The Distributional Consequences of Real Exchange Rate Adjustment

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IMF Working Paper
IMF Institute

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June 2003

Abstract

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The paper focuses on distributional consequences of macroeconomic adjustment. The preferences of economic agents over the level of the real exchange rate derived from standard models are monotonic, with agents favoring either an infinitely appreciated or depreciated rate. To generate less extreme preferences, a model is presented where appreciation would depress economic activity, while a large depreciation would hit the tradable sector by limiting the availability of labor, offsetting the favorable price effect. The model is in the spirit of the dependent economy model, but built on explicit microfoundations. The results can be used to analyze political economy aspects of macroeconomic adjustment.

JEL Classification Numbers:F41, P16

Keywords: real exchange rate, political economy

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1 I would like to acknowledge with gratitude the helpful comments of Eric Clifton, Andrew Feltenstein, Jeffry Frieden, Peter Montiel, Alexandros Mourmouras, and the participants at the IMF Institute Seminar Series.
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I. INTRODUCTION

Assessing whether the level of the real exchange rate (RER)\(^2\) is appropriate plays an important role in evaluating an economy's health. The adjustment of the real exchange rate, along with appropriate adjustment of aggregate demand, is frequently key to the correction of macroeconomic imbalances. The standard approach, which emphasizes the use of real exchange rate and aggregate demand management to attain simultaneously full employment of domestic factors of production in a noninflationary environment and sustainable external position, is summarized in the internal and external balance (IB-EB) model.\(^3\)

The IB-EB model prescribes adjustments appropriate to restore economy-wide balances. These adjustments are likely to affect different groups in society asymmetrically. The distributional issues that arise are important to recognize, as real life adjustments are outcomes of political process rather than mere embodiments of prescriptions of economic models.

While political economy literature has made considerable progress in analyzing the implications of the struggle over the sharing of the burden of fiscal adjustment (e.g., Alesina and Drazen, 1991), the analysis of the distributional consequences of real exchange rate adjustment has not gone beyond the recognition that the producers of tradable goods prefer relatively more depreciated RER while the producers of nontradable goods prefer appreciated RER. Switching from sectoral to factorial differences, confusion seems to exist as to whether labor should favor a strong or weak currency.

As Frieden and Stein (2001) emphasize, "political economy of exchange rate policy is barely in its infancy." They contrast this situation with the voluminous literature on the political economy of trade and suggest that the reason for the difference is the existence of "well-developed theories of the distributional implications of different trade policies" and the absence of such background for exchange rate policy.\(^4\) Indeed, there have been very few attempts to model the real exchange rate as an outcome of a distributional conflict. Dornbusch and Edwards (1993) show how in an open economy political populism may lead to an unsustainable increase in the real wage. Alfaro (2002) offers a political economy explanation for temporary exchange-rate-based stabilization programs by focusing on the distributional effects of RER appreciation. Some other papers (e.g., Ghezzi and others

\(^2\) We define the real exchange rate as the price of nontradables in terms of tradables.

\(^3\) This model, pioneered by Salter (1959) and Swan (1960), is also known as the Salter-Swan model, the Australian model, or the dependent economy model.

\(^4\) Using the rich, though inconclusive, economic literature on the choice of exchange rate regimes as a foundation, political scientists have produced a fair amount of work on the political determinants of exchange rate arrangements. See, for example, Eichengreen (1992), Simmons (1994), Bernhard and Leblang (1999).
(2000), Stein and Streb (1999) relate cycles in the real or nominal exchange rates to political events, such as elections, but their models are based on the representative consumer paradigm.

Our paper seeks to fill the gap identified above. We believe that the development of the political economy of real exchange rate adjustment has been hindered by the lack of clear identification of sectoral preferences over the level of the real exchange rate. Our paper focuses precisely on this issue. We do not construct a political economy model devoted to analyzing a particular aspect of policies affecting the RER in a particular group of countries. Rather we make a necessary step that can provide a foundation for constructing a variety of such models – we consider how sectoral preferences over the RER can be derived from such primitives as utility functions, production functions, factor endowments, and budget constraints. Particular attention is paid to the question whether sectoral preferences should be modeled as monotonic or single-peaked. This question is important, as political economy literature frequently makes the assumption that individual or group preferences over economic policies are single-peaked, so substantiating this assumption would greatly expand the class of political economy models applicable to RER issues.

In this paper we propose a model that allows government policy to influence the real exchange rate and gives rise to single-peaked preferences. The model is similar in spirit to the IB-EB framework, but extends it in a number of important ways. First, the model incorporates explicitly preferences and constraints of different agents in the economy, setting the stage for a distributional struggle. Second, the incorporation of preferences and constraints puts the IB-EB model on microfoundations. Third, the model allows explicit intertemporal analysis.

To preview some results of the model, the workers would like an appreciation of real exchange rate brought about by an expansion of aggregate demand (fueled by fiscal deficits), while the capitalists in the tradable sector would oppose it. Deviations of the RER from its short-term equilibrium value (i.e., the value consistent with market clearing at a given level of aggregate demand) in either direction hurt labor. Capitalists would prefer a slightly undervalued currency. The model highlights the interdependence between aggregate demand and exchange rate policies in agents’ preferences.

The paper is organized as follows. The next section sets the stage by exploring how one might think of derived preferences of economic actors over the level of the real exchange rate in various standard models. The following section introduces our model and derives some implications. The objective of Section IV is to suggest how these results can be taken into the realm of political economy. Section V discusses limitations and possible extensions of the model. The last section concludes.

II. WHAT DO THEY CARE ABOUT?

An appreciated currency raises the relative price of nontradables, while a depreciated currency raises the relative price of tradables, so the producers of nontradables should favor a more appreciated real exchange rate than the producers of tradables. This much seems clear and noncontroversial. A more precise specification of preferences is more complicated. An
implicit assumption appears to be that the preferences of both groups are single-peaked, with the peak (bliss point) of the producers of tradables found at a more depreciated level. This specification, however, invites a question—why would the producers of tradables ever be satisfied with the level of the RER? Would they not always prefer a more depreciated rate to any given level? And wouldn’t the producers of nontradables always want more appreciation? We will consider the sectoral preferences over RER derived from fundamental preferences over consumption and from budget constraints in several basic models.

Another question has to do with the preferences of labor. A claim is made sometimes that since labor is (largely) nontradable across borders, its preferences over the level of the real exchange rate should be aligned with the preferences of the producers of nontradables, i.e., workers should stand for an appreciated real exchange rate. We will evaluate the validity of that claim.

To think of the preferences of labor over the level of the RER, a useful starting point is the basic Heckscher-Ohlin model of trade. In that model, two factors, mobile within the economy but not across borders, are employed to produce two tradable goods. The fundamental theorem of the model\(^5\) is that if the relative price of one good rises, the return to the factor used intensively in the production of that good increases. But this theorem is not specific to trade—it is derived from the production side and applies to any 2x2 system (two goods, two factors of production).\(^6\) If the two goods are tradables and nontradables, and the two factors are labor and, for example, capital, then when the price of nontradables rises (the real exchange rate appreciates), the wage will go up if labor is used intensively in the production of the nontradables and go down otherwise. If the nontraded good is labor-intensive, then an increase in its relative price will raise the wage relative to the price of capital, and relative to the prices of both goods (the magnification effect). If the traded good is labor intensive, the opposite will happen. Usually we believe that the production of nontradables is more labor-intensive than the production of tradables (think of nontradables as mostly services), so labor would benefit from a real appreciation in that framework. The reason, however, would not be the fact that labor is not tradable across borders, but rather that it is used intensively in the production of nontradables.

Now let us think of the preferences of producers over the level of the RER. We start with a very simple two-sector model. Each sector produces its output by combining a tradable and nontradable input. We can introduce net production functions relating the net output \(q\) of the sector (the difference between gross output and the input of the good that is produced in the sector) to the input \(i\) from the other sector.

\[
q_T = f_T(i_N) \quad q_N = f_N(i_T)
\]

\(^5\) The Stolper-Samuelson theorem (Stolper and Samuelson, 1941).

