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Is There a Novelty Premium on New Financial Instruments? The Argentine Experience with GDP-Indexed Warrants

Alejo Costa, Marcos Chamon, and
Luca Antonio Ricci

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Prepared by Alejo Costa, Marcos Chamon, and Luca Antonio Ricci[1]

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Abstract

This paper examines the Argentine experience with GDP-indexed warrants in order to gauge the existence of a novelty premium on new financial instruments. It develops a Monte Carlo pricing exercise to calculate the expected net present value of payments, on the basis of various forecast assumptions. The results show that the residual premium paid by these warrants over standard bonds declined significantly by about 600 basis points between December 2005 and July 2007. This suggests that financial innovation may be associated with premia, which decay reasonably fast.

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I. INTRODUCTION

The purpose of this paper is to shed light on the cost of financial innovation in international financial markets, by analyzing the introduction of a new instrument of sizable issuance: the Argentine GDP-indexed warrants. New financial instruments can play a key role in improving international risk sharing. Indeed, the emerging market crises of the 1990s/early 2000s have stimulated—both among policymakers and the academic circle—an ongoing reflection on how international financial market development could help prevent financial crises, or at least attenuate their impact. Substantial emphasis has been placed on the need to reduce foreign currency denomination of external liabilities (Eichengreen and Hausmann, 1999; Eichengreen, Hausmann and Panizza, 2005), which played a central role in those crises. The large build-up in reserves in the recent tranquil years has generated renewed interest in the effectiveness of self-insurance through reserve accumulation (Jeanne and Ranciere, 2006; Aizenman and Lee, 2007). Some observers have also suggested a role for contingent financial products, whose payment would depend on the realization of, for example, commodity prices (Caballero 2000; Becker et al 2007), GDP (Borensztein et al, 2004; Borenszstein and Mauro, 2004; Griffith-Jones and Sharma, 2006) or indices of market volatility (Caballero and Panageas, 2004).

Since the last round of crises, we have indeed observed a gradual shift towards local currency denomination of liabilities and a substantial increase in reserve holdings in many emerging market countries. However, by and large there have been no developments of radically new instruments, with the exception of Argentina's GDP-linked warrants. Several reasons may explain why financial innovation is limited (for a survey, see Shiller, 1993), such as the presence of innovation costs combined with the limited ability to appropriate the corresponding benefits (Schroth, 2006), or the potentially limited liquidity of the prospective markets (e.g. Brorsen and Fofana, 2001, on failures in the agricultural futures market). Overall, these arguments would suggest the presence of a "novelty premium" (i.e. an interest premium with respect to similar existing financial products) even if the new market is sufficiently liquid. There is, however, little empirical analysis on financial innovation, especially at the international level (see Frame and White, 2004, for a survey).

The contribution of our paper is to help fill that gap, by quantifying the evolution of a (theoretically estimated) novelty premium following the introduction of Argentina's GDP-linked warrants. The Argentine experience with the GDP-indexed warrants provides an interesting example for this analysis. Similar bonds had been issued before in Costa Rica, Bulgaria, Bosnia and Herzegovina, and Singapore, but in more moderate amounts.[2] The

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[2] In the case of Bulgaria the bond was callable. In the example of Bosnia and Herzegovina, trading activity has been limited and prices are rarely published. The government of Singapore issued shares—which are neither tradable nor transferable—with payments indexed to GDP, as part of a redistribution scheme to compensate for a Goods and Service Tax; a second series was issued targeting mainly lower-income households.

dimension of the Argentine case, in terms of volume and market liquidity, makes it a more suitable experience to evaluate how these new instruments are received by the market.

In order to quantify the premium over traditional securities (e.g., "plain-vanilla" bonds), we build a pricing model for the theoretical value of a warrant. There are four key elements in this exercise: a discount rate and three stochastic processes: for GDP, inflation, and the exchange rate (whose mean and volatility are based on market expectations and historical evidence, respectively). We simulate 5,000 paths for the (jointly-distributed) stochastic processes, computing the implied payments by the GDP-warrant in each path. Consistent with market valuations, our model indicates a sharp increase in the value of the warrants following its introduction. Much of that increase can be traced to the Argentina's strong recovery—in light of the high sensitivity of the price to economic growth in the initial years of the warrant—but also to a decline in the novelty premium.

The novelty premium is then estimated by comparing our derived theoretical price with the market's valuation of the warrant. In all scenarios explored, the theoretical price is always higher than the market price. By reversing the calculation, we evaluate the path for the discount rate that would make the theoretical price match the market price at any point in time (for given stochastic processes for GDP, inflation, and the exchange rate). As we will argue later, the gap between such discount rate and the interest rate on traditional Argentine bonds could at least in part be attributed to the novelty of the instrument.

Our findings point to a significant novelty premium, which declined rapidly by about 600 basis points during the first year and a half of trading. In the first month of trading (December 2005), the premium was estimated in the order of 800-1200 basis points, declining to about 200-550 basis points by July 2007 (the wide range of these estimates reflects the sensitivity of the valuation to the underlying assumptions, particularly those related to the growth and volatility of GDP). These results confirm the presence of substantial first-mover costs, which can help explain the limited financial innovation observed. Almost two thirds of the decline in the premium occurred within the first three months, a result that is consistent with the evidence offered by Schroth (2006) for equity-linked and derivative securities. It is important to bear in mind that the introduction of the GDP-warrants may have lowered the overall default risk of the country, thus reducing the interest rate paid on standard bonds (note that this would not affect our estimate of the novelty premium, as the latter is derived as a differential).

The remainder of the paper is organized as follows. Section II describes the introduction of the GDP warrants at the time of the debt exchange and the details of this security. Section III depicts the market response. Section IV presents our valuation method. Section V illustrates the baseline results. Section VI considers alternative scenarios. Finally, Section VII concludes.

II. THE ARGENTINE GDP-WARRANTS

Following three years of negative growth (Table 1), Argentina declared a moratorium on its debt at the end of 2001, and in early 2002 the "convertibility regime"—a currency board arrangement adopted in 1991—was abandoned.[3] The run-up to the default was a period of economic, financial, and social instability, marked by bank deposit freezes. After the default and the banking crisis erupted, output collapsed and inflation soared, while the debt-to-GDP ratio jumped from 45% to 139%, in large part because of the depreciation of the peso. In the following years, Argentina experienced an impressive recovery, whose extent was unanticipated by both sides of the debt negotiation and by financial markets in general.

Table 1. Main Economic Indicators

	1999	2000	2001	2002	2003	2004	2005	2006
The Economy								
Real GDP (in billions of 1993 pesos)	278.4	276.2	264.0	235.2	256.0	279.1	304.8	330.6
GDP growth (in percent)	-3.4	-0.8	-4.4	-10.9	8.8	9.0	9.2	8.5
CPI inflation (in percent)	-1.8	-0.7	-1.5	40.9	3.7	6.1	12.3	9.8
Unemployment rate (in percent)	14.5	15.4	16.4	21.5	14.5	12.1	10.1	8.7
Balance of Payments (In billions of US$)								
Current Account	-12.0	-9.0	-3.8	8.8	8.1	3.2	5.7	8.0
Capital and Financial Account	13.7	8.7	-5.4	-11.4	-3.2	1.5	3.1	-6.1
Public Finance								
Primary Fiscal Balance to GDP ratio (in percent)	0.3	1.0	0.5	0.7	2.2	3.3	2.6	2.8
Public Debt to GDP ratio (in percent)	43.0	45.0	53.8	129.7	139.4	127.0	84.8	64.0

Source: Republic of Argentina, Prospectus Supplement and Ministry of Economy and Production, and INDEC

In June 2003, the government started a debt renegotiation process with bond holders, which lasted two years. By the end of 2003, the stock of debt in default was US$101.2 billion, corresponding to 48.5% of the total public debt.[4] In the proposal, investors were offered new bonds in exchange for the defaulted debt and unpaid interest up to December 2001, lowering the value of debt eligible for the exchange to US$81.8 billion. The exchange was finally concluded in June 10, 2005 with a creditor participation of 76%. The participating creditors swapped their claims, with combined face-value of about US$62 billion, for a new set of bonds with face-value of US$35.3 billion. The new bonds came in three varieties: a par, a discount, and a quasipar bond (all of which had pre-set interest rates, varying only with the time horizon). Each variety was issued in four different currencies: Argentine Peso, U.S. Dollar, Euro, and Yen.

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[3] For a detailed discussion of Argentina's debt crisis and restructuring experience, please refer to chapter 8 of Sturzenegger and Zettelmeyer (2006).

[4] December 31st 2003 was the day chosen by Argentine authorities to define the original principal amount of new securities in the renegotiation process. New bonds would start paying interests on that day, and the accrued interest until the end of the renegotiation would be paid in cash to bond holders at the time of settlement.

Argentina placed great emphasis on debt sustainability at the time of the debt exchange offer. A GDP-linked security, making annual payments contingent on Argentina's growth performance, was attached to each restructured bond.[5] This GDP-linked security not only raised the expected value of the offer, but also reduced the likelihood of a crisis and improved debt sustainability (by shifting the fiscal burden of debt obligation to the more favorable growth scenarios, and thus acting as an automatic-stabilizer). Weak fiscal sustainability was indeed perceived by observers as one of the main reasons for the crisis.

In November 2005, these GDP-linked security became detachable, and could be traded separately from their underlying bond. Overall, five types of GDP-linked securities were issued (based on the denomination of the original defaulted bonds): one in local currency, i.e., Argentine pesos (under Argentine law); two in dollars (one under Argentine law, the other one under New York law); one in Euros (under British law); and the last one, with a low volume, in Yen (under Japanese law) (see Figure 1).

Figure 1. GDP Warrants - Currency Distribution
(In percentage of 62bn Total*)

Ar$ (Arg Law)
47%

Yen (Japan Law)
1%

Euro (British Law)
24%

US$ (NY Law)
23%

US$ (Arg Law)
5%
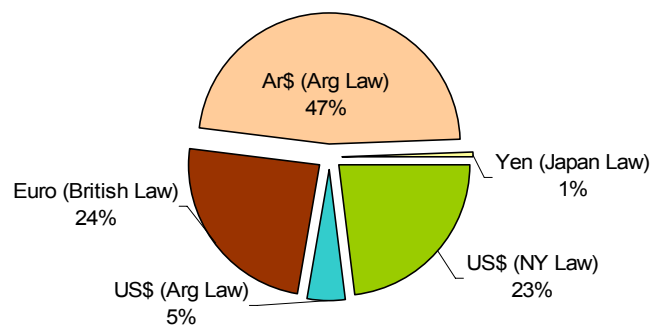
Source: Ministry of Economy, Rep. of Argentina
*Measured in Dec. 2003 Exchange Rates

The Argentine GDP-indexed warrant provides the holder with a payment if the following three conditions are met: (1) actual real GDP must exceed baseline real GDP in the reference year;[6][7] (2) growth in actual real GDP must exceed growth in baseline real GDP in the reference year; (3) the cumulative amount of past payments should not exceed 0.48 per unit of security (in its corresponding currency). If all these conditions are met, the total payment on all warrants is a fraction of the excess GDP in the reference year (i.e. the difference

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[5] For a general description of GDP-indexed bonds, see Borenszstein and Mauro (2004)

[6] The reference year is the year before the one in which payments occur. It is also the year on the basis of which payments are calculated.

