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Lower for Longer:
Neutral Rates in the United States

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Neutral Rates in the United States

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Abstract

We use a semi structural model to estimate neutral rates in the United States. Our Bayesian estimation incorporates prior information on the output gap and potential output (based on a production function approach) and accounts for unconventional monetary policies at the ZLB by using estimates of "shadow" policy rates. We find that our approach provides more plausible results than standard maximum likelihood estimates for the unobserved variables in the model. Results show a significant trend decline in the neutral real rate over time, driven only in part by a decline in potential growth whereas other factors (including excess global savings) matter. Neutral rates likely turned negative during the Global Financial Crisis and are expected to increase only gradually looking forward.

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Keywords: Neutral interest rate, monetary policy

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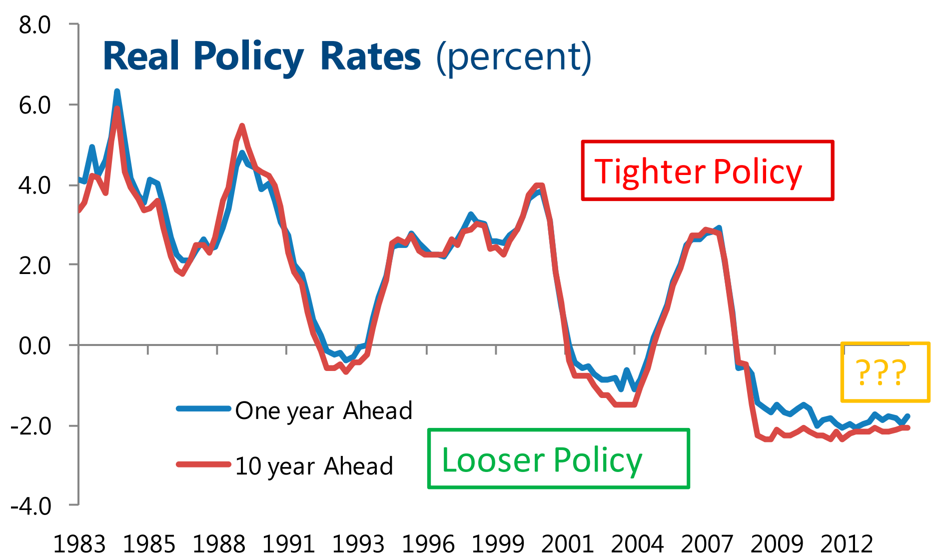
"*The natural rate is an abstraction; like faith, it is seen by its works.*" (Williams, J., 1931)

"*There is a certain rate of interest on loans which is neutral in respect to commodity prices [...] This is necessarily the same as the rate which would be determined by supply and demand if no use were made of money*." (Wicksell, 1898)

I. Introduction

Real interest rates in the United States have been declining for some time, reflecting a trend decline that took shape already before the global financial crisis (GFC) and policy rates that have been close to zero over the past several years, bringing current real policy rates to negative territory (at around minus 2 percent) (Chart 1). Long-term interest rates have seen a similar decline over time. This trend decline is likely to reflect, in part, global factors, such as higher savings in emerging markets, higher demand for safe assets as well as lower investment in advanced economies (IMF, 2014). Following the GFC it may also reflect persistent "headwinds" from the crisis: tighter underwriting standards, restricted access to credit (in particular mortgage credit), deleveraging by households, contractionary fiscal policy, and increased uncertainty about the economic outlook (Yellen, 2014 and CBO, 2014).

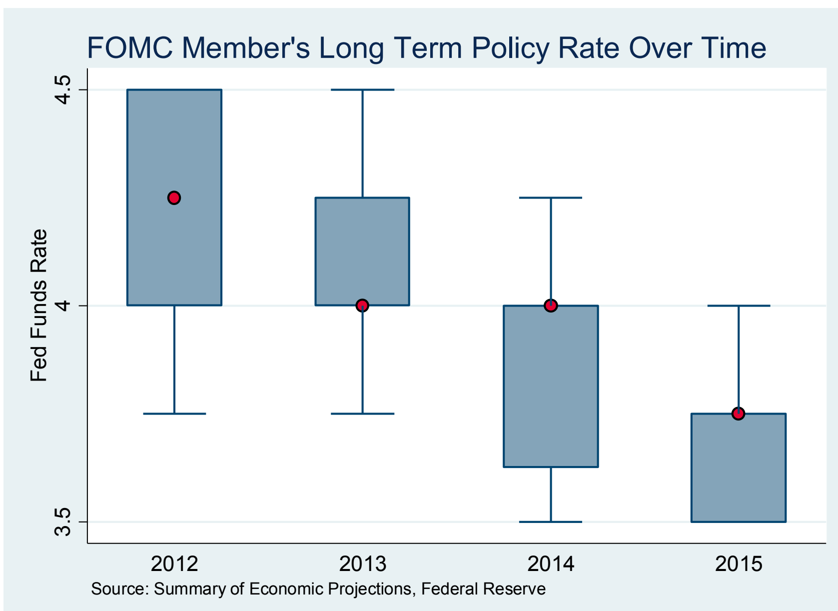
Chart 1.



The same factors that have contributed to the trend decline in observed real rates may also have resulted in a decline in unobserved "neutral" or equilibrium policy rates. Indeed, citing persistent "headwinds" Federal Reserve Chair Yellen has argued that: *"…the equilibrium real federal funds rate is at present well below its historical average and is anticipated to rise only gradually over time as the various headwinds that have restrained the economic recovery continue to abate. If incoming data support such a forecast, the federal funds rate should be normalized, but at a gradual pace*" (Chair Yellen, March 27, 2015).

Going beyond the current neutral rate, professional forecasters and the FOMC members have been revising down their estimate of the long-term expectations of the fed funds rate (Chart 2). As recently as in 2012, the median of the FOMC member's median projection for the long-term fed funds was 4.25 percent, with relatively little dispersion in individual views. The median has since then declined and is now (March 2015) at 3.75 percent in nominal terms.

