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Abstract

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We study a monetary, general equilibrium economy in which banks exist because they provide intertemporal insurance to risk-averse depositors. A "banking crisis" is defined as a case in which banks exhaust their reserve assets. Under different model specifications, the banking industry is either a monopoly bank or a competitive banking industry. If the nominal rate of interest (rate of inflation) is below (above) some threshold, a monopolistic banking system will always result in a higher (lower) crisis probability. Thus, the relative crisis probabilities under the two banking systems cannot be determined independently of the conduct of monetary policy. We further show that the probability of a "costly banking crisis" is always higher under competition than under monopoly. However, this apparent advantage of the monopoly bank is due strictly to the fact that it provides relatively less valuable intertemporal insurance. These theoretical results suggest that banking system structure may matter for financial stability.

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I. Introduction and Summary

Banking panics and crises have been relatively frequent and costly events. In the United States, for instance, banking panics occurred in 1819, 1837, 1857, 1873, 1893, 1907, and 1930–33. These crises were not only frequent but nearly always were associated with major recessions. In order to prevent altogether—or, at least, to reduce the frequency of banking crises—many countries created lenders of last resort and deposit insurance systems. Early in their inception, such banking system "safety nets" were often accompanied by considerable regulation. However, as bank regulation started to be relaxed in the 1970s, banking crises reemerged as frequent and costly phenomena. Caprio and Klingebiel (1997), for instance, document the occurrence of over 80 banking crises during the last three decades. These crises occurred throughout the globe and in all types of economies: developed, developing, and transitional. Modern banking crises often involve a different type of cost than historical banking panics—in particular, the government often bails out the banking system rather than allowing depositors to bear loses directly—they have nonetheless often been costly.

Interestingly, there is a variety of experience with respect to the output losses associated with modern banking crises. In a sample of 23 economies that experienced a single postwar banking crisis, Boyd, Kwak, and Smith (2002) argue that four of these economies experienced no output losses whatsoever. However, of the remaining countries in the sample, the median value of lost output is 7 percent of the discounted present value of current and future GDP. For the seven sample economies with the largest output losses, the median value of lost output is more than 10 percent of the discounted present value of current and future GDP. Thus some banking crises are associated with no real resource losses whatsoever, while others seem to be accompanied by quite large resource losses.

Among macroeconomic factors that help to predict the occurrence of banking crises, inflation is by far the most prominent. Demirguc-Kunt and Detragiache (1997) show that higher rates of inflation are one of the few macroeconomic factors that robustly increase the probability of a banking crisis. Boyd, Gomis, Kwak, and Smith (2001) argue that economies that fail to reduce their inflation rates during and after a banking crisis have a much higher probability of experiencing subsequent crises.

There is a considerable theoretical literature that has examined the relationship between banking industrial organization and the risk (probability of failure) of individual banks. This literature has produced somewhat mixed findings[2] and suffers from several major limitations. The first is that this work is mostly partial equilibrium and therefore ignores possibly important feedbacks from the banking industry (or banking regulation) to the macro

[2]For example, Helmann, Murdoch, and Stiglitz (2000), Allen and Gale (2001), or Boyd and De Nicoló (2003). The only empirical research we have seen on this topic is by De Nicoló et al. (2003) and Beck, Demirgüç-Kunt, and Levine (2003).

economy, (Boyd, Chang, and Smith (2002)). Another limitation is that this research has not considered inflation—or the possibility that inflation is related to banking crises—even though the existence of such a relationship has strong empirical support. To our knowledge, none of this literature addresses the following basic issues.

1. What are the relative probabilities of banking crises in competitive versus monopolistic banking systems (ceteris paribus)?

2. What is the relative probability that a banking crisis will involve some real output losses in competitive versus monopolistic banking systems?

3. What are the expected output losses from a crises in competitive versus monopolistic banking systems?

This paper addresses each of these issues. In addition, we introduce a government that chooses a steady-state inflation rate (nominal rate of interest). We can then ask:

4. How does the inflation rate affect the probability of a banking crisis under the two systems? How does it affect the probability that some resource losses will be observed?

To that end, we need a monetary model with banks that are potentially subject to crises, and here we draw on work by Champ, Smith, and Williamson (1996) and Smith (2002). Those analyses, however, consider only economies with competitive banking systems. In this study, we compare the situation with monopolistic versus competitive banking systems. As will be shown, the nature of banking system competition has significant implications for the probability that a banking crisis will occur, and for what happens if it does.

To explain our main results, it is useful to start with a description of what we mean by a "banking crisis." The specific notion of a banking crisis that we employ is motivated primarily by the historical experiences of banking panics. For instance, Noyes (1909) listed the following distinguishing features of a banking panic: (a) the suspension of cash payments to depositors; (b) the depletion of cash reserves; (c) the emergence of a premium on currency; and (d) the use of "emergency expedients" to provide substitutes for media of exchange. This is how we will think about a banking crisis. We employ a model that is explicit about the role of currency in transactions, and in which banks arise to insure agents against what amounts to "liquidity preference shocks." In addition, the withdrawal demand confronted by banks—that is, the demand to convert deposits into currency—is subject to aggregate randomness. When withdrawal demand is high enough, banks will exhaust their (optimally chosen level of) cash reserves. As noted by Champ, Smith, and Williamson (1996), when banks exhaust their reserves this can be associated with a suspension of convertibility of deposits, currency premia will be observed, and substitutes for explicit media of exchange may be used. Thus the exhaustion of cash reserves potentially involves all of the features of historical banking crises noted by Noyes. We then analyze the potential for, and the nature of, such banking crises under competitive versus monopolistic banking systems. We now briefly describe our major results.[3]

First, the relative probability of a banking crisis under competition versus monopoly cannot be inferred independently of monetary policy. If the nominal rate of interest (the rate of inflation) is below some threshold, a monopolistic banking system will involve a higher crisis probability of a panic than a corresponding competitive banking system. However, if the nominal interest rate (the rate of inflation) is above that threshold, the crisis probability will be higher under competition than under monopoly. Intuitively, a monopolistic bank can generate higher expected profits by limiting its holdings of cash reserves. Other things being equal, this will raise the probability of a banking crisis (reserve exhaustion) relative to a competitive banking system. However, a monopolistic bank also offers depositors relatively lower returns, ceteris paribus. This factor tends to reduce the probability of reserve exhaustion. The relative probability of panic under monopoly versus competition depends on the strengths of these two forces. At low (high) levels of nominal interest rates, the former (latter) force will dominate, and a monopolistic banking system will confront a higher (lower) probability of a crisis than a competitive banking system.

Second, we show that the probability of some output loss due to a banking crisis is always higher under competition than under a monopolistic banking system. This occurs because the monopoly bank has a strong (profit) motive to economize on the liquidation of any asset except cash. As will be seen, however, this is a sort of "mixed blessing" with a monopoly banking system. The monopoly bank is able to avoid some real output losses only because it provides relatively poor intertemporal insurance to depositors. This is an important point because it reflects another way that a banking industrial organization may be evaluated—in this case, their efficiency in providing intertemporal insurance. This link has received no attention we are aware of in either the theoretical or empirical literature.

Finally, increases in the nominal rate of interest (the rate of inflation) increase the probability of a banking crisis under both competitive and monopolistic banking systems. This is consistent with the empirical evidence on inflation and the probability of a banking crisis cited above.[4]

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[3]Admittedly, in the environment we study, there is no distinction between a (historical) banking panic and a (modern) banking crisis. That is because we do not allow for the existence of the regulatory interventions lender of last resort, and deposit insurance—that have arguable turned the former into the latter. Our thinking is to study the effects of bank competition in the simplest general equilibrium environment possible. Adding a discount window and deposit insurance to the model are works in progress.

[4]In addition, we show that in economies with competitive banking systems, increases in the nominal rate of interest (the rate of inflation) raise the probability that some output losses will occur associated with a banking crisis. However, when the banking system is

The remainder of the paper proceeds as follows. Section II describes the general environment analyzed. Section III introduces a competitive banking system, and describes the equilibrium behavior of competitive banks. This section draws heavily on Smith (2002) and is presented in cursory manner, for completeness. Section IV conducts a comparable analysis for a monopolistic banking system. Specifically, this section describes the behavior of agents who do not use banks. Such an analysis is necessary since our monopoly banks can extract all the surplus depositors might earn from having access to a banking system. Section V then considers how the probabilities of a banking crisis, the probabilities of output losses due to a crisis, and expected output losses due to a crisis differ under competition versus monopoly in banking. Proofs of all propositions are detailed in the Appendix.

II. The Environment

We consider a discrete time economy populated by an infinite sequence of two-period-lived, overlapping generations. Let $t = 1, 2, ...$ index time.

The economy consists of two islands. At each date a new young generation is born on each island comprised of two types of agents. One type is potential bank depositors. In each generation there is a continuum of young depositors on each island with unit mass. The other type is potential bank operators. There are N of these agents born on each island at each date. Setting $N = (>) 1$ allows us to consider a monopolistic (competitive) banking system.

There is a single good at each date. All depositors are endowed with $w > 0$ units of this good when young, and no endowment when old. In addition, for simplicity, depositors care only about second period consumption, denoted by c. Then depositors have the lifetime utility level $u(c) = \ln c$. Bankers, on the other hand, have no endowment of goods in either period of life. They also care only about second period consumption, and are risk neutral.

All agents—bankers and depositors—have access to a technology for storing the consumption good. One unit of the good stored at t yields $R > 1$ units of consumption at $t+1$. In addition, a storage investment initiated in any period can be "scrapped" later in the same period. Scrapped storage investments yield $r < 1$ units of consumption.

Following Townsend (1980, 1987), we generate a transactions role for money by emphasizing limited communication across spatially separated markets. In particular, at each date agents can trade and communicate only with other agents who inhabit the same location. The nature of trade is as follows. Young agents can sell consumption goods in exchange for currency held initially by old agents. In addition, young agents can store the good. These transactions may or may not be intermediated, as described in further detail below.

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monopolistic, the probability of some such output loss is independent of the nominal interest (inflation) rate, if the nominal rate of interest is below some threshold.

After young agents allocate their savings between currency and storage investments at t, a fraction π_t of young agents is selected at random to be moved to "the other" location. The value π_t is the same in each location. However, at each date π_t is itself a random variable drawn from the distribution F. Let f denote the *pdf* associated with this distribution, let $[0,1]$ be the support of f, and let $f(\pi) > 0$ hold $\forall \pi \in (0,1)$. The distribution F is known by all agents, but the value π_t is not known at the time agents allocate their portfolios.

The significance of random relocation is as follows. If agents invest directly (that is, if savings are not intermediated), then they can leave storage investments in place until maturity. However, agents who relocate will have no access to any investments left in storage.[5] Thus relocated agents scrap any storage investments and carry the goods obtained along with any currency in their possession to their new location. Currency held can then be used to purchase additional consumption goods. Thus relocation acts like a liquidity preference shock that forces agents to liquidate higher yielding in favor of lower yielding assets. Agents would therefore like to be insured against the event of being relocated.