[7] The baseline GDP over the horizon of the warrant (until 2034) is implicitly defined by the baseline GDP growth rate, set by the authorities over that horizon. Such a growth rate gradually converges from 4.3 percent in 2005 to its long run level (of 3 percent) in 2014 (see Figure 2).

between actual GDP and baseline GDP) [8]. Payments are made on December 15 of the year following the reference year (the first payment was made in 2006).[9] Figure 2 shows two paths for both the level and growth of GDP: the first path represents the baseline assumptions chosen by the Argentine authorities; the second path shows both realized values (for 2005 and 2006) and the forecasts entering our calculations (after 2007; see Section IV.2 for the corresponding assumptions).

Figure 2. Base Level GDP and GDP Growth

Source: Latin American Consensus Forecast and Prospectus

Source: Latin American Consensus Forecast and Prospectus
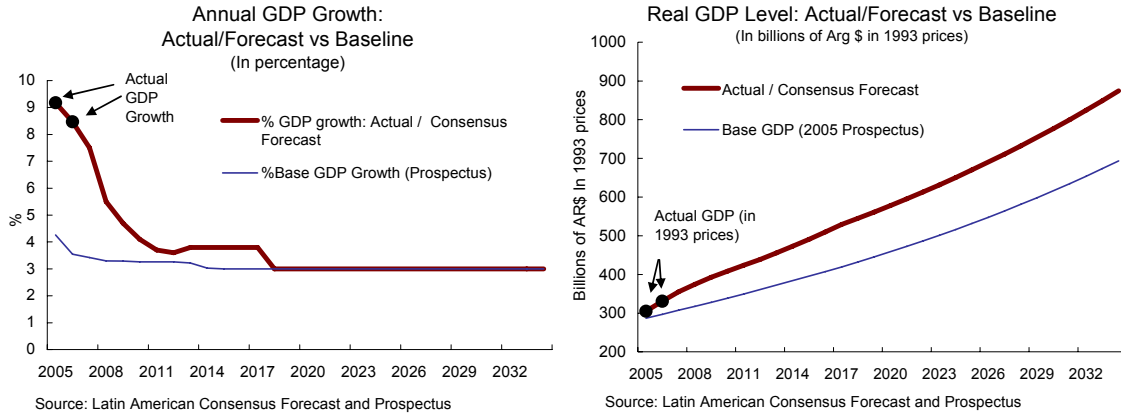
The formula for the payment ($C(j)_{t+1}$) per unit of currency j in period $t+1$ (with respect to reference year t) is:

$$C(j)_{t+1} = \frac{0.05(Y_t - y_t^b D_t)}{ER_t(AR\$/j)} U(j) I_1(y_t > y_t^b) I_2(\Delta y_t > \Delta y_t^b) I_3\left(\sum_{\tau=1}^{t} C(j)_\tau < 0.48\right) \quad (1)$$

Where:

$C(j)_{t+1}$: Coupon payment at time $t+1$ of the warrant in currency j.

j: Type of currency denomination: Argentine peso, U.S. dollar, Euro, or Yen.

Y_t: Nominal GDP in period t, in current prices.

y_t^b: Real GDP in baseline case (from the Debt Exchange Prospectus) for period t.

D_t: GDP price deflator in period t, base 1993=1.

ER_t: Exchange rate in Arg\$ per unit of currency j in period t.

$U(j)$: Unit of currency coefficient (see below).

y_t: Real GDP in period t, in 1993 prices.

Δ: Represents the growth rate.

The three $I(.)$ terms are indicator functions for the three conditions that must hold to trigger a payment. They have values of 1 when the inequality holds, and zero otherwise. The constant $U(j)$ is the *unit of currency coefficient* and "represents the proportion that one GDP-linked

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[8] The fraction is 3.8 percent of excess GDP. It would have been 5 percent if participation in the debt exchange had been 100 percent. Since participation was 76 percent, the fraction is 76 percent of 5 percent, i.e. 3.8 percent.

[9] Trades are ex-dividend starting December 12, so the price should reflect the payment on that day.

security with a notional amount of one unit of currency bears to the aggregate eligible amount of all eligible securities outstanding" (Debt Exchange Prospectus) by December 24, 2004 (see Table 2). This constant *U(j)* implicitly determines the share of "excess GDP" that the holder of one unit of each security of currency *j* is entitled to. The total amount of eligible securities at the time of the exchange was 81.8 billion dollars. In order to allocate the securities among the participating creditors, each GDP-linked security received its corresponding share, with the different currencies being converted into the same currency based on the exchange rate at the time of the offer. As only 76 percent of creditors participated in the exchange, the sum of all notional amounts of all warrants adds up to US$62bn. The unit of currency *U(j)* (for billions of currency *j*) is hence defined as 1/81.8 multiplied by the inverse of the exchange rate in terms of currency *j* per dollar. The sum of these units multiplied by their corresponding outstanding amounts is equal to the ratio of US$62bn to US$ 81.8bn, i.e. 76 percent.

Table 2. GDP-linked Unit: Outstanding Value, Units of Currency and Payments
(In units of each currency)

Currency	Outstanding value in millions of original currency	Unit of Currency Coefficient ($U(j)$) (for billions of currency j)	First coupon in millions of original currency	First coupon as a percent of outstanding value
US$ (NY Law)	14,386	1/81.8	89.84	0.6245
US$ (Arg Law)	2,925	1/81.8	18.27	0.6245
Euro (British Law)	11,808	(1/81.8)*(1/0.7945)	78.15	0.6618
Ar$ (Arg Law)	86,154	(1/81.8)*(1/2.9175)	559.22	0.6491
Yen (Japan Law)	38,369	(1/81.8)*(1/107.42292)	261.20	0.6808
Total (In Dollars*)	62,318			

Source: Republic of Argentina, Ministry of Economy and Production
*Using Exchange Rates by Dec 31st 2003

There are a number of features of the payment formula that are worth emphasizing:

- *Importance of level of real GDP:* Once the three payments conditions are met, the payment is based on the level of GDP relative to a baseline value, and not on the growth rate per se (i.e. current growth only affects the payment through its effect on the level of GDP). The high growth experienced following the debt exchange has substantially raised the theoretical value of the security, by placing the level of GDP well above baseline GDP and creating the potential for large payments. Future revisions of GDP or changes to the base year can change the value of the GDP-linked security. These are typically associated with upward revisions in the level of GDP, which would automatically widen the gap between GDP and its baseline value, thus increasing the value of GDP-linked security.
- *Discontinuity of the growth effect:* Since the payments only occur when growth is above a threshold (about 3 percent), regardless of the level of GDP, there is an important discontinuity in the effect of growth on payments. For example, a growth rate of 3.1 percent could imply a fairly substantial payment, whereas a growth rate of 2.9 percent will imply none. This discontinuity is perceived by investors to contribute to moral hazard concerns related to data accuracy.

- *Role of growth volatility:* Volatility affects the value of the GDP-linked security through two different channels:
  - *Effect of volatility on payment triggers:* When the expected level and growth rates of GDP are sufficiently above the baseline, higher growth volatility decreases the value of the GDP-linked security (since it would increase the chances of a growth rate below the threshold for payment). Conversely, when they are sufficiently below their baseline, higher volatility increases the value of the security (since it increases the likelihood of the level and growth rate of GDP surpassing the threshold for payments). When growth rates are normally distributed, this channel weakens as GDP growth rates and levels get further from their baseline, because the channel would be operating at the tails of the distribution, where density is small. The channel includes implicitly two effects: one on the level and one on growth rates.
  - *Effect of volatility on skewness of GDP:* When growth rates are normally distributed, the level of GDP is log-normal, and hence a higher variance will imply higher right-skewness for GDP (thicker density towards the right tail of the GDP distribution), a property of the log-normal distribution. Higher right skewness implies that, when the level of GDP is close to the trigger level, payments can be expected to be higher on average, because higher growth rates of GDP are associated with a higher distribution density.. In this case, a higher variance would be associated with a higher price (similarly to the case of a standard call option).
  - These two channels have opposite directions for levels close to the baseline imposed by the contract. And both channels will be more powerful the closest the path is to the baseline path (as higher variance will imply more zero-cash flows from the first channel, but also more right skewness from the second). For paths that are sufficiently far from the trigger rate and level, the second effect would tend to dominate. In the range of exercises we performed, the first channel generally dominates, so that a higher variance would typically imply a lower value.
- *Role of inflation:* There is a one-year lag between the reference period for the determination of the payment in nominal terms (for all currencies) and the time it is actually paid. This generates two effects:
  - First, the lag reduces the real value of the securities in the respective currency, and the reduction would be stronger for securities denominated in higher inflation currencies; indeed the lag acts as an inflation tax. When compared in the same currency, the securities denominated in higher inflation currencies would loose value relative to the others, as long as higher inflation currencies do not appreciate.
  - Second, to the extent that the sum of total repayments is capped at 48 cents of the respective currency, higher inflation can further reduce the real value of the payments if that cap becomes binding (it would also reduce the foreign currency

value of the securities denominated in higher inflation currencies relative to the value—in the same currency—of securities denominated in lower inflation currencies).

- *Role of real exchange rate:* The excess GDP is computed in constant peso terms. If there is a real appreciation of the peso, the value of the constant peso payment in foreign currency terms will increase (and vice-versa if there is a real depreciation). Consequently, in the respective currency, the value of the foreign currency warrants will increase relative to the value of the Peso warrant. When measured in the same currency, as a first approximation, the payment and value of warrants denominated in different currencies would be unaffected by a permanent change in the real exchange rate, if it were not for the cap on nominal payments cap and for the one-year lag between the determination of the payment and the actual payment (similarly to what is discussed above).

III. MARKET RESPONSE

On November 30th 2005—180 days after the settlement of the debt exchange—the GDP-linked securities became detachable, leading to a one time adjustment in the prices of the underlying bonds and to a market for the warrants. Although no official market was available until that time, there had already been some pricing indications. At the time of the exchange offer, major investment banks were suggesting a price for the US$ warrants of about US$2 per 100 dollar of notional value (as common practice, all prices and values will continue to be reported per 100 units of notional value of own currency). In the few months prior to detachment (but after the conclusion of the debt exchange on June 10th), there was a small and illiquid futures market, with prices ranging from US$3 to US$5.85.[10] At the time of detachment, the US$ warrant was trading at US$4.25. Market analysts continued to recommend a price about 50 percent lower than their estimated theoretical value (close to US$8), due to the instrument's novelty and other issues.

Since then, the market price of these instruments has more than doubled due to improved near-term growth outlooks and what appear to be pricing corrections. In particular, a sizable contribution came from Argentina's strong recovery, as the theoretical value is very sensitive to growth in the first years. Asset management agencies recommended long positions on the GDP Warrants until June of 2007, when news about an energy crisis induced a reduction in growth forecasts: after reaching a peak of US$15.82 in early June 2007, the US$ warrant experienced a decline (Figure 3). By mid-January 2008, it was trading at around US$12.50. A substantial increase in the Argentine risk spread (from about 300 basis points in June 2007 to about 450 in January 2008) and the warrant payment for December 2007 contributed to this decline.

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[10] Data on the futures market is from Merrill Lynch.

