Optimal Central Bank Conservatism and Monopoly Trade Unions

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Abstract

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The "conservative central banker" has come under attack recently. On the basis of models in which there is explicit interaction between trade union behavior and monetary policy, it has been argued that if 'trade unions' are averse to inflation, welfare will be lower with a conservative than with a liberal central bank. We reframe this discussion in a standard trade union model. We show that the case against the conservative central banker rests exclusively on the assumption of a strictly nominal outside option (for instance, unemployment benefits) for the union. There is no welfare gain associated with making the central bank less conservative than society, however, if the outside option is in real terms. As the nominal components of the trade union's outside option are mainly public transfers, we also show that the conservative central banker is always optimal if the government can choose the level of nominal unemployment benefits as well as the degree of central bank conservatism.

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I. INTRODUCTION

The "conservative central banker" has come under attack lately. In an important paper, Rogoff (1985) suggested reducing the inflationary bias of monetary policy by delegating monetary policy to an independent and conservative central bank, which cares less about unemployment than the government does. Changing the preferences would reduce the expected rate of inflation and thus the actual rate. While other solutions to this problem have been suggested (see, among others, Lohmann 1992, Walsh 1995), the conservative central banker remains perhaps the most popular point of reference regarding institutional remedies against inflation suggested by economists. And indeed, a numerous and still growing empirical literature often finds a negative relationship between central bank independence and inflation across countries and time (for surveys, see e.g., Eijffinger and de Haan 1996, Berger and others 2001).

Recently, however, a series of papers has questioned the theoretical foundation of the conservative central banker solution. One strand links labor market reform with monetary policy. The basic argument is that while labor market reforms might be politically costly, they will help to lower the inflationary bias (e.g., Calmfors 1998). A second strand tries to endogenize the inflation bias by allowing for direct interaction of non-atomistic trade unions and monetary policy (Cubitt 1992, Grüner and Hefeker 1999, Cukierman and Lippi 1999, Guzzo and Velasco 1999, Lawler 2000). The argument builds on a non-atomistic trade union model of the labor market, where nominal wage setters take into account the reaction of the central bank to the implied real wage. What sets these models apart from the standard labor market literature is that they assume that trade unions are "inflation averse," i.e., that their target functions include not only some real wage and employment targets but also costs of inflation. The assumption is often justified by non-indexed nominal components in a trade union's outside option.

The effect of this change can be quite dramatic: because trade unions dislike inflation, they moderate their wage claims to limit the central bank's incentives for an inflationary policy. This has two important consequences for the traditional monetary policy model. First, the behavior of inflation-averse trade unions establishes a direct link between central bank characteristics and real labor market outcomes—a link that does not exist in the standard framework. Second, a more conservative central bank will prefer tolerating higher unemployment to increasing inflation. The more nominal wage increases raise inflation, the more moderate an inflation-averse trade union will be. Hence, a liberal central bank rather than a conservative central bank will induce trade unions to moderate wages. In fact, there even seems to be a case for an "ultra-liberal" or "populist central banker" (see Guzzo and Velasco 1999, Cukierman and Lippi 1999, Lippi 1999a, and Berger and others 2001 for a survey).

The case against the conservative central banker is clearly strongest in a single trade union model. Obviously, a very small or atomistic trade union will disregard any effect wages have on inflation. Also, as stressed by Lippi (1999b) and Coricelli and others (2000), in the intermediate case of multiple but large trade unions, the effect of inflation on the
relative real wage set by a trade union might produce a "competition effect" that qualifies the case for a liberal central banker. An increase in the nominal wage—ceteris paribus—implies a higher real wage for all trade unions in the economy and thus increases the outside option for the particular trade union. Given the other trade unions' nominal wage rates, the trade union will demand higher nominal wages, which, in turn, will lead to a lower level of production and labor demand in the overall economy from the perspective of the individual union. The more conservative the central bank is, the larger the moderating effect of this mechanism will be.

It would seem, however, that the argument against the conservative central banker is still very much alive in the "special case" of a monopoly trade union setting, where such moderating effects are absent. Clearly, the result here hinges on the assumption of inflation aversion or "money illusion" (cf. Soskice and Iversen 2000, p.266) on the trade union's side. Most of the literature so far relies on an exogenous and ad hoc specification of the trade union's target function that simply assumes that trade union utility is decreasing in deviations of inflation from a target level of zero. Such a specification is clearly at odds with more standard models of trade union behavior (compare Oswald 1982). Therefore, the question arises as to how the inflation aversion of a monopoly trade union could come about.

Probably the most natural way to model the dislike of inflation is by introducing nominal income components in a standard trade union optimization problem. To follow up on this notion, we will contrast the behavior of a trade union with an outside option defined in real terms with the behavior of a trade union with a nominal outside option. Building on a simple model (presented in Section II) of the goods and labor market with decreasing returns to scale in which the price level is controlled by the central bank, we discuss how inflation aversion affects trade unions by looking at two benchmark cases in Section III. In Section IV we then show that the conservative-central-banker result is socially optimal when the outside option is defined in real terms. The opposite might be true, however, when the trade union's...

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2 Another moderating effect of central bank conservatism is discussed in Soskice and Iversen (2000) and Coricelli and others (2000). In their monopolistic competition frameworks, a conservative central bank will run a less accommodative policy when unions raise nominal wages and, consequently, firms' prices. This policy change decreases employment and induces a more cautious wage-setting behavior. This moderating effect is increasing in central bank conservatism. Lawler (2000), in addition, shows that in a stochastic environment, central banks should not be ultra-liberal because they would produce excessively high inflation variance.

3 Note that this assumption is needed for making the case for the liberal central banker. The real non-neutrality of monetary policy as such does not depend on the trade union's aversion to inflation (Lippi 1999b, Soskice and Iversen 2000). For examples from the earlier literature that already derived the non-neutrality result from the interaction of monopoly trade unions with (fiscal as well as monetary) policy regimes, see Drifill (1985) and Jensen (1993).
outside option, for instance, the unemployment benefit payments, is defined in nominal terms. In this case, the trade union will enforce a higher real wage if the real outside option faced by its members improves due to a more conservative monetary policy. Section V generalizes these results and discusses the extent to which the latter result is a consequence of restricting the government's set of policy instruments. We show that the Rogoff solution always prevails if the government can choose the level of unemployment benefits as well as the degree of central bank conservatism and if there is a minimum real living standard. Section VI concludes.

II. MODEL

The model considers four stages. In the first stage the government chooses the degree of conservatism $c$ of the central bank, i.e., the weight the central bank gives to inflation relative to unemployment in its objective function. In Section V we will, in addition, allow the government to choose unemployment benefits. In the second stage we assume a single monopolistic trade union in the economy, which maximizes the income of its members by fixing the wage rate $w$.