As in Diamond and Dybvig (1983), such insurance can be provided through banks. If banks operate, at the beginning of a period depositors deposit all their funds with a bank.[6] The bank uses deposits to acquire primary assets: currency and storage investments. After banks allocate their portfolios between these assets, the value π_t is realized, and the specific identities of the agents to be relocated are revealed.

For agents who are relocated, they contact their bank in a decentralized manner,[7] and make withdrawals. When a withdrawal is made, a depositor might receive cash—enabling him to purchase goods in his new location—or liquidated storage investments—which can be carried to the agent's new location and consumed. However, spatial separation and limited communication imply that agents do not remain in contact with their bank after they physically relocate. Hence they cannot use checks, credit cards, or other private credit instruments in their new location. On the other hand, agents who are not relocated do remain in contact with their bank, and hence do not need cash in order to transact.

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[5]Goods in storage are not transportable. Also, goods cannot be kept out of storage and transported across locations—goods must go through the storage process in order to be carried into future periods.

[6]As in Diamond and Dybvig (1983), if banks operate, all savings will be intermediated.

[7]In particular, no markets operate after it is revealed who is to be relocated.

Bankers differ from depositors not only in their preferences and endowments. They also differ in that bankers are never relocated, so that they can always be contacted by their depositors.

In addition to bankers and depositors there is a government, which injects or withdraws fiat money. Let M_t be the time t money stock per depositor. Then the nominal money stock evolves according to $M_{t+1} = \sigma M_t$. The gross rate of money creation, σ, is selected once and for all in the initial period. Monetary injections or withdrawals are accomplished via lump-sum transfers to young agents.

III. A Competitive Banking System

A. The Competitive Bank Problem

In this section we assume that the number of bankers, *N*, exceeds one. In this context this is sufficient to guarantee that the banking system is competitive.

With competitive banks, at each date *t* each young depositor deposits his entire after-tax endowment, $w+\tau_t$, with a bank. Banks use their deposits to acquire the economy's primary assets: currency—which banks hold as reserves to pay relocated agents—and storage investments. Let m_t denote the real value of cash reserves held by a representative bank (per depositor), and let s_t denote the real value of storage investments. Since bankers have no resources of their own, the bank faces the balance sheet constraint.

(3.1) $$m_t + s_t \le w + \tau_t$$

In addition to choosing reserve and investment levels, m_t and s_t, the bank makes several other choices. Among these, a bank chooses a schedule of gross, real returns paid to depositors who do [do not] relocate at *t*. We denote these returns by $d^m(\pi_t)\left[d(\pi_t)\right]$, and note that these returns will be a function of the aggregate state, π_t.[8] Since banks must give relocated agents either cash or the proceeds of liquidated storage investments to finance consumption in their new locations, there are several constraints that a bank faces on its choice of deposit return schedules. In order to describe these constraints, let $\alpha(\pi_t) \in [0,1]$ denote the fraction of its cash reserves that the bank pays out at *t* as function of the time *t* state, let $\delta(\pi_t) \in [0,1]$ denote the fraction of its storage investments that the bank scraps at *t*.

[8]To be more specific, we allow banks to optimally insure individuals against the event of relocation and also against the realization of π_t. In particular, banks are not subject to a sequential service constraint here.

Finally, define $\gamma_t \equiv m_t / (w_t + \tau_t)$ to be a representation bank's reserve-deposit ratio. It is then possible to write the bank's remaining resource constraints as follows:

(3.2) $$\pi_t d^m(\pi_t) \leq \alpha(\pi_t)\gamma_t\left(\frac{p_t}{p_{t+1}}\right) + \delta(\pi_t) r(1-\gamma_t)$$

(3.3) $$(1-\pi_t)d(\pi_t) \leq [1-\alpha(\pi_t)]\gamma_t\left(\frac{p_t}{p_{t+1}}\right) + [1-\delta(\pi_t)]R_t(1-\gamma_t)$$

Expression (3.2) asserts that the bank must pay the fraction π_t of agents who withdraw early by liquidating its own cash reserves and storage investments, so long as the bank does not exhaust its own reserves. Expression (3.3) says that the fraction $1-\pi_t$ of agents who are not relocated at *t* are paid out of any unliquidated cash reserves and the proceeds of unliquidated storage investments.

With two or more bankers, banks compete against each other for deposits. This competition takes the following form: each bank announces a set of schedules $[d^m(\pi_t), d(\pi_t), \alpha(\pi_t), \delta(\pi_t), b(\pi_t)]$, and a portfolio allocation (summarized by its reserve-deposit ratio γ_t), taking the choices announced by other banks as given. In a Nash equilibrium, the result is that the objects just described must be chosen to maximize the expected utility of a representative depositor, $\int_0^1 [\pi \ln d^m(\pi) + (1-\pi)\ln d(\pi)] f(\pi)\, d\pi$, subject to the constraints (3.2) and (3.3). We now describe the solutions to this maximization problem. A formal derivation of the solution appears in Appendix A. Define $I_t \equiv R\frac{p_{t+1}}{p_t}$ to be the gross nominal rate of interest.

Define by the variable π_t^* by

(3.4) $$\pi_t^* \equiv \frac{\gamma_t}{\gamma_t + (1-\gamma_t)I_t} \equiv H(\gamma_t, I_t).$$

Then, for $\pi_t \leq \pi_t^*$, $\delta(\pi_t) = 0$, and

(3.5) $$\alpha(\pi_t) = \pi_t / \pi_t^*.$$

In particular, if the fraction of agents withdrawing early is less than π_t^*, then banks do not exhaust their reserves. Or, in other words, π_t^* is the critical level of withdrawal demand above which banks do exhaust their reserves. Moreover, since scrapping storage investments involves an opportunity cost, storage investments are never scrapped if a bank has unliquidated cash reserves. Finally, for $\pi_t \leq \pi_t^*$, $d^m(\pi_t) = d(\pi_t)$ holds. That is, if banks do not exhaust their reserves, they provide complete insurance against the event of relocation.

Once $\pi_t > \pi_t^*$ holds, banks exhaust their reserves. As discussed in the introduction, we associate this event with a banking crisis.

Next, define $\bar{\pi}_t$ by

$$(3.6) \qquad \bar{\pi}_t \equiv \frac{\gamma_t}{\gamma_t + (1-\gamma_t)(r/R)I_t} \equiv Q(\gamma_t, I_t).$$

Then, for $\pi_t \in \left[\pi_t^*, \bar{\pi}_t\right]$, $\alpha(\pi_t) = 1$, $\delta(\pi_t) = 0$,

$$(3.7) \qquad d^m(\pi_t) = \frac{\gamma_t \left(\frac{p_t}{p_{t+1}}\right)}{\pi_t},$$

and

$$(3.8) \qquad d(\pi_t) = \frac{R(1-\gamma_t)}{1-\pi_t} > d^m(\pi_t)$$

all hold. For $\pi_t \in [\bar{\pi}_t, 1]$ we have $\alpha(\pi_t) = 1$,

$$(3.9) \qquad d(\pi_t) = (R/r)d^m(\pi_t),$$

and

$$(3.10) \qquad \delta(\pi_t) = \left(\frac{R}{rI_t}\right)\left(\frac{\gamma_t}{1-\gamma_t}\right)\left(\frac{\pi_t - \bar{\pi}_t}{\bar{\pi}_t}\right).$$

In short, for $\pi_t \in \left[\pi_t^*, \bar{\pi}_t\right]$, banks exhaust their reserves. However, the opportunity cost of scrapping capital investments makes it optimal not to do so. In addition, there is now a positive opportunity cost of providing complete insurance against the event of relocation, and banks cease to do so at sufficiently high levels of withdrawal demand.

When $\pi_t > \bar{\pi}_t$ is satisfied, the fraction of relocated agents is so large that banks are willing to scrap storage investment in order to augment the consumption of agents withdrawing early. However, the opportunity cost of doing so is positive, and this again prevents banks from offering complete insurance against the risk of relocation.

For our purposes, the values π_t^* and $\bar{\pi}_t$ are of particular importance. When $\pi_t > \pi_t^*$ banks exhaust their reserves, and a banking crisis results. Note that, with a competitive banking system, the probability of a banking crisis is $1 - F(\pi_t^*)$.

However, for $\pi_t \in \left[\pi_t^*, \bar{\pi}_t\right]$, even though a banking crisis is under way, banks do not liquidate investments. An implication is that, while some agents may suffer from the effects of a crisis (relocated agents have $d^m(\pi_t) < d(\pi_t)$ if $\pi_t > \pi_t^*$), there are no real resource losses from a crisis.

Once $\pi_t > \bar{\pi}_t$ holds, however, the liquidation of socially productive investment occurs. Since this liquidation is costly, there is not just incomplete insurance—there is an actual physical resource loss. Such a loss occurs with probability $1-F(\bar{\pi}_t)$. As we have noted, in practice some banking crises occur with no associated resource losses, while other crises are associated with quite large losses. Thus our model can confront these observations.

We have yet to determine the bank's optimal choice of a reserve-deposit ratio. We next turn our attention to this task.

B. The Optimal Reserve-Deposit Ratio

The analysis above implies that $\bar{\pi}_t < 1$ and $\pi_t^* < \bar{\pi}_t$ hold if $I_t > 0$. Assuming $I_t > 0$, we now analyze the bank's equilibrium choice of its reserve-deposit ratio.
The observations in the previous section imply that the expected utility of a representative depositor—as a function of γ_t—is

$$
\begin{aligned}
(3.11)\quad & \int_0^{H(\gamma_t,I_t)} \ln\left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+(1-\gamma_t)R\right]f(\pi)\,d\pi \\
&+\int_{H(\gamma_t,I_t)}^{Q(\gamma_t,I_t)}\left\{\pi\ln\left[\frac{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{\pi}\right]+(1-\pi)\ln\left[\frac{R(1-\gamma_t)}{1-\pi}\right]\right\}f(\pi)\,d\pi \\
&+\int_{Q(\gamma_t,I_t)}^{1}\ln\left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+r(1-\gamma_t)\right]f(\pi)\,d\pi \\
&+\ln\left(\frac{R}{r}\right)\int_{Q(\gamma_t,I_t)}^{1}(1-\pi)f(\pi)\,d\pi \equiv M(\gamma_t,I_t).
\end{aligned}
$$

Appendix A establishes that, at an interior optimum satisfying $I_t > 0$, a competitive bank's equilibrium reserve-deposit ratio satisfies.

$$
\begin{aligned}
(3.12)\quad & M_1(\gamma_t,I_t)= \\
&-\left[\frac{I_t-1}{\gamma_t+(1-\gamma_t)I_t}\right]F\left[H(\gamma_t,I_t)\right] \\
&+\left[\frac{1-\left(\frac{r}{R}\right)I_t}{\gamma_t+(1-\gamma_t)\left(\frac{r}{R}\right)I_t}\right]\left\{1-F\left[Q(\gamma_t,I_t)\right]\right\}+
\end{aligned}
$$

$$\int_{H(\gamma_t,I_t)}^{Q(\gamma_t,I_t)}\left\{\left(\frac{\pi}{\gamma_t}\right)-(1-\pi)\left[\frac{I_t}{(1-\gamma_t)I_t}\right]\right\}f(\pi)\,d\pi \geq 0.$$

To characterize solutions to (3.12), we first assume that π_t is uniformly distributed, so that $f(\pi)=1\forall\pi\in[0,1]$.