Chart 2



Note: Red dots indicate medians; the box represents the interquartile range and "whiskers" represent the bottom and top 10 percent. 2012 is January 2012 (first publication of "dots"), remaining years are based on March observations.

To better assess monetary policy stance, in this paper, we use a semi-structural model to provide estimates of how the neutral rate has evolved over time and construct interest rate gaps to assess policy stance. While there are subtle conceptual differences between terms used in the literature, e.g. the "equilibrium", "natural" or "neutral" rates, we consider the neutral rate as a measure of the real rate that, broadly speaking, is consistent with output at potential and price stability. This is true in benchmark new Keynesian models that do not include a policy trade-off between stabilization of inflation and the output gap, while it can be argued that the neutral rate also provides a useful benchmark measure of policy stance in more general models (Curdia *et al.*, 2015 and Barsky *et al.*, 2014).[1] We consider our empirical measures of interest rate gaps, i.e. the difference between observed real rates and estimated neutral rates as a useful, albeit incomplete, summary indicator of monetary policy stance.[2]

The empirical framework we use is based on the seminal work of Laubach and Williams (2003). They apply a Kalman filter to jointly estimate several unobserved variables – the neutral interest rate, potential output and trend growth – linking neutral rate closely to estimated trend growth. They find significant variation in the estimated neutral rate over time, consistent with variation in trend growth, but also note that estimates are imprecise and subject to potential mis-measurement in real time (a point echoed by Clark and Kozicki, 2005, based on a similar model. See also Trehan and Wu, 2007). Alternatives to using a semi-structural model, a la Laubach and Williams, include measures based on simpler, including uni-variate, time-series methods, measures based on full-blown Dynamic Stochastic General Equilibrium (DSGE) models; and measures that use additional information from the yield curve and other financial variables (see Giammarioli and Valla, 2004). The main advantage of a semi-structural model over a simple statistical approach (such as a uni-variate filter) is that by imposing mild theoretical restrictions the model is able to exploit information from other variables, most notably inflation and output, in the estimation. The advantage over a DSGE model, such as in Barsky et al. (2014) and Curdia et al. (2015), is that the semi-structural models does not impose strong theoretical restrictions that are more prone to misspecification, especially in the presence of near-nonstationarity in observed real rates. Moreover, neutral rates derived from DSGE models also tend to show more variability than is commonly attributed to neutral rates. Hamilton et al. (2015) take a narrative approach and use international real rate data over a long time period to study the link between the equilibrium real rate and trend growth, concluding that the relationship is not empirically as strong as predicted by theory. Their results confirm significant uncertainty about the equilibrium real rate.

Our contribution to the literature is twofold. First, we use a Bayesian approach, which allows us to incorporate prior information on the output gap and potential output (based on a production function approach). We find that our approach provides more plausible results than standard maximum likelihood estimates for the unobserved variables in the model,

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[1] It is important to note that further complications arise in the presence of the zero lower bound (ZLB), where considering the neutral rate requires ability of the central bank to influence inflation expectations through e.g. forward guidance.

[2] Important dimensions that may matter for policy stance that are not explicitly considered in our empirical model include measures of monetary and financial conditions. The paper also abstracts from possible policy responses to address financial stability risks.

including trend potential growth, output gap and the neutral rate. In related research, Clark and Kozicki (2005) compare estimates from the Laubach and Williams (2003) model to those from a simplified model that is based on the potential output as estimated by the Congressional Budget Office (CBO, 2001) and find that while results are broadly comparable, there are periods when differences across the two sets of estimates can be large. However, instead of taking an outside estimate of potential output as is, we use that information to constrain our estimates for potential growth and the output gap to a range of reasonable values.

Second, we account for additional monetary policy accommodation through unconventional policies by using “shadow” policy rates. Observed policy interest rates have been constrained by the zero lower bound (ZLB) since the onset of the GFC and the Federal Reserve has employed unconventional policies, such as forward guidance and asset purchases to provide further policy accommodation. As a result, the gap between observed policy rates and the neutral rate derived from a model that relies on them may not be sufficient to describe the overall stance of monetary policy. The rate gap constructed using the real shadow policy rate, instead, provides an alternative measure of monetary policy stance that captures the impact of unconventional policies. To account for the sensitivity to specific modeling assumptions involved in the construction of shadow policy rates we take an average of shadow rates from three studies that employ different methods to estimate shadow rates. Lombardi and Zhu (2014) construct a model free, dynamic factor model based estimate using several indicators related to monetary policy stance (such as interest rates, monetary aggregates, and components of the Federal Reserve’s balance sheet). Both Krippner (2013) and Wu and Xia (2014) use a dynamic term structure model that relaxes the ZLB constraint and thus obtain policy rates that account for the compression of term premia in longer term rates attributable to Federal Reserve asset purchases and allow the policy interest rate to be negative. While the results vary across models, all models point to strongly negative shadow policy rates during the GFC, averaging about minus two percent at the end of 2014.

We find three main results. First, together with a decline in the trend potential growth rate and a significant fall in the output gap during the GFC, the neutral rate has also declined over time. While the range of estimates is relatively large, estimates suggest that the neutral real rate at the end of 2014 was close to zero. Projections of the neutral rate, conditional on the WEO forecast for output, inflation and policy rates for the next five years, suggest that the neutral rate is likely to increase only very gradually looking forward and to stay well below the FOMC participants’ median forecast for the long term real policy rate (at about 1.75 percent). Second, despite the decline in the neutral rate, we assess that monetary policy has been strongly accommodative, especially when taking unconventional monetary policy into account. Re-estimating the model with a shadow policy rate suggests a lower neutral rate, but given the larger implied interest rate gaps, also more policy accommodation, during the GFC. Third, the results show that the significant trend decline in the neutral real rate over time was driven only in part by a decline in the trend growth rate. Other factors, including excess global savings (proxied by current account surpluses in emerging markets) and an increase in the equity premium after the GFC, appear to have played an important role in explaining the decline.