Figure 3. GDP Warrants - Market Price
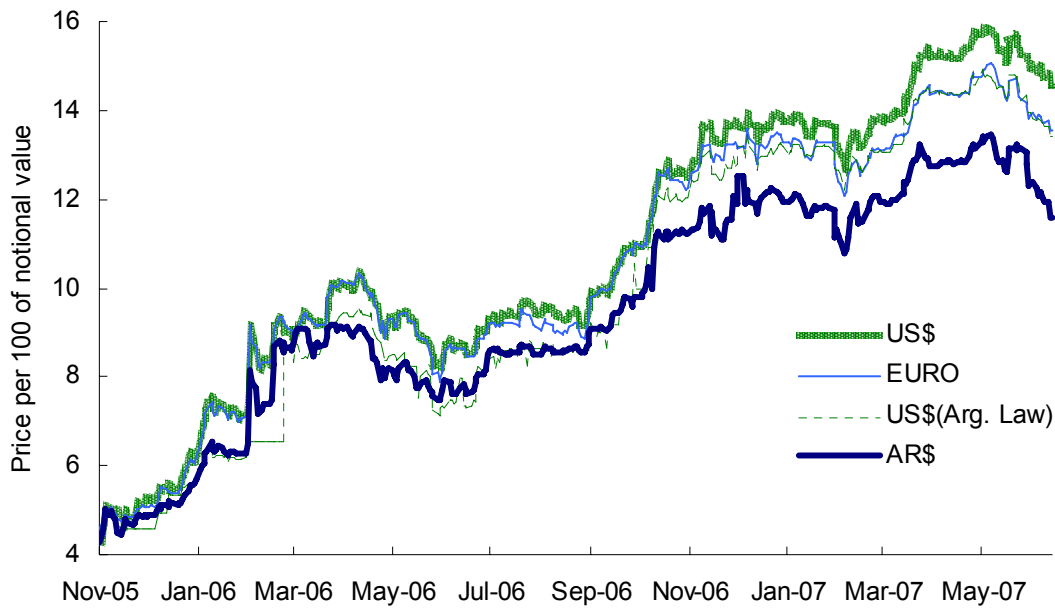
Source: Bloomberg

The increase in value due to the recovery of the economy has also imposed a higher fiscal cost for the government. In 2005, with the economy growing 9.2 percent, the amount of payments associated with the different GDP-linked securities was US$387 million, or 0.22 percent of GDP. The particular structure of these instruments—with payments based on the level of real GDP rather than its growth rate—implies that even though the growth rate in 2006 was lower than in 2005 (8.5 percent), payments in 2007 more than doubled compared to those in 2006, reaching US$811 millions or 0.33 percent of GDP. While rising payments do not compromise debt sustainability, they do impose a higher burden on the Treasury than they would in a counterfactual scenario where payments depended exclusively on the reference year's growth performance (as opposed to the level of GDP). At the same time, as a result of the increasing market valuation for the warrant, the ex-post market value of Argentina's debt exchange offer has increased substantially.

IV. THEORETICAL VALUE

Different methods have been developed to price GDP-linked securities (see, for example, Chamon and Mauro, 2006). [11] In this note, we use the one most commonly used among

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[11]The properties of the instrument are close to those of "path-dependent" options, and hence numerical techniques are necessary for its valuation, as standard calculations based on the formula obtained by Black and Scholes would not be appropriate. For an analytical treatment, see Pernice and Fagundes (2005).

market participants, based on Monte Carlo simulations. Since this is an instrument that only pays coupons and no principal, its price can be determined by the discounted value of all expected coupon payments, using an appropriate discount rate. The theoretical value of the currency *j* warrant is therefore:

$$V(j)_t = E_t \sum_{k=t}^{T} \frac{C(j)_k}{\left(1 + r(j)_k\right)^{k-t}} \tag{2}$$

where $r(j)_k$ is the discount rate associated with the warrant of currency *j* and $C(j)_k$ is given by equation (1). As discussed below we will draw 5000 paths for GDP, inflation, and the exchange rate for each time t. The theoretical price at time t would then be the average of the 5000 resulting prices.

A. Discount Rate

The choice of the discount rate has been the subject of intense debate among market participants and has changed often since the introduction of the warrant. First, given that the instrument only pays during periods of good economic growth, and that countries are less likely to declare default on those years, the instrument should carry less default risk than standard bonds. However, this effect would be dampened by a persistently poor growth performance, as the latter would worsen public finances and would affect future payments on the GDP-linked securities even after growth eventually resumes. Second, moral hazard can raise the premium on this type of contingent claims. Indeed market surveys suggest that data accuracy was one of the concerns raised by potential investors (see Appendix 1). These concerns would be stronger when growth is near the critical threshold for repayments. Third, risk aversion may contribute to a higher premium on GDP-indexed bonds than on plain vanilla bonds, as the return on the former is likely to be more volatile. Fourth, the appropriate discount rate should also take into account the correlation between the market portfolio and the coupon payments on the GDP-linked security. Borensztein and Mauro (2004) report that Argentina's GDP growth shows very little correlation with international stock market indices. Thus, international investors should not charge a premium for the easily diversifiable idiosyncratic risk associated with Argentina's growth. However, Argentina's GDP growth is highly correlated with Argentina's stock market indices (e.g. the Merval or the Burcap) and with domestic consumption and labor income. This implies that local investors should use a discount rate larger than foreigners (since they face a significantly positive beta on the GDP-linked security).

Financial markets have used discount rates at least as high as those from Argentine plain-vanilla bonds denominated in U.S. dollar. As the arguments above do not give a clear indication to deviate, we will follow a similar approach, noting, however, that this discount rate is likely to constitute an upper bound. In particular, we construct the discount rate for the U.S. GDP-warrant by adding up the yield curve of the zero-coupon Treasury Bill and Coupon Strips, and the curve for the default risk implied by the Argentine Credit Default

Swap (CDS). This curve for the discount rate is modestly shifted so as to match the discount rate of the Argentine US$ Discount bond under New York law at the duration of the latter.[12] This adjustment has been extensively used by market analysts, because the default risk associated with the Argentine US$ Discount bond is considered more precise than the one reflected in the CDS, due to the higher liquidity of the Discount bond. However, such a bond is available only for a particular duration, while the CDS can provide a whole curve of default risk.[13] Hence, while we use the CDS to construct the curvature, our estimate of the level of the default risk is just the duration-adjusted spread of the Argentine US$ Discount bond on the US Treasury bond matching that duration. Finally, we add 100 basis points, a common practice among investment banks, usually justified as a way to account for the market's concerns about data accuracy and uncertainty in macroeconomic forecasts. Note that this additional spread of 100 basis points will not enter our calculation of the residual premium paid by the warrants.

For the warrant in pesos, we have two possibilities: either discount projected cash flows using the domestic interest rate curve, which would require additional assumptions,[14] or convert the cash flows into U.S. dollars and then discount the expected dollar value of those cash flows using the discount curve discussed just above for the U.S. dollar warrant. Appendix 2 shows that in theory the two approaches should deliver the same result under the uncovered interest parity assumption. We will use the second method.[15]

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[12] The shift necessary to match the two discount rates is very small, so that it is basically irrelevant which proxy is used. Note that these yields are not available for the period prior to the detachment of the GDP-linked securities from the underlying bonds: following common market behavior, for that period, we use an Ecuadorian bond with a similar maturity, as Ecuador was perceived to have—in 2005—a sovereign risk similar to that of Argentina.

[13] The CDS contracts have maturities up to 10 years. We extend their default curve to the maturity of the warrant on the basis of a regression of the CDS spread on the lag of maturity.

[14] All Argentine bonds are indexed to the CPI. The warrants instead adjust to inflation using the GDP deflator (which can be substantially different from the CPI, although they tend to co-move in the long-run) up to the reference year. However, the year lag between reference year and the payment is not inflation adjusted. Hence, a nominal interest rate needs to be used to discount the payment year (obtained by adding expected inflation to the real interest rate).

[15] Note that one key difference in the payment structure (the payment on the foreign currency warrants occurs with a one year lag at the exchange rate of the previous year) implicitly adds another component to the effective discount. Such a component will be separated from the discount rate and will be taken into account via exchange rate expectations, as discussed later.

B. GDP

The GDP process is assumed as follows:

$$y_{t+1} = y_t + \mu_{yt} + (1-\theta)(y_t - y_{t-1} - E_{t-1}(y_t - y_{t-1})) + \varepsilon_{yt} \qquad (3)$$

where y_t is the log of GDP (in constant local currency prices), μ_{yt} is the expected growth rate (which will be time-varying), 1-θ measures persistence (and multiplies the previous period deviation of the growth rate from its expected value), and ε_{yt} is a residual error term. The main difference among alternative growth scenarios will be given by the process for the error term.

The main scenario, or baseline, is based on random sampling of residuals. First, the above equation is estimated for the period 1981-2006 using OLS (imposing a constant mean growth rate). This provides the historical distribution of the error terms (with an estimated mean μ and variance σ^2), as well as the estimated persistence (θ =0.65, similar to what used in market valuations by major investment banks; see Deutsche Bank, 2006). The GDP projections until 2034 are then derived on the basis of: expectations for mean growth from the Latin America Consensus Forecast (LACF); the estimated persistence; and random sampling with replacement from the estimated distribution of residuals (5000 different paths of GDP are generated, obtaining a shock distribution that resembles Argentina's past, with occasionally large negative shocks).[16]

Alternative GDP processes are based on error terms modeled as geometric Brownian motion with a given variance. These correspond to scenarios where large crises are considered unlikely. Two different values for volatility are used. The historical standard deviation for Argentina, at around 6 percent, and the one for emerging markets, at around 4 percent. The latter, also used by market analysis, would capture the view that Argentina's outlook has improved with respect to its history.

For all processes, mean growth expectations for the next two years are derived from mean forecast in the monthly issues of the LACF. Forecasts for the 3rd to the 10th year are based on the biannual LACF issues, while forecasts for the remaining years until 2034 are based on the original long run projections of the Argentine authorities (3 percent, which is also what is used by investment banks). All forecasts that are not available monthly are interpolated linearly in order to convert them to monthly frequency. Also, volatility in the first year of the projection is reduced linearly from January to December, in order to reflect lower uncertainty

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[16] Note that, given the nonlinearity associated with the log transformation, in order to ensure that the process has a mean equal to the imposed one (chosen from LACF), one needs to make a standard correction to the GDP process described in equation (3), by adding the term $(-\frac{1}{2})\sigma^2$, where σ^2 is the estimated variance.

as more information becomes available during the year and GDP growth estimates are updated.

C. Inflation and the Exchange Rate

Finally, we turn to the processes for inflation and the exchange rate. The shocks to the macroeconomic variables are jointly-distributed.[17] The correlations are set to the values prevailing for 1991-2006, which are -0.13 (GDP and real exchange rate), 0.03 (GDP and inflation), and 0.24 (real exchange rate and inflation). Note that these correlations were computed excluding data prior to 1991, in order to remove the distortions created by hyperinflation prior to the 1991 stabilization[18].

We define two processes for inflation: GDP deflator inflation (D_t in equation (1)) and CPI inflation. Since inflation rates have been positive in most emerging countries—with the exception of some very isolated cases—we use a chi-square distribution for the shock to both inflation processes, setting the mean of the distributions at their expected value. In the same way as for GDP, we obtained mean inflation forecasts from the Latin American Consensus forecast for CPI, using monthly forecasts for 2-year ahead horizons, bi-annual forecasts for the 3-10 year ahead, and constant after that. Again, these forecasts are interpolated linearly to derive monthly figures. For the GDP deflator, we use the information provided by the IMF World Economic Outlook (WEO). The variance for the inflation processes was chosen so as to match the standard deviation of emerging economies in the last 10 years (about 5.7 percent). This reflects the assumption that Argentine history of hyperinflation or currency board is unsuited to provide a forecast of inflation volatility under the current exchange rate regime.

