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## A Latent Factor Model with Global, Country, and Industry Shocks for International Stock Returns

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**IMF Working Paper**

Research Department

**A Latent Factor Model with Global, Country, and Industry Shocks for International Stock Returns<sup>1</sup>**

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**Abstract**

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We estimate a latent factor model that decomposes international stock returns into global, country-, and industry-specific shocks and allows for stock-specific exposures to these shocks. We find that across stocks there is substantial dispersion in these exposures, which is partly explained by the extent to which firms operate across countries. We show that portfolios consisting of stocks with low exposures to country shocks achieve substantial variance reduction relative to the global market, both in- and out-of-sample. The shock exposures are thus a stock-selection device for international portfolio diversification.

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## I. INTRODUCTION

We investigate the exposures of individual stocks to global, country-, and industry-specific shocks. The methodological contribution of this paper is to estimate a latent factor model that allows for stock-specific exposures to these shocks for a large panel of international stock returns. Allowing for stock-specific exposures has intuitive appeal. After all, it seems plausible that multinationals have higher exposures to global shocks, for example, than firms that operate only domestically. Allowing for stock-specific exposures has practical appeal as well. The exposures represent a stock-selection device that is potentially useful for international diversification. If—as has been documented by Heston and Rouwenhorst (1994) and Griffin and Karolyi (1998) among others—country-specific shocks are on average the most important source of international return variation, why not construct portfolios consisting of stocks with low country shock exposures in the first place? To the extent that country exposures differ significantly from stock to stock, these portfolios may deliver substantial reductions in volatility relative to the global benchmark.

Building on Heston and Rouwenhorst (1994), our model decomposes stock returns into (i) a global factor common to all stocks; (ii) country-specific factors that capture common variation within countries; (iii) industry-specific factors that capture common variation within industries; and (iv) idiosyncratic variation in each stock. As in the latent factor model literature (see Cho and others, 1986, and Heston and others, 1995), our factors are unobserved. However, borrowing from the identified vector autoregression (VAR) literature (see Sims, 1986, and Bernanke, 1986) in macroeconomics, we identify our factors as global, country-, and industry-specific by imposing restrictions on the factor exposures of individual stocks, called exposures below. For example, we identify the U.S. country factor by assuming that only U.S. stocks have nonzero exposures to that factor. In other words, we sort stocks into groups according to observable characteristics (country and industry affiliation) and identify a given factor by assuming that it affects only stocks in that group. Conceptually, our identifying assumption, which groups stocks according to observed characteristics, is similar to that in Heston and Rouwenhorst (1994) who create portfolios of stocks based on their country and industry affiliations. We depart from their approach in that we estimate the degree to which stocks belong to their assigned portfolios. Our shock exposures capture both observed and unobserved firm-level characteristics that determine the true exposure to a factor.

We estimate our model for 1,965 stocks from 21 countries in the period January 1985 to February 2002, and for a larger sample of 3,939 stocks in 33 countries from January 1990 to February 2002. Our results can be summarized as follows. First, we document that dispersion in exposures to global, country, and industry shocks is economically and statistically significant. Second, we show that shock exposures relate to observed firm-level characteristics, such as the degree to which companies operate internationally and their size. We find, however, that observables explain only a small fraction of the cross-section of the exposures, suggesting that the shock exposures capture more information than that captured by our list of firm-level variables. Third and most importantly, we show that a portfolio consisting of stocks with low exposure to country shocks delivers substantial variance reduction relative to the global market portfolio, both in- and out-of-sample. Countries in this portfolio are present in the same

proportion as in the global market portfolio, so that variance reduction results not from a particular country tilt, but purely from lower exposure to country-specific shocks. We construct similar low country exposure portfolios for individual countries and show that in virtually all cases these low exposure portfolios are less volatile than the respective country benchmark, again both in- and out-of-sample.

The remainder of the paper is organized as follows. Sections II and III describe the model and the data, respectively. Section IV contains the results. Section V concludes.

## II. THE MODEL

This section describes our model. Let us denote by  $R_{nt}$  the excess return over a riskless benchmark on stock  $n$  in period  $t$ , where  $n$  ranges from 1 to  $N$  and  $t$  from 1 to  $T$ . Let us index countries with the letter  $c$  ( $c = 1, \dots, C$ ) and industries with the letter  $i$  ( $i = 1, \dots, I$ ). The model can then be written as:

$$R_{nt} = \mu_n + \beta_n^G f_t^g + \sum_{c=1}^C \beta_{nc}^C f_t^c + \sum_{i=1}^I \beta_{ni}^I f_t^i + \epsilon_{nt}, \quad (1)$$

where  $\mu_n$  represents the expected excess return on stock  $n$ ,  $f_t^g$ ,  $f_t^c$  and  $f_t^i$  represent the global market factor, the country factor  $c$ , the industry factor  $i$ , and  $\epsilon_{nt}$  represents the idiosyncratic shock to the return of stock  $n$ , all in period  $t$ . The factors are unobservable random variables, i.e., latent factors. In order to identify the factors as global, country, and industry shocks we impose the following set of zero restrictions on the exposures ( $\beta$ s):

$$\begin{aligned} \beta_n^G &= \text{unconstrained}, \\ \beta_{nc}^C &= \begin{cases} \text{unconstrained} & \text{if stock } n \text{ belongs to country } c, \\ 0 & \text{otherwise,} \end{cases} \\ \beta_{ni}^I &= \begin{cases} \text{unconstrained} & \text{if stock } n \text{ belongs to industry } i, \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \quad (2)$$

As Chan and others (1998) argue, latent factor model have the disadvantage that it is difficult to give economic interpretation to purely statistical factors. Our model attempts to address this deficiency via the identifying restrictions.

Model (1) can be written in stacked form as:

$$\mathbf{R}_t = \boldsymbol{\mu} + \mathbf{B}\mathbf{f}_t + \boldsymbol{\epsilon}_t, \quad (3)$$

where  $\boldsymbol{\mu}$  denotes the  $N \times 1$  vector of mean returns, and  $\mathbf{f}_t$  the  $K \times 1$  vector of factors ( $K$  is the total number of factors),  $\mathbf{B}$  is a  $N \times K$  matrix of  $\beta$ s, and  $\boldsymbol{\epsilon}_t$  the  $N \times 1$  vector of idiosyncratic shocks. Without restrictions (2), model (3) is not identified: if we replace  $\mathbf{B}$  with  $\mathbf{B}\boldsymbol{\Omega}$  and  $\mathbf{f}_t$

with  $\Omega' \mathbf{f}_t$ , where  $\Omega$  is an orthonormal matrix, the likelihood is unchanged. However, in our model the zero restrictions (2) on  $\mathbf{B}$  are enough to pin down the rotation matrix  $\Omega$ : only if  $\Omega$  is equal to the identity matrix, will  $\mathbf{B}\Omega$  have the same zero restrictions as  $\mathbf{B}$ . With  $\Omega$  pinned down, the factors are “identified.”

We estimate model (1) via maximum likelihood and make the following distributional assumptions:

$$\mathbf{f}_t \rightarrow N(0, \mathbf{I}), \quad (4)$$

where  $\mathbf{I}$  is a  $K \times K$  identity matrix, and:

$$\epsilon_t \rightarrow N(0, \Sigma) \text{ for all } t \text{ and } n, \quad (5)$$

where  $\Sigma$  is diagonal (idiosyncratic shocks are cross-sectionally uncorrelated) with elements  $\sigma_n^2$ . In expression (4), the assumption that the factors have unitary variance is purely a normalization assumption. However, the assumption that the factors are uncorrelated is not without loss of generality here, given that the rotation matrix  $\Omega$  has already been pinned down by the zero restrictions (2). We discuss this issue later in greater detail.

The use of zero restrictions implies that we cannot use standard packages to obtain the maximum likelihood estimates. The restrictions also imply that other methods for estimating APT models (Connor and Korajczyk, 1986) are not immediately applicable. A value-added of this paper is that we provide a method for estimating model (1) via maximum likelihood for large cross-sections, using the EM algorithm.<sup>2</sup> The first step of the EM algorithm follows the intuition that if the factors were observable the exposures could be estimated by means of OLS, equation by equation (see also Marsh and Pfleiderer, 1997). In the next step, an estimate of the factors is obtained by taking their conditional expectation given the data and the parameters from the previous step. Specifically, at each step  $q$  of the algorithm, the estimate of the non-zero loadings for stock  $n$  are given by the formula:

$$\beta_q^n = \left[ \sum_{t=1}^T M^n E_{q-1} [f_t f_t'] M^{n'} \right]^{-1} \sum_{t=1}^T M^n E_{q-1} [f_t] (R_{nt} - \mu_n), \quad (6)$$

where the matrix  $M^n$  simply selects the factors that are relevant for stock  $n$  (i.e., the appropriate country and industry factor). Details on the derivation of expression (6) and of the conditional

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<sup>2</sup> Lehmann and Modest (1985) also use the EM algorithm to estimate factor models. Our model extends their work to the case where the restrictions (2) are present.

expectations  $E_{q-1}[f_t]$  and  $E_{q-1}[f_t f_t']$  are given in the appendix. This iterative procedure is guaranteed to increase the likelihood at each step and hence delivers maximum likelihood estimates. Since formula (6) is computed stock by stock, the approach is computationally feasible even for very large cross-sections.

One may question the orthogonality assumption (4): For example, are the country factors for Germany and France really uncorrelated? Note that from a Bayesian point of view, assumption (4) can be seen as a prior on the vector of factors  $\mathbf{f}_t$ . Upon convergence, the EM algorithm delivers a (conditional) posterior distribution for the factors, which is normal with first and second moments equal to  $\mathbf{E}[\mathbf{f}_t | \mathbf{B}, \Sigma, \mathbf{R}_t]$  and  $\mathbf{E}[\mathbf{f}_t \mathbf{f}_t' | \mathbf{B}, \Sigma, \mathbf{R}_t]$ , respectively. From the average of the posterior second moments matrix of the factors, computed as

$$\sum_{t=1}^T \mathbf{E}[\mathbf{f}_t \mathbf{f}_t' | \mathbf{B}, \Sigma, \mathbf{R}_t] / T, \quad (7)$$

one can assess whether the orthogonality assumption is borne out by the data or not.

### III. THE DATA

Our data are based on Brooks and Del Negro (2004) who collect monthly U.S. dollar-denominated returns for 9,679 stocks in 42 countries from January 1985 to February 2002 from Datastream International. They show that the sample provides a good depiction of the global stock market, both in terms of coverage within and across countries and in terms of overall coverage, which comes close to matching the market capitalization of the known universe of stocks.<sup>3</sup>

We modify this data in two ways. First, we balance the data because our maximum likelihood algorithm does not allow for missing observations. Since this step may result in survivorship bias and reduce coverage, we do this at two points in time, to ensure that our results are robust to this step. One balanced sample consists of stocks with continuous returns for the entire sample. It contains 1,965 stocks in 21 countries. Another begins in January 1990 and contains stocks with continuous returns for the rest of the sample. It covers 3,939 stocks in 33 countries. Second, we follow common practice in the literature, see Ferson and Harvey (1994) and Heston and others (1995), and estimate our model for excess returns. We compute these as the difference between individual stock returns and the return on a three month U.S. Treasury Bill, which we obtain from Datastream International. Although both samples cover fewer stocks than the original data, Table A.1 in appendix II shows that they are comparable in terms of the means and standard deviations of the equal-weighted excess return across all stocks. In particular, there

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<sup>3</sup> Using U.S. dollar-denominated returns may lump nominal currency movements into the country-specific shocks, potentially biasing up the importance of these shocks in our model. We investigate the magnitude of this bias by using local currency returns instead and find it to be negligible, consistent with Heston and Rouwenhorst (1994).

is little indication of a systematic bias in the standard deviation of excess returns, which is important because we focus on return variability and comovement.