Second, we observe that the definition of π_t^* implies that

$$\text{(3.1)} \qquad \gamma_t=\frac{\pi_t^* I_t}{\pi_t^* I_t+(1-\pi_t^*)}$$

Appendix B proves the following result.

Proposition 1.

(a) if $I_t>0$ and (a.1) holds, then the equilibrium value of π_t^* satisfies

$$\text{(3.14)} \quad -\left(\frac{I_t-1}{I_t}\right)\left[\frac{r}{R}+\left(1-r/R\right)\pi_t^*\right]+$$
$$\left(\frac{1-\pi_t^*}{\pi_t^*}\right)\left[\frac{r/R}{r/R+\left(1-r/R\right)\pi_t^*}\right]\left[\left(\frac{1}{I_t}\right)-\frac{r}{R}\right]-$$
$$(1-r/R)+\left(\frac{1}{2}\right)(1-r/R)\left[\frac{1+(I_t-1)\pi_t^*}{I_t}\right]\times$$
$$\left[1+\left(\frac{1}{r/R+\left(1-r/R\right)\pi_t^*}\right)\right]\geq 0.$$

(3.14) holds as an equality if $\pi_t^*>0$.

(b) $M_1(0,I_t)>0$ holds if $I_t<R/r$ is satisfied;

(c) If $I_t<R/r$, equation (3.14) at equality has a unique solution for π_t^*. Once the equilibrium value of π_t^* is obtained, the bank's equilibrium reserve-deposit ratio can be deduced from equation (3.13).

Suppose that $\bar{\pi}_t<1$ holds (there is a positive probability that capital investments will be scrapped). Then, when (a.1) holds (π_t is uniformly distributed), we have the following result.

Proposition 2.

(a) The equilibrium value of $\bar{\pi}_t$ for a competitive bank satisfies the condition

$$(3.15)\quad 2\left[1-\left(\frac{r}{R}\right)I_t\right]=\left(\frac{\bar{\pi}_t}{1-\bar{\pi}_t}\right)\left[\bar{\pi}_t I_t\left(\frac{r}{R}\right)+\left(1-\bar{\pi}_t\right)\right]\left\{1-\left[\frac{\frac{r}{R}}{1-\bar{\pi}_t\left(1-\frac{r}{R}\right)}\right]^2\right\}\equiv G\left(\bar{\pi}_t\right).$$

(b) $G(0)=0$, $G(1)=2I_t\left(1-\frac{r}{R}\right)\geq 2\left[1-\left(\frac{r}{R}\right)I_t\right]$, and $G'(\bar{\pi})>0$ hold.

The proof of proposition 4 appears in Appendix C. The proposition implies that there is always a unique solution for $\bar{\pi}_t$ in the unit interval so long as $I_t \leq \frac{R}{r}$. The equilibrium values of γ_t and π_t^* can then be recovered from the definitions (3.4) and (3.5):

$$\gamma_t \equiv \frac{\bar{\pi}_t I_t (r/R)}{\bar{\pi}_t I_t (r/R)+1-\bar{\pi}_t},$$

$$\pi_t^* \equiv \frac{(r/R)\bar{\pi}_t}{1-\bar{\pi}_t\left[1-(r/R)\right]}.$$

IV. A Monopolistic Bank

We now consider an economy identical in all respects to the one discussed thus far, except that now we set $N=1$. Thus there is a monopoly in banking.

With the same notation as previously, a bank receiving all deposits earns an ex post profit of $\left[1-\alpha(\pi_t)\right]m_t\left(\frac{p_t}{p_{t+1}}\right)+\left[1-\delta(\pi_t)\right]Rs_t-(1-\pi_t)d(\pi_t)(w-\tau_t)$, measured in units of date $t+1$ consumption. That is, the second period real profits of a monopolistic bank consist of the real value of unliquidated cash reserves, plus the value of unliquidated storage investments, less payments to (non-relocated) depositors in the second period.

As before, let $\gamma_t \equiv m_t/(w_t+\tau_t)$ denote the bank's reserve deposit. Then constraints (3.2)–(3.3) continue to apply to the bank's choices of $\alpha(\pi_t)$, $\delta(\pi_t)$, $d^m(\pi_t)$, $d(\pi_t)$, and γ_t. In addition, we allow the bank to impose a minimum deposit requirement—which in this case will clearly be $(w_t+\tau_t)$. This allows the bank to extract the maximum possible surplus from depositors (in effect the bank can impose a two-part tariff). Since depositors always have the option of investing autarkically, it follows that the bank faces an additional constraint, the participation constraint of depositors. We derive autarkic agents' expected utility next.

A. Autarkic Agents

We begin by describing the behavior of agents if there are no banks in operation or, equivalently, if agents choose not to save through intermediaries. We refer to agents whose savings are not intermediated as autarkic.

Let $\hat{\pi} \equiv \int_0^1 \pi f(\pi) d\pi$ be the expected value of π_t, let γ_t^a be the fraction of savings an autarkic agent holds in the form of cash reserves (so that $1-\gamma_t^a$ is the fraction invested in storage), let p_t be the time t price level in each location, and let τ_t be the real value of the lump-sum tax/transfer paid/received by an agent when young.[9] Then an autarkic agent who is relocated will liquidate his storage investments, carry the goods obtained to his new location, and use the currency he holds to purchase additional consumption goods. Since the gross real return on currency between t and $t+1$ is p_t / p_{t+1}, the old age consumption of an agent who is relocated at t is $\left[\gamma_t^a\left(\frac{p_t}{p_{t+1}}\right)+\left(1-\gamma_t^a\right)r\right](w+\tau_t)$.

For an agent who is not relocated, storage investments can be left in place until maturity. Hence agents who remain in their original location have second period consumption equal to $\left[\gamma_t^a\left(\frac{p_t}{p_{t+1}}\right)+\left(1-\gamma_t^a\right)R\right](w-\tau_t)$. It follows that, for a given portfolio allocation, the expected utility of a young depositor at t is given by

$\hat{\pi}\ln\left[\gamma_t^a\left(\frac{p_t}{p_{t+1}}\right)+\left(1-\gamma_t^a\right)r\right]+\left(1-\hat{\pi}\right)\ln\left[\gamma_t^a\left(\frac{p_t}{p_{t+1}}\right)+\left(1-\gamma_t^a\right)R\right]$. Young depositors then choose $\gamma_t^a \in [0,1]$ to maximize this expression.

It is evident that storage will occur at all only if $\hat{\pi}r+\left(1-\hat{\pi}\right)R > \frac{p_t}{p_{t+1}}$, and that currency will be held at all only if $p_t / p_{t+1} > r$. Both conditions are assumed to hold for the remainder of this section. When they do, the solution to the depositor's problem is to set

$$\gamma_t^a = \max\left\{0, \frac{\hat{\pi}\left(\frac{p_t}{p_{t+1}}\right)(R-r)-r\left(R-\frac{p_t}{p_{t+1}}\right)}{\left(R-\frac{p_t}{p_{t+1}}\right)\left(\frac{p_t}{p_{t+1}}-r\right)}\right\} \tag{4.1}$$

[9] Since agents' portfolios are allocated prior to the realization of π_t, the equilibrium price level and the equilibrium transfer display no aggregate randomness.

Defining $I_t \equiv R\frac{p_{t+1}}{p_t}$ to be the gross nominal rate of interest, clearly $\gamma_t^a > 0$ holds if and only if

(4.2) $$\hat{\pi}R + (1-\hat{\pi})r > rI_t$$

In short, if the nominal rate of interest (the opportunity cost of holding money) is too high, agents will not hold it. Moreover, if (4.2) holds, (4.1) implies that .

(4.3) $$\gamma_t^a = \frac{\hat{\pi}(R-r) - r(I_t - 1)}{(I_t - 1)\left(\frac{R}{I_t} - r\right)}$$

Appendix D establishes the following result.

Proposition 3. Suppose that

(4.4) $$\left(\frac{R}{r}\right)\min\left\{1, \hat{\pi} + (1-\hat{\pi})\left(\frac{r}{R}\right)\right\} > I_t$$

holds. Then $\gamma_t^a > 0$ and $\frac{\partial \gamma_t^a}{\partial I_t} < 0$.

For values of I_t satisfying (4.4), let $\gamma^a(I_t)$ denote the optimal choice of an autarkic agent's ratio of cash-reserves to total savings. Then define

$$v(I_t) \equiv \hat{\pi}\ln\left[\gamma^a(I_t) + (1-\gamma^a(I_t))\left(\frac{r}{R}\right)I_t\right] +$$

(4.5) $$(1-\hat{\pi})\ln\left[\gamma^a(I_t) + (1-\gamma^a(I_t))I_t\right] + \ln(R/I_t) \equiv \hat{\pi}\ln\left[\gamma^a(I_t)\left(\frac{p_t}{p_{t+1}}\right) + (1-\gamma^a(I_t))r\right]$$

$$+(1-\hat{\pi})\ln\left[\gamma^a(I_t)\left(\frac{p_t}{p_{t+1}}\right) + (1-\gamma^a(I_t))R\right]$$

to be the indirect utility function of an autarkic agent. Appendix E demonstrates

Proposition 4. For values of I_t satisfying (4.4), $v'(I_t) < 0$ holds.

Clearly $v(I_t) + \ln(w + \tau_t)$ gives the maximum expected utility attainable by an autarkic agent, given the transfer $w + \tau_t$. For a given value of τ_t, this utility level is decreasing in I_t.

B. The Monopolist Bank Problem

The depositors' participation constraint is

(4.6) $$\int_0^1 \left[\pi \ln d^m(\pi) + (1-\pi)\ln d(\pi)\right] f(\pi)\, d\pi \geq v(I_t)$$

The bank then maximizes its expected profits,

$\int_0^1 \left\{ \left[1-\alpha(\pi)\right]\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+\left[1-\delta(\pi)\right]R(1-\gamma_t)-(1-\pi)d(\pi)\right\} f(\pi)d(\pi)$, subject to the constraints (3.2), (3.3), (4.6), and non-negativity.