Real exchange rate (RER) projections for the first two year of the horizon are based on forward nominal exchange rates—as these are the relevant exchange rates concerning arbitrage opportunities–and CPI inflation forecasts from LACF (foreign inflation is assumed to be 2.3 percent, from the 2006 consensus forecast for the U.S.). For the remaining horizon, we use a mean reverting process towards the average real exchange rate that has prevailed over the past fifty years[19]. We calibrate the standard deviation of the process to match the standard deviation of the RER in the period 1991-2006 (on the basis of data from the

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[17] A different way to construct processes for all three variables, while keeping our assumption of monthly updates based on market expectations, would be to construct a Bayesian VAR. This, however, would constitute a task of its own, and is left for future work.

[18] These correlations are likely to be different in different phases of the business cycle, but their effect is quite small, so we do not consider necessary to add such an additional feature.

[19] This level is 23 percent above the one in 2005.

Argentine central bank, BCRA) and use a coefficient of mean reversion of 0.15—which implies a half life of 5 years—as estimated by Rogoff (1996).

Nominal exchange rate projections are based on the forward exchange rate for the first two years of forecasts. For the following years, we derive the projections from the processes for RER and CPI inflation.

Our main assumptions are summarized in the Table 3. We later discuss the sensitivity of results to alternative assumptions.

Table 3. Baseline Assumptions*

Variable	Mean at t ($\mu(t)$)	Standard Deviation at t ($\sigma(t)$)
GDP	Market Expectations	Random Sampling of Residuals 81/07
Inflation (Deflator)	IMF (WEO)	Emerging Mkts. Avg. (1997-2006): 5.73
RER (t:t+2)	Forward ER*Inf. Forecast Ratio	Historical (1991-2007): 0.23
RER (t+3:T)	$R_{t+1} = R_t + 0.15(R_t - R^*)$	Historical (1991-2007): 0.23
Nominal ER	Forward (t+1: t+2) & Endog. (t+3: T)	Endog.: $\sqrt{Var(RER) + Var(\pi) - cov(RER, \pi)}$

Variable	Stochastic Process
ln(GDP)	$y_{t+1} = y_t + \mu_{yt} + (1-\theta)(y_t - y_{t-1} - E(y_t - y_{t-1})) + \varepsilon_{yt}$
Inflation (Deflator)	$\pi_t = \mu_{\pi t} + \varepsilon_{\pi t}, \quad \varepsilon_{\pi t} \sim \chi^2$
ln(RER) (t:t+2)	$r_t = r_{t-1} + \ln(F_t / F_{t-1}) + \ln\left(\dfrac{1+\pi_{US}}{1+\pi_t}\right) + \varepsilon_{et}$
RER (t+3:T)	$R_{t+1} = R_t + 0.15(R_t - R^*) + \varepsilon_{et+1}$
Nominal ER (t:t+2)	$er_t = er_{t-1} + \ln(F_t / F_{t-1}) + \varepsilon_{et}$
Nominal ER (t+3:T)	$er_t = er_{t-1} + r_t - r_{t-1} - \ln\left(\dfrac{1+\pi_{US}}{1+\pi_t}\right)$

Source: Authors' estimates
* In this table, time is denoted in annual terms. R denotes the RER, and R* is defined as the long run RER, set to 0.77. Small caps denotes natural logarithms. F denotes the forward exchange rate.

V. MAIN BASELINE RESULTS FOR THE US$ GDP WARRANT

For each time *t*, we simulate five thousand paths for the time series of these variables up to 2034. For each path, we calculate the payment per unit of currency in each period, and the value of the corresponding GDP-linked unit. To obtain the final price for each time *t*, we average the five thousand values obtained from different paths. Theoretical values were computed daily, by combining the slow-moving forecast data (GDP growth, and inflation) with daily market data (Treasury yields, CDS risk premia, Argentine US$ discount bond yields, and exchange rates).

For the baseline scenario, our calculations indicate a theoretical value of US$9.99 per 100$ of the US$ warrant on December 12th 2005, increasing to US$18.61 by the 24[th] of July 2007. Figure 4 presents the evolution of our theoretical value versus the market prices for the US$ warrants under both laws (available through Bloomberg from December 2005 onwards).

Figure 4. US$ GDP Warrant Theoretical Values
Using Random Sampling from 1981-2007 Residuals
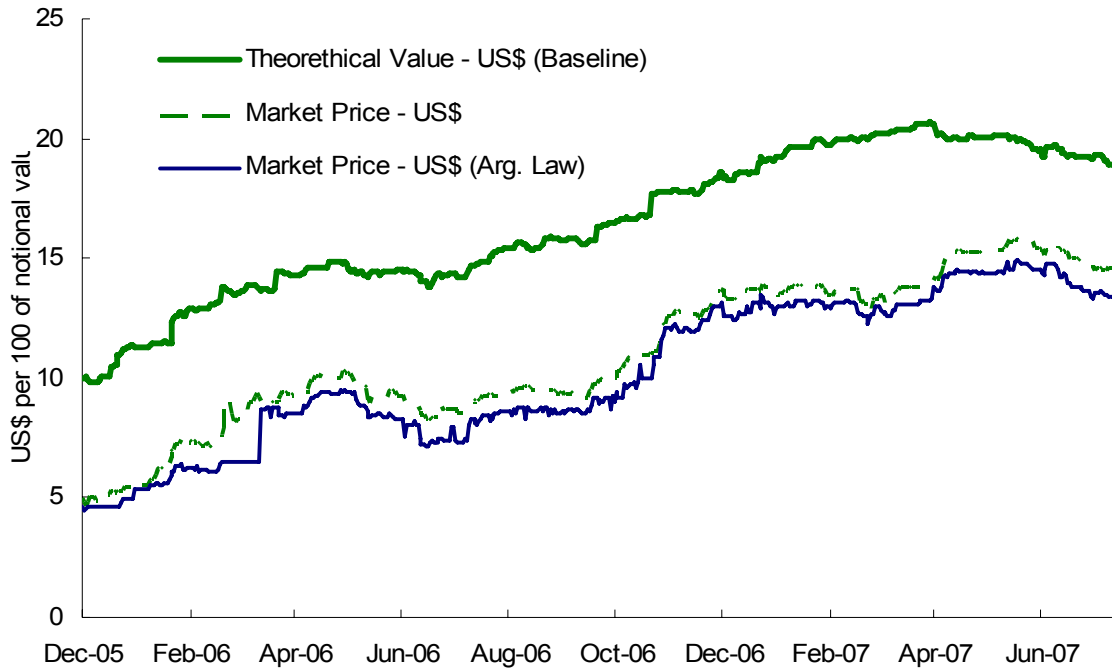
Source: Bloomberg and authors' estimates

We can also estimate the yield at which the market is implicitly discounting these securities, by calculating the discount rate that would generate a theoretical value equal to the market price (holding all other assumptions constant). This implied yield can be broken down in three components: (1) the risk free rate, (2) a default premium, (3) a residual premium. The residual premium is therefore obtained by subtracting, from the implied yield, two components: the default premium discussed in the previous section, and the risk free rate of a US treasury bond, all of a similar duration to the warrant (duration goes from 7 years at detachment to around 6 years in July 2007). [20] This residual premium captures the discount due to the novelty and potential illiquidity of the GDP-linked security, the market's concerns about data accuracy, risk aversion, and comovement of returns with those on other securities/consumption.[21] Thus, this residual premium provides an upper bound on the novelty premium, which should in theory decline as the warrant's market develops.

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[20] Similarly to what done before, we obtain the implicit default premium by shifting the CDS default risk curve by the amount necessary to match the yield of the Discount bond minus the yield on the U.S. Treasury Bill for the same the duration (the GDP-linked security has a shorter expected duration than the bonds due to the cap on total payments).

[21] The large volumes for the dollar, peso, and Euro warrants might indicate that liquidity is likely to be a minor issue for these warrants. Liquidity could instead be particularly relevant for the Yen denominated instrument, for which, however, we could not gather price data.

Implied yields were more than 20 percent when the instruments were detached at the end of 2005, and have been decreasing ever since, reaching values around 12 percent by mid-2007 (Figure 5 presents the two largest components of the implied yield, excluding the risk free rate). The reasons for this decline have laid mainly on updates of macroeconomics forecasts and a reduction in the residual premium. If one considers the default premium on alternative Argentine U.S. dollar-denominated bonds as applying also to the GDP-warrant, the residual premium was almost 1200 basis points initially (Figure 5). It quickly declined by about one third within two months. Towards the end of the sample, the residual premium remained close to around 550 basis points.

The apparent surge in the yield (and the associated the price dip) in the first few months of 2007 can in part be explained by two factors that are not fully captured by our valuation method: an increase in international risk aversion that led to a reduction in the price of most emerging market bonds; and possible moral hazard concerns related to a methodological change in the construction of the CPI in early 2007.[22] While GDP warrants were greatly affected by these factors (especially by the first factor, which induced price reductions of up to 6 percent in one day), the US$ Discount bond—used to derive the discount rate in the baseline scenario—seem to have suffered a more muted effect. In a separate exercise, we employed the Arg$ Discount bond—whose price fell more than the one of the US$ Discount bond in the early part of 2007—to derive an alternative proxy for the discount rate: the resulting theoretical price better matched the movements in the actual price, thus reducing the surge in the implied yield evident in the baseline. The Arg$ Discount bond should be more sensitive to these factors due to its currency risk and CPI indexation.

While yields can provide information in terms of returns to maturity of expected cash flows, expected returns give a better measure of risk-adjusted returns for comparison with other risky assets.[23] Expected returns on the warrant have hovered around 11 percent during our sample. At the same time, the volatility of returns decreased during this interval, pushing the

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[22] It is worth noting that only the GDP deflator—and not the CPI—enter directly the formula for the coupon payments.

[23] The expected return is defined as the average of the yields implied by the theoretical values corresponding only to simulated paths when payment occurs. Hence, it can be much lower than the average implied yield.

Figure 5. US$ GDP Warrant Premiums
Using Random Sampling from 1981-2007 residuals
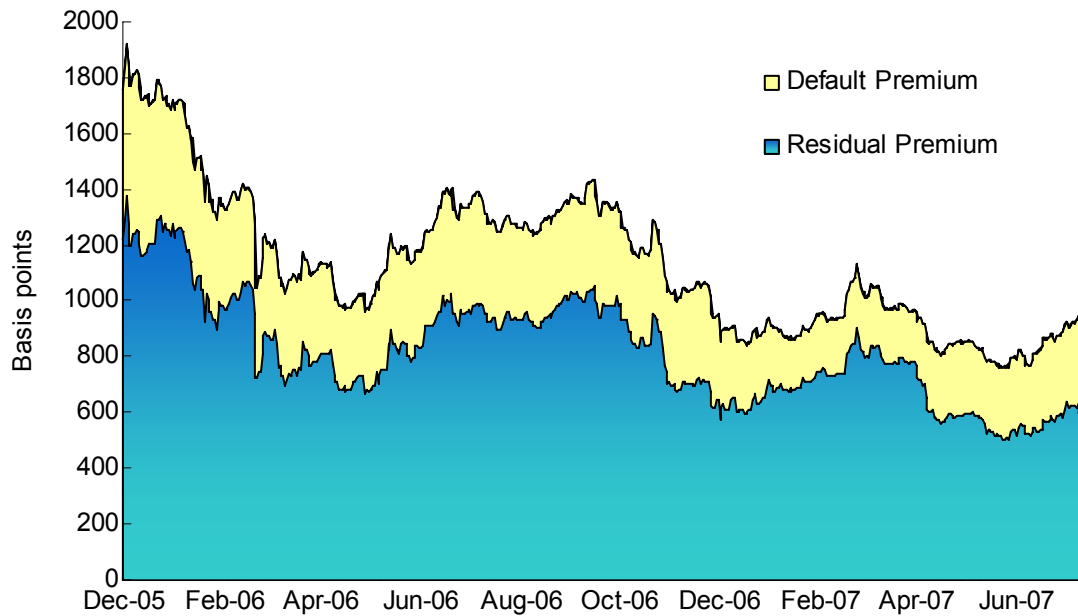
Source: Authors' estimates

Sharpe ratio up from 0.22 to 0.35. [24] These levels for the ratio are comparable to that of the S&P 500 (which is around 0.3).

