Competition Among Regulators

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IMF Working Paper

Research Department

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Authorized for distribution by Eduardo Borensztein

May 2001

Abstract

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This paper shows that competition among regulators reduces regulatory standards relative to a centralized solution. It suggests that a central regulator is more likely to emerge for homogeneous and financially integrated countries. The paper proves these results in a model where regulators concerned with their banking system's stability and efficiency and with their banks' profitability set their regulatory policy non-cooperatively. Externalities in bank regulation make the independent solution collectively inefficient. These externalities and the benefits of centralized regulation increase with financial integration, while the costs associated with the loss of independence decrease with the homogeneity of the countries involved.

JEL Classification Numbers: G21, G28

Keywords: Externalities, banking regulation, financial integration

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1 Robert Marquez is an assistant professor at University of Maryland. We thank Tito Cordella, Rafael Repullo, Bruce Smith, and Javier Suarez for helpful comments, as well as participants at the conference on Financial Supervision of Banks and Specialized Banks in the EU held at the European University. Institute, Florence, Italy
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I. Introduction

In recent years, technological progress and regulatory changes have led to the progressive integration of international financial markets. In that context, banks' cross-border activities have become increasingly important, raising new problems for regulators that have remained nationally bounded. This trend has spurred a debate on the costs and benefits of the international harmonization of bank regulation. As a specific example, in the EU the introduction of the Euro and the single market have raised the question of whether a continental regulatory agency would be necessary or desirable.

This paper examines how competition among regulators affects regulatory standards and which factors favor the emergence of "regulatory unions". The paper presents a model where national regulators concerned with the stability and efficiency of their country's banking system as well as the profitability of their domestic banks set their regulatory policies non-cooperatively. It compares such a setup with one where an international regulator sets standards for all the banks and shows that competition among regulators leads to lower regulatory standards than under a unified regulator. The paper then analyzes under which conditions a centralized international regulator is more likely to emerge, i.e., when would domestic regulators find it preferable to surrender their authority to a supranational regulator in order to reap the benefits from centralization. In particular, the paper examines how asymmetries across countries and the degree of international financial integration affect the incentives of multiple independent regulators to merge.

The paper finds that independent domestic regulators will spontaneously agree to merge into a central regulator only if countries are not too dissimilar in their individual regulatory standards or objectives. In that context, one of the main results in the paper is that, with asymmetric countries, a centralized regulator will be unanimously preferred to independence only if such regulator were to choose regulatory standards higher than those of the country with the highest individual standards. Finally, the paper suggests that an increase in banks' cross-border activity may increase individual regulators' incentives to merge in a centralized regulatory agency. This will be true when the negative impact of increased regulation for a regulator's own country is not too large. This suggests that countries that are currently too dissimilar to exploit the benefits of a centralized regulator may over time choose to form a regulatory union as they become more financially integrated.

The intuition for the main results is straightforward. Independent regulators do not internalize all the benefits stemming from higher regulatory requirements. Essentially, in financially integrated economies, banking regulation introduces an externality: higher regulatory standards in one country not only make the domestic banking system more stable, they also benefit foreign systems where the domestic country's banks operate by lowering the probability of bank failure in those markets. Independent regulators do not internalize this positive spillover, so that each regulator will "under-regulate" relative to a unified regulatory agency.
Moreover, there is a second effect that also arises. To the extent that regulators are concerned about the profitability of their banks, regulators are effectively competing with each other to increase the profitability of their banks at the expense of banks in other countries. This causes them to further lower their regulatory standards in order to provide their domestic banks with an advantage over foreign banks.

In light of these considerations, increases in the degree of financial integration across countries magnify the impact of forming a regulatory union. More cross-border banking means more international competition and more interdependence of financial stability. Hence, greater financial integration implies greater externalities in regulation, increasing the benefit of coordinating regulatory policy when countries are similar.  

There are costs, however, associated with forming a regulatory union. These costs are mainly related to a loss of flexibility and are higher for more asymmetric countries. Indeed, it would be politically difficult, if not impossible, for a central regulator to impose different standards across countries. Hence, more similar and more integrated countries are those more likely to opt for a centralized solution, while peripheral and asymmetric countries will probably choose to maintain independent regulators. However, even if countries are similar, regulation still has to increase relative to the highest level with independent regulators in order for both countries to be better off. This occurs because increasing the level of regulation hurts domestic banks. If this effect is to be compensated, it must be that regulation also increases for foreign banks.

Most of the insights and implications of this paper can be applied to the broad issue of bank regulation in a financially integrated international economy or to a single economy with competing regulatory agencies, such as the U.S. For the European case, for instance, this analysis is relevant for a number of reasons. First, the Single Market Act has created an environment where banks chartered in any EU member country are allowed to open branches anywhere in the Union while remaining under the supervisory authority of their country of origin. Second, the introduction of the Euro has intensified the linkages among financial markets. Finally, Euroland is characterized by a unique situation of separation between the geographic domain of monetary and regulatory authorities. Indeed, since the inception of the Euro, the European Central Bank (ECB) has assumed full authority over monetary policy, while bank regulation and supervision has remained in the hands of independent national agencies (often the national central banks).

At the same time, regulatory and supervisory practices vary across countries. Although the Basle Accord has gone a long way towards harmonizing capital standards at an international level, individual countries have maintained large discretion on the determination of which assets can

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2 Similar insights have been applied to the analysis of optimum currency areas. Recently, Bencivenga, Huybens, and Smith (1999) have argued that the benefit or cost of a “dollarization” depends greatly on the extent to which credit markets are integrated internationally. Here we offer a similar focus in the context of banking regulation by demonstrating that centralized regulation is only beneficial if banking markets are well integrated.

3 See Belaisch, Kodres, Levy, and Ubide (2001) for a comprehensive analysis of the banking industry in Europe.
be used to meet capital requirements. Information disclosure, inspection procedures, accounting systems, and limits on the scope of banks' activities are also heterogenous across countries, as well as rescue policies designed to manage banking crises. In this paper, we work on the basis of the view that even in the presence of international agreements, countries maintain sufficient autonomy to compete over regulatory standards. Our results apply to this setting.

This paper proceeds as follows. Section 2 briefly review some related literature. Section 3 describes the main model. Section 4 considers the case of asymmetric countries. Section 5 extends the results of the previous sections by analyzing the impact of increased financial integration. Section 6 concludes.

II. RELATED LITERATURE

Traditionally, regulation has been justified as an attempt to provide protection for depositors from the risk of failure of their bank. Moreover, bank failures also create negative externalities that can adversely affect the economy, hurting their customers, both depositors and borrowers, and possibly spreading to other banks (see Bhattacharya and Thakor, 1993, for a survey of these issues). Therefore, regulation has focused on promoting the safety and soundness of the banking system.