The rest of the paper is organized as follows. In Section II we describe the underlying model and our Bayesian estimation. In Section III we report on the results, first based ont eh baseline model, followed by the model with shadow rates, and discuss both implications for assessing monetary policy stance using rate gaps and determinants of trends in estimated neutral rates. We summarize our results and conclude with policy implications in Section IV.

II. Empirical Framework

In this section, we re-estimate and extend the baseline Laubach and Williams (2003) model. The core system includes an IS-curve which relates output gap to interest rates gaps, a backward looking Phillips curve which relates core inflation to the output gap; and an equation that links the neutral rate to its determinants:

(IS equation) $$x_t = a_1 x_{t-1} + a_2 x_{t-2} - a_r(r_{t-1} - r^n_{t-1} + r_{t-2} - r^n_{t-2}) + \epsilon^s_t$$

(Phillips curve) $$\pi_t = \sum_{j=1}^{8} b_j \pi_{t-j} + b_y x_{t-1} + b_i \pi^m_{t-1} + b_o \pi^o_{t-1} + \epsilon^p_t$$

where x is the output gap defined as the difference between actual and potential log-output $(y_t - y^n_t)$; r is the real (policy) interest rate, r^n is the neutral rate, π is core inflation; π^m, π^o are oil and non-oil import relative price inflation, respectively, while $\epsilon^s_t, \epsilon^p_t$ are i.i.d. shocks. Potential output growth is a random walk $g_t = g_{t-1} + \epsilon^g_t$ (trend potential output growth) plus a noise component ϵ^n_t.[3] The evolution of the neutral rate is governed by trend potential output growth g_t and an exogenous process z_t

(Neutral rate) $$r^n_t = c g_{t-1} + z_t.$$

where c is a positive constant. The exogenous autoregressive process z_t has the difficult task of capturing all possible neutral rate determinants that are not captured by trend growth. While some factors are clearly unobservable or hard to capture (such as shifts in time preferences due to demographics, aggregation effects, or financial frictions) other factors that have potentially affected the neutral rate—but that are approximately orthogonal to inflation and output growth—are observable.[4] Hence, we postulate a process for z that includes a

[3] When the real policy rate is equal to the neutral rate at all times, in absence of transitory disturbances, the output gap will be closed and inflation will be stable.

[4] In order to determine what other variables to include in the z process, we ran preliminary regressions with several candidate variables, including measures of global savings (current account surplus of emerging economies in terms of US GDP and global official reserves), uncertainty (realized and implied equity volatility and policy uncertainty) , as well as preference for safe assets (equity premium). Three variables are statistically

(continued…)

measure of global savings S (proxied by the current account surplus of emerging and developing economies in terms of US GDP) and the equity premium E_t and a measure of policy uncertainty P_t (both trying to capture increased demand for safe assets owing to tighter regulatory requirements, portfolio shifts, or higher consumption uncertainty):

(z-process) $$z_t = d_1 z_{t-1} + d_2 z_{t-2} - d_c \Delta S_t - d_e \Delta E_t - d_p \Delta P_t + \epsilon_t^z.$$

This formulation broadly reflects some of the most important determinants of the decline in the global real rate found by the literature (Blanchard et al. 2015, IMF 2014).

A maximum likelihood estimation of the above system, in its basic version without observable determinants for z, has three key shortcomings. First, estimates of the output gap would differ substantially from the ones derived by alternative approaches such as production function approaches used by the CBO. In particular, updated estimates from the original Laubach and Williams work give a positive output gap already in 2014 and a negative one for the majority of the '80s and '90s—in contrast with most alternative estimates of the output gap (see Chart 2).[5] Second, it does not account for the ZLB on nominal policy interest rates. Nominal rates at zero underestimate actual monetary policy accommodation in the aftermath of the GFC and are likely to induce an upward bias to estimates of neutral rates that are based on observed correlations between nominal interest rates and activity in the recent period. Three, the model relates the neutral rate closely to the trend growth rate while allowing for other factors (z) to play a role. At the same time, the system in its basic form may be not sufficiently informative to back-out, with a satisfactory degree of precision, the process z.

We use Bayesian methods to estimate the model's parameters (see Appendix B). Specifically, we set our prior density function on the smoothed estimate of the (unobservable) state vector of the model by incorporating prior information on the output gap and potential output growth based on a production function approach (CBO, 2001). Since there is a mapping between the inferred-state-vector space and the parameter space we can sample from the posterior using standard methods (i.e., through Monte Carlo Markov Chain). In practice, the prior density on the smoothed state vector is a mixture of uninformative priors and uniform distributions, which is equivalent to imposing bounds on the evolution of the smoothed estimates of the output gap over certain historical time periods (Chart 3).