In order to simplify this problem, we make the following observations. First, if $\alpha(\pi_t)<1$ holds (the bank does not exhaust its cash reserves), it does not take the costly action of liquidating its storage investments $\left[\delta(\pi_t)=0\right]$. Thus, if $\alpha(\pi_t)<1$,

$$d^m(\pi_t)=\alpha(\pi_t)\gamma_t\left(\frac{p_t}{p_{t+1}}\right)\Big/\pi_t \tag{4.7}$$

Second, if $\alpha(\pi_t)=1$ (the bank exhausts its reserves) and it does not liquidate its storage investments, it sets $\delta(\pi_t)=0$. Hence,

$$d^m(\pi_t)=\left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right)\right]\Big/\pi_t\,. \tag{4.8}$$

It is possible that the bank will liquidate some storage investments. Hence, in this situation,

$$d^m(\pi_t)=\left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+\delta(\pi_t)r(1-\gamma_t)\right]\Big/\pi_t\,. \tag{4.9}$$

We now anticipate a result: that $\alpha(\pi_t)<1$ will hold if π_t lies below some threshold. For a monopoly bank we denote this threshold by ζ_t^*. Similarly, $\delta(\pi_t)>0$ holds if π_t exceeds an additional threshold, which we denote by $\bar{\zeta}_t$. Hence there are two possibilities: (a) $\bar{\zeta}_t<1$ holds, or (b) $\bar{\zeta}_t=1$. We now consider each case in turn.

C. The Case $\bar{\zeta}_t<1$

We begin by using (4.7)–(4.9) to rewrite the depositor's participation constraint as

$$\int_0^{\zeta_t^*} \pi \ln\left[\frac{\alpha(\pi)\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{\pi}\right] f(\pi)d\pi + \tag{4.10}$$

$$\int_{\zeta_t^*}^{\bar{\zeta}} \pi \ln\left[\frac{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{\pi}\right] f(\pi)d\pi + \int_{\bar{\zeta}_t}^1 \pi \ln\left[\frac{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+\delta(\pi)r(1-\gamma_t)}{\pi}\right] f(\pi)d\pi$$

$$+\int_0^1 (1-\pi)\ln d(\pi) f(\pi)d\pi \geq v(I_t)$$

Similarly, the bank's objective function can be written as

$$\int_0^{\zeta_t^*}\left[1-\alpha(\pi)\right]\gamma_t\left(\frac{p_t}{p_{t+1}}\right)f(\pi)d\pi$$
$$+\int_{\bar{\zeta}_t}^1 R(1-\gamma_t)\left[1-\delta(\pi)\right]f(\pi)d\pi -$$
$$\int_0^1(1-\pi)d(\pi)f(\pi)d\pi + R(1-\gamma_t)\int_0^{\bar{\zeta}_t} f(\pi)d\pi .$$

A monopoly bank maximizes this expression, subject to the constraint (4.10).
Let λ_t be the Lagrange multiplier associated with (4.10). Appendix F proves that the solution to the bank's maximization problem has the following properties.
Define ζ_t^* and $\bar{\zeta}_t$ by

(4.11) $$\zeta_t^* \equiv \frac{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{\lambda_t},$$

and

(4.12) $$\bar{\zeta}_t = \left(R/r\right)\zeta_t^* .$$

Then

(4.13) $$\alpha(\pi_t) = \begin{matrix} \pi_t/\zeta_t^* & ; & \pi_t \le \zeta_t^* \\ 1 & ; & \pi_t \ge \zeta_t^* \end{matrix}$$

(4.14) $$\delta(\pi_t) = 0;\ \pi_t \in \left[\zeta_t^*, \bar{\zeta}_t\right]$$

(4.15) $$\delta(\pi_t) = \left[\frac{\gamma_t}{1-\gamma_t}\right]\left(\frac{R}{rI_t}\right)\left(\frac{\pi_t - \bar{\zeta}_t}{\bar{\zeta}_t}\right);\ \pi_t \in \left[\bar{\zeta}_t, 1\right].$$

Note that (4.13)–(4.15) are the exact counterparts of equations (3.7)–(3.10). However, the determination of ζ_t^* and $\bar{\zeta}_t$ is considerably different than in the competitive case. We also note that a monopoly bank chooses its deposit return schedules as follows:

(4.16) $$d(\pi_t) = \lambda_t;\ \forall \pi_t$$

(4.17) $$d^m(\pi_t) = \lambda_t;\ \pi_t \in \left[0, \zeta_t^*\right]$$

(4.18) $$d^m(\pi_t) = \frac{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{\pi_t};\ \pi_t \in \left[\zeta_t^*, \bar{\zeta}_t\right]$$

(4.19) $$d^m(\pi_t) = \left(r/R\right)\lambda_t;\ \pi_t \in \left[\bar{\zeta}_t, 1\right].$$

It remains to determine ζ_t^*, γ_t and λ_t. Appendix F demonstrates that a monopoly bank's optimal choice of ζ_t^* satisfies

$$I_t = F\left[\zeta_t^*\right] + \left(R/r\right)\left\{1 - F\left[\left(R/r\right)\zeta_t^*\right]\right\} + \int_{\zeta_t^*}^{\left(\frac{R}{r}\right)\zeta_t^*} \left[\frac{\pi}{\zeta_t^*}\right] f(\pi)\, d\pi \qquad (4.20)$$

The values γ_t and λ_t are then determined by using all of these conditions in (4.10), and solving the resulting condition at equality for λ_t. γ_t can then be deduced from (4.11). If assumption (a.1) holds (that is, if π_t is uniformly distributed), then a particularly simple expression for ζ_t^* .obtains. In this case (4.20) reduces to

$$\zeta_t^* = \frac{2\left[\left(\frac{R}{r}\right) - I_t\right]}{\left[\left(\frac{R}{r}\right)^2 - 1\right]} \qquad (4.21)$$

A banking crisis occurs (the monopoly bank exhausts its reserves) if $\pi_t > \zeta_t^*$ holds. This occurs with probability $1 - F\left(\zeta_t^*\right)$. Banks liquidate storage investments—so that banking crises involve a social resource loss—if $\pi_t > \bar{\zeta}_t$. This occurs with probability $1 - F\left(\bar{\zeta}_t\right)$. Finally, we can state precisely when $\bar{\zeta}_t < 1$ holds. Since $\bar{\zeta}_t = \left(R/r\right)\zeta_t^*$, equation (4.21) implies that $\bar{\zeta}_t < 1$ will hold if

$$I_t > \left(\frac{1}{2}\right)\left[\frac{R}{r} + \frac{r}{R}\right] > 1 \qquad (4.22)$$

obtains. Thus, liquidation of storage investments will occur at all under a monopolistic banking system if and only if the nominal rate of interest (the rate of inflation) is sufficiently high. We now consider what happens when (5.19) fails to hold.

D. The Case $\bar{\zeta}_t = 1$

When $\bar{\zeta}_t = 1$ the bank's objective function reduces to

$$\int_0^{\zeta_t} \left[1 - \alpha(\pi)\right]\gamma_t \left(\frac{p_t}{p_{t+1}}\right) f(\pi)\, d\pi$$

$$+ R\left(1 - \gamma_t\right) - \int_0^1 (1 - \pi) d(\pi) f(\pi)\, d\pi$$

and the depositors' participation constraint becomes

$$(4.23)\qquad \int_0^{\zeta_t} \pi \ln\left[\frac{\alpha(\pi)\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{\pi}\right] f(\pi)\,d\pi +$$

$$\int_{\zeta_t}^1 \pi \ln\left[\frac{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{\pi}\right] f(\pi)\,d\pi + \int_0^1 (1-\pi)\ln d(\pi) f(\pi)\,d\pi \ge v(I_t)$$

Appendix G shows that the bank's optimal choice of ζ_t^* in this case is given by

$$(4.24)\qquad I_t = F\left[\zeta_t^*\right] + \int_{\zeta_t^*}^1 \left[\frac{\pi}{\zeta_t^*}\right] f(\pi)\,d\pi$$

Again λ_t can be deduced from the depositors' participation constraint, and γ_t can be recovered from (4.11).

V. Monopoly Versus Competition, the Probability of a Banking Crisis, and the Resource Losses Associated with a Banking Crisis

In this section we consider three issues. First, what is the probability that costly liquidation of investments occurs in monopolistic versus competitive banking systems? As we show, the probability that some resource losses arise due to investment liquidation is unambiguously higher in competitive than in monopolistic banking systems, other conditions equal. Thus competition leads to a higher probability of some resource loss in each period.

Second, we consider the probability of banking crises—which may or may not involve investment liquidation—under monopoly versus competition in banking. Here the results are more ambiguous. If the nominal rate of interest (the rate of inflation) is sufficiently low, the probability of a banking crisis is higher in monopolistic than in competitive banking systems (other factors being equal). However, if the nominal interest rate (the rate of inflation) is sufficiently high this ranking is reversed. In particular, when the nominal rate of interest exceeds some critical level, banking crises occur with higher probability under competition than under monopoly in banking. Thus the conduct of monetary policy interacts with the industrial organization of the banking system in influencing the relative likelihood of banking crises.

Finally, we compare the expected output losses—conditional on a crisis occurring—under monopolistic and competitive banking systems.

A. Investment Liquidation

Our first result is that the probability of some investment liquidation is higher under competition $\left[1-F(\bar{\pi}_t)\right]$ than under monopoly $\left[1-F(\bar{\zeta}_t)\right]$ in banking.

Proposition 5. $\bar{\pi}_t \leq \bar{\zeta}_t$ holds. The inequality is strict if $I_t > 1$.

The proof of proposition 5 is given in Appendix H. Intuitively, monopolistic banks earn higher expected profits to the extent that they can avoid liquidating storage investments. Thus a monopoly bank has less incentive to liquidate investments early than does a competitive bank. In particular, competitive banks earn zero profits in any event, and they face strong incentives to liquidate some storage investment as a way of enhancing insurance when withdrawal demand is sufficiently high. Thus the probability of a banking crisis entailing a social output loss is higher when the banking system is competitive than when it is monopolistic.

B. The Probability of a Banking Crisis

Our next result concerns the relative probability of a banking crisis under monopoly versus competition.

Proposition 6.

(a) Suppose that $I_t = 1$. Then $\pi_t^* = \zeta_t^* = 1$.

(b) Suppose that $I_t \in \left(1, \dfrac{1+\left(r/R\right)^2}{2\left(r/R\right)}\right)$. Then $\bar{\zeta}_t = 1 > \bar{\pi}_t$, and $\pi_t^* > \zeta_t^*$ holds.

(c) Suppose that $I_t > \dfrac{1+\left(r/R\right)^2}{2\left(r/R\right)}$ holds. Then there exists a value $\tilde{I}_t \in \left(\dfrac{1+\left(r/R\right)^2}{2\left(r/R\right)}, \left(R/r\right)\right)$ such that $\pi_t^* > (<) \zeta_t^*$ holds if $I_t < (>) \tilde{I}_t$.

