The U.S. Dollar and the Trade Deficit: What Accounts for the Late 1990s?

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Abstract

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

Based on a version of the IMF's new Global Economic Model (GEM), calibrated to analyze macroeconomic interdependence between the United States and the rest of the world, this paper asks to what extent an asymmetric productivity shock in the tradable sector of the economy may account for real exchange rate and trade balance developments in the United States in the second half of the 1990s. The paper concludes that the Balassa-Samuelson effect of such a productivity shock is only part of the story. A second shock, a broadly defined "risk premium" shock, and some uncertainty about the persistence of both shocks are needed to match the data more satisfactorily.

JEL Classification Numbers: F43, F47, O41

Keywords: Balassa-Samuelson Effect, Learning, Productivity Shocks, Real Exchange Rate, Risk Premium Shocks, Trade Balance, U.S. Economy

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1 The Global Economic Model (GEM) was developed by a team in the IMF's Research Department led by Paolo Pesenti and Douglas Laxton. The authors wish to thank Chris Erceg, Chris Gaust, and Luca Guerrieri at the Federal Reserve Board for countless discussions and comments, and Tam Bayoumi, Peter Isard, and Ken Rogoff for encouraging this project. The authors also acknowledge the invaluable input of Michel Juillard and Susanna Mursula in developing some of the numerical procedures used in the quantitative analysis of the model, and thank Tam Bayoumi, Matthew Canzoneri, Giancarlo Corsetti, Luca Dedola, Margarida Duarte, Jordi Gali, Fabio Ghironi, Mads Kieler, Jaewoo Lee, Hali Edison, Dong Laxton, Paolo Pesenti, and seminar participants at the Bank of Canada, Bank of Italy, Georgetown University, the IMF, and the 2003 Summer Meeting of the North American Econometric Society for their useful comments. Remaining errors are of the authors.
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I. Introduction

The late 1990s witnessed a coincidence of events in the United States striking enough to require an explanation. After a long period of modest growth, productivity accelerated markedly. The acceleration in productivity was accompanied by an investment boom, a gradual appreciation in the real exchange rate and a marked deterioration in the external balance financed by large foreign capital inflows (Figure 1). Optimists saw the U.S. economy entering a new era of prosperity driven by technology, while pessimists pointed to growing financial imbalances characteristic of an unsustainable boom.

The subsequent recession has tempered the wings of both camps. It has taken some of the shine off the optimists’ “new-economy” paradigm. At the same time, the slowdown has been less steep and less U.S. specific, and the rate of increase in productivity has remained more buoyant, than the pessimists had predicted. Indeed, to some extent the jury remains out on the underlying causes, and hence consequences, of the acceleration of growth in the late 1990s.

This paper uses the IMF’s new Global Economic Model (GEM) to examine what may account for the real exchange rate appreciation, external balance deterioration, and broader business cycle developments in the United States in the second half of the 1990s. The main result of the analysis is that both sides of the “new economy” debate had a point: the behavior of the US economy in the late 1990s likely reflected both a genuine productivity improvement as well as some “irrational exuberance”.

GEM is a New-Open-Economy-Macroeconomics (NOEM) model. NOEM models feature short-run demand determination of output and international trade in goods, services, and assets derived completely within a choice-theoretic framework. Short-run demand determination of output originates from costly price and wage adjustment and monopolistic competition, leading to an explicit stabilization role for policy. International trade originates from specialization, tastes and technology, allowing for a thorough analysis of the international transmission mechanism of shocks and policies. Because of their explicit mico-foundations, NOEM models are less vulnerable to the Lucas Critique than traditional, Open-Economy-Macroeconomics models (e.g., Dornbusch, 1980) and also allow for welfare-based analysis of different distortions and alternative policies.

While not unique in its class, GEM’s structure is richer than the typical NOEM model, both in terms of potential sources of disturbance as well as in terms of the complexity of their endogenous propagation mechanisms. Specifically, GEM combines many features and properties of models

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2 See the seminal contribution of Obstfeld and Rogoff (1996, Chapter 10). See Lane (2001) and Samo (2002) for surveys of this recent literature.
that have been designed to shed light on specific policy or theoretical issues. By combining features that have been shown to matter for a good fit of the data, GEM is among a handful of large, theory-consistent models capable of delivering a satisfactory representation of trade and financial interdependence.\footnote{The one-good, noEM model developed by Erceg, Guerrieri, and Gaust (2002) at the Federal Reserve Board and the closed economy model of the euro area developed by Smets and Wouters (2002) at the ECB are the closest comparators of GEM.} A contribution of this paper, therefore, is to show that a rigorous, theory-based framework of macroeconomic interdependence may be useful also for actual policy purposes.

To cite only the most closely related references in the literature, GEM is a multi-good model with a distribution sector, as in the multi-good framework proposed by Corsetti and Dedola (2002), inducing market segmentation and endogenous incomplete pass-through. As in the dynamic framework with endogenous specialization of Fitzgerald (2003), or in the static framework with an explicit role for net foreign assets of Lee (2003), in GEM, the emergence and/or the strength of the Balassa-Samuelson effect depends on the specific parametrization of the model. As in the analyses of McCallum and Nelson (1999), Duarte and Stockman (2001), and Kollmann (2001), in GEM, uncovered interest rate parity shocks have an important role for exchange rate and broader business cycle dynamics. As illustrated by Erceg, Guerrieri, and Gaust (2002), in a comparably rich NOEM model, and by Hunt (2002) in the context of the IMF’s MULTIMOD, some departure from full rational expectations is needed also in GEM to produce realistically shaped model responses to productivity shocks as those considered in this paper.

This paper examines the extent to which a permanent, asymmetric productivity shock in the tradable sector of the economy—possibly associated with the so-called information technology (IT) revolution—may explain the behavior of the U.S. dollar and the trade balance in the late 1990s. As we shall see, the Balassa (1964)-Samuelson (1964) effect of such a productivity shock allows GEM to explain about half of the dollar appreciation in second half of the 1990s.\footnote{See Lee and Tang (2003) for a fresh look at the empirical evidence on productivity and the real exchange rate.} A productivity shock is not the end of the story, though. To match the data more satisfactorily, we need to add a second shock—namely, a reduction in the perceived riskiness of U.S. assets—and a learning mechanism about the persistence of these two shocks.\footnote{While GEM has more sources of potential disturbance than the two analyzed in the paper, note that no other shock in addition to the two mentioned underpins the simulation results reported in this paper.}

The rest of the paper is organized as follows. Section II provides an overview of GEM and some detail on two of its features, which are key to generate the simulation results. Section III discusses the model baseline calibration and its numerical solution. Section IV reports on the simulation results. Section V concludes. The model is fully described in the Appendix.

II. THE MODEL

This section outlines the basic structure of GEM, illustrated in Figure 1, and focuses on two
model features that are central to our results: the production of final, nontraded goods and the distribution sector and its effect on price setting in the intermediate goods sector. All optimization problems underlying the model’s structure and the resulting first order conditions are reported in the Appendix. The two shocks considered, as well as others potentially operating in GEM, are also discussed in the Appendix.

A. Model overview

The world economy consists of two countries, Home (identified with the United States) and Foreign (identified with the rest of the world). Foreign variables are indexed with a star. In each country there are households, firms, and a government.

Each household is infinitely lived, consumes a single nontradable final good ($A$), and is the monopolistic supplier of a differentiated labor input ($\ell$) to all domestic firms. Wage contracts are subject to adjustment costs, which give rise to nominal wage rigidities.

Each household owns all domestic firms and the domestic capital stock ($K$), which it rents to domestic firms. The market for capital is competitive, but capital accumulation is subject to adjustment costs. Labor and physical capital are immobile internationally. Households trade internationally a short-term nominal bond, denominated in Foreign currency and issued in zero net supply worldwide. There are intermediation costs for Home households entering the international bond market that assure model stationarity and determinacy of the steady-state net foreign asset distribution. No other asset is traded internationally.

Firms produce a continuum of nontradable final goods, a continuum of differentiated nontradable intermediate goods ($N$), a continuum of differentiated tradable intermediate goods ($T$), and provide distribution and financial intermediation services.

The final good is produced by perfectly competitive firms that use all intermediate goods as inputs: nontradable goods ($N_A$), domestic tradables ($Q$) or imported tradables ($M$). The final good can be consumed by domestic households ($C$) or the government ($G_A$), or used for investment ($I$).

Each intermediate good is produced by a single firm under conditions of monopolistic competition by using labor and capital. Prices of intermediate goods are subject to adjustment costs, which give rise to nominal price rigidities. Each nontradable intermediate good is either used directly in the production of the nontradable final good or used indirectly in the distribution sector to make tradable intermediate goods available to firms producing the final good. Each tradable intermediate good is used either in the production of the domestic nontradable final good or in the production of the foreign final good.

Firms in the distribution sector operate under perfect competition. They purchase tradable intermediate goods worldwide (at the producer price) and distribute them to firms producing

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the final good \textit{(at the consumer price)}. Local nontradables are the only input in the provision of distribution services.

Government spending \((G_A)\) falls exclusively on the final nontradable good. Government spending is financed through tax and seigniorage revenues. The government controls the national short-term nominal interest rate. Monetary policy is specified in terms of an interest rate rule.

