IMF Working Paper

Does Easing Monetary Policy Increase Financial Instability?

by Ambrogio Cesa-Bianchi and Alessandro Rebucci
Does Easing Monetary Policy Increase Financial Instability?

Prepared by Ambrogio Cesa-Bianchi and Alessandro Rebucci1

Authorized for distribution by Prakash Loungani

June 2015

Abstract

This paper develops a model featuring both a macroeconomic and a financial friction that speaks to the interaction between monetary and macro-prudential policies. There are two main results. First, real interest rate rigidities in a monopolistic banking system have an asymmetric impact on financial stability: they increase the probability of a financial crisis (relative to the case of flexible interest rate) in response to contractionary shocks to the economy, while they act as automatic macro-prudential stabilizers in response to expansionary shocks. Second, when the interest rate is the only available policy instrument, a monetary authority subject to the same constraints as private agents cannot always achieve a (constrained) efficient allocation and faces a trade-off between macroeconomic and financial stability in response to contractionary shocks. An implication of our analysis is that the weak link in the U.S. policy framework in the run up to the Global Recession was not excessively lax monetary policy after 2002, but rather the absence of an effective regulatory framework aimed at preserving financial stability.

JEL Classification Numbers: E44, E52, E61

Keywords: Macro-prudential policies, monetary policy, financial crises, credit frictions, interest rate rigidities

Authors E-Mail Addresses: ambrogio.cesa-bianchi@bankofengland.co.uk; arebucci@jhu.edu

1 Bank of England and Johns Hopkins University, respectively. This paper was previously circulated under the title “Coordinating Monetary and Macro-Prudential Policies.” We would like to thank Jihad Dagher and the seminar participants at the 2013 Macro Banking and Finance Workshop, the 2013 MMF Conference, the Cattolica University, the Bank of Italy, the EPFL, the Bank of Portugal, the Banque de France, the Bank of England, the IADB, LACEA 2012 Annual Meetings, and EEA 2012 Annual Meetings for useful discussions and helpful comments. The information and opinions presented in this paper are entirely those of the authors, and not necessarily those of the Bank of England.
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Introduction</td>
<td>3</td>
</tr>
<tr>
<td>II. The Model</td>
<td>7</td>
</tr>
<tr>
<td>A. Consumers and Loan Demand</td>
<td>9</td>
</tr>
<tr>
<td>B. Banks and Loan Supply</td>
<td>11</td>
</tr>
<tr>
<td>C. Shocks and Parameter Values</td>
<td>13</td>
</tr>
<tr>
<td>III. Decentralized Equilibrium</td>
<td>15</td>
</tr>
<tr>
<td>A. Financial Friction</td>
<td>15</td>
</tr>
<tr>
<td>B. Macroeconomic Friction</td>
<td>17</td>
</tr>
<tr>
<td>C. The Interaction between Financial Friction and Macroeconomic Friction</td>
<td>18</td>
</tr>
<tr>
<td>IV. Restoring Efficiency</td>
<td>20</td>
</tr>
<tr>
<td>A. Addressing the Pecuniary Externality</td>
<td>21</td>
</tr>
<tr>
<td>B. Addressing Monopolistic Competition and Interest Rate Stickiness</td>
<td>24</td>
</tr>
<tr>
<td>C. Addressing Both Frictions with Two Instruments</td>
<td>25</td>
</tr>
<tr>
<td>D. The Trade-off: Addressing Both Frictions with One Instrument</td>
<td>28</td>
</tr>
<tr>
<td>V. Implications for US Monetary Policy and Financial Stability</td>
<td>29</td>
</tr>
<tr>
<td>VI. Conclusions</td>
<td>35</td>
</tr>
<tr>
<td>References</td>
<td>37</td>
</tr>
<tr>
<td>Tables</td>
<td></td>
</tr>
<tr>
<td>1. Calibration of Model's Parameter</td>
<td>13</td>
</tr>
<tr>
<td>Figures</td>
<td></td>
</tr>
<tr>
<td>1. A Counterfactual Path for the U.S. Policy Rate</td>
<td>4</td>
</tr>
<tr>
<td>2. Model Equilibrium with Financial Friction</td>
<td>16</td>
</tr>
<tr>
<td>3. Model Equilibrium with Both Frictions: Positive Shock to the Interest Rate</td>
<td>19</td>
</tr>
<tr>
<td>4. Model Equilibrium with Both Frictions: Negative Shock to the Interest Rate</td>
<td>21</td>
</tr>
<tr>
<td>5. Efficient Allocation with Both Frictions: Positive Shock to the Interest Rate</td>
<td>26</td>
</tr>
<tr>
<td>6. Efficient Allocation with Both Frictions: Negative Shock to the Interest Rate</td>
<td>28</td>
</tr>
<tr>
<td>7. Alternative Path of the Lending Interest Rate under Different Assumptions about the Number of Instruments at the Policy-maker Disposal</td>
<td>31</td>
</tr>
<tr>
<td>8. Monetary Policy and the U.S. Housing Sector</td>
<td>34</td>
</tr>
<tr>
<td>Appendix A. Numerical Solution</td>
<td>41</td>
</tr>
</tbody>
</table>
I Introduction

The global financial crisis and ensuing Great Recession of 2007-09 have ignited a debate on the role of policies for the stability of the financial system or the economy as a whole (i.e., so called macro-prudential policies). In advanced economies, this debate is revolving around the role of monetary and regulatory policies in causing the global crisis and how the conduct of monetary policy and the supervision of financial intermediaries should be altered in the future to avoid the recurrence of such a catastrophic event.

In this paper we develop a simple model featuring both a macroeconomic and a financial friction—i.e., a real interest rate rigidity that give rise to a traditional macroeconomic stabilization objective and a pecuniary externality that give rise to a more novel financial stability objective—which speaks to the necessity to complement monetary policy with macro-prudential policies in response to contractionary shocks.