To promote the stability of this system, a number of regulatory instruments have been identified, ranging from portfolio restrictions to capital adequacy standards. Moreover, while some, such as deposit insurance, have been inducted in order to benefit depositors directly, others, such as deposit interest rate ceilings, are believed to benefit depositors only indirectly by increasing bank profits and thus strengthening the system. Much of the recent literature has focused on the optimal assignment of these instruments to address the concerns highlighted above. For example, as a response to increased risk-taking by banks resulting from deposit insurance, regulators have instituted capital requirements to control the risk in banks' portfolios.

In this paper, we abstract from that debate and simply assume, as in much of the theoretical as well as the policy driven literature, that domestic regulators implement "in isolation" whatever policy is optimal for their country. We use the idea that while regulation may be beneficial for bank customers it tends to be disliked by banks. As such, much of the recent deregulation in banking can be seen as driven by banks' desire to improve their competitive position vis-à-vis

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4Gorton and Winton (1999) discuss how this issue has led to significant heterogeneity in the definition of tier 1 capital across countries. Also see Prati and Schinasi (1999) for a detailed examination of the differences across European countries.

5See White (1993) for a discussion of this issue.

6See Santos (2001) for a recent survey.

7There is considerable debate as to the ultimate effect of many of these instruments. In particular, much of the focus has been placed on whether capital adequacy ratios can in fact be successful at curbing risk-taking behavior. For example, see the discussion in Rochet (1992) or more recently in Besanko and Kanatas (1996), Calem and Rob (1999), and Hellmann, Murdock, and Stiglitz (2000).
other financial institutions that are competing in banks' traditional markets.\textsuperscript{8} It is in this context that we examine the issue of competition in regulation.

The notion of competition among regulators has been recently analyzed in a context where multiple regulators have authority over the same group of agents. Kane (1984) argues that competition among financial regulators is beneficial as it fosters the production of more efficient regulatory services.\textsuperscript{9} Laffont and Martimort (1999) demonstrate that, in the presence of non benevolent regulators, splitting regulatory authorities among different regulatory bodies limits their discretion in engaging in socially wasteful activities by making collusion between regulators and the regulated firm more difficult. Here, we focus on an alternative situation where multiple regulators have authority over different groups of agents that in turn compete with each other over common markets. From that point of view, our paper is closer to White (1994), who argues that cooperation among independent regulators may not be sustainable by drawing a parallel with the well-known problems of cartel stability, and to Santos and Scheinkman (2000), who examine whether competing exchanges have incentives to lower their standards and demand fewer contractual guarantees to their traders.

III. A MODEL OF EXTERNALITIES IN REGULATION

Following the discussion above, we assume that a bank regulator cares about the efficiency and the safety and soundness (stability) of the banking sector, which would be the case if the primary motivation for regulation was to protect depositors. We also allow for the possibility that the regulator may care about the well-being of other claimants on the banking firm, such as bank shareholders. We do this by assuming that the regulator may care about bank profits directly.

We assume that the regulator may have a number of different instruments at its disposal, and will choose some combination of these instruments optimally, given its regulatory objective, which is described below.\textsuperscript{10} Examples of such instruments are the imposition of a capital adequacy ratio, portfolio requirements, or any of the other possible tools referred to in the previous section. The common thread among these instruments is that they help alleviate agency problems in lending and so reduce the risk of bank failure, or lower the expected cost of a bank bailout. At the same time, the combined use of these instruments is disliked by banks, so that banks would always prefer that regulations be lowered. We refer to the set of these instruments for country $i$ as the variable $k_i$, which we assume is a variable summarizing the effects of many possible regulatory instruments.

\textsuperscript{8}See Gehrig (1998) for a related discussion about stock exchange markets.
\textsuperscript{9}Similarly, Dermine (1991) argues that competition among exchanges would improve their quality if participants were able to choose the market in which to operate and investors were able to judge the quality of regulation.
\textsuperscript{10}We abstract from the question as to what the optimal design of the regulation might be, and assume that given its objective function, the regulator is able to find a solution to maximize its objective. This is equivalent to assuming that the regulator is able to determine the impact of imposing certain regulatory standards.
We take a reduced form approach to our model. Specifically, assume that there are two countries, each with a banking system and a regulator for the banks chartered in the country. Profits for banks chartered in country \(i\) depend both on the level of regulations imposed on them by their regulator, as well as the level imposed on their foreign competitors chartered in country \(j\): \(\Pi^i(k_i, k_j)\). The assumption that banks dislike the imposition of regulatory standards can be summarized as follows:

\[
\frac{\partial \Pi^i(k_i, k_j)}{\partial k_i} < 0, \quad \frac{\partial \Pi^i(k_i, k_j)}{\partial k_j} > 0
\]

The first expression says that the bank’s profit is lower for higher regulatory standards. In other words, the more regulated is the bank, the lower is its profit.\(^{11}\) The second expression just says that a bank benefits when its competitors, or at least some of them, are more heavily regulated.

Regulators are concerned about the efficiency and the safety and soundness of their banking system, which is summarized by a function \(F^i(k_i, k_j)\), which for low values of regulation is increasing in both \(k_i\) and \(k_j\), but is decreasing for values close to 1, and is concave in both of these variables: \(\frac{\partial^2 F^i}{\partial k_i^2}, \frac{\partial^2 F^i}{\partial k_j^2} < 0.\(^{12}\) This function represents the net benefits of optimal regulation in promoting stability of the banking system and protecting depositors.\(^{13}\) However, regulators also care to some extent about shareholders of the bank, which is modeled by assuming that they care about the profitability of their banks. This is achieved by assuming that the regulator puts a weight \(\alpha\) on bank profits and a weight \(1 - \alpha\) on the stability function \(F^i\).

We consider two separate cases, one of independent national regulators, where each bank is regulated only by its home country, and the other of a central regulator that sets regulatory standards for all banks in the system. We assume that each independent national regulator has the ability to set regulations for the banks chartered in that country, and so can only control \(k_i\). In the case of a centralized regulator, we assume that this regulator can set both \(k_i\) and \(k_j\), but must treat all banks equally, so it must set both to be the same \((k = k_i = k_j)\). Indeed, it seems reasonable to assume that a centralized regulator would find it politically infeasible to impose different

\(^{11}\) It should be emphasized that we are talking about regulations that impose a constraint on a bank’s operations, and so are costly to the bank. Historically, many countries have also adopted regulations designed to increase bank profitability by increasing their market power, in an attempt to lower systemic risk. An example of this is deposit rate ceilings. However, this type of regulation is only effective when regulatory barriers to entry into the banking system exist.