Figure 1: Sequence of the Model

<table>
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<th>Stage 1</th>
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<td>Government</td>
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<td>sets price ($p$)</td>
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<td>central bank</td>
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The focus on a single trade union allows us to identify the assumptions underlying the unambiguous result that a benevolent government should choose an “ultra-liberal” central bank. Given the nominal wage rate and the predetermined degree of conservatism in the third stage, the central bank then chooses the price level and therefore the inflation rate. In the fourth and final stage, profit-maximizing firms determine output levels and employment levels. The sequencing is illustrated in Figure 1. The model is solved in reverse order.

Stage 4: Firms

In Stage 4, profit-maximizing firms decide upon output and employment levels. To keep the model as simple as possible, we focus on the strongest case in favor of an “ultra-liberal” central bank: centralized wage setting. Therefore, the economy comprises one sector only in which firms produce a consumer good $Y$ with a Cobb-Douglas technology

$$Y = AN^\alpha,$$
with \( A > 0 \) and \( 0 < \alpha < 1 \) being parameters of production and \( N \) denoting labor demand. It is convenient to express \( N \) as a percentage of the total labor supply \( M \). So employment is \( n = N/M \), with \( n \in (0,1) \).

Firms are price takers. Labor demand thus becomes

\[
n = \left( \frac{d \cdot p}{w} \right)^{\delta},
\]

with \( d = \alpha A \) and \( \delta = 1/(1-\alpha) > 1 \). Without loss of generality, we can normalize \( A = 1/\alpha \) so that \( d = 1 \). Unemployment is \( u = 1 - n \). Note that a real wage of \( w = p \) would ensure full employment (or zero unemployment) in the economy.

Output prices and the nominal wage rate are determined by the central bank and the trade union, respectively. To see the impact their decisions have on employment, note that employment is decreasing in the nominal wage

\[
(1) \quad n_w = -\frac{\delta}{w} n < 0,
\]

but increasing in the output price

\[
(2) \quad n_p = \frac{\delta}{p} n = -\frac{w}{p} n_w.
\]

We use sub-indices to indicate partial derivatives.

**Stage 3: Central Bank**

In Stage 3 the central bank determines the price level, taking into account the nominal wage set by the trade union in Stage 2. The central bank’s loss function has the standard quadratic form

\[
L = 0.5 \cdot u^2 + 0.5 \cdot c \pi^2,
\]

where \( \pi \) is the rate of inflation and \( c > 0 \) is the weight attached to the inflation target. (Note that an ultra-liberal central bank with \( c = 0 \) would always set \( w = p \).) The parameter \( c \) measures the bank’s degree of conservatism. The central bank aims at minimizing deviations of average unemployment and inflation from their target levels set to zero in both cases. It is convenient to rewrite \( \pi = (p_t - p_{t-1})/p_{t-1} \). Normalizing \( p_{t-1} = 1 \) and dropping the time index this alters the appearance of the loss function to

\[
(3) \quad L = 0.5 \cdot u^2 + 0.5 \cdot c (p - 1)^2.
\]
The central bank, which has complete control over the output price level, will set \( p \) (and thus inflation) to minimize (3). It takes into account the labor demand behavior of firms, but it will take nominal wages set by the trade unions in stage 2 as given. That is, the central bank acts as Stackelberg leader vis-à-vis firms, but is pricetaker (Stackelberg follower) vis-à-vis trade unions. Taking the derivative of (3) with respect to \( p \) yields the following first order condition

\[
L_p = -\frac{\delta}{p} n (1 - n) + c (p - 1) = 0 .
\]

(4)

The central bank will set \( p \) in such a way that the marginal benefit of a higher price level (first term) equals the marginal cost (last term). While the latter is strictly increasing in \( p \), the marginal benefit, however, is hump-shaped in \( p \) with a maximum at \( n = (\delta - 1)/(2\delta - 1) \). The reason is that a higher price level changes both the weight attached to a further change in the unemployment level and the impact a change in the price level has on unemployment. The first effect is due to the quadratic nature of the loss function, while the second effect stems from the decreasing marginal returns of the Cobb-Douglas technology. While both effects decline in \( p \) and (via (1)) \( n \), the concavity of the production function ensures that the hump-shape disappears for average employment levels larger than \( n > (\delta - 1)/(2\delta - 1) \). This condition is always fulfilled if average employment \( n > 1/2 \). We will realistically assume that the employment level is beyond that threshold in what follows. For \( 0.5 < n < 1 \) we have

\[
L_{pp} = \frac{1}{p} \left[ \frac{\delta^2}{p} (2n - 1)n + c(2p - 1) \right] > 0 ,
\]

(5a)

\[
L_{pc} = (p - 1) \geq 0 ,
\]

(5b)

and

\[
L_{pw} = -\frac{\delta^2}{pw} (2n - 1)n < 0 .
\]

(5c)

Thus comparative statics yield

\[
p_c = -\frac{L_{pc}}{L_{pp}} \leq 0 ,
\]

(6)

i.e., the more conservative the central bank is (for all \( p > 1 \)), the lower the price level will be. With respect to the nominal wage, we obtain
(7) \[ p_w = \frac{L_{pw}}{L_{pp}} > 0 , \]

i.e., the price level increases in the nominal wage.

A convenient way to summarize the behavior of the central bank as described in equation (4) is to look at the nominal wage elasticity of the price level. Differentiating the first-order condition of the central bank with regard to \( w \) and \( p \) and rearranging yields

(8) \[ \theta \equiv p_w \frac{w}{p} = \frac{\delta^2 (2n-1)n}{\delta^2 (2n-1)n + cp(2p-1)} . \]

The wage elasticity of the price level is less than unity if the central bank is concerned about inflation because, in this case, it is not willing to accommodate a nominal wage increase completely. Instead, it is willing to accept some unemployment in order to keep the inflation rate low. This follows directly from the concavity of the utility function.

**Stage 2: Trade Union**

In Stage 2 the trade union is fixing the nominal wage rate to maximize a (utilitarian) welfare function incorporating the disposable real income of employed and unemployed members

(9) \[ V = n \frac{w}{p} + (1 - n) \left( b^{real} + \frac{b^{nom}}{p} \right) . \]

The variable \( b^{real} \) can be interpreted as real unemployment benefits or real opportunity costs of labor supply. An example of the latter would be the real income in the shadow economy forgone by entering the labor market. More importantly, Blanchard and Katz (1999) argue that at least part of public unemployment benefits might be defined in real terms, too. To the extent that unemployment benefits are instead fixed in nominal terms, they are covered by the term \( b^{nom} \). Other than the distinction between nominal and real outside options, the model is quite standard in the labor market literature (see Oswald 1982). Note that we abstain, for instance, from the introduction of nominal wealth in the trade union’s target function. One reason is that the economic consequences of nominal wealth and a nominal outside option supplied through, say, a public unemployment benefit scheme are similar. Moreover, the role publicly provided nominal unemployment schemes play in the
trade union’s decision will probably dominate possible nominal wealth effects. On a more fundamental note, one could add that, in a model based on rational choice, the very existence of nominal, non-indexed wealth would have to be justified either by non-rational behavior or an incomplete markets argument.\footnote{This might be the case because the stock of wealth is too low to matter for many trade union members or because state-supplied nominal benefits are means tested, making possibly existing private assets de facto a part of the public support scheme.}

The wage rate cannot fall short of the outside option, i.e., \( w \geq b^{\text{real}} \frac{p}{p} + b^{\text{nom}} \), because otherwise, trade union members would refuse to work and prefer being unemployed. To restrict unemployment to occurrences of involuntary unemployment, in the sense that at the given wage rate unemployed workers strictly prefer to work, we assume that full employment is reached at a real wage rate that exceeds the real outside option, i.e., \( w/p = 1 \geq b^{\text{real}} + b^{\text{nom}}/p \).