The prime objective of macro-prudential policy is to limit build-up of system-wide financial risk in order to reduce the frequency and mitigate the impact of a financial crash. Most commonly used prudential tools, however, interact with other policy instruments. The overlap between different policy areas is a major challenge for policy-makers, who have to consider the unintended impact of their instruments on other policy objectives as well as the unintended impact of other policy-makers’ instruments on their own policy objective (Svensson, 2012).

For instance, monetary policy can affect financial stability: investors may be induced to substitute low-yielding, safe assets for higher-yielding, riskier assets (Rajan, 2005, Dell’Ariccia, Laeven, and Marquez, 2011); investors may also be encouraged to take greater risks if they perceive that monetary policy is being used asymmetrically to support asset prices during downturns (Issing, 2009); and asset price increases induced by falling interest rates might cause banks to increase their holdings of risky assets through active balance sheet management (Adrian and Shin, 2009, 2010). On the other hand, macro-prudential policy instruments can affect macroeconomic stability. In fact, by affecting variables such as asset prices and credit, macro-prudential policy is likely to affect a key transmission mechanism of monetary policy (see e.g., Ingves, 2011). This overlap in the respective areas of influence entails the possibility of the instruments having offsetting or amplifying effects on their objectives if they are implemented in an uncoordinated manner, possibly leading to worse outcomes than if the instruments had been coordinated (see Bean, Paustian, Penalver, and Taylor, 2010, Angelini, Neri, and Panetta, 2011, between others).

Against this background, some observers have assigned to monetary policy a key role in exacerbating the severity of the global financial crisis of 2007-09. Taylor (2007), in particular, noticed

---

that during the period from 2002 to 2006 the U.S. federal funds rate was well below what a good
rule of thumb for U.S. monetary policy would have predicted. Figure 1 displays the actual fed-
eral funds rate (solid line) and the counterfactual policy rate that would have prevailed if mon-
etary policy had followed a standard Taylor rule (dashed line). Indeed, the interest rate implied
by the Taylor rule is well above the actual federal funds rate, starting from the second quarter of
2002. Taylor (2007) argues that such a counterfactual policy rate would have contained the hous-
ing market bubble; moreover, Taylor also supports the idea that deviating from this rule-based
monetary policy framework has been a major factor in determining the likelihood and the severity
of the 2007-09 crisis (Taylor, 2010).

Despite a somewhat widely shared common sentiment that the Federal Reserve is partly to blame
for the housing bubble, the issue is highly controversial in academia and the policy community.
Besides Taylor (2007, 2010), Borio and White (2003), Gordon (2005), and Borio (2006) support
the idea that monetary policy contributed significantly to the boom that preceded the global fi-
nancial crisis. In contrast, Posen (2009), Bean (2010), and Svensson (2010) argue against this
thesis.2

---

2Bernanke (2010) recently said that “the best response to the housing bubble would have been regulatory, rather
To address some of these issues, we develop a simple model of consumption-based asset pricing with collateralized borrowing, monopolistic banking, real interest rates rigidities and pecuniary externalities. The presence of real and financial frictions give rise to both a traditional macroeconomic stabilization role for policy and a more novel financial stability objective.

The macroeconomic stabilization objective arises from the presence of monopolistic competition and real interest rates rigidities in the banking sector. Due to monopolistic power, banks apply a markup on lending rates. Moreover, when banks cannot fully adjust their lending rates in response to macroeconomic shocks, the economy displays distortions typical of models with staggered price setting, generating equilibrium allocations that are not Pareto efficient (Hannan and Berger, 1991, Kwapil and Scharler, 2010, Gerali, Neri, Sessa, and Signoretti, 2010).

The financial stability objective stems from the fact that the model endogenously generates financial crises when the borrowing constraint occasionally binds. When access to credit is subject to an occasionally binding collateral constraint, a pecuniary externality arises. Private agents do not internalize the effect of their individual decisions on the market price of collateral, thus borrowing and consuming more than socially efficient, and increasing the frequency and the severity of financial crises.

The main results of the analysis are two. First, the analysis of our model economy shows that real interest rate rigidities have a different impact on financial stability depending on the sign of the shock hitting the economy. In response to positive shocks to interest rates, aggregate lending rates rise, too. However, because of interest rate stickiness, they increase less than in the flexible interest rate case. This affects next period net worth through two effects. On the one hand, lower lending rates prompt consumers to borrow more than in the flexible rate case, and thus lowering next period net worth; on the other hand, interest rate repayments are lower relative to the flexible case, thus increasing next period net worth. As the second effect dominates the first one in equilibrium (for a wide range of parameter values), the probability of a crisis is lower with interest rate stickiness. Thus, interest rate rigidity acts as an automatic macro-prudential stabilizer in response to shocks that require interest rates to increase. In contrast, when interest rates are hit by a negative shock, aggregate lending interest rates decrease relatively less. Because of the same mechanisms working in reverse, real interest rate rigidity leads to a higher probability of a crisis in response to shocks that lower interest rates (relative to the flexible interest rates case).

Second, the model shows that a policy authority, facing the same constraints faced by private agents and with only one policy instrument (namely, the policy interest rate), may not achieve efficiency when a shock that lowers interest rates hits the economy. Specifically, in response to
shocks that lower interest rates, achieving both macroeconomic and financial stability entails a trade-off because the two objectives require interventions of opposite direction on the same policy tool; in our case, the interest rate. However, when two different instruments are at the policymaker’s disposal (as, for example, a tax on debt and the policy interest rate), efficiency can be achieved in response to both positive and negative shocks to the risk-free interest rate.