A. Main Factors Behind Valuation Changes

It is interesting to get an idea of the relative importance of the factors that determined the large increase in the valuation of the warrant. In order to do that, we build the path for the theoretical value that matches exactly the actual market prices: such a path is based on the implied yield we just discussed. We then construct five counterfactual theoretical values. Four scenarios are derived by holding constant all variables but one at the level prevailing in December 2005 (exchange rate and inflation are allowed to move together given their limited impact). Hence, we obtain four scenarios, where only one of the following variables changes at the time: (i) GDP, (ii) inflation/exchange rate, (iii) discount rate (U.S. interest rate plus the proxy for the default risk), and (iv) residual premium. In a fifth counterfactual scenario we hold all of these variable constant at the their initial value in order to gauge the effect of the passage of time (due to interest rate arbitrage, any asset price increases with the proximity

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[24] The Sharpe ratio is a measure of the mean excess return over the risk free rate per unit of risk, where risk is measured as the standard deviation of mean returns. It is defined as the ratio between expected excess returns and excess returns volatility.

to payment)[25]. The changes in price indicated by each of these five counterfactuals approximately adds up to the change in the actual price (the approximation being due to the fact that this accounting would neglect the effect of interaction among variables).

The contributions to the changes in the actual price between December 2005 and July 2007 is presented in Figure 6. Around 65 percent of the increment was due to updates in macroeconomic forecasts for the whole horizon. The main contribution is assigned to pure GDP growth realizations and forecast updates (42 percent), while realizations and forecast updates in inflation and exchange rate explained about 5 percent and the interaction among these macro variables about 18 percent. The reduction in the residual premium explains 23 percent of the change. The remaining change is attributed to passage of time (9 percent) and changes in the discount rate (3 percent).

Figure 6. Factors Explaining Price Changes
Dec05/Jul07 - US$ Warrant
(In percentage)
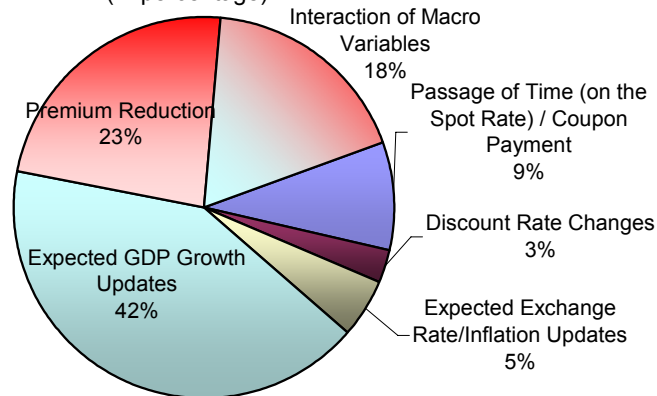
Source: Authors' estimates

B. Comparisons with Investment Bank Valuations

Investors priced this instrument in March 2005 in order to calculate the value of the debt exchange offer. At that time, estimated market values for the US$ warrant were around US$2.15, while our corresponding theoretical value is US$6.5. The main reason for this large difference—as discussed in Section III—is that many investment banks applied a 50 percent haircut on the theoretical value they found, which was then around US$4.3 (Credit Suisse, 2005; JP Morgan, 2005). The remaining difference is due to macroeconomic assumptions, mainly related to long run growth and convergence: many agencies priced the warrants assuming growth would converge to 3 percent by 2008, even though LACF indicated higher

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[25] For a definition of the passage of time see Elton and Gruber (1995).

forecasts for the coming years.[26] Such a fast convergence makes a large difference: the fact that the coupon pays on accumulated GDP implies that changes in expected growth early on in the sample have a large impact on expected cash flows.

As the moment of detachment got closer, investment banks' calculations were refined. While our calculations give a valuation of US$9.32 for the US$ warrant for the end of November 2005, market reports mention values slightly above US$8, on the basis again of somewhat more pessimistic growth assumptions. Deutsche Bank developed a robust model for the pricing of the warrants, indicating a theoretical value of US$8.64 for January 2006.

The most remarkable change occurred in the first week of March 2006, when major investment banks (Credit Suisse, HSBC, and Morgan Stanley, among others) updated their theoretical values not only by using new growth forecasts but also by using lower discount rates. This boosted the demand for these instruments, whose price increased by 27 percent in a single week. Since then, valuations have been updated mainly because of new growth forecasts and investment banks have indicated theoretical values slightly below our calculations. For instance, while JP Morgan obtained a theoretical value of US$17.73 for December 6th 2006, our theoretical value is 18.22 (our theoretical valuation would match quite close that of JP Morgan or Deutsche Bank, if we were to allow growth to converge to its long run value by 2008).

Overall the discrepancy between our theoretical values and those suggested by major investment banks—for periods subsequent to detachment—is mainly due to differences in assumptions regarding the growth process, as investment banks' pricing models tend to exhibit more conservative growth forecasts than the average of the Consensus Forecast sample (which underlies our calculation). Indeed, they assume growth converges to its long run level of 3 percent by 2008-2009 (depending on the year in which the forecast was made). In addition to this early convergence, they also assume: growth is normally distributed with standard deviation of 4 percent; inflation is assumed to be 4 percent in the long run; and the equilibrium real exchange rate is assumed to be 19 percent above the level prevailing in 2005[27]. As previously explained, these other assumptions play a smaller role, because the last two variable do not affect much the valuation (see Section V.1) and volatility has a small impact for GDP levels far from the trigger point (see Section II).
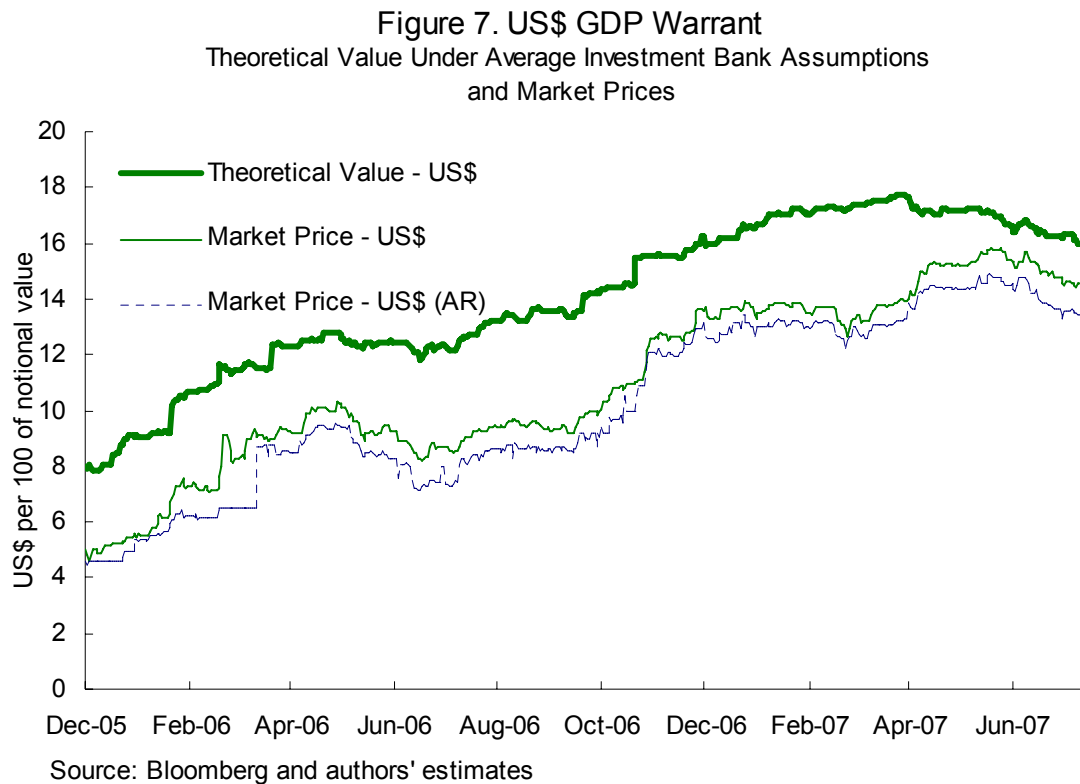
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[26] Most observers failed to predict Argentina's impressive recovery.

[27] We average assumptions from 5 main investment banks that publicly disclosed analytical valuations of the Argentine GDP-warrant (Barclays, CSFB, Deutsche Bank, Dresdner Kleinwort and Wassertein, and JP Morgan Chase) .

In Figure 7, we present theoretical values calculated using assumptions similar to those used by the main investment banks.[28] On average these calculations match reasonably well investment banks' valuations. Since the long run 3 percent growth rate is near the threshold for one of the condition on payments (and lower than the short- to medium-term forecasts), accelerating the speed of convergence reduces payments. Indeed, these conservative assumptions have a large negative impact on estimated theoretical values, which however remain above market prices. The reduction in the theoretical values implies a considerable reduction in the estimated residual premium, which reaches an average of 292 basis points by July-07 (see Figure 8).