### B. Key model features

#### Final goods production

Home firm \(x\)'s output of final goods at time (quarter) \(t\) is denoted \(A_t(x)\). The final good is produced with the following CES technology:

\[
A_t(x) \equiv \left[ \gamma^\frac{1}{\varepsilon} \left( \nu^\frac{1}{\varepsilon} Q_t(x)^{1 - \frac{1}{\varepsilon}} \right) + (1 - \nu)^\frac{1}{\varepsilon} M_t(x) (1 - \Gamma_{M,t}(x)) \right]^{\frac{\varepsilon}{\varepsilon - 1}} \left[ \xi_{QM} \right]^{1 - \frac{1}{\varepsilon}},
\]

where imports are subject to adjustment costs,\(^7\)

\[
\Gamma_{M,t}(x) = \frac{\phi_M}{2} \left( \frac{M_t(x)/A_t(x)}{M_{t-1}/A_{t-1}} - 1 \right)^2.
\]

Three intermediate inputs are used in the production of the final good: a basket \(Q\) of Home tradable goods, a basket \(M\) of imported Foreign tradable goods, and a basket \(N_N\) of Home nontradable goods. The elasticity of substitution between tradables and nontradables is \(\varepsilon\), while the elasticity of substitution between domestic and imported tradable goods is \(\varepsilon_{QM}\). The parameters \(\gamma\) and \(\nu\) determine the degree of openness of the economy and the extent to which Home preferences are skewed toward home goods (i.e., home bias).\(^8\)

As we shall see, the use of nontradable intermediate goods in the production of the final nontradable good is an essential feature of the model. This allows for a Balassa-Samuelson effect on the real exchange rate of asymmetric productivity shocks in the tradable sector. With higher productivity in the tradable sector, the marginal product of labor is higher and so must be the producer real wage. To achieve a higher producer real wage, with labor freely mobile across sectors, the price of tradables relative to the price of nontradables must decline. For an elasticity of substitution between the Foreign and Home tradable good above a certain threshold (e.g., above one for the comparative static results reported in Figure 5 and discussed below), the decline

\(^7\) Following Ercig, Guerrieri, and Gaust (2002), we assume that it is costly to change the share of the imported goods in total production. Unlike them, we assume that it is costly for the firm to adjust its current imports/output ratio relatively to the past aggregate imports/output ratio.

\(^8\) The model can be easily re-interpreted as if households consumed directly a basket \(A\) of domestic and imported final goods. In this case \(\gamma\) and \(\nu\) would be interpreted as preference parameters.
in the Home country relative price of tradables necessitates a real exchange rate appreciation to moderate demand for the Home tradable good and achieve trade balance equilibrium in the long run. Then, a specific value of the elasticity of substitution between the Home and Foreign tradables above this threshold determines the magnitude of this Balassa-Samuelson effect on the real exchange rate.\textsuperscript{9}

The distribution sector and price setting in the intermediate goods sector

As in the model of Corsetti and Dedola (2002), the distribution sector purchases tradables both domestically and abroad and distributes them to the firms producing the final good. There are no distribution costs for nontradables. The distribution technology is Leontief: To produce one unit of an intermediate good available to downstream producers, firms in the distribution sector require $\eta \geq 0$ units of the nontradable basket $N$. Firms in the distribution sector are perfectly competitive.

Distribution costs create a wedge between producer and consumer prices (that is, between wholesale and retail prices). Denoting with $p$ the consumer price, and with $\bar{p}$ the Home-currency producer price of an intermediate good, for each good $n$, $h$, and $f$, we have, respectively:

\begin{align}
    p_t(n) &= \bar{p}_t(n), \\
    p_t(h) - \bar{p}_t(h) &= \eta P_{N,t}, \\
    p_t(f) - \bar{p}_t(f) &= \eta P_{N,t}.
\end{align}

(3) \quad (4) \quad (5)

To see the impact of distribution costs on the pricing behavior of producers, consider profit maximization in the Home country's tradable sector. Firm $h$ takes into account demand for its product—equation (A-13) in the Appendix and its equivalent in the Foreign market—and sets nominal prices $\bar{p}_t(h)$ and $\bar{p}^*_t(h)$ to maximize the present discounted value of real profits, given sluggish price adjustments due to resource costs measured in terms of total profits.

Denoting the nominal exchange rate with $E$ (i.e., the Home currency price of one unit of Foreign currency), firm $h$'s price setting problem in the Home tradable intermediate sector is:

\begin{align}
    \{\bar{p}_t(h), \bar{p}^*_t(h)\}_{t=1}^{\infty} = \arg \max E_t \sum_{t=1}^{\infty} D_{t,t} \left\{ \left[ \bar{p}_t(h) - MC_t(h) \right] \right. \\
    \left. \times \left( \frac{\bar{p}_t(h) + \eta P_{N,t}}{P_{Q,t}} \right)^{-\theta} Q_t \left[ 1 - \Gamma_{PQ,t}(h) \right] + \left[ E_t \bar{p}^*_t(h) - MC_t(h) \right] \right. \\
    \left. \times \left( \frac{\bar{p}^*_t(h) + \eta^* P_{N,t}^*}{P_{M,t}^*} \right)^{-\theta} M_t^* \frac{1 - s}{s} \left[ 1 - \Gamma_{PM,t}^*(h) \right] \right\}. 
\end{align}

(6)

\textsuperscript{9} For the Balassa-Samuelson effect to operate, analogous conditions must be met in the framework of Corsetti and Dedola (2002), Fitzgerald (2003), and Lee (2003).
The adjustment costs, denoted $\Gamma_{PQ,t}(h)$ and $\Gamma^*_{PM,t}(h)$, are:

$$\Gamma_{PQ,t}(h) = \frac{\phi_{Q1}}{2} \left( \frac{\bar{p}_t(h)}{\pi \bar{p}_{t-1}(h)} - 1 \right)^2 + \frac{\phi_{Q2}}{2} \left( \frac{\bar{p}_t(h)}{\bar{p}_{t-1}(h) - 1} \right)^2 \left( \frac{\bar{p}_{t-1}(h)}{\bar{p}_{t-2}(h)} - 1 \right)^2$$

(7)

$$\Gamma^*_{PM,t}(h) = \frac{\phi^*_{M1}}{2} \left( \frac{\bar{p}_t(h)}{\pi \bar{p}_{t-1}(h)} - 1 \right)^2 + \frac{\phi^*_{M2}}{2} \left( \frac{\bar{p}_t(h)}{\bar{p}_{t-1}(h) - 1} \right)^2 \left( \frac{\bar{p}_{t-1}(h)}{\bar{p}_{t-2}(h)} - 1 \right)^2$$

(8)

Profit maximization yields the following two pricing conditions:

$$0 = [1 - \Gamma_{PQ,t}(h)] \frac{\bar{p}_t(h)}{\bar{p}_t(h) + \eta P_{N,t}} \left[ \bar{p}_t(h) (1 - \theta) + \eta P_{N,t} + \theta MC_t(h) \right]$$

$$- \left[ \bar{p}_t(h) - MC_t(h) \right] \frac{\phi_{Q1} \bar{p}_t(h)}{\pi \bar{p}_{t-1}(h)} \left( \frac{\bar{p}_t(h)}{\pi \bar{p}_{t-1}(h)} - 1 \right)$$

$$- \left[ \bar{p}_t(h) - MC_t(h) \right] \frac{\phi_{Q2} \bar{p}_t(h)}{\bar{p}_{t-1}(h) - 1} \left( \frac{\bar{p}_t(h)}{\bar{p}_{t-1}(h) - 1} \right)^2 \left( \frac{\bar{p}_{t-1}(h)}{\bar{p}_{t-2}(h)} - 1 \right)^2$$

$$+ E_t D_{t+1} \left[ \bar{p}_{t+1}(h) - MC_{t+1}(h) \right] \frac{Q_{t+1}(h)}{Q_t(h)} \left( \frac{\bar{p}_{t+1}(h)}{\bar{p}_t(h)} - 1 \right) \left[ \frac{\phi_{Q1} \bar{p}_t(h)}{\pi \bar{p}_t(h)} - 1 \right]$$

$$\left( \frac{\bar{p}_{t+1}(h)}{\pi \bar{p}_t(h)} - 1 \right) + \frac{\phi_{Q2} \bar{p}_{t+1}(h)}{\bar{p}_t(h)} \left( \frac{\bar{p}_{t+1}(h)}{\bar{p}_t(h)} - 1 \right)$$

(9)

and

$$0 = [1 - \Gamma^*_{PM,t}(h)] \frac{\bar{p}_t(h)}{\bar{p}_t(h) + \eta^* P_{N,t}^*} \left[ \bar{p}_t(h) (1 - \theta) + \eta^* E_t P_{N,t}^* + \theta MC_t(h) \right]$$

$$- \left[ \bar{p}_t(h) - MC_t(h) \right] \frac{\phi^*_{M1} \bar{p}_t(h)}{\pi \bar{p}_{t-1}(h)} \left( \frac{\bar{p}_t(h)}{\pi \bar{p}_{t-1}(h)} - 1 \right)$$

$$- \left[ \bar{p}_t(h) - MC_t(h) \right] \frac{\phi^*_{M2} \bar{p}_t(h)}{\bar{p}_{t-1}(h) - 1} \left( \frac{\bar{p}_t(h)}{\bar{p}_{t-1}(h) - 1} \right)^2 \left( \frac{\bar{p}_{t-1}(h)}{\bar{p}_{t-2}(h)} - 1 \right)^2$$

$$+ E_t D_{t+1} \left[ \bar{p}_{t+1}(h) - MC_{t+1}(h) \right] \frac{M_{t+1}(h)}{M_t(h)} \left( \frac{\phi^*_{M1} \bar{p}_t(h)}{\pi \bar{p}_t(h)} - 1 \right)$$

$$\left( \frac{\bar{p}_{t+1}(h)}{\pi \bar{p}_t(h)} - 1 \right) + \frac{\phi^*_{M2} \bar{p}_{t+1}(h)}{\bar{p}_t(h)} \left( \frac{\bar{p}_{t+1}(h)}{\bar{p}_t(h)} - 1 \right)$$

(10)

In the absence of price stickiness, these last two equations become:

$$\bar{p}_t(h) = \frac{\theta}{\theta - 1} MC_t(h) + \frac{\eta}{\theta - 1} P_{N,t}$$

(11)

and

$$E_t \bar{p}_t^*(h) = \frac{\theta}{\theta - 1} MC_t(h) + \frac{\eta^*}{\theta - 1} E_t P_{N,t}^*$$

(12)
As stressed by Corsetti and Dedola (2002), in the presence of distribution costs, exchange rate pass-through is less than perfect (i.e., \( \partial \log \bar{p}_t(h) / \partial \log \bar{E}_t \leq 1 \)) and the law of one price does not hold (i.e., \( \bar{p}_t(h) \neq \bar{E}_t \bar{p}_t^*(h) \)) even if the prices of tradable goods are flexible (and hence even in steady state).