Our analysis has important implications regarding the role of U.S. monetary policy for the stability of the financial system in the run-up to the Great recession. In particular, we show that Taylor’s argument —i.e., that higher interest rates would have reduced both the probability and the severity of the great recession— is supported by our theoretical model only if we make the auxiliary assumption that the Fed had to address all distortions in the economy with only one instrument, namely the policy interest rate. However, Taylor’s argument cannot be rationalized in the context of our model when the policy authority has two different instruments. In this case, in response to a negative aggregate demand shock, interest rates ought to be lowered as much as needed without concerns for financial stability. As suggested by Bernanke (2010) and Blanchard, Dell’Ariccia, and Mauro (2010), this implies that the same monetary policy stance as the one adopted by the Fed during the 2002-06 period, accompanied by stronger regulation and supervision of the financial system, might have been more effective in reducing the likelihood and the severity of the crisis, relative to a tighter monetary policy stance with the same financial supervision and regulation observed during the 2002-06 period.

This paper is related to several strands of literature. The first is the branch of the New Keynesian literature that considers financial frictions and Taylor-type interest rate rules (see Angelini, Neri, and Panetta, 2011, Beau, Clerc, and Mojon, 2012, Kannan, Rabanal, and Scott, 2012, for example). These papers consider either interest rules augmented with macro-prudential arguments — such as credit growth, asset prices, loan-to-value limits— or a combination of interest and macro-prudential rules in order to allow monetary policy to “lean against financial winds.” However, in this class of models, macro-prudential regulation is taken for granted, in the sense that it does not target a clearly identified market failure giving rise to a well defined financial stability objective. In our model, there is a well defined pecuniary externality that justify government intervention for financial stability purposes.

The second is a growing literature on pecuniary externalities that interprets financial crises as episodes of financial amplification in environments where credit constraints are only occasionally binding (see, between others, Korinek, 2010, Bianchi, 2011, Jeanne and Korinek, 2010a,b, Benigno, Chen, Otrok, Rebucci, and Young, 2013). In this class of models the need for macro-prudential policies stems from a well-defined market failure: a pecuniary externality originating from the presence of the price of collateral in the aggregate borrowing constraint faced by private
agents. However, in all these models, the financial friction is the only distortion in the economy. The question of how the pursuit of financial stability may affect macroeconomic stability is therefore novel relative to this literature.

The third and final is a small, but growing literature that considers both macroeconomic and financial frictions at the same time. Benigno, Chen, Otrok, Rebucci, and Young (2011) analyze a fully specified new open economy macroeconomics 3-period model that features the same financial friction analyzed here and Calvo-style nominal rigidities. The solution of the fully non-linear version of that model (i.e., without resorting to approximation techniques) shows that there is a trade-off between macroeconomic and financial stability, but it is quantitatively too small to warrant the use of a second policy instrument in addition to the interest rate. Kashyap and Stein (2012) use a modified version of the pecuniary externality framework of Stein (2012) where the central bank has both a price stability and a financial stability objective. Similar to our findings, a trade-off emerges between the two objectives when the policy interest rate is the only instrument and it disappears when there is a second instrument (a non-zero interest rate on reserves, in their case). However, they do not model the price stability objective explicitly. Woodford (2012), in contrast, sets up a New Keynesian model with credit frictions, where the probability of a financial crisis is endogenous (i.e., it is a regime-switching process that depends on the model variables). Woodford characterizes optimal policy in this environment, showing that —under certain circumstances— the central bank may face a trade-off between macroeconomic and financial stability. However, he does not explicitly model financial stability.

In contrast, in our paper, both the macroeconomic and the financial stability objective are well defined and each objective originates from a friction that we model explicitly. The interaction between the macroeconomic and the financial friction delivers a stark trade-off between macroeconomic and financial stability, that helps rationalize the role of monetary policy and macro-prudential policy (or the lack of thereof) in the run-up to the Great Recession in the United States.

The rest of the paper is organized as follows. Section 2 describes the model economy. Sections 3 and 4 characterize the decentralized and the socially planned equilibrium of the economy, respectively. In Section 5 we discuss the implications of our model in terms of the role played by U.S. monetary policy for the stability of the financial system in the run-up to the Great Recession. Section 6 concludes.

II The Model

We include monopolistic banking and real interest rate rigidities in the pecuniary externality framework of Jeanne and Korinek (2010a). In Jeanne and Korinek (2010a)’s set up, consumers
borrow directly from international capital markets (or foreign banks). In our model, consumers must borrow from a stylized monopolistic banking sector that intermediates foreign saving. Assuming that some of the changes in US interest rates ultimately originate abroad is in line with the view that the global external imbalances (i.e., the behavior of foreign savings) influenced significantly the US economy in the run-up to the Great Recession. Alternatively, borrowers could be interpreted as entrepreneurs/households in a closed economy enjoying a comparative advantage in owning certain assets.

The financial friction is given by the presence of collateralized borrowing. Strictly speaking, the real frictions are two: the first is the presence of market power in loan markets, exercised by monopolistically competitive banks, and the second is infrequent adjustment of interest rates by banks.

The economy is populated by two sets of agents: a continuum of monopolistically competitive banks and a continuum of identical atomistic individuals who borrow from banks and consume. Each set of agents has a mass normalized to one. There are only three periods, denoted $t = 0, 1, 2$.

At the beginning of period 0 consumers own an asset whose available stock is normalized to 1. In order to consume they can either sell a fraction of the asset $(1 - \theta_{i,1})$ at market prices or borrow from banks $(b_{i,1})$. They have a well-defined demand function for loans which is decreasing in the lending interest rate $(R_{L1})$. Monopolistic banks freely borrow from foreign lenders at the risk-free interest rate $(R^*)$ and —given loans demand— optimally set their lending rates. The risk-free interest rate can be hit by a temporary shock $(R^* \pm \nu)$ at the beginning of period 0. We assume that only a fraction of banks $(\mu)$ can reset their lending rates conditional on this shock, while the remaining banks $(1 - \mu)$ need to keep their lending rates fixed. The purpose of this assumption is to introduce macroeconomic stabilization considerations in relatively simple manner. The credit market clears after the realization of the shock, which is observed by all agents. At the end of period 0, households consume $(c_{i,0})$.