To identify country and industry shocks, we use the country and industry affiliations for each stock from Datastream International. With regard to industry affiliation, Datastream has six levels of disaggregation. At each level, there are more disaggregated industry definitions, up to the most disaggregated classification, level 6. We use the most disaggregated level in our benchmark estimation.<sup>4</sup> To estimate the country and industry shocks in a meaningful way, our balanced data must provide a reasonable representation of international stock markets, both in terms of country and industry coverage. Appendix II reviews this coverage and shows that it is representative.<sup>5</sup>

#### IV. RESULTS

To our knowledge, this is the first paper that studies firm-level exposures to global, country, and industry shocks. Since firms differ in the degree to which they operate across countries and industries, it seems a priori plausible that such differences be reflected in their exposures to common shocks. The next section investigates whether firm-level heterogeneity in the exposures is statistically and economically significant. Section IV-B will show how the exposures relate to observable firm-level information, such as size, book-to-market, and the degree to which firms operate internationally. In section IV-C we show that these exposures represent valuable information for bottom-up international diversification strategies: knowledge of the exposures makes it possible to construct global and country portfolios that are less volatile than their respective benchmarks, both in- and out-of-sample. Finally, section IV-D documents that our model reproduces well-known findings in terms of variance decomposition for international stock returns, and considers some robustness checks.

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<sup>4</sup> Griffin and Karolyi (1998) argue that broad industrial classifications may lump together heterogeneous industries, which may bias downward the importance of industry-specific shocks. Against this background, Griffin and Stulz (2001) use the level 6 industry definitions from Datastream in their study on the importance of industry-specific and exchange rate shocks in stock returns. See <http://www.datastream.com/product/investor/index.htm> for a description of the Datastream Global Market Indices

<sup>5</sup> Ince and Porter (2004) report that individual equity return data for the United States from Datastream is sometimes inaccurate. However, they find that these problems are concentrated among smaller size deciles. They are thus not likely to materially affect our results. Moreover, as we document below, our variance decomposition results are consistent with earlier papers that use different data sources.



### A. Firm-Level Heterogeneity in the Exposures to Global, Country, and Industry Shocks

We first test whether allowing for stock-specific exposures to global, country-, and industry-specific shocks helps significantly in explaining comovement in international stock returns. We implement this test by estimating a restricted version of our model, in which all stocks within a country (industry) are restricted to have the same exposure to that country (industry) shock. In addition, we impose the restriction that all stocks (across countries and industries) have the same exposure to the global shock. By testing the restricted versus the unrestricted model we learn whether stock-level heterogeneity in shock exposures is statistically important, i.e., whether the benefit in terms of fit outweighs the cost of estimating additional parameters.<sup>6</sup> We compare the goodness of fit across the restricted and unconstrained models using both the likelihood ratio statistic and the Bayesian Information Criterion (BIC), which we compute following Kass and Raftery (1995):

$$BIC = L - \frac{\ln(T)}{2} \times (\# \text{ of free parameters}) \quad (8)$$

where  $L$  is the log-likelihood at the peak.<sup>7</sup>

The value of the likelihood ratio statistic, equal to twice the difference between the log-likelihood values at the peak for the unrestricted and the restricted models, is equal to 38,475.8 and 59,394.8 for the 1985–2002 and 1990–2002 samples, respectively. The degrees of freedom are 5,768 and 11,677, respectively. In both samples, the difference in likelihoods is large enough

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<sup>6</sup> Formally, in the restricted version of our model the restrictions (2) are replaced with:

$$\begin{aligned} \beta_n^G &= \beta^G \\ \beta_{nc}^C &= \begin{cases} \beta_c^C & \text{if stock } n \text{ belongs to country } c, \\ 0 & \text{otherwise,} \end{cases} \\ \beta_{ni}^I &= \begin{cases} \beta_i^I & \text{if stock } n \text{ belongs to industry } i, \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \quad (11)$$

<sup>7</sup> Recall that in the factor model the variance-covariance matrix of excess returns is a sufficient statistics for the likelihood. Also, at the peak of the likelihood the variance of the idiosyncratic term is by construction such that the model exactly fits the variance of each individual stock. Hence the model that maximizes the BIC is the one that best captures the co-movements among stocks, and does so parsimoniously.

that the restricted model is soundly rejected, with the p-value essentially zero. We reach the same conclusions using the more conservative BIC. In summary, we find strong statistical evidence in favor of the unrestricted model.

We now investigate whether the differences across stocks in their exposures to global, country- and industry-specific shocks are economically large. Table 1 describes the distribution of global, country, and industry exposures within each portfolio (global, country and industry). The first column shows the number of stocks in each portfolio. The second column shows the mean exposures, while the third shows their standard deviations. The mean exposure to global and industry shocks amounts to 2 percent, while it measures 6 percent on average across countries. This result points to the fact that country shocks are the dominant source of variation for international stock returns—a result that we will revisit in section IV-D. The standard deviation of the exposures measures the degree of firm-level heterogeneity. The standard deviations of exposures to global and industry shocks are about 2 percent, compared with 1.5 percent on average for country exposures. However, these standard deviations may be biased upward, because the factor exposures are estimated with error. To address this question, we decompose the dispersion of the exposures into that associated with estimation error and that associated with variation in the underlying “true” betas. The expected value of the standard deviation of estimated exposures for a portfolio of  $N$  stocks is equal to:

$$E \left[ \frac{1}{N} \sum_{i=1}^N \hat{\beta}_i^2 - \left( \frac{1}{N} \sum_{i=1}^N \hat{\beta}_i \right)^2 \right] = E \left[ \frac{1}{N} \sum_{i=1}^N \beta_i^2 - \left( \frac{1}{N} \sum_{i=1}^N \beta_i \right)^2 \right] + E \left[ \frac{1}{N} \sum_{i=1}^N \eta_i^2 - \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \eta_i \eta_j \right] \quad (9)$$

where  $\hat{\beta}$  and  $\beta$  are the estimated and true exposure, respectively, and  $\eta$  is the sampling error. Expression 9 shows that the estimated standard deviation is equal to the true standard deviation plus a bias term. Using the asymptotic (normal) distribution of our maximum likelihood estimator, we can compute this bias and correct for it, which we do in the fourth column of Table 1. This column shows that stock-level heterogeneity in the exposures persists after the bias-correction: heterogeneity in our shock exposures is not simply a result of sampling error.

To investigate whether the dispersion of shock exposures is economically large, we divide the global, country and industry portfolios into high and low exposure portfolios. A high country (global, industry) exposure portfolio, for example, is a portfolio with stocks whose exposure to that country (global, industry) shock is higher than the mean exposure. Low country (global, industry) exposure portfolios are portfolios whose stocks have exposures to country (global, industry) shocks below their respective means. We then contrast the differences in exposure of high and low exposure portfolios to the respective factor. Assuming that the distributions of the betas is normal—the skewness and kurtosis numbers in columns (7) and (8) show that this is approximately the case—this difference equals:

$$\int_{\mu}^{\infty} \beta \varphi(\mu, \sigma) d\beta - \int_{-\infty}^{\mu} \beta \varphi(\mu, \sigma) d\beta = 2 \frac{\sigma}{\sqrt{2\pi}}, \quad (10)$$

where  $\varphi$  indicate the Normal distribution with mean  $\mu$  and standard deviation  $\sigma$ . The results of

this exercise are reported in columns (5) and (6), using the actual and bias-corrected standard deviations, respectively. After the bias-correction, this difference is 1.4, 1, and 1.3 percent on average for global, country, and industry portfolios. This implies, for instance, that a high country exposure portfolio will have an exposure to country shocks that is one percent higher—after bias-correction—than the exposure of a low-beta-portfolio (on average across countries). In summary, we have found that differences across individual stocks in their exposures to global, country- and industry-specific shocks are both statistically significant and economically relevant.

## **B. What Are Stock-Specific Exposures Capturing?**

We have shown that stock-specific exposures to global, country- and industry-specific shocks are empirically and economically important. We now investigate what these exposures are capturing, by relating them to observable characteristics that measure (i) the extent to which firms operate across countries; (ii) their size; (iii) their book-to-market ratio; and (iv) the presence of financial constraints.

To measure the extent to which firms operate across countries, we use two variables. First, we construct a dummy variable for traded versus non-traded goods firms using the classification in Griffin and Karolyi (1998). This classification uses industrial affiliation as opposed to firm-specific information. To address this deficiency, we construct an additional variable that measures the average importance of foreign sales for each company over our sample. To this end, we download annual firm-level data from Worldscope on the percentage of foreign sales and make sample averages.

Separately, we download annual data from Worldscope for year-end market capitalizations and the book-to-market ratio for each firm and construct sample averages for these variables. To measure financial constraints, we follow Lamont and others (2001) and Kaplan and Zingales (1997) and use the following five variables: cash flow to total capital ratio, Tobin's Q, debt to total capital ratio, dividends to total capital ratio, and cash holdings to capital ratio. We download annual data for each variable from Worldscope and construct sample averages of the KZ index for each stock.

Table 2 reports multivariate regressions of the global (Panel A), country-specific (Panel B) and industry-specific (Panel C) stock market exposures (in percent) from the 1985 to 2002 sample on our list of observables. The regression results using the shock exposures from the 1990 to 2002 sample are qualitatively similar and therefore omitted for brevity. In place of the KZ index, which is based on regression results for the United States, we adopt the more general specification of including each of the constituent variables separately. We omit Q from the list of KZ constituent variables because the book-to-market ratio enters our regression separately. Because Table 1 and work by Ferson and Harvey (1998) suggest that there are substantial differences across countries in mean shock exposures, we also include country dummies in our regressions. We report t-ratios in parentheses, which we compute using robust standard errors (White 1980), the associated adjusted  $R^2$ , and the number of observations in each regression.

Panel A shows that global shock exposures are positively and significantly related to measures of international activity. For traded goods companies, exposure to the global shock is on average 0.62 percent per month higher than for non-traded goods firms. Separately, a company raising its foreign sales ratio by 10 percentage points raises its exposure to the global shock by 2 percentage points per month.<sup>8</sup> Among the other explanatory variables, exposure to global shocks is negatively and significantly related to size. Since we take the natural log of market capitalization, the size coefficient is readily interpretable: if market capitalization doubles in size, the exposure to the global shock falls by 0.21 percent per month. This negative relation could be an indication that large firms are less risky, *ceteris paribus*. Meanwhile, there is little evidence of a systematic link between the book-to-market ratio and the KZ index constituents, with the exception of the cash flow to total capital ratio and the dividends to total capital ratio. Global shock exposures are significantly positively related to the former, while they are significantly negative related to the latter. Since these variables have opposite signs, there is little indication that global shock exposures are systematically related to financial constraints.

Panel B shows that there is little evidence that country shock exposures are systematically related to measures of international activity. The traded goods dummy has a positive coefficient, while the foreign sales ratio has a negative coefficient that is borderline significant. Instead, country exposures are significantly negatively related to size, while they are positively and significantly related to book-to-market ratio. In this dimension, our results parallel Avramov and Chordia (2001) who find that comovement among U.S. stocks is negatively associated with size but positively associated with the book-to-market ratio. Among the KZ constituents, there is evidence that country exposures are increasing with leverage (the debt to capital ratio has a significant positive coefficient) and declining with liquidity (the cash to capital ratio has a significant negative coefficient), suggesting that more financially constrained companies are more exposed to these shocks. However, the coefficient on cash flow to total capital ratio is significant and goes in the opposite direction, which suggests that the link between country exposures and measures of liquidity should be interpreted with caution.

Panel C shows that there is little systematic association between industry shock exposures and observables. The one robust pattern that stands out is that industry shock exposures are positively and significantly associated with firm size, consistent with Chan and others (1999) who for U.S. data find that correlations across stocks within industries are higher among large firms.