This condition rules out voluntary unemployment, which would also occur in a perfect Walrasian labor market, and thus should not be part of the loss function of either the government or the central bank.

The trade union maximizes (9) taking unemployment benefits and the degree of conservatism as given. The trade union takes into account the reaction of the central bank and, by extension, of labor demand of the firms, when setting its nominal wage rate \( w \). Hence, in line with the standard literature, the trade union is acting as Stackelberg leader vis-à-vis the central bank and the firms. An inner solution requires that \( w \geq b^{\text{real}} p + b^{\text{nom}} \).

**Stage 1: Government**

In Stage 1 the government determines the degree of conservatism \( c \) and (see Section V) the unemployment benefits \( b \). Conceptually, the existence of Stage 1 allows us to consider the effects of variations in \( c \) or \( b \) on the equilibrium of the model. We do not explicitly consider the objective function of the government at this point, but will return to the government’s decision below.

**Solving for the equilibrium**

In order to describe the equilibrium where both the trade union and the central bank have made their optimal decisions given the degree of conservatism \( c \), we need to take

\footnote{A trade union consisting of members capable of calculating the optimal wage-setting policy—including the inflation implied by any nominal wage rate set—in a multi-stage, multi-actor model is difficult to bring in line with the existence of non-indexed wealth. After all, a rational trade union member will not invest in, say, a nominal government bond without taking into account expected inflation.}
explicit account of the first-order conditions of both the central bank and the trade union. Using the partial characteristics of the two first-order conditions we get the following linear equation system

\[
\begin{vmatrix}
0 & V_{ww} \\
L_{pp} & L_{pw}
\end{vmatrix} \frac{dp}{dc} \begin{vmatrix}
V_{ww} \\
L_{pc}
\end{vmatrix} = - \begin{vmatrix}
V_{wc} \\
L_{pc}
\end{vmatrix},
\]

which combines the second-order conditions for the central bank and the trade union. Using the signs of the partial derivatives in (10), as derived before, we find that the system has a positive determinant \( D = -L_{pp}V_{ww} > 0 \). Applying Cramer’s Rule, we obtain

\[
\frac{dp}{dc} = \frac{1}{D} \left( \frac{-V_{wc}L_{pw} + L_{pc}V_{ww}}{V_{ww}L_{pp} - L_{pc}} \right) = \frac{V_{wc}L_{pw} - L_{pc}}{V_{ww}L_{pp}} = p_w w_c + p_c.
\]

The result implies that the equilibrium price change due to an increase in the degree of the central bank’s conservatism, \( dp/dc \), is the result of two—possibly opposing—effects. On the one hand, there is the direct effect on central bank behavior induced by the changed weights in its loss function. This unambiguously tends to lower the price level, i.e., \( p_c < 0 \). On the other hand, there is the implied change in the wage rate, \( w_c \), and its indirect effect on the central bank’s price setting. From the optimal reaction of the central bank we know that higher wages are unambiguously translated into higher prices, i.e., that \( p_w > 0 \). However, the change in the nominal wage rate depends on trade union behavior and might go either way. If the trade union decreases the nominal wage when the central bank becomes more conservative, \( w_c < 0 \), it follows that the observed equilibrium price level is decreasing as well: \( dp/dc < 0 \). By contrast, if the trade union increases the nominal wage when the central bank becomes more conservative, \( w_c > 0 \), the overall price decrease becomes smaller or might even turn into an equilibrium price increase. Appendix I sheds some further light on the relation between \( c \) and \( p \). It can be shown that the—intuitively appealing—negative relation \( dp/dc < 0 \) always holds if the outside option were defined strictly in real terms. What is more, even if the outside option was defined in nominal terms only, \( dp/dc < 0 \) would prevail as long as inflation is not too high, i.e., \( p < (\delta - (1 - n))/(\delta - 1) \) (see Appendix I). For instance, in the case of a strictly nominal outside option, if the labor share is \( \alpha = 2/3 \) and the unemployment rate is at 20 percent, a moderate inflation rate below 40 percent would still guarantee \( dp/dc < 0 \). Note that inflation could be even higher if part of the outside option were defined in real terms. This result is independent of the level of \( c \). To summarize

**PROPOSITION 1 (inflation):** An increase in the central bank’s conservatism decreases the equilibrium price level (or inflation) if and only if the direct effect on the central bank’s price-setting behavior is not overcompensated by the incentive to respond to trade union behavior, i.e., if and only if \( w_c p_w < -p_c \). This
is always the case for moderate price levels (or moderate levels of inflation) 
\[ p < \frac{(\delta - (1 - n))}{(\delta - 1)} < \frac{(\delta - 0.5)}{(\delta - 1)}. \]

In what follows, we focus on the normal reaction that an increase in the central bank’s 
conservatism decreases the equilibrium price level. This seems to be well in line with the 
inflation rates and unemployment rates we observe in unionized countries as well as with the 
stylized fact that an increase in central bank conservatism empirically reduces (rather than 
increases) inflation both across time and countries (Eijffinger and de Haan 1996, Berger and 
others 2001).

But how will the trade union react to a variation in the central bank’s conservatism? 
The change in the nominal wage rate can also be derived from (10):

\[ \frac{dw}{dc} = -\frac{V_{wc}}{V_{ww}} = w_c \begin{cases} > 0 & \text{if } \begin{bmatrix} > \\ < \end{bmatrix} \\ \begin{bmatrix} > \\ < \end{bmatrix} & \text{if } \begin{bmatrix} > \\ < \end{bmatrix} \end{cases} \iff V_{wc} \begin{bmatrix} > \\ < \end{bmatrix} = 0. \]

As the trade union is a Stackelberg leader with respect to the central bank, the 
equilibrium effect is equal to the partial derivative \( w_c \), which cannot be signed \textit{a priori}. We 
will return to this below. The employment effect has the opposite sign of the real wage effect, 
i.e., \( n_{wp} < 0 \). Therefore, we have to derive the effect on the real wage. This is

\[ \frac{d\frac{w}{p}}{dc} = \frac{dw}{dc} - \frac{dp}{p} w. \]

(13)

Substituting in equations (8), (11), and (12) gives

\[ \frac{d\frac{w}{p}}{dc} = \frac{w_c(1-\theta) - p_c w}{p}. \]

(14)

Note that \( w_c > 0 \) is a sufficient condition for the equilibrium real wage to increase in \( c \), too. It 
is therefore necessary to determine the conditions under which this effect becomes positive 
and the conditions under which the reverse occurs.