In period 1, consumers are endowed with a stochastic endowment $(e)$, they repay their debt $(b_{i,1}R_{L1})$, borrow an additional amount from banks $(b_{i,2})$, realize banks profits $(\pi_{i,1})$, and consume $(c_{i,1})$. Note that debt rollover is subject to a collateral constraint: additional borrowing $(b_{i,2})$ is limited to a fraction of consumers’ assets (at their market value). The purpose of this assumption is to introduce financial stability considerations. If hit by a shock in period 0, the level of the risk-free interest rate returns to its pre-shock value $(R^*)$ in period 1.

---

3This assumption may be justified by both theoretical and empirical findings (see, for example, Hannan and Berger, 1991, Neumark and Sharpe, 1992, Kwapił and Scharler, 2010, Gerali, Neri, Sessa, and Signoretti, 2010).
Period 2 represents the long run. Consumers get a deterministic return on the asset that they own ($y$), repay their debt ($b_{i,2}R_{L2}$), realize banks profits ($\pi_{i,2}$), and consume ($c_{i,2}$).

We now discuss the consumers’ and banks’ problems in turn.

### A Consumers and Loan Demand

The utility of each consumer, indexed by $i \in [0, 1]$, is given by:

$$u(c_{i,0}) + u(c_{i,1}) + c_{i,2},$$

where, for simplicity, we assume a unitary discount factor. The period utility function, $u(\cdot)$, is a standard CES function:

$$u(c) = \frac{c^{1-\rho}}{1-\rho}.$$  

The budget constraint can be written as:

$$\begin{cases} 
    c_{i,0} = b_{i,1} + (1 - \theta_{i,1})p_0, \\
    c_{i,1} + b_{i,1}R_{L1} = e + b_{i,2} + (\theta_{i,1} - \theta_{i,2})p_1 + \pi_{i,1}, \\
    c_{i,2} + b_{i,2}R_{L2} = \theta_{i,2}y + \pi_{i,2}. 
\end{cases}$$

Initially, each consumer owns $\theta_{i,0} = 1$ unit of the asset, where the price of the asset in period $t$ is denoted by $p_t$. Consumers can buy or sell the asset in a perfectly competitive market, but they cannot sell it to the lenders and rent it back. As in Jeanne and Korinek (2010b), we assume that consumers derive some important benefits from owning the asset.\(^4\) Note that consumers are identical and, in a symmetric equilibrium, we must have $\theta_{i,0} = \theta_{i,1} = \theta_{i,2} = 1$.

As it is evident from the budget constraint, in order to consume in period 0, consumers need to either sell a fraction of their assets $(1 - \theta_{i,1})$ or borrow from banks ($b_{i,1}$). Moreover, each consumer, in period 1, faces a collateral constraint of the form:

$$b_{i,2} \leq \theta_{i,1}p_1,$$

where $\theta_{i,1}$ is the quantity of domestic collateral held by the consumer at the beginning of period 1.

The microfoundation of the collateral constraint follows the spirit of Kiyotaki and Moore (1997). However, for tractability, while in Kiyotaki and Moore (1997) borrowing capacity is an increas-

\(^4\)Consumers can be interpreted as households owning durable assets (such as their homes, for example).
ing function of the future value of the collateral, we assume that borrowing capacity is an increasing function of the current value of the collateral. The same modelling choice has been made by Mendoza (2010), Jeanne and Korinek (2010b) and Mendoza and Smith (2006), and is justified by the work of Cordoba and Ripoll (2004) and Kocherlakota (2000) who show that collateral constraints specified with next-period price of collateral asset do not yield quantitatively significant differences in response to shocks.

Consumers maximize (1) subject to the budget constraint (3) and the collateral constraint (4). The utility maximization problem of the representative consumer (i.e., variables without the subscript \(i\)) can be written as:

\[
\max_{b_1, b_2, \theta_2} \left\{ u\left(b_1 + (1-\theta_1)p_0\right) + \mathbb{E}_0 \left[ u\left(e + b_2 + (\theta_1 - \theta_2)p_1 + \pi_1 - b_1 R_L\right) + \theta_2 y + \pi_2 - b_2 R_L - \lambda (b_2 - \theta_1 p_1) \right] \right\}.
\]

(5)

Solving this problem backwards, the first order conditions are:

\[
p_1 = \frac{y}{u'(c_1)},
\]

\[
u'(c_1) = R_L + \lambda,
\]

\[
u'(c_0) = R_L \mathbb{E}_0 [u'(c_1)].
\]

(6)

The first equation represents the asset pricing condition for the economy. The second and third equations are the Euler equation for consumption in period 1 and 0, respectively.\(^5\)

**Consumers’ demand of loans**

In order to allow for market power in the banking sector, we model the market for loans in a Dixit and Stiglitz (1977) framework.\(^6\) That is, we assume that loan contracts bought by consumers are a constant elasticity of substitution composite basket of slightly differentiated financial products —each supplied by a bank \(j\) — with an elasticity of substitution \(\zeta\) (which will be the main determinant of the spread between bank rates and the risk-free rate).

In particular, the consumer \(i\), in order to obtain a loan of a given size \(b_{i,t}\), needs to take out a continuum of loans \(b_{ij,t}\) from all existing banks \(j\), such that:

\[
b_{i,t} \leq \left( \int_0^1 b_{ij,t}^\xi \, dj \right)^{\frac{\zeta}{\xi - 1}}
\]

(7)

\(^5\)Details on the the derivation of the equilibrium conditions are reported in Appendix A.

where $\zeta > 1$ is the elasticity of substitution between differentiated loans (or banking services, in general). Demand by consumer $i$ seeking a real amount of loans equal to $b_{i,t}$ can be derived by minimizing the total repayment due to the continuum of banks $j$ over $b_{j,t}$. Aggregating over symmetric households, this minimization problem yields downward-sloping loans demand curves of the kind:

$$b_{j,t} = \left( \frac{R_{L,j,t}}{R_{Lt}} \right)^{-\zeta} b_{i,t}.$$  

with the aggregate interest rate on loans given by:

$$R_{Lt} = \left( \int_0^1 R_{L,j,t}^{1-\zeta} d_j \right)^{\frac{1}{1-\zeta}}.$$  

**B BANKS AND LOAN SUPPLY**

There is a continuum of monopolistically competitive domestic banks indexed by $j \in [0, 1]$ owned by households. Microeconomic theory typically considers market power as a distinctive feature of the banking sector (Freixas and Rochet, 2008). In particular, we assume that each bank $j$ supplies slightly differentiated financial products, and no other bank produces the same variety: each bank has, therefore, some monopoly power over its product. However, each bank competes with all other banks, since consumers consider each bank’s variety as imperfect substitutes. As banks have market power over the supply of their variety, they set prices to maximize profits, taking into account the elasticity of demand for their variety.

