Measures of Fiscal Risk in Hydrocarbon-Exporting Countries

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IMF Working Paper

Middle East and Central Asia Department

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Authorized for distribution by Paul Cashin

October 2012

Abstract

The recent relatively high levels of global oil prices have led to a significant improvement in the public finances of several hydrocarbon-exporting countries. However, despite the increase in fiscal buffers, medium-term risks remain high. Fiscal vulnerabilities have increased as a consequence of the substantial spending packages that have been implemented in recent years. This has raised break-even prices—that is, the price levels that ensure that fiscal accounts are in balance at a given level of spending—in these countries. This study analyses such risks and develops measures of fiscal risk stemming from oil price fluctuations. An empirical application to hydrocarbon-exporting countries from the Middle East and North Africa region is included. Additionally, it is noted that countries with large net assets and proven oil reserves are much less vulnerable to fiscal risk than is indicated by standard measures based on break-even prices.

JEL Classification Numbers: C53, G17, H60, Q38.

Keywords: Fiscal risk; stochastic simulations; oil prices; volatility; break-even prices.

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1 Without any implication, the authors wish to thank Alberto Behar, Paul Cashin, Serhan Cevik, Raphael Espinoza, Davide Furceri, Joong Shik Kang, Maxym Kryshko, Aiko Mineshima, Tigran Poghosyan, Christine Richmond, and Pedro Rodriguez for constructive discussions and helpful comments.
I. INTRODUCTION

“What commodity prices lack in trend, they make up for in variance”. Angus Deaton (1999)

The recent relatively high levels for commodity (in particular, hydrocarbons) prices have led to a significant improvement in headline fiscal aggregates and strengthened public finances in many oil-exporting countries. However, despite the increase in fiscal buffers in oil-exporting countries, fiscal stances have been loosened in recent years owing to new spending pressures. This is particularly the case in the Middle East and North Africa (MENA) region.

The substantial spending packages that have been implemented in MENA countries over the last few years have led to an increase in fiscal vulnerability. Consequently, fiscal break-even oil prices—the price level that ensures that fiscal accounts are in balance given existing levels of spending—have been trending upward in most countries, and in some cases, are near or exceed the actual spot market oil price (Figure 1). Furthermore, the high volatility of global oil prices could pose serious challenges to fiscal balances throughout the region over the medium-term. In particular, the increases in recurrent expenditure, such as wages and salaries, in several of these countries could prove difficult to reverse, even in the case where oil prices were to fall significantly.

Figure 1: Real Break-Even Prices in Selected Middle East and North African Countries


Source: IMF staff calculations. See Table 1 for listing of country acronyms.

This study seeks to analyze such risks in a quantitative manner. For this purpose, we will develop measures of fiscal risk emanating from oil price fluctuations. These measures attempt to quantify the probability of oil prices falling below the break-even price for each individual oil-exporting country. This is done through modeling the distribution of oil prices based on

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2 These buffers reflect the significant increase in foreign assets and noticeable reduction in public debt.
their historical behavior (in particular, their volatility) and then generating stochastic simulations of the possible future paths of crude oil prices. It is worth mentioning that we do not intend to forecast any specific future price of oil, but rather to describe in a probabilistic manner the likely distribution of the path of oil prices over the coming years.

Break-even prices only consider the fiscal stance in a particular year, as it represents the crude oil price that would ensure that the level of expenditures in that year are balanced by the level of total revenues in that same year. However, the break-even price does not take into account the past accumulation (or depletion) of net assets in a country, and thus per se cannot be used as a complete measure of solvency risk. That said, for countries with limited access to global financial markets where an overall deficit cannot be easily financed, the break-even price could play a very crucial role. Despite its limitations, the break-even price is a simple yardstick of the fiscal position that is regularly used by market analysts, and is frequently picked up by the media.

Episodes of booms and busts in commodity prices generate volatility in the fiscal revenues of commodity-exporting countries. The existing literature has studied at great length the cyclical behavior of fiscal policy. Theoretically, in the absence of uncertainty and restrictions on capital mobility, an optimal fiscal policy will entail fully smoothing spending and taxes over time in order to avoid introducing intertemporal distortions. Riascos and Végh (2003), solving a Ramsey problem, show that under complete markets, the optimal path of public consumption and tax rates is constant across states of nature. Therefore, the correlation between fiscal policy (both expenditures and taxes) and output is zero.