Figure 7. US$ GDP Warrant
Theoretical Value Under Average Investment Bank Assumptions and Market Prices

Source: Bloomberg and authors' estimates

In order to have a better sense of the role of the GDP process in this comparison, the effect of alternative assumptions is illustrated in Figure 9, which presents the residual premium for: our baseline scenario (the dark line, based on 1981-2007 distribution of residuals and 3 percent long run growth), the investment bank assumptions scenario, and three additional ones. In one alternative scenario, we modify our baseline by assuming that long run growth is given by the last observation in the mean forecast from LACF. In the other two, we depart from the baseline scenario, by assuming the alternative growth process based on a Brownian
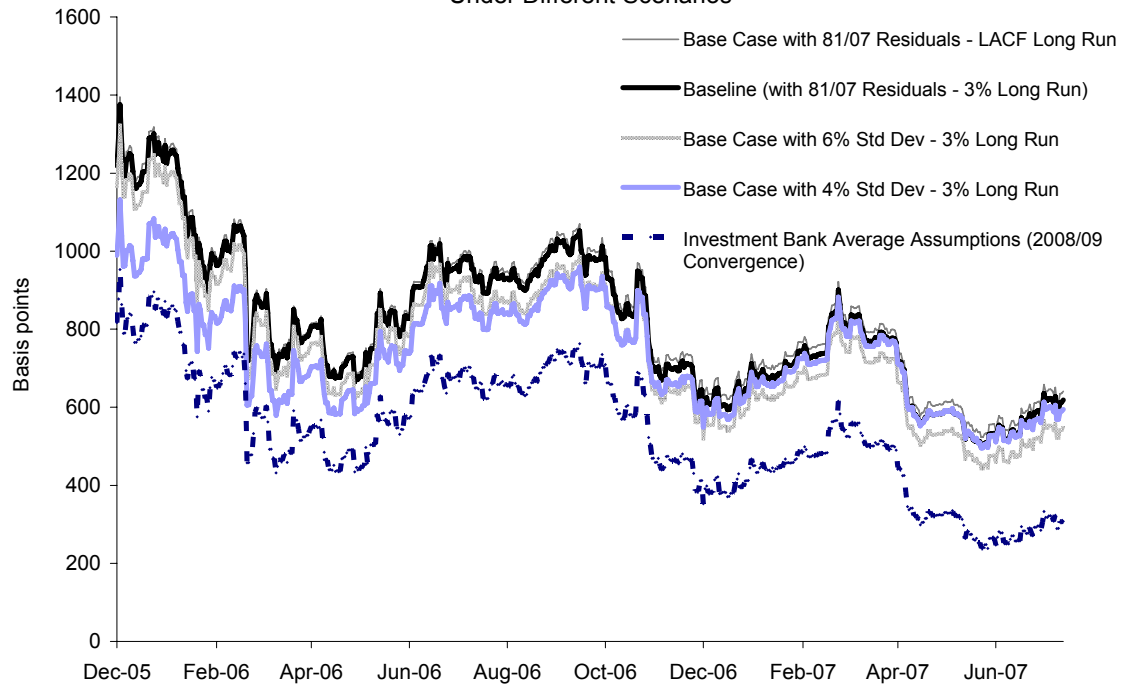
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[28] Regarding GDP, we feed their assumptions in our formula for the GDP process (hence allowing for a difference due to the persistence of GDP).

Figure 8. US$ GDP Warrant Premiums
Using Average Investment Banks Assumptions
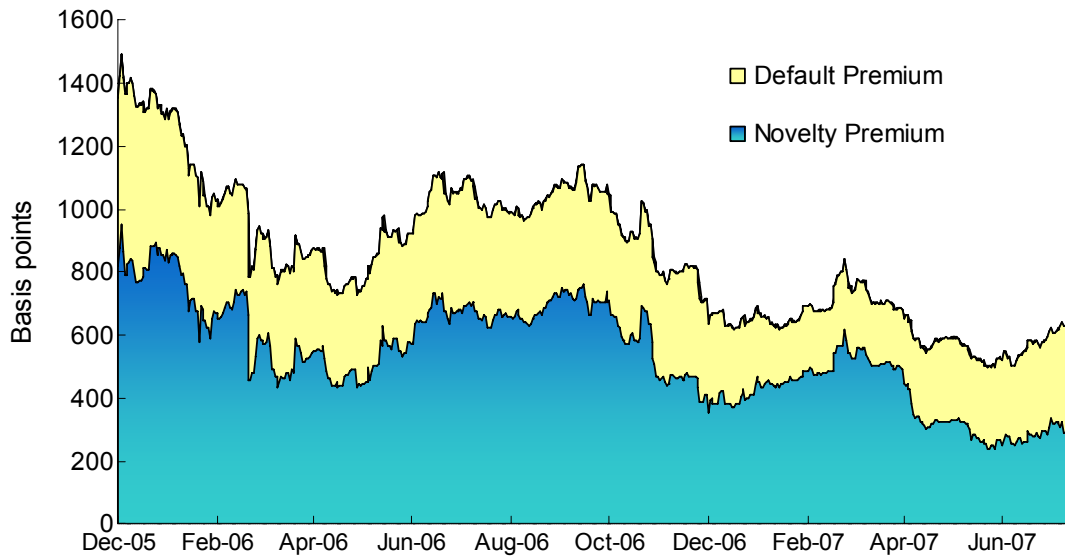
Source: Authors' estimates.

Figure 9. U.S.$. Warrant Residual Premium
Under Different Scenarios

Source: Authors' estimates

motion and employing two different levels for the standard deviation of GDP growth: 4 percent, as under investment banks' assumptions, and 6 percent, which is close to the standard deviation in our baseline. As expected, the more optimistic the growth scenario is, the larger is the residual premium required to justify observed market prices. However, the scenario under investment bank assumptions shows a premium which is much lower than all the others: because of the discounting, assuming a fast convergence to long run growth plays a much larger role than changing long run growth or the distribution for the shocks. One noteworthy feature is that this residual premium has declined over time across all scenarios.

This sensitivity of theoretical valuations to underlying assumptions highlights the main challenge inherent to this type of exercise: the difficulty of disentangling the novelty premium from other factors such as different assumptions regarding the macroeconomic outlook.

VI. OTHER VALUATION EXERCISES

A. Theoretical Values for the Arg$ GDP Warrant

This Section calculates the theoretical value for the Arg$ security by converting the expected cash flows to dollar terms using the exchange rate at the time of payment (see Section IV.1). We then use the dollar discount rate to compute their net present value, and transform them back to pesos based on the exchange rate of the valuation day.[29]

Our theoretical values for this instrument show an increase from Arg$8.6 by December 2005 to Arg$16.5 by July 2007 (Figure 10). The forces behind this increment are almost identical to the ones behind the increase in the US$ warrant. Market prices, as well as our calculations, show a higher value for the dollar warrant than for the peso warrant per unit of notional value in the respective currency. This is mainly due to the relatively high expected inflation in Argentina as well as and the cap on cumulative nominal payments at 48 cents of the respective currency (see Section II).

Since the peso warrant is under Argentine law, its yield may also include a small legal premium, which must be separated from the residual premium for comparison with the previous results. The legal premium was estimated by comparing yields from the warrants in dollars under two different jurisdictions: New York Law and Argentine Law. This spread captures potential differences in litigation opportunities and recovery values in the event of a default. However, the lower liquidity of the dollar instrument in the Argentine market and the high participation of local investors imply that much of this spread could also reflect liquidity concerns as well as the higher covariance of returns with those of Argentine equities and

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[29] The dollar discount rate may slightly underestimate the appropriate discount rate in this valuation method, because the Ars$ warrant carries an extra year of currency risk.

Figure 10. Ars$ GDP Warrant
Using Random Sampling from 1981-2007 Residuals
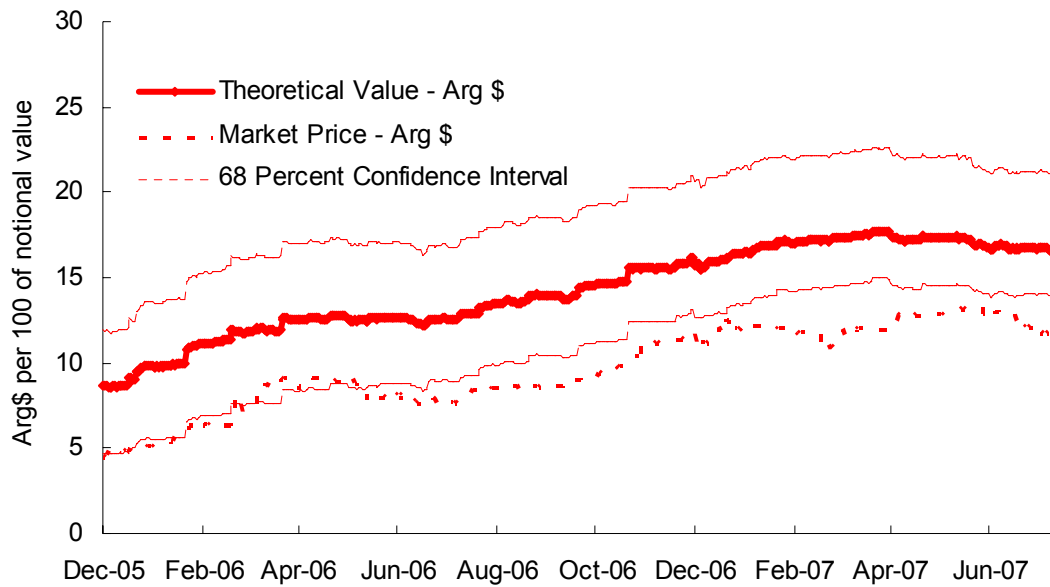
Source: Bloomberg and authors' estimates.

household income. Hence, the gap between the yields from the warrants in dollars under the New York Law and Argentine Law should be considered an upper bound of the legal premium.

Similarly to the previous exercise, we derive a residual premium by subtracting from the total discount rate three factors: the risk free rate, the default premium, and the legal premium. Again, the residual premium would be underestimated to the extent the proxy for legal premium reflects limited liquidity of the US$ warrant under Argentine law and/or the default risk of the Arg$ warrant is smaller than the one for ordinary bonds.[30]

We observed a reduction in the Arg$ securities' residual premium, from around 910 basis points in the first two months to 650 basis points on average afterwards (see Figure 11). As in the case of the US$ security, there is a sharp decline in March 2006. The legal premium hovered around 150 basis points. Thus, once we account for the legal premium, the residual premium in the Arg$ and US$ securities is fairly comparable.

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[30] The residual premium may also include a small currency premium component (due to the one year lag in payments, which has a larger effect on the real value of peso cashflows than on the dollar ones).

Figure 11. Ars$ GDP Warrant Premiums
Using Random Sampling from 1981-2007 residuals
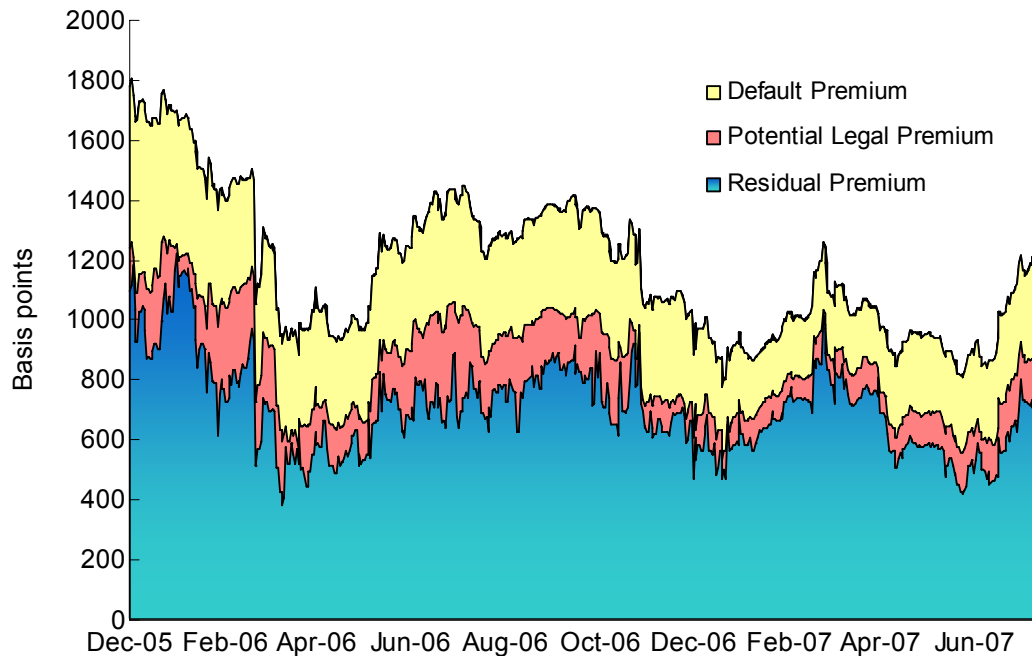
Source: Authors' estimates

B. Theoretical Values for the Euro GDP Warrant

We use the same valuation method presented in the previous Section, to derive a theoretical price for the GDP warrant denominated in Euros. We then calculate the implied residual and compare it with the one associated with the US$ GDP warrant. The two residuals are remarkably close, which also confirms the robustness of the exercise (Figure 12). This results should not be surprising also because the liquidity of the instruments is very similar.