Overall, we find that (i) global shock exposures are positively associated with measures of international activity and negatively related to size; (ii) country shock exposures are negatively related to size, positively related to the book-to-market ratio, and increasing in leverage; and (iii) industry shock exposures are positively associated with size. However, it is important to

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<sup>8</sup> The positive association between the global shock exposures and the measures of international activity is consistent with Bodnar and others (2002) who argue that foreign sales tend to increase a firm's exposure to international shocks.

note that the observable characteristics fail by a substantial margin to explain these exposures: the adjusted  $R^2$  of the regressions in panels A, B, and C measure only 27, 57, and 15 percent respectively, and about half of the explanatory power is due to country fixed effects. This suggests that the shock exposures contain information that is not readily available: there is substantial heterogeneity in the exposures which cannot be explained with observable characteristics. To put it differently, it is not the case that the low (high) country exposure portfolios described in the previous section are simply portfolios containing large (small) firms, or firms that are not (are) financially constrained. Indeed, if we divide firms within each country into large and small firms (firms with market cap higher—or lower—than the median), we find that on average low and high country exposure portfolios contain the same proportion of large and small firms. The same finding roughly applies to the other observables as well.

### **C. Diversification Strategies Using Stock-Specific Exposures**

We have shown that there is heterogeneity across stocks in their exposures to common shocks, which in part reflects differences in observed characteristics of the underlying companies. We now explore whether we can use these exposures for risk reduction strategies. We do this by constructing bottom-up low exposure portfolios with respect to global, country and industry-specific shocks. We then examine how effective these portfolios are relative to their respective benchmark in terms of volatility and returns, both in- and out-of-sample.

We construct low exposure portfolios in the following way. With respect to the global shock, we construct an equal-weighted portfolio of stocks with global shock exposures below the median, which we call the low global exposure (LGE) portfolio. By construction this portfolio contains half the stocks in our sample. With respect to country shocks, within each country we determine which stocks have country exposures above and below their respective country median. We form a global low country exposure (LCE) portfolio that equal-weights across countries all stocks that have shock exposures below the country median. This is a global portfolio consisting of half the stocks from each country. We compare the performance of the LGE and global LCE portfolios to that of the equal-weighted global portfolio. Furthermore, we construct country portfolios that include all stocks within a given country with country exposures below the respective country median, which we call local low country exposure (LCE) portfolios. We compare the performance of these portfolios to the respective equal-weighted country portfolio. With respect to industry-specific shocks, we follow the same procedure and form global and local low industry exposure (LIE) portfolios. For reasons of symmetry, we also construct the respective high exposure portfolios at the global, country and industry levels—the HGE, the global and local HCE portfolios, and the global and local HIE portfolios.

Our global LCE and LIE portfolios share an important characteristic: in each we impose that countries and industries be present in the same proportion, respectively, as in the overall sample. With respect to the global LCE portfolio, this means that risk reduction relative to the global market is not the result of a particular country tilt, but reflects the incremental gain from picking stocks with low country exposures. The same holds with respect to the global LIE portfolio.

This is of course not true for our LGE portfolio. Here, country and industry composition depend entirely on the distribution of the global shock exposures.

Based on shock exposure estimates for the 1985 to 2002 sample, Table 3 shows in- and out-of-sample volatility of the equal-weighted global market portfolio, and of the respective low and high exposures portfolios. The results for the shorter sample from 1990 to 2002 are qualitatively similar and therefore omitted for brevity. Column (1) shows the variance of the benchmark (B), column (2) shows the variance of the corresponding low exposure portfolio (L), while the number in parentheses in column (3) shows the variance reduction (increase, if positive) relative to benchmark  $\left(\frac{L-B}{B}\right)$ . All figures (variances and variance reduction) are in percent. Column (4) shows the variance of the corresponding high exposure portfolio (H), with column (4) showing the variance increase over the respective benchmark  $\left(\frac{H-B}{B}\right)$ . Columns (6) through (10) repeat this format for out-of-sample results, based on reestimating our model from January 1985 through February 2000 and examining the risk reduction properties of the low exposure portfolios in the period March 2000 to February 2002.

The first three rows of Table 3 compare the volatility of the global market to that of the LGE and HGE portfolios (row 1), the global LCE and HCE portfolios (row 2), and the global LIE and HIE portfolios (row 3). The table shows that in-sample the global LCE portfolio performs best in terms of risk reduction: its variance is 27 percent below that of the global market portfolio. Next in line is the LGE portfolio, whose variance is 17 percent below that of the global market, while no risk reduction is associated with the global LIE portfolio. Why is it that the global LCE portfolio does best in terms of risk reduction? Section IV-A sheds some light on this: the means of country shock exposures are higher than those for the global or industry shocks. This means that country-specific shocks are more important drivers of international return variation than global or industry shocks, a result we will confirm in the next section using variance decompositions. Taken together with the fact that there is substantial heterogeneity in country shock exposures, as demonstrated in Section IV-A, the global LCE portfolio is best positioned to reduce volatility relative to the global market portfolio. Importantly, these results carry over to out-of-sample. Relative to the market during the period from March 2000 to February 2002, the global LCE portfolio yields a reduction in volatility of 31 percent, while the LGE portfolio delivers a reduction in volatility of 26 percent. Again, no risk reduction is associated with the global LIE portfolio.

The remaining rows of Table 3 compare the volatility of our low and high exposure portfolios with that of the respective country or industry benchmarks. For countries, we show the volatilities of the local LCE and HCE portfolios for each country, as well as the equal-weighted average across countries. For industries, we only show the equal-weighted average across industries to save space. We have seen that at the global level our LCE portfolio deliver risk reduction relative to the global market. Do this result carry over to the country level? The answer is yes, both in- and out-of-sample: our local LCE portfolios have lower variance than the respective country benchmarks. Only for a handful of countries for which we have very few

stocks in our sample (like South Africa) this is not the case. On average across countries, local LCE portfolios deliver an in-sample reduction in volatility of 26 percent relative to the country portfolio, while high exposure portfolios boost volatility by 43 percent. Out-of-sample, there is a 25 percent reduction in volatility associated with local LCE portfolios on average. Again, note that this finding is due to the heterogeneity in exposures: under the null that all stocks have the same "true" country exposure, we would simply be randomly picking half the stocks in each country. It would be quite surprising to find that both in- and out-of-sample (almost) all of these portfolios have lower variance than their respective benchmark. Finally, the last row in Table 3 confirms that there is little if any systematic risk reduction associated with the local LIE portfolios.

We next examine the robustness of our risk reduction result over different sub-periods. Table 4 shows the variance reduction relative to the global market portfolio of the LGE portfolio (column 1), the global LCE portfolio (column 2) and the global LIE portfolio (column 3) for eight two-year sub-periods, working backward from the end of the sample. This exercise shows that the global LCE portfolio is the most robust in terms of risk reduction. For all subsamples the variance of this portfolio is less than that of the global market, though the gains range from 37 to 15 percent. Not so for the LGE portfolio where much of the risk reduction appears to be driven by the later part of the sample.

Figure 1 examines the risk reduction properties of our low exposure portfolios using the graphical representation in Heston and Rouwenhorst (1994) and Solnik (1975). It gives the average portfolio variance as the number of stocks in a given portfolio increases from one to forty, expressed as a percentage of the average variance of all individual stocks in our sample. The solid lines in Figure 1 show the diversification benefit associated with the global market portfolio (line 3), the average country portfolio (line 5), and the average industry portfolio (line 6). All averages are equal-weighted. The global benchmark has a variance of 16 percent relative to the average stock. The country benchmark has a variance of 34 percent (on average across countries) relative to the average stock. The industry benchmark delivers a risk reduction of 27 percent (on average across industries) relative to the average stock. These figures restate the well-known finding that diversifying across countries (within an industry) is more promising for risk reduction than diversifying across industries (within a country). The value added of this paper is represented by the dashed lines—lines (1), (2), and (4). Line (4) shows that risk reduction for the local LCE portfolios (on average across countries) is 25 percent. This shows that one does not need to diversify internationally to achieve higher risk reduction than that delivered by the average country portfolio. Just by selecting stocks with low country exposure one obtains roughly the same level of volatility. Lines (1) and (2) show that the LGE and global LCE portfolios have a lower variance than the global benchmark, consistently with Table 3. A bottom-up strategy that picks stocks according to their country or global exposures beats the global benchmark in terms of risk reduction. Of course, for this to be the case the exposures need to be estimated quite precisely. Using the shorter sample (1990-2002) we find that all the above results hold, except for the out-of-sample performance of the LGE portfolio. This is perhaps because global exposures, which are harder to estimate than country exposures (due to the lower variability of global relative to country shocks), are imprecisely estimated in the

shorter sample. The global and local LCE portfolios however still outperform their respective benchmarks for the 1990-2002 sample, both in- and out-of-sample.

We have shown that global and local LCE portfolios and the LGE portfolio successfully achieve risk reduction relative to the global market portfolio. But does this risk reduction come at the cost of lower expected returns? In other words, is there no difference in risk-adjusted terms between holding low exposure portfolios and holding the market? Table 5 examines this question. The rows of Table 5 correspond to those in Table 3. Column (1) and (2) show the in-sample mean monthly return in percent for the benchmark (B) and for the corresponding low exposure portfolio (L), while the number in parentheses in column (3) shows the difference between the two (L-B). Columns (4) and (5) show the corresponding figures for the high exposures portfolios. Columns (6) through (10) give the in-sample Sharpe ratios, while columns (11) through (15) show out-of-sample means for the period March 2000 to February 2002, based on shock exposure estimates for the shortened sample from January 1985 to February 2000. The in-sample mean returns on the LGE, global LCE, and global LIE portfolios are 7, 5 and 1 basis points per month in excess of the global benchmark, while the HGE, global HCE, and global HIE all do worse than the market. The in-sample Sharpe ratios of these portfolios are 2, 3, and 0 basis points per month over the global market. On a risk-adjusted basis, the LGE and global LCE portfolios thus outperform the market. Out-of-sample the low global and country exposure portfolios do better than the market, to the tune of 64 and 10 basis points per month. This is noteworthy given that the out-of-sample period spans the bursting of the tech-bubble and is thus a challenging period during which to examine out-of-sample behavior of our low exposure portfolios. This is evidence that our bottom-up approach to construct international portfolios provides diversification gains, even on a risk-adjusted basis.

#### **D. Variance Decomposition Results and Robustness Tests**

Are country- or industry-specific shocks more important in explaining the behavior of national stock markets? Heston and Rouwenhorst (1994) and Griffin and Karolyi (1998) among others conclude that country shocks are much more important, using cross-sectional regressions of international returns on country and industry dummies. We use our methodology to re-examine this result. Although we allow for firm-specific exposures to the different shocks, the impact of country shocks on a country portfolio, for example, depends on the average country exposure, which should roughly correspond to the standard deviation of the country shock in the Heston and Rouwenhorst model. Hence we expect to find a similar answer for the variance decomposition as in the existing literature. However, in contrast to the existing literature, we control for global shocks in addition to country- and industry-specific shocks. Does this throw off the balance between country- and industry-specific shocks? Finally, the literature focuses on the relative importance of country versus industry shocks in country and industry portfolios. We ask how important these shocks are for the average stock and for the global market portfolio.

Based on the 1985 to 2002 sample, the first row in Table 6 reports the average variance decomposition across country portfolios, while the second row shows the average variance decomposition across industry portfolios. The first column gives the actual variance in percent per month squared of the average country and industry portfolios. The second column gives the



percentage of that variance explained by the global shock, the third column gives the percentage due to country-specific shocks, while the fourth column gives the percentage associated with industry-specific shocks. On average, we find that country shocks explain 81 percent of variation in our country portfolios, while industry shocks explain barely one percent. Meanwhile, we find that country shocks explain 32 percent of the volatility for the average industry portfolio, while industry shocks explain only 11 percent. In short, we replicate the result in the literature: country shocks are much more important than industry shocks for international returns. That said, Heston and Rouwenhorst (1994) and Griffin and Karolyi (1998) find that country shocks explain virtually all of the variation in national stock markets. Our number is lower because model (1) includes a global shock, which explains close to 12 percent of the variation in the average country portfolio. In summary, we replicate the existing result, with the caveat that accounting for global shocks reduces the importance of country shocks in national stock markets somewhat.