With respect to the literature on political distortions as a source of fiscal variability, we identify the following, Tornell and Lane (1999) present the so-called “voracity effect”, in which powerful groups, when competing for a common pool of funds via the fiscal process, have access to the capital of other groups. Talvi and Végh (2005) introduce a political distortion which makes it costly for the government to run budget surpluses due to the pressures they create on public spending, thereby generating procyclical fiscal policy responses by lowering the tax rate in good times to fend off spending pressures. Alesina and Tabellini (2005) explain that political agency can lead to excessive debt accumulation when voters are uninformed. Similarly, Ilzetzki (2008) argues that polarized political environments may yield procyclical fiscal policies.

The empirical literature has typically focused on documenting the reaction of fiscal positions to the output cycle rather than directly examining commodity-price cycles, i.e., only indirectly linking commodity price fluctuations with fiscal outcomes, and looking at the impact of commodity prices solely through their possible effect on GDP. Only a few papers discuss the fiscal risk from commodity-price shocks.³ ⁴
It is generally agreed that unexpected large and persistent fluctuations in the real price of oil are detrimental to the welfare of countries that are oil-importers and, in particular, oil-producers. Therefore, attempting to forecast the price of oil has become a key issue in order to assess the medium-term macroeconomic frameworks of these countries. However, empirical evidence shows that changes in oil prices have been very difficult to predict, owing to volatile trends and increasing variances.

Much of the existing work on forecasting the price of oil has focused on testing for the existence of a predictive relationship between macroeconomic aggregates and the price of oil. Other models, such as “short-horizon forecasts” treat the price of the oil futures contract of maturity $t$ as the $t$-period forecast of the price of oil. Cheng (2010) uses a multi-lognormal approach, building on the methodology of Bahra (1997), to extract risk neutral density functions using futures and futures option pricing data. However, some recent evidence points to the lack of predictive ability of future contracts when forecasting future oil prices. Indeed, Hamilton (2008) notes that several studies found that the current spot price of oil provides as good or an even better forecast of the future actual price of oil than does the futures price.

The literature has also studied the cycles in commodities in general and oil in particular. Those studies conveyed that the behavior of commodities is better described by their variance rather than their deterministic trend; for example, Cashin and McDermott (2002) examine long-run trends in the price of different commodities, as well as the length and duration of commodity price cycles. They particularly analyze, first whether there have been changes in the trend growth rate of these prices, and second whether there have been changes in the volatility of price movements, finding that developments in commodity prices are highly volatile, making short-run movements highly unpredictable. They conclude that, in terms of its economic and statistical significance, the variability of prices completely dominates any long-run trend. Similarly, Cashin, Liang and McDermott (2000) study the persistence of commodity price shocks; they find that shocks are rather persistent for many commodities, and that the variability is quite high.

This paper implements an empirical application that quantifies the risks to the fiscal stance in several Middle East and North African hydrocarbon-exporting countries stemming from oil price volatility over the short- to medium term. This analysis can also be easily replicated for other commodity-exporting countries. This paper presents two measures of fiscal risk stemming from crude-oil-price fluctuations, based on estimating the probability that oil prices

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5 See, for example, Barsky and Kilian (2002), and Kilian and Vega (2010).

6 See, for example, International Monetary Fund (2005), p. 67; International Monetary Fund (2007), p. 42.

7 See, for example, Knetsh (2007) and Alquist and Kilian (2010). Also, for a more detailed discussion on forecasting the price of oil see Hamilton (2008) and Alquist et al. (2011).

8 These include Bopp and Lady (1991), Abosedraa and Baghestani (2004), Chinn, LeBlanc and Coibion (2005), and Alquist and Kilian (2010).
fall below the break-even price in each country. Note that here break-even prices are taken as given, and their estimation is outside of the scope of this study.

This paper is organized as follows: Section II introduces the dataset and focuses on the empirical methodology used to model and simulate the likely distribution of future oil prices. Section III presents the results and assesses the main findings, while Section IV concludes.

II. DATA AND EMPIRICAL METHODOLOGY

The main objective of this study is to construct a measure of fiscal risk stemming from oil-price volatility. For this purpose, our sample includes 11 hydrocarbon-exporting countries from the Middle East and North Africa region. Historical data ranging from 1980 to 2011, at annual frequency, are used for estimation purposes. Annual projections over the period 2012–17 are also used in parts of the analysis.