\(^6\) Appendix I provides an explicit derivation of this statement.
Let us normalize the price of tradables to unity. We will denote the price of nontradables $p$, which will also be the real exchange rate. Profit maximization will require that the marginal product of the input be equal to its relative price.

$$\max f_r(i_N^*) - pi_N^* \Rightarrow f_r'(i_N^*) = p$$
$$\max pf_N(i_T^*) - i_T^* \Rightarrow f_N'(i_T^*) = 1/p$$

The asterisks denote the optimal employment of inputs. Profit functions will look as follows:

**Tradables:**

$$\pi_T(p) = f_r(i_N^*(p)) - pi_N^*(p)$$
$$\frac{d\pi_T(p)}{dp} = -i_N^*(p) < 0$$

**Nontradables:**

$$\pi_N(p) = pf_N(i_T^*(p)) - i_T^*(p)$$
$$\frac{d\pi_N(p)}{dp} = f_N(i_T^*(p)) > 0$$

Profits of the producers of tradables monotonically decrease with real appreciation, and those of the producers of nontradables monotonically increase with real appreciation. This is consistent with the notion that producers of tradables prefer a more depreciated real exchange rate than the producers of nontradables. But this says more—whatever the level of the real exchange rate, the producers of tradables would prefer some depreciation, and the producers of nontradables would like some appreciation. There is no “bliss point” or “optimal RER” from the point of view of either sector—one would like the RER to go down all the way to zero, and the other one would like it to go all the way up to infinity. The fact that net production functions were used disposes of the argument that the preferences would not be so extreme if, for example, the producers of tradables were to use some imported inputs.

One might think the producers would moderate their preferences if they also cared about the price of their consumption basket. But in fact one can show (see Appendix II) that the impact on income will always dominate, so real appreciation will increase the purchasing power of producers of nontradables and decrease that of producers of tradables.

An extension of the above model is a model with two specific factors of production (say, sector-specific capital) and one general factor of production (labor). We can simply assume two sectoral production functions with labor as a sole input. On the production side, enterprise owners maximize profits by setting the marginal product of labor equal to the product wage in their sectors. On the consumption side, firm owners as well as workers will set the marginal rate of substitution between nontradables and tradables equal to their relative price (the real exchange rate).
In Appendix III we show that in this model the producers benefit from an increase in the price of the good they produce and from a decrease in the price of the other good and in the wage rate. The workers benefit from a rise in wages and from a reduction in the goods prices. These results, of course, are intuitive, even pedestrian. They are derived from optimization problems for each group, without imposing market-clearing conditions.

An important question is—what comparative statics are reasonable in this model? The first-order conditions and the budget constraints impose eight conditions on eleven variables (consumption of each commodity by each group (six variables), labor input in each sector (two variables), two prices, and the wage rate). We can express the eight quantity variables as functions of the three price variables. Of course, only relative prices matter, so one price can be set equal to one. This leaves us with two parameters, and they will be determined once we impose two market-clearing conditions—for nontradedables and for labor. Hence, there does not seem to be any leeway in setting these parameters arbitrarily. In that context, the question “how would the welfare of group X be affected by a change in the real exchange rate” does not appear to be meaningful, since there is only one real exchange rate consistent with the parameters of the model.

This analysis yields two important conclusions. The first is that for a given wage rate and the price of the other commodity, the producers would always prefer a higher price for the good that they produce to a lower price. Second, market-clearing conditions actually dictate all the relative prices, so in a world of flexible prices and market equilibrium the derived preferences over the real exchange rate do not matter—the level of the RER will be determined by the market.

These conclusions may appear bland. In what direction should one search for more exciting results, such as non-monotonicity of derived sectoral preferences over the level of the real exchange rate and the ability to manipulate that level? Three possibilities come to mind. The first is to impart dynamics to the model—allowing agents to borrow and lend, and having them maximize the present discounted value of their utility. After all, the RER is linked with the current account of the balance of payments, which, in turn, comes from intertemporal optimization. Another possibility is to move away from competitive markets to monopoly or monopsony. Unlike a perfectly competitive producer, a monopolist is aware of the demand for its product, so its profit maximization calculus would lead to a finite price for its product. A third approach is to do away with price flexibility and market clearing. Let us explore these possibilities one by one.

Appendix IV sets out a framework with the production side analogous to the model just considered (two productive sectors with specific capital and labor as a non-specific factor), but allowing the capitalists and the workers to borrow and lend in international markets. The agents will maximize the present discounted value of their utilities subject to the flow budget constraint and a no-Ponzi-game condition. From utility maximization, we will have an equality between the marginal rate of substitution between tradables and nontradables for each group and the real exchange rate. From profit maximization we will have an equality between the marginal value product of labor in each sector and the wage rate. These are intratemporal conditions. In addition, for each of the three types of agents we will have the
Euler equation (first-order differential equation relating the growth rate of consumption to the difference between the discount rate and the interest rate) and the integral budget constraint (the present discounted value of consumption equals the present discounted value of profits or wages plus initial assets). From profit maximization, labor demand for each sector will be found as a function of the real exchange rate and the wage rate. The Euler equations and the marginal conditions from the consumer problems will determine the time paths of consumption of tradables and nontradables for each group up to an arbitrary constant (three free parameters—one per group) as functions of the real exchange rate, the wage rate, and the interest rate. The integral budget constraints will help find the constants. Thus, we will have the time paths of consumption of each agent and labor inputs in each sector, and those will be determined by the time paths of the real exchange rate, the wage rate, and the exogenously given interest rate. Finally, we have to satisfy two market-clearing conditions—for labor and for the nontraded good—at each point in time. Hence, the time paths of wage and real exchange rate cannot be arbitrary—they have to be such as to clear the markets. We conclude that adding the time dimension does not help to get around the fact that the real exchange rate cannot be set arbitrarily—unless we allow for disequilibrium in some of the markets.

Introducing elements of monopoly is now standard in the new open economy macroeconomics. These elements, for example, help explain the transmission of monetary shocks across countries. We do not believe, however, that these elements are key to understanding the attitude of different sectoral actors toward the level of the real exchange rate. Economic agents will be concerned about the prices set by the government only to the extent that they are price takers in the relevant markets.

The final possibility is to allow deviations from flexible price equilibria. It is the approach we take in our model, which is presented in the next section.

III. Model

The model we develop is in the general spirit of the IB-EB model, but it is built on explicit microfoundations, introduces sectoral actors, and incorporates intertemporal constraints. Similar to the IB-EB model, our framework emphasizes complementarity between aggregate demand (expenditure-reduction) and exchange rate (expenditure-switching) policies in bringing about internal and external balances. But it goes beyond that and derives the implications of various combinations of aggregate demand, exchange rate, and income redistribution policies for the welfare of different groups in society.

A. Setup

The economy produces two goods, one that is tradable across its border and one that is not. Both goods are produced utilizing constant-returns-to-scale technology, the tradable sector using labor and sector-specific capital and the nontradable sector using only labor. The latter assumption captures, in extreme form, the idea that the nontradable sector is labor-intensive and the tradable sector is capital-intensive. The stock of capital in the tradable sector is fixed.
Aggregating across firms, we can express output in each sector as a function of labor input:

\[ q_T = f(l_T) \quad q_N = a l_N \]

By appropriate choice of units we can set the technology parameter \( a \) equal to one. Individual producers are assumed to be small, so they act as price takers in their output and input markets. Labor can move across sectors, so the wage rate must be the same in both sectors. Competition among producers must eliminate all profits in the nontradable sector, so the sales revenue will equal the wage bill and, with our choice of units, the price of the nontradable good will equal the wage rate.

\[ p_N = w \]

The return on capital, or profit in the tradable sector, equals:

\[ \pi = p_T q_T - w l_T \]

As usual, profit maximization implies that the marginal product of labor equals the real product wage:

\[ f'(l_T) = \frac{w}{p_T} \]

On the consumption side, we assume identical Cobb-Douglas preferences for both capitalists and workers. These preferences imply that a constant share \( \alpha \) of total expenditure is spent on tradables; the rest is spent on nontradables.

