( s \to s^* \) then the inequality (17) becomes \( 0 > \lambda \), which is not true since by assumption \( 0 < \lambda < 1 \). Therefore,

\[ \lim_{s \to s^*} \frac{\partial G_B}{\partial s} < 0 \]

2. If \( s \to f^{-1}(\theta) \) then the inequality (17) becomes \( 2\theta(1 - \lambda) > -\lambda \), which is always true. Accordingly,

\[ \lim_{s \to f^{-1}(\theta)} \frac{\partial G_B}{\partial s} > 0 \]

3. If \( s \to 0 \) then the inequality (17) becomes

\[ \theta < \frac{1}{2} + \frac{1}{2\lambda} \]

Therefore,

\[ \lim_{s \to 0} \frac{\partial G_B}{\partial s} > 0 \quad \text{if and only if} \quad \theta < \frac{1}{2} + \frac{1}{2\lambda} \]

\[ \lim_{s \to 0} \frac{\partial G_B}{\partial s} < 0 \quad \text{if and only if} \quad \theta > \frac{1}{2} + \frac{1}{2\lambda} \]

Chart 2 demonstrates these results.
IMPACT OF SANCTIONS ON THE CHALLENGER’S EXPECTED CONSUMPTION

In this section the challenger’s indifference map presented in Chart 3 is derived. To this end, the signs of \( \frac{\partial C_B}{\partial s} \) and \( \frac{\partial C_B}{\partial \theta} \) for various combinations of \( s \) and the offense efficiency parameter \( \theta \) are derived. It is also demonstrate that the optimal expected consumption of the challenger \( C_B \) is a continuous function of \( s \) for \( s \in [0,1] \).

To complete this analysis the exact assumption of how the burden of sanctions is shared among the incumbent and the challenger (equations (4) and (5)) is not required. Instead, the less restrictive and less analytically cumbersome formulation is adopted:

*The burden of sanctions falls on both groups, i.e. victory income of group A, \( Y_A \), and defeat income of group B, \( y_B \), are decreasing linear functions of \( s \). Defeat income of group A, \( y_A \), and victory income of group B, \( Y_B \), are independent of \( s \).*

This implies that \( Y_A - y_A \) depends negatively on \( s \) and \( Y_B - y_B \) depends positively on \( s \). Then the properties of the deterrence / engagement threshold value of \( \theta \), \( f'(s) = \frac{Y_A - y_A}{2(Y_B - y_B)} \), are unchanged: \( f(0) = 1/2 \), \( f(s^*) = 0 \), \( f'(s) < 0 \), and \( f''(s) > 0 \).

**Deterrence** is an equilibrium for \( s \in [0, f^{-1}(\theta)] \). Then for \( s \) in that interval

\[
C_B^{det} = y_B
\]

Hence, in case of deterrence the challenger’s consumption depends negatively on \( s \), since sanctions reduce the defeat income of the challenger, and is independent of \( \theta \). The indifference map consists of straight vertical lines with lower expected utility for higher \( s \).

The dominant group selects **engagement** if \( s \in (f^{-1}(\theta), s^*) \) such that, from (8), (11), and (14), the challenger invests in offensive activities

\[
G_B = \frac{1}{4} \frac{Y_A - y_A}{\theta} \left[ 2 - \frac{Y_A - y_A}{\theta(Y_B - y_B)} \right]
\]

and wins with probability

\[
1 - P = 1 - \frac{1}{2\theta} \left[ \frac{Y_A - y_A}{Y_B - y_B} \right]
\]

Then, the expected consumption of the challenger is

\[
C_B^{eng} = y_B + \frac{(Y_B - y_B - \frac{Y_A - y_A}{2\theta})^2}{Y_B - y_B}
\]

According to the above expression, the expected consumption of the challenger depends positively on \( \theta \).
How does the expected consumption depends on $s$ in case of engagement? Taking the partial derivative of $C_B$ from equation (18) yields

$$\frac{\partial C_B(s)}{\partial s} = -\frac{\dot{Y}_A}{\theta} + \frac{2(Y_A - y_A)(Y_B - y_B)\dot{Y}_A + (Y_A - y_A)^2\dot{y}_B}{4\theta^2(Y_B - y_B)^2}$$