Each bank $j$ collects fully insured deposits $d_{j,t}$ from foreign investors at the risk-free interest rate $R_t = R^*$, where $R^*$ is exogenous and given. We further assume that foreign lenders have an infinite supply of deposits, so that banks can satisfy any demand for loans. Finally, banks use deposits to supply loans to consumers with the following constant return to scale production function:

$$b_{j,t} = d_{j,t}.$$  

---

7The presence of market power can be justified by the existence of switching costs which lead to long-term relationships between banks and borrowers (see Diamond (1984) for example). Empirically, the presence of market power in the banking sector, as well as its determinants over the business cycle, are well documented. See, for example, Berger, Demirguc-Kunt, Levine, and Haubrich (2004) and Degryse and Ongena (2008).
In each period, bank $j$ maximizes its profits choosing prices and quantities:

$$\max_{R_{Lj,t}, b_{jt}} b_{jt} R_{Lj,t} - d_{jt} R_{t},$$

subject to the demand schedule in (8) and to the production function in (10). The first order condition for this problem implies that the optimal lending rate applied by banks is a positive constant gross markup ($\mathcal{M}$) over the marginal cost:

$$R_{Lj}(j) = \frac{\zeta}{\zeta - 1} R_{t} = \mathcal{M} R_{t}. \tag{11}$$

Note that, together with consumers’ optimality conditions, equation (11) determines the equilibrium of the economy. That is, once the lending rate has been set by banks, households make their consumption (and, therefore, borrowing) decisions. The market clearing in the loan market closes the model.

We also assume that the banking sector displays short-run interest rate stickiness. In particular, we assume that banks cannot immediately adjust their lending rates in response to macroeconomic developments. The presence of interest rate stickiness in the banking sector can be justified by the presence of adjustment costs of and monopolistic power. For example, Hannan and Berger (1991) show that, in the presence of fixed adjustment costs, banks re-set their lending rates only if the costs of changing the interest rate are lower than the costs of maintaining a non-equilibrium rate (see also Neumark and Sharpe, 1992). Empirically, it is a well documented fact that the adjustment of banks lending rates to changes in the risk-free rate is only partial and heterogeneous, in particular in the short run. For example, Kwapił and Scharler (2010) show that interest rate pass-through of consumer loans in the U.S. can be as low as 0.3, implying that interest rates charged on consumer loans are smoothed heavily by banks. For tractability, we implement interest rate stickiness by means of a simple one-period real rigidity —while we assume that in the long-run interest rates are fully flexible.

In particular we assume that, if the risk-free interest rate is hit by a temporary shock ($\upsilon$) in period 0, only a fraction $\mu$ of the banks can reset their rates, whereas the remaining $1 - \mu$ banks cannot. This entails that, following a shock to the risk free interest rate, the aggregate lending rate will be in general different from the one desired by banks: remembering that consumers are price takers and that their loans demand depends on the average interest rate in the economy, this friction will lead to a distortion in the competitive equilibrium and will create the policy scope for restoring efficiency. Moreover, given that the incomplete pass-through of changes in the risk-free rate on lending rates is a realistic assumption only in the short run, we assume that from period 1 interest rates are again fully flexible.
Finally, note that shocks to the interest rate in period 0 are observed by all agents before they make their decisions. We will consider three different scenarios: no shock to the risk-free interest rate ($\nu = 0$), a temporary increase in the risk-free rate ($\nu > 0$), and a temporary reduction in the risk-free rate ($\nu < 0$). We can interpret these three scenarios as the result of a realized temporary “shock” to the risk-free interest rate at the beginning of period 0. Specifically, shock $\nu$ can be interpreted as a demand shock —such as a preference shock or a government spending shock—in a closed economy or as a foreign demand shock in a small open economy (see Harrison and Oomen (2010) and Cook and Devereux (2011), for example).

**C Shocks and Parameter Values**

To be able to solve and simulate the model we need to make assumptions about key parameters: the distribution of the stochastic endowment ($e$), the return of the asset ($y$), households’ preferences ($\rho$), the degree of monopolistic competition in the banking sector ($\zeta$), the risk-free interest rate ($R^*$), the degree of interest rates stickiness ($\mu$), and the size of the shocks to the interest rate ($\nu$). Table 1 summarizes the assumptions we make on these processes and parameters.

<table>
<thead>
<tr>
<th>Table 1 Calibration of Model’s Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General</strong></td>
</tr>
<tr>
<td>Average Endowment $\bar{e}$</td>
</tr>
<tr>
<td>Asset return $y$</td>
</tr>
<tr>
<td>Risk free rate $R^*$</td>
</tr>
<tr>
<td>Elasticity of Subst. (Loans) $\zeta$</td>
</tr>
<tr>
<td>Risk Aversion Coefficient $\rho$</td>
</tr>
<tr>
<td>Interest rate stickiness $\mu$</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
</tr>
<tr>
<td>Shock to the endowment $\tilde{e}$</td>
</tr>
<tr>
<td>Shock to the interest rate $\nu$</td>
</tr>
</tbody>
</table>

Note. 3M US T-Bill is the the average 3-Month Treasury Bill deflated with US CPI; $R_L$ is the 15-Year mortgage fixed rate deflated with US CPI. U.S. monthly data from 1985:M1 to 2007:M3.