Noting that oil prices are inherently uncertain and hard to predict, this approach uses historical oil price data (in particular the mean and standard deviation) to estimate the future distribution of oil prices. For this purpose, we use the price of Brent crude oil (London market price). The latter is then deflated using the U.S. CPI series so that our entire Brent price series is in 2011 U.S. dollars per barrel. The resulting real Brent price series is presented in Figure 2.

For each country, the break-even price — that is, the price of crude oil at which the overall fiscal deficit would be equal to zero in any given year — is presented in Tables 1 and 2 in Annex I.

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9 This includes the six GCC countries (Bahrain, Kuwait, Oman, Qatar, Saudi Arabia and the United Arab Emirates) as well as Algeria, Iran, Iraq, Libya, and Yemen.

10 Alternative oil prices, such as West Texas Intermediate (WTI) and Fateh Dubai, could be easily analyzed in the same way if break-even prices for those series were available. Note, however, that these oil prices tend to exhibit a similar pattern in terms of historic volatility, and thus the results would not vary considerably from those of Brent prices presented here.

11 The U.S. CPI series was rebased so that the index is set to 100 in 2011.

12 These break-even prices are estimated by IMF country teams, covering each of the respective countries, and are the projected prices (for 2012 – 17) as of July 2012.
In order to simulate the possible distribution of future oil prices, we use a geometric Brownian motion model to describe the behavior of (the logarithm of) Brent oil prices. This type of model is also widely used to characterize the behavior of equity prices. Indeed, this model seems to capture well the ‘roughness’ observed in the path of several asset prices (such as equity and commodity prices). Nevertheless, one important drawback of this technique is that the volatility parameter — which is a key element in the quantification of the risk emanating from fluctuations in the asset in question — is assumed constant throughout the sample. In reality, it appears that oil prices tend to experience periods of relatively high volatility and periods of relatively low volatility. However, accounting for this feature adds a significant degree of complexity into the model and could be computationally costly.

Algebraically, the model can be expressed using the following stochastic differential equation:

$$dy_t = \alpha y_t dt + \gamma_t \sigma dB_t$$  \hspace{1cm} (1)

where $y_t$ is the log-price of Brent oil at time $t$, $B_t$ is a standard Brownian motion (or Wiener process), and $\alpha$ and $\sigma$ are the ‘drift’ and ‘volatility’ parameters, respectively.

The parameters $\alpha$ and $\sigma$ in equation (1) can be estimated using maximum likelihood estimation (see Annex II for details). Thus, given an initial log-price of Brent oil $y_0$ at time $t_0$, the

\[13\] For instance, it is used to model stock prices in the seminal Black-Scholes option pricing model. See Black and Scholes (1973) for details.
distribution for the future path of Brent oil prices can be generated iteratively using Monte Carlo simulations based on the formula:

\[ y_{t_n} - y_{t_{n-1}} = \left( \hat{\alpha} - \frac{1}{2} \hat{\sigma}^2 \right) (t_n - t_{n-1}) + \hat{\sigma} (B_{t_n} - B_{t_{n-1}}) \]  

(2)

where \( \hat{\alpha} \) and \( \hat{\sigma} \) are the maximum likelihood estimates of \( \alpha \) and \( \sigma \), respectively.

Then, after stochastically simulating the future distribution of crude oil prices, we estimate the probability that these Brent prices fall below the break-even price for each of the countries, for each year in our simulation horizon (2012–17). In other words, our measures of fiscal risk, stemming from the fluctuations in oil prices, are based on the probabilistic quantification of the risk of global oil prices falling below the level that would ensure a balanced budget.

Two measures of fiscal risk are constructed in this study. The first assumes that for each of the countries in our sample, the break-even price remains constant in real terms from 2011 onwards. The probabilities of the price of Brent oil falling below the break-even price (assumed constant in real terms) are presented in Table 3 (risk measure I). The advantage of such an assumption is its simplicity, as only the break-even price for 2011 is needed, and thus there is no need to forecast future break-even prices over the simulation horizon. However, historically break-even prices have tended to vary quite a deal over time, even in real terms, reflecting the obvious fact that the main budgetary aggregates (e.g. total expenditures) do vary over time according to the decisions made by policy-makers, macro-economic conditions, and oil production and extraction constraints.

A second risk measure is constructed using the break-even prices estimated by IMF country teams over our simulation horizon (2012–17) for each of the countries. This measure can be seen as a robustness check of our previous risk measure. Implicitly, this second measure depends, of course, on the assumptions regarding the path of total revenues and expenditures over this period. These break-even prices are summarized in Table 2. For each year, we calculate the probability of the price of Brent falling below these break-even price paths. The results of these simulations—which yield an alternative risk measure (measure II)—are presented in Table 4.