**III. The Role Of The Trade Union’s Outside Option**

In this section we discuss two benchmark cases to illuminate the role of the trade 
union’s outside option in the model. First, we consider a trade union, which faces a pure real 
outside option. Then, we analyze the case of a trade union whose outside option is
determined in nominal terms only so that the outside option is negatively affected by a price increase.

A. Real Outside Option for the Trade Union

Let us start with the case where there is no nominal element in the outside option \((b_{\text{nom}} = 0)\). In this case, the trade union's objective function (9) becomes

\[
V = n \frac{w}{p} + (1 - n) b^{\text{real}},
\]

and the first-order condition is

\[
V_w = \frac{n(1 - \theta)}{p_w} \left[ -w(\delta - 1) + \delta p b^{\text{real}} \right] = 0 \iff -w(\delta - 1) + \delta p b^{\text{real}} = 0,
\]

where we used the fact that the wage elasticity of labor demand is \(n_w/n = -\delta\). A nominal wage that maximizes the objective function of the trade union monopoly exists if and only if the second derivative is negative at \(V_w = 0\). This is always true for the case of a real outside option, i.e., \(V_{ww} = -(\delta - 1) + \delta p_b^{\text{real}} < 0\).²

The first-order condition does not depend on \(\theta\), i.e., the nominal wage elasticity of the price level. The change in the nominal wage with respect to a change in the degree of conservatism is given by

\[
V_{ww} dw + V_{wc} dc = 0 \iff \\
\left[ -(\delta - 1)w + \delta p b^{\text{real}} - \delta p b^{\text{real}} + \delta p b^{\text{real}} \frac{w}{p} \right] dw + \left[ \delta p b^{\text{real}} \frac{w}{p} \right] dc = 0.
\]

Using the definition of \(\theta\), applying the first-order condition (15) and rearranging yields

\[
w_c = p_c \frac{w}{p} \frac{1}{1 - \theta} < 0.
\]

² To see this, substitute for \(n\) from the first order condition (15) and rearrange to find \(V_{ww} = -(1 - \theta) \delta b^{\text{real}} p / w\).
From the discussion of equation (11) above, we already know that \( w_c < 0 \) is a sufficient condition for equilibrium inflation to decrease in \( c \): \( dp/dc < 0 \). Substituting (16) in (14) also yields the following result for the implied change in the real wage rate:

\[
\begin{align*}
    d \frac{w}{p} &= \frac{w}{p (1-\theta)} - \frac{w}{p} \\
    \frac{dp}{dc} &= \frac{w}{p (1-\theta)} - \frac{w}{p} = 0.
\end{align*}
\]  

(14a)

As the trade union has complete control over the real wage, it will set the real wage equal to the real unemployment benefit payments times a mark up, independently of the price level. Hence, the degree of conservatism does not affect the real wage nor employment.

**PROPOSITION 2 (real outside option):** If the outside option of the monopoly trade union is defined in real terms only, the real wage and employment are independent of the degree of central bank conservatism. The price level is decreasing in the degree of central bank conservatism.

The intuition behind this result is straightforward. If the trade union is not concerned about nominal values (and in the absence of uncertainty or shocks), it can always enforce its preferred real wage. Since a variation in nominal values such as the price level does not affect the trade union’s outside option, the trade union will change the nominal wage in response to changes in \( c \) only in order to keep the optimal real wage constant.

**B. Nominal Outside Option for the Trade Union**

Now let us turn to the case where the unemployment benefit payments are nominally fixed only (\( b^{real} = 0 \)). Obviously, the trade union is now inflation averse, as a higher price level implies a lower income for all unemployed for any given real wage rate. Equation (9) then becomes

\[
(9b) \quad V = n \frac{w}{p} + (1-n) \frac{b^{nom}}{p},
\]

and the derivative with respect to the wage rate is for \( c > 0 \) and thus \( \theta < 1 \) (see Appendix I):

\[
(17) \quad V_w = \frac{n}{pw} \left[ -w(1-\theta)(\delta-1) + b^{nom} \left[ \delta(1-\theta) - \frac{1-n}{n} \right] \right] = 0.
\]

As mentioned before, an ultra-liberal central bank with \( c = 0 \) will always choose \( w = p \) and thus guarantee full employment, i.e., \( n = 1 \). In this case, the trade union cannot influence the real wage by its choice of the nominal wage rate and its rent is always equal to
\( V = 1 \), that is, the rent it would obtain in a perfect Walrasian labor market. In this case, the nominal wage rate and thus the price level would remain undetermined. This is because, with full employment guaranteed by monetary policy, the nominal nature of the trade union's outside option ceases to be relevant for its decision making. It would remain relevant, though, if we allowed for some nominal wealth that enters the trade union objective function.

Assuming that the second-order condition is negative, i.e., \( V_{ww} < 0 \), the sign of \( w_c \) is given by the sign of \( V_{wc} \). As is shown in Appendix I, the partial derivative of the first-order condition with respect to \( c \) is for \( b^{real} = 0 \):

\[
V_{wc} = \frac{b^{nom}}{2(p-1)(\delta-1)} p_c \left[ -\frac{\delta}{p} (p-1) + \frac{1-2n}{2(p-1)} \right].
\]

As \( p_c < 0 \), \( p > 1 \) and \( n > 1/2 \) the sign is unambiguously positive. Hence we have

\[
(19) \quad w_c > 0.
\]

A more conservative central bank reduces the negative effect that higher nominal wages have on the outside option and thus makes the trade union more demanding. What does this imply for inflation? Proposition I already established that the change in \( p \) is ambiguous when the change in the nominal wage implied by a higher degree of central bank conservatism is positive, i.e., if \( w_c > 0 \). Although we cannot sign the change in the price level, we can sign the change in the real wage. From \( w_c > 0 \) it follows immediately that

\[
(14b) \quad \frac{dP}{dc} = \frac{w_c(1-\theta) - p_c w}{p} > 0.
\]

This can be summed up as follows:

**PROPOSITION 3 (nominal outside option):** If the outside option of the monopoly trade union is defined in nominal terms only, the nominal wage and the real wage are both increasing and employment is decreasing in the degree of central bank conservatism.

In Appendix II we show that this result can be generalized to the case where the trade union has to consider both a real and a nominal outside option. In this case the nominal wage
may increase or decrease depending on the relative weights of the nominal and the real outside options. However, the real wage is always increasing as soon as there is a nominal outside option \( b_{nom} > 0 \) for the trade union to consider.

IV. THE GOVERNMENT DECISION

So far the discussion of the role of government has been limited to comparative statics concerning the union’s outside option and central bank characteristics. As shown, a change in central bank conservatism imposed by the government has different repercussions for inflation and unemployment, depending on whether the trade union’s outside option is defined in nominal or real terms. However, the government might also have a significant influence on the nature of the trade union’s outside option. Given this possible menu of policy tools and policy effects, how will the government set its instruments?