The third row in Table 6 gives the variance decomposition for the average across individual stocks. Country shocks account for 32 percent at this level, followed by industry shocks at 7 percent and global shocks at 6 percent. The fourth row in Table 6 gives the variance decomposition for the global market portfolio. Here country shocks are still the most important source of variation, at 28 percent, followed by 21 percent for the global shock.

In Table 6 we show the results for the ex-ante variance decomposition, that is, assuming that the factors are orthogonal. As discussed in Section II, ex-post the factors may not turn out to be orthogonal. Table 7 shows the median ex-post correlations among factors for the 1985-2002 sample (the 1990-2002 results are very similar). The diagonal elements of the matrix show the average correlation within each class of factors and the off-diagonal elements show the average correlation across different classes of factors. The average correlation among country factors is 0.3, and all other average correlations are less than 0.1. These results do not point out substantial mis-specifications in the baseline model. Most importantly, the off-diagonal elements are virtually zero, suggesting that the orthogonality assumption used in the variance decomposition is not far-fetched.

Our decision to focus on global, country- and industry-specific shocks is based on Heston and Rouwenhorst (1994) and Griffin and Karolyi (1998). We now test the robustness of our results with respect to other sources of common variation. We augment our model with: (i) an additional global factor—orthogonal to the first one—which may capture different kinds of global shocks, e.g. oil versus interest rate shocks; (ii) an emerging markets factor, to allow for comovement associated with emerging markets; (iii) an ADR factor that captures common variation associated with stocks that have ADRs; (iv) regional factors for Developed Europe, Emerging Europe, Developed Americas, Emerging Americas, Developed Asia, Emerging Asia, to capture regional comovement; and (v) factors associated with small cap stocks (a size factor), high book-to-market ratio stocks (value stocks) and high KZ index stocks (financing constrained stocks). In some instances (depending on the sample) regional factors, the size factor, book-to-market factor and the KZ factor add significantly to the explanatory power of the model: they have higher BICs than our baseline model. However, they do not add significantly to explaining international return variation according to additional variance

decomposition results (available upon request). In other words, our model consisting of global, country- and industry-specific shocks provides a parsimonious specification with which to investigate international return comovement.

We consider one final robustness test, which is motivated by recent papers that argue that the relative importance of country versus industry shocks in international returns has been changing over time.<sup>9</sup> We modify the model to allow the variances of global, country and industry shocks to vary over four pre-specified sub-periods of equal length. Estimating this version of the model for the 1985 to 2002 sample, variance decompositions show that the global shock increases in importance during the last sub-period (1997:11 to 2002:02), while country and industry shocks are roughly stable over time. Overall, however, these changes leave the results qualitatively unchanged.

## V. CONCLUSION

This paper presents a methodological contribution, as it estimates a latent factor model that allows for stock-specific exposures to global, country, and industry shocks on a large panel of international stock returns. These firm-level exposures capture observed and unobserved heterogeneity across firms. We make the following empirical contributions.

First, we quantify the extent to which stock-specific exposures to global, country, and industry shocks are important, both statistically and economically. Second, we explore whether exposures to global, country, and industry shocks are systematically related to observed characteristics of firms, such as their extent of their international operations, size, book-to-market ratio, and whether they are financially constrained. Finally, we explore whether stock-specific shock exposures can serve as a stock-selection device to construct portfolios that successfully diversify risk. We find that this is the case at the global as well as the country level, both in- and out-of-sample.

Our model can be extended in several ways, including: (i) having regime-switching in the variance of the factors (see Ang and Bekaert, 2002); and (ii) having time variation in the exposures. We leave these extensions for future research.

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<sup>9</sup> See Brooks and Del Negro (2004) for a review of this literature.

Table 1. Distribution of the Exposures to Global, Country-, and Industry-Specific Shocks

|                           | (1)         | (2)   | (3)      | (4)              | (5)  | (6)              | (7)      | (8)      |
|---------------------------|-------------|-------|----------|------------------|------|------------------|----------|----------|
|                           | # of stocks | Mean  | St. Dev. | (Bias corrected) | H-L  | (Bias corrected) | Skewness | Kurtosis |
| Global Factor             | 1965        | 2.04  | 1.93     | 1.78             | 1.54 | 1.42             | 0.92     | 4.4      |
| Country Factors           |             |       |          |                  |      |                  |          |          |
| Australia                 | 41          | 5.87  | 1.09     | 0.95             | 0.87 | 0.76             | -0.80    | 3.3      |
| Austria                   | 9           | 5.85  | 1.35     | 1.21             | 1.07 | 0.97             | -0.23    | 1.6      |
| Belgium                   | 26          | 4.74  | 0.85     | 0.72             | 0.68 | 0.57             | -0.45    | 1.8      |
| Canada                    | 89          | 3.79  | 1.14     | 0.94             | 0.91 | 0.75             | -0.20    | 3.8      |
| Denmark                   | 20          | 4.67  | 0.85     | 0.67             | 0.68 | 0.54             | -0.90    | 4.4      |
| France                    | 54          | 5.33  | 1.38     | 1.23             | 1.10 | 0.98             | -0.53    | 2.5      |
| Germany                   | 100         | 4.49  | 1.04     | 0.92             | 0.83 | 0.73             | -0.13    | 2.6      |
| Hong Kong SAR             | 51          | 8.80  | 2.39     | 2.30             | 1.91 | 1.84             | -0.36    | 2.8      |
| Ireland                   | 14          | 6.06  | 1.10     | 0.66             | 0.88 | 0.52             | 0.56     | 2.5      |
| Italy                     | 31          | 6.88  | 1.29     | 1.16             | 1.03 | 0.93             | -0.59    | 2.9      |
| Japan                     | 529         | 7.23  | 1.89     | 1.79             | 1.51 | 1.43             | -0.10    | 2.9      |
| Korea                     | 45          | 10.31 | 2.44     | 2.19             | 1.95 | 1.75             | 0.38     | 2.8      |
| Malaysia                  | 18          | 9.45  | 2.18     | 2.09             | 1.74 | 1.67             | 1.72     | 7.1      |
| Netherlands               | 56          | 4.03  | 1.27     | 1.18             | 1.01 | 0.94             | -0.33    | 2.3      |
| Norway                    | 16          | 5.82  | 1.60     | 1.28             | 1.28 | 1.02             | 0.10     | 2.5      |
| Singapore                 | 54          | 8.73  | 2.61     | 2.54             | 2.08 | 2.02             | -0.35    | 3.1      |
| South Africa              | 23          | 5.60  | 1.52     | 1.32             | 1.21 | 1.05             | -0.24    | 1.8      |
| Sweden                    | 16          | 5.92  | 1.29     | 1.16             | 1.03 | 0.92             | -0.92    | 3.4      |
| Switzerland               | 42          | 4.33  | 0.89     | 0.71             | 0.71 | 0.57             | -0.37    | 2.2      |
| U.S.                      | 451         | 4.07  | 1.35     | 1.21             | 1.08 | 0.97             | -0.16    | 2.7      |
| U.K.                      | 280         | 4.95  | 1.21     | 1.05             | 0.96 | 0.84             | 0.18     | 3.1      |
| Average across countries  | 94          | 6.04  | 1.46     | 1.30             | 1.17 | 1.04             | -0.18    | 3.0      |
| Average across industries | 22          | 2.14  | 2.02     | 1.68             | 1.61 | 1.34             | 0.42     | 2.8      |

Notes: The Table describes the distribution of global, country, and industry exposures within the global, country and industry portfolios. The columns show: (1) number of stocks in each portfolio; (2) mean ; (3) standard deviations; (4) biased-corrected standard deviations (see formula (9) for bias-correction); (5) difference in exposure of high and low exposure portfolios to the respective shock, computed using the formula  $2 \frac{St. Dev.}{\sqrt{2\pi}}$ ;

(6) same as previous column using bias-corrected standard deviations; (7) skewness; (8) kurtosis. The table is based on estimates for the 1985 to 2002 sample.

Table 2. Regressions of Exposures on Observables

|                         | A                  |          | B                  |          | C                  |          |
|-------------------------|--------------------|----------|--------------------|----------|--------------------|----------|
|                         | Global Exposures   |          | Country Exposures  |          | Industry Exposures |          |
|                         | (1)                | (2)      | (3)                | (4)      | (5)                | (6)      |
|                         | Coeff.             | t-ratios | Coeff.             | t-ratios | Coeff.             | t-ratios |
| Cash flow/total capital | 0.018              | 4.39     | 0.008              | 2.30     | -0.006             | -1.265   |
| Cash/total capital      | 0.001              | 1.24     | -0.002             | -2.10    | 0.001              | 1.296    |
| Debt/total capital      | -0.006             | -1.50    | 0.029              | 8.26     | -0.008             | -1.500   |
| Dividends/total capital | -0.045             | -3.25    | 0.014              | 1.27     | -0.008             | -0.373   |
| Book-to-market          | -0.014             | -0.06    | 1.479              | 6.20     | 0.439              | 1.346    |
| Market-capitalization   | -0.209             | -3.47    | -0.157             | -3.43    | 0.326              | 4.463    |
| International sales     | 0.024              | 8.20     | -0.005             | -1.66    | 0.004              | 1.195    |
| Traded/non traded dummy | 0.618              | 5.58     | 0.076              | 0.80     | 0.473              | 3.050    |
|                         | adj R <sup>2</sup> | n        | adj R <sup>2</sup> | n        | adj R <sup>2</sup> | n        |
|                         | 0.251              | 946      | 0.555              | 946      | 0.158              | 945      |

*Notes:* Columns (1), (3), and (5) report the OLS coefficients for the cross-sectional regressions where the independent variable is global, country, and industry exposures, respectively. All regressions include country fixed effects. The exposures are measured in percent. Columns (2), (4), and (5) report the associated t-ratios in parentheses, which we compute using robust standard errors (White 1980). The last row in the table reports for each regression the adjusted R<sup>2</sup> and the number of observations. The table is based on estimates for the 1985 to 2002 sample. The regressors are constructed as follows. We download annual data from Worldscope for year-end market capitalizations, book-to-market ratio, capital, debt, cash holdings, cash flow, and dividends for each firm. The variables *Cash Flow/Total Capital*, *Cash/Total Capital*, *Debt/Total Capital*, *Dividends/Total Capital*, *Book-to-Market*, and *Market-Capitalization* are constructed as the average for each company over the sample period of ratio of cash flow over total capital, cash holdings over total capital, debt over total capital, dividends over total capital, book-to-market, and market capitalization respectively. The variable *Market-Cap* enters the regression in logs. The *International Sales* variable is constructed as the average over the sample period of ratio of foreign sales over total sales, which is downloaded from Worldscope. The *Traded/Non Traded Dummy* dummy is constructed by aggregating up the Datastream level 6 industry dummies, following the classification in Griffin and Karolyi (1998), and is equal to one for traded goods industries and zero otherwise.