III. MAIN RESULTS AND DISCUSSION

Our two fiscal risk measures relate to the probability of crude oil prices falling below the break-even price for each of the countries in our sample over the period 2011–17. The main results obtained from our simulations can be summarized using Figures 3 and 4 below. The full set of results, for every year of the period 2011–17, are presented in Tables 3 and 4 in Annex II.

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14 Note that we are using annual data, thus our risk measure relates to the probability of the annual average of crude oil prices in a given year falling below the break-even price.

15 The full set of results, for every year of the period 2011–17, are presented in Tables 3 and 4 in Annex II.
(i.e. 2012) and over the medium-term (2017), assuming that break-even prices are to remain constant in real terms from 2011 onwards (measure I). Figure 4 represents the corresponding probabilities of the price of crude oil falling below the break-even prices in 2012 and 2017, but this time using the break-even prices provided by the IMF country teams (measure II).

Figure 3 shows that fiscal risks stemming from crude oil price fluctuations tend to differ considerably across the countries in the region. Furthermore, this observation remains valid throughout our simulation horizon. Indeed, the risk of Brent oil prices falling below the break-even prices in the near-term in countries such as Kuwait, Oman, Qatar, and Saudi Arabia is fairly low. Medium-term risks also remain relatively muted in these countries, with the probability of crude oil prices falling below break-even prices in 2017 still lower than 6 percent in both Kuwait and Qatar.

**Figure 3: Probability that Brent Oil Prices Fall below the Break-Even Price (measure I)**

![Bar graph showing probability of Brent oil prices falling below the break-even price for different countries in 2012 and 2017.](image)

Source: IMF staff calculations.
Note: Break-even prices are assumed to remain constant in real terms from 2011 onwards.

However, in countries such as Yemen or Libya, where Brent oil prices were already below their break-even price in 2011, the probability that this occurs again remains quite high throughout the simulation horizon. In other words, these countries could face sizeable fiscal challenges over the near– and medium-terms owing to their relatively high break-even prices.

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15 Note that we are dealing here in terms of *annual averages* (and thus these results do not refer to the probability of Brent oil prices falling below the break-even price on any given day, or during a portion of the year).

16 Note that in those countries, that probability was equal to 100 percent in 2011 (see Table 3). The average price of Brent in 2011 was US$111.32 per barrel.
Interestingly, in countries where the price of Brent was markedly above the break-even price in 2011 (e.g. Qatar, Kuwait, and Saudi Arabia), the probability that Brent prices fall below the break-even price increases through time. Conversely, in countries where the break-even prices were above the price of Brent in 2011 (e.g. Bahrain, Libya and Yemen), such a probability tends to fall over time (Figure 3). These results are both a consequence of the fact that uncertainty increases with time in our model—and thus, as expected, the distribution of the possible paths for the price of Brent oil widens throughout our simulation horizon.

Turning to our second measure of fiscal risk stemming from oil price fluctuations (Figure 4), it seems that the short-term results are broadly similar to those obtained under our first measure. Kuwait, Qatar, and Saudi Arabia once again appear to be the most resilient to possible fluctuations in the price of oil in 2012, whilst Bahrain and Yemen appear to be among the most vulnerable countries in our sample. This result is a direct consequence of the fact that the 2012 break-even prices estimated by IMF country teams do not differ substantially from the 2011 break-even prices in real terms. Libya is, however, a noticeable exception. Mainly driven by the political situation in the country, the break-even price in 2011 in Libya was US$183.5 per barrel, whilst the break-even price is estimated to fall to US$88.5 per barrel in 2012 as domestic conditions stabilize.

**Figure 4: Probability that Brent Oil Prices Fall below the Break-Even Price (measure II)**

![Figure 4: Probability that Brent Oil Prices Fall below the Break-Even Price (measure II)](source: IMF staff calculations.
Note: These simulations use the break-even prices provided by IMF country teams (presented in Table 2). *Break-even prices for Iran and Yemen were only available up to 2012.