A natural assumption is that the government values both price stability and employment. Then the benchmark results discussed above have a straightforward policy implication if the only policy option of the government is to choose the degree of central bank conservatism. Assume that the social loss function of the government is given by a standard quadratic loss function

\[
L_{gos} = 0.5 \cdot u^2 + 0.5 \cdot g \pi^2,
\]

where \( 0 < g < +\infty \) is the weight the government attaches to losses from inflation. The derivative with respect to the degree of central bank conservatism is

\[
L_{c}^{gov} = L_{p}^{gov} \frac{dp}{dc} + L_{w}^{gov} \frac{dw}{dc},
\]

where the change in the price level and the nominal wage is determined by the equation system (10). Substituting in the partial derivatives \( L_{p}^{gov} = \delta(1-n)np^{-1} + g(p-1) \), \( L_{w}^{gov} = \delta(1-n)nw^{-1} \) and making use of the first-order condition of the central bank (4), we can rewrite the first order derivative of the government in the following way:

\[
L_{c}^{gov} = \frac{(p-1)}{w} \left[ (g-c)w \frac{dp}{dc} + cp \frac{dw}{dc} \right] = \frac{(p-1)}{w} \left[ gw \frac{dp}{dc} + c \left( p \frac{dw}{dc} - w \frac{dp}{dc} \right) \right].
\]

Consider first the case of a trade union which faces a real outside option only. Substituting in equations (13) and (14a) gives

\[
L_{c}^{gov} = (p-1)g \frac{dp}{dc}.
\]
From (16) and (11) we can infer that $dp/dc < 0$ and hence $L_c^{\text{opt}} < 0$. It is therefore optimal for the government to choose an ultra-conservative central bank with $g < c \to +\infty$. This can be summarized as follows:

**PROPOSITION 4a (ultraconservative central bank):** If the outside option of the monopoly trade union is fixed in real terms only, the central bank should be ultra-conservative, i.e., the government should set $c$ such that $g < c \to +\infty$.

This confirms the well-known result that, if the trade union’s objective function is not affected by nominal values (and in the absence of shocks or uncertainty), the government should credibly commit itself to a non-inflationary policy to minimize the inflationary bias.

Next, consider the case where the trade union has a nominal outside option. In this case, both the nominal and real wages increase in $c$, i.e., $dw/dc > 0$ [cf. equations (12) and (19)] and $d w/p/dc > 0$ [cf. equation (14b)]. By inspection of equation (22) one can infer from the last equation that an internal solution requires $dp/dc < 0$. If this is the case, the first equation shows that the optimal degree of central bank conservatism is in the interval $0 < c^* < g$. This leads to

**PROPOSITION 4b (liberal central bank):** If the outside option of the monopoly trade union is fixed in nominal terms only, and if the price level decreases in central bank conservatism, the central bank should be liberal in the sense that $0 \leq c^* < g$.

How can one interpret Propositions 4a and 4b? From the perspective of the government, setting $g < c \to +\infty$ if the outside option of the monopoly trade union is fixed in real terms ($b'^{\text{real}}$) follows directly from Proposition 2 and the government loss function (21). Since $c$ has no influence on the real economy but an increase in $c$ unambiguously lowers inflation, making the central bank infinitely conservative will ensure a second-best welfare optimum. It is second best since the real wage set by the monopoly trade union is too high to allow full employment. Things change, however, if the trade union is inflation averse. If the outside option of the monopoly trade union is defined in nominal terms ($b'^{\text{nom}}$) only, the

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7 Note that for $dp/dc > 0$, we have no inner solution. If increasing $c$ unambiguously increases inflation, the outcome is completely undetermined. As long as $p > 1$, there is no reason for a conservative central bank, as both government objectives are served best by a completely permissive monetary policy. Choosing $c = 0$, i.e., the case for an “ultra-liberal” central bank, made by Cukierman and Lippi (1999) and Guzzo and Velasco (1999), implies that prices are determined by the trade union rather than the central bank, which sets $p = w$. $c = 0$ is only optimal for the government, however, as long as $p > 1$. If $p < 1$ at $c = 0$, we will have an inner solution for $dp/dc > 0$ with $c > 1$. 
government's best choice will always be \( c^* < g \). The reason is that an increase in \( c \) will now have adverse real effects on unemployment (Proposition 3). Committing to a policy which puts less weight on inflation will therefore be beneficial.

V. **Do We Need a Liberal or a Conservative Central Bank?**

So far we have considered two benchmark cases, the case of a trade union which faces a real outside option only and a trade union which faces a nominal outside option only. As has been pointed out at the end of Section III, however, if the outside option of the trade union consists of both nominally fixed and real values, the change in the nominal wage due to a change in the degree of central bank conservatism is undetermined (compare Appendix II). As before, one can see from inspecting equation (22) that an internal solution requires \( dp/dc < 0 \). For this case, the following condition for the optimal degree of central bank conservatism applies:

\[
c^* \begin{cases} > & g \\ < & 0 \\ > & 0 \end{cases} \iff \begin{cases} dw & < \\ dc & > 0 \end{cases}.
\]

If the nominal wage does not react to a change in the degree of central bank conservatism at all, there is no incentive for the government to commit itself to a more conservative monetary policy compared with its own preferences. The reason is simply that with inflation-invariant wage setting there is no time-inconsistency problem to deal with. If instead a conservative central bank forces the trade union to moderate the nominal wage, the government gains from making the central bank more concerned about inflation. If the nominal outside option dominates, however, inflation aversion may make the trade union more moderate. The more permissive monetary policy is, the stronger the degree of moderation will be. In this case, the government should commit to a central bank which is more liberal than the government itself in order to exploit the trade union’s dislike of inflation.

**PROPOSITION 5 (general case):** If the outside option of the monopoly trade union consists of both nominal and real elements, the central bank should be **conservative** in the sense that \( c^* > g \) if the nominal wage is decreasing in central bank conservatism. It should be **liberal** in the sense that \( 0 < c^* < g \) if the nominal wage is increasing in central bank conservatism.

Whether the government should choose a conservative or liberal central bank critically hinges on the composition of the trade union’s outside option. In particular, a liberal central bank is justified only if a sufficiently large nominal outside option exists for the trade union. The question is thus, what can be said about the possible sources of an outside option in nominal terms?
As argued earlier, it would seem that the most likely candidate for introducing a nominal element in the outside option of the trade union is the *government* itself. Assume for a moment that government provisions for unemployment relief are not indexed to inflation and thus encompass a nominal part. Then, if the nominal element is sufficiently large, the general case would indeed allow an argument in favor of a "liberal" central bank in the sense of Proposition 5. Or does it?

Careful consideration shows that the argument focusing on the government itself as source of the nominal element in the trade union's outside option is potentially inconsistent. A crucial observation in this regard is that de facto the government will always determine a real rather than a nominal outside option for the union, even when de jure unemployment benefits are defined in nominal terms. This is because the government acts as *Stackelberg leader* vis-à-vis the central bank. Therefore it implicitly determines the price level (or inflation) by setting the degree of the central bank conservatism and hence also the level of real unemployment benefits. As a consequence, from the viewpoint of the government, determining the trade union's real outside option *directly* (for instance, by setting \( b^{\text{real}} > 0, b^{\text{nom}} = 0 \)) or *indirectly* (for instance, by setting \( b^{\text{real}} = 0, b^{\text{nom}} > 0 \) and taking into account the implied price level) is perfectly equivalent with respect to the resulting real transfers to the unemployed. This is a relevant result in its own right, since there will often be a constitutional requirement to guarantee a minimum standard of living. What is more, however, is that—given such an implicitly or explicitly defined level of unemployment benefits—the type of transfer has important consequences for welfare.