Table 3. Portfolio Variances of Low versus High Exposure Portfolios Relative to Benchmark

|                          | (1)              | (2)  | (3)     | (4)   | (5)     | (6)                  | (7)  | (8)     | (9)   | (10)    |
|--------------------------|------------------|------|---------|-------|---------|----------------------|------|---------|-------|---------|
|                          | <i>In-sample</i> |      |         |       |         | <i>Out-of-sample</i> |      |         |       |         |
|                          | B                | L    | (L-B)/B | H     | (H-B)/B | B                    | L    | (L-B)/B | H     | (H-B)/B |
| Global Portfolios        |                  |      |         |       |         |                      |      |         |       |         |
| Global                   | 19.4             | 16.1 | -17     | 27.9  | 44      | 19.9                 | 14.8 | -26     | 30.7  | 54      |
| Country                  | 19.4             | 14.2 | -27     | 26.3  | 35      | 19.9                 | 13.8 | -31     | 28.3  | 42      |
| Industry                 | 19.4             | 19.3 | 0       | 20.0  | 3       | 19.9                 | 20.0 | 0       | 20.5  | 3       |
| Country Portfolios       |                  |      |         |       |         |                      |      |         |       |         |
| Australia                | 48.7             | 39.3 | -19     | 61.9  | 27      | 32.6                 | 31.1 | -4      | 38.0  | 17      |
| Austria                  | 43.0             | 40.5 | -6      | 60.9  | 42      | 26.0                 | 37.0 | 43      | 38.0  | 46      |
| Belgium                  | 26.0             | 23.0 | -12     | 34.5  | 33      | 18.1                 | 16.0 | -11     | 25.8  | 43      |
| Canada                   | 23.7             | 19.3 | -18     | 32.2  | 36      | 22.0                 | 20.1 | -8      | 32.9  | 49      |
| Denmark                  | 26.1             | 23.6 | -10     | 35.4  | 35      | 21.1                 | 22.3 | 6       | 36.3  | 72      |
| France                   | 35.2             | 25.9 | -27     | 50.1  | 42      | 30.9                 | 21.3 | -31     | 46.8  | 52      |
| Germany                  | 22.6             | 15.8 | -30     | 33.8  | 50      | 16.3                 | 11.1 | -32     | 28.9  | 78      |
| Hong Kong SAR            | 96.9             | 66.5 | -31     | 138.9 | 43      | 46.9                 | 35.8 | -24     | 63.7  | 36      |
| Ireland                  | 54.6             | 40.2 | -26     | 95.8  | 75      | 47.8                 | 47.4 | -1      | 61.1  | 28      |
| Italy                    | 57.9             | 43.9 | -24     | 78.9  | 36      | 44.0                 | 37.3 | -15     | 52.8  | 20      |
| Japan                    | 56.9             | 42.0 | -26     | 79.8  | 40      | 54.4                 | 36.0 | -34     | 80.7  | 48      |
| Korea                    | 122.3            | 94.3 | -23     | 172.3 | 41      | 141.3                | 98.3 | -30     | 202.5 | 43      |
| Malaysia                 | 104.0            | 82.8 | -20     | 143.8 | 38      | 35.2                 | 28.5 | -19     | 48.3  | 37      |
| Netherlands              | 20.4             | 14.4 | -30     | 31.0  | 52      | 16.6                 | 10.8 | 35      | 26.5  | 59      |
| Norway                   | 53.7             | 44.8 | -17     | 84.8  | 58      | 24.2                 | 20.3 | -16     | 34.8  | 44      |
| Singapore                | 90.6             | 56.8 | -37     | 138.4 | 53      | 46.8                 | 33.0 | -29     | 71.2  | 52      |
| South Africa             | 56.3             | 73.3 | 30      | 66.1  | 17      | 54.8                 | 73.9 | 35      | 58.7  | 7       |
| Sweden                   | 51.9             | 51.9 | 0       | 64.1  | 23      | 64.1                 | 86.6 | -35     | 50.8  | -21     |
| Switzerland              | 25.2             | 20.4 | -19     | 34.6  | 38      | 28.5                 | 18.8 | -34     | 47.9  | 68      |
| U.S.                     | 21.8             | 13.7 | -37     | 34.1  | 56      | 21.7                 | 16.5 | -24     | 37.3  | 72      |
| U.K.                     | 31.3             | 23.2 | -26     | 42.9  | 37      | 33.6                 | 26.7 | -21     | 44.2  | 31      |
| Average across countries | 42.5             | 31.4 | -26     | 60.6  | 43      | 37.9                 | 28.3 | -25     | 55.6  | 47      |
| Industry Portfolios      |                  |      |         |       |         |                      |      |         |       |         |
| Average across countries | 33.2             | 36.0 | 8       | 48.3  | 45      | 40.4                 | 44.1 | 9       | 55.6  | 38      |

Notes: Column (1) shows the variance of the corresponding benchmark portfolio (B) – that is, the global portfolio for rows 1 through 3, country portfolios for the following twenty-two rows, industry portfolios for the last row. Column (2) shows the variance of the corresponding low exposure portfolio (L). The global low exposure portfolios are considered in rows 1-3 and are constructed as follows (see also text): The low global exposure (LGE) portfolio contains half the stocks in the sample, those with global shock exposures below the median. The global low country exposure (LCE) portfolio also contains half the stocks in the sample, those with country shock exposures below the median for each country. The global low industry exposure (LIE) portfolio contains stocks with industry shock exposures below the median for each industry. The local low exposure portfolios are considered in the remaining rows and are constructed as follows (see also text): For countries, the local low country exposure (LCE) portfolio contains half the stocks in each country, those with country shock exposures below the median. For industries, the local low industry exposure (LIE) portfolio contains half the stocks in each industry, those with industry shock exposures below the median. Column (3) shows the variance reduction (increase, if positive) relative to the corresponding benchmark [(L-B)/B]. Column (4) shows the variance of the corresponding high exposure portfolio (H). High exposure portfolios are constructed as the mirror image of low exposure portfolios. Column (5) shows the variance increase over the respective benchmark [(H-B)/B]. Columns (6) through (10) repeat this format for out-of-sample results. Portfolios are constructed based on estimates from January 1985 through February 2000 and the variances are computed for the period March 2000 to February 2002. All figures (variances and variance reduction) are in percent. The table is based on estimates for the 1985 to 2002 sample.

Table 4. Variance Reduction (Increase) Relative to Global Benchmark Portfolio:  
Subsample Analysis

| Portfolios:  | LGE    | LCE    | LIE   |
|--------------|--------|--------|-------|
| Sub-Periods: |        |        |       |
| 3/00-2/02    | -38.00 | -27.59 | 2.83  |
| 3/98-2/00    | -23.94 | -37.36 | 2.57  |
| 3/96-2/98    | -25.73 | -32.95 | -5.92 |
| 3/94-2/96    | -17.44 | -14.71 | -4.22 |
| 3/92-2/94    | -8.80  | -20.27 | -5.88 |
| 3/90-2/92    | -7.35  | -27.91 | -8.34 |
| 3/88-2/90    | 3.35   | -22.15 | -1.85 |
| 3/86-2/88    | -33.69 | -25.80 | 6.65  |

*Notes:* The table show the variance reduction in percent relative to the global benchmark for three low exposure portfolios during eight different subsamples. The low global exposure (LGE) portfolio contains half the stocks in the sample, those with global shock exposures below the median. The global low country exposure (LCE) portfolio also contains half the stocks in the sample, those with country shock exposures below the median for each country. The global low industry exposure (LIE) portfolio contains stocks with industry shock exposures below the median for each industry. The table is based on estimates for the 1985 to 2002 sample.

Table 5. Portfolio Mean Returns and Sharpe Ratios of Low versus High Exposure Portfolios Relative to Benchmark

|                            | <i>In-sample Mean</i> |      |       |       |       | <i>In-sample Sharpe Ratio</i> |      |       |       |       | <i>Out-of-sample Mean</i> |       |       |       |       |
|----------------------------|-----------------------|------|-------|-------|-------|-------------------------------|------|-------|-------|-------|---------------------------|-------|-------|-------|-------|
|                            | (1)                   | (2)  | (3)   | (4)   | (5)   | (6)                           | (7)  | (8)   | (9)   | (10)  | (11)                      | (12)  | (13)  | (14)  | (15)  |
|                            | B                     | L    | (L-B) | H     | (H-B) | B                             | L    | (L-B) | H     | (H-B) | B                         | L     | (L-B) | H     | (H-B) |
| <b>Global Portfolios</b>   |                       |      |       |       |       |                               |      |       |       |       |                           |       |       |       |       |
| Global                     | 0.35                  | 0.42 | 0.07  | 0.28  | -0.07 | 0.08                          | 0.10 | 0.02  | 0.05  | -0.03 | -0.71                     | -0.08 | 0.64  | -1.35 | -0.64 |
| Country                    | 0.35                  | 0.40 | 0.05  | 0.30  | -0.05 | 0.08                          | 0.11 | 0.03  | 0.06  | -0.02 | -0.71                     | -0.61 | 0.10  | -0.81 | -0.10 |
| Industry                   | 0.35                  | 0.36 | 0.01  | 0.33  | -0.02 | 0.08                          | 0.08 | 0.00  | 0.07  | 0.00  | -0.71                     | -0.70 | 0.01  | -0.71 | 0.00  |
| <b>Country Portfolios</b>  |                       |      |       |       |       |                               |      |       |       |       |                           |       |       |       |       |
| Australia                  | 0.45                  | 0.42 | -0.03 | 0.47  | 0.02  | 0.06                          | 0.07 | 0.00  | 0.06  | 0.00  | 0.12                      | 0.68  | 0.57  | -0.42 | -0.54 |
| Austria                    | 0.56                  | 0.39 | -0.17 | 0.70  | 0.14  | 0.09                          | 0.06 | -0.02 | 0.09  | 0.00  | -0.16                     | 0.14  | 0.30  | -0.40 | -0.24 |
| Belgium                    | 0.63                  | 0.49 | -0.13 | 0.76  | 0.13  | 0.12                          | 0.10 | -0.02 | 0.13  | 0.01  | -0.36                     | -0.33 | 0.03  | -0.39 | -0.03 |
| Canada                     | 0.38                  | 0.49 | 0.10  | 0.28  | -0.10 | 0.08                          | 0.11 | 0.03  | 0.05  | -0.03 | -0.15                     | -0.14 | 0.01  | -0.15 | -0.01 |
| Denmark                    | 0.49                  | 0.52 | 0.03  | 0.46  | -0.03 | 0.10                          | 0.11 | 0.01  | 0.08  | -0.02 | 0.00                      | -0.22 | -0.22 | 0.21  | 0.22  |
| France                     | 0.62                  | 0.56 | -0.06 | 0.68  | 0.06  | 0.10                          | 0.11 | 0.01  | 0.10  | -0.01 | -0.93                     | -0.44 | 0.49  | -1.41 | -0.49 |
| Germany                    | 0.42                  | 0.51 | 0.09  | 0.33  | -0.09 | 0.09                          | 0.13 | 0.04  | 0.06  | -0.03 | -0.47                     | -0.09 | 0.38  | -0.85 | -0.38 |
| Hong Kong SAR              | 0.51                  | 0.52 | 0.01  | 0.51  | -0.01 | 0.05                          | 0.06 | 0.01  | 0.04  | -0.01 | -0.57                     | -0.64 | -0.07 | -0.51 | 0.07  |
| Ireland                    | 0.91                  | 1.32 | 0.41  | 0.50  | -0.41 | 0.12                          | 0.21 | 0.09  | 0.05  | -0.07 | -0.76                     | -0.86 | -0.10 | -0.66 | 0.10  |
| Italy                      | 0.22                  | 0.49 | 0.27  | -0.03 | -0.25 | 0.03                          | 0.07 | 0.04  | 0.00  | -0.03 | -1.78                     | -1.89 | -0.11 | -1.67 | 0.11  |
| Japan                      | -0.14                 | 0.00 | 0.13  | -0.27 | -0.13 | 0.02                          | 0.00 | 0.02  | -0.03 | -0.02 | -1.65                     | -1.77 | -0.12 | -1.53 | 0.12  |
| Korea                      | 0.50                  | 0.44 | -0.06 | 0.56  | 0.06  | 0.05                          | 0.05 | 0.00  | 0.04  | 0.00  | 0.32                      | 0.40  | 0.08  | 0.24  | -0.08 |
| Malaysia                   | 0.11                  | 0.15 | 0.05  | 0.06  | -0.05 | 0.01                          | 0.02 | 0.01  | 0.01  | -0.01 | -1.54                     | -1.10 | 0.44  | -1.98 | -0.44 |
| Netherlands                | 0.77                  | 0.84 | 0.07  | 0.70  | -0.07 | 0.17                          | 0.22 | 0.05  | 0.13  | -0.04 | -0.78                     | -0.62 | 0.16  | -0.94 | -0.16 |
| Norway                     | 0.18                  | 0.49 | 0.31  | -0.13 | -0.31 | 0.02                          | 0.07 | 0.05  | -0.01 | -0.04 | -0.53                     | -0.85 | -0.32 | -0.21 | 0.32  |
| Singapore                  | 0.11                  | 0.24 | 0.13  | -0.03 | -0.13 | 0.01                          | 0.03 | 0.02  | 0.00  | -0.01 | -0.99                     | -0.63 | 0.36  | -1.35 | -0.36 |
| South Africa               | 0.07                  | 0.07 | 0.00  | 0.07  | 0.00  | 0.01                          | 0.01 | 0.00  | 0.01  | 0.00  | -0.88                     | 0.63  | 1.51  | -2.27 | -1.39 |
| Sweden                     | 0.73                  | 0.68 | -0.04 | 0.77  | 0.04  | 0.10                          | 0.10 | -0.01 | 0.10  | 0.00  | -1.64                     | -2.54 | -0.90 | -0.75 | 0.90  |
| Switzerland                | 0.37                  | 0.15 | -0.22 | 0.59  | 0.22  | 0.07                          | 0.03 | -0.04 | 0.10  | 0.03  | -2.14                     | -1.20 | 0.94  | -3.08 | -0.94 |
| U.K.                       | 0.52                  | 0.58 | 0.06  | 0.46  | -0.06 | 0.09                          | 0.12 | 0.03  | 0.07  | -0.02 | -0.97                     | -1.12 | -0.15 | -0.82 | 0.15  |
| U.S.                       | 0.68                  | 0.66 | -0.02 | 0.70  | 0.02  | 0.15                          | 0.18 | 0.03  | 0.12  | -0.03 | 0.46                      | 0.76  | 0.30  | 0.17  | -0.29 |
| Average across countries   | 0.35                  | 0.40 | 0.05  | 0.30  | -0.05 | 0.07                          | 0.09 | 0.02  | 0.05  | -0.02 | -0.71                     | -0.61 | 0.10  | -0.81 | -0.10 |
| <b>Industry Portfolios</b> |                       |      |       |       |       |                               |      |       |       |       |                           |       |       |       |       |
| Average across countries   | 0.35                  | 0.36 | 0.02  | 0.33  | -0.01 | 0.07                          | 0.07 | 0.00  | 0.06  | -0.01 | -0.71                     | -0.71 | 0.00  | -0.71 | 0.00  |