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17 The correlation between measures I and II for the year 2012 is very high at 0.93.
Nevertheless, when looking at the medium-term, the picture appears to be slightly different. For instance, our second risk measure suggests that countries such as Iraq and the United Arab Emirates (U.A.E) are likely to be less vulnerable in 2017 compared to countries such as Qatar and Saudi Arabia (Figure 4). This was not the case under risk measure I (Figure 3). In fact, in both the U.A.E. and Iraq, break-even prices are estimated (by the IMF country teams) to fall markedly – even in nominal terms – between 2011 and 2017, hence resulting in a much lower probability of oil prices falling below the break-even prices relative to measure I (which assumes break-even prices constant in real terms from 2011 to 2017). Though, in general, it seems that countries that had a relatively high break-even price in 2011 are likely to have a relatively high break-even price in 2017.18 Thus, on the whole, the results obtained using our second risk measure tend to be in agreement with those obtained using our first risk measure.

Although the aim of this paper is not to assess the level of break-even prices, including their estimation and their usefulness since such prices are taken here as given, it is nevertheless worth mentioning a couple of issues that could be relevant when using the fiscal risks measures developed in this paper. First, as currently defined, the break-even price only takes into account the fiscal deficit in a single year. It does not take into account the dynamics of the deficit (i.e. flows) in previous years or in years to come, and more importantly, it neglects the accumulation of assets or liabilities (i.e. stocks) to date. This is a crucial point as two countries might have the same break-even prices for a given year, however one of them might have significantly higher (already accumulated) wealth combined with, possibly, much larger oil reserves in the ground that can be exploited over many years to come, rendering it more sustainable fiscally than the other country – despite both having the same “fiscal risk measure” in that particular year.19 In addition, by virtue of only considering one period at the time, break-even prices do not take into account the temporal nature of the oil price shock —that is, whether the price change is temporary or permanent—which in principle could entail a different policy response.

Secondly, as currently estimated, break-even prices assume that fiscal expenditures and non-oil revenues will remain on a pre-determined path independently of the actual path of oil prices. In a sense, these budgetary aggregates are treated as exogenous to the price of oil. In reality, although some of these variables (e.g. current expenditures) tend to exhibit some degree of rigidity, one would expect that policymakers would adjust their fiscal plans if oil prices turn out to be much lower (or higher) than previously expected, hence affecting the break-even price. In other words, break-even prices should be seen as endogenous to the actual price of crude oil, but estimating how the former should respond to the latter is not a straight forward exercise, and is beyond the scope of this paper.

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18 The correlation between the estimated break-even prices in 2011 and 2017 is positive and equal to 0.54.

19 In our sample, Oman and Saudi Arabia illustrate this point well. Although Oman appears to be just slightly more vulnerable than Saudi Arabia on our two risk measures, Saudi Arabia has significantly larger net assets, and its proven oil reserves are almost 50 times larger than those of Oman. Hence, in reality, one would expect Saudi Arabia to be much less vulnerable (when compared with Oman) than what our risk measures based on break-even prices would suggest.
Overall, it seems that some countries are relatively more resilient than others to fluctuations in global oil prices, both in the short- and in the medium-term. For instance, Kuwait and Qatar appear to be among the most resilient countries in our sample, whilst Yemen, Algeria and Bahrain appear to be among the most vulnerable. Under both our risk measures, the latter three countries exhibit a probability higher than 40 percent of oil prices falling below their break-even prices throughout the entire simulation horizon (2012–17).

**IV. CONCLUSION**

This paper presented two measures of fiscal risk for hydrocarbon-exporting countries stemming from crude oil price fluctuations. These measures are based on estimating the probability that crude oil prices fall below the break-even prices in each country. The first measure assumes that crude oil prices remain constant in real terms from 2011 onwards, whilst the second measure utilizes the break-even prices estimated by IMF country teams over the period 2012–17.

Importantly, and as mentioned earlier, our analysis does not intend to forecast a specific path for the price of oil in the future. Instead, it uses a probabilistic approach in order to quantify the future distribution for all the possible future paths of crude oil prices based on their historical pattern, in particular their volatility. For this purpose, we used a geometric Brownian motion model. Furthermore, for each country the break-even price — that is the price of crude oil that renders the fiscal balance equal to zero on a particular year — is taken as given.

Our study finds that, under our two measures of fiscal risk, some countries are relatively more resilient than others to fluctuations in global oil prices, both in the short and in the medium-term. For instance, Kuwait and Qatar appear to be among the most resilient countries in our sample, whilst Yemen, Algeria and Bahrain appear to be among the most vulnerable.