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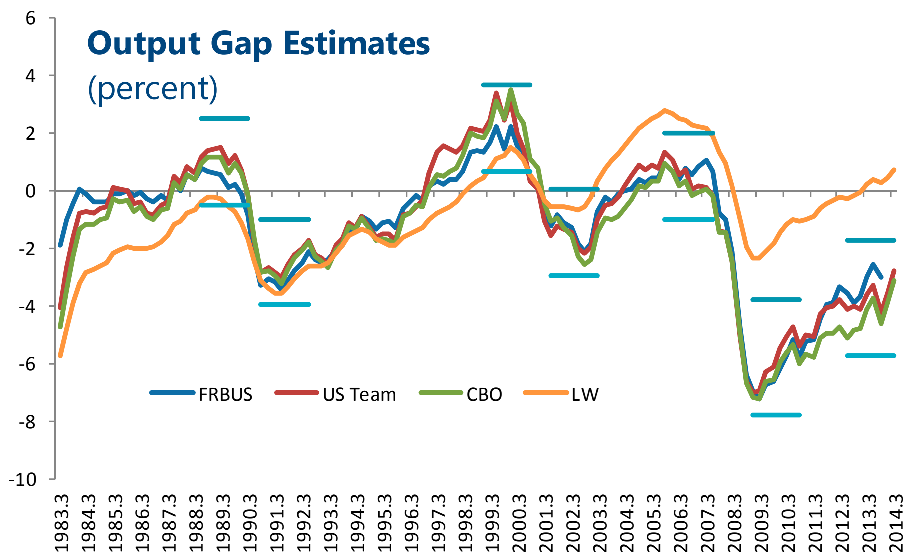
significant and are included in the estimation: (1) current account surplus of emerging economies in terms of US GDP; (2) policy uncertainty as measured by the Bloom index; and (3) a summary indicator of equity premia as estimated by Duarte and Rosa (2015).

[5] Updated estimates of the original Laubach and Williams (2003) paper are available at http://www.frbsf.org/economicresearch/economists/john-williams/Laubach_Williams_updated_estimates.xlsx

The constraints we impose are relatively minor. For the output gap they reflect bounds of +/- 1.5 percentage point around the average output gap during 2 year periods around NBER peaks and troughs. Reflecting higher uncertainty about potential growth during GFC we allow for larger (+/- 2 percentage point) bands during and after the crisis. The constraints exclude updated Laubach and Williams estimates during the pre-crisis boom, the GFC and in post crisis periods. However, the bands always include FRB/US estimates of the output gap. We also impose mild constraints on potential growth that reflects the prior that potential growth was lower in the 2000s than in the 1990s.

Chart 3.



Note: LW refers to updated Laubach and Williams (2001) estimates available from the San Francisco Federal Reserve website. Blue lines reflect the constraints.

The parameter estimates for the baseline model show that trend growth rates is associated with the neutral rate even though the c coefficient is not precisely estimated. The z-process is highly persistent especially when estimated without observable factors—the sum $d_1 + d_2$ is about 1. Indeed, both the global saving proxy and the equity premium are significantly different from zero and, in part, help explain the trend decline in the neutral rate.

Table 1. Parameter estimates

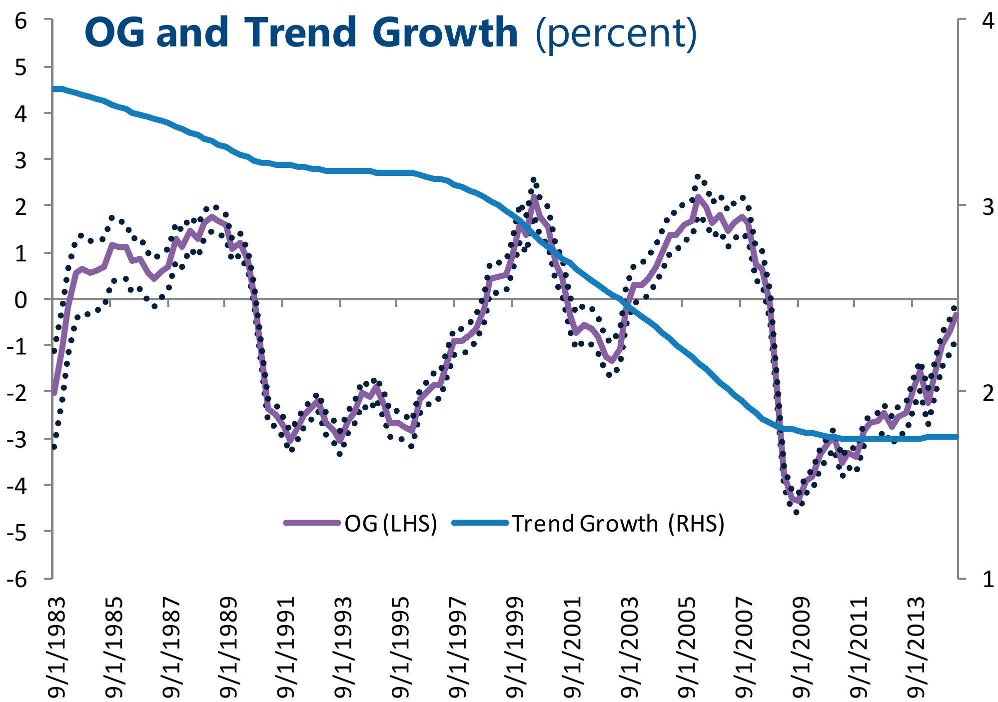
	with z-determinants			without z-determinants		
		percentile			percentile	
Parameter	10th	Median	90th	10th	Median	90th
c	0.23	0.58	0.98	0.21	0.90	1.68
d_1	0.58	1.19	1.47	1.01	1.27	1.47
$d_1 + d_2$	0.959	0.989	0.998	0.990	0.997	1.000
a_1	1.23	1.25	1.26	1.24	1.26	1.26
a_r	0.05	0.06	0.08	0.04	0.05	0.07
d_c	0.20	0.32	0.41	-	-	-
d_e	0.01	0.07	0.21	-	-	-
d_p	-0.01	0.00	0.01	-	-	-
$\sigma(\varepsilon^n)$	0.25	0.27	0.29	0.23	0.27	0.29
$\sigma(\varepsilon^s)$	0.44	0.49	0.53	0.48	0.52	0.58
$\sigma(\varepsilon^p)$	1.57	1.69	1.83	1.58	1.70	1.81
$\lambda_g = \sigma(\varepsilon^g)/\sigma(\varepsilon^n)$	0.23	0.26	0.29	0.27	0.29	0.30
$\lambda_z = \mathrm{a}_\mathrm{r}\sigma(\varepsilon^z)/\sigma(\varepsilon^s)$	0.010	0.013	0.018	0.011	0.013	0.027