To focus on the government's ability to control economic outcomes, it is assumed that private agents in this economy have no access to lending or borrowing facilities, domestic or international, so their expenditures equal their income at each point in time. Aggregating across individual capitalists or workers, we can express each group's demand for the two commodities as a function of its income and prices:

\[ p_T c_T^K = \alpha y^K \quad p_N c_N^K = (1 - \alpha) y^K \quad p_T c_T^L = \alpha y^L \quad p_N c_N^L = (1 - \alpha) y^L \]

The income of a group consists of profit for capitalists and wage income for labor plus group-specific lump-sum government transfer (a negative transfer would represent a lump-sum tax):

\[ y^K = \pi + t_K \quad y^L = w(l_T + l_N) + t_L \]

We will assume that the law of one price holds in the tradable sector, so the domestic price of tradables is linked to their international price via the nominal exchange rate:

\[ p_T = e p_T^* \]
By normalizing the exchange rate and the price of foreign goods we can set

\[ e = 1; \quad p_T^* = 1 \Rightarrow p_T = 1 \]

With the normalizations we have made and with the assumption of zero profits in the nontradable sector, the real exchange rate—the ratio of nontradable to tradable goods prices—will be represented by the wage rate in our model.

In the model the government is the only agent in the economy whose expenditures are not constrained by current income. The government can borrow abroad at interest rate \( r \), and it has to borrow if transfers to one group exceed taxes collected from the other. The government’s budget constraint is:

\[ t_K + t_L = \dot{b} - rb, \]

where \( b \) is the current debt level. It can be re-written in the integral form:

\[ b_0 + \int_0^\infty (t_K + t_L) \exp(-rs) ds = 0. \]

The government performs four essential functions in the economy. It can redistribute income between capitalists and labor through taxes and transfers. It can stimulate or suppress aggregate demand by changing the sum of transfers to both groups. It sets the numeraire price (we have chosen to set the nominal exchange rate equal to one, but we could have assigned a value to any other nominal variable). Finally, we will allow the government to interfere with the market and set the wage rate, throwing the economy off the full employment equilibrium.

**B. Flexible-Price Equilibrium**

Let us first consider the benchmark where wages are allowed to adjust and all markets clear. The flexible-price equilibrium is characterized by the following three conditions:

1. Labor-market clearing. Labor demands from two sectors add up to a fixed labor supply \( I \):

\[ l_T + l_N = l \]

2. Nontradable market clearing. Demands from the two groups add up to supply:

\[ \frac{(1-\alpha)(f(l_T) - wI_T + t_K)}{w} + \frac{(1-\alpha)(wI_T + wI_N + t_l)}{w} = l_N \]

3. Profit maximization in the tradable sector:

\[ f'(l_T) = w \]
Equation (3) determines \( I_r \) as a function of \( w \). Substituting this relationship and (1) into (2), we obtain an equation for equilibrium wage:

\[
(1 - \alpha) \left( f \left[ I_r \left[ w \right] \right] + t_w + t_L \right) = \alpha w \left( I - I_r \left[ w \right] \right)
\]

This equation could also be derived by equating the share \((1 - \alpha)\) of the disposable income of the private sector, which is the sum of the income generated in the tradable sector \( f(I_r) \), income generated in the nontradable sector \( w I_N \), and government transfers \( t_T + t_N \), to the value-supply of nontradable goods \( w I_N \).

Since the private sector cannot save or dissave in our model, the difference between the consumption and production of tradables, or the trade deficit, equals the primary fiscal deficit,\(^7\) i.e., the sum of transfers to the two sectors:

\[
\begin{align*}
& c^x_T + c^x_L - f(I_r) = \alpha \left( f(I_r) - w I_T + t_T \right) + \alpha \left( w I_T + w I_N + t_L \right) - f(I_r) = \\
& = t_T + t_L - (1 - \alpha) \left( f(I_r) + t_T + t_L \right) + \alpha w I_N = t_T + t_L
\end{align*}
\]

We can see that in this model the wage rate and, by extension, output in each sector depend only on the sum of transfers, but not on their allocation.\(^8\) Redistribution will affect income and consumption of each of the two groups, but not aggregate income or consumption.

An increase in the sum of transfers will increase the demand for both tradables and nontradables at a given wage rate. To equilibrate the market for nontradables, the wage rate must go up, which, on the one hand, will drive up the marginal cost in the tradable sector and force it to release some labor (which can be employed in the nontradable sector) and, on the other hand, will suppress demand for nontradables through the substitution effect. The profits in the tradable sector will fall. The real exchange rate (the ratio of nontradable to tradable prices) will appreciate. Because of the higher demand for and lower supply of tradables, the trade deficit will increase.\(^9\) This is similar to moving along the IB schedule away from the external balance in the IB-EB model by expanding aggregate demand and appreciating the real exchange rate.

Who is the recipient of the transfer will, of course, affect the welfare of the two groups. In Appendix V we demonstrate that workers benefit not only from a transfer given to that group, but also from a transfer provided to capitalists, as the resulting increase in the

\(\text{\footnotesize 7 Adding external interest payments to both sides of the equation will yield an equality between the current account deficit and the fiscal deficit.}\)

\(\text{\footnotesize 8 This result holds because of Cobb-Douglas preferences.}\)

\(\text{\footnotesize 9 Of course, this also follows directly from the fact that the sum of transfers equals the trade deficit in this model.}\)
aggregate demand pushes up their income. Capitalists are negatively affected by a transfer to labor, as the wage increase that will stem from an expansion of the aggregate demand will depress their profits and push up the prices they are facing. Capitalists will benefit from a transfer provided to them, but the expansion of aggregate demand will offset some of the gain. A purely redistributive action—taxing one group and transferring income to the other—will naturally increase the welfare of the group receiving the transfer and hurt the other group.

So far no intertemporal elements have been brought into the analysis. It is important to fill that gap, as the aggregate transfers can only be provided by the government from borrowed resources that eventually have to be repaid with interest. A natural question to ask is what is the optimal level of transfers at a given point in time? In Appendix VI we show that if the government wants to maximize, subject to its intertemporal budget constraint, a linear combination of the present discounted values of the utilities of the two groups, the transfers to each group must be constant over time. This is easy to understand, as all the parameters of the economy—labor supply, production functions, tastes—are time-invariant. The aggregate level of transfers will also be constant and will be determined by the foreign asset position of the government at the beginning of the analysis. If at time zero the government has no foreign assets or liabilities, optimal transfers will sum to zero at each subsequent point in time. Since the economy is not growing and all borrowed resources are consumed, borrowing, which will require subsequent repayment, interferes with consumption smoothing. Depending on which sector of the economy the government favors, it can force internal resource transfer from one sector to the other, but it should run a balanced budget.

C. Manipulating RER

In the previous subsection the economy was assumed to be free of rigidities or government intervention in price-setting, so the real exchange rate adjusted smoothly to clear the markets. In what follows we allow the real exchange rate to deviate from its market-clearing value. We simply postulate this possibility rather than incorporating the reasons for the imperfect flexibility explicitly into the model. One can come up with numerous explanations as to how the RER may end up on one or the other side of its equilibrium value.

Among the most relevant ones for explaining an overvalued RER would be an unsustainable expansion of aggregate demand, for example, through lax fiscal policy, which appreciates the equilibrium RER, followed by a contraction. If wages are rigid downward, the RER will be slow to depreciate to the new equilibrium value, unless aided by a nominal devaluation. An alternative story would be the government yielding to workers' demands for higher wages. We show below that the wage in excess of the equilibrium value—the wage rate that leads to unemployment—is likely to hurt the workers. But the political calculus may well change in the presence of labor unions.

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10 This result holds if the rate of time preference equals the interest rate.

11 Each story, of course, would have to rely on some kind of market imperfections.
Temporarily undervalued currency is frequently produced by an upstart devaluation at the beginning of exchange-rate-based stabilization programs, where the exchange rate is fixed to anchor inflation expectations, but recognizing inflation inertia the authorities devalue the rate before fixing it, to build in a cushion for subsequent real appreciation. An alternative scenario is a story of macroeconomic mismanagement, where the government is unable to bring down the fiscal deficit or check the rate of credit expansion, and tries to fight the resulting current account deficits by continually devaluing the currency. Another possibility is a devaluation combined with restrictions on wage growth (incomes policies) to gain external competitiveness.

With this preamble we move to the analysis of disequilibrium outcomes.

1. **Wages that are too high**

What will be the allocation of resources if the government sets the wage higher than the market-clearing wage for a given level of transfers? The producers of tradables will hire fewer workers. Their profits will go down, and their welfare will decrease because of lower disposable income and higher price of nontradables.