This relative quantification of fiscal risks suggests that those countries that appear to be more vulnerable to oil price volatility should pay greater attention to the conduct of fiscal policy in the short- to medium term. Such fiscal consolidation could come about chiefly by reducing expenditure pressures that might have arisen following temporary events, and should aim at building larger fiscal buffers in the event that some of these fiscal risks do materialize in the future.
# ANNEX I: SUMMARY TABLES

## Table 1: Projected Nominal Break-Even Prices in Selected MENA Oil-Exporting Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>ALG</th>
<th>BHR</th>
<th>IRN</th>
<th>IRQ</th>
<th>KWT</th>
<th>LBY</th>
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<th>SAU</th>
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<tr>
<td>Break-even prices (in U.S. dollars per barrel):</td>
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<tr>
<td>2011</td>
<td>104.7</td>
<td>113.7</td>
<td>102.0</td>
<td>95.0</td>
<td>43.8</td>
<td>183.5</td>
<td>75.5</td>
<td>38.0</td>
<td>70.0</td>
<td>92.3</td>
<td>195.0</td>
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<td>2012</td>
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<td>119.1</td>
<td>117.0</td>
<td>112.0</td>
<td>47.2</td>
<td>88.5</td>
<td>82.0</td>
<td>40.4</td>
<td>74.0</td>
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<td>237.0</td>
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<td>93.9</td>
<td>69.3</td>
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</tbody>
</table>

Source: IMF staff calculations.  
Note: ALG denotes Algeria; BHR (Bahrain); IRN (Iran); IRQ (Iraq); KWT (Kuwait); LBY (Libya); OMN (Oman); QAT (Qatar); SAU (Saudi Arabia); UAE (United Arab Emirates); YMN (Yemen). Nominal Brent oil prices averaged US$111.32 per barrel in 2011.

## Table 2: Projected Real Break-Even Prices in Selected MENA Oil-Exporting Countries

<table>
<thead>
<tr>
<th>Country</th>
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<td>-</td>
<td>71.1</td>
<td>51.8</td>
<td>91.5</td>
<td>94.8</td>
<td>75.5</td>
<td>82.5</td>
<td>65.6</td>
<td>-</td>
</tr>
<tr>
<td>2017</td>
<td>99.5</td>
<td>114.2</td>
<td>-</td>
<td>67.2</td>
<td>50.5</td>
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<td>100.4</td>
<td>77.0</td>
<td>83.4</td>
<td>61.5</td>
<td>-</td>
</tr>
</tbody>
</table>

Source: IMF staff calculations.  
Note: 2011 is taken as the base year; U.S. CPI inflation is assumed to be 2.0 percent annually over the period 2012-17. Real Brent oil prices averaged US$ 111.32 per barrel in 2011 (in 2011 U.S. dollars).
### Table 3: Probability that Brent Oil Prices Fall below the Break-Even Price (measure I)

<table>
<thead>
<tr>
<th>Country</th>
<th>ALG</th>
<th>BHR</th>
<th>IRN</th>
<th>IRQ</th>
<th>KWT</th>
<th>LBY</th>
<th>OMN</th>
<th>QAT</th>
<th>SAU</th>
<th>UAE</th>
<th>YMN</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>0.0</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>39.8</td>
<td>52.8</td>
<td>35.7</td>
<td>25.7</td>
<td>0.0</td>
<td>97.7</td>
<td>5.7</td>
<td>0.0</td>
<td>3.0</td>
<td>22.3</td>
<td>98.8</td>
</tr>
<tr>
<td>2013</td>
<td>42.5</td>
<td>51.6</td>
<td>39.4</td>
<td>31.9</td>
<td>0.4</td>
<td>92.0</td>
<td>13.0</td>
<td>0.1</td>
<td>9.0</td>
<td>29.1</td>
<td>94.2</td>
</tr>
<tr>
<td>2014</td>
<td>43.5</td>
<td>51.0</td>
<td>40.9</td>
<td>34.7</td>
<td>1.4</td>
<td>87.2</td>
<td>17.7</td>
<td>0.6</td>
<td>13.5</td>
<td>32.4</td>
<td>89.9</td>
</tr>
<tr>
<td>2015</td>
<td>44.0</td>
<td>50.6</td>
<td>41.9</td>
<td>36.5</td>
<td>2.8</td>
<td>83.6</td>
<td>20.9</td>
<td>1.4</td>
<td>16.8</td>
<td>34.4</td>
<td>86.4</td>
</tr>
<tr>
<td>2016</td>
<td>44.4</td>
<td>50.3</td>
<td>42.5</td>
<td>37.6</td>
<td>4.3</td>
<td>80.8</td>
<td>23.3</td>
<td>2.4</td>
<td>19.3</td>
<td>35.7</td>
<td>83.6</td>
</tr>
<tr>
<td>2017</td>
<td>44.7</td>
<td>50.1</td>
<td>42.9</td>
<td>38.4</td>
<td>5.9</td>
<td>78.5</td>
<td>25.1</td>
<td>3.6</td>
<td>21.3</td>
<td>36.8</td>
<td>81.2</td>
</tr>
</tbody>
</table>

Source: IMF staff calculations.