Proposition 6 is proved in Appendix I. The intuition underlying the proposition is as follows. First, if $I_t = 1$, then currency is as good an asset as storage. If follows that, even if $N = 1$, a bank has no monopoly power. Hence monopolistic and competitive banks behave identically. And, with $I_t = 1$, competitive banks set $\gamma_t = 1$.[10] In particular, doing so allows banks to provide complete insurance against relocation risk, and there is no opportunity cost to foregoing storage investments.

[10]See Smith (2002) for a proof.

If $I_t \in \left(1, \dfrac{1+\left(r/R\right)^2}{2\left(r/R\right)}\right)$, equation (4.22) implies that $\bar{\zeta}_t = 1 > \bar{\pi}_t$. Here a monopoly bank never liquidates investments. Moreover, their incentive to invest in storage is strong; they earn greater expected profits by storing than by not storing goods. Hence monopoly banks have relatively low cash reserve holdings, and consequently they have a relatively high probability of exhausting cash reserves.

When $I_t > \left[1+\left(r/R\right)^2\right] / 2\left(r/R\right)$ holds, however, the above argument is too simple.

Monopoly banks not only hold low cash reserves, but they promise low rates of interest to depositors who withdraw early. The probability of reserve exhaustion depends on the relative strength of these two forces. Proposition 6 shows that, when I_t is sufficiently high, monopoly banks offer sufficiently low deposit returns that the latter effect dominates. In this case a monopolistic banking system has a lower probability of a banking crisis than does a competitive banking system (given the prevailing equilibrium value of I_t).

Note that monetary policy (the choice of I_t or σ) interacts with the structure of the banking system to determine which type of banking system faces a higher probability of a crisis. It is evident that $\partial\zeta_t^* / \partial I_t < 0$ holds. And Smith (2002) shows that $\partial\pi_t^* / \partial I_t < 0$ holds. Thus increases in the nominal rate of interest (the rate of inflation) raise the probability of a banking crisis. This is true whether or not the banking system is competitive or monopolistic. Demirguc-Kunt and Detragiache (1997) and Boyd, Gomis, Kwak, and Smith (2001) show that, empirically, higher rates of inflation do increase the probability that banking crises will occur.

C. Expected Output Losses

We now compute the expected output loss for economies with monopolistic and competitive banking systems, conditional on some output loss occurring. We consider first a monopolistic banking system.

If $\pi_t > \bar{\zeta}_t$ at date *t*, the quantity of investment liquidated equals $(1-\gamma_t)\delta(\pi_t)$. This yields $r(1-\gamma_t)\delta(\pi_t)$ units of consumption at *t*.

At the same time, $R(1-\gamma_t)\delta(\pi_t)$ units of time $t+1$ output is forgone in the process of this liquidation. Discounting this loss to date t implies a discounted present value of lost output in the amount $(1-r)(1-\gamma_t)\delta(\pi_t)$. Using equation (4.15), the expected output loss—conditional on some loss occurring—is given by

$$\frac{(1-r)(1-\gamma_t)\int_{\bar{\zeta}_t}^{1}\delta(\pi)d\pi}{1-\bar{\zeta}_t}=\frac{(1-r)(R/r\,I_t)(\gamma_t/\bar{\zeta}_t)\int_{\bar{\zeta}_t}^{1}(\pi-\bar{\zeta}_t)d\pi}{1-\bar{\zeta}_t}.$$

$$=\left[\frac{(1-r)(R/r\,I_t)\gamma_t}{2}\right]\left(\frac{1-\bar{\zeta}_t}{\bar{\zeta}_t}\right)$$

An analogous expression for an economy with a competitive banking system yields an expected output loss—conditional on some output loss occurring—of

$$\left[\frac{(1-r)(R/r\,I_t)\gamma_t}{2}\right]\left(\frac{1-\bar{\pi}_t}{\bar{\pi}_t}\right).$$

In comparing these expressions, it must be kept in mind that the reserve-deposit ratios under monopoly and under competition will generally be different.

Proposition 5 implies that $(1-\bar{\zeta}_t)/\bar{\zeta}_t<(1-\bar{\pi}_t)/\bar{\pi}_t$. Clearly, if $\bar{\zeta}_t=1$ conditional expected losses under competition are higher than those under monopoly. However, if $\bar{\zeta}_t<1$ holds, conditional expected output losses under the two banking systems cannot be compared without knowledge of the relevant reserve-deposit ratios.

MATHEMATICAL APPENDIX

A. The Maximization Problem of a Competitive Bank

Consider first the problem of choosing schedules $d^m(\pi_t)$, $d(\pi_t)$, $\alpha(\pi_t)$, $\delta(\pi_t)$ and $b(\pi_t)$ to maximize $\int_0^1 \left[\pi \ln d^m(\pi) + (1-\pi)\ln d(\pi)\right] f(\pi)\, d\pi$, subject to constraints (3.2) and (3.3).[11] Let λ_i, i=1,3 be the Lagrange multiplier associated with constraints (3.2) and (3.3). Then, if $\alpha(\pi) < 1$ holds, the first order condition for the choice of $\alpha(\pi)$ is $\lambda_1 = \lambda_3$.

The first order condition for the choice of $d^m(\pi)$ is $\lambda_1 d^m(\pi) = f(\pi)$. And, the first order condition for the choice of $d(\pi)$ is $\lambda_3 d(\pi) = f(\pi)$. In addition, it is easy to verify that $\delta(\pi) = 0$.[12]

From these conditions, it follows that $d^m(\pi) = d(\pi)$ if $\alpha(\pi) < 1$. Then, setting $\delta(\pi) = 0$ in (4.2) and (4.4) and setting $d^m(\pi) = d(\pi)$ yields

(E.1) $$\alpha(\pi) = \pi\left[1 + \left(\frac{1-\gamma_t}{\gamma_t}\right) I_t\right] = \pi / \pi_t^*$$

In addition

(E.2) $$d^m(\pi) = d(\pi) = \gamma_t\left(\frac{p_t}{p_{t+1}}\right) + (1-\gamma_t) R.$$

Clearly $\alpha(\pi) \le 1$ holds if $\pi \le \pi_t^*$.

If $\alpha(\pi) = 1$ holds, then the first order conditions for $d^m(\pi)$ and $d(\pi)$ continue to be given by $\lambda_2 d^m(\pi) = f(\pi)$ and $\lambda_3 d(\pi) = f(\pi)$.

The first order condition for $\delta(\pi)$ is

(E.3) $$r\lambda_2 \le R\lambda_3,$$

with equality if $\delta(\pi) > 0$ holds.

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[11]Notice that, at this point, we take $\gamma_t \in (0,1)$ to be an arbitrary number. Below we analyze the optimal choice of γ_t.

[12]So long as banks do not exhaust their cash reserves, there is no opportunity cost to further reserve liquidation. There is an opportunity cost to scrapping storage investments. Hence $\alpha(\pi) < 1$ implies $\delta(\pi) = 0$.

If $\delta(\pi)=0$, then the preceding conditions imply that $d^m(\pi)\leq\left(r/R\right)d(\pi)$ must be satisfied. Thus banks now provide incomplete insurance against the event of relocation. Moreover, setting $\alpha(\pi)=1$ and $\delta(\pi)=0$ in (3.2), and (3.3), $d^m(\pi)\leq\left(r/R\right)d(\pi)$ holds if $\pi\leq\bar{\pi}_t$, as defined in the text. (3.7) and (3.8) then give the deposit return schedules.

If $\delta(\pi)>0$ holds $(\pi>\bar{\pi}_t)$, then the bank's first order conditions imply that (3.9) holds. Setting $\alpha(\pi)=1$ in (3.2) and (3.3), and using (3.2), (3.3), and (3.9) yields (3.10).

B. The Equilibrium Choice of Reserve-Deposit Ratio

Substituting the previous results into the bank's objective function give the expression in (3.11) for the maximized value of depositor expected utility as a function of γ_t and I_t. In addition, if $I_t>0$ holds, then $\pi_t^*<\bar{\pi}_t<1$ is satisfied.

It is straightforward to show that

(E.4) $$M(\gamma_t,I_t)\equiv\ln\left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+R(1-\gamma_t)\right]F\left[H(\gamma_t,I_t)\right]$$

$$+\ln\left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+r(1-\gamma_t)\right]\left\{1-F\left[Q(\gamma_t,I_t)\right]\right\}+\ln\left(R/r\right)\left\{1-F\left[Q(\gamma_t,I_t)\right]\right\}$$

$$-\ln\left(R/r\right)\left\{1-Q(\gamma_t,I_t,\mu)F\left[Q(\gamma_t,I_t)\right]\right\}+\ln\left(R/r\right)\int_{Q(\gamma_t,I_t)}^{1}F(\pi)d\pi+$$

$$\int_{H(\gamma_t,I_t)}^{Q(\gamma_t,I_t)}\left\{\pi\ln\left[\frac{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{\pi}\right]+(1-\pi)\ln\left[\frac{R(1-\gamma_t)}{1-\pi}\right]\right\}f(\pi)d\pi$$

It then follows that

(E.5) $$M_1(\gamma_t,I_t)=-\left[\frac{I_t-1}{\gamma_t+(1-\gamma_t)I_t}\right]F\left[H(\gamma_t,I_t)\right]+$$

$$\left[\frac{1-\left(\frac{r}{R}\right)I_t}{\gamma_t+(1-\gamma_t)\left(\frac{r}{R}\right)I_t}\right]\left[1-F\left[Q(\gamma_t,I_t)\right]\right]+$$

$$\int_{H(\gamma_t,I_t)}^{Q(\gamma_t,I_t)}\left\{\frac{\pi}{\gamma_t}-(1-\pi)\left[\frac{I_t}{I_t(1-\gamma_t)}\right]\right\}f(\pi)d\pi+$$

$$H_1(-)\{\ln\left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+R(1-\gamma_t)\right](\ )f[H(-)]-$$

$$H(\gamma_t,I_t)\ln\left[\frac{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{H(\gamma_t,I_t)}\right](\ )f[(1+\mu)H(-)]$$

$$-[1-H(\gamma_t,I_t)]\ln\left[\frac{R(1-\gamma_t)}{1-H(\gamma_t,I_t)}\right](\ )f[H(-)]\}$$

$$+Q_1(-)f[Q(\gamma_t,I_t,\mu)]\{Q(\gamma_t,I_t)\ln\left(R/r\right)-\ln\left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+r(1-\gamma_t)\right]$$

$$-\ln\left(R/r\right)+Q(\gamma_t,I_t)\ln\left[\frac{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{Q(\gamma_t,I_t)}\right]+[1-Q(\gamma_t,I_t)]\ln\left[\frac{R(1-\gamma_t)}{1-Q(\gamma_t,I_t)}\right]\}$$

However,

(E.6) $$\gamma_t/H(\gamma_t,I_t)=\gamma_t+(1-\gamma_t)I_t$$

and

(E.7)
$$\frac{R(1-\gamma_t)}{1-H(\gamma_t,I_t)}=\frac{R(1-\gamma_t)}{1-\left[\frac{\gamma_t}{\gamma_t+(1-\gamma_t)I_t}\right]}=$$

$$\left[\frac{R(1-\gamma_t)}{I_t(1-\gamma_t)}\right][\gamma_t+(1-\gamma_t)I_t]=\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+R(1-\gamma_t)$$

both hold, as do

(E.8) $$\frac{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}{Q(\gamma_t,I_t)}=\left(\frac{p_t}{p_{t+1}}\right)\gamma_t+(1-\gamma_t)r$$

and

$$\text{(E.9)}\qquad \begin{aligned}&\frac{R(1-\gamma_t)}{1-Q(\gamma_t,I_t)}=\left[\frac{R(1-\gamma_t)}{(1-\gamma_t)\left(\frac{r}{R}\right)I_t}\right]\bullet\\ &\left[\gamma_t+(1-\gamma_t)\left(\frac{r}{R}\right)I_t\right]=\\ &\left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+r(1-\gamma_t)\right]\left[\frac{I_t(1-\gamma_t)}{\left(\frac{r}{R}\right)I_t(1-\gamma_t)}\right]\\ &=\left(\frac{R}{r}\right)\left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right)+r(1-\gamma_t)\right]\end{aligned}$$

using (A.13)–(A.16) in (A.12) gives the expression for $M_1(\gamma_t,I_t)$ in equation (3.12).