The market-clearing condition in the nontradable sector will determine the number of workers employed in that sector at a given wage:

\[
\frac{(1-\alpha)(f(l_T) - w l_T + t_T)}{w} + \frac{(1-\alpha)(w l_T + w l_N + t_N)}{w} = l_N
\]

\[
(l-\alpha)(f(l_T) + t_T + t_L) = \alpha w l_N
\]

\[
l_N = \frac{1-\alpha}{\alpha} \frac{f(l_T) + t_T + t_L}{w}
\]

Compared with the flexible-wage equilibrium, \( w \) is higher, \( l_T \) is lower, and \( f(l_T) \) is lower. It can be see from the equation above that \( l_N \) is also lower. Hence, employment shrinks both in the tradable and in the nontradable sector, and overall labor demand falls short of the available supply. Will the shortfall in employment be compensated by the higher wage? Let us consider the change in the labor income.

\[
dy_T = d(w l_T) = d(w l_T + w l_N) = d\left[ w l_T + \frac{1-\alpha}{\alpha} (f(l_T) + t_T + t_L) \right] =
\]

\[
= l_T d w + \left( w + \frac{1-\alpha}{\alpha} f'(l_T) \right) d l_T = l_T d w + \frac{w}{\alpha} \frac{d l_T}{d w} d w = l_T \left( 1 + \frac{f'(l_T)}{\alpha f''(l_T)} \right) d w = l_T \left( 1 + \frac{d l_T}{\alpha l_T} \right) d w
\]

The fraction in the parentheses is the ratio of the wage elasticity of labor demand in the tradable sector to the weight of the tradables in the utility function. It is a negative number.
How big it is depends on the form of the production function. As long as the elasticity is greater than \( \alpha \)—the share of tradables in the utility function—labor income will fall if the wage is raised above its equilibrium level.\(^{12}\) Even if the production function is such that the labor income goes up when the wage is increased,\(^{13}\) the welfare of labor may still deteriorate, as the higher income is offset by a higher price of nontradables.

So, just raising the wage and appreciating the real exchange rate out of equilibrium hurts capital and is likely to hurt labor as well. If this result is unexpected, we should note that in the experiment the RER was appreciated while holding the transfers constant. Hence, despite the real appreciation, the country’s position vis-à-vis the outside world did not change. In terms of the traditional IB-EB model, the economy moved along the EB curve in the direction of RER appreciation. The fall in real income and in real domestic demand offset the appreciation so that the external position did not change, but the internal balance was disrupted. The government could add transfers to stimulate demand and move the economy away from the EB schedule and toward the IB schedule.

This result has important positive and normative implications. It shows that a real appreciation that pushes the economy off internal balance is in nobody’s interest. Analysts may conclude that a country has an overly appreciated RER by observing that its current account deficit is not sustainable. Such a country will also have a level of domestic demand that is too high relative to the gross national disposable income. This external imbalance arises quite frequently, and adjustment imposes costs on the population. Liquidating an internal imbalance, on the other hand, is Pareto-improving. Internal imbalances should, therefore, be a much more rare occurrence than external imbalances. If they do arise, it could be for two reasons. First, there might be interest groups omitted from our model. For example, all labor was assumed uniform in the model. In reality, unionized labor may push for higher wages, even at the cost of lower employment.\(^{14}\) Second, prices may not adjust to clear the markets because of rigidities or because of the government’s ineptness in managing the prices. Hence, the flexibility of the labor market and the goods markets, as well as appropriate exchange rate policies come to the forefront.

---

\(^{12}\) For example, for the Cobb-Douglas production function, the elasticity equals minus the reciprocal of the capital share, so the expression in the parentheses will be negative.

\(^{13}\) This can only be true for large enough \( l_r \). For certain production functions labor may favor a wage rate that is above the market-clearing value, but the most preferable wage is always finite.

\(^{14}\) The owners of sector-specific capital in the nontradable sector may also gain from real appreciation, although this does not have to be true, as the example of labor in our model shows. A real appreciation that pushes the economy off the internal balance will create excess supply of nontradables, so while the price of nontraded goods will be higher, not all of them will be sold.
2. Wages that are too low

If the government sets the wage at a rate that is lower than the market-clearing rate for the given level of transfers, the sum of demands for labor coming from the tradable and nontradable sectors will exceed available labor supply. This will create upward pressure on the wage and on the price of nontradables. As long as the government is able to resist the pressure and maintain the wage rate at the low level, available labor will have to be rationed. If we do not allow the nontradable sector to get enough labor to satisfy the demand for its goods, we will have to think of a mechanism to allocate nontraded output. To avoid this complication, we assume that the nontradable sector gets all the labor it needs, and the tradable sector gets the residual labor.

The allocation of resources will be driven by the market clearing condition in the nontradable sector:

\[(1 - \alpha)(f(l - l_N) + t_r + t_l) = \alpha w l_N\]

A decline in wage will move resources into the nontradable sector:

\[
dl_N = \frac{\alpha l_N}{(1 - \alpha) f'(l - l_N) + \alpha w} dw
\]

The reason for this is that as the wage goes down, so does the price of nontradables; the quantity demanded goes up, and the supply is assumed to accommodate demand.

The workers clearly lose when the wage rate goes below the equilibrium value—the level of employment does not change, but compensation for work is lower.

Even being rationed, the capitalists in the tradable sector would benefit from a wage rate that is set somewhat below the equilibrium level. This can be seen by calculating the impact of a wage decline on the capitalists' income:

\[
dy^T = d\pi = d\left[ f - w(l - l_N) \right] = -f dl_N + w dl_N - (l - l_N) dw =
\]

\[
= -\left[ (l - l_N) + (f' - w) \frac{dl_N}{dw} \right] dw = -\left[ (l - l_N) - \frac{\alpha l_N (f'(l - l_N) - w)}{(1 - \alpha) f'(l - l_N) + \alpha w} \right] dw
\]

At the flexible wage equilibrium the marginal product of labor in the tradable sector equals the wage rate, so the second expression in the brackets equals zero. In other words, in the vicinity of the equilibrium the inability of capitalists to adjust the level of employment has a second-order negative effect, while the decline in the price of the labor input has a first-order positive effect.

We can see, however, that as the wage deviates more from the equilibrium, the employment in the tradable sector shrinks further, and the divergence between the wage and the marginal product of labor grows. At some point, the second term will come to dominate the
expression, and the impact of further wage decline on the capitalists' profit will turn negative. The turning point in the welfare of the capitalists will be at a somewhat lower wage, since over a certain range the negative impact on profits will be more than compensated by the decline in the consumption price of nontradables, but there has to be a turning point eventually, as in the limit the fall in the wage will completely wipe out the tradable sector.

How will these results be affected by alternative assumptions about rationing? If the nontradable sector does not receive all the labor it requires, the output of nontradables will fall short of demand. Obviously, for any given wage rate below the equilibrium level, the capitalists in the tradable sector will be better off the more labor is allocated to that sector (up to the point of equality between the wage rate and the marginal product of labor) and the greater share of their demand for nontradables is satisfied. If the employment shares in the two sectors are fixed (which fixes the output in the nontradable sector) and the capitalists are entitled to a constant share of the nontradable output, they will prefer as low a wage rate as possible, so that their profit, hence the consumption of tradable, hence welfare increases. Workers are indifferent as to in which sector they are to be employed, so the mechanism of allocating labor between tradables and nontradables does not affect their welfare. But if the output of nontradables has to be rationed, it makes workers worse off the smaller their share is.

An important issue is the cost of maintaining the wage rate below the market equilibrium. The government is certain to incur administrative costs of policing and enforcing the wage ceiling and maintaining the rationing mechanism. In addition, if the wage rate slips upward despite the wage controls, the government will have to devalue the currency to maintain the relative prices at the chosen level, igniting an inflationary spiral. While an explicit incorporation of these factors into the model is outside the scope of this paper, it is clear that their impact on the welfare of both groups is negative. Moreover, the impact will be small for a small disequilibrium and grow large for big deviations. For labor, these costs will just add insult to injury. For capitalists, they should dampen somewhat their enthusiasm for low wages, but they would not extinguish it completely, since for a small reduction in the wage rate the effect on profit will be first order, while the cost is likely to be small.\textsuperscript{15}

To sum up the main results of this subsection, an overly appreciated real exchange rate (given the level of aggregate demand) is not in anybody's interests and may result from lack of wage and price flexibility and/or from mismanagement of the economy on the part of the government. An overly depreciated real exchange rate would hurt labor, which also stands to represent the interests of the nontradable sector in our model. It would benefit the capitalists in the tradable sector, but only up to a limit.