\(^{12}\) This assumption guarantees a unique optimum for the level of regulation imposed on each bank and an interior solution. If \(F^i\) were increasing in \(k_i\) for all values of \(k_i \in (0, 1)\), regulators would have a tendency to impose extreme regulatory standards. For example, it is quite likely that a system with a 100% capital requirements would not be particularly desirable as it would involve a very high cost of credit.

\(^{13}\) Each specific benefit of regulation could be modeled independently, with a benefit function for each possible instrument at the regulator’s disposal. \(F\) stands for the aggregate benefit of all of these instruments combined, aggregated across all the regulator’s possible objectives. To the extent that achieving some goals may hinder the attainment of others, we assume that \(F\) represents the regulator’s optimal balance between these possibly conflicting goals.
regulatory standards for each country. In addition, we assume that the central regulator's objective function is the sum of the objective functions of the national regulators. Here we abstract from more complex considerations about how the objective function of a centralized regulator may differ from that of the original domestic regulators.

In the case of independent national regulators the maximization problem for each regulator is

$$\max_{k_i} U^i(k_i, k_j) = \alpha \Pi^i(k_i, k_j) + (1 - \alpha) F^i(k_i, k_j)$$

(1)

while the central regulator would maximize

$$\max_k U(k) = \alpha \left( \Pi^i(k) + \Pi^j(k) \right) + (1 - \alpha) \left( F^i(k) + F^j(k) \right)$$

(2)

where $k = (k, k)$, so that the central regulator must choose the same level of regulation for both countries. If $\Pi^i(k) = \Pi^j(k)$ and $F^i(k) = F^j(k)$, so that the two countries are symmetric, this can be more succinctly written as

$$\max_k U(k) = 2\alpha \Pi(k) + 2(1 - \alpha) F(k)$$

We will concentrate on the symmetric case. It should be pointed out that symmetry in this context refers to the impact on profits and on regulatory benefits, but not necessarily on the weights placed on each of these terms. Later in the paper we will explicitly consider the case where each domestic regulator assigns a different weight $\alpha$ to bank profits.

The set of first order conditions (FOC) for the case of independent national regulators, equation (1), is

$$\alpha \frac{\partial \Pi^i(k_i, k_j)}{\partial k_i} + (1 - \alpha) \frac{\partial F^i(k_i, k_j)}{\partial k_i} = 0$$

$$\alpha \frac{\partial \Pi^j(k_j, k_i)}{\partial k_j} + (1 - \alpha) \frac{\partial F^j(k_j, k_i)}{\partial k_j} = 0$$

(3)

The solution to this maximization problem is a pair $(k^*_i, k^*_j)$ satisfying these two equations simultaneously. Since the countries are symmetric, an equilibrium exists with $k^*_i = k^*_j$. We focus on this case.

The case of the central regulator is given as follows. The FOC for equation (2) is

$$\alpha \left( \frac{\partial \Pi(k_i, k_j)}{\partial k_i} + \frac{\partial \Pi(k_i, k_j)}{\partial k_j} \right) + (1 - \alpha) \left( \frac{\partial F(k_i, k_j)}{\partial k_i} + \frac{\partial F(k_i, k_j)}{\partial k_j} \right) = 0,$$

(4)

The solution to this problem yields a regulatory standard $k^*$.

Compare now the solutions to the two problems. If we substitute the solution of the Nash game between independent regulators, the pair $(k^*_i, k^*_j)$, into the first order conditions for the
central regulator, equation (4), and rearrange slightly, we obtain

\[
\left[ \alpha \frac{\partial \Pi(k^*_i, k^*_j)}{\partial k_i} + (1 - \alpha) \frac{\partial F(k^*_i, k^*_j)}{\partial k_i} \right] + \alpha \frac{\partial \Pi(k^*_i, k^*_j)}{\partial k_j} + (1 - \alpha) \frac{\partial F(k^*_i, k^*_j)}{\partial k_j}
\]

(5)

The first term in equation (5) is zero as it is identical to the first order condition for national regulator \(i\). The second term is positive as both its components are positive. Then, by concavity of \(\Pi\) and \(F\), we must have a higher regulatory standard in order to satisfy equation (4). In other words, it must be \(k^*_i > k^*_i, k^*_j\).

This result shows that competing regulatory agencies will “under-regulate” relative to a centralized solution, as they fail to fully internalize the benefits stemming from their activity. Moreover, regulators also try to provide their banks with an advantage over competitors, further lowering the regulatory standard.\(^{14}\) Consequently, we have \(U^i(k^*, k^*) > U^j(k^*_i, k^*_j)\) for \(i, j = 1, 2\).

It is clear that the externality in regulation exists only to the extent that the banking systems of the two countries are somewhat integrated. Without this, regulation in one country would have no impact on the stability of the banking system in another country. In a later section, we show that this effect is more pronounced in more financially integrated countries, so that the impact of coordination is magnified.

IV. REGULATORY UNIONS AMONG ASYMMETRIC REGULATORS

So far, we have analyzed the difference between having centralized banking regulation and having independent regulators for banks that are already allowed to compete in each other’s markets, under the assumption that the banks in question, as well as the regulators, were identical and had similar preferences. In that case, we find that lack of coordination among regulators leads each regulator to impose more lax regulatory standards than if they could coordinate their actions.

However, there are a number of reasons to believe that the competition either among banks or among regulators need not be symmetric. Banks in some countries might be more efficient than in others, and the marginal impact of increasing regulations might be more or less deleterious for those banks. In fact, one of the strongest arguments for the opening of both product and financial markets in Europe was the promotion of competition and its impact on improving the efficiency of domestic firms. Similarly, the banking system in one country might be more stable than in another, necessitating lower regulatory standards, all things equal. This can occur either because financial institutions are more sound, financial markets are older and better developed, or there is more credibility in the government’s role as a prudential regulator.