This second model feature is particularly important in tracking the evolution of the U.S. real exchange rate and trade balance in the presence of a decline in the risk premium demanded on US assets. This is because distribution costs lower the trade balance's response to relative price changes, and forces the real exchange rate to move more than otherwise it would have to do. As we explain in more detail below, a muted expenditure switching effect of the exchange rate, in turn, helps in improving the model's overall performance considerably.

III. Model calibration and solution

A. Baseline calibration

The model is calibrated to simulate interdependence between the United States and the rest of the world. The strategy to develop a baseline calibration is to rely on parameter values supported by available empirical evidence whenever possible, or values that have already been used in the literature.

We work with a standard, simple specification for preferences and technologies by assuming that the production functions for both traded and non-traded goods have a Cobb-Douglas functional form and that the period utility function is additive-separable in consumption and labor effort, and logarithmic in consumption (i.e., unitary intertemporal elasticity of substitution). These assumptions guarantee that the model converges to a steady state consistent with balanced growth following persistent shocks to the rate of growth of total factor productivity (TFP)—that imply a permanent change in the level of TFP.

In general, we retain as much symmetry as possible between the Foreign and the Home economy, except for those parameters that need to be set asymmetrically to characterize the U.S. economy. These include country size and home bias in consumption.

Home country size \((s)\) relative to the rest of the world is measured in terms of GDP. We set \(s\) at 0.25 percent. For the home bias, we adopt parameter values that yield import and export ratios consistent with the data. The choice of the relative weight of the nontraded sector \((\gamma)\) and the home bias parameter \((\nu)\), and their foreign equivalents, implies that there is no home bias in the Foreign economy and some home bias in the Home economy compared to country size. Specifically, in the baseline, we set \(\gamma = \gamma^* = 0.67, \nu = 0.98 \) and \(\nu^* = 0.75\).

We assume that the weight of capital in the production of traded goods is about a third \((\alpha_T = \alpha_T^* = 0.36)\). In the non-traded good sector, we assume that the capital share is slightly smaller—i.e., \(\alpha_N = \alpha_N^* = 0.30\). Together, these result in an investment-to-GDP ratio of about 20
percent and a capital-to-GDP ratio of about 2.\textsuperscript{10}

The habit persistence parameter ($b$) is set to 0.931, to achieve the appropriate relative variability of aggregate demand components. The marginal disutility of labor effort is $V' = \ell'$. Estimates of $\zeta$ based on micro-data consider anything between 3 and 20 as a reasonable range. For instance, Gali, Gertler, and Lopez-Salido (2002) take $\zeta = 5$ as their baseline, but Kollmann (2001) chooses $\zeta = 1$ (linear disutility of labor), following the real business cycle literature. We assume $\zeta = \zeta^* = 3$.

The discount rate, $\beta$, is the same in the two countries. The steady-state real interest rate is $1/\beta = (1 + i)/\pi$. A typical yearly calibration for the real interest rate is 3-4 percent. We follow Christiano, Eichenbaum, and Evans (1999) and set $\beta = 1.03^{-0.25}$.

The elasticity of substitution between traded and non-traded goods in final consumption, $\varepsilon$, is set at a relatively low level, 0.44 in both countries, following Stockman and Tsar (1995). The elasticity of substitution between home and foreign tradable goods, instead, is assumed to be much higher—i.e., $\varepsilon_{QM} = \varepsilon_{QM} = 3$. This is higher than in other studies, although the range of plausible options is large and our sensitivity analysis suggests that all we need to generate an appreciation in response to a permanent, country-specific productivity shock is a value larger than one.\textsuperscript{11}

Aggregate data suggest an annual depreciation rate for capital of about 10 percent, so $\delta = \delta^* = 0.025$. The adjustment cost parameters for capital accumulation, $\phi_{t1}$ and $\phi_{t2}$, are set at 50 and 4, respectively, following the same logic as in the case of the habit persistence parameter.

Burstin, Neves and Rebelo (2000) highlight the link between the distribution cost coefficient, $\eta$, and the wholesale/retail margin and set the parameters $\eta$ and $\eta^*$ equal to unity. However, in our model the wholesale/retail margin is a function of other structural parameters such as the demand elasticities, and the choice of the distribution parameter $\eta$ also affects the degree of exchange rate pass-through. Our baseline is $\eta = \eta^* = .33$ in both the Home and the Foreign economy, implying a moderate degree of pass-through consistent with empirical evidence—see Campa and Goldberg (2001).

The elasticities of substitution among differentiated intermediate goods, $\theta$ and $\theta^*$, are chosen by considering the steady-state price markups. For instance, the markup is $\theta/ (\theta - 1)$ in the tradable sector, if there are no distribution costs. Martins, Scarpetta and Pilat (1996) estimate the average markup for manufacturing sectors at around 1.2 in most OECD countries over the period 1980-92. However, some authors rely on lower estimates: for instance, Chari, Kehoe, and McGrattan (2001) choose 1.1, while Morrison (1990), and Domowitz, Hubbard, and Petersen (1988) suggest that a range between 1.2 and 1.7 may be plausible. Benigno and Theorissen (2001) estimate a parameter value yielding a mark up of 1.16 for non-traded goods and 1.18 for traded goods in the

\textsuperscript{10} Because GEM's steady state has no growth, these ratios are lower than they would normally be in the presence of growth.

\textsuperscript{11} Empirical studies of the price elasticity of import demand such as Hooper and Marquez (1995) report a median value of 0.6 for Japan, Germany, and the United Kingdom. Gali and Monacelli (2002) choose a unitary value as their baseline. Others, including Chari, Kehoe and McGrattan (2001) and Smets and Wouters (2002), set the elasticity of substitution between Home and Foreign goods at 1.5.
United Kingdom. We set $\theta / (\theta - 1) = 1.2$ or $\theta = 6$ in both countries.

The elasticities of substitution among differentiated labor inputs, $\phi$ and $\phi^*$, are related to the wage markup. According to Gali, Gertler, and Lopez-Salido (2002), values between 1.15 and 1.4 for the sum of the steady-state wage and price markups can be thought of as “falling within a plausible range.” For instance, Benigno and Thoenissen (2001) estimate $\phi$ to imply a mark up of 1.24 for the United Kingdom, and 1.33 for the euro area. We set $\phi = \phi^* = 6$ to yield a markup of 1.2.

The transaction cost parameters in the bond market are $\phi_{B1} = 0.01$ and $\phi_{B2} = 0.01$, leading to a very slow reversion of the net foreign asset position between Home and Foreign to its steady-state value of zero. This guarantees that, in the short- to medium-term, the properties of the model—as especially the degree of persistence in the current account dynamics—are largely unaffected by the asymptotic convergence condition.

Money demand plays a residual role in our model. We follow Schmitt-Grohe and Uribe (2001) and set $\phi_{S1} = 0.011$ and $\phi_{S2} = 0.075$ in both countries, consistent with their estimates of money demand in the US.

The baseline calibration of the model assumes a significant degree of structural inflation persistence in wages and prices in the tradable sector (controlled by $\phi_{T2}$ and $\phi_{W2}$), but it does not assume any adjustment costs for changes in the level of prices or wages ($\phi_{T1} = \phi_{W1} = 0$). The adjustment cost parameters that determine the degree of structural inflation persistence were calibrated to be consistent with a sacrifice ratio of 1.2 in the Home and Foreign economy. This assumption implies values for $\phi_{N2}$, $\phi_{T2}$, and $\phi_{W2}$ equal to 400. The baseline calibration of the model also assumes that the prices of imported goods respond instantaneously to changes in exchange rates ($\phi_{M1} = \phi_{M2} = \phi^*_{M1} = \phi^*_{M2} = 0$).

**B. Solution**

The steady state of the non-linear model is solved numerically by using an algorithm designed by Douglas Laxton and Michael Juillard to deal with large, non-linear models. This routine splits a complex non-linear problem into a number of simpler steps and applies the Newton-Raphson algorithm iteratively to each step. Breaking down the complex, non-linear problem in simpler steps allows the algorithm to treat the sub-problems as approximately linear without breaking down. We also rely on a variant of this algorithm to compute the forward-looking solution of the non-linear dynamic model under the assumption of either perfect foresight or Kalman-filter-based learning about the underlying realizations of the stochastic forcing processes.

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12 The sacrifice ratio is defined as the cumulative annual output gap that is required to permanently reduce inflation by one percentage point. A sacrifice ratio of 1.2 is consistent with that implied by the FRB/US model under rational expectations.

13 These algorithms were programmed in portable TROLL by Susanna Mursula, at the IMF. See Juillard and Laxton (2003) for a more detailed description of these algorithms.

14 For a discussion of the efficiency and robustness properties of these latter algorithm see Armstrong and others (1998) and Juillard and others (1998).
IV. WHAT ACCOUNTS FOR THE LATE 1990s?