We assume that endowment $e$ has a deterministic component $\bar{e}$ and a stochastic component $\tilde{e}$:

$$ e = \bar{e} + \tilde{e}, $$

where $\tilde{e}$ is uniformly distributed over the $[-\varepsilon, +\varepsilon]$ interval. This implies that the endowment $e$ is uniformly distributed over the $[\bar{e} - \varepsilon, \bar{e} + \varepsilon]$ interval. We will analyze the model’s properties for
different values of the maximum size of the shock to the endowment ($\varepsilon$). In particular, we will consider values for $\varepsilon$ such that the economy may be constrained for a sufficiently large negative realization of the shock, but would not be constrained in the absence of disturbances. As shown in Appendix A, under these assumptions the model can be solved largely in closed form.

While it is possible to make reasonable assumptions for the majority of parameters, two degrees of freedom are left for the solution of the model: the return of the asset ($y$) and the expected value of the endowment ($\bar{e}$). Following Jeanne and Korinek (2010a), we assume $\bar{e} = 1.3$ and $y = 0.8$. Jeanne and Korinek (2010a) choose these two parameters jointly with the maximum size of the shock to the endowment ($\varepsilon$) to control when the borrowing constraint binds. Note also that, given the stylized nature of the model, we do not use it for quantitative analysis. The parametrization chosen is to study the solution in the case in which the borrowing constraint does not bind today, but can bind tomorrow. The analysis of a model with an occasionally binding constraint adds a layer of complexity that is not present in standard New Keynesian models and banking models, and thus justifies simplifying in other dimensions. In the last section of the paper we shall use the qualitative predictions of the model to interpret the recent U.S. experience in the run-up to the Great Recession.

We calibrate the remaining parameters using U.S. data from 1985 to 2007, i.e., from the beginning of the Great Moderation to the beginning of the Great Recession. The gross risk-free real interest rate is set to $R^* = 1.015$ in order to match the average yield of the 3-Month Treasury Bill (deflated with US CPI) over the period 1985-2007. We set the elasticity of substitution between financial products to $\zeta = 33.3$, which implies a gross markup of $M \simeq 1.03$. This markup yields approximately a spread of 250 basis points over the risk-free interest rate, which is consistent with the average spread of the 15-year mortgage fixed rate over the 3-Month Treasury Bill rate.\footnote{Notice here that Gerali, Neri, Sessa, and Signoretti (2010) set the elasticity of loan contracts to about 2.5, to match an average spread of 170 basis points of deposit rates on the policy rate. Our number differs from theirs because we assume that the markup is applied to the gross interest rate (i.e., $M R$) instead of the net interest rate (i.e., $1 + M r$).}

Household preferences are given by a constant elasticity of substitution utility function, with a relative risk aversion coefficient $\rho = 2$, which is a conventional value.

Under these assumptions, the model economy is never constrained when $\varepsilon \leq \varepsilon^b = 0.095$. That is, the constraint never binds below the threshold $\varepsilon^b$, and the probability of observing a crisis in period 1 is zero. In this case, the model has a closed-form solution given by optimality conditions (6) together with $\lambda = 0$. In contrast, when $\varepsilon > 0.095$ there exists a positive probability that the constraint will bind in period 1: in this case the model does not have a closed-form solution and, therefore, the levels of debt and consumption have to be solved numerically (as shown in Appendix A).
The calibration of the degree of interest rate stickiness (µ) is more difficult. Even if there is compelling evidence on the imperfect adjustment of retail interest rates rate to movements in the risk free rate, the degree of such rigidity is not consistently quantified. For the U.S., Kwapil and Scharler (2010) estimate a short-run pass through of 0.3 for consumer loans. Based on this evidence we assume that, in the short run, only 50 percent of the banks can adjust their lending rates conditional to a movement in the interest rate. In the long-run, in contrast, pass through is assumed to be complete.

Finally, we assume that the risk-free interest rate is affected by a shock in period 0, such that:

\[ R_1 = R^* + \nu, \]

where \( \nu \) can take three values, namely \( \nu = \{0, +0.02, -0.02\} \). The size of the shock matches the standard deviation of the yield on the U.S. 3-Month Treasury Bill over the 1985-2007 period.

### III Decentralized Equilibrium

We can now analyze the decentralized equilibrium of the economy. In order to build intuition, we will consider first the effects of the financial friction (which manifests itself conditional on shocks to the endowment) by comparing the allocation in our model economy with an economy in which the collateral constraint is never binding. Second, we will analyze the effect of the macroeconomic friction (which manifests itself conditional on shocks to the risk free interest rate) by comparing the allocation in our model economy with an economy with fully flexible interest rates. Third, and finally, we will analyze the full model, when both frictions are at work simultaneously.

#### A Financial Friction

The financial friction affects the economy only when the collateral constraint is not binding today but can bind tomorrow with a positive probability. In particular, a shock (\( \tilde{\epsilon} \)) to the endowment received by households, if large enough to make the collateral constraint binding, will lead to a downward spiral of declining consumption, falling asset prices, and tighter borrowing constraints typical of financial accelerator models, such as Bernanke, Gertler, and Gilchrist (1996) and Kiyotaki and Moore (1997). We label states in which the collateral constraint is binding as “crisis

---

9These estimates are in line with older studies on interest rate pass-through in the U.S.. For example, Cottarelli and Kourelis (1994) estimate a short run pass through of 0.32 and a long run pass through of 1; Moazzami (1999) and Borio and Fritz (1995) report a short run coefficient of 0.4 and 0.34, respectively.