\(^{14}\) It is worth pointing out that this “laxity” in regulation is measured only relative to the optimal regulatory standard that would be imposed if regulation were instead coordinated across countries. We take no stance on whether regulation should optimally be lax or tight, assuming that any incentive to loosen or tighten regulations should be reflected in the regulator’s objective function. For example, one reason to want regulation to be lax may be to promote financial innovation.
A further reason why competition among regulators can be asymmetric is that regulators in different countries might have different institutional arrangements with the banks under their control, or may have different concerns over the trade-off between bank profits and systemic stability. In some contexts, this has been referred to as the degree to which regulators are captured by the financial institutions under their control. To the extent that regulators in different countries may exhibit differing degrees of regulatory capture, the competition among these regulators will also be asymmetric. Yet another reason why each country might treat its banks differently is that in some countries there may be greater foreign ownership of domestic banks. If so, we would expect domestic regulators to be relatively less concerned about bank shareholders if a significant fraction of them are foreigners.\footnote{Though we do not focus explicitly on this issue, for countries with highly captured regulators, centralizing bank regulation could be a way of decreasing the influence banks yield over their regulators. For countries that exhibit a degree of regulatory capture that is not optimal for the eventual well-being of its constituents, centralizing regulation takes control out of the hands of local regulators and makes it more difficult for local banks to exert influence on the central regulators.}

Under any of these scenarios, we may well expect the regulatory standards set by competing regulators to be different. Moreover, we should expect that a central regulator with the ability to discriminate across countries and apply different standards to each would also choose different levels of regulation for each country. However, as we have argued in a previous section, one of the characteristics of a central regulator is that it is typically unable to apply different standards to each country. The imposition of equal regulatory standards in this case may entail losses to each country that could dominate any gains that would be obtained under (symmetric) coordination. In this case, coordination, to the extent that it imposes the same standard to each country, might be more harmful than beneficial, as there are strong economic reasons for maintaining separate regulators. We therefore investigate the conditions under which countries will endogenously choose to form a regulatory union.

We model asymmetry as follows. Suppose there are two countries, 1 and 2, that bank profits, $\Pi$, and regulatory benefit, $F$, are the same in both countries, but that regulators exhibit different degrees of regulatory capture. Specifically, assume that $\alpha_1 > \alpha_2$, so that we can think of country 1’s regulator as being more captured.\footnote{This way of modeling asymmetry places the focus on regulatory capture rather than on the impact of regulation on bank profits or stability directly. This allows for a tractable analysis without having to impose additional structure on $\Pi$ and $F$.} Following the notation of the previous section, let the regulatory standard chosen by competing (independent) regulators be given by $k_1^*, k_2^*$. We now have the following proposition.

**Proposition 1** If $\Pi$ and $F$ are symmetric across countries, and $\alpha_1 \neq \alpha_2$, then $k_1^* \neq k_2^*$.

**Proof.** Assume WLOG $\alpha_1 > \alpha_2$. Consider the first order conditions for regulator 1:
\[
\alpha_1 \left. \frac{\partial \Pi(k_1, k_2^*)}{\partial k_1} \right|_{k_1 = k_1^*} + (1 - \alpha_1) \left. \frac{\partial F(k_1, k_2^*)}{\partial k_1} \right|_{k_1 = k_1^*} = 0.
\]

By the symmetry of the profit function and the regulatory benefit function, and considering that \(\left. \frac{\partial \Pi(k_1, k_2^*)}{\partial k_1} \right|_{k_1 = k_1^*} < 0\) and \(\left. \frac{\partial F(k_1, k_2^*)}{\partial k_1} \right|_{k_1 = k_1^*} > 0\), we can write
\[
\alpha_2 \left. \frac{\partial \Pi(k_2, k_1^*)}{\partial k_2} \right|_{k_2 = k_2^*} + (1 - \alpha_2) \left. \frac{\partial F(k_2, k_1^*)}{\partial k_2} \right|_{k_2 = k_2^*} > 0.
\]

That proves that it cannot be \(k_2^* = k_1^*\). □

To establish a relationship between the degree of regulatory capture and the equilibrium level of regulation in each country, we impose the following standard stability conditions:
\[
\left| -\frac{\partial^2 V}{\partial k_i \partial k_j} \right| < 1 \text{ for } i = 1, 2. \tag{6}
\]

This assumption is closely related to conventional stability conditions used in the industrial organization literature (see for example Dixit, 1986). There is a broad class of functions that satisfies this assumption. In the Appendix, we provide a concrete example of the model based on a specific objective function and demonstrate that our assumptions are not overly restrictive.

We can now state the following proposition:

**Proposition 2** Under condition (6), \(\alpha_i > \alpha_j\) implies \(k_i^* < k_j^*\).

**Proof.** See the Appendix. □

The intuition for this result is straightforward. A regulator that places a greater emphasis on bank profits (that is more "captured") will choose a lower level of regulation than one that places a relatively greater emphasis on the safety of the banking system.

Now consider a centralized regulator that maximizes the joint benefit to both countries but has to impose the same regulatory standards across both countries. Under what conditions will both countries be better off under a centralized regulator? Another way of phrasing this question is whether there exists a common level of regulation, \(k\), such that a central regulator imposing that regulation uniformly on both countries can make both individual regulators better off.

The following proposition demonstrates that the answer to this question is not entirely obvious. This result extends to the asymmetric case our finding that competition among regulators reduces regulatory standards.
Proposition 3 Any common regulatory standard \( k \) such that both regulators prefer it to the outcome under independent regulators will have to be larger than the largest of the two independent levels of regulation.

Proof. Assume WLOG that \( \alpha_1 > \alpha_2 \), or \( k^*_1 < k^*_2 \). Define the reaction function \( \hat{k}_i(k_j) \) as \( \hat{k}_i = \arg \max_k U^i(k, k_j) \), so that \( k^*_i = \hat{k}_i(k^*_j) \). By the envelope theorem, we know that

\[
\frac{\partial U^i}{\partial k_j}(\hat{k}_i(k_j), k_j) > 0.
\]

Now, consider \( k = k^*_2 \). By (7), we have \( U^2(k^*_2, k^*_2) > U^2(k^*_1, k^*_1) \). However, by definition \( U^1(k^*_2, k^*_2) < U^1(k^*_1, k^*_1) \), so that a uniform regulation with \( k = k^*_2 \) would not be accepted by regulator 1. Furthermore, because of (7), for any \( k < k^*_2 \), we have \( U^1(k, k) < U^1(\hat{k}_1(k), k) < U^1(k^*_1, k^*_2) \). Therefore no \( k \in [0, k^*_2] \) will be preferred to the Nash solution by regulator 1.

This result says that, for both countries’ regulators to be better off under centralized regulation, at a minimum the level of regulation chosen by the central regulator must be greater than the level either regulator would choose independently. In order to renounce its independence, a regulator has to be given compensation that comes in the form of a higher level of regulation for its competitor. Then, the choice of each regulator is between the independent solution where the level of regulation is optimal given the opponent’s choice, and the unified regime, where the level of regulation is individually sub-optimal, but the opponent’s level is higher than under independence.