This section asks how well can GEM match the stylized facts of the U.S. business cycle in the late 1990s briefly presented in the introduction. We consider first the effect of a fully anticipated, country-specific shock to productivity. We assume this shock originates in the tradable sector of the economy (possibly emanating from IT goods), thus leading to a Balassa-Samuelson effect through the change in the relative price of tradable goods in terms of nontradable goods. We then add a learning process about this shock to generate a path for its expected persistence that is more in line with that observed in forecasts of real growth in the United States over the relevant period. Finally, we add a broadly defined risk premium shock to improve the model’s ability to match the data.

As we shall see, the Balassa-Samuelson effect is crucial to correctly predicting the sign of the exchange rate movement following such a productivity shock. Adding a second shock and some uncertainty about their persistence, however, buys us a better match of the shape of this response as well as a more realistic magnitude. The presence of endogenous incomplete exchange rate pass-through due to distribution costs also helps fitting remarkably well the broader macroeconomic picture in the United States in the late 1990s.

A. Increased Productivity

The shock we consider is a long-lived, one percentage point per year acceleration in the Home tradable sector total factor productivity (TFP). This acceleration lasts for five years, subsequently decaying slowly with a quarterly auto-regressive coefficient of 0.975, implying a half life of ten years once it starts to decay. This implies a permanent increase in the TFP level of about 15 percentage points in steady state.

A shock of this magnitude is broadly consistent with the available evidence on the acceleration in productivity growth in the United States in the second half of the 1990s. Estimates of the acceleration in labor productivity growth during the second half of the 1990s vary, but generally point to an increase somewhat in excess of one percent per year, on average, broken down between capital deepening, TFP, and other factors (e.g., October 2001 WEO, Table 3.4). Most of these estimates attribute about a third of this increase to capital deepening and two thirds to TFP and other factors. The evidence also suggests that the acceleration in productivity growth was primarily in the tradable sector (see Figure 3), including IT but possibly also the broader manufacturing sector. Thus, we ignore the possibility that the shock originated partly from autonomous capital deepening or innovation in the semi-finished-good sector.15

Figure 4 reports a first set of baseline results. The solid line reports again the key stylized facts referred to in the introduction (i.e., in Figure 1) for ease of reference. The dashed line represents the impulse response of the model calibrated as in the baseline case described in the previous

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15 Breaking down the productivity shock analyzed in intermediate and semi-finished good components and investigating its diffusion to the whole economy is beyond the scope of this paper, but would be feasible in GEM and a natural extension of this work.
section, assuming that the shock started in the first quarter of 1996. The dotted line represents the response to the same shock in a version of the model calibrated without nontradable goods.\textsuperscript{16} This highlights the key role of the Balassa-Samuelson effect in explaining the real exchange rate response to the productivity shock.

The baseline calibration of the model presented in the previous section predicts the sign of the exchange rate movement correctly, both on impact and in the long run, and, to a lesser extent, also its magnitude. The real exchange rate appreciates by roughly 7 percent on impact and then gradually appreciates further over the following five years, reaching a level roughly 10 percent above its initial-steady state level. In the long run, it appreciates by about 15 percent. Instead, in the model without nontradables, the real exchange rate exhibits only a mild appreciation on impact and then depreciates gradually to a long-run value 4 percent below baseline.

As we noted earlier, the appreciation of the exchange rate owing to the Balassa-Samuelson effect stems from the presence of the nontradable sector combined with perfect labor mobility between the two sectors which equalizes their nominal wages. Figure 5 elaborates further on this point by tracing out the comparative static property of the model. The figure plots the relation between the steady-state change in the real exchange rate and different values of the elasticity of substitution between the Foreign and Home tradable good, when the Home economy experiences a positive tradable sector productivity shock. For this experiment, the model is calibrated so that the steady-state effect of a productivity shock on the real exchange rate is zero when the elasticity of substitution between the Foreign and Home tradable good is unity.\textsuperscript{17} As the elasticity of substitution increases above unity, the productivity shocks leads to a steady-state appreciation of the real exchange rate. Conversely, as the elasticity of substitution falls below unity, the steady-state real exchange rate depreciates in the face of the productivity shock.

In the model calibrated without nontradable goods, instead, the real exchange rate must depreciate to create the demand needed to absorb the additional supply of Home tradable goods. With no fall in the Home price of the tradeable good after the increase in productivity, the Home economy has an excess supply of tradable goods. To increase demand for these goods in both Foreign and Home, the real exchange rate must depreciate, thereby lowering the relative price of the Home tradables in terms of the Foreign tradables in both the Foreign and Home economy. A closely related point is that without nontradables, and thus without a real appreciation, the model does not track the historical path of the trade balance nearly as well.

Nonetheless, as it is often the case with rational expectations, the model with nontradables exhibits too rapid a response to the productivity shock and does not track the other variables considered

\textsuperscript{16} Some minor changes to the baseline calibration are required to eliminate non-tradable goods from the model. Specifically, $\gamma$ is increased to unity and $\eta$ is set to zero in both the Home and the Foreign economy. The home bias parameter, $\nu$, is set to 0.958 to obtain an import-to-GDP ratio of 0.13. The Home country’s capital share $\alpha_T$ is set to 0.33 to achieve an investment-to-GDP ratio of 20 percent. The shock is also scaled appropriately so that it has the same long-run effect on the level of output.

\textsuperscript{17} The model is calibrated with no distribution sector ($\eta = \eta^* = 0$) and no home bias ($\nu = 0.25$ and $\nu^* = 0.75$) to achieve this.
very well. In particular, consumption responds immediately and quite strongly, as rational consumers understand fully how the acceleration in productivity growth will affect their wealth. This drives demand above supply, leading to a pickup in inflation and a tightening of monetary policy, which in turn dampens the response of investment, subsequently further constraining supply. Moreover, even the exchange rate jumps excessively in the baseline model compared to historical data.

To improve upon this first set of baseline results, we now consider how expectations of the persistence in the productivity shock might have affected the dynamics. In the last subsection, we then consider the value added of a second shock.

B. Uncertainty about the shock’s persistence

To capture uncertainty about the shock’s persistence, we augment the model with the signal extraction mechanism proposed by Erceg, Guerrieri, and Gust (2002), whereby agents learn gradually about the persistence of the shock. This gradual learning is consistent with the historical evolution of the five-year-ahead forecasts of the acceleration in real growth in the United States plotted in Figure 6. These forecasts (from the Congressional Budget Office and the consensus of Blue Chip Forecasters) suggest that over the first three years of the acceleration in productivity growth, expectations about its persistence were quite low. In the fourth and fifth years, expectations about the persistence of this acceleration increased sharply.

This uncertainty is incorporated by integrating the following signal extraction problem into the simulation analysis:

\[
\begin{align*}
\Delta Z_t &= P_t + T_t, \\
P_t &= \rho \cdot P_{t-1} + \varepsilon_t, \\
T_t &= \kappa \cdot T_{t-1} + \nu_t,
\end{align*}
\]

where \( \Delta Z_t \) is the observed growth in productivity, \( P_t \) is the unobserved persistent component, \( T_t \) is the unobserved temporary component, \( \rho \) is the autoregressive coefficient on the persistent component (set to 0.975), \( \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \), and \( \nu_t \sim N(0, \sigma_\nu^2) \).

Given the learning model (13) of productivity growth and observed growth in productivity for period \( t \), the Kalman filter generates optimal estimates of the persistent and temporary components of productivity growth for period \( t \). These estimates are then used along with the model to generate forecasts of productivity growth beyond period \( t \). At period \( t + 1 \), a new observation on productivity growth is received and estimates of the persistent and temporary components of productivity growth for period \( t + 1 \) are generated. These in turn are used to generate new

---

18 We are grateful to Christopher Erceg, Luca Guerrieri and Christopher Gust for providing us with their TROLL code for the implementation of the Kalman filter to solve their signal extraction problem. Note that Hunt (2002) also considers how slowly evolving expectations about the persistence of the shock affect the model’s response to a productivity disturbance. Hunt (2002), however, does not embed a signal extraction mechanism in the model, but rather hard codes alternative expectation paths.
forecasts of future productivity growth beyond period $t + 1$ and so on.

In this simple model, the speed with which agents learn about the persistent component of the shock depends on their view of the relative magnitude of the variances of the persistent and temporary components. If agents expect the variance of the persistent component to be high relative to that of the temporary component, they will learn quickly. If, on the other hand, this expected relative variance is small, they will learn rather slowly.

Figure 6 contains three alternative paths for the expectations of the five-year-ahead acceleration in real growth in addition to the five-year-ahead forecasts of the acceleration in U.S. real growth. These paths are generated as outlined above. The path labeled "high-variance" represents expectations of the five-year-ahead growth rate based on a high expected variance of the persistent component. In this case, the Kalman filter is able to perfectly forecast the persistent component. The path labeled "low variance" reflects expectations of the five-year-ahead growth rate based on a low expected variance of the persistent component, as in the case analyzed by Erceg, Guerrieri, and Gust (2002).

While, as expected, the "low-variance" path results in slower learning about the persistent component of the shock than under "high-variance," it still does not capture the trends in the growth forecasts. This suggests that very little learning occurred in the early years, with a rapid learning catch up in the last two years of the period considered. To capture this non-linearity in the learning process, we extend the signal extraction model allowing for expectations of the variance of the persistent component to be time varying. Thus, we now assume that $\varepsilon_t \sim N(0, \sigma^2_{\varepsilon_t})$ and $\nu_t \sim N(0, \sigma^2_{\nu_t})$, respectively.