C. Proof of Proposition 1

In order to prove proposition 1, we begin by noting that, by definition,

$$\text{(E.10)}\qquad \bar{\pi}_t\equiv\frac{\pi_t^*}{\left(\frac{r}{R}\right)+\left(1-\frac{r}{R}\right)\pi_t^*}\equiv\phi\left(\pi_t^*\right)$$

Note that $\phi(0)=0$, $\phi(1)=1$, and $\phi'\left(\pi_t^*\right)>0$ hold. Note further that the definition of π_t^* implies that (4.18) holds. Finally, all of these observations imply that

$$\text{(E.11)}\qquad R(1-\gamma_t)=\frac{R\left(1-\pi_t^*\right)}{\pi_t^*I_t+\left(1-\pi_t^*\right)},$$

$$\text{(E.12)}\qquad \begin{aligned}&\frac{R(1+\mu)\pi_t^*+r\left(1-\pi_t^*\right)}{\pi_t^*I_t+\left(1-\pi_t^*\right)}=\\ &\left(\frac{p_t}{p_{t+1}}\right)\gamma_t+r(1-\gamma_t),\end{aligned}$$

and

$$\text{(E.13)}\qquad \gamma_t\left(\frac{p_t}{p_{t+1}}\right)+R(1-\gamma_t)=\frac{R}{I_t\pi_t^*+\left(1-\pi_t^*\right)}.$$

Using these observations and assumption (a.1) in (3.12), it follows that the condition $M_1(\gamma_t,I_t)\geq 0$ is equivalent to

(E.14)
$$-\left(\frac{I_t-1}{I_t}\right)\pi_t^* + (1-\bar{\pi}_t)\left[\frac{(1+\mu)\left(\frac{p_t}{p_{t+1}}\right)-r}{\pi_t^*(R-r)+r}\right]$$
$$+\int_{\pi_t^*}^{\bar{\pi}_t}\left\{\left(\frac{\pi}{I_t\pi_t^*}\right)-\left(\frac{(1-\pi)}{(1-\pi_t^*)}\right)\right\}d\pi \geq 0$$

Now observe that

(E.15)
$$\int_{\pi_t^*}^{\bar{\pi}_t}\left\{\pi\left(\frac{1}{I_t\pi_t^*}\right)-\left[\frac{(1-\pi)}{(1-\pi_t^*)}\right]\right\}d\pi =$$
$$\left[\frac{(I_t-1)\pi_t^*}{I_t\pi_t^*(1-\pi_t^*)}\right](1/2)\left[(\pi_t)^2-(\pi_t^*)^2\right]-\left[\frac{1}{(1-\pi_t^*)}\right](\bar{\pi}_t-\pi_t^*)$$

Moreover, (A.17) implies that

(E.16)
$$\bar{\pi}_t-\pi_t^* = \frac{\left[1-(r/R)\right]\pi_t^*(1-\pi_t^*)}{(r/R)+(1-r/R)\pi_t^*}$$

and

(E.17)
$$(\bar{\pi}_t)^2-(\pi_t^*)^2 = (\pi_t^*)^2\left[\frac{(1-r/R)(1-\pi_t^*)}{r/R+(1-r/R)\pi_t^*}\right]\left[\frac{1+r/R+(1-r/R)\pi_t^*}{r/R+(1-r/R)\pi_t^*}\right].$$

Finally, using (A.17),

(E.18)
$$1-\bar{\pi}_t = \frac{(r/R)(1-\pi_t^*)}{(r/R)+(1-r/R)\pi_t^*}$$

Using (A.15)–(A.18) in (A.10), it follows that $M_1(\gamma_t, I_t)\geq 0$ is equivalent to

(E.19)
$$-\left(\frac{I_t-1}{I_t}\right)\pi_t^* + \left[\frac{(r/R)(1-\pi_t^*)}{\left[(r/R)+(1-r/R)\pi_t^*\right]^2}\right]\left[\left(\frac{1}{I_t}\right)-(r/R)\right]$$
$$-\left[\frac{(1-r/R)\pi_t^*}{r/R+(1-r/R)\pi_t^{**}}\right]+$$
$$\left[\frac{1+(I_t-1)\pi_t^*}{2I_t}\right]\frac{(1-r/R)\pi_t^*\left[1+r/R+(1-r/R)\pi_t^*\right]}{\left[r/R+(1-r/R)\pi_t^*\right]^2}\geq 0$$

Rearranging terms in (A.19) yields (3.14)

(b) Setting $\pi_t^* = 0$ (and hence $\gamma_t = 0$) in (3.14), we have $M_1(0, I_t) > 0$ if

$$\left(\frac{R}{I_t}\right) - r > 0.$$

Rearranging terms gives equation (4.20) in the text.

(c) Define

$$Q(\pi_t^*) \equiv \left[\frac{1+(I_t-1)\pi_t^*}{I_t}\right]\left\{1+\left[\frac{1}{r/R+(1-r/R)\pi_t^*}\right]\right\}$$

Then

$$Q'(\pi_t^*) = \left(\frac{I_t-1}{I_t}\right)\left\{1+\left[\frac{1}{r/R+(1-r/R)\pi_t^*}\right]\right\} - \left[\frac{1+(I_t-1)\pi_t^*}{I_t}\right]\frac{(1-r/R)}{\left[r/R+(1-r/R)\pi_t^*\right]^2}$$

It is then apparent that the left-hand side of (3.14) is decreasing in π_t^* if

$$-\left(\frac{I_t-1}{I_t}\right)(1-r/R)+(1/2)(1-r/R)Q'(\pi_t^*) =$$

(E.20)
$$-\left(\frac{I_t-1}{I_t}\right)(1-r/R)+\left(\frac{1}{2}\right)(1-r/R)\left(\frac{I_t-1}{I_t}\right)\left\{1+\left[\frac{1}{r/R+(1-r/R)\pi_t^*}\right]\right\}$$
$$-\left(\frac{1}{2}\right)(1-r/R)^2\left[\frac{1+(I_t-1)\pi_t^*}{I_t}\right]\left[\frac{1}{r/R+(1-r/R)\pi_t^*}\right]^2 \leq 0.$$

is satisfied. Rearranging terms in (A.20) yields that the left-hand side of (3.14) is decreasing in π_t^* if

(E.21)
$$(I_t-1)(r/R)(1-\pi_t^*)-\pi_t^*(I_t-1)\left[r/R+(1-r/R)\pi_t^*\right] \leq 1.$$

We now observe that if $\left(\frac{R}{I_t}\right) - r > 0$ holds,

$$I_t - 1 \leq \left(\frac{R-r}{r}\right)$$

It is then immediate that (A.21) is satisfied, the left-hand side of (3.14) is decreasing in π_t^* and that $M_1(0, I_t) > 0$. $I_t > 0$ implies that $\pi_t^* < 1$ holds, so (3.14) has a unique solution.

D. Proof of Proposition 2

(a) Smith (2001), proposition 3, shows that $M_1(\gamma_t, I_t) = 0$ if

(E.22) $$\left[1-\left(r/R\right)I_t\right]\bar{\pi}_t\left(1-\gamma_t\right)=\int_{\pi_t^*}^{\bar{\pi}_t}F(\pi)\,d\pi .$$

Using the assumption that π_t is uniformly distributed, and that by definition

(E.23) $$\pi_t^* \equiv \frac{\left(r/R\right)\bar{\pi}_t}{1-\bar{\pi}_t\left[1-\left(r/R\right)\right]}$$

holds, (A.22) reduces to

(E.24) $$2\left[1-\left(r/R\right)I_t\right]\left(1-\gamma_t\right)=\bar{\pi}_t\left[1-\left(\frac{\left(r/R\right)}{1-\bar{\pi}\left(1-r/R\right)}\right)^2\right]$$

In addition, by definition,

(E.25) $$\gamma_t=\frac{\bar{\pi}_t I_t\left(r/R\right)}{\bar{\pi}_t I_t\left(r/R\right)+1-\bar{\pi}_t}.$$

Substituting (A.25) into (A.24) and rearranging terms gives equation (3.15) in the text.
(b) That $G(0)=0$ is obvious. To evaluate $G(1)$, note that, by L'Hopital's Rule,

$$\lim_{\bar{\pi}_t\to 1}\frac{1-\left[\frac{\left(r/R\right)}{1-\bar{\pi}_t\left[1-\left(r/R\right)\right]}\right]^2}{1-\pi_t}=\frac{2\left[1-\left(r/R\right)\right]}{\left(r/R\right)}$$

Thus $G(1)=2I_t\left(1-r/R\right)$.

In order to show that $G'>0$ holds, note that we can write

(E.26) $$G(\bar{\pi}_t)\equiv\left\{1+\left[\frac{\left(r/R\right)}{1-\bar{\pi}_t\left[1-\left(r/R\right)\right]}\right]\right\}\frac{\bar{\pi}_t\left[\bar{\pi}_t I_t\left(\frac{r}{R}\right)+\left(1-\bar{\pi}_t\right)\right]\left(1-r/R\right)}{\left[1-\bar{\pi}_t\left(1-r/R\right)\right]}$$

Next, define

(E.27) $$T(\bar{\pi}_t)\equiv\frac{\bar{\pi}_t\left\{1-\bar{\pi}_t\left[1-\left(r/R\right)I_t\right]\right\}}{1-\bar{\pi}_t\left(1-r/R\right)}$$

Clearly, if $T'(\bar{\pi}_t)\geq 0$, then $G'(\bar{\pi}_t)>0$. Differentiating (A.34) gives

$$\frac{\bar{\pi}_t T'(\bar{\pi}_t)}{T(\bar{\pi}_t)}=\frac{1}{1-\bar{\pi}_t\left(1-r/R\right)}-\frac{\left[1-\left(r/R\right)I_t\right]\bar{\pi}_t}{1-\bar{\pi}_t\left[1-\left(r/R\right)I_t\right]}>0.$$

This establishes the proposition.