This result is somewhat surprising and puts in question the possibility of obtaining a centralized solution. It seems natural to expect that the creation of a centralized regulator would be a negotiated outcome, and as such would probably involve an intermediate result, such as the centralized regulator choosing the average level of regulation that would be chosen by independent regulators. However, this ignores the result that lack of coordination leads to under regulation. Forming a regulatory union may in fact push regulation above the level imposed by either independent regulator, which the proposition demonstrates is a necessary condition for a centralized regulator to emerge endogenously.

It is worth noting that this result holds for any situation where \( k^*_1 \neq k^*_2 \). The asymmetry need not stem from different degrees of regulatory capture. It may descend from completely different objective functions, as long as the main properties \( \frac{\partial^2 U^i}{\partial k^i_j} < 0 \) and \( \frac{\partial U^i(k, k)}{\partial k_j} > 0 \) hold.

This result helps explain the current situation with multiple regulators. More precisely, we can prove that, under certain conditions, only countries with sufficiently similar degrees of regulatory capture will be able to reach an agreement on a regulatory union. In order to do so, we first need the following preliminary result.
Lemma 1  For any regulatory standard \( k \) such that \( U^1 (k, k) > U^1 (k^*_1, k^*_2) \), we have \( U^2 (k, k) > U^2 (k^*_2, k^*_1) \).

Proof. See the Appendix. ■

This corollary implies that any level of common regulation preferred to the independent solution by regulator 1 would also be preferred by regulator 2. Hence, a sufficient condition for the emergence of a central regulator to be feasible is

\[
\max \limits_k U^1 (k, k) \geq U^1 (k^*_1, k^*_2).
\]

The net benefit to regulator 1 from choosing to renounce independence and merge in a unified regulatory agency can be written as \( U^1 (\tilde{k}, k) - U^1 (k^*_1, k^*_2) \), where \( \tilde{k} = \arg \max \limits_k U^1 (k, k) \).

The next step consists of showing that the net benefit to regulator 1 from choosing to renounce independence and merge in a unified regulatory agency is decreasing in the degree of asymmetry between the two regulators. By Proposition 3, we can restrict attention to values of \( \tilde{k} > k^*_2 \), since otherwise we know that this net benefit to regulator 1 must be negative. To simplify notation, we use the variable \( s \) to represent a measure of the difference between the degrees of regulatory capture as follows: \( s = \frac{\alpha_1 - \alpha_2}{2} \). This allows us to write \( \alpha_1 = \alpha - s \) and \( \alpha_2 = \alpha + s \), where \( \alpha = \frac{\alpha_1 + \alpha_2}{2} \). Note that, as \( 0 \leq \alpha < 1 \), we have \( 0 \leq s < \frac{1}{2} \). For the following result, we need to assume that \( \frac{d \alpha_2}{ds} \geq 0 \), so that the regulator in country 2 would optimally (weakly) increase its level of regulation when the weight it places on the benefit function \( F \) increases (and the difference between \( \alpha_1 \) and \( \alpha_2 \) increases as well).\(^{17}\)

Proposition 4  The maximum net benefit to regulator 1 of a unified regime relative to the solution with independent regulators is decreasing in the difference \( s \).

Proof. See the Appendix. ■

This proposition states that, while very similar countries may benefit from having a centralized regulator, this benefit decreases as the countries become more dissimilar, i.e., as one country’s regulator becomes more captured relative to its peer. This is clear given our earlier discussion. Centralizing regulation eliminates the ability to impose different regulatory standards in each country. However, when countries are very different, optimal regulation may in fact call

\(^{17}\)The condition for this to hold is that

\[
\frac{\partial \tilde{k}_2}{\partial k_1} \leq - \frac{\frac{\partial U^2 (k^*_1, k^*_2)}{\partial s} U^1 (k^*_1, k^*_2)}{\frac{\partial U^1 (k^*_1, k^*_2)}{\partial s} U^2 (k^*_2, k^*_1)}
\]

This puts a restriction on the slope of the reaction functions which is similar, though somewhat stronger, than what is typically required for stability (see Dixit, 1986). This condition is always satisfied if the regulatory instruments are strategic substitutes for the regulators. See the appendix for a simple example that satisfies \( \frac{d \alpha_2}{ds} \geq 0 \).
for unequal standards in regulation. Therefore, as the differences between countries magnify, the cost of this loss of flexibility increases.

Finally, we can state the following corollary. The proof follows as a direct consequence of Proposition 4.

**Corollary 1** If there exists an $\tilde{s} \in \left[0, \frac{1}{2}\right)$ such that $U^1(\tilde{k}, \tilde{k}) - U^1(k_1^*, k_2^*) < 0$, then there will always exist an $\tilde{s} \in [0, \tilde{s})$ such that, for $s \leq \tilde{s}$, there exists a level of common regulation $k$ that both regulators prefer to the Nash solution, and for $s > \tilde{s}$, such level of $k$ does not exist.

This result demonstrates that more symmetric countries are the most likely to end up forming regulatory unions. As long as the differences between the countries are not too large, regulators in these countries can find a mutually agreeable level of regulation such that the benefits of coordination outweigh the loss of flexibility of imposing equal regulatory levels in each country. Figure 1 summarizes this result: for high (low) levels of asymmetry the costs from losing regulatory independence are higher (lower) than the benefits from fully exploiting cross-country externalities in regulation and avoiding regulatory competition.

**V. FINANCIAL INTEGRATION**

The existence of externalities in regulation stems from international financial integration. Identical countries with no domestic banks operating abroad and no foreign bank presence would choose the same level of capital requirement as a centralized regulator. In other words, when financial markets are not integrated, there are no regulatory spillovers across countries and the debate over the superiority of a single centralized regulator versus individual country-level regulators becomes moot.

The discussion in this paper therefore raises a number of issues on the impact of increasing the level of financial integration across countries. One obvious question is whether increased
financial integration may provide for greater incentives for individual countries to join in a regulatory union. This might occur because the externality stemming from the regulatory constraints placed on one country’s banks increases with the degree of financial integration, and can imply that the benefit of forming a regulatory union increases as markets become more integrated. With symmetric banks, this suggests that the difference in the levels of regulation under alternative systems should be increasing in the level of integration. In the context of asymmetric countries, this implies that countries that are more dissimilar may nevertheless choose to form a regulatory union in order to internalize the increased externality.

As a countervailing force, greater financial integration can also mean that the impact of the loss of flexibility from imposing uniform standards is magnified. This can be particularly troublesome for a country that would, under independent regulation, choose a lower regulatory standard. Forcing that country to raise its standard hurts the domestic banking system. This effect will be greater when markets are more integrated. Therefore, which effect dominates is ambiguous.