III. Results

A. Baseline estimates

We find that our approach provides more plausible results than standard maximum likelihood estimates for the unobserved variables in the model, including trend potential growth, output gap and the neutral rate. In particular, the results confirm a decline in the trend potential growth rate and a significant fall in the output gap during the GFC (Chart 4). The output gap is estimated to be negative at the end of 2014, ranging from -0.4 and -1.2 percent, which is broadly consistent with outside estimates (e.g. CBO, WEO and FRB/US model estimates). The estimated output gap during the GFC is significantly lower than the updated Laubach and Williams estimates and somewhat higher than outside estimates of the output gap. The decline in trend growth moderated in the late 1990s, consistent with the technology driven growth expansion before the GFC (see Fernald, 2014), before continuing to fall further below 2 percent during the crisis and its aftermath. We believe these results are more plausible than updated Laubach and Williams estimates, giving us more confidence in the neutral rate estimates coming out of our estimation.

Chart 4.



The results show a significant trend decline in the neutral real rate, especially in the mid 2000s (Chart 5). We compare results from two estimations: one that uses only data up to 2015Q1 and another that in addition draws on IMF's WEO projections for observed variables five years ahead (up to end 2020).[6] While the range of estimates is relatively large, confirming that neutral rates are estimated imprecisely, estimates suggest that the neutral real rate in early 2015 was likely positive, but close to zero. The estimates bottomed out as low as -1.5 percent in 2008 with even the top centile of all estimates falling close to or below zero. The baseline estimates (without projections) show a somewhat stronger decline during the GFC and more negative current neutral rates. Importantly the range of estimates widens during the GFC period, pointing to more significant uncertainty about the neutral rate during and in the aftermath of the GFC.

These results are broadly in line with the results in Williams (2015), based on an update of the Laubach and Williams (2003) model, but point to a more significant decline in the neutral rates than is suggested by the narrative approach in Hamilton et al. (2015). However, their estimates of an equilibrium real rate in the 1-2 percent range are based on

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[6] Both estimates are so-called smoothed estimates. Laubach and Williams (2003) discuss a difference between one-sided and two-sided (smoothed) estimates and show that the former, also referred to as "real-time" estimates, are estimated less precisely. As in Laubach and Williams, this discussion abstracts from data revisions. Clark and Kozicki (2005) show that data revisions add another layer of uncertainty on neutral rate estimates available for the policymaker.

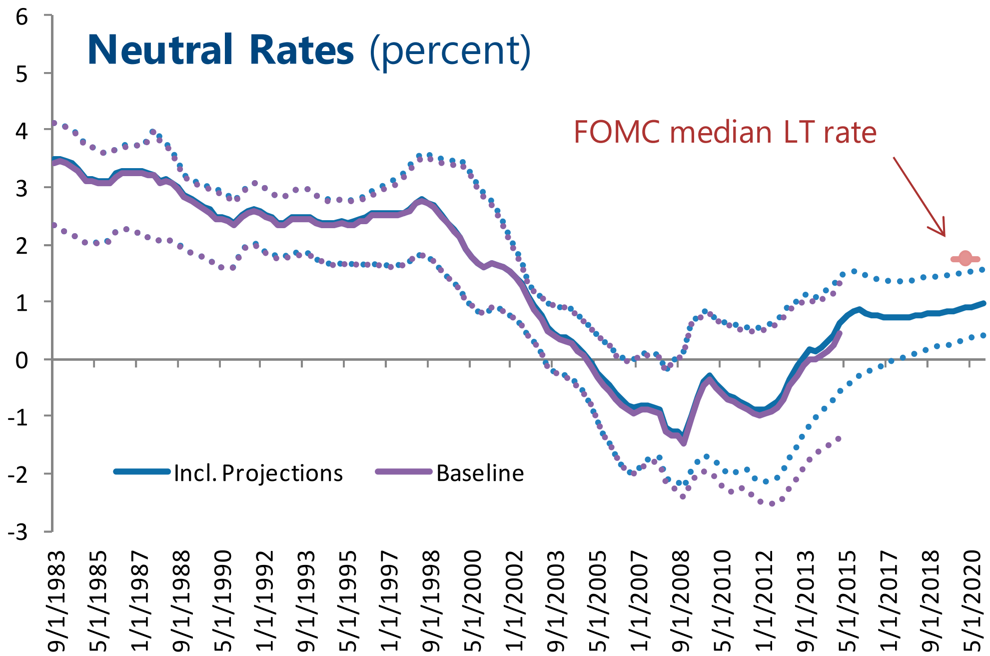
moving averages that span business cycles and thus refer to a medium term neutral rate concept.

The baseline results suggest that neutral rates bottomed out shortly after the crisis hit, have been trending upwards thereafter and likely turned positive in the course of 2014. While these results do not provide definite evidence on the debate, they nevertheless point to temporary headwinds rather than a persistent secular stagnation scenario involving persistently negative equilibrium real rates in the aftermath of the GFC. The estimation that draws also from WEO projections shows a smaller decline and a more pronounced increase, driven by the projected continuation of the recovery in real GDP growth and inflation over the next few years that will eventually drive up the real federal funds rate towards its longer term equilibrium value.[7]

Projections of the neutral rate, conditional on the WEO forecast for output, inflation and policy rates for the next five years, suggest that the neutral rate is likely to increase only very gradually looking forward. With estimates ranging from 0.4 to 1.6 percent the projected neutral rate at the end of the projection horizon stays well below the FOMC participants' median forecast for the long term real policy rate (at about 1.75 percent). This is consistent with the estimated persistence in trend growth and other factors that determine the neutral rate.