\textsuperscript{15} The cost may be first-order if there is a fixed cost of instituting wage controls.
IV. Political Economy Connection

The purpose of this section is to suggest how the above results can be used for political economy analysis. Any political economy model consists of the following essential ingredients: preferences of heterogeneous agents; an economic trade-off between the variables over which the preferences are defined; and a political mechanism that translates the individual preferences subject to the constraints into economic policy. The political mechanism stipulates institutional features of a polity (e.g., the electoral system) and the motivation of politicians (which could include winning or retaining office, furthering the interests of a particular group or those of society as whole, earning rents, and various combinations of these and other motives). The same economic setup— for example, the Phillips curve and the assumption that, relative to capital, labor is more averse to unemployment and more tolerant of inflation—can generate a multitude of political economy models featuring different political mechanisms.

Indeed, there is voluminous literature on the political economy of such areas as monetary policy, fiscal policy, and trade policy. Much less has been written on the political economy of exchange rate policy, even though the issue is not less important for an open economy as those aforementioned. We believe that this dearth can be attributed primarily to the fact that the preferences of economic agents over the level of real exchange rate have not been well understood and adequately modeled. Therefore, an essential building block for constructing political economy models has been missing in this field.

Our model seeks to fill this gap. It shows what the preferences over the level of the RER look like in a fully-specified economic model. Such preferences can be combined with a variety of political mechanisms to produce political economy models that would analyze specific issues of importance for particular countries. Of course, in principle one can define preferences over consumption and build a mega-model that would specify both the economic and political structure. Methodologically, this may sound like a cleaner way to proceed. There are, however, at least three practical considerations that suggest that splitting the task into two steps may be expedient. First, the mega-models may be just too complicated to solve analytically, unless exceedingly simplified assumptions are made either about the economy or about the polity. Second, the logic of comparative advantage suggests that it would be reasonable for economists to focus on modeling the economy and deriving the preferences of agents over economic policies from the fundamental structure, while political scientists are better equipped for analyzing how these preferences are aggregated and translated into policy. Third, while political mechanisms are immensely diverse, the basic laws of economics do not change from country to country. Therefore, the results derived from a particular economic model can form the basis for a variety of political economy models differing in their political superstructure.

Hence, the objective of this paper is not to present a model of exchange rate politics for a particular country or group of countries. It is, rather, to make the fundamental first step that would allow the construction of such models. To repeat, this step consists of deriving what the preferences of workers and of capitalists in the tradable sector over the level of the real exchange rate and the level of aggregate demand will be, given their preferences over consumption and their budget constraints. For a given level of aggregate demand, the
preferences of both groups of agents over the RER are shown to be single-peaked, with capitalists favoring a more depreciated rate. The single-peakness result will facilitate the construction of political economy models, as most such models rely on this property.

We finish this discussion with three examples that illustrate how these derived preferences can be incorporated into political economy models.

*Pre-election uncertainty about the exchange rate.* Alesina and Gatti (1995) present a model where uncertainty about the outcome of an election, with competing candidates having different preferences over output and inflation, translates into uncertainty about post-election levels of these variables. In a similar vein, electoral competition between a pro-labor party, which favors high real wages and a strong currency, and a party representing the tradable sector, which favors lower wages and a weaker currency, would produce uncertainty about the future level of the real exchange rate. If the left wins the election, it may raise government wages and set an example for the rest of the economy. If its competitor wins, it may devalue the currency. This uncertainty complicates wage bargaining and may engender currency speculation. In Alesina and Gatti, delegating monetary policy to an independent central bank helps eliminate the politically induced variability of output and inflation.\(^\text{16}\) In a similar fashion, eliminating election-related exchange rate uncertainty requires taking real exchange rate management out of the realm of partisan politics.\(^\text{17}\)

*Timing of devaluation.* An empirical regularity (see, e.g., Frieden et al. (2001), for evidence on Latin America) is that devaluations tend to be delayed in the run-up to elections, but are likely to happen after elections. As a result, the post-election real exchange rate is more depreciated on average than before elections. A political economy model that would generate this outcome could rely on our result that workers prefer a strong currency combined with an assumption that the incumbent government places more weight on the welfare of workers (the more numerous class) in the run-up to elections, while heeding more to the interests of capitalists or to the general welfare after the elections.\(^\text{18}\) The plausible idea that voters reward

\(^{16}\) This advantage of a politically independent central bank is different from the reduction in the inflation bias that is afforded by a conservative central banker in Rogoff's (1985) model.

\(^{17}\) If labor and product markets are sufficiently flexible in the medium term, either a strong institutional commitment to a fixed nominal rate or a floating currency would do the job. If the economy is rigid, a hard peg would not only isolate the real exchange rate from political interference but also preclude necessary adjustments in response to shocks; therefore, a floating rate is recommended in these circumstances. It should also be noted that a government can affect the real exchange rate not only through direct manipulation of prices but also by controlling aggregate demand.

\(^{18}\) Even if the weights that the government places on the welfare of the two groups do not change after the election, if the strong currency and high level of aggregate demand were maintained by external borrowing before the election, the level of aggregate demand will have to be brought down and the currency depreciated after the elections to repay the debt incurred in the run-up.
the incumbent for favorable economic outcomes typically implies that voters are myopic. A more sophisticated setup would involve forward-looking but incompletely informed voters.\textsuperscript{19}

\textit{Lobbying.} If politicians value campaign contributions, either because they see them as rents or because the contributions help the politicians win elections, then organized interest groups can influence economic policies by conditioning their contributions on policies. One could construct a model in the spirit of the Grossman and Helpman (2001) lobbying models and see how the (political) equilibrium real exchange rate would vary with the weights the politicians assign to contributions relative to the general welfare and with the relative ability of capitalists and workers to come up with contributions. Contributions could be made in the form of effort (campaign work) rather than monetary transfers – that would circumvent the complication of having to trace their effects on consumption in a general equilibrium setup.

V. Discussion

The model presented in Section III can serve as a building block for political economy models of real exchange rate determination. In terms of preferences over the level of RER, the model gets past the simple statement that the producers of tradables would prefer a more depreciated rate and the producers of nontradables prefer a more appreciated rate. It shows that it is important to specify what will happen to aggregate demand as we manipulate the real exchange rate. It makes a step toward understanding the preferences of labor. It shows how non-monotonic preferences over the RER level may arise. The model helps evaluate the impact of exchange rate misalignment on the welfare of economic agents by translating this impact into equivalent taxes and transfers. It introduces intertemporal elements into analysis. In principle, it could be used to assess the impact of any policy (defined as a time path of $w$, $t_k$, and $t_L$ satisfying the intertemporal budget constraint) on the welfare of the two groups. Conducting thought experiments like that may be the most productive way of deriving testable implications from the model.

One can also mesh the indifference curves of capital and labor with the traditional IB-EB diagram of the four combinations of macroeconomic imbalances.\textsuperscript{20} This will allow better understanding of which groups benefit from a particular imbalance and which groups would oppose to a particular adjustment. Such analysis is valuable both for the development of political economy models and for practical design of adjustment programs.

There are several issues in this paper that require further elaboration. To begin with, one might want to introduce sector-specific capital in the nontradable sector. Assuming linear technology in that sector simplifies the algebra, but it equates wage with the price of

\textsuperscript{19} Ghezzi and others (2000) present a model of political RER cycles driven by opportunistic politicians and forward-looking, incompletely informed voters. They abstract from distributional issues in their paper.

\textsuperscript{20} One can find the diagram, for example, in Clark and others (1994).
nontradables, which makes it difficult to differentiate between sectoral issues (tradables vs. nontradables) and factorial ones (capital vs. labor).

With or without nontradable sector capital, the issue of specifying disequilibrium allocation of resources should be explored further. In the model, unemployment naturally arises when the wage rate is set too high, and that is probably the more important case from a practical point of view. Still, the treatment on the other side of the equilibrium, when the wage is too low, does not appear quite satisfactory. It is not clear whether the product market or the labor market or both should be out of equilibrium and which sector should be thought of as having preferential access to labor. The issue of how the available resources will be rationed (which may vary across countries depending on institutional arrangements) requires more thorough investigation.