While in equilibrium the real allocation of trade union members between employment and unemployment will be the same for a given ex post level of real unemployment benefits, inflation will be strictly higher when the trade union ex ante perceives its outside option as being defined (mainly) in nominal terms. As shown above in Propositions 4b and 5, when unemployment benefits are defined strictly in nominal terms or when the nominal element in a benefit system is large enough, the government will opt to decrease \( c^* \) to moderate the trade union's wage demands. As discussed, the effect works through the resulting "liberal" monetary policy stance. However, such a "liberal" central bank policy will also increase inflation compared with a scenario in which the trade union's outside option is defined in real terms ex ante. As already discussed in Propositions 4a and 5, the government will decide to nominate a "conservative" central bank if unemployment benefits are (predominantly) real. The reason is that in this scenario, the trade union lacks a sufficiently strong incentive to moderate its wage claims and only a strong anti-inflationary monetary policy can achieve price stability. As a consequence, if the government itself is the source of a possibly nominal outside option for the trade union, welfare maximization would imply that it chooses unemployment benefits to be in strictly real terms only.
An illustrative example is the case when we acknowledge that, perhaps for social reasons, the government pledges to secure a certain minimum real living standard, say, $b_{nom} / p = \bar{b} > 0$. Since then

$$db_{nom} = \bar{b} dp \Leftrightarrow \frac{p}{dp} = 0,$$

such a policy would effectively reintroduce a real outside option for the trade union. With a real outside option set at its minimum, the trade union will decide to set its wage rate such that employment is maximized relative to a scenario in which the government would allow the outside option of the trade union to contain nominal elements. As Proposition 2 shows, setting $c \to +\infty$ will, in addition, achieve zero inflation. Using its two instruments to tackle its two policy targets, zero inflation and minimum unemployment, the government can unambiguously improve welfare compared with the initial equilibrium with ($b_{nom} > 0$, $c < +\infty$).

A consequence of this thought experiment is that the use of both policy instruments will reintroduce the Rogoff solution even to the single monopoly trade union case with a de jure nominal outside option. Consequently, setting $c \to +\infty$ would be the government’s preferred choice. A similar reasoning would apply if the monopoly trade union’s outside option would be forgone real income in the shadow economy.

**Proposition 6 (government and outside option):** If the nominal outside option of the monopoly trade union can be set by the government but there is a real floor (a social minimum) limiting the choice of the nominal outside option, the government will choose the minimum real option and resurrect the conservative-central-bank solution ($c \to +\infty$).

**VI. CONCLUSION**

Rogoff’s (1985) conservative central banker, an important point of reference for economists thinking about institutional remedies against inflation, has come under attack lately. A series of influential papers has questioned the very core of the Rogoff argument, that is, whether making the central bank more conservative than society will indeed help to reduce inflation at no real cost. The standard monetary policy model supports this view, simply because a more inflation-averse central bank will be less tempted to trade off higher

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8 Alternatively, we could argue that the government might not be able to commit itself to a zero level of nominal unemployment benefits in a time-consistent way. Then $\bar{b}$ might be the real outcome of the underlying political economic equilibrium.
inflation for (short-term only) gains in output and employment. Lower unemployment is a worthwhile policy target in the standard model because rigidities such as trade union market power render equilibrium employment too low. A major drawback of this argument is, however, that this incentive is introduced as an exogenous assumption rather than an outcome of, for instance, monopolistic trade union behavior.

Explicitly modeling the behavior of a monopoly trade union and its interaction with monetary policy, Cukierman and Lippi (1999) and Guzzo and Velas (1999) have argued that a conservative central bank might actually be welfare reducing. The reason is that, if a conservative central bank keeps prices in check even when nominal wages rise, the trade union will not have to suffer the same inflationary consequences as with a less conservative monetary policy. Because more aggressive wage demands will also drive up real wages, an important consequence of this interaction between the central bank and the trade union is that now monetary policy also has real effects. The more conservative the central bank, the less moderate wage claims are and the higher unemployment is. As a result, an “ultra-liberal” rather than a Rogoff-type central bank will maximize welfare in such a model.

So, is the institutional remedy for inflation suggested by Rogoff (1985) erroneous in the presence of strong labor unions? Our answer is no. The present paper shows that the “ultra-liberal” central bank result is based on a specific assumption about the nature of the monopoly trade union’s outside option. In fact, it is only if parts of the outside option of the trade union are defined in strictly nominal terms that the case against the conservative central bank can be made. Only then will the threat that wage-induced price increases pose to unemployed trade union members effectively moderate trade union wage demands. If, however, the outside option of the trade union is defined in real terms, trade union behavior and monetary policy are no longer interconnected. In this case, the incentive to trade off inflation against employment is again exogenous from the perspective of monetary policy makers—central bank characteristics no longer matter for trade union behavior. Consequently, there is no welfare gain associated with making the central bank less conservative than society—quite the contrary. An important question raised by this dichotomy is, which scenario is more likely?

Consider the possible sources of a nominal outside option for trade union members, i.e., the assumption that lies at the heart of the “liberal” central bank result. Probably the most likely reason for the existence of a nominal outside option is the government itself. While perhaps not a particularly plausible assumption, it might be argued that unemployment benefits are sometimes specified in strictly nominal terms. Other important outside options for trade union members, for instance, leisure or black market activities, are almost exclusively defined in real terms. But, is the government actually free to leave the real benefits of unemployed trade union members in the hands of the central bank and the trade union? Most likely there will be an explicit or implicit guarantee of a minimum real standard of living. Such a real floor to the government-provided outside option has important consequences.
As we argue in the paper, if the government is to guarantee a certain ex post real outside option for the unemployed, it is always better off by announcing ex ante that, for instance, unemployment benefits are defined in real terms. The reason is that, while the ex post real wage and thus employment would be similar under both real and nominal outside options, inflation would be higher in the latter case. This is because with a nominal outside option, the government would choose a more “liberal” central bank to run monetary policy in order to moderate trade union wage claims. This will raise inflation above the level that would prevail with the same (ex post) real outside option pre-specified ex ante. In other words, a government that values employment and stable prices is always better off fixing the level of unemployment benefits and social transfers in real terms ex ante and, at the same time, choosing a conservative, Rogoff-type central banker. Once both instruments of government policy are taken into account, the standard solution is resurrected.