[7] The projections assume liftoff in policy rates in the second half of 2015 and a gradual path towards an equilibrium real federal funds rate of 1.5 percent (25 basis points below the median FOMC projection for the appropriate longer term policy rate). While using projections of real GDP, inflation and policy interest rates to back out current neutral rates is influenced by judgment included in the forecast, it is worth noting that the neutral rate based on the model does not converge to the projected equilibrium real rate within the forecast horizon. That is to say that while there is a degree of circularity in this approach the model provides additional information about the likely path of the neutral real rate.

Chart 5.

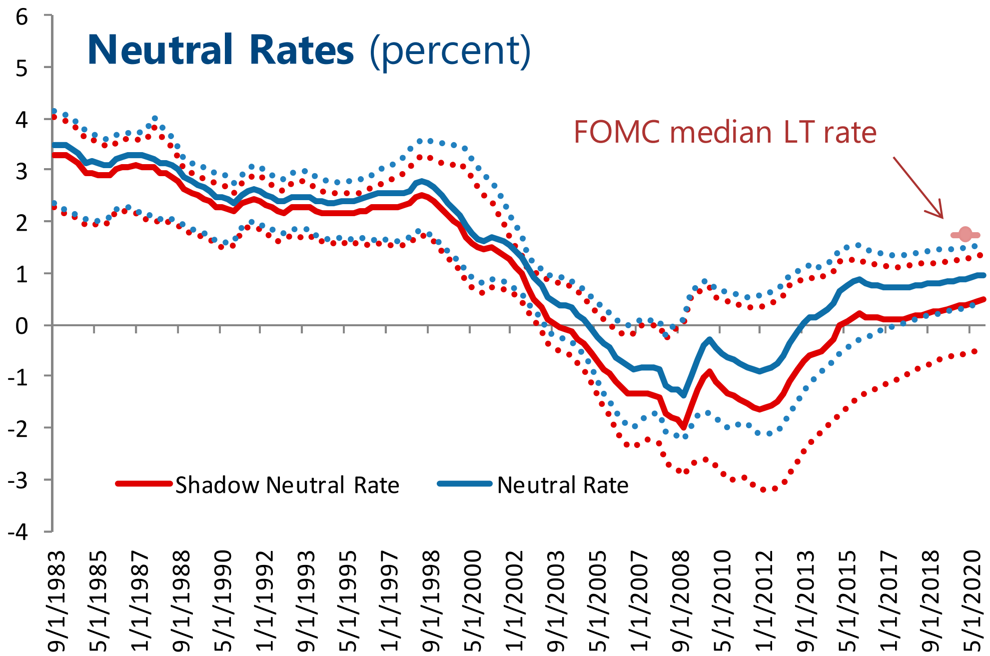


B. Results with shadow rates

The baseline model uses observed policy rates as an input and is therefore constrained by the ZLB in constructing estimates of the neutral policy rate. In addition, several studies suggest that significant asset purchases by the Federal Reserve have lowered term premia on long-term bonds and contributed to stronger growth in the aftermath of the GFC (Engen et al., 2015). There is therefore little doubt that policy has been more accommodative than suggested by real policy rates alone. To account for these effects we re-estimate the model using shadow policy rates that account for this additional policy accommodation.[8] Our estimates using a shadow policy rate suggests a neutral rate that is lower (close to zero in early 2015 compared to the comparable positive neutral rate in the estimation based on observed real rates) (Chart 6). The "shadow" neutral rate is also estimated to increase in line with the gradual normalization in policy interest rates and the Federal Reserve's balance sheet over time. However, the shadow neutral rate remains below the neutral rate based on observed real rates.

[8] An alternative approach would be to use observed long-term interest rates instead of the policy rate. However, while monetary policy effects go beyond the short term rate, and include expectations of the policy rate over time that are reflected in longer term interest rates, long-term rates are also influenced by a number of financial market factors that do not capture policy effects.

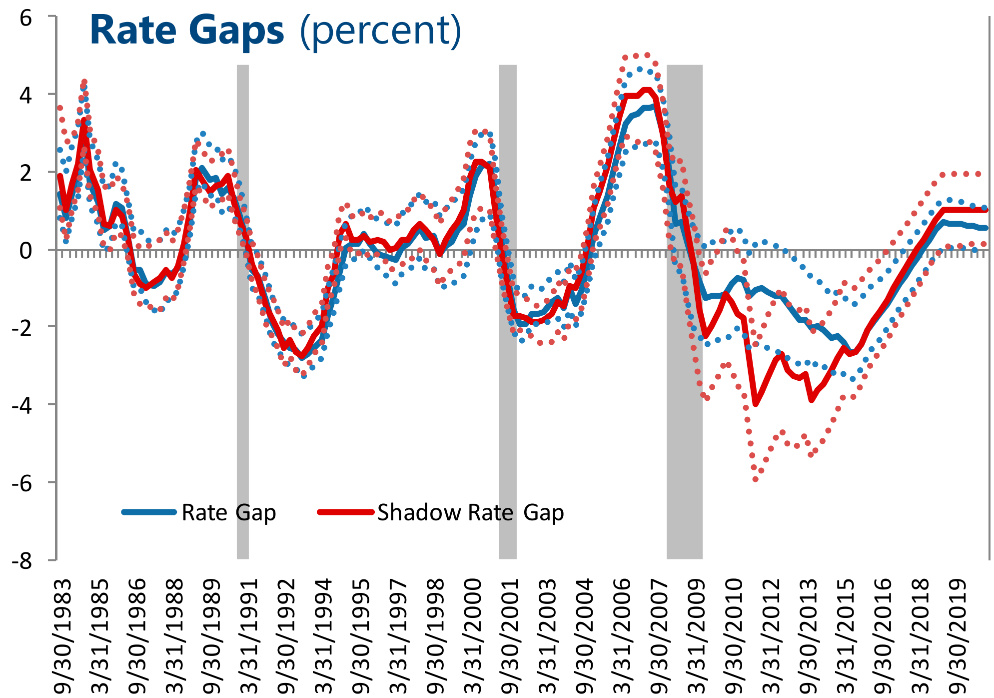
Chart 6.