One might criticize the model on the grounds that it provides no vehicles for saving or dissaving to the private sector. The purpose here, or course, was to focus on the role of the government in managing the economy. At the opposite extreme, granting private agents the same access to credit as the government will lead us to the regime of Ricardian equivalence, where the government cannot affect aggregate demand. One could augment the model by allowing the capitalists to borrow while leaving the workers credit-constrained. That would leave a channel for government tax policy to affect the economy. Of course, then the views of capitalists and workers on economic policies would reflect not only their factor endowments but also the degree of access to credit.

The intertemporal aspects of the model could be exploited to a greater extent. In particular, it would be interesting to explore what happens when the rate of time preference is different from the interest rate. Intuitively it seems that a government with a high discount rate (say, a government facing an imminent election) would want to borrow up front and stimulate the economy with transfers, and then start taxing to repay the debt. Another important intertemporal aspect is capital accumulation. It was omitted from our model for reasons of tractability and to focus on factor specificity, but its inclusion would certainly enrich the model.

VI. CONCLUSIONS

This paper introduces an open-economy model where the government can affect the incomes of capitalists and workers and the prices they are facing. Given this environment, the economic agents set the levels of production and consumption of tradable and nontradable goods. The model allows to analyze not only the aggregate effects of government actions, but also their impact on the welfare of the two groups.

In the model, a higher level of aggregate transfers will increase domestic demand and lead to a higher wage rate, which will depress the profits of capitalists in the tradable sector. For that reason, labor would benefit from higher transfers even if all of them went to capitalists. Capital, on the other hand, is negatively affected by a transfer going to labor.

These are “point-in-time” considerations, ignoring the issue of how the international debt that the government needs to incur to increase aggregate transfers will be repaid. If the
government's intertemporal budget constraint is taken into account, it emerges that for the case where the rate of time discounting equals the interest rate the optimal policy is to keep transfers constant over time. If the government's initial net foreign assets are zero, the aggregate transfers should be zero as well—the government should run a balanced budget. Zero-sum transfers can be used to pursue the government's distribution agenda. In the model, the government will set those transfers in such a way as to make each group's disposable income proportional to the weight the government assigns to the welfare of that group.

If the government is allowed to manipulate directly (rather than through aggregate transfers) the real exchange rate, which is equivalent to the wage rate in the model, it turns out that appreciating the real exchange rate relative to the equilibrium level serves nobody's interest. The capitalists lose as their input price goes up, and the workers lose as employment goes down. An overly depreciated RER would hurt the workers, who would work as much for less money. Capitalists in the tradable sector would benefit from a moderate real depreciation, but after a certain threshold further depreciation would hurt their interests. Thus, holding the level of fiscal stimulus constant, the preferences of the two groups over the level of the real exchange rate are shown to be single-peaked – a property that will facilitate construction of political economy models of exchange rate policies.

We view this analysis as a contribution to better understanding of interests of economic actors, which would allow political economy models to move from ad hoc specifications of preferences to a more solid foundation. From the point of view of economics, the main contribution is that this paper provides microfoundations for the widely used IB-EB model, building the model from such primitives as factor endowments, utility functions, and production functions and incorporating intertemporal budget constraints. Finally, the design of macroeconomic adjustment programs can be enhanced by improved understanding of how the adjustment will affect different interests in the economy.
Appendix I. Wages and RER in the Two-Sector Two-Factor Model

Suppose we have two goods—$T$ and $N$—and two inputs—$k$ and $l$. Let $p^T, p^N, r,$ and $w$ be the prices of the goods and the factors, respectively. Assume constant returns to scale and perfect competition. Then the price of each commodity will equal its unit cost (the cost of producing one unit of the commodity in the cheapest way given the prices of the factors).

$$p^T = u(r, w) \quad p^N = v(r, w)$$

We can fully differentiate these equations.

$$dp^T = u_r dr + u_w dw \quad dp^N = v_r dr + v_w dw$$

The subscripts denote partial derivatives. By Shephard’s lemma, the partial derivative of a cost function with respect to the price of an input equals the factor demand for that input. Since we are differentiating a unit cost function, the derivatives will give us the amount of input required to produce one unit of output. Given that we have constant returns to scale, it is also the ratio of the amount of input actually employed to the amount of output actually produced.

$$dp^T = \frac{k^T}{q^T} dr + \frac{l^T}{q^T} dw \quad dp^N = \frac{k^N}{q^N} dr + \frac{l^N}{q^N} dw$$

Here the superscripts denote in which sector a factor is employed. Solving for $dr$ and $dw$, we obtain:

$$dr = \frac{\frac{l^N}{q^N} dp^T - \frac{l^T}{q^T} dp^N}{\frac{k^T}{q^T} - \frac{k^N}{q^N}} = \frac{\frac{q^T}{l^T} dp^T - \frac{q^N}{l^N} dp^N}{\frac{k^T}{k^N} - \frac{l^T}{l^N}}$$

$$dw = \frac{\frac{k^T}{q^T} dp^N - \frac{k^N}{q^N} dp^T}{\frac{l^T}{q^T} - \frac{l^N}{q^N}} = \frac{\frac{q^T}{k^T} dp^T - \frac{q^N}{k^N} dp^N}{\frac{l^T}{k^T} - \frac{l^N}{k^N}}$$

If $p^N$ goes up while $p^T$ does not change, $w$ (the price of factor $l$) will go up if and only if sector $N$ is more labor-intensive than sector $T$ (i.e., if sector $N$ has a higher labor-to-capital ratio). Assuming that is the case, the price of factor $k$ ($r$) will go down if $p^N$ goes up and $p^T$ does not change. Obviously, $w$ will increase in relation to $p^T$ and to $r$. It can be shown that $w$ will increase in relation to $p^N$ as well (the magnification effect—if the price of a good goes up, the price of the intensive factor goes up more than proportionately). Indeed,
\[ \frac{dw}{dp^N} = \frac{dw}{dp^N} \frac{p^N}{w} = \frac{q^N}{k^N} \frac{p^N}{w} \frac{p^N}{l^N} - \frac{l^N}{k^N} > 1 \]

Thus, if the price of a commodity increases, the price of the intensive factor increases in relation to the prices of both commodities, while the price of the other factor goes down.
Appendix II. Two-Sector Model: Consumption Side

Producers in each sector want to maximize their utility, with production generating their income. The problem can be split into profit maximization (considered in the main text) and utility maximization given profit. The latter problem takes the form:

\[
\begin{align*}
\max & \quad u(c_T, c_N) \\
\text{s.t.} & \quad c_T + p c_N = \pi
\end{align*}
\]

Let \( c_T(p, \pi), c_N(p, \pi) \) denote the solution. Now we recognize that the profit itself is a function of the relative price \( p \) and introduce the indirect utility function:

\[
V(p) = u(c_T(p, \pi(p)), c_N(p, \pi(p)) )
\]

How does it vary with the real exchange rate?

\[
\frac{dV(p)}{dp} = \frac{\partial u}{\partial c_T} \left[ \frac{\partial c_T}{\partial p} + \frac{\partial c_T}{\partial \pi} \frac{\partial \pi}{\partial p} \right] + \frac{\partial u}{\partial c_N} \left[ \frac{\partial c_N}{\partial p} + \frac{\partial c_N}{\partial \pi} \frac{\partial \pi}{\partial p} \right]
\]

\[
= \frac{\partial \pi}{\partial p} \left[ \frac{\partial u}{\partial c_T} \frac{\partial c_T}{\partial \pi} + \frac{\partial u}{\partial c_N} \frac{\partial c_N}{\partial \pi} \right] + \left[ \frac{\partial u}{\partial c_T} \frac{\partial c_T}{\partial p} + \frac{\partial u}{\partial c_N} \frac{\partial c_N}{\partial p} \right]
\]

The first bracket is the derivative of a standard indirect utility function\(^{21}\) with respect to income, so it is positive. The second bracket is the derivative of a standard utility function with respect to the price of the nontradable good. By Roy's identity, the second bracket equals the first bracket multiplied by minus the demand for nontradables. Replacing the price derivatives of the profit functions with their corresponding expressions, we can rewrite the price derivatives of the indirect utility functions for the two sectors in the following way.

\[
\frac{dV_T}{dp} = -(c_T^* + c_N^*) \frac{\partial \tilde{V}_T}{\partial \pi}
\]

\[
\frac{dV_N}{dp} = (q_N^* - c_N^*) \frac{\partial \tilde{V}_N}{\partial \pi}
\]

Here tilde denotes the "standard" indirect utility function. The first expression is clearly negative, so the producers of tradables are hurt by real exchange rate appreciation from any level. The second expression is positive, since the producers of nontradables must produce

\(^{21}\) That is the indirect utility function from a consumer problem where income does not depend on prices.
more nontradables than they themselves consume so as to leave something for the other agents in the economy. Hence, the producers of nontradables will unambiguously benefit from real appreciation.