10Note that the calibration of this parameter does not affect the qualitative behavior of our model.
states” and define the probability that the constraint will bind in period 1 (i.e., the crisis probability) as our measure of financial stability.\(^{11}\)

We consider different values of the maximum size of the shock (\(\varepsilon\)) so that i) the collateral constraint never binds (i.e., the shock \(\tilde{\varepsilon}\) is never large enough to push the economy in the constrained region); and ii) the collateral constraint is occasionally binding (i.e., for large enough realizations of the shock \(\tilde{\varepsilon}\) the economy can enter the constrained region and experience a financial crisis).

As we discussed earlier, the threshold for \(\varepsilon\) that makes the collateral constraint bind with positive probability is \(\varepsilon^b \approx 0.095\).

Figure 2 displays the behavior of some endogenous variables in our model for different values of the maximum size of the shock (\(\varepsilon\), displayed on the horizontal axis). The upper-left panel of Figure 2 plots the equilibrium level of borrowing in period 0 (\(b_1\)). Conditional on \(b_1\), it is possible to compute net worth (\(e - b_1 R_{L1}\)), consumption (\(c_1\)), and the probability of observing a crisis (\(\pi\)) in period 1.

![Graphs showing the behavior of endogenous variables](image)

**Figure 2** Model Equilibrium with Financial Friction. On the horizontal axis is the maximum size of the endowment shock (\(\varepsilon\)).

\(^{11}\)Note that both Woodford (2012) and Benigno, Chen, Otrok, Rebucci, and Young (2013) define financial stability in these terms.
When $\varepsilon \leq \varepsilon^b$ the economy is never constrained, households’ decisions are not affected by the size of the shock $\varepsilon$: if hit by a negative endowment shock, households can borrow from banks to smooth consumption. In contrast, when the maximum size of the shock is above its threshold ($\varepsilon^b = 0.095$), consumers take into account that there is a positive probability that the constraint will bind in period 1. They insure by reducing borrowing in period 0 (so that their net worth next period will be higher) and by reducing their consumption in period 1. The probability of a crisis ($\pi$) is positive and increases in a non-linear way with the maximum size of the shock to the endowment.

The intuition for the comparative statics in Figure 2 is the following. The Lagrangian multiplier ($\lambda$) in the Euler equation (6) represents the shadow value of the collateral constraint. When the shock to the endowment is not large enough to push the economy in the constrained region, $\lambda = 0$ and the economy achieves its efficient allocation (as we are not explicitly considering here other distortions). In contrast, when the maximum size of the shock ($\varepsilon$) is large enough, $\lambda$ may be positive and increasing in $\varepsilon$. Therefore, the larger $\varepsilon$, the larger is $\lambda$ and the level of precautionary savings undertaken by consumers.

**B Macroeconomic Friction**

Let us now analyze how the macroeconomic friction affects our model economy. As it is well known from the standard New Keynesian literature, there are two potential distortions in models with monopolistic competition and staggered pricing. First, monopolistic power forces average output below the socially optimal level. Second, staggered pricing implies that both the economy’s average markup and the relative price of different goods will vary over time in response to shocks, violating efficiency conditions.\(^{12}\) As we shall see below, our model displays a similar behavior.

Let us assume for the moment that interest rates can freely adjust and that lending rates at the beginning of period 0 are set at the desired level, as a markup over the marginal cost ($R_{L1} = M R^*$). If a positive shock $\upsilon > 0$ hits the economy, banks face a new, higher marginal cost and update their lending interest rates such that $R_{L1} = M (R^* + \upsilon)$. Households update their loans demand accordingly and the loans market clears at a higher lending rate. In response to the higher interest rate, consumption and borrowing in period 0 fall relative to the case in which $\upsilon = 0$. This allocation (henceforth “flex-rates” allocation) is efficient, conditional on the shock $\upsilon$.

\(^{12}\)Note here that, if no shock pushes the economy away from its equilibrium, the average markup would be equal to the constant frictionless markup and the price of all goods in the economy would be the same, implying that no efficiency condition would be violated.
In a sticky-rates environment, not all banks can reset their lending rate so as to be consistent with the new marginal cost. The fraction $\mu$ of banks that can reset lending rates will set:

$$R_{L1}^\mu = \mathcal{M} (R^* + \upsilon).$$

In contrast, the remaining $1 - \mu$ banks will not be allowed to reset their lending rates, implying that:

$$R_{L1}^{1-\mu} = \mathcal{M} R^* < R_{L1}^\mu.$$

As a consequence, the aggregate lending rate in the economy would differ from its flex-rates counterpart. According to equation 9, the aggregate lending rate in the sticky-rates economy becomes:

$$R_{L1} = \mathcal{M} (R^* + \mu \upsilon),$$

which is higher than the lending rate prevailing under flex-rates in the case of positive shocks to the interest rate. A similar gap of opposite sign emerges when the $\upsilon$ shock is negative.

The model properties analyzed in this section can be summarized as follows. In general, interest rates stickiness results in an average interest rate, $R_{L1}$, which differs from the one required to obtain the flex-rates allocation, therefore affecting the aggregate level of borrowing and consumption. More specifically, when a positive shock hits the interest rate, debt and consumption are higher than in the flex-rates economy, because interest rates increase by less than they would in a fully flexible world. But, when a negative shock hits the economy, debt and consumption are lower than in the flex-rates economy, because interest rates decrease by less than they would in a fully flexible world. As we shall see, this property has crucial implications for the results of our analysis when the macroeconomic frictions interact with the financial friction.

### C The Interaction between Financial Friction and Macroeconomic Friction

In this section we show that the impact of staggered interest rates setting on the crisis probability (our measure of financial stability, i.e. the crisis probability) depends on the sign of the shock hitting the economy. Specifically, we shall see that in response to positive shocks to the risk-free interest rate, the probability of a crisis in the sticky price economy is lower (increases less) than in the in the flex-price economy. Instead, in response to negative shocks to the risk-free interest rate, the crisis probability is higher (it falls less) than in the flex-rates economy. In this sense, we say that interest rate rigidities have an asymmetric impact on financial stability.