(19)

where

$$\dot{Y}_A = \frac{dY_A}{ds}$$
$$\dot{y}_B = \frac{dy_B}{ds}$$

By assumption, $\dot{Y}_A$ and $\dot{y}_B$ are negative constants. Then,

$$\lim_{s \to f^{-1}(\theta)} \frac{\partial C_B}{\partial s} = \dot{y}_B < 0$$

The expected utility of the challenger depends positively on $\theta$ and depends negatively on $s$ if $s$ is sufficiently close to the deterrence/engagement threshold. The indifference curves are sloping upward.

$$\lim_{s \to s^*} \frac{\partial C_B}{\partial s} = -\frac{\dot{Y}_A}{\theta} > 0$$

The expected utility of the challenger depends positively on $\theta$ and depends positively on $s$ if $s$ is sufficiently close to $s^*$. Consequently, the indifference curves are sloping downward for $s$ close to $s^*$.

What happens if $\theta$ is sufficiently large such that deterrence is costly and engagement is the equilibrium for any $s \in [0, s^*)$, as shown in the upper portion of Chart 1? To derive the slope of the indifference curves in this case we need to evaluate the limit of $\frac{\partial C_B}{\partial s}$ as $s$ approaches zero.

Since by assumption $\lim_{s \to 0}(Y_A - y_A) = \lim_{s \to 0}(Y_B - y_B)$, then from (19)

$$\lim_{s \to 0} \frac{\partial C_B}{\partial s} = -\frac{\dot{Y}_A}{\theta} + \frac{2Y_A + y_B}{4\theta^2}$$

Accordingly, $\lim_{s \to 0} \frac{\partial C_B}{\partial s}$ is negative if and only if the offense efficiency parameter is sufficiently low, i.e.

$$\theta < \frac{1}{2} \frac{\dot{y}_B}{4Y_A}$$
From (4) and (5)

\[ \dot{Y}_A = -\lambda (y_A + Y_B) \]

\[ \dot{Y}_B = -(1 - \lambda)(y_A + Y_B) \]

Then, \(\lim_{s \to 0} \frac{\partial C_B}{\partial s}\) is negative (and, consequently, the indifference curves are upward-sloping) if and only if \(\theta\), the effectiveness of the challenger against the incumbent, is low and/or \(1 - \lambda\), the burden of sanctions borne by the challenger, is high:

\[ \theta < \frac{1}{2} + \frac{1 - \lambda}{4\lambda} \]

In other words, high value of \(1 - \lambda\) (or low value of \(\lambda\)), stretches out the indifference curves in Chart 3 along the vertical axis. Thus, as stated in Proposition 3, low value of \(\lambda\) makes it more likely that weak sanctions will hurt the challenger.

The dominant group **surrenders** if \(s \in [s^*, 1]\). The expected consumption of the challenger is then given by

\[ C_B^{sur} = Y_B \]

It depends on neither \(s\) nor \(\theta\). The rectangle corresponding to \(s \in [s^*, 1]\) is the bliss area for group \(B\) since in that case the initially dominant group surrenders and the challenger is able to appropriate victory income \(Y_B\) with probability 1 and negligible offense expenditure.

To demonstrate the continuity of \(C_B\) in \(s\), we need to show that for any given \(\theta\)

\[ \lim_{s \to f^{-1}(\theta)^+} C_B^{eng} = C_B^{det} \]

\[ \lim_{s \to s^*} C_B^{eng} = C_B^{sur} \]

From the definition of \(f(s)\) it follows that if \(s \to f^{-1}(\theta)\) then \(Y_B - Y_B \to \frac{Y_A - y_A}{2\theta}\) and, using (18),

\[ \lim_{s \to f^{-1}(\theta)^+} C_B^{eng} = y_B(f^{-1}(\theta)) = C_B^{det}(f^{-1}(\theta)) \]

Also, if \(s \to s^*\) then \(Y_A - y_A \to 0\). As a result,
\[
\lim_{s \to s^*} C_B^{en} = Y_B = C_B^{eq}\]

Hence, \(C_B\) is a continuous function for \(s \in [0, 1]\).

Now it is easy to show that the challenger prefers sufficiently strong sanctions to no sanctions even if the incumbent does not surrender. Comparison between the expected consumption values in cases of deterrence and surrender indicates that \(C_B(s^*) > C_B(0)\). By continuity of \(C_B\) it means that there exist values of \(s < s^*\) such that \(C_B(s) > C_B(0)\).
References