A simple graph can demonstrate the same point. For a given profit, the budget constraint for the choice of a consumption basket for each group is a straight line with that crosses the axes at points whose distance from the origin equals to the ratio of the profit to the price of the respective good. It is easy to see that the budget constraint for the producers of nontradables will shift out if $p$ goes up, even if they do not expand their production, so real appreciation will allow them to consume more and increase their welfare. The same holds for the producers of tradables if $p$ goes down.

![Budget constraint for producers of tradables. Budget set expands as $p$ goes down.](image1)

![Budget constraint for producers of nontradables. Budget set expands as $p$ goes up.](image2)
Appendix III. Two-Sector Model with Labor Input

Output in the two sectors depends on labor input.

\[ q_T = f(l_T) \quad q_N = g(l_N) \]

Assuming inelastic supply of labor normalized to one, the labor problem takes the form:

\[
\max \quad u(c_T, c_N) \\
\text{s.t.} \quad p_Tc_T + p_Nc_N = w
\]

According to standard microeconomic theory, the indirect utility function \( V_L(p_T, p_N, w) \) is non-increasing in prices and nondecreasing in wage.

Producers in the tradable sector solve the following problem:

\[
\max u(c_T, c_N) \\
\text{s.t.} \quad p_Tc_T + p_Nc_N = p_Tf(l_T) - wI_T
\]

Optimization yields two first-order conditions, one on the consumption side and one on the production side:

\[
\frac{\partial u}{\partial c_N} = \frac{p_N}{p_T} \quad p_Tf'(l_T) = w
\]

Obviously, an increase in wage or in the price of nontradables will shrink the feasible set and decrease attainable utility. An increase in the price of tradables will expand the feasible set and improve the welfare of producers of tradables.

Similarly, the producers of nontradables would benefit from an increase in the price of nontradables and would be hurt by an increase in wages or in the price of tradables.

Finally, to pin down the relative prices, we introduce market clearing conditions for labor:

\[ l_T + l_N = 1 \]

and for nontradables:

\[ c_T^T + c_N^N + c_N^L = g(l_N) \]

The subscript identifies the good consumed, and the superscript denotes who consumes the good. So, for example, \( c_N^T \) is the consumption of (or demand for) the nontradable good by the producers of tradables. Labor inputs into each sector are functions of that sector's real
product wage. Consumption demand for each good is a function of the two prices and (through income) of wages. Due to homogeneity properties, demand can be expressed as a function of relative prices. Hence, the two market clearing conditions fix the relative prices and wages.
Appendix IV. Adding Intertemporal Elements

On the production side, we will have the same setup as in Appendix III. We will now allow domestic agents to borrow and lend abroad at the interest rate \( r \), so their demand functions will be derived from intertemporal optimization.

The optimization problem for the producers of tradables takes the following form:

\[
\max \int_0^\infty u(c_t, c_N)e^{-\rho t}dt \\
\text{s.t. } c_t + pc_N + rb = f(l_t) - wl_t + b \\
b(0) = b_0^T
\]

Here the price of tradables is set to unity. The price of nontradables is now also the relative price of nontradables and tradables, or the real exchange rate. We denote it \( p \). The level of debt is \( b \). All variables are functions of time. In addition to identifying the initial endowment (or initial indebtedness) and the flow-budget constraint, we should also impose a no-Ponzi-game condition. We can combine these constraints into an integral budget constraint that says that the present discounted value of consumption must equal the present discounted value of profit minus the initial debt:

\[
\int_0^\infty (c_t + pc_N)e^{-\int_{t_0}^t r(s)ds}dt = \int_0^\infty (f(l_t) - wl_t)e^{-\int_{t_0}^t r(s)ds}dt - b_0^T
\]

To solve the problem, we set up a Hamiltonian:

\[
H = u(c_t, c_N)e^{-\rho t} + \mu[c_t + pc_N + rb - f(l_t) + wl_t]
\]

The first-order conditions are as follows:

\[
\frac{\partial H}{\partial c_t} = \frac{\partial u}{\partial c_t}e^{-\rho t} + \mu = 0 \\
\frac{\partial H}{\partial c_N} = \frac{\partial u}{\partial c_N}e^{-\rho t} + \mu p = 0 \\
\frac{\partial H}{\partial l_t} = \mu[w - f'(l_t)] = 0 \\
\frac{\partial H}{\partial b} = \mu r = -\dot{\mu}
\]

These four equations can be manipulated to exclude \( \mu \) and obtain the standard intratemporal marginal conditions on the consumption side and on the production side and the Euler equation:
\[
\frac{\partial u}{\partial c_n} = p \\
\frac{\partial u}{\partial c_r} = w
\]

Analogous conditions can be derived for the producers of nontradables and for labor, except that for labor the present discounted value of consumption must equal the present discounted value of their labor income (wages) rather than profits minus the initial level of debt, and they won’t have a “supply-side” condition as they supply their labor inelastically. For the producers of nontradables, the supply-side condition takes the form:

\[
p_g \frac{d}{dt} (l_N) = w
\]

The intratemporal and intertemporal optimization on the consumption side (the equality between the marginal rate of substitution and the relative price and the Euler equation) are identical for each group.

To solve the model means to determine the time paths of the eight quantity variables (the consumption of each of the two commodities by each of the three groups and the employment in each of the two sectors) and the two price variables (the real exchange rate and the wage rate). The five marginal conditions (three on the consumption side and two on the production side), the three Euler equations, and the three integral budget constraints allow one to express the time paths of the eight quantity variables as functionals of initial levels of debt and the time paths of the two price variables and the interest rate. Finally, if we demand that at each point in time the employment in the two sectors add up to the fixed labor supply and that the demand for nontradables equal the supply of nontradables, these conditions will pin down the time paths of the real exchange rate and the wage rate. Hence, consumption, production, sectoral employment, wages, and relative prices in this model are fully determined by initial conditions and the time path of the interest rate.
Appendix V. Welfare Evaluation

We start out by deriving the indirect utility function. The Cobb-Douglas utility function has the following form:

\[ u(c_T, c_N) = \alpha \ln(c_T) + (1 - \alpha) \ln(c_N) \]

Given income \( y \) and the prices, the consumer purchases the following basket:

\[ c_T = \alpha \frac{y}{p_T}, \quad c_N = (1 - \alpha) \frac{y}{p_N} \]

The indirect utility function takes the form

\[
\begin{align*}

v(p_T, p_N, y) &= \alpha \ln\left(\alpha \frac{y}{p_T}\right) + (1 - \alpha) \ln\left((1 - \alpha) \frac{y}{p_N}\right) = \\
&= \alpha \ln \alpha + (1 - \alpha) \ln(1 - \alpha) + \ln y - \alpha \ln p_T - (1 - \alpha) \ln p_N \\

\end{align*}
\]

In our case \( p_T \) is fixed and \( p_N = w \). So the variable portion of the indirect utility function is:

\[ v(w, y) = \ln y - (1 - \alpha) \ln w \]

The impact of any change on the welfare of a group can be evaluated by looking at the difference between the percentage change in the group’s income and the fraction of nontradables in the utility function times the percentage change in the wage rate (which proxies for the price of nontradables in this equation). For any parameter \( x \), the impact of its change on the welfare can be computed as:

\[ \frac{\partial v}{\partial x} = \frac{1}{y} \frac{\partial y}{\partial x} - (1 - \alpha) \frac{1}{w} \frac{\partial w}{\partial x} \]