Notes: Column (1) shows the mean return for the corresponding benchmark portfolio (B) – that is, the global portfolio for rows 1 through 3, county portfolios for the following twenty-two rows, industry portfolios for the last row. Column (2) shows the mean return of the corresponding low exposure portfolio (L). The *global* low exposure portfolios are considered in rows 1-3 and are constructed as follows (see also text): The low global exposure (LGE) portfolio contains half the stocks in the sample, those with global shock exposures below the median. The global low country exposure (LCE) portfolio also contains half the stocks in the sample, those with country shock exposures below the median for each country. The global low industry exposure (LIE) portfolio contains stocks with industry shock exposures below the median for each industry. The *local* low exposure portfolios are considered in the remaining rows and are constructed as follows (see also text): For *countries*, the local low country exposure (LCE) portfolio contains half the stocks in each country, those with country shock exposures below the median. For *industries*, the local low industry exposure (LIE) portfolio contains half the stocks in each industry, those with industry shock exposures below the median. Column (3) shows the difference between columns (2) and (1) (*L-B*). Column (4) shows the mean return of the corresponding high exposure portfolio (H). High exposure portfolios are constructed as the mirror image of low exposure portfolios. Column (5) shows the difference between columns (4) and (1) (*H-B*). Columns (6) through (10) are constructed analogously for the in-sample Sharpe Ratios. Columns (11) through (15) are constructed analogously for the out-of-sample mean returns. Portfolios are constructed based on estimates from January 1985 through February 2000 and the mean returns are computed for the period March 2000 to February 2002. Mean returns figures are in percent. The table is based on estimates for the 1985 to 2002 sample.

Table 6. How Much Do Global, Country, and Industry Factors Matter? Variance Decomposition

|                                    | <i>Actual Variance</i> | Global | Country | Industry-6 |
|------------------------------------|------------------------|--------|---------|------------|
| Average across country portfolios  | 42.5                   | 11.86  | 81.16   | 0.81       |
| Average across industry portfolios | 33.2                   | 15.03  | 31.88   | 10.73      |
| Average across stocks              | 124.7                  | 5.82   | 32.09   | 7.31       |
| Global market portfolio            | 19.4                   | 21.44  | 28.34   | 0.29       |

*Notes:* Sample: 1985-2002. All portfolios, as well as the averages, are constructed using equal weights. All figures are in percent. Actual variances are: averages of the variances of individual stocks, variance of the global market portfolio, and averages of the variances of country and/or industry portfolios. The variance decomposition figures are obtained as follows. Eq. (3) implies that the return for any portfolio with a  $N \times 1$  vector of weights is given by

$$\omega' \mathbf{R}_t = \omega' \mu + \omega' \mathbf{B}^G \mathbf{f}_t^g + \sum_{c=1}^C \omega' \mathbf{B}_c^C \mathbf{f}_t^c + \sum_{i=1}^I \omega' \mathbf{B}_i^I \mathbf{f}_t^i + \omega' \epsilon_t,$$

Using the factor orthogonality assumption, the variance of the portfolio is:

$$\text{var}(\omega' \mathbf{R}_t) = (\omega' \mathbf{B}^G)^2 + \sum_{c=1}^C (\omega' \mathbf{B}_c^C)^2 + \sum_{i=1}^I (\omega' \mathbf{B}_i^I)^2 + \text{var}(\omega' \epsilon_t),$$

where  $\mathbf{B}^G$  is the column of the  $\mathbf{B}$  matrix corresponding to the global factor,  $\mathbf{B}_c^C$  is the column corresponding to the country factor  $c$ , and so on. The variance decomposition obtains by dividing the summands in the right hand side of the last equation by the actual (sample) variance of the portfolio. We discuss in the text the fact that the factors may not be *ex post* orthogonal.

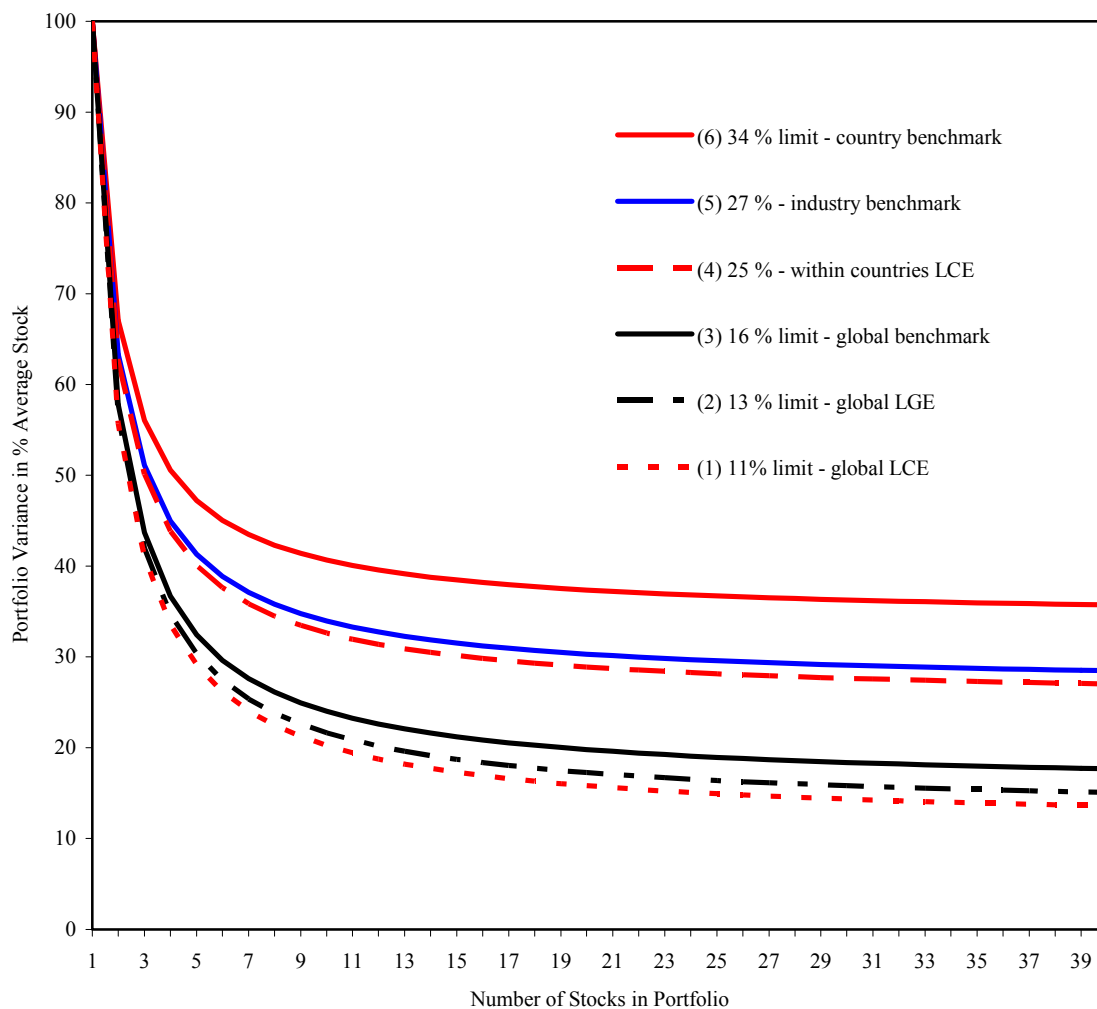


Table 7. Factor Correlation (median)

|            | Global | Country | Industry-6 |
|------------|--------|---------|------------|
| Global     | 1.000  | 0.000   | -0.000     |
| Country    |        | 0.343   | 0.027      |
| Industry-6 |        |         | 0.006      |

*Notes:* Sample: 1985-2002. The table shows the ex-post factor correlation averaged across periods (see formula (7)) for each class of factors (global, country, and industry). The diagonal elements show the median covariance across factors within the same class, and the off-diagonal terms show the median correlation across classes. The correlations are computed fixing the mean of the factor at zero.

Figure 1. The Diversification Gains Associated with Low Global and Country Exposure Portfolios



Notes: The figure examines the risk reduction properties of the low exposure portfolios using the graphical representation in Heston and Rouwenhorst (1994) and Solnik (1975). It gives the average portfolio variance as the number of stocks in a given portfolio increases from 1 to 40, expressed as a percentage of the average variance of all individual stocks in our sample. The solid lines in Figure 1 show the diversification benefit associated with the global market portfolio (line 3), the average country portfolio (line 5), and the average industry portfolio (line 6). The dashed lines – lines (1), (2), and (4) – show the diversification benefit associated with the low exposure portfolios. The low global exposure (LGE) portfolio contains half the stocks in the sample, those with global shock exposures below the median (line 2). The *global* low country exposure (LCE) portfolio also contains half the stocks in the sample, those with country shock exposures below the median for each country (line 1). The *country* low country exposure (LCE) portfolio contains half the stocks in each country, those with country shock exposures below the median. Line (4) is computed using the equal-weighted average of the variance of country LCE portfolios across countries.