A key insight given by the discussion above is that important institutions governing labor market performance and inflation are not independent, but rather are connected by the interaction of monetary policy and trade union behavior. The present paper has shown that economic policy might combine fiscal measures and institutional design to achieve a desired outcome in the presence of trade union monopoly power. Future research should find it interesting to combine this line of thought with earlier work, for instance, by Agell and Ysander (1993), which investigated the role of progressive income taxes for trade union behavior.
OPTIMAL CONSERVATISM IN THE GENERAL CASE

The sign of (11) is given by

\[
\text{sign} \left( \frac{dp}{dc} \right) = \text{sign} \left( -V_w L_{pw} + L_{pc} V_{ww} \right) = \text{sign} \left( -V_w \frac{L_{pw}}{L_{pc}} + V_{ww} \right) = \text{sign} \left( -V_w \frac{P_w}{P_c} + V_{ww} \right)
\]

The first-order condition of the trade union’s maximization problem is given by (using (8)):

\[
V_w = \frac{n}{pw} \left[ -w(1 - \theta)(\delta - 1) + b_{\text{real}} p \delta(1 - \theta) + b_{\text{nom}} \left[ \delta (1 - \theta) - \frac{1 - n}{n} \theta \right] \right] = 0,
\]

or

(A-1) \quad V_w = \frac{n(1 - \theta)}{pw} \left[ -w(\delta - 1) + b_{\text{real}} p \delta + b_{\text{nom}} \left[ \delta - \frac{1 - n}{n} \frac{\theta}{(1 - \theta)} \right] \right] = 0,

The term

\[
\frac{1 - n}{n} \frac{\theta}{(1 - \theta)}
\]

in the first-order condition (A-1) can be simplified by substituting in equation (8):

\[
\frac{1 - n}{n} \frac{\theta}{(1 - \theta)} = \frac{\delta^2 (2n - 1)(1 - n)}{cp(2p - 1)} < 1.
\]

The inequality must hold for condition \( V_w = 0 \) to be fulfilled throughout. Applying the first-order condition of the central bank (4) we can show that:

\[
\delta - \frac{1 - n}{n} \frac{\theta}{(1 - \theta)} = \frac{\delta cp(2p - 1) - \delta^2 (2n - 1)(1 - n)}{cp(2p - 1)} = \frac{\delta [cp^2 + cp(p - 1) + \delta (2n - 1)n - \delta (2n - 1)]}{cp(2p - 1)}
\]

\[
= \frac{\delta \left[ cp^2 + \delta n^2 - \delta (2n - 1) \right]}{cp(2p - 1)} \frac{\delta + \delta (1 - n) \frac{\delta}{(p - 1)}}{cp(2p - 1)} = \frac{\delta n + (p - 1)}{(2p - 1)n}.
\]

Hence (A-1) becomes

(A-2) \quad V_w = \frac{n(1 - \theta)}{pw} \left[ -w(\delta - 1) + b_{\text{real}} p \delta + b_{\text{nom}} \left[ \delta (p - 1) + n \right] \right] = 0.
The first order condition can thus be rewritten as

\[(A-3) \quad -wn + b^{real} p \frac{\delta}{(\delta - 1)} n + b^{nom} \left[ \frac{\delta}{(\delta - 1)} \left( \frac{p - 1}{(2p - 1)} + n \right) \right] = 0.\]

From this it follows that

\[(A-4) \quad V_{wc} |_{r_{w0}} = -wn_p p_c + b^{real} \frac{\delta}{(\delta - 1)} (np_c + n_p p_c p) + b^{nom} \frac{\delta}{(\delta - 1)} \left[ \frac{1}{(2p - 1)^2} + \frac{n_p (2p - 1) - 2n_p}{(2p - 1)^2} \right] p_c \]

\[= -\frac{\delta}{p} wnp_c + b^{real} \delta np_c \frac{(1 + \delta)}{(\delta - 1)} + b^{nom} \frac{\delta}{(\delta - 1)} \left[ \frac{1}{(2p - 1)^2} + \frac{\delta}{(2p - 1)^2} \right] p_c \]

\[= \frac{\delta}{p} p_c \left[ -wn + \frac{b^{nom}}{(\delta - 1) (2p - 1)} \frac{n}{(2p - 1)} \right] + b^{real} \delta np_c \frac{(1 + \delta)}{(\delta - 1)} + b^{nom} \frac{\delta}{(\delta - 1)} \left[ \frac{1 - 2n}{(2p - 1)^2} \right] p_c .\]

Applying the first-order condition (A-3), the term in the first brackets changes so that

\[(A-5) \quad V_{wc} |_{r_{w0}} = -\frac{\delta}{p} p_c \left[ \frac{b^{nom}}{(\delta - 1) (2p - 1)} \frac{\delta}{(2p - 1)} \frac{(p - 1)}{2p - 1} \right] + b^{nom} \frac{\delta}{(\delta - 1)} \left[ \frac{1 - 2n}{(2p - 1)^2} \right] p_c + b^{real} \delta np_c \frac{(1 + \delta)}{(\delta - 1)} \]

Thus, applying the conditions for \(p\) and \(n\), we have \(V_{wc} > 0\) for \(b^{real} = 0\) and \(V_{wc} < 0\) for \(b^{nom} = 0\).

Next, calculate the second-order condition \(V_{ww}\). From (A-3), it follows:

\[(A-6) \quad V_{ww} |_{r_{w0}} = -n \cdot wn_w + b^{real} \frac{\delta}{(\delta - 1)} n_p p + b^{nom} \frac{\delta}{(\delta - 1)} \frac{1}{(2p - 1)} n_w \]

\[= -wn_p p_w + b^{real} \frac{\delta}{(\delta - 1)} (np_w + n_p p_w p) + b^{nom} \frac{\delta}{(\delta - 1)} \left[ \frac{1}{(2p - 1)^2} + \frac{n_p (2p - 1) - 2n_p}{(2p - 1)^2} \right] p_w \]

Substituting the first line of (A-4) in the second and third line of (A-6), and rearranging the first line, we obtain:

\[(A-7) \quad V_{ww} |_{r_{w0}} = -n(1 - \delta) - \frac{b^{real} p}{w} \frac{\delta^2}{(\delta - 1)} n - \frac{b^{nom} p}{w} \frac{\delta^2}{(\delta - 1) (2p - 1)} n_p + \frac{p_w}{p_c} V_{wc} |_{r_{w0}} \]
Thus we have at the optimum:

$$sign\left(-V_{wc} \frac{P_w}{p_e} + V_{ww}\right) = sign\left(n \left(\delta - 1 - \frac{b_{real} p}{w} \frac{\delta^2}{(\delta - 1)} - \frac{b_{nom}}{w} \frac{\delta^2}{(\delta - 1) (2p - 1)}\right)\right)$$

From (A-2), we have

$$w = \frac{\delta}{(\delta - 1)} \left(\frac{b_{real} p + b_{nom} \frac{(p - 1) + n}{(2p - 1)n}}{1}\right).$$