C. The Effect of Exchange Rates and Inflation

This Section analyzes the robustness of the results to changes in the assumptions related to the exchange rate and CPI inflation. First, we maintain the real exchange rate constant at its projected 2008 level. This modification implies a more depreciated real exchange rate, which reduces the theoretical value of US$ warrant, as expected. Second, in the case of inflation, we lower our baseline estimates by 1 percentage point at all horizons. The value of the dollar warrant remains unaffected, due to our PPP assumption, while the peso warrant increases in value due to the lower inflation during the one year lag for its payment (see Figure 13).

Figure 12. Euro GDP Warrant Premiums
Using Random Sampling from 1981-2007 residuals
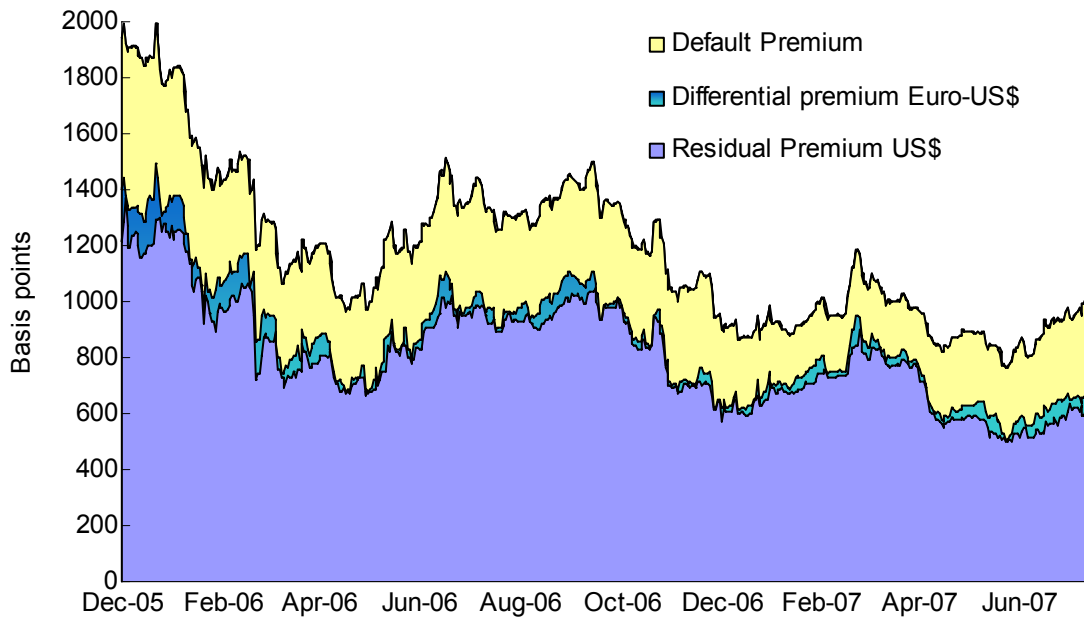
Source: Authors' estimates

Figure 13. Arg $ GDP Warrant Theoretical Values
Robustness With Respect to Inflation
Baseline Inflation -1% vs Baseline
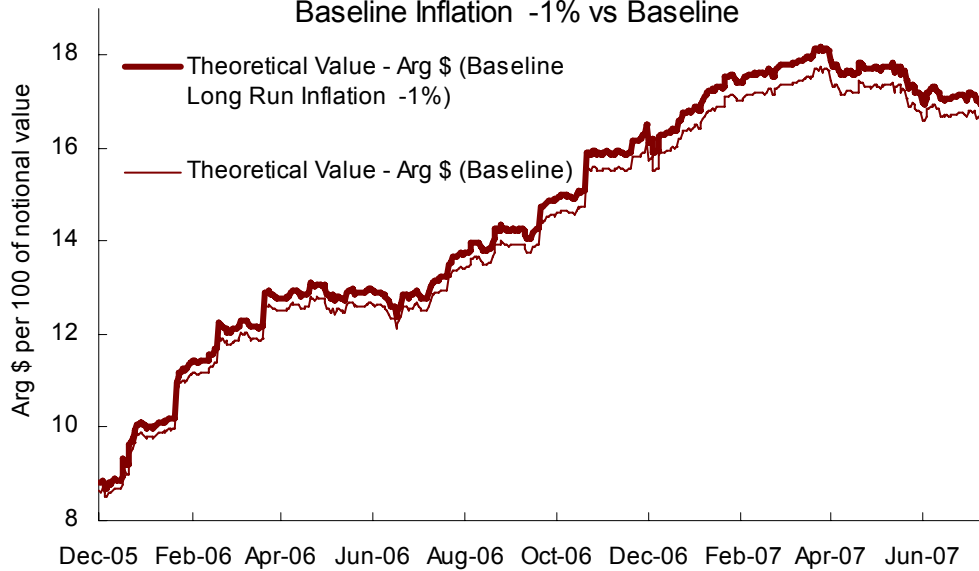
Source: Authors' estimates

D. The Effect of Growth

This Section shows additional analysis of the sensitivity of the theoretical values to changes in the assumptions relative to the most important variable: growth. The comparisons are based on theoretical values calculated for a given day (July 24, 2007). Table 4 compares the effect of changing mean growth forecasts from the baseline values: changes in the GDP growth in closer years (2007) matter more than changes in further ones (2008). The effect of a given percentage point deviation from the baseline is larger if occurring in 2007 (differences across rows) than in 2008 (differences across columns)—despite the fact that it implies a larger percent change in growth in 2008. Table 5 shows the effect of altering assumptions related to the convergence to long-run growth: as expected, faster convergence lowers the theoretical valuation. Table 5 is also useful to illustrate that even a long-term growth of 2.5 percent—which is below the payment threshold—can still yield positive payments, due to the large variance of the GDP process which allows for paths where payments can still occur.

Table 4. Sensitivity on 2007/2008 Baseline Growth +/- 1.5%*

Theoretical Values: U.S.$ Warrant (In US$ per 100 of notional value)

2007 Growth (In percentage points)	2008 Growth (In percentage points)		
	7.00	5.5 (Baseline)	4.00
9.00	19.40	18.70	17.98
7.5 (Baseline)	18.62	17.93	17.21
6.00	17.82	17.13	16.41

Source: Authors' estimates
* Baseline by July 24th, 2007, based on Consesus Forecast Expectations

Table 5. Sensitivity on Convergence Year

Theoretical Values: U.S.$ Warrant (In US$ per 100 of notional value)

Long Run Expected Growth	Year of Convergence		
	2009	2014	2018 (Baseline)
2.50	14.60	17.07	17.75
3.00 (Baseline)	15.87	17.52	17.93
3.50	17.10	17.96	18.11

Source: Authors' estimates
* Baseline by July 24th, 2007, based on Consesus Forecast Expectations

E. The Effect of Uncertainty

To capture the effect of uncertainty on the value of the instrument, we calculate the theoretical value under the hypothesis that forecasted values of GDP, exchange rates, and inflation are realized with probability one (i.e. there are no shocks to their processes). This perfect foresight situation gives us an idea of the effect of volatility on prices and returns. In this case, the theoretical values tend to be higher by more than 30 percent, on average. The implied yield under the no uncertainty scenario was about 2000 basis points (or only 75 basis points higher than in the baseline case) in December 2005. However, it declined at a smaller pace decreasing by only 140 basis points in our sample. This higher yield under the no uncertainty scenario can be seen as an uncertainty premium. Investors' risk aversion can affect this premium, as higher risk aversion will increase the return demanded on volatile financial assets.

F. Expected Value for the Remaining Maturity

To conclude our analysis on the theoretical value of the warrants, we performed a forecast on the theoretical price of the instrument from 2007 until maturity, on the basis of macroeconomic forecasts available in July 2007. Future discount factors are obtained from the yield curve using the "expectations hypothesis" imbedded in the term structure of the interest rate. While the expectations hypothesis does not typically hold empirically,[31] it provides a reasonable first approximation for horizons of four years or more. For the remaining variables, we use the latest forecast path available in July 2007.

While in the theoretical calculation there is always an expected positive measure of payments in all periods (as there are always some projections where payments occur), in reality the warrant may at some point stop paying as the 48 percent payment cap is reached and the third condition is no longer met. Argentina's inflation projections imply that the peso warrant is likely to expire sooner than the US$ warrant (as it might reach the nominal payment cap sooner). In fact, payments in the respective currency can be expected to be higher for the Ars$ warrant than for the US$ one (see Figure 14a), since the nominal exchange rate can be expected to neutralize most of the inflation differential. This is reflected in the shorter duration of the Ars$ instrument: while 32 percent of our simulated paths for the US$ warrant reach maturity, only 26 percent does for the Arg$ warrant.[32] The peso warrant has a 50 percent probability of reaching the cap before 2020 and 70 percent of reaching it before

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[31] For example, in the U.S. the historical return on a long-term bond is higher than the total return from holding a succession of short-term ones. For a more detailed and deeper discussion of the expectations hypothesis, see Cochrane (2001).

[32] Our calculations give a duration of 5.8 for the Arg$ warrant and of 6.5 for the US$ warrant by Dec-2005. By definition, this implies that a 100 basis point increment in the discount rate reduces the value of the warrant by 5.8 and 6.5 percent approximately.

2030, while those probabilities for the U.S. dollar warrant are only 37 and 63 percent respectively (see Figure 14b). The shorter duration of the Ars$ warrant is reflected in the fact that its value is expected to peak sooner—in the forecast period—than the expected value of the US$ warrant (2008 versus 2009).

Figure 15 shows the forecasted theoretical values for the warrants in pesos and dollars. These are values ex-coupon after December 12 of each year (as coupon payments are due on December 15 of each year, starting 2006). This means that discrete jumps arise from coupon payments, and that a progressive increase within each interval is due only to the passage of time on the spot rate.