E. Proof of Proposition 3

That $\gamma_t^a > 0$ holds follows from the discussion in the text. Differentiating (4.3) yields

$$\text{(E.28)} \qquad \frac{1}{\gamma_t^a}\frac{\partial \gamma_t^a}{\partial I_t} = \frac{\left(\frac{R}{I_t^2}\right)}{\left(\frac{R}{I_t} - r\right)} - \frac{\hat{\pi}(R-r)}{(I_t - 1)\left[\hat{\pi}(R-r) - r(I_t - 1)\right]}$$

Thus $\partial\gamma_t^a/\partial I_t < 0$ holds if

$$\text{(E.29)} \qquad \left(\frac{I_t - 1}{I_t}\right)\left[\frac{1}{1-\left(r/R\right)I_t}\right]\left[\hat{\pi}(R-r) - r(I_t - 1)\right] < \hat{\pi}(R-r)$$

Rearranging terms in (A.2), one obtains that $\partial\gamma_t^a/\partial I_t < 0$ if

$$\text{(E.30)} \qquad r > \left[\frac{\left(r/R\right)I_t^2 - 1}{(I_t - 1)^2}\right]\hat{\pi}(R-r)$$

is satisfied. It is readily verified that the right-hand side of (A.30) is increasing in I_t, $\forall I_t$ satisfying (4.4). Thus, since $I_t < R/r$, (A.30) holds $\forall I_t$ satisfying (4.4) if

$$\text{(E.31)} \qquad r > \left[\frac{\frac{R}{r} - 1}{\left(\frac{R}{r} - 1\right)^2}\right]\hat{\pi}(R-r) = \hat{\pi}r \, .$$

But (A.31) is obviously satisfied, establishing the result.

F. Proof of Proposition 4

Differentiating (4.5) and using the envelope theorem yields

$$\text{(E.32)} \quad I_t v'(I_t) = \hat{\pi}\left[\frac{(1-\gamma_t^a)\left(r/R\right)I_t}{\gamma_t^a + (1-\gamma_t^a)\left(r/R\right)I_t}\right] + (1-\hat{\pi})\left[\frac{(1-\gamma_t^a)I_t}{\gamma_t^a + (1-\gamma_t^a)I_t}\right] - 1 =$$

$$-\hat{\pi}\left[\frac{\gamma_t^a}{\gamma_t^a + (1-\gamma_t^a)\left(r/R\right)I_t}\right] - (1-\hat{\pi})\left[\frac{\gamma_t^a}{\gamma_t^a + (1-\gamma_t^a)I_t}\right] < 0 . \; \square$$

G. The Optimal Behavior of a Monopolistic Bank; $\bar{\zeta}_t < 1$

The bank's first order condition for $\alpha(\pi_t)$ is

$$\lambda\pi_t \Big/ \alpha(\pi_t) = \gamma_t\left(\frac{p_t}{p_{t+1}}\right)$$

Rearranging terms and using the definition of ζ_t^* in the text gives equation (4.13). Note that $\alpha(\pi_t) < 1$ holds if and only if $\pi_t < \zeta_t^*$.

The bank's first order condition for $\delta(\pi_t)$ is

$$R \le \lambda\pi_t r \Big/ \left[\gamma_t\left(\frac{p_t}{p_{t+1}}\right) + \delta(\pi_t) r (1-\gamma_t)\right],$$

with equality if $\delta(\pi_t) > 0$. This equation implies that $\delta(\pi_t) = 0$ if (4.14) is satisfied. For $\pi_t > \bar{\zeta}_t$, solving the above expression for $\delta(\pi_t)$ and using the definition of $\bar{\zeta}_t$ gives equation (4.15).

The first order condition for $d(\pi_t)$ is $d(\pi_t) = \lambda$. This is equation (4.16) in the text.

Equations (4.17)–(4.19) are derived from (4.7)–(4.9) and (4.11)–(4.15).

The bank's first order condition for γ_t is

$$\text{(E.33)} \quad \left(\frac{p_t}{p_{t+1}}\right)\int_0^{\zeta_t^*}\left[1-\alpha(\pi)\right] f(\pi)\,d\pi - RF(\bar{\zeta}_t)$$

$$-R\int_{\bar{\zeta}_t}^{1}\left[1-\delta(\pi)\right] f(\pi)\,d\pi + \lambda\int_0^{\zeta_t^*}\left(\frac{\pi}{\gamma_t}\right) f(\pi)\,d\pi +$$

$$\lambda\int_{\zeta_t^*}^{\bar{\zeta}_t}\left(\frac{\pi}{\gamma_t}\right) f(\pi)\,d\pi +$$

$$\lambda\int_{\bar{\zeta}_t}^{1}\left\{\frac{\pi\left[\left(\frac{p_t}{p_{t+1}}\right) - r\delta(\pi)\right]}{\gamma_t\left(\frac{p_t}{p_{t+1}}\right) + \delta(\pi) r(1-\gamma_t)}\right\} f(\pi)\,d\pi$$

$$f(\zeta_t^*)\frac{\partial\zeta_t^*}{\partial\gamma_t}\left\{\left[1-\alpha(\zeta_t^*)\right]\gamma_t\left(\frac{p_t}{p_{t+1}}\right) + \right\}$$

$$-f(\bar{\zeta}_t)\frac{\partial\bar{\zeta}_t}{\partial\gamma_t} R(1-\gamma_t)\left\{1-\left[\delta(\bar{\zeta}_t)\right]\right\} = 0$$

Now note that $\alpha(\zeta_t^*) = 1$ and $\delta(\bar{\zeta}_t) = 0$. It follows that the first order condition for γ_t—equation (A.35)—reduces to

$$\text{(E.34)}\quad 0=\int_0^{\zeta_t^*}\left(\frac{\zeta_t^*-\pi}{\zeta_t^*}\right)f(\pi)\,d\pi-I_tF(\bar{\zeta}_t)-$$

$$I_t\int_{\bar{\zeta}_t}^{1}\left\{1-\left[\frac{\gamma_t}{1-\gamma_t}\right]\left(\frac{R}{rI_t}\right)\left(\frac{\pi-\bar{\zeta}_t}{\bar{\zeta}_t}\right)\right\}f(\pi)\,d\pi+$$

$$\lambda\,\{\frac{p_{t+1}}{p_t}\int_0^{\zeta_t^*}\left(\frac{\pi}{\gamma_t}\right)f(\pi)\,d\pi+$$

$$\left(\frac{p_t}{p_{t+1}}\right)\int_{\zeta_t^*}^{\bar{\zeta}_t}\left(\frac{\pi}{\gamma_t}\right)f(\pi)\,d\pi+\left(\frac{p_{t+1}}{p_t}\right)\int_{\bar{\zeta}_t}^{1}\left[\frac{\left(\frac{p_t}{p_{t+1}}\right)-r\delta(\pi)}{\left(r/R\right)\lambda}\right]f(\pi)\,d\pi\,\}$$

where we have used (4.11)–(4.15) to substitute $\alpha(\pi)$, $b(\pi)$, and $\delta(\pi)$ out of (A.34). Now note that

$$\text{(E.35)}\qquad \int_0^{\zeta_t^*}\left[\frac{\pi\lambda}{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}\right]f(\pi)\,d\pi=\int_0^{\zeta_t^*}\left(\frac{\pi}{\zeta_t^*}\right)f(\pi)\,d\pi\,,$$

$$\text{(E.36)}\qquad \int_{\zeta_t^*}^{\bar{\zeta}_t}\left[\frac{\pi\lambda}{\gamma_t\left(\frac{p_t}{p_{t+1}}\right)}\right]f(\pi)\,d\pi=\int_{\zeta_t^*}^{\bar{\zeta}_t}\left(\frac{\pi}{\zeta_t^*}\right)f(\pi)\,d\pi\,,$$

and

$$\text{(E.37)}\qquad \begin{aligned}&R\int_{\bar{\zeta}_t}^{1}\left\{1-\left[\frac{\gamma_t}{1-\gamma_t}\right]\left(\frac{R}{rI_t}\right)\left(\frac{\pi-\bar{\zeta}_t}{\bar{\zeta}_t}\right)\right\}f(\pi)\,d\pi+\\&\int_{\bar{\zeta}_t}^{1}\left[\frac{r\delta(\pi)}{\left(r/R\right)}\right]f(\pi)\,d\pi=R\left[1-F(\bar{\zeta}_t)\right]\end{aligned}$$

Substituting (A.35)–(A.37) into (A.34) and rearranging terms we obtain the equivalent condition

$$\text{(E.38)}\qquad I_t=\int_{\zeta_t^*}^{\bar{\zeta}_t}\left(\frac{\pi}{\zeta_t^*}\right)f(\pi)\,d\pi+F(\zeta_t^*)+\left(\frac{R}{r}\right)\left[1-F(\bar{\zeta}_t)\right]$$

Rearranging terms in (A.39) yields equation (4.20) in the text.

H. Optimal Behavior of a Monopoly Bank; $\bar{\zeta}_t = 1$

Here the first-order conditions for $\alpha(\pi)$, $d(\pi)$, and $b(\pi)$ are exactly as in Appendix F. The first order condition for γ_t is

(E.39) $$I_t = \int_0^{\zeta_t^*}\left[1-\alpha(\pi)\right]f(\pi)d\pi + \lambda\left\{\left(\frac{p_{t+1}}{p_t}\right)\int_0^{\zeta_t^*}\left(\frac{\pi}{\gamma_t}\right)f(\pi)d\pi + \left(\frac{p_{t+1}}{p_t}\right)\int_{\zeta_t^*}^{1}\left(\frac{\pi}{\gamma_t}\right)f(\pi)d\pi + \right.$$

Using the first order conditions for $\alpha(\pi)$ and $b(\pi)$ (see Appendix F) in (A.40) yields the equivalent condition

(E.40) $$I_t = F(\zeta_t^*) + \int_{\zeta_t^*}^{1}\left(\frac{\pi}{\zeta_t^*}\right)f(\pi)d\pi$$

Rearranging terms in (A.41) gives equation (4.24) in the text.

I. Proof of Proposition 5

If $I_t = 1$, then $\bar{\pi}_t = \bar{\zeta}_t = 1$. We now consider what happens if $I_t > 1$ holds. There are two cases to consider.