This third, "time-varying relative variance" line is calibrated to match the path of the five-year-ahead expectation of real growth. Although this path continues to incorporate more learning in the early years than is suggested by the evolution of the actual growth forecasts, it captures the basic non-linearity in the learning process in a better manner.

As Figure 7 shows, when agents learn about the persistence of the productivity shock in this non-linear manner, there is a much milder initial increase in consumption. Given the increase in productivity, this slower response of consumption implies more spare capacity that puts downward pressure on inflation and allows nominal interest rates to fall. A lower cost of capital results in a more rapid increase in investment than under certainty about the persistence of the shock. Further, the resulting increase in real GDP growth is also much closer to the historical acceleration. In addition, the real exchange rate does not jump on impact following the shock and appreciates gradually in a manner which is more consistent with the historical data. Nonetheless, the slower appreciation of the real exchange rate results in a notably smaller deterioration in the trade balance.

In sum, integrating a model of learning into the simulation of the productivity shock goes some way toward allowing the baseline model to match not only the sign of the exchange rate response but also its shape. In addition, it helps match in a better manner the broader picture of key stylized
facts. Yet, the magnitude of the real exchange rate appreciation, the trade balance deterioration, the inflation decline, and consumption and investment responses remain smaller than those observed in the data. It is thus natural to consider an additional shock, which is what we now turn to.

C. Decreased risk premium

The reduction in the perceived riskiness of investments in U.S. assets is a factor often mentioned as a possible explanation of the business cycle developments of the late 1990s, in addition to faster productivity growth (or possibly because of the productivity acceleration mentioned above). In particular, this is an obvious way of explaining the large appreciation of the dollar over that period and the significant deterioration in the trade balance.

While the assertion that productivity growth accelerated during the second half of the 1990s is well supported by rather hard evidence, the proposition that investors' subjective evaluation of the riskiness of US assets changed in the second part of 1990s is more tentative because estimates of the unobserved risk premium are difficult to generate. Wadhwani (1999) provides some evidence that the equity risk premium declined in the United States in the latter half of the 1990s, although Jagannathan and others (2001) provide estimates that suggests that the decline in the risk premium is evident over the last three decades. Nonetheless, economic developments in south east Asia, Latin America, Japan, and Europe in the latter half of the 1990s provide some support for considering the possibility of shift in investor portfolio preferences toward U.S. assets. In fact, the Asian currency crisis and its spillover effects to Latin America, the stagnant economic performance of Japan, and uncertainties and frictions related to the introduction of euro could all have contributed to investors viewing U.S. assets as being less risky in the latter half of the 1990s.

On the background of this evidence (or lack thereof), we add to our simulation analysis a shock to the risk premium demanded on U.S. assets, as in the analysis of Duarte and Stockman (2001) and Kollmann (2001). This shock consists of a 100 basis point decline in the home country risk premium that lasts for five years and then subsequently decays with an autoregressive parameter of 0.95.

As in the case of the productivity shock, we assume that agents learn over time about its persistence. We achieve this by adding a second signal extraction problem to the model. Further, we assume that the two learning processes are correlated, and thus expectations about the persistence of the risk premium shock evolves in a (non-linear) fashion similar to that incorporated for the productivity shock.¹⁹

Incorporating both a productivity and risk premium shock into the analysis yields very good results, which are reported in Figure 8. The combination of these two shocks and slowly evolving

¹⁹ It is appealing to assume that the two shocks are correlated. If we interpret the shock to the uncovered interest parity as capturing “capital flows” factors that would have obtained following a productivity shock in a model with international trade in equities, then agents’ expectations about the persistence of the two shocks should be correlated. On equity trade in a NOEM model, see, for instance, Ghironi, Lee, and Rebucci (2003).
expectations about their persistence enables the model to closely reproduce the key stylized facts of the U.S. business cycle in second half of the 1990s. In particular, the real exchange rate now jumps slightly on impact, but its subsequent path tracks the evolution of actual data very closely both in terms of shape and magnitude. The deterioration in the trade balance is also virtually identical to that in the data, while the paths of investment, consumption, real growth, and inflation are matched in a satisfactory manner.

Key to this last set of results is not only the mere addition of learning and a second source of disturbance to the model, but also the model’s intrinsic propagation mechanism. Specifically, the presence of the distribution sector results in endogenous incomplete pass-through of changes in the exchange rate to final goods prices, thereby muting the expenditure switching impact of a nominal depreciation. With incomplete pass-through, the net export position in the Home country remains stronger than it otherwise would have been, supporting domestic demand and avoiding too much deterioration in the trade balance. Without the distribution sector and incomplete pass-through, imports would rise too much, exports decline too much, and demand for Home output would need to be supported by a decline in nominal interest rates. This decline in nominal interest rates, in turn, would lead to a weaker real exchange rate than in the data. The weaker demand for Home tradables would also result in a worse trade balance, lower inflation and slower growth than in the data.

V. Conclusions

The evolution of the U.S. economy in the late 1990s presents economists with an interesting puzzle. Unlike the (monetary) business cycles in the previous two decades, the period of accelerating growth of the late 1990s was not quickly followed by a significant increase in inflation, tightening of monetary policy, and reversal in real activity. Instead, inflation remained subdued, and interest rates remained stable, the U.S. exchange rate appreciated and capital flowed into the United States, financing a significant deterioration in the trade balance. Moreover, there is now growing evidence that the acceleration in real activity was driven largely by an acceleration in productivity growth.

In this paper we have used GEM to investigate how much of these developments may be explained by a temporary but persistent acceleration in tradables sector productivity growth. We find that the presence of both tradable and nontradable goods in the model, through the Balassa-Samuelson effect, is essential for a productivity shock to start reproducing the extent of the appreciation in the dollar and the deterioration in the trade balance in the second half of the 1990s. Allowing agents to learn only slowly about this shock’s persistence is also important to capture the responses of real growth, investment, consumption, and inflation as well as to slow the response of the exchange rate to the shock.

Even after integrating learning into the model, however, a productivity shock alone does not appear sufficient to explain the data fully. Adding to the productivity shock a temporary but persistent decline in the perceived riskiness of U.S. assets, together with a learning mechanism
correlated with that on the persistence of the productivity increase, enables the model to track the historical evolution of the real exchange rate, the trade balance, real growth, investment, consumption, and inflation very closely. As in the case of the productivity shock, the presence of nontradable goods remains key for the model's ability to replicate the historical data; in the case of the risk premium shock, through their use in the distribution sector resulting in incomplete pass-through.

The paper also shows that a fully theory-consistent framework for international macroeconomic analysis and policy evaluation, like GEM, may help to shed some light on actual international policy issues.
THE MODEL’S DETAILS

Here we present model details only for the Home economy. The structure for the Foreign country is analogous to that for the Home economy except for the balance of payments determining the real exchange rate.

A. Final goods production

There is a continuum of symmetric Home firms producing the Home final good, indexed by \( x \in [0, s] \), where \( 0 < s < 1 \) measures country size. World size is normalized to 1, and Foreign firms producing the Foreign final good are indexed by \( x^* \in (s, 1] \).

The production function of the final, non-traded good is described in the main text. Home firm \( x \) takes the prices of the intermediate baskets \( P_Q, P_M \) and \( P_N \) as given, and solves the following minimization problem:

\[
\{ Q_t(x), M_t(x), N_{N,t}(x) \} = \arg \min \{ P_{Q,t}Q_t(x) + P_{M,t}M_t(x) + P_{N,t}N_{N,t}(x) \\
+ P_t \left( A_t(x) - \left[ \gamma \left( \nu^{1 - \epsilon_{QM}} Q_t(x)^{1 - \epsilon_{QM}} + (1 - \nu)^{1 - \epsilon_{QM}} [M_t(x)(1 - \Gamma_{M,t}(x))]^{1 - \epsilon_{QM}} \right) \\
+ (1 - \gamma)^{1 - \epsilon_{QM}} N_{N,t}(x)^{1 - \epsilon_{QM}} \right]^{1 - \epsilon_t} \right) \},
\]

where \( P_t \) is the consumption price of the final good, equal to its marginal cost.

Cost minimization in Home final good production yields the following demands by firm \( x \):

\[
Q_t(x) = \gamma \nu \left( \frac{P_{Q,t}}{P_t} \right)^{-\epsilon_{QM}} \left( \frac{P_{X,t}}{P_t} \right)^{\epsilon_{QM}-\epsilon} A_t(x) \tag{2}
\]

\[
N_{N,t}(x) = (1 - \gamma) \left( \frac{P_{N,t}}{P_t} \right)^{-\epsilon} A_t(x) \tag{3}
\]

\[
M_t(x) = \gamma (1 - \nu) \left( \frac{P_{M,t}}{P_t} \right)^{-\epsilon_{QM}} \left( \frac{P_{X,t}}{P_t} \right)^{\epsilon_{QM}-\epsilon} A_t(x)^* \tag{4}
\]

\[
\left[ 1 - \Gamma_{M,t}(x) - \phi_M \left( \frac{M_t(x)}{A_t(x)} / \frac{M_{t-1}}{A_{t-1}} - 1 \right) \left( \frac{M_t(x)}{A_t(x)} / \frac{M_{t-1}}{A_{t-1}} \right)^{\epsilon_{QM}} \right]^{1 - \epsilon_{QM}} \left[ 1 - \Gamma_{M,t}(x) \right]
\]

where the shadow price of the basket of tradables is:

\[
\frac{P_{X,t}}{P_t} = \nu \left( \frac{P_{Q,t}}{P_t} \right)^{1-\epsilon_{QM}} + (1 - \nu) \left( \frac{P_{M,t}}{P_t} \right)^{1-\epsilon_{QM}} \tag{A-5}
\]

\[
* \left[ 1 - \Gamma_{M,t} - \phi_M \left( \frac{M_t(x)}{A_t} / \frac{M_{t-1}}{A_{t-1}} - 1 \right) \left( \frac{M_t(x)}{A_t} / \frac{M_{t-1}}{A_{t-1}} \right)^{\epsilon_{QM}} \right]^{1 - \epsilon_{QM}}.
\]
B. Intermediate goods

Demand

The basket of Home intermediate nontradable goods $N_N$ is a CES index of differentiated goods produced by Home firms, indexed by $n \in [0, s]$. The baskets $Q$ and $M$ are similarly defined. Intermediate tradables Home firms producing $Q$ are indexed by $h \in [0, s]$, while Foreign firms producing $M$ are indexed by $f \in (s, 1]$.