C. Policy stance

Real interest rate gaps (i.e. the difference between observed real rates and estimated neutral rates—values below zero indicate the observed policy rates were below neutral rates, suggesting accommodation, and vice versa) confirm that policy has been accommodative since the crisis started. Furthermore, the estimated gap based on baseline estimates is relatively small, and comparable to the gap observed during the early 2000s slow growth period (when the output gap was just barely negative). These results confirm that unconventional policies were therefore needed to bring extraordinary accommodation as is shown by the more negative gaps based on shadow interest rates. The more negative shadow rate gap suggests that those unconventional policies added between 1-3 percentage points of policy accommodation during the GFC. Looking forward, assuming policy rate lift-off takes place this year, policy rates increase only gradually and the Federal Reserve's balance sheet normalizes gradually (as assets mature), monetary policy is likely to remain accommodative for some time (Chart 7). It is important to note that while the rate gaps are a useful summary indicator of stance, they are incomplete, and given uncertainties associated with estimation of the neutral rate, imprecise measures. For example, while the rate gaps suggest relatively tight monetary policy before the GFC, other factors, such as bank lending conditions and a relatively flat yield curve, likely contributed to looser financing conditions during that period.

Chart 7.



D. Determinants

The neutral rate equation above shows that the neutral rate depends on trend growth and other factors. Our results show that trend growth indeed is an important, but not the only driver of neutral rates over time (Chart 8). Trend growth was high in the pre-crisis period, moving up during late 1990s boom and declining in the 2000s and during the GFC. The decline since the 2000s is an important determinant of the trend decline in neutral rates. However, as discussed above, in addition to the change in trend growth, other factors (*z*'s) are likely to play a key role. Indeed, Hamilton et al. (2015) argue that "other factors play a large, indeed dominant, role in the determination of average real rates" (page 16). We find that the decline in neutral rates observed since the early 2000s is consistent with a significant increase in demand for U.S. assets owing to substantial increases in emerging market current account surpluses during this time period (Chart 9). The results suggest that other factors, such as increased risk aversion, as well as preference for safer assets, may have further amplified the decline in neutral rates in the 2000s and during the GFC. Looking forward the expected increase in neutral rates is driven by a gradual recovery in trend growth, which recovers to just above 2 percent, and declining downward pressure from other factors.

Chart 8.

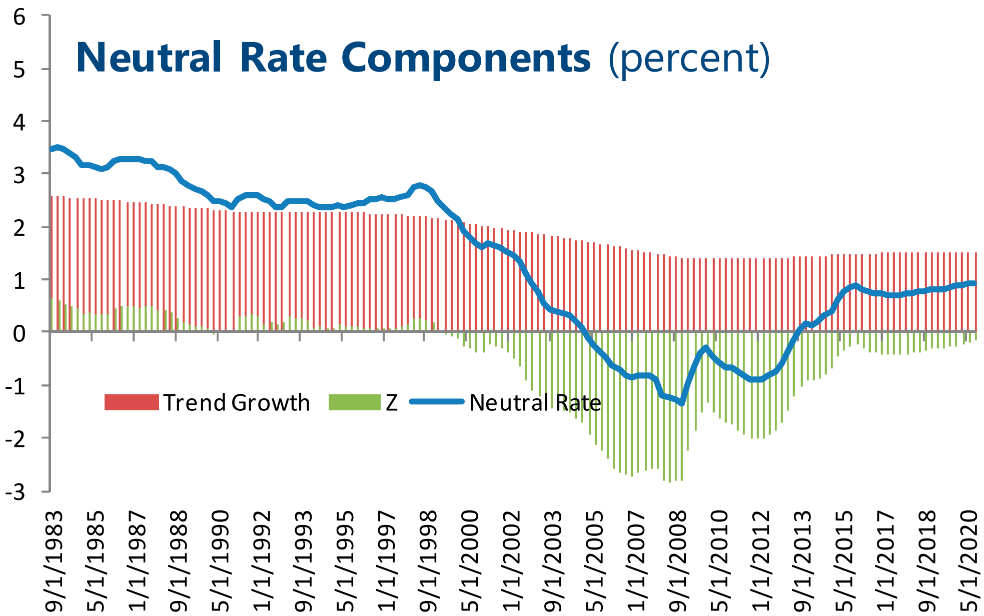
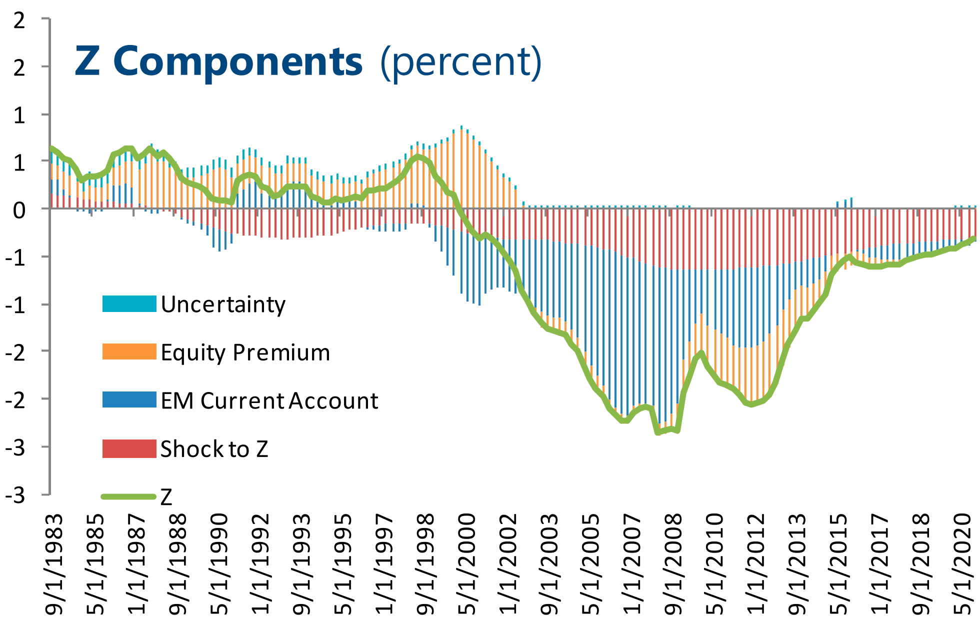


Chart 9.