Thus

$$sign\left(\frac{dp}{dc}\right) = sign\left(\delta - 1 - \frac{\delta^2}{(\delta - 1)} \left(\frac{b_{real} p}{w} + \frac{b_{nom}}{w} \frac{1}{(2p - 1)}\right)\right)$$

$$= sign\left(\delta - 1 - \delta \left(\frac{b_{real} p(2p - 1)n + b_{nom} n}{b_{real} p(2p - 1)n + b_{nom} ((p - 1) + n)}\right)\right)$$

$$= sign\left(-1 + \delta \left(\frac{b_{nom} (p - 1)}{b_{real} p(2p - 1)n + b_{nom} ((p - 1) + n)}\right)\right)$$

Consider the case with $b_{nom} = 0$. In this case, we can see immediately that $dp/dc < 0$. In the case $b_{real} = 0$, we have:

$$sign\left(\frac{dp}{dc}\right) = sign\left(\delta - 1 - \frac{n\delta}{(p - 1) + n}\right) = sign\left((\delta - 1)(p - 1) - n\right).$$

Given the assumption about the labor market, $n > 0.5$, we have

$$\frac{p}{\delta - 1} < \frac{1 - n}{\delta - 1} \Rightarrow \frac{dp}{dc} < 0.$$ 

Note that the higher $b_{real}$, the higher $p$ can be without changing the sign.
CENTRAL BANK CONSERVATISM AND INFLATION

It is shown that Proposition 5 also holds for the more general case where the trade union has to consider both a real and a nominal outside option:

\[ V = n \frac{w}{p} + (1 - n) \left( b_{\text{real}} + \frac{b_{\text{nom}}}{p} \right) \]

The first-order condition is given by:

\[ (A-4) \quad V_w = \frac{n}{pw} \left[ -w(1 - \delta)(\delta - 1) + b_{\text{real}} p \delta (1 - \theta) + b_{\text{nom}} \left[ \delta (1 - \theta) - \frac{1 - n}{n} \theta \right] \right] = 0 \]

or, by following the steps for \( b_{\text{nom}} \) above;

\[ V_w = \frac{n(1 - \theta)}{pw} \left[ -w(\delta - 1) + b_{\text{real}} p \delta + b_{\text{nom}} \left[ \delta \frac{(p - 1) + n}{(2p - 1)n} \right] \right] = 0. \]

Hence we have

\[ (A-5) \quad -w(\delta - 1) + \delta \left[ b_{\text{real}} p + b_{\text{nom}} \frac{(p - 1) + n}{(2p - 1)n} \right] \equiv -w(\delta - 1) + \delta \left[ b_{\text{real}} p + b_{\text{nom}} Z \right] = 0, \]

where we have defined \( Z \equiv [(p - 1) + n]/[(2p - 1)n] \). Note that the second part of \( (A-4) \) consists simply of the sum of the changes in the outside option in the two special cases of \( b_{\text{nom}} = 0 \) and \( b_{\text{real}} = 0 \) for a marginal increase in \( w \). In the latter case this is obvious from a comparison with \( (A-1) \). In the former case just multiply \( (15) \) through by \( 1/(1 - \delta) \). The derivative of \( (A-5) \) with regard to \( w \) is

\[ V_{ww} = -\delta b_{\text{real}} p_w + \delta b_{\text{nom}} Z_w, \]

where the second and third term can be interpreted in a similar fashion as \( (A-5) \) above as the sum of the second derivatives of the outside option at the extremes \( b_{\text{nom}} = 0 \) and \( b_{\text{real}} = 0 \). As \( V_{ww} < 0 \) must hold if it is optimal for the trade union to raise the wage rate above the outside option, we can again concentrate on the sign of \( V_{wc} \) at \( V_w = 0 \).

\[ (A-6) \quad V_{wc} = \delta \left[ b_{\text{real}} p_c + b_{\text{nom}} Z_c \right] \]

with \( Z_c > 0 \). Using the first-order condition for \( V_w = 0 \), \( (A-3) \), we can solve for

\[ w_c = -V_{wc}/V_{ww} : \]
\[ w_c = -\frac{\delta [b^{\text{real}} p_c + b^{\text{nom}} Z_c]}{(1 - \delta) + \delta b^{\text{real}} p_w + \delta b^{\text{nom}} Z_w}. \]

Substituting in the first-order condition yields

\[ (A-7) \quad w_c = -\frac{\delta [b^{\text{real}} p_c w + b^{\text{nom}} Z_c w]}{-\delta b^{\text{real}} (p + b^{\text{nom}} Z) + \delta b^{\text{real}} p_w w + \delta b^{\text{nom}} Z_w w}. \]

Defining \( A \equiv -\delta b^{\text{real}} p + \delta b^{\text{real}} p_w w < 0 \) since \( \theta < 1 \) and \( B \equiv -\delta b^{\text{nom}} Z + \delta b^{\text{nom}} Z_w w < 0 \)\(^9\), we can split (A-7) in the following way:

\[ w_c = -\frac{\delta b^{\text{real}} p_c w}{-\delta b^{\text{real}} p + \delta b^{\text{real}} p_w w} \quad \frac{A}{(A + B)} \quad -\frac{\delta b^{\text{nom}} Z_c w}{-\delta b^{\text{nom}} Z + \delta b^{\text{nom}} Z_w w} \quad \frac{B}{(A + B)}. \]

Rearranging the first term yields:

\[ (A-8) \quad w_c = p_c \frac{w}{p(1 - \theta)} \quad \frac{A}{(A + B)} \quad -\frac{\delta b^{\text{nom}} Z_c w}{-\delta b^{\text{nom}} Z + \delta b^{\text{nom}} Z_w w} \quad \frac{B}{(A + B)}. \]

As \( p_c < 0 \), the sign of the first term is negative, but the second is positive as \( Z_c > 0 \) and \( B < 0 \). Thus, as was to be expected, the influence of the nominal and real outside options determine the reaction of wages to an increase in \( c \). If the absolute size of the first term is smaller than that of the second, an increase in \( c \) leads to higher wage demands. This result shows that the change in nominal wage can go either way.

Using (A-8) allows us to rewrite equation (14) as follows:

\[ \frac{d}{dc} \left( \frac{w}{p} \right) = p_c \frac{1}{p(1 - \theta)} \left( 1 - \frac{p_c}{p} \right) \quad \frac{A}{(A + B)} \]

\[ (A-9) \quad -\frac{\delta b^{\text{nom}} Z_c w}{p} \quad + \quad \frac{\delta b^{\text{nom}} Z_c w}{p} \quad \frac{A}{(A + B)} \quad \frac{B}{(A + B)}. \]

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\(^9\) \( B < 0 \) if \( n > (p - 1)(\delta - 1) \). Otherwise, the second-order condition is not fulfilled for the case with the trade union facing a nominal outside option only.
As the first term is zero, this reduces to:

\[
\frac{d}{dc} \frac{w}{p} = \frac{-\delta b_{\text{nom}} Z_c w}{p} (1 - \theta) - \frac{p_c w}{p} \frac{B}{(A + B)}
\]

As \( p_c < 0, B < 0, \) and \( A + B < 0 \) the sign of the second term is positive. The first term is also positive as \( Z_c > 0. \) Hence, if the nominal outside option is positive, the real wage always increases in the degree of central bank conservatism.
REFERENCES