Now, defining $N_N(n, x)$ as the demand of the intermediate good $n$ by firm $x$, the basket of intermediate nontradable goods demanded by firm $x$, $N_N(x)$, is defined as:

$$N_{N,t}(x) = \left[ \left( \frac{1}{s} \right) \int_0^s N_{N,t}(n, x)^{1-\frac{1}{\theta}} \, dn \right]^{\frac{\theta}{\theta-1}},$$  \hspace{1cm} (A-6)

where $\theta > 1$ denotes the elasticity of substitution among intermediate nontradables. Thus:

$$Q_t(x) = \left[ \left( \frac{1}{s} \right) \int_0^s Q_t(h, x)^{1-\frac{1}{\theta}} \, dh \right]^{\frac{\theta}{\theta-1}},$$  \hspace{1cm} (7)

$$M_t(x) = \left[ \left( \frac{1}{1-s} \right) \int_s^1 M_t(f, x)^{1-\frac{1}{\theta^*}} \, df \right]^{\frac{\theta^*}{\theta^*-1}},$$  \hspace{1cm} (8)

where $\theta^* > 1$. Hence, the cost-minimizing prices of the baskets of intermediate goods $P_N$, $P_Q$ and $P_M$ are given by:

$$P_{N,t} = \left[ \left( \frac{1}{s} \right) \int_0^s p_t(n)^{1-\theta} \, dn \right]^{\frac{1}{1-\theta}},$$ \hspace{1cm} (A-9)

$$P_{Q,t} = \left[ \left( \frac{1}{s} \right) \int_0^s p_t(h)^{1-\theta} \, dh \right]^{\frac{1}{1-\theta}},$$ \hspace{1cm} (A-10)

$$P_{M,t} = \left[ \left( \frac{1}{1-s} \right) \int_s^1 p_t(f)^{1-\theta^*} \, df \right]^{\frac{1}{1-\theta^*}}.$$ \hspace{1cm} (A-11)

Aggregating across $x$, it can be shown that total demand for $n$ is:

$$N_t^D(n) = \left( \frac{p_t(n)}{P_{N,t}} \right)^{-\theta} \left[ N_{N,t} + \eta \left( Q_t + M_t \right) \right].$$ \hspace{1cm} (A-12)
Similarly, Home demands for the tradable intermediate goods $h$ and $f$ are given by:

$$Q_t(h) = \left( \frac{p_t(h)}{P_{Q,t}} \right)^{-\theta} Q_t,$$  \hspace{1cm} (A-13)

$$M_t(f) = \frac{s}{1-s} \left( \frac{p_t(f)}{P_{M,t}} \right)^{-\theta^*} M_t.$$  \hspace{1cm} (A-14)

**Supply**

Home nontradables are produced by symmetric firms using labor $\ell(n)$ and capital $K(n)$. The technology for production of good $n$ is Cobb-Douglas:

$$N_t^S(n) = Z_{N,t} \left[ \ell_t(n)^{(1-\alpha_N)} K_t(n)^{\alpha_N} \right],$$  \hspace{1cm} (A-15)

where $Z_N$ is a common total factor productivity (TFP) shock.

Denoting $W$ the wage index and $R$ the Home nominal rental price of capital, firms solve the following cost-minimization problem:

$$\{\ell_t(n), K_t(n)\} = \text{arg min}\{W_t \ell_t(n) + R_t K_t(n) + MC_t(n) (N_t^S(n) - Z_{N,t} \ell_t(n)^{(1-\alpha_N)} K_t(n)^{\alpha_N})\},$$

where the $MC(n)$ is the nominal marginal cost. Cost minimization yields:

$$\ell_t(n) = (1 - \alpha_N) \left( \frac{W_t}{MC_{N,t}(n) Z_{N,t}} \right) \frac{N_t^S(n)}{Z_{N,t}},$$  \hspace{1cm} (16)

$$K_t(n) = \alpha_N \left( \frac{R_t}{MC_{N,t}(n) Z_{N,t}} \right) \frac{N_t^S(n)}{Z_{N,t}}.$$  \hspace{1cm} (17)

The production of Home tradables, $T^S(h)$ is similarly characterized. The productivity shock discussed in the paper is the common total factor productivity (TFP) in the production function of tradable goods.

**The distribution sector and price setting in the intermediate goods sector**

The distribution sector and price setting in the intermediate tradable goods sector is described in the main text. Here we describe price setting in the intermediate nontradable goods sector.

Consider profit maximization in this sector. Each firm $n$ takes into account demand for its product (equation A-12) and sets the nominal price $p(n)$ by maximizing the present discounted value of real profits, in the presence of price adjustment costs measured in terms of total profits.
The adjustment cost is denoted $\Gamma_{PN,t}$:

$$
\Gamma_{PN,t}(n) = \frac{\phi_{N1}}{2} \left( \frac{p_t(n)}{\pi p_{t-1}(n)} - 1 \right)^2 + \frac{\phi_{N2}}{2} \left( \frac{p_t(n)/p_{t-1}(n)}{P_{N,t-1}/P_{N,t-2}} - 1 \right)^2,
$$

(A-18)

where $\phi_{N1}, \phi_{N2} \geq 0$ and $\pi > 0$, with $\pi$ denoting the gross steady-state rate of inflation. This specification generalizes Rotemberg's (1982) quadratic cost of price adjustment. Drawing from Ireland (2001), the adjustment cost has two components. The first component is related to changes in the nominal price relative to steady-state inflation. The second component is related to changes in firm $n$'s price inflation relative to the past observed inflation rate in the whole nontradables sector.

Thus, the price setting problem is:

$$
\{p_t(n)\}_{t=\tau}^{\infty} = \arg \max_{D_{t,\tau}} E_t \sum_{\tau=\tau}^{\infty} \{D_{t,\tau}[p_t(n) - MC_t(n)] N_t^D(n) [1 - \Gamma_{PN,t}(n)]\},
$$

(A-19)

where the discount rate $D_{t,\tau}$ is the intertemporal marginal rate of substitution in consumption of the representative household, to be defined below. The first order condition for this problem is:

$$
0 = [1 - \Gamma_{PN,t}(n)] [p_t(n) (1 - \theta) + \theta MC_t(n)]
$$

$$
- [p_t(n) - MC_t(n)] \frac{\phi_{N1} \pi p_t(n)}{\pi p_{t-1}(n)} \left( \frac{p_t(n)}{\pi p_{t-1}(n)} - 1 \right)
$$

$$
- [p_t(n) - MC_t(n)] \frac{\phi_{N2} p_t(n)/p_{t-1}(n)}{P_{N,t-1}/P_{N,t-2}} \left( \frac{p_t(n)/p_{t-1}(n)}{P_{N,t-1}/P_{N,t-2}} - 1 \right)
$$

$$
+ E_t D_{t,t+1} [p_{t+1}(n) - MC_{t+1}(n)] \frac{N_{t+1}(n)}{N_t(n)} \left( \frac{\phi_{N1} p_{t+1}(n)}{\pi p_t(n)} \left( \frac{p_{t+1}(n)}{\pi p_t(n)} - 1 \right) \right)
$$

$$
+ \frac{\phi_{N2} p_{t+1}(n)/p_t(n)}{P_{N,t}/P_{N,t-1}} \left( \frac{p_{t+1}(n)/p_t(n)}{P_{N,t}/P_{N,t-1}} - 1 \right),
$$

(A-20)

which, in the absence of price stickiness ($\phi_{N1} = \phi_{N2} = 0$), comes down to the simple markup rule:

$$
p_t(n) = \frac{\theta}{\theta - 1} MC_t(n).
$$

(A-21)

### C. Labor market

Differentiated labor inputs in the Home country are indexed by $j \in [0, s]$. The elasticity of substitution within this continuum of differentiated labor inputs is $\phi > 1$.

Firm $n$ takes the price of labor and capital as given. Thus, cost minimization implies that the demand for labor input $j$ by firm $n$, $\ell_t(n, j)$, is a function of the relative wage and its total labor demand:

$$
\ell_t(n, j) = \left( \frac{1}{s} \right) \left( \frac{W_t(j)}{W_t} \right)^{-\phi} \ell_t(n),
$$

(A-22)
where
\[ \ell_t(n) = \left[ \left( \frac{1}{s} \right)^{\phi} \int_0^s \ell(n, j)^{1 - \frac{1}{\phi}} dj \right]^{\frac{1}{\phi - 1}}, \] (A-23)

is total labor demand by firm \( n \), and
\[ W_t = \left[ \left( \frac{1}{s} \right) \int_0^s W_t(j)^{1 - \phi} dj \right]^{\frac{1}{1 - \phi}}, \] (A-24)

is the wage index, with \( W(j) \) denoting the nominal wage (equal across sectors) paid to Home labor input \( j \). Firm \( h \) in the tradable sector of the economy faces the same problem, and thus:
\[ \ell_t(h, j) = \left( \frac{1}{s} \right) \left( \frac{W_t(j)}{W_t} \right)^{-\phi} \ell_t(h). \] (A-25)

Using (A-22) and (A-23), total demand of type \( j \) input is:
\[ \ell_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\phi} \ell_t, \] (A-26)

where \( \ell_t \) is per-capita total labor in the Home economy.