Note: Break-even prices are assumed to remain constant in real terms from 2011 onward.

### Table 4: Probability that Brent Oil Prices Fall below the Break-Even Price (measure II)

<table>
<thead>
<tr>
<th>Country</th>
<th>ALG</th>
<th>BHR</th>
<th>IRN</th>
<th>IRQ</th>
<th>KWT</th>
<th>LBY</th>
<th>OMN</th>
<th>QAT</th>
<th>SAU</th>
<th>UAE</th>
<th>YMN</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>0.0</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>2012</td>
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<td>57.1</td>
<td>54.3</td>
<td>47.3</td>
<td>0.0</td>
<td>15.5</td>
<td>9.3</td>
<td>0.0</td>
<td>4.1</td>
<td>10.4</td>
<td>99.8</td>
</tr>
<tr>
<td>2013</td>
<td>56.9</td>
<td>50.7</td>
<td>-</td>
<td>27.1</td>
<td>1.3</td>
<td>31.9</td>
<td>17.1</td>
<td>6.3</td>
<td>17.2</td>
<td>15.3</td>
<td>-</td>
</tr>
<tr>
<td>2014</td>
<td>49.7</td>
<td>48.1</td>
<td>-</td>
<td>23.8</td>
<td>4.5</td>
<td>33.7</td>
<td>24.5</td>
<td>5.7</td>
<td>21.0</td>
<td>17.4</td>
<td>-</td>
</tr>
<tr>
<td>2015</td>
<td>45.4</td>
<td>47.5</td>
<td>-</td>
<td>22.1</td>
<td>5.7</td>
<td>35.1</td>
<td>34.5</td>
<td>14.1</td>
<td>24.2</td>
<td>16.6</td>
<td>-</td>
</tr>
<tr>
<td>2016</td>
<td>42.7</td>
<td>48.4</td>
<td>-</td>
<td>20.1</td>
<td>8.0</td>
<td>35.0</td>
<td>37.5</td>
<td>23.3</td>
<td>28.5</td>
<td>16.2</td>
<td>-</td>
</tr>
<tr>
<td>2017</td>
<td>41.5</td>
<td>50.3</td>
<td>-</td>
<td>19.4</td>
<td>9.2</td>
<td>40.4</td>
<td>42.0</td>
<td>26.2</td>
<td>30.6</td>
<td>15.6</td>
<td>-</td>
</tr>
</tbody>
</table>

Source: IMF staff calculations.

Note: Break-even prices as estimated by IMF country teams (presented in Table 2).
ANNEX II: MODELING OIL PRICES USING GEOMETRIC BROWNIAN MOTION

To characterize the behavior of the Brent oil price we use a geometric Brownian motion model. Here, we assume that the log-price of Brent satisfies the following stochastic differential equation:

$$dy_t = \alpha y_t dt + y_t \sigma dB_t$$  \[A1\]

where $y_t$ is the log-price of Brent oil at time $t$, $B_t$ is a standard Brownian motion (or Wiener process).

In order to estimate the parameters $\alpha$ and $\sigma$ above through Maximum Likelihood Estimation (MLE) we need to specify the corresponding likelihood function. For that purpose, and for a given set of time series data of log-prices $y_{t_1}, ..., y_{t_N}$, we have that:

$$y_{t_n} - y_{t_{n-1}} = \left(\alpha - \frac{1}{2} \sigma^2\right) (t_n - t_{n-1}) + \sigma (B_{t_n} - B_{t_{n-1}}).$$  \[A2\]