In this section, we try to determine whether the inefficiency characterizing the Nash equilibrium in the model already presented is increasing in the degree of market integration (cross-border banking). For this, we define a (stylized) measure of market integration as follows. We assume that an increased presence of domestic banks in foreign markets increases the sensitivity of bank profits to regulation. Indeed, the more domestic banks are exposed to banks subject to a different regulatory regime, the more they suffer from an increase in their own regulatory standards, and the more they benefit from an increase in the regulatory standards of competing banks. Formally, this means that, if we define $b$ as the degree of internationalization of domestic banks, we can write $\frac{\partial^2 \Pi(k_i, k_j)}{\partial k_i \partial b} < 0$, and $\frac{\partial^2 \Pi(k_i, k_j)}{\partial k_j \partial b} > 0$. By analogy, an increased presence of foreign banks on the domestic market has similar effects.

On the stability side, greater penetration by foreign banks also affects the effectiveness of domestic regulation. More precisely, the regulator's ability to increase the stability of its own banking system decreases with the market share of foreign banks, which means $\frac{\partial^2 F(k_i, k_j)}{\partial k_i \partial b} < 0$, and $\frac{\partial^2 F(k_i, k_j)}{\partial k_j \partial b} > 0$. Note that we use the same parameter, $b$, to represent the foreign activities of domestic banks as well as the market share of foreign banks in domestic markets. While in principle one could separate out these effects, their implications are similar so that we focus on only one measure of integration.

First, we show that the marginal benefit of having centralized regulation is increasing in the level of integration. Using again equation (5), the first order condition for a centralized regulator evaluated at the Nash equilibrium regulation levels,

$$\alpha \frac{\partial \Pi(k_i^*, k_j^*)}{\partial k_j} + (1 - \alpha) \frac{\partial F(k_i^*, k_j^*)}{\partial k_j},$$

we see that both terms, which are both positive, are increasing in $b$. Therefore, the benefit
of increasing the regulatory level is greater when markets are more integrated.\footnote{This need not imply, however, that the difference between the two levels of regulation, $k^* - k^*_i$, is increasing with the level of integration. This depends on the specific functional forms for $\pi$ and $F$.} Under some regularity conditions, this implies that the difference $U^C - U^I$ also increases with the degree of financial integration, where $U^C$ represents the equilibrium utility of a regulator under a centralized system, and $U^I$ that under an independent system.

Furthermore, it can be shown that greater integration actually \textit{lowers} the level of regulation for the case of independent regulators. This occurs because, with greater integration, not only does increased regulation hurt the domestic bank more, but the regulator is less able to control the stability of the system in his own country, so that the regulatory benefit is lower. This can be seen directly from the first order conditions for independent regulators, equation (3), which is reproduced here.

$$\alpha \frac{\partial \Pi^I(k_i, k_j)}{\partial k_i} + (1 - \alpha) \frac{\partial F^I(k_i, k_j)}{\partial k_i} = 0$$

The first term is negative and decreasing in $b$ (becoming more negative), while the second term is positive and decreasing in $b$. Since, by assumption, $U$ is increasing and concave, a lower equilibrium level of regulation, $k^*_i$, will be chosen by the regulator for country $i$.\footnote{Some observers have suggested that allowing domestic banks to compete with foreign banks imposes discipline on the domestic banking system. This effect is ignored here since we are focusing on the marginal impact of regulation under the maintained hypothesis that some measure of financial integration is already in effect. Therefore, the disciplinary impact should already be observed on the level of profits and (possibly) on the level of stability of the banking system.}

To show that the net benefit of coordination becomes greater when countries are more integrated, we need to show that $\frac{d[U^C - U^I]}{db} > 0$. By the envelope theorem,

$$\frac{d[U^C - U^I]}{db} = \frac{\partial U^C}{\partial b} - \frac{\partial U^I}{\partial b} - \frac{\partial U^C}{\partial k_i} \frac{\partial k_i}{\partial b}.$$  

Unfortunately, this expression cannot be unambiguously signed without imposing some further condition. The last term, $\frac{\partial U^C}{\partial k_i} \frac{\partial k_i}{\partial b}$, is negative, which goes in the right direction. We then only need for the difference of the first two terms, $\frac{\partial U^C}{\partial b} - \frac{\partial U^I}{\partial b}$, to not be “too” negative. One condition that guarantees that the entire expression will be positive is\footnote{This condition, while sufficient, is clearly much too strong and is not necessary. A much weaker condition can be obtained by imposing a restriction on the cross partial of $U^I$ with respect to $k_i$, $k_j$, and $b$.}

$$\frac{\partial^2 U^I(k_i, k_j)}{\partial k_i \partial b} + \frac{\partial^2 U^I(k_i, k_j)}{\partial k_j \partial b} > 0, \quad \forall k_j \geq k_i$$

This condition is simply that the marginal impact of increased domestic regulation be no more sensitive to increased integration than the impact of increased foreign regulation. With this condition, it becomes clear that the benefit of integration increases as countries become more integrated.
The question that remains to be answered is whether the increase in the net benefit of centralizing regulation also implies that the incentive to form a regulatory union increases. While the analysis of this section has been applied only to the case of symmetric countries, it can be extended to the case of asymmetric countries covered in the previous section. As argued at the beginning of this section, however, the answer is not clear cut. Increasing the level of financial integration also amplifies the negative impact on the domestic banking system of imposing uniform standards, which may make coordination less likely to be acceptable to both parties.

What can be shown, however, is that, in a two-country model, the benefit to the more regulated (less captured) country will always be increasing in the level of integration. For the less regulated (more captured) country, coordination will be more beneficial as long as the impact on bank profits of increasing regulation does not overwhelm the benefits on stability (and the internalization of the externality).

VI. Conclusions

This paper has shown that competition among regulators reduces regulatory standards relative to a centralized solution. In addition, it has proven, under some broad regularity conditions, that centralizing regulation is more likely to be beneficial for countries that are homogeneous and financially integrated. The implications of this paper are relevant for regulatory policy in an increasingly integrated world. In particular, the outcome of our model suggests that as financial integration develops and the cross-border activities of banks become more important there will be an increasing number of countries who will find regulatory unions attractive. At the same time, the potential inefficiencies from maintaining a system of national regulators will also become more relevant.