Each household monopolistically supplies its labor input \( j \) and sets the nominal wage by taking into account (A-26) and its cost of adjusting wages measured in terms of the total wage bill.\(^{20}\)

Following Kim (2000), we specify the latter as:
\[ \Gamma_{W_t}(j) \equiv \frac{\phi_{W_1}}{2} \left( \frac{W_t(j)}{\pi W_{t-1}(j)} - 1 \right)^2 + \frac{\phi_{W_2}}{2} \left( \frac{W_t(j)/W_{t-1}(j)}{W_{t-1}/W_{t-2} - 1} \right)^2 \] (A-27)

where \( \phi_{W_1}, \phi_{W_2} \geq 0 \).

### D. Households' budget constraint and asset menu

The individual flow budget constraint for agent \( j \) in the Home country is:
\[ \mathcal{M}_t(j) + B_{t+1}(j) + \mathcal{E}_t B^*_t(j) \leq \]
\[ \mathcal{M}_{t-1}(j) + (1 + \delta_t) B_t(j) + (1 + \delta^*_t) \left[ 1 - \Gamma_{B_t}(j) \right] \mathcal{E}_t B^*_t(j) + R_t K_t(j) + P_L^t L_t(j) + W_t(j) \ell_t(j) \left[ 1 - \Gamma_{W_t}(j) \right] - P_t C_t(j) \left[ 1 + \Gamma_{S_t}(j) \right] \]
\[ - P_t I_t(j) + \Pi_t - \text{NETT}_t(j). \]

Home agents hold domestic money \( \mathcal{M} \) and two bonds, \( B \) and \( B^* \), denominated in Home and Foreign currency, respectively. The short-term nominal rates \( \delta_{t+1} \) and \( \delta^*_{t+1} \) are paid at the beginning of period \( t + 1 \) and are known at time \( t \).\(^{21}\) Only the Foreign-currency bond is traded.

\(^{20}\) The Home household maximization problem is spelled out below.

\(^{21}\) We adopt the notation of Obstfeld and Rogoff (1996, chapter 10). Specifically, our timing convention has \( B_t(j) \) and \( B^*_t(j) \) as agent \( j \)'s nominal bonds accumulated during period \( t - 1 \) and
internationally: the Foreign bond is in zero net supply worldwide, while the Home bond is in zero net supply at the domestic level.

While markets are internally complete, guaranteeing equalization of the marginal utility across each country’s residents, markets are not complete internationally and the marginal utilities of Home and Foreign agents need not be equal.

Drawing on Benigno (2001), when they take a position in the Foreign bond, market Home agents face a transaction cost, $\Gamma_B$ given by:

$$\Gamma_{B,t+1} = \phi_{B1} \frac{\exp\left(\frac{\phi_{B2}}{\phi_{B2}} \frac{\mathcal{E}_{t} B_{H,t+1}^*}{P_t} \right) - 1}{\exp\left(\frac{\phi_{B2}}{\phi_{B2}} \frac{\mathcal{E}_{t} B_{H,t+1}^*}{P_t} \right) + 1} + Z_{E,t} \quad (A-29)$$

with $0 < \phi_{B1} < 1$, $\phi_{B2} > 0$ and $B_{H,t}^* \equiv (1/s) \int_0^s B_{t}^*(j) dj$ and where $Z_{B,t}$ is a shock term.

Ignoring $Z_B$, when average Home holdings of the Foreign bond are zero, $\Gamma_B = 0$. When Home is a net lender and holdings of the Foreign bond go to infinity, $\Gamma_B$ raises from zero to $\phi_{B1}$, implying that Home households lose an increasing fraction of their Foreign bond returns to financial intermediaries. The parameter $\phi_{B2}$ controls the flatness of the $\Gamma_B$ function, hence the speed of convergence to the steady state with zero net asset positions worldwide.\(^{22}\)

As for $Z_B$, the broadly defined “risk premium” shock discussed in the paper, we note that uncertainty in international financial intermediation plays in the GEM the same role that “uncovered interest parity shocks” or risk-premium fluctuations play in other open-economy models such as McCallum and Nelson (1999), Duarte and Stockman (2001), or Kollmann (2001).

Consumption spending is subject to a proportional transaction cost $\Gamma_S$ that depends on the household’s money velocity $v$, where:

$$v_t(j) = \frac{P_t C_t(j)}{\mathcal{M}_t(j)} \quad (A-30)$$

Following Schmitt-Grohe and Uribe (2001), the particular functional form for the transaction cost is:

$$\Gamma_S(v_t) = \phi_{S1} v_t + \frac{\phi_{S2}}{v_t} - 2\sqrt{\phi_{S1}\phi_{S2}} \quad (A-31)$$

Agents choose their stock of real money holdings $\mathcal{M}/P$ so that at the margin shopping costs are equal to the benefits from investing in yield-bearing assets.

\(^{22}\)Alternative approaches to guarantee stationarity rely on parametric assumptions as in Corsetti and Pesenti (2001a,b), time-varying discount rates or demographic dynamics as discussed by Benigno (2001) and Ghironi (2003).
Home agents also accumulate physical capital which they rent to Home firms at the nominal rate $R$. The law of motion of capital is:

$$K_{t+1}(j) = (1 - \delta) K_t(j) + \Psi_t K_t(j) \quad 0 < \delta \leq 1$$

(A-32)

where $\delta$ is the depreciation rate.

Capital accumulation, denoted $\Psi_t K_t(j)$, is subject to adjustment costs. The function $\Psi(.)$ is increasing, concave, and twice-continuously differentiable in the investment/capital ratio, $I_t(j)/K_t(j)$, such that $\Psi(\delta) = \delta$ and $\Psi'(\delta) = 1$, which entails no adjustment costs in steady state. The specific functional form we adopt is quadratic:

$$\Psi_t \equiv \frac{I_t(j)}{K_t(j)} - \frac{\phi_{I1}}{2} \left( \frac{I_t(j)}{K_t(j)} - \delta (1 + Z_{I,t}) \right)^2 - \frac{\phi_{I2}}{2} \left( \frac{I_t(j)}{K_t(j)} - \frac{I_{t-1}}{K_{t-1}} \right)^2$$

(A-33)

where $\phi_{I1}, \phi_{I2} \geq 0$ and $Z_{I,t}$ is a shock, essentially equivalent to an increase in the rate of capital depreciation.

Home agents own all Home firms and there is no international trade in claims on firms’ profits. The variable $\Pi$ in equation (A-28) includes all profits accruing to Home households, all Home-currency revenue from nominal and real adjustment costs, and all revenue from financial intermediation.

Finally, Home agents pay lump-sum (non-distortionary) net taxes that we denote with $NETT_t(j)$.

**E. Consumer preferences and optimization**

Denoting with $\mathcal{W}_t(j)$ the lifetime expected utility of Home agent $j$, we have:

$$\mathcal{W}_t(j) \equiv E_t \sum_{\tau=t}^{\infty} \beta^{r-\tau} [U(C_\tau(j)) - V(\ell_\tau(j))]$$

(A-34)

where $\beta$ is the discount rate, identical across countries.

There is habit persistence in consumption according to the specification:

$$U_t(j) = Z_{U,t} \log (C_t(j) - bC_{t-1})$$

(A-35)

where $C_{t-1}$ is past per-capita Home consumption and $0 < b < 1$. The term $Z_{U,t}$ is a preference shock common to all Home residents.

The parametric specification of $V$ is:

$$V_t(j) = Z_{V,t} \frac{\ell_t(j)^{1+\zeta}}{1+\zeta}$$

(A-36)

where $\zeta > 0$ and $Z_{V,t}$ is a shock to labor disutility.

The representative Home household chooses bond and money holdings, capital and consumption
paths, and sets wages to maximize its expected lifetime utility (A-34) subject to the budget constraints (A-28) and (A-32), and taking into account (A-26) and (A-27).

The maximization problem of Home agent $j$ can be written in terms of the following Lagrangian:

$$\left\{ C_t(j), I_t(j), M_t(j), B_{t+1}(j), B^*_t(j), K_{t+1}(j), W_t(j) \right\}_{r=t}^{\infty} = \arg \max E_t \sum_{r=t}^{\infty} \beta^{r-t} \left\{ U[C_t(j)] - V[W_t^{-\phi}(j)W_t^{\phi}i_r] + (1/\mu_r(j)) \left[ -M_t(j) - B_{t+1}(j) - \mathcal{E}_rB^*_t(j) + M_{t-1}(j) + (1 + i_r)B_r(j) \right] + (1 + i_r^*) \left[ 1 - \Gamma_{B,r} \right] \mathcal{E}_rB^*_t(j) + R_rK_r(j) + P_{L,r}L_r(j) + W_t(j)^{1-\delta}W_t^{\phi}i_r \left[ 1 - \Gamma_{W,r} \{W_t(j), W_{t-1}(j)\} \right] - P_rC_r(j) \left[ 1 + \Gamma_{S,r} \{M_t(j), C_t(j)\} \right] - P_rI_r(j) + \Pi_r - \text{NETT}_r(j) + \lambda_r \left[ -K_{t+1}(j) + (1 - \delta)K_r(j) + \Psi_r \{I_r(j)/K_r(j)\} K_r(j) \right] \right\} \quad (37)$$

where $1/\mu$ and $\lambda$ are the multipliers associated with the budget constraint and the law of motion for capital, respectively.