IV. Conclusions

We use a semi-structural model to estimate time-varying neutral rates. Our empirical framework is based on the seminal work of Laubach and Williams (2003). However, we use a Bayesian approach to estimation of the model which incorporates prior information on the output gap and potential output, account for additional monetary policy accommodation through unconventional policies and include other observed determinants such as excess global savings as potential determinants of neutral rates.

Our results show a significant trend decline in the neutral real rate over time driven by both a decline in the trend growth rate and other factors, including excess global savings. While the range of estimates is relatively large, estimates suggest that the neutral real rate fell below zero during the GFC before turning positive in the course of 2014. Projections suggest that the neutral rate is likely to increase only very gradually looking forward. Re-estimating the model with a shadow policy rate suggests a lower neutral rate, but given the larger implied interest rate gaps also more policy accommodation during the GFC. Projected interest rate gaps show that monetary policy is likely to remain accommodative for some time. Finally, our results confirm results elsewhere in the literature that point to significant uncertainty about estimates of time-varying neutral rates (Laubach and Williams, 2003; Clark and Kozicki, 2005, and Hamilton et al., 2015). This uncertainty appears to be particularly high during periods of exceptional volatility, such as during and immediately after the GFC further complicating the use of neutral rates to assess future policy stance.

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Appendix A: Data definitions

Two endogenous variables are observable, core PCE inflation and log-GDP, while other various observable variables are treated as exogenous: real federal funds rate (deflated using one year ahead inflation expectations), the oil and import prices, the emerging market and developing economies current account as a share of US GDP, the equity premium, and a measure of policy uncertainty.

- Real GDP (loggdp): 100*ln(real GDP), where real GDP is SAAR, Billions of Chained 2009 dollars. WEO projections.

- Core inflation (coreinfl): 400*ln(P(t)/P(t-1)), where P is PCE less Food and Energy Chain Price Index (SA, 2009=100). WEO projections.

- Oil import price gap (oilgap): oilinfl-coreinfl. Oilinfl is 400*ln(P(t)/P(t-1)), where P is Petroleum & Products Imports Price Index (SA, 2009=100). Gaps are assumed to equal zero for the projection period.

- Import (ex-oil) price gap (impgap): impinfl-coreinfl. Impinfl is 400*ln(P(t)/P(t-1)), where P is Nonpetroleum Goods Imports: Chain Price Index (SA, 2009=100). Gaps are assumed to equal zero for the projection period.

- Federal funds rate (ffr): Effective Federal Funds Rate (% p.a.). WEO projections.

- Real federal funds rate (realffrrate): ffr-expinfl. Expinfl is Median 1-Year-Ahead CPI Inflation Expectation (%) from Survey of Professional Forecasters. Assumed to equal latest value for the projection period.

- Shadow federal funds rate: simple average of estimates in Krippner (2013), Lombardi and Zhu (2014) and Wu and Xia (2014). Shadow rates are projected to normalize (i.e. approach projected policy interest rates) gradually over the projection horizon.

- Potential output (based on CBO, 2001).

- Emerging market current account surplus as a share of US GDP. WEO projections.

- Policy uncertainty index. Based on Baker, Bloom and Davis (2013). http://www.policyuncertainty.com/

- Equity risk premium. Principal component derived from several models. Source: Duarte and Rosa (2015)

Appendix B. Model estimation details

The model described in section 2 can be written in a state space form which allows us to construct the likelihood of observing the data (y) for a given set of parameters (θ): $p(y|\theta)$. To exploit prior information, I_o, on some unobservable smoothed series we first define the mapping that from the parameter space Θ to the smoothed series as $\varphi(\theta)$. Hence a prior on the smoothed series (e.g., the output gap) can be written as $p(\varphi(\theta)|I_o)$. By letting $\hat{p}(\theta|I_o) = p(\varphi(\theta)|I_o)$ be a probability density function defined in the parameter space Θ we can write the posterior density function as:

$$p(\theta|y) = \hat{p}(\theta|I_o)p(y|\theta).$$

Since the priors are uniform distributions the mapping from p to $\hat{p}$ can be trivially implemented as imposing restrictions. Some additional restrictions on the parameter space are also included to avoid some parameters from taking negative values and/or in order to ensure a stable (non-explosive) system of equations. Finally, in light of the pile-up problem discussed in Stock (1994) we impose a lower bound of 0.10 to λ_z.

A sample from the posterior is drawn using the Metropolis-Hasting algorithm based on 100,000 draws from a symmetric proposal density.

In the estimation some parameters were held fixed following Trehan and Wu (2007) to reduce computational intensity. We also constrain coefficients in the Phillips curve such that $b_2 = b_3 = b_4$, $b_6 = b_7 = b_8 = 1 - b_1 - b_2$, and $b_4 = 1 - b_1 - b_2 - b_0$. While the neutral rate is sensitive to the parameter on the relative oil import price variable in the Phillips curve, it is relatively insensitive to the other parameters.

Table C1.

parameter	value
a_2	-0.27
b_y	0.15
b_i	0.004
b_o	0.03
b_1	0.47
b_2	0.35
b_0	0.05