A question that has not been analyzed, however, is whether a regulatory union is actually likely to emerge, even if one is feasible. The analysis in the paper focused on a situation where coordinating regulatory policy leads to a Pareto improvement. However, what was ignored is that the formation of a central regulatory agency would probably be the outcome of negotiation between all the relevant parties, particularly the individual regulators. This is an important issue in this paper, since our model demonstrated that in order for centralized regulation to be Pareto improving it must raise the level of regulation of all banking systems involved. We may expect that the inherent tensions of a negotiated outcome may push the centralized institution to choose a level of regulation somewhere between the levels that would be chosen by independent regulators. Even if the outcome could be successfully negotiated, it is still possible that after the centralized regulator is formed there might be strong pressure put on it to choose a moderate level of regulation. The study of this issue is beyond the scope of this paper.

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21 For example, the negotiated outcome may lead to a centralized regulator that assigns a weight \( \alpha = \frac{\alpha_1 + \alpha_2}{2} \) to bank profits. There may then be situations where the level of regulation chosen by this centralized regulator is below \( k_2^* \), even if a level exists greater than \( k_2^* \) that would make both countries better off.
The analysis in this paper can be extended in a number of directions. One natural extension is to allow banks to change the location of their headquarters, and hence, implicitly allow them to choose their regulator. Under those circumstances, the race-to-the-bottom effect of competition described in this paper would be even stronger as regulators would compete to prevent banks from leaving their regulatory domain, and our conclusions would most likely be, if anything, reinforced.

On the other hand, one extension that would radically change our welfare analysis is to consider non-benevolent regulators. In that case, the welfare consequences of competition could be radically different if one took into account beneficial effects like those analyzed by Laffont and Martimort (1999) and Kane (1984). Finally, this paper abstracts from any cost involved in the transition from a decentralized to a centralized regulatory system, like for example the potential loss of country specific information and expertise. These costs, although likely to be transitory, if large enough would help explain the reluctance of countries with relatively similar characteristics to surrender national authority to a centralized regulator. We do not believe that taking into account transition costs would dramatically change the main conclusions in this paper.
EXAMPLE

While in the main text the analysis was presented in a very general setting, various regularity conditions were imposed in order to obtain clear-cut results. In this appendix, we provide a concrete example of an objective function that respects those restrictions.

Assume that \( \Pi^t = -bk_t^2 + 1 + bk_{j} \) and that \( F^t = 1 - bk_t^2 + bk_{i} + \frac{1}{2}(-bk_j^2 + bk_{j}) \). These are just standard quadratic objective functions. Note that both \( \Pi \) and \( F \) satisfy the properties we require in the general model: \( \frac{\partial \Pi}{\partial k_i} < 0, \frac{\partial \Pi}{\partial k_j} < 0, \frac{\partial F}{\partial k_i} > 0, \frac{\partial F}{\partial k_j} > 0 \) for \( 0 < k_i, k_j < \frac{1}{2} \). They also satisfy our conditions on the cross partials with regard to financial integration: \( \frac{\partial^2 \Pi(k_i,k_j)}{\partial k_i \partial k_j} < 0 \), \( \frac{\partial^2 F(k_i,k_j)}{\partial k_i \partial k_j} > 0 \), and \( \frac{\partial^2 F(k_i,k_j)}{\partial k_i \partial k_j} < 0 \), for the same values of \( k_i \) and \( k_j \).

The independent regulator’s problem can be written as

\[
\max_{k_i} \alpha \left(-bk_i^2 + 1 + bk_j \right) + (1 - \alpha) \left(1 - bk_i^2 + bk_i + \frac{1}{2}(-bk_j^2 + bk_j) \right).
\]

This yields the first order condition

\[-2\alpha k_i + (1 - \alpha)(-2k_i + 1) = 0,
\]

with solution

\[k^*_i = \frac{1 - \alpha}{2}.
\]

Note that this particular case is easy to solve since strategic interaction does not play any role.

To solve the case for the centralized regulator, we maximize the equation above, subject to the constraint that \( k_i = k_j \).

\[
\max_k \alpha \left(-bk^2 + 1 + bk \right) + (1 - \alpha) \left(1 - bk^2 + bk + \frac{1}{2}(-bk^2 + bk) \right).
\]

This yields the first order condition

\[-2\alpha k + \alpha + (1 - \alpha) \left(-2k + 1 - k + \frac{1}{2} \right) = 0,
\]

and the solution

\[k^C = \frac{1}{2}.
\]

This confirms our first result: \( k^C > k^*_i \), so that independent regulators under-regulate relative to the centralized solution. This also trivially confirms that, in the asymmetric case, a centralized regulator would impose a higher capital requirement than the domestic regulator with the lowest \( \alpha \).

By substituting the equilibrium values in the objective functions for the centralized and the
independent regulators, we obtain

\[ U(k_i^*) = -\frac{1}{8} \alpha b + \frac{1}{8} b \alpha^3 + 1 + \frac{3}{8} b - \frac{3}{8} b \alpha^2, \]

\[ U(k^C) = -\frac{1}{8} \alpha b + 1 + \frac{3}{8} b. \]

Hence, we can write

\[ U(k^C) - U(k_i^*) = \frac{1}{8} b \alpha^2 (3 - \alpha) > 0, \]

that is increasing in the degree of financial integration, \( b \), confirming our result from section 4.

Finally, for the asymmetric case with \( \alpha_i \neq \alpha_j \), consider what happens if we let regulator \( i \) choose the level of regulation for both countries. In that case the problem becomes

\[ \max_k \alpha_i \left( -bk^2 + 1 + bk \right) + (1 - \alpha_i) \left( 1 - bk^2 + bk + \frac{1}{2}(-bk^2 + bk) \right). \]

This still has solution \( \hat{k} = \frac{1}{2} \), and

\[ U_i(k) - U_i(k_i^*, k_j^*) = \left( \alpha_j - \frac{1}{2} \alpha_i \right) \frac{1}{2} \alpha_i b + \frac{1}{8} b \alpha^2 (1 - \alpha_i). \]

This difference is always positive for \( \alpha_i < \alpha_j \). That is, the “less captive” regulator always benefits from being able to coordinate the actions of both regulators. For the more captive regulator, the case where \( \alpha_i > \alpha_j \), the expression is negative if

\[ \alpha_j < \frac{1}{(1 - \alpha_i)} \left( \sqrt{(6 - 2\alpha_i)} - 2 \right) \alpha_i. \]

In addition, if we express

\[ \alpha_i = \alpha + s, \]
\[ \alpha_j = \alpha - s, \]

we can write

\[ \frac{d}{ds} \left( U_i(k) - U_i(k_i^*, k_j^*) \right) = \frac{1}{4} \left( b \alpha \left( -3 + \frac{1}{2} \alpha + s \right) - bs \left( 5 + \frac{3}{2} s \right) \right) < 0, \]

which demonstrates that the benefits of integration shrink as countries become more dissimilar.
Proofs