With complete domestic markets, in a symmetric equilibrium, $U'(C_t(j)) = U'(C_t)$ for all agents $j$, where $C$ is per-capita Home consumption. Now define:

$$D_{t,r} \equiv \frac{\beta P_t U'(C_t)}{P_t U'(C_t)} \left[ 1 + \Gamma_{S,t} + \Gamma_{S,t}v_t \right] \quad (A-38)$$

which is the Home agents' stochastic discount rate or Home agents' pricing kernel.

Then the first order conditions with respect to $B_{t+1}(j)$ and $B^*_{t+1}(j)$ imply the following no-arbitrage condition:

$$1 = (1 + i_{t+1})E_t D_{t,t+1} = (1 + i^*_{t+1}) \left( 1 - \Gamma_{B,t+1} \right) E_t \left( \frac{D_{t,t+1}E_{t+1}}{E_t} \right), \quad (A-39)$$

which is a risk-adjusted uncovered interest parity condition. In the non-stochastic steady state, $1 + i = \pi/\beta$, where $\pi$ is the gross steady-state inflation rate and $1/\beta$ is the gross rate of time preference, and the interest differential $(1 + i) / (1 + i^*)$ is equal to the nominal depreciation rate of the Home currency (if any) in steady state.

The first order conditions with respect to $M_t(j)$, $K_{t+1}(j)$ and $W_t(j)$ are, respectively:

$$1 - \Gamma'_{S,t}v_t^2 = E_t D_{t,t+1} \quad (A-40)$$

$$\frac{1}{\Psi_t'} = E_t \left\{ D_{t,t+1} \frac{P_{t+1}}{P_t} \left( \frac{P_{t+1}}{P_{t+i}} + \frac{1}{\Psi_{t+1}'} \right) \right\} \quad (A-41)$$
\[
\phi V_t(j) \frac{P_t}{U_t(j) W_t(j)} = (\phi - 1) [1 - \Gamma_{W,t}(j)] \\
+W_t(j) \left[ \frac{\phi w_1}{\pi W_{t-1}(j)} \left( \frac{W_t(j)}{\pi W_{t-1}(j)} - 1 \right) + \frac{\phi w_2 W_{t-2}}{W_{t-1}(j) W_{t-1}} \left( \frac{W_t(j)/W_{t-1}(j)}{W_{t-1}/W_{t-2}} - 1 \right) \right] \\
-E_t \left\{ D_{t,t+1} \frac{\ell_t(j)}{W_t(j)} \frac{\phi w_1 W_{t+1}(j)}{\pi W_t^2(j)} \left( \frac{W_{t+1}(j)}{W_t(j)} - 1 \right) \right\} \\
-E_t \left\{ D_{t,t+1} \frac{W_{t+1}(j)}{W_t(j)} \frac{\ell_t(j)}{\ell_t^*(j)} \frac{\phi w_2 W_{t+1}(j)/W_t(j)}{W_t/W_{t-1}} \left( \frac{W_{t+1}(j)/W_t(j)}{W_t/W_{t-1}} - 1 \right) \right\}
\] (A-42)

Expression (A-40) defines real money balances, \(M/P\), as a positive function of consumption and a negative function of the nominal interest rate. Expression (A-41) links capital accumulation to the behavior of the real price of capital \(R/P\). In the non-stochastic steady state, \(1 + R/P\) is equal to the sum of the rate of time preference \((1/\beta)\) and the rate of capital depreciation \((\delta)\).\(^{23}\) Finally, expression (A-42) describes the dynamics of real wages. In the non-stochastic steady state, the real wage \(W/P\) is equal to the marginal rate of substitution between consumption and leisure, \(V'/U'\), augmented by the markup \(\phi/ (\phi - 1)\) which reflects monopoly power in the labor market.

Optimization also implies that households exhaust their intertemporal budget constraint: the flow budget constraint (A-28) holds as equality, and the following transversality condition must be satisfied:

\[
\lim_{\tau \to \infty} E_t D_{t,\tau} [M_{t-1}(j) + (1 + i_t) B_t(j) + (1 + i_t^*) (1 - \Gamma_{B,t}) B_t^*(j)] = 0.
\] (A-43)

F. Government

Public spending falls entirely on the nontradable final good, \(G_A\). Governments finance public expenditure through net lump-sum taxes and seigniorage revenue. Thus, the budget constraint of the Home government is (in per capita terms):

\[
s P_t G_A, t \leq \int_0^{\delta} NETT_t(j) dj + \int_0^{\delta} [M_t(j) - M_{t-1}(j)] dj.
\] (A-44)

The government controls the short-term rate \(i_{t+1}\). Monetary policy is specified in terms of generalized interest-rate rules of the form:

\[
(1 + i_{t+1})^4 - 1 = \omega_t \left[ (1 + i_t)^4 - 1 \right] + (1 - \omega_t) \left[ (1 + i_{t+1})^4 - 1 \right] + \omega_t E_t \left[ \frac{P_{t+\tau}}{P_{t-4+\tau}} - \Pi_{t+\tau} \right]
\] (A-45)

\(^{23}\)If the shocks \(Z_t, Z_t^*\) were permanent, \(1 + R/P = 1/\beta + \delta (1 + Z_t)\) in steady state.
where \(0 < \omega_t < 1\), and \(\bar{i}_{t+1}\) is the desired interest rate, defined as:

\[
(1 + \bar{i}_{t+1})^4 = \frac{1}{\beta^4} \frac{P_{t+\tau}}{P_{t-4+\tau}}. \tag{A-46}
\]

In this expression, \(P_{t+\tau} / P_{t-4+\tau}\) is the year-on-year gross CPI inflation rate \(\tau\) quarters into the future, and \(\Pi_{t+\tau}\) is the year-on-year gross inflation target \(\tau\) quarters into the future. In a steady state, in which a constant inflation target \(\Pi\) is achieved, it must be the case that:

\[
1 + i_{t+1} = 1 + \bar{i}_{t+1} = \frac{1}{\beta} \left( \frac{P_t}{P_{t-4}} \right)^{0.25} = \frac{\Pi^{0.25}}{\beta} = \frac{\pi}{\beta}. \tag{A-47}
\]

**G. Market clearing**

The Home nontradable final good can be used for private consumption \(G_A\) and investment \(I\):

\[
\int_0^s A_t(x) dx \geq \int_0^s C_t(j)[1 + \Gamma S_t(j)]dj + sG_A + \int_0^s I_t(j) dj \tag{A-48}
\]

The resource constraint for good \(n\) is \(N^S(n) \geq N^D(n)\). In equilibrium this constraint holds as an equality and implies \(p(n) = P_N\). Similar considerations hold for the other price indexes.

The Home tradable \(h\) can be used by Home firms or imported by Foreign firms, so that \(T(h) \geq Q(h) + M^*(h)\) and, in aggregate, \(sT \geq sQ + (1 - s) M^*(h)\).

The resource constraint for factor markets are:

\[
\int_0^s e_t(j) dj = \int_0^s e_t(n) dn + \int_0^s e_t(h) dh \tag{49}
\]

\[
\int_0^s K_t(j) dj = \int_0^s K_t(n) dn + \int_0^s K_t(h) dh. \tag{50}
\]

Market clearing in the asset market requires:

\[
\int_0^s B_t(j) dj = 0, \quad \int_0^s B_t^*(j) dj + \int_0^1 B_t^*(j^*) dj^* = 0. \tag{A-51}
\]

Finally, the market clearing condition above imply that changes in real net foreign assets, equal the current account balance, which in turn equals the income balance plus the trade balance. Thus:

\[
\left( E_t D_t, t+1 \frac{F_{t+1}}{P_{t+1}} \frac{\pi_{t+1}}{P_t} - \frac{F_t}{P_t} \right) = \]

\(^{24}\)This specification facilitates the comparison with Taylor-style rules in log-linearized models.
\begin{align*}
&\quad + (1 + i_t^*) \frac{\Gamma_{B,t}}{\pi_t^*} \frac{B_{H,t}^*}{P_{t-1}^*} \frac{\xi_t^*}{P_t^*} \\
&\quad + \frac{1 - s}{s} \frac{\xi_t^*}{P_t^*} \left( \frac{\bar{P}_{M,t}^*}{P_t^*} M_t^* \right) - \frac{\bar{P}_{M,t}}{P_t} M_t,
\end{align*}

where \( \frac{\bar{P}_t}{P_t} \)

\[
\frac{\bar{P}_t}{P_t} = (1 + i_t^*) (1 - \Gamma_{B,t}) \frac{1}{\pi_t^*} \frac{\xi_t^*}{P_t^*} \frac{B_{H,t}^*}{P_{t-1}^*}.
\]
Figure 1. United States Stylized Facts (1994:Q1-2000:Q4)
Figure 2. GEM Overview
Figure 3. United States Stylized Facts (1994:Q1-2000:Q4)

Productivity Shock

(Solid - Actual data, Dashed - Tradable good only version, Dotted - Tradable plus nontradable good version)
Figure 4. Baseline Simulation Results

Productivity - Tradable and Non-Tradable

(Indices: 1996=1)

Relative Price of Tradables

(Indices: 1996=1)
Figure 5. Real Exchange Rate Response to a Productivity Increase

(*Increase indicates appreciation; Percentage deviation from baseline,*

Elasticity of Substitution Between Home and Foreign Tradable Good)
Figure 6. Expected GDP Growth Rate Five Years Ahead

(Year-on-year; In percent)
Figure 7. Simulation Results with Learning

Productivity Shock

(Solid - Actual data, Dashed - Tradable plus nontradable good version under uncertainty about persistence of shock)
Figure 8. Simulation Results with Learning and Two Shocks

Productivity Plus Risk Shock

(Solid - Actual data, Dashed - Tradable plus nontradable good version under uncertainty about persistence of shocks)
References