(a) $\bar{\zeta}_t = 1$. Here $\bar{\pi}_t < \bar{\zeta}_t$ necessarily holds if $I_t > 1$, since $\bar{\pi}_t < 1$ is implied by (3.15), $G(1) > 2\left[1-\left(r/R\right)I_t\right]$ (see part (b) of proposition 4), and $G'(\bar{\pi}_t) > 0$.

(b) $\bar{\zeta}_t < 1$. If $\bar{\zeta}_t < 1$ holds, then $\bar{\zeta}_t = \left(R/r\right)\zeta_t^* = 2\left[1-\left(r/R\right)I_t\right]\Big/\left[1-\left(r/R\right)^2\right]$. Moreover, the definition of the function G implies that

$$G\left\{\frac{2\left[1-\left(r/R\right)I_t\right]}{1-\left(r/R\right)^2}\right\} = 2\left[1-\left(r/R\right)I_t\right]\left[\frac{1-\bar{\pi}_t+\bar{\pi}_t\left(r/R\right)I_t}{1-\bar{\pi}_t+\bar{\pi}_t\left(r/R\right)}\right]\bullet$$

$$\left\{\frac{1+\dfrac{\left(r/R\right)}{1-\left[1-\left(r/R\right)\right]\bar{\pi}_t}}{1+\left(r/R\right)}\right\} > 2\left[1-\left(r/R\right)I_t\right]$$

Thus (3.15) and part (b) of proposition 2 imply that $\bar{\pi}_t < \bar{\zeta}_t$ holds.

J. Proof of Proposition 6

It is easy to verify that a competitive banking system has

(E.41) $$\bar{\pi}_t = \frac{\pi_t^*}{\left(r/R\right) + \left[1-\left(r/R\right)\right]\pi_t^*} \equiv J\left(\pi_t^*\right).$$

Clearly $J(0)=0$, $J(1)=1$, and $J'\left(\pi_t^*\right)>0$ hold.

Now, define the function H by

(E.42) $$H\left(\pi_t^*\right) \equiv G\left[J\left(\pi_t^*\right)\right].$$

Proposition 2 implies that $H(0)=0$ and that $H(1)=2I_t\left[1-\left(r/R\right)\right]$. Moreover, clearly $H'>0$ holds.

(a) If $I_t=1$, then (4.22) is violated and $\bar{\zeta}_t=1$. Moreover, if (4.22) is violated it is easy to verify that

(E.43) $$\zeta_t^* = I_t - \sqrt{I_t^2-1}.$$

Thus, when $I_t=1$, $\zeta_t^*=1$. The result that $\pi_t^*=1$ is implied by $J(1)=1$, and $G(1)=2I\left[1-\left(r/R\right)\right]=2\left[1-\left(r/R\right)I_t\right]$ when $I_t=1$ holds.

(b) We first consider the case in which $I_t>\left[1+\left(r/R\right)^2\right]/2\left(r/R\right)$ holds. This implies that $\bar{\zeta}_t<1$.

We begin by observing that the equilibrium value π_t^* satisfies

(E.44) $$H\left(\pi_t^*\right)=G\left(\bar{\pi}_t\right)=2\left[1-\left(r/R\right)I_t\right].$$

In addition, given the restriction on I_t,

(E.45) $$\zeta_t^* = \left(r/R\right)\bar{\zeta}_t = \frac{2\left(r/R\right)\left[1-\left(r/R\right)I_t\right]}{1-\left(r/R\right)^2}$$

holds.

The observation that $H'>0$ holds implies that $\pi_t^*>(<)\zeta_t^*$ holds if

(E.46) $$G\left[J\left(\zeta_t^*\right)\right]<(>)2\left[1-\left(r/R\right)I_t\right]$$

since π_t^* satisfies $G\left[J\left(\pi_t^*\right)\right]=2\left[1-\left(r/R\right)I_t\right]$. We now note that

(E.47) $$J\left(\zeta_t^*\right)=\frac{2\left(r/R\right)\left[1-\left(r/R\right)I_t\right]}{\left(r/R\right)\left[1-\left(r/R\right)\right]\left\{1+\left(r/R\right)+2\left[1-\left(r/R\right)I_t\right]\right\}}$$

and that

(E.48)
$$H\left(\zeta_t^*\right)=\left\{\frac{2\left[1-\left(r/R\right)I_t\right]}{1+\left(r/R\right)+2\left[1-\left(r/R\right)I_t\right]}\right\}\left\{\frac{1-J\left(\zeta_t^*\right)\left[1-\left(r/R\right)I_t\right]}{1-J\left(\zeta_t^*\right)\left[1-\left(r/R\right)\right]}\right\}\times$$
$$\left\{1+\frac{\left(r/R\right)}{1-J\left(\zeta_t^*\right)\left[1-\left(r/R\right)\right]}\right\}$$

Moreover,

(E.49)
$$1-J\left(\zeta_t^*\right)\left[1-\left(r/R\right)\right]=\frac{1+\left(r/R\right)}{1+\left(r/R\right)+2\left[1-\left(r/R\right)I_t\right]}$$

and

(E.50)
$$1-J\left(\zeta_t^*\right)\left[1-\left(r/R\right)I_t\right]=1-\frac{2\left[1-\left(r/R\right)I_t\right]^2}{\left[1-\left(r/R\right)\right]\left\{1+\left(r/R\right)+2\left[1-\left(r/R\right)I_t\right]\right\}}$$

Substituting (A.50) and (A.51) into (A.47) and rearranging terms yields

(E.51)
$$H\left(\zeta_t^*\right)=\left\{\frac{2\left[1-\left(r/R\right)I_t\right]}{1+\left(r/R\right)}\right\}\left\{1+\frac{\left(r/R\right)\left\{1+\left(r/R\right)+2\left[1-\left(r/R\right)I_t\right]\right\}}{1+\left(r/R\right)}\right\}-$$
$$\left\{\frac{4\left[1-\left(r/R\right)I_t\right]^3}{\left[1-\left(r/R\right)\right]\left[1+\left(r/R\right)\right]}\right\}\left\{\frac{\left(r/R\right)}{1+\left(r/R\right)}+\frac{1}{1+\left(r/R\right)+2\left[1-\left(r/R\right)I_t\right]}\right\}$$

We now note that $H\left(\zeta_t^*\right)<(>)2\left[1-\left(r/R\right)I_t\right]$ holds if

(E.52)
$$1>(<)\left[\frac{1}{1+\left(r/R\right)}\right]\left\{1+\left[\frac{1+\left(r/R\right)+2\left[1-\left(r/R\right)I_t\right]}{1+\left(r/R\right)}\right]\left(r/R\right)\right\}$$
$$-\left\{\frac{2\left[1-\left(r/R\right)I_t\right]^2}{\left[1-\left(r/R\right)\right]\left[1+\left(r/R\right)\right]}\right\}\left\{\frac{\left(r/R\right)}{1+\left(r/R\right)}+\frac{1}{1+\left(r/R\right)+2\left[1-\left(r/R\right)I_t\right]}\right\}$$

Moreover, algebraic manipulation establishes that (A.53) is equivalent to

(E.53)
$$\frac{\left(r/R\right)^2(I_t-1)}{1+\left(r/R\right)}<(>)\frac{1-\left(r/R\right)I_t}{1+\left(r/R\right)+2\left[1-\left(r/R\right)I_t\right]}$$

Now define the function $Z\left(I_t\right)$ by

(E.54) $$Z(I_t) \equiv \frac{\left(\frac{r}{R}\right)^2 (I_t - 1)}{1 + \left(\frac{r}{R}\right)} - \frac{1 - \left(\frac{r}{R}\right) I_t}{1 + \left(\frac{r}{R}\right) + 2\left[1 - \left(\frac{r}{R}\right) I_t\right]}$$

It is easy to verify that $Z'(I_t) > 0 \forall I_t > 1$. Moreover, clearly $Z\left(\frac{r}{R}\right) > 0$ holds. Finally, one can check that $Z\left[\frac{1+\left(\frac{r}{R}\right)^2}{2\left(\frac{r}{R}\right)}\right] < 0$ holds if $\left(\frac{r}{R}\right)\left[1-\left(\frac{r}{R}\right)\right]\left[2-\left(\frac{r}{R}\right)\right] < 1 + \left(\frac{r}{R}\right)$, which obviously holds. It follows that there is a unique value $\tilde{I}_t \in \left(\left[1+(r/R)^2\right]/2(r/R), (R/r)\right)$ satisfying $Z(\tilde{I}_t) = 0$. In addition, $Z(I_t) < (>) 0$ holds for $I_t < (>) \tilde{I}_t$. It is then immediate that $H(\zeta_t^*) < (>) 2\left[1-(r/R)I_t\right]$ for $I_t < (>) \tilde{I}_t$. Hence $\pi_t^* > (<) \zeta_t^*$ holds if $I_t < (>) \tilde{I}_t$.

It remains to consider that case in which $I_t \in \left(1, \left[1+(r/R)^2\right]/2(r/R)\right]$. Here $\bar{\zeta}_t = 1$ holds, and ζ_t^* is given by

(E.55) $$\zeta_t^* = I_t - \left(I_t^2 - 1\right)^{0.5} < 1.$$

It is easy to check that, for $I_t \leq \left[1+(r/R)^2\right]/2(r/R)$,

$\frac{2\left[1-\left(\frac{r}{R}\right)I_t\right]}{1-\left(\frac{r}{R}\right)^2} \geq 1$ holds. It then follows that

(E.56) $$I_t - \sqrt{I_t^2 - 1} \leq \frac{2\left[1-\left(\frac{r}{R}\right)I_t\right]}{1-\left(\frac{r}{R}\right)^2}$$

is satisfied for all $I_t \leq \left[1+(r/R)^2\right]/2(r/R)$.

It is now immediate that $G\left[J(\zeta_t^*)\right] < G\left[J\left\{\frac{2\left[1-\left(\frac{r}{R}\right)I_t\right]}{1-\left(\frac{r}{R}\right)^2}\right\}\right]$ holds for all $I_t < \left[1+(r/R)^2\right]/2(r/R)$. Moreover, since $\left[1+(r/R)^2\right]/2(r/R) < \tilde{I}_t$,

$G\left[J(\zeta_t^*)\right] < G\left[J\left\{\frac{2\left[1-\left(\frac{r}{R}\right)I_t\right]}{1-\left(\frac{r}{R}\right)^2}\right\}\right] < 2\left[1-\left(\frac{r}{R}\right)I_t\right]$ for all $I_t \in \left(1, 1+(r/R)^2/2(r/R)\right)$.

Thus $H(\zeta_t^*) < 2\left[1-(r/R)I_t\right] = H(\pi_t^*)$ for all relevant values of I_t, and $\pi_t^* > \zeta_t^*$ holds.

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