Proof of Proposition 2: Start from a symmetric equilibrium with \( \alpha_1 = \alpha_2 \) and \( k_1^* = k_2^* \), and consider a mean preserving spread over \( \alpha \), so that \( \alpha_1 = \alpha + s \) and \( \alpha_2 = \alpha - s \). We need to show that \( \frac{d(k_1^* - k_2^*)}{ds} < 0 \). Define

\[
\begin{align*}
a_i &= \frac{\partial^2 U^i}{\partial k_i^2} \\
b_i &= \frac{\partial^2 U^i}{\partial k_i \partial k_j} \\
\mu_{\alpha_i} &= \frac{\partial^2 U^i}{\partial k_i \partial \alpha_i}
\end{align*}
\]

Note that \( a_i \) and \( \mu_{\alpha_i} \) are both negative, while the sign of \( b_i \) can be either positive or negative and determines the slope of the reaction functions. Totally differentiating the first order conditions and rearranging, we have

\[
\begin{bmatrix} a_1 & b_1 \\
a_2 & b_2 \end{bmatrix} \begin{bmatrix} dk_1^* \\
dk_2^* \end{bmatrix} = - \begin{bmatrix} \mu_{\alpha_1} \frac{\partial \mu_{\alpha_1}}{\partial s} ds \\
\mu_{\alpha_2} \frac{\partial \mu_{\alpha_2}}{\partial s} ds \end{bmatrix},
\]

that solving gives

\[
\begin{bmatrix} dk_1^* \\
dk_2^* \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -a_2 & b_1 \\
b_2 & -a_1 \end{bmatrix} \begin{bmatrix} \mu_{\alpha_1} ds \\
-\mu_{\alpha_2} ds \end{bmatrix},
\]

where \( \Delta \) is the determinant of the matrix. Now, we can write

\[
\frac{d(k_1^* - k_2^*)}{ds} = -\frac{1}{\Delta} [(a_2 + b_2) \mu_{\alpha_1} + (a_1 + b_1) \mu_{\alpha_2}] ds.
\]

Condition (6) implies \( \Delta > 0 \), and together with \( a_1 < 0, a_2 < 0 \) implies that \( a_i + b_i < 0 \). Then, as \( \mu_{\alpha_1} < 0, \mu_{\alpha_2} < 0 \), we have \( \frac{d(k_1^* - k_2^*)}{ds} < 0. \)

Proof of Lemma 1: Assume \( U^1(k, k) > U^1(k_1^*, k_2^*) \). As \( k_1^* < k_2^* \), by the envelope theorem we know that \( U^1(k_1^*, k_2^*) > U^1(k(k_1^*), k_2^*) \), and by definition \( U^1(k(k_1^*), k_1^*) > U^1(k_2^*, k_1^*) \). Hence, we can write

\[
U^1(k, k) - U^1(k_1^*, k_2^*) = \alpha_1 [\Pi(k, k) - \Pi(k_2^*, k_1^*)] + (1 - \alpha_1) [F(k, k) - F(k_2^*, k_1^*)] > 0.
\]

As \( k > k_2^* \) (by Proposition 3), we know that the second term, \( F(k, k) - F(k_2^*, k_1^*) \), has to be positive. Given \( \alpha_1 > \alpha_2 \), the equivalent expression with \( \alpha_2 \) instead of \( \alpha_1 \) must also be positive, since we are putting more weight on the component we know is positive. Therefore \( U^2(k, k) > U^2(k_2^*, k_1^*) \). \( \square \)

Proof of Proposition 4: Our goal is to show that the following expression is negative.

\[
\frac{d}{ds} \left[ U^1(\tilde{k}, \tilde{k}) - U^1(k_1^*, k_2^*) \right]
\]
This can be written as (the subscripts on the utility functions refer to the first derivative with respect to that argument)

\[
U_1 \frac{\partial \bar{k}}{\partial s} + U_2 \frac{\partial \bar{k}}{\partial s} + \frac{\partial U^1}{\partial k_1} \bigg|_{\bar{k}, \bar{k}} - U_1 \left[ \frac{\partial k_1^*}{\partial s} + \frac{\partial k_1}{\partial k_2} \frac{\partial k_2^*}{\partial s} \right] \\
- U_2 \left[ \frac{\partial k_2^*}{\partial s} + \frac{\partial \hat{k}_2}{\partial k_1} \frac{\partial k_1^*}{\partial s} \right] - \frac{\partial U^1}{\partial s} \bigg|_{k_1^*, k_2^*}
\]

Applying the envelope theorem, this reduces to

\[
\frac{\partial U^1}{\partial s} \bigg|_{\bar{k}, \bar{k}} - U_2 \left[ \frac{\partial k_2^*}{\partial s} + \frac{\partial \hat{k}_2}{\partial k_1} \frac{\partial k_1^*}{\partial s} \right] - \frac{\partial U^1}{\partial s} \bigg|_{k_1^*, k_2^*}
\]  \hspace{1cm} (A-1)

Now, remember that

\[
\frac{\partial U^1}{\partial s} \bigg|_{\bar{k}, \bar{k}} = \Pi(\bar{k}, \bar{k}) - F(\bar{k}, \bar{k})
\]

\[
\frac{\partial U^1}{\partial s} \bigg|_{k_1^*, k_2^*} = \Pi(k_1^*, k_2^*) - F(k_1^*, k_2^*)
\]

So that the difference is

\[
\frac{\partial U^1}{\partial s} \bigg|_{\bar{k}, \bar{k}} - \frac{\partial U^1}{\partial s} \bigg|_{k_1^*, k_2^*} = \left[ \Pi(\bar{k}, \bar{k}) - \Pi(k_1^*, k_2^*) \right] + \left[ F(k_1^*, k_2^*) - F(\bar{k}, \bar{k}) \right]
\]

Since, by assumption, \( \bar{k} > k_2^* \), the second term must be negative. The first term must also be negative since \( \Pi \) is concave and is decreasing in its first argument.

What remains is to show that the second term in equation (A-1) is non-negative so that once it is subtracted from the rest, the whole expression is negative. First, it is clear that \( U_2^1 \) is positive. To show that the whole thing is positive, it is then sufficient to show that \( \frac{\partial k_2^*}{\partial s} + \frac{\partial \hat{k}_2}{\partial k_1} \frac{\partial k_1^*}{\partial s} \) is positive as well. But this is just \( \frac{\partial k_2^*}{\partial s} \), which is (weakly) positive by assumption. Therefore, the net benefit is decreasing in \( s \) for values of \( \bar{k} > k_2^* \), as desired. \( \square \)
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