### Computational Details

Let us rewrite model (1) as:

$$\bar{R}_t = Bf_t + \epsilon_t \quad (A1)$$

where  $\bar{R}_t = R_t - \mu$  are the de-meanded returns. The variance-covariance matrix of  $\epsilon_t$ , called  $\Sigma$ , is diagonal, following assumption (4). The log-likelihood function of  $B$  and  $\Sigma$  is given by:

$$L(B, \Sigma | \bar{R}) = -\frac{T}{2} \ln |\Omega| - \frac{T}{2} \text{tr} \{ \Omega^{-1} S \} \quad (A2)$$

where  $\Omega = BB' + \Sigma$  and  $S = \sum_{t=1}^T \frac{\bar{R}_t \bar{R}_t'}{T}$ . It is hard to maximize the likelihood for (A1) by brute force when  $N$  is very large, given that the number of parameters is  $4 \times N$ . The application of the EM algorithm implies the problem considerably from a numerical point of view (see Lehmann and Modest 1985). The EM follows the intuition that if the factors were observable  $B$  could be estimated by means of OLS - equation by equation (given that  $\Sigma$  is diagonal). The EM algorithm is an iterative procedure where at each step the factors  $f_t$  are replaced by their conditional expectations given the observations and the value of the parameters obtained at the end of the previous step, and then applies standard regression tools to obtain a new estimate of the parameters until convergence. Each iteration of the algorithm is bound to increase the likelihood, so that convergence to a -possibly local- maximum is guaranteed.

Given the distributional assumptions, and assuming a flat prior on all the parameters of interest (except for the factors), the logarithm of the joint posterior distribution of  $B$ ;  $\Sigma$  and  $f$ , given the observations  $\bar{R}$ , is equal to the likelihood of the model if the factors were observable

$$\ln \text{pdf} (B, \Sigma, f | \bar{R}) \propto -\frac{T}{2} \ln |\Sigma| - \frac{1}{2} \sum_{t=1}^T (\bar{R}_t - Bf_t)' \Sigma^{-1} (\bar{R}_t - Bf_t) - \frac{1}{2} \sum_{t=1}^T f_t' f_t \quad (A3)$$

Let us call  $(B^q, \Sigma^q)$  the values of the parameters obtained at the end of the previous ( $q^{\text{th}}$ ) iteration of the algorithm. The first step of the EM algorithm involves taking the expectation (E) of (A3) with respect to the conditional distribution of  $f$  given  $(B^q, \Sigma^q)$  and  $\bar{R}$ . If we denote by  $E_q[.]$  the expectation taken with respect to the conditional distribution of  $f$  given  $(B^q, \Sigma^q)$  and  $y$ , we can write the E-step as:

$$E_q [\ln \text{pdf} (B, \Sigma, f | \bar{R})] = -\frac{T}{2} \{ \ln |\Sigma| + \text{tr} \{ \Sigma^{-1} [S - 2B (\sum_{t=1}^T \frac{E_q [f_t] \bar{R}_t'}{T}) + B (\sum_{t=1}^T \frac{E_q [f_t f_t']}{T}) B'] \} \}. \quad (A4)$$

One obtains that (see Lehman and Modest 1985):

$$\sum_{t=1}^T \frac{E_q[f_t] \bar{R}_t'}{T} = (I_K + B' \Sigma^{-1} B)^{-1} B' \Sigma^{-1} S \quad (A5)$$

$$\begin{aligned} \sum_{t=1}^T \frac{E_q[f_t f_t']}{T} &= (I_K + B' \Sigma^{-1} B)^{-1} \\ &+ (I_K + B' \Sigma^{-1} B)^{-1} B' \Sigma^{-1} S \Sigma^{-1} B (I_K + B' \Sigma^{-1} B)^{-1}, \end{aligned} \quad (A6)$$

where we drop the superscript  $q$  for convenience. The second step consists in maximizing (M) the resulting expression with respect to  $(B; \Sigma)$ . Since the joint maximization is complicated, we split the (M) step into a number of conditional maximization (CM) (see Gelman and others 1994). Each substep consists in maximizing (A4) with respect to a first set of parameters, keeping all other parameters constant at the level attained at the end of the previous substep. When  $N$  is large and restrictions (2) are present, it is convenient to divide the maximization or (A4) with respect to  $B$  into  $N$  maximizations - equation by equation. In order to see how this can be accomplished, let us note that A4 can be rewritten as (considering the case  $N = 2$  to simplify the notation):

$$\begin{aligned} E_q[\ln pdf(B, \Sigma, f | \bar{R})] &= tr\{\Sigma^{-1} [S - 2BN_1 + B \sum_{t=1}^T \frac{E_q[f_t f_t']}{T} B']\} \\ &= tr\{\Sigma^{-1} \left[ \begin{array}{cc} S^{11} & S^{12} \\ S^{12} & S^{22} \end{array} \right] - 2 \begin{bmatrix} B^1 \\ B^2 \end{bmatrix} \left[ \sum_{t=1}^T \frac{E_q[f_t] \bar{R}_{1,t}'}{T} \quad \sum_{t=1}^T \frac{E_q[f_t] \bar{R}_{2,t}'}{T} \right] + \begin{bmatrix} B^1 \\ B^2 \end{bmatrix} \sum_{t=1}^T \frac{E_q[f_t f_t']}{T} \begin{bmatrix} B^1 & B^2 \end{bmatrix} \right\} \\ &= tr \left[ \begin{array}{cc} \left( S^{11} - 2B^1 \sum_{t=1}^T \frac{E_q[f_t] \bar{R}_{1,t}'}{T} + B^1 \sum_{t=1}^T \frac{E_q[f_t f_t']}{T} B^1 \right) / \sigma_1^2 & \dots \\ \dots & \left( S^{22} - 2B^2 \sum_{t=1}^T \frac{E_q[f_t] \bar{R}_{2,t}'}{T} + B^2 \sum_{t=1}^T \frac{E_q[f_t f_t']}{T} B^2 \right) / \sigma_2^2 \end{array} \right] \end{aligned}$$

where  $B^n$  and  $\bar{R}_{n,t}$  are the  $1 \times K$  vector of loadings and the demeaned return, respectively, for stock  $n$ . Since the likelihood depends only on the trace, we can ignore the off-diagonal terms. Call  $\beta^n$  the  $K^n \times 1$  vector of non-zero parameters in  $B^n$ , and  $M^n$  the  $K^n \times K$  matrix that maps  $B^n$  into  $\beta^n$  (i.e., the matrix that “picks” the non-zero loadings):  $M^n B^n = \beta^n$ .

Then:

$$\beta_{q+1}^n = \left[ \sum_{t=1}^T \frac{M^n E_q [f_t f_t'] M^{n'}}{T} \right]^{-1} \sum_{t=1}^T \frac{M^n E_q [f_t] \bar{R}_{n,t}'}{T} \quad (A7)$$

Note that  $M^n$  simply selects the factors that are relevant to stock  $n$ . Call  $B_{q+1}$  the outcome of the  $N$  maximizations. The last CM step consists in maximizing (A4) with respect to  $\Sigma$ , for given  $B_{q+1}$ , obtaining:

$$diag(\Sigma) = diag(S - 2B_{q+1}N_1 + B_{q+1} \sum_{t=1}^T \frac{E_q [f_t f_t']}{T} B_{q+1}'). \quad (A8)$$

Convergence is reached whenever the mean squared gradient is less than  $10^{-4}$ . Note that the (M) step is essentially OLS equation by equation, as in Marsh and Pfliederer (1997). But the (E) step - where one obtains the estimates of  $f_t$  given the betas - is different from the cross-sectional GLS estimator used in Marsh and Pfliederer (1997). The former is  $E_q[f_t] = (I_K + B'\Sigma^{-1}B)^{-1} B'\Sigma^{-1}\bar{R}_t$  while the latter is  $f_{tGLS} = (B'\Sigma^{-1}B)^{-1} B'\Sigma^{-1}\bar{R}_t$ . Hence the two algorithms will not in general converge to the same estimator. The difference between the (E)-step estimator and the GLS estimator arises because the (E)-step estimator treats  $f_t$  as a random variable with prior variance  $I_K$  while the GLS estimator treats the factor(s) as unknown- but fixed- coefficients.

Next we discuss the computation of the variance covariance matrix of the estimates. The information matrix, written for ease of notation as a function of  $\Omega = BB' + \Sigma$ , is equal to:

$$E[d^2L] = -\frac{T}{2} vec(d\Omega)'(\Omega^{-1} \otimes \Omega^{-1})vec(d\Omega). \quad (A9)$$

Given that  $d\Omega = dBB' + BdB' + d\Sigma$ , and that we know the mapping between  $\beta$  (the non-zero parameters in  $B$ ) and  $B$ , we can use formula (A9) to compute the  $(i^{th}, j^{th})$  element of the information matrix with respect to the unrestricted elements of  $B$  and  $\Sigma$ . The inverse of the information matrix is the asymptotic variance-covariance matrix of our estimates.

### The Data

This appendix explains construction of the data for which we estimate the model and review its country and industry composition. The purpose of this exercise is to show that our data provide a reasonable representation of international equity markets, so that our model delivers meaningful estimates of country- and industry-specific shocks.

We derive our data from Brooks and Del Negro (2004, BD hereafter) who cover monthly total US dollar denominated stock returns and market capitalizations for 9,769 stocks in 42 developed and emerging markets from January 1985 to February 2002, downloaded from Datastream International. Their sample includes all constituent stocks in the Datastream Global Market Indices for these countries as of March 2002 and is augmented with a list of active and inactive stocks for each country derived from Worldscope. This step ensures that the data include stocks that become inactive over time, due to bankruptcy or merger for example. The data include dummy variables that identify the country affiliation of each stock. It also includes 110 industry dummies, which measure industry affiliation according to the most disaggregated industry definition provided by Datastream International, level 6. We balance the data at two different points in time, in order to check the robustness of our results to changes in data coverage. The first sample consists of all stocks with continuous returns for the entire sample period (1,965 stocks in 21 countries). The second begins in January 1990 and contains stocks with continuous returns through the remainder of the sample (3,939 stocks in 33 countries).

Table B-2 gives a snapshot of the data as of December 2000. For each of the balanced panels it provides by country the number of stocks, the market capitalization in billions of U.S. dollars, and the capitalization share in percent of the sample total. It gives the same breakdown by industry, using the level 3 industry classification, which groups level 6 industries into 10 broad sectors. Table A1 provides the same breakdown for the original data by BD, to put into perspective how balancing affects coverage. Finally, it provides the same decomposition for the known universe of stocks in each country, according to the Global Stock Markets Factbook (2001), to give an assessment of the degree to which the various datasets provide a realistic representation of international stock markets. In December 2000, the smaller of our balanced panels has a total market capitalization of US\$15,334 billion, while this number amounts to US\$21,834 billion for the balanced panel that begins in January 1990. This compares to a total market capitalization of 31,486 billion U.S. dollars in the original BD data, almost 99 percent of total market capitalization in our 42 countries, according to the Factbook. While the original data thus provide fairly exhaustive coverage in capitalization terms, the balanced data are clearly more limited in this regard. However, although balancing shrinks coverage within countries and industries, the balanced data remain fairly representative in their coverage across countries and industries. For example, in the smaller of our balanced samples, the U.S. makes up 52 percent of total market capitalization, marginally above the 50 percent in the original BD data and the same order of magnitude as the 47 percent in the Factbook. The same goes for other advanced countries, whose relative importance in the balanced data is in line with the Factbook. The major omission from the balanced data are emerging markets. With the exception of Korea, Malaysia and South Africa, for example, the smaller of our balanced samples contains

no emerging markets. Coverage of emerging markets is somewhat better in the balanced sample that begins in January 1990. In terms of coverage across industries, the balanced data are comparable to the original BD dataset. The largest level 3 sector in their sample is made of financial sector stocks, which account for almost 24 percent of their sample in market capitalization terms. This is also true for the balanced data, where financials make up 23 and 24 percent of market capitalization, respectively. While balancing the original BD data thus clearly reduces coverage, the balanced data still provide a reasonable representation of international stock markets, in terms of the relative importance of countries and industries.