The likelihood function $f(y_{t_1}, ..., y_{t_N}; \alpha, \sigma^2)$ can be defined by:

$$f(y_{t_1}, ..., y_{t_N}; \alpha, \sigma^2) = \prod_{n=1}^{N} f(y_{t_n}|y_{t_{n-1}}; \alpha, \sigma^2)$$  \[A3\]

using the convention $f(y_{t_n}; \alpha, \sigma^2) = f(y_{t_n}|y_{t_0}; \alpha, \sigma^2)$, and where the conditional probability in equation [A3] is given by:

$$f(y_{t_n}|y_{t_{n-1}}; \alpha, \sigma^2) = [2 \pi \sigma^2 (t_n - t_{n-1})]^{-1/2} \exp\left\{\frac{-(y_{t_n} - y_{t_{n-1}} - (\alpha - \frac{1}{2} \sigma^2) (t_n - t_{n-1}))^2}{2 \sigma^2 (t_n - t_{n-1})}\right\}.$$  \[A4\]

Thus, the resulting log-likelihood becomes:

$$L(\alpha, \sigma^2) = -\frac{1}{2} \sum_{n=1}^{N} \left\{ \log[2\pi \sigma^2 (t_n - t_{n-1})] + \frac{[y_{t_n} - y_{t_{n-1}} - (\alpha - \frac{1}{2} \sigma^2) (t_n - t_{n-1})]^2}{2 \sigma^2 (t_n - t_{n-1})}\right\}.$$  \[A5\]

In particular, if $t_n = n$, equation [A5] can be simplified as:

$$L(\alpha, \sigma^2) = -\frac{1}{2} \sum_{n=1}^{N} \left\{ \log[2\pi \sigma^2] + \frac{[y_n - y_{n-1} - (\alpha - \frac{1}{2} \sigma^2)]^2}{\sigma^2}\right\}.$$  \[A6\]

Now, differentiating the above log-likelihood function with respect to $\alpha$ and $\sigma^2$, we obtain:

$$\frac{\partial L(\alpha, \sigma^2)}{\partial \alpha} = \frac{1}{\sigma^2} \sum_{n=1}^{N} \left[ y_n - y_{n-1} - \left(\alpha - \frac{1}{2} \sigma^2\right) \right]$$  \[A7\]

and
\[
\frac{\partial L(\alpha, \sigma^2)}{\partial \sigma^2} = \frac{1}{\sigma^2} \sum_{n=1}^{N} \left\{ 1 - \frac{1}{\sigma^2} \left[ y_n - y_{n-1} - \left( \alpha - \frac{1}{2} \sigma^2 \right) \right]^2 + \left[ y_n - y_{n-1} - \left( \alpha - \frac{1}{2} \sigma^2 \right) \right] \right\} \text{.} \tag{A8}
\]

Hence, the values of to $\alpha$ and $\sigma^2$ ($\hat{\alpha}$ and $\hat{\sigma}^2$, respectively) which maximize the log-likelihood function [A6] satisfy the first order conditions:

\[
\frac{\partial L(\hat{\alpha}, \hat{\sigma}^2)}{\partial \alpha} = 0 \quad \text{and} \quad \frac{\partial L(\hat{\alpha}, \hat{\sigma}^2)}{\partial \sigma^2} = 0 \quad \text{[A9]}
\]

Therefore, solving the system defined by equations [A7], [A8] and [A9], we obtain that:

\[
\hat{\alpha} = \frac{1}{2} \hat{\sigma}^2 + \frac{1}{N} \sum_{n=1}^{N} (y_n - y_{n-1}) \quad \text{[A10]}
\]

and

\[
\hat{\sigma}^2 = \frac{1}{N} \sum_{n=1}^{N} (y_n - y_{n-1})^2 + \left[ \frac{1}{N} \sum_{n=1}^{N} (y_n - y_{n-1}) \right]^2 \quad \text{[A11]}
\]

Note that the expression for $\hat{\alpha}$ and $\hat{\sigma}^2$ in [A10] and [A11] are functions of the time series data $y_1, y_2, \ldots, y_N$ only, and can be easily computed. Based on our sample, the estimated values of these two parameters are $\hat{\alpha} = 0.0342$ and $\hat{\sigma}^2 = 0.0617$. The results of the simulations for the real log-price of Brent oil using these parameters are summarized in Annex III.
Figure 5 summarizes the distribution of possible paths for the real log-price of Brent oil, obtained through Monte Carlo simulations based on the methodology described in Annex II.

**Figure 5: Historic and Simulated Real Price of Brent Oil (in logarithm)**

Source: IMF staff calculations.
Note: The simulation starts in 2012, and the confidence intervals (percentiles) shown above are based on one million repetitions.
REFERENCES