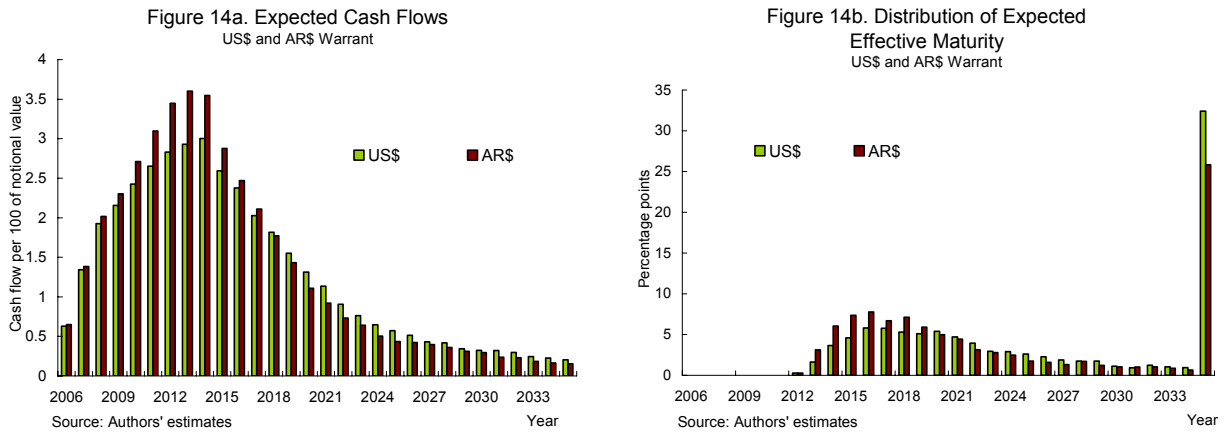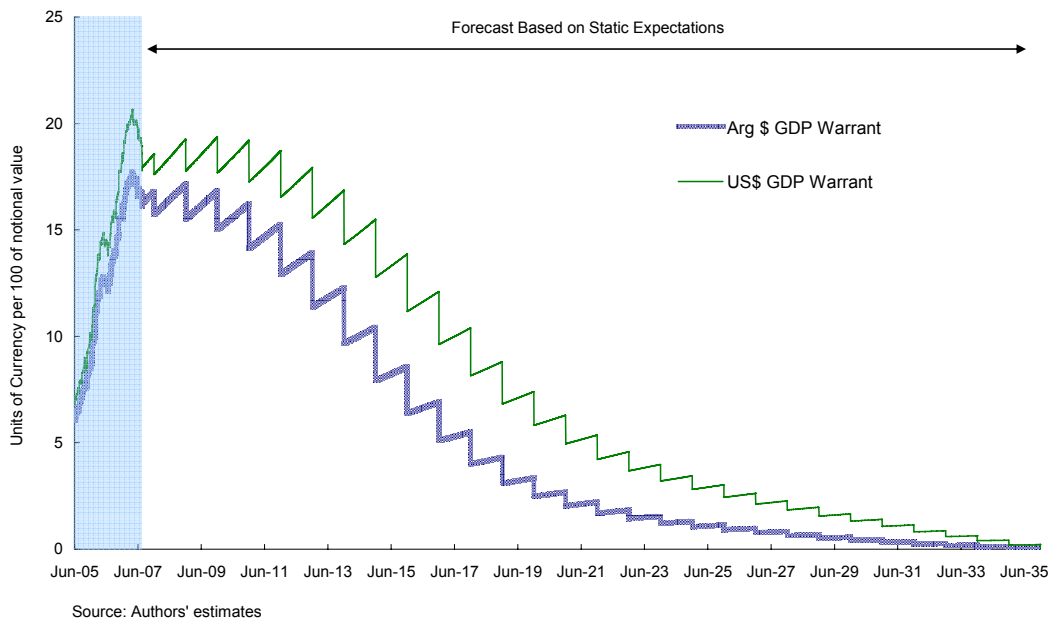Figure 14a. Expected Cash Flows
US$ and AR$ Warrant

Figure 14b. Distribution of Expected Effective Maturity
US$ and AR$ Warrant

Source: Authors' estimates

Figure 15. Expected Theoretical Values for the Warrants Over Time

Source: Authors' estimates

VII. CONCLUSIONS

We calculated the theoretical price for the Argentine GDP warrant as the expected net present value of potential payments, on the basis of a Monte Carlo exercise applied to daily, monthly, and annual forecast for the determinants of the payments. We also derive the implied discount rate, as the one that would make the theoretical price match the actual price. Our baseline model suggests a sizable reduction in the premium associated with GDP-linked securities over the standard Argentine bonds.

This premium is likely to be due to the novelty of the instrument, the complexity of the pricing valuation, its liquidity, market concerns about the accuracy of variables that affect the payment, comovements of return with other assets, the difficulty of fitting this new financial product in the overall asset-liability management, and potentially risk aversion (indeed investor surveys point to the relevance of such factors, as discussed in Appendix 1). Most of these factors are likely to decline over time, and as such they can be considered as contributing to a "novelty premium". Our estimates suggest that the premium declined on average by about 600 basis points during the first year and a half of trading (from a range of 800-1200 basis points in December 2005 to a range of 200-550 in July 2007), with most of the reduction taking place in the first three months of trading.[33] The decline in the estimated premium is a very robust result, even though—as discussed—the estimated level of the premium is sensitive to the specific assumptions.

Several caveats are in order. To the extent the default risk on the GDP warrants should be expected to be smaller than for standard bonds, the residual premium of the GDP-linked securities over standard Argentine bonds would be underestimated. Also, we have shown that the theoretical price and the residual premium are highly sensitive to the specific forecast assumptions, especially for GDP growth. Indeed, part of the premium in the GDP-linked securities may be related to the possibility of a sufficiently sharp economic contraction. Introducing the possibility of sharp crises in our baseline growth process can lower our theoretical valuation to the observed market prices for the GDP-linked security. It is possible that the market has been extremely cautious, and has implicitly assigned a probability for sharp crises in the future, although such probability is not visible in the available forecasts, which mainly report only the first moment of the expectations. The large sensitivity of our theoretical valuation to the specific assumptions made for the growth process suggest that all these results should be read with extreme caution.

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[33] Note that the introduction of the GDP-warrant may have lowered the overall default risk of the country, thus reducing the default premium imbedded in interest rates paid on plain-vanilla bonds (this effect would not affect our estimate of the novelty premium, as the latter is derived as a spread).

To the extent that a declining residual premium is due to the novelty of the instrument, and that the Argentine experience has made international financial markets more familiar with the instrument, future issues of GDP-linked securities could face a lower novelty premium. This could encourage other countries to contemplate issuing this type of instruments, and Argentina would have provided a positive externality by incurring first-mover costs. It is beyond the scope of this paper to compute the welfare implications of this security. Overall, the results might indicate that financial innovation may be associated with significant premia, which however decline reasonably fast.

APPENDIX **I.** INVESTORS' SURVEY

One of the main concerns at the time of the introduction of the Argentine GDP warrants was how investors would react to an instrument barely used before, whose value was difficult to gauge, and payments depended on data provided by the issuer. Fernandez et. al. (2006) provide an interesting investors survey (mainly bond holders participating in the debt exchange) implemented in mid 2005, prior to the detachment of the GDP warrants from the plain vanilla bonds. The survey focuses on the Argentine warrants and is based on a previous survey by Borensztein et al (2004), which is related to GDP-indexed bonds in general.[34] This Appendix gives a short summary of the answers, emphasizing the implications for our analysis.

The first question was about the existence of regulatory restrictions on holding the GDP warrants in their portfolio. Among thirteen respondent, six indicated the presence of regulatory restrictions (mainly pension funds). Three answers were negative (all from banks) and four did not know.

The second question aimed at evaluating investors' understanding of the instrument, and asked what premium investors would require relative to plain-vanilla Argentine bonds offering the same expected coupon payment. As discussed in the main text, the choice of an adequate discount factor is critical for an appropriate assessment of the theoretical value of the warrant. Hence, knowing market demand for a premium is fundamental in understanding the observed return on the instrument. Similarly to the results in Borensztein et al (2004), the median answer was a discount rate premium between 100 and 200 basis points above the one used for the valuation of a standard Argentine bond. This is particularly interesting, because theory suggests that GDP-indexed bonds should be subject to a smaller default risk than regular bonds, as the country does not need to pay a coupon on GDP-indexed bonds in bad times. At the time of the survey market agencies were imposing premia of much larger amounts, on the basis of the novelty of the instrument and concerns related to lower liquidity, higher volatility, and possible data mishandling.

The third question aimed at assessing the reasons behind the higher discount rate, focusing on two main aspects: the understanding of the instrument and its pricing, and the determination of the data used to calculate its payments. Specifically, the question asked under what conditions (within a list of non-mutually exclusive possibilities) the investor would demand a lower rate of return on the instrument. In order of importance, the proposed answers (and the percentage of positive responses) were:

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[34] For another survey related to GDP-indexed bonds in general, see Schroder, Heinemann, Kruse, and Meitner (2004).

- Argentina's government accepts to base GDP-coupon payments on growth data provided by the IMF (57%).
- An international agency announces that their research shows the reliability of Argentine GDP data and commits to supervise the quality of future data (50%).
- A consultant agency announces that it's going to provide a free software with a formula to value the growth coupon (21%).
  Other five important emerging countries plan to issue a GDP-indexed bond at the time (21%).
- The United States plans to issue a GDP-indexed bond (14%).

From the above answers we can see that more than half of the respondent were concerned with moral hazard issues related to data inaccuracies. The second issue is pricing: given the novelty of the instrument, markets were not comfortable with the methodologies being used to obtain theoretical values.

The fourth question asked more generally about the main factors that provide disincentives for investors to hold the instrument in their portfolio. Among the (non-mutually exclusive) choices, the order of importance was:

- Complexity of the instrument and/or difficulties in the valuation (92%).
- Uncertainty about market liquidity in the future (57%).
- Uncertainty about GDP data provided by the government (57%).
- Volatility of coupon payments (29%).
- Concerns about poorer growth incentives (7%).

The above results show that the lack of an established valuation method was a very important issue. Consistently with the previous results, data reliability appears again as an important issue. Finally, liquidity seems another major concern: even though the size of the issuance is reasonably large, potential restrictions in the participation of key players such as pension funds may adversely affect the instrument's liquidity. It is important to note that there is virtually no indication of moral hazard concerns related to poorer growth incentives: the overall benefits of higher growth are likely to exceed by large margins the higher fiscal cost due to the indexation of the warrant.

The last question compares the default risk of the GDP warrants and the one of a plain vanilla bond. It asks how likely it is that Argentina will also honor payments on the GDP warrant, assuming it honors its conventional debt. Only the first two answers, either that definitely would honor (58%) or most likely would honor (42%), were chosen by investors. Hence, the GDP warrants were not seen as a riskier instrument than conventional bonds in terms of default risk. This result implies that the premium differential observed on the warrants with respect to standard bonds is not likely to reflect default risk.

Overall, these results suggest that the residual premium identified in the paper is likely to stem from concerns related to the novelty of the instrument, its complexity of pricing, liquidity, and data accuracy. An improvement in the perception of these factors—coupled with the improved economic outlook—is likely to be behind the rapid increase in the price observed in the last year.

APPENDIX II. DISCOUNTING CASH FLOWS

As discussed in Section IV, we have two possibilities for pricing the peso GDP-linked security: either discounting cash flows using the domestic interest rate curve, or convert the peso cash flows into dollars, and then use the dollar discount rate plus a risk premium. This section shows that we the two possibilities are equivalent under standard assumptions. Let's define as $R(t)$ and $R^*(t)$ as the real interest rate in Argentina and the U.S. respectively, while $r(t)$ and $r^*(t)$ denote the respective nominal interest rates. In the first method, the price in pesos in period t is defined as:

$$P_t(AR\$) = \sum_{k=t-1}^{T} \frac{\overline{C}(Arg\$)_k}{\left(1 + r_{k+1}\right)^{k+1}}$$

While the second method simply uses the local currency nominal rate:

$$P_t(AR\$) = \sum_{k=t-1}^{T} \frac{\overline{C}(Arg\$)_k}{\left(1 + r^*_{k+1}\right)^{k+1}} \cdot \frac{ER_t}{ER_{k+1}}$$

where $C(Arg\$)_k$ is defined in the text.

If uncovered interest parity holds:

$$\left(1 + r_{t+1}^*\right)\frac{E_{t+1,t}}{ER_t} = \left(1 + r_{t+1}\right)$$

and in expectations both formulas are equivalent. However, as mentioned in the text, we do not have a curve for the nominal rate in Argentina for the long run, and the second method is preferred.

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