Table A.1. Descriptive Statistics for Equal-Weighted Excess Return Across Stocks

**Panel A. Balanced Sample 1985 - 2002**

|                    | Full Sample | Balanced Sample |
|--------------------|-------------|-----------------|
| Time-Series Mean   | 0.3         | 0.34            |
| Standard Deviation | 4.32        | 4.38            |

**Panel B. Balanced Sample 1990 - 2002**

|                    | Full Sample | Balanced Sample |
|--------------------|-------------|-----------------|
| Time-Series Mean   | -0.18       | -0.13           |
| Standard Deviation | 4.18        | 4.02            |

*Notes:* The table shows that the balanced samples used for estimation are comparable to the original data in Brooks and Del Negro (2004), in terms of the time-series means and standard deviations of the equal-weighted excess return across all stocks. In particular, there is little indication of a systematic bias in the standard deviation of excess returns, important because we focus on the covariance structure of excess returns. The original data from Brooks and Del Negro (2004) cover 9,679 stocks in 42 countries from January 1985 to February 2002. The Balanced Sample 1985 – 2002 consists of stocks with continuous returns for the entire sample and contains 1,965 stocks in 21 countries. The Balanced Sample 1990 - 2002 begins in January 1990 and contains stocks with continuous returns for the rest of the sample. It covers 3,939 stocks in 33 countries.



Table A.2. Comparing Our Balanced Data to Brooks and Del Negro (2004) and Reality

|                             | Balanced Sample<br>1985 - 2002 |                       |              | Balanced Sample<br>1990 - 2002 |                       |              | Full Sample     |                       |              | S&P Stock Market<br>Factbook 2001 |                       |              |
|-----------------------------|--------------------------------|-----------------------|--------------|--------------------------------|-----------------------|--------------|-----------------|-----------------------|--------------|-----------------------------------|-----------------------|--------------|
|                             | No. of<br>Firms                | Mkt Cap<br>(Bill. \$) | Share<br>(%) | No. of<br>Firms                | Mkt Cap<br>(Bill. \$) | Share<br>(%) | No. of<br>Firms | Mkt Cap<br>(Bill. \$) | Share<br>(%) | No. of<br>Firms                   | Mkt Cap<br>(Bill. \$) | Share<br>(%) |
| World                       | 1965                           | 15334                 | 100          | 3939                           | 21834                 | 100          | 8969            | 31486                 | 100          | 37188                             | 31852                 | 100          |
| Argentina                   | 0                              | 0                     | 0.00         | 9                              | 5                     | 0.02         | 73              | 51                    | 0.16         | 127                               | 166                   | 0.52         |
| Australia                   | 41                             | 159                   | 1.04         | 81                             | 196                   | 0.90         | 207             | 323                   | 1.02         | 1330                              | 373                   | 1.17         |
| Austria                     | 9                              | 4                     | 0.03         | 28                             | 12                    | 0.06         | 82              | 33                    | 0.11         | 97                                | 30                    | 0.09         |
| Belgium                     | 26                             | 59                    | 0.39         | 59                             | 73                    | 0.33         | 127             | 131                   | 0.41         | 174                               | 182                   | 0.57         |
| Brazil                      | 0                              | 0                     | 0.00         | 0                              | 0                     | 0.00         | 189             | 102                   | 0.32         | 459                               | 226                   | 0.71         |
| Canada                      | 89                             | 418                   | 2.73         | 198                            | 564                   | 2.58         | 424             | 792                   | 2.52         | 3977                              | 841                   | 2.64         |
| Chile                       | 0                              | 0                     | 0.00         | 50                             | 33                    | 0.15         | 92              | 54                    | 0.17         | 258                               | 60                    | 0.19         |
| China                       | 0                              | 0                     | 0.00         | 0                              | 0                     | 0.00         | 152             | 48                    | 0.15         | 1086                              | 581                   | 1.82         |
| Colombia                    | 0                              | 0                     | 0.00         | 0                              | 0                     | 0.00         | 31              | 3                     | 0.01         | 126                               | 10                    | 0.03         |
| Czech Republic              | 0                              | 0                     | 0.00         | 0                              | 0                     | 0.00         | 59              | 13                    | 0.04         | 131                               | 11                    | 0.03         |
| Denmark                     | 20                             | 51                    | 0.33         | 52                             | 57                    | 0.26         | 109             | 116                   | 0.37         | 225                               | 108                   | 0.34         |
| Finland                     | 0                              | 0                     | 0.00         | 27                             | 196                   | 0.90         | 106             | 250                   | 0.79         | 154                               | 294                   | 0.92         |
| France                      | 54                             | 505                   | 3.30         | 167                            | 963                   | 4.41         | 363             | 1438                  | 4.57         | 808                               | 1447                  | 4.54         |
| Germany                     | 100                            | 696                   | 4.54         | 181                            | 773                   | 3.54         | 404             | 1043                  | 3.31         | 1022                              | 1270                  | 3.99         |
| Greece                      | 0                              | 0                     | 0.00         | 31                             | 41                    | 0.19         | 109             | 86                    | 0.27         | 329                               | 111                   | 0.35         |
| Hong Kong, SAR              | 51                             | 378                   | 2.46         | 96                             | 442                   | 2.02         | 208             | 669                   | 2.13         | 779                               | 623                   | 1.96         |
| India                       | 0                              | 0                     | 0.00         | 0                              | 0                     | 0.00         | 153             | 110                   | 0.35         | 5937                              | 148                   | 0.46         |
| Indonesia                   | 0                              | 0                     | 0.00         | 0                              | 0                     | 0.00         | 77              | 6                     | 0.02         | 290                               | 27                    | 0.08         |
| Ireland                     | 14                             | 35                    | 0.23         | 45                             | 59                    | 0.27         | 79              | 81                    | 0.26         | 76                                | 82                    | 0.26         |
| Italy                       | 31                             | 258                   | 1.68         | 104                            | 406                   | 1.86         | 219             | 750                   | 2.38         | 291                               | 768                   | 2.41         |
| Japan                       | 529                            | 2002                  | 13.06        | 783                            | 2516                  | 11.52        | 1189            | 3052                  | 9.69         | 2561                              | 3157                  | 9.91         |
| Korea                       | 45                             | 52                    | 0.34         | 61                             | 82                    | 0.38         | 168             | 136                   | 0.43         | 1308                              | 172                   | 0.54         |
| Luxembourg                  | 0                              | 0                     | 0.00         | 0                              | 0                     | 0.00         | 31              | 41                    | 0.13         | 54                                | 34                    | 0.11         |
| Malaysia                    | 18                             | 19                    | 0.12         | 98                             | 53                    | 0.24         | 168             | 97                    | 0.31         | 795                               | 117                   | 0.37         |
| Mexico                      | 0                              | 0                     | 0.00         | 19                             | 24                    | 0.11         | 117             | 106                   | 0.34         | 179                               | 125                   | 0.39         |
| Netherlands                 | 56                             | 412                   | 2.68         | 87                             | 437                   | 2.00         | 183             | 688                   | 2.18         | 234                               | 640                   | 2.01         |
| New Zealand                 | 0                              | 0                     | 0.00         | 27                             | 5                     | 0.02         | 69              | 23                    | 0.07         | 144                               | 19                    | 0.06         |
| Norway                      | 16                             | 21                    | 0.14         | 27                             | 27                    | 0.12         | 103             | 69                    | 0.22         | 191                               | 65                    | 0.20         |
| Peru                        | 0                              | 0                     | 0.00         | 0                              | 0                     | 0.00         | 60              | 4                     | 0.01         | 230                               | 11                    | 0.03         |
| Philippines                 | 0                              | 0                     | 0.00         | 16                             | 13                    | 0.06         | 71              | 24                    | 0.08         | 230                               | 52                    | 0.16         |
| Poland                      | 0                              | 0                     | 0.00         | 0                              | 0                     | 0.00         | 60              | 18                    | 0.06         | 225                               | 31                    | 0.10         |
| Portugal                    | 0                              | 0                     | 0.00         | 21                             | 21                    | 0.10         | 59              | 76                    | 0.24         | 109                               | 61                    | 0.19         |
| Singapore                   | 54                             | 61                    | 0.40         | 77                             | 84                    | 0.38         | 164             | 164                   | 0.52         | 418                               | 153                   | 0.48         |
| South Africa                | 23                             | 36                    | 0.24         | 46                             | 70                    | 0.32         | 189             | 153                   | 0.49         | 616                               | 205                   | 0.64         |
| Spain                       | 0                              | 0                     | 0.00         | 73                             | 323                   | 1.48         | 135             | 387                   | 1.23         | 1019                              | 504                   | 1.58         |
| Sweden                      | 16                             | 172                   | 1.12         | 50                             | 198                   | 0.91         | 145             | 305                   | 0.97         | 292                               | 328                   | 1.03         |
| Switzerland                 | 42                             | 579                   | 3.78         | 105                            | 710                   | 3.25         | 185             | 830                   | 2.64         | 252                               | 792                   | 2.49         |
| Taiwan, Province of China   | 0                              | 0                     | 0.00         | 44                             | 87                    | 0.40         | 165             | 270                   | 0.86         | 531                               | 248                   | 0.78         |
| Thailand                    | 0                              | 0                     | 0.00         | 35                             | 13                    | 0.06         | 78              | 29                    | 0.09         | 381                               | 29                    | 0.09         |
| Turkey                      | 0                              | 0                     | 0.00         | 9                              | 10                    | 0.04         | 72              | 223                   | 0.71         | 315                               | 70                    | 0.22         |
| United Kingdom              | 280                            | 1440                  | 9.39         | 530                            | 2121                  | 9.71         | 986             | 3114                  | 9.89         | 1904                              | 2577                  | 8.09         |
| United States               | 451                            | 7977                  | 52.02        | 703                            | 11220                 | 51.39        | 1309            | 15577                 | 49.47        | 7524                              | 15104                 | 47.42        |
| Basic Industries            | 287                            | 714                   | 4.65         | 558                            | 998                   | 4.57         | 1059            | 1254                  | 3.98         |                                   |                       |              |
| General Industries          | 320                            | 1876                  | 12.23        | 581                            | 2208                  | 10.11        | 1140            | 2838                  | 9.01         |                                   |                       |              |
| Cyclical Consumer Goods     | 119                            | 557                   | 3.63         | 215                            | 689                   | 3.16         | 457             | 853                   | 2.71         |                                   |                       |              |
| Non-Cyclical Consumer Goods | 226                            | 3124                  | 20.37        | 422                            | 3730                  | 17.08        | 990             | 4558                  | 14.48        |                                   |                       |              |
| Cyclical Services           | 306                            | 1610                  | 10.5         | 594                            | 2148                  | 9.84         | 1455            | 3724                  | 11.83        |                                   |                       |              |
| Non-Cyclical Services       | 42                             | 814                   | 5.31         | 102                            | 1611                  | 7.38         | 376             | 2891                  | 9.18         |                                   |                       |              |
| Utilities                   | 92                             | 525                   | 3.43         | 160                            | 730                   | 3.34         | 345             | 1107                  | 3.52         |                                   |                       |              |
| Information Technology      | 73                             | 1522                  | 9.92         | 178                            | 2956                  | 13.54        | 815             | 4933                  | 15.67        |                                   |                       |              |
| Financials                  | 406                            | 3443                  | 22.45        | 935                            | 5236                  | 23.98        | 1925            | 7414                  | 23.55        |                                   |                       |              |
| Resource Industries         | 94                             | 1150                  | 7.5          | 194                            | 1527                  | 6.99         | 407             | 1915                  | 6.08         |                                   |                       |              |

*Notes:* The table provides by country and by industry (Level 6) the number of stocks, the market capitalization in billions of U.S. dollars, and the capitalization share in percent of the sample total as of December 2000 for each balanced sample (1985–2002 and 1990–2002), for the full sample of Brooks and Del Negro (2004), and the known universe of stocks from the S&P Stock Market Factbook (2001).

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