The incomes of the two groups equal, respectively:

\[ y^K = \pi + t_K = f(l_T) - w l_T + t_K \]

\[ y^L = w(l_T + l_N) + t_L \]

In the flexible-wage equilibrium, \( t_L \) and \( t_K \) are controlled by the government, while \( w \) adjusts so that total employment equals total labor supply. The impacts of small changes in transfers on the welfare of the two groups can be calculated as follows:
\[
\frac{\partial v^L}{\partial t_L} = \frac{1}{wl + t_L} \left(1 + \frac{\partial w}{\partial t_L}\right) - \frac{1 - \alpha}{w} \frac{\partial w}{\partial t_L} = \frac{1}{wl + t_L} - \frac{1 - \alpha}{w} \left[\frac{1}{w + \frac{t_L}{l}} - \frac{1 - \alpha}{f''[\ell_T]} \frac{f''[\ell_T]}{f''[\ell_T]} \right]
\]

\[
\frac{\partial v^L}{\partial t_K} = \left[\frac{1}{w + \frac{t_L}{l}} - \frac{1 - \alpha}{w} \frac{\partial w}{\partial t_K}\right] - \frac{1 - \alpha}{w} \frac{\partial w}{\partial t_K} = \left[\frac{1}{w + \frac{t_L}{l}} - \frac{1 - \alpha}{f''[\ell_T]} \right] \frac{f''[\ell_T]}{f''[\ell_T]}
\]

\[
\frac{\partial v^K}{\partial t_K} = \frac{1}{\pi + t_K} \left(1 + \frac{\partial \pi}{\partial t_K}\right) - \frac{1 - \alpha}{\pi + t_K} \frac{\partial w}{\partial t_K} = \frac{1}{\pi + t_K} - \left[\frac{l_T}{\pi + t_K} + \frac{1 - \alpha}{w} \right] \frac{1 - \alpha}{f''[\ell_T]} \frac{f''[\ell_T]}{f''[\ell_T]}
\]

\[
\frac{\partial v^K}{\partial t_L} = \left[-\frac{l_T}{\pi + t_K} + \frac{1 - \alpha}{w} \right] f''[\ell_T] \frac{\partial t_T}{\partial t_K} = \left[-\frac{l_T}{\pi + t_K} + \frac{1 - \alpha}{w} \right] \frac{1 - \alpha}{f''[\ell_T]} \frac{f''[\ell_T]}{f''[\ell_T]}
\]

As is noted in the main text, the wage rate in the flexible price equilibrium depends on the aggregate level of transfers and not on their distribution, so the positive impact of a unit increase in transfer to labor on the wage rate is the same as that of a unit increase in transfer to capital. It is interesting to note that, unless transfers per worker are comparable in size to wages, the bracket in the expressions for labor utility will be positive, so that labor will benefit from an expansion in aggregate demand even if it comes completely from supplementing the incomes of capitalists. Of course, workers would benefit even more from a transfer provided to them. In contrast, capitalists would unambiguously lose out if aggregate demand were stimulated by providing transfers to labor. The benefit to the capitalists of a transfer given to them is partially offset by an increase in wage, which depresses their profit by raising the price of the labor input and makes their consumption basket more expensive by raising the price of nontradables.

A redistribution—an increase in transfer to one group paid for by a decrease in the amount of transfer provided to the other group—would leave the wage rate unchanged and obviously benefit the group receiving the increase and hurt the group that pays for the increase:

\[
\frac{\partial v^L}{\partial t_L} \bigg|_{\ell_T = 0} = \frac{1}{wl + t_L}
\]

\[
\frac{\partial v^K}{\partial t_L} \bigg|_{\ell_T = 0} = -\frac{1}{\pi + t_K}
\]

22 To be precise, unless they exceed \(\alpha/(1-\alpha)\) times the wage rate.
Appendix VI. Intertemporal Analysis

Suppose the government wants to maximize a weighted sum of present discounted values of the capitalists’ and labor’s utility functions subject to its budget constraint:

\[
\max \int_0^\infty \left\{ \beta u(c^c_T, c^k_T) + (1 - \beta) u(c^l_T, c^k_N) \right\} \exp(-\rho s) ds \\
\text{s.t. } b_0 + \int_0^\infty (t_k + t_L) \exp(-rs) ds = 0
\]

Each group’s consumption will depend on its disposable incomes and the prices it faces. We will assume that the wage rate adjusts to clear the labor market, so the allocation of resources at each moment of time is a function of the government’s transfers to each group. Hence we can replace the direct utility functions in the maximand with indirect utility functions, whose functional form was derived in Appendix V:

\[
\beta u(c^k_T, c^k_N) + (1 - \beta) u(c^l_T, c^k_N) = \beta \left[ \ln(\pi + t_k) - (1 - \alpha) \ln w \right] + (1 - \beta) \left[ \ln(\omega l + t_L) - (1 - \alpha) \ln w \right] = \\
= \beta \ln(\pi + t_k) + (1 - \beta) \ln(\omega l + t_L) - (1 - \alpha) \ln w
\]

where \( \pi \) and \( w \) depend on aggregate transfers.

To solve the optimization problem, we form a Hamiltonian:

\[
H = \left[ \beta \ln(f(l_T) - f'(l_T)l_T + t_k) + (1 - \beta) \ln(f'(l_T)l_T + t_L) - (1 - \alpha) \ln f''(l_T) \right] e^{-\rho s} + \mu(t_k + t_L + rb)
\]

and apply first-order conditions:

\[
\frac{\partial H}{\partial t_k} = \beta \frac{1 - f''(l_T)l_T}{f(l_T) - f'(l_T)l_T + t_k} \frac{dl_T}{dt} + (1 - \beta) \frac{f''(l_T)l_T}{f'(l_T)l_T + t_L} - (1 - \alpha) \frac{f''(l_T)}{f'(l_T)} \ln f'(l_T) \right] e^{-\rho s} + \mu = 0
\]

\[
\frac{\partial H}{\partial t_L} = \beta \frac{-f''(l_T)l_T}{f(l_T) - f'(l_T)l_T + t_k} \frac{dl_T}{dt} + (1 - \beta) \frac{1 + f''(l_T)l_T}{f'(l_T)l_T + t_L} - (1 - \alpha) \frac{f''(l_T)}{f'(l_T)} \ln f'(l_T) \right] e^{-\rho s} + \mu = 0
\]

\[
\frac{\partial H}{\partial b} = r \mu = -\dot{\mu}
\]

With constant \( r \), the co-state variable \( \mu \) will decline exponentially over time:

\[
\mu(s) = \mu_0 e^{-rs}
\]
The first two first-order conditions can be rearranged in the following way:\(^{23}\)

\[
\frac{\beta}{y_K} \frac{dw}{dt} \left[ -\frac{\beta l_t}{y_K} + \frac{(1-\beta)l}{y_L} - \frac{1-\alpha}{w} \right] = \mu e^{\alpha s} \quad \frac{dw}{dt} = f^*[l_t] \frac{dl_t}{dt} = \frac{1-\alpha}{\alpha l_n - \frac{f^*[l_t]}{f^*[l_t]}}
\]

\[
\frac{1-\beta}{y_L} \frac{dw}{dt} \left[ -\frac{\beta l_t}{y_K} + \frac{(1-\beta)l}{y_L} - \frac{1-\alpha}{w} \right] = \mu e^{\alpha s}
\]

Comparing the two equations, we can see that they can hold simultaneously only if

\[
\frac{\beta}{y_K} = \frac{1-\beta}{y_L}
\]

This intratemporal condition says that at each point in time the government will see to it that each group’s disposable income be proportional to the weight the government puts on the welfare of that group.

Regarding intertemporal optimization, we can note that in case \(\rho = r\), i.e., when the rate of time preference equals the interest rate, the right-hand side in the equations above is constant, and so should be the left-hand side. This means that the level of transfers should be constant over time. We can then take the aggregate transfers outside the integral side in the government’s budget constraint and obtain the following result:

\[
t_K + t_L = -rb_0
\]

The sum of transfers equals the annuity value of the country’s initial assets. If at time zero the country’s net foreign asset position is zero, the government should run a balanced budget.

---

\(^{23}\) Just to avoid confusion, please note that \(t\) denotes the level of aggregate transfers, not time. The time variable is \(s\).
REFERENCES