We first analyze the effect of a positive shock to the risk-free interest rate (Figure 3). The bench-
mark is the economy with both frictions but no interest rate shocks (solid line, i.e., the same allocation as in Figure 2). The thin line with asterisk markers and the thin line with circle markers display the equilibrium after the interest rate shock has hit, under flexible and sticky interest rates respectively.

Figure 3 MODEL EQUILIBRIUM WITH BOTH FRICTIONS: POSITIVE SHOCK TO THE INTEREST RATE. On the horizontal axis is the maximum size of the endowment shock ($\varepsilon$). The thick solid line displays the equilibriums when no shock hits the risk-free interest rate; the thin line with asterisk markers and the thin line with circle markers display the equilibrium after a positive shock hits the risk-free rate under flex-rates and sticky rates, respectively.

As we showed above, under the assumption of sticky interest rates, the aggregate lending rate in the economy does not increase as much as the risk-free rate following a positive shock. What are the implications for the probability of a crisis, our measure of financial stability? On the one hand, lower lending rates —relative to the flex-rates case— prompt consumers to borrow more ($b_1$) in period 0 and to consume more ($c_1$) in period 1, as shown by the difference between the circles line and the asterisks line. All else equal, this implies higher expected next-period refinancing needs ($b_2$) and, therefore, a higher expected probability that the constraint will be binding in period 1. On the other hand, and despite the higher level of borrowing in period 0, expected net worth ($e - b_1 R_{L1}$) in period 1 is larger under sticky rates than under flex rates, because
of lower interest rate repayments. All else equal, this implies a relaxation of the borrowing constraint in period 1. The net effect is displayed in the bottom right-hand panel of Figure 3: when a positive shock hits the economy, with sticky interest rates the probability that the constraint will bind in period 1 increases by less than in the flex-rates case. This is because, as long as the coefficient of relative risk aversion ($\rho$) is larger than 1, the effect of the lower interest rates on net worth dominates the effect on borrowing and consumption. Note that this result is robust to assuming different values for all other parameters of the model, including the size of the shock to the interest rate ($\nu$) and the degree of interest rate stickiness ($\mu$). Changing these parameters does not affect the mechanisms driving the result, but only the magnitude of the effects. In other words, for every possible value of $\nu$ and $\mu$ the allocation under sticky-rates (circles line) is bounded between the allocation under flex-rates (asterisks line) and the allocation where no shock hits the economy (solid line).

Consider now a negative shock. In the case of a negative shock, sticky interest rates exacerbate the effects of the financial friction rather than dampening it. To see that, Figure 4 displays how the model equilibrium and the crisis probability vary in response to a negative shock to the risk-free interest rate.

Under interest rate stickiness (circles line), the average lending rate now falls by less than the risk-free interest rate. In the sticky rate economy, consumption and borrowing are lower (or increase less) than in the flex-rate economy (asterisks line) in response to the shock, but next period interest payments are higher. As a result next-period net worth in the sticky rates economy is lower than in the flex-rate economy, and the crisis probability is higher (or it falls less).

In conclusion, the analysis of the interaction between the macroeconomic and financial friction can be summarized as follows: when both the macroeconomic and the financial friction are present, interest rate stickiness, conditional on positive shocks to the interest rate reduces the crisis probability relative to the flex-rate equilibrium; conditional on negative shocks to the interest rate, it increases the crisis probability relative to the flex-rate equilibrium. In this sense, interest rate rigidities have an asymmetric impact on financial stability.

IV RESTORING EFFICIENCY

In this section we discuss how policy intervention, and in particular monetary policy, can address the market failures of our model economy.
Figure 4 MODEL EQUILIBRIUM WITH BOTH FRICTIONS - NEGATIVE SHOCK TO THE INTEREST RATE. On the horizontal axis is the maximum size of the endowment shock (\( \varepsilon \)). The thick solid line displays the equilibriums when no shock hits the risk free interest rate; the thin line with asterisk markers and the thin line with circle markers display the equilibrium after a negative shock hits the risk free rate under flex-rates and sticky-rates, respectively.

To build understanding and intuition for the main results, we first analyze the case in which there is only the financial friction or the macroeconomic friction. Then, we consider the case in which the policy authority faces both frictions with either one or two policy instruments.

A key result is that a policy-maker with a macro-prudential instrument (a tax on borrowing) and a monetary policy instrument (the policy interest rate) can address both distortions induced by the financial friction and the macroeconomic friction. In contrast, if the interest rate is the only available instrument, the policy-maker faces a trade-off between macroeconomic and financial stability when the economy is hit by negative shocks.

A ADDRESSING THE PECUNIARY EXTERNALITY

As it is well known, the occasionally binding constraint that is in our model generates a pecuniary externality. This pecuniary externality drives a wedge between private and socially optimal outcomes because private agents do not internalize the effect of their decisions on the asset price.
that enters the specification of the borrowing constraint. A social planner, unlike private agents, can internalize that consumption decisions affect the asset price—as shown by the asset price equation in (6)—which, in turn, affects the aggregate collateral constraint in (4).

Following Jeanne and Korinek (2010a), the planner’s problem for this economy can be written as:

\[
\max_{b_1, b_2} \left\{ u(b_1) + \mathbb{E}_0 \left[ u(e + b_2 + \pi_1 - b_1 R_{L1}) + y - b_2 R_{L2} - \lambda^{sp} (b_2 - p_1(e + b_2 - b_1 R_{L1})) \right] \right\},
\]

where the maximization is subject to the budget constraint (3), the aggregate borrowing constraint (4), and the pricing rule of the competitive equilibrium allocation:

\[
p_1(c_1) = \frac{y}{u'(c_1)},
\]

where the asset price, \( p_1(c_1) \), is now a function of aggregate consumption.

The corresponding first order conditions are:

\[
\begin{align*}
  u'(c_0) &= R_{L1} \mathbb{E}_0 [u'(c_1) + \lambda^{sp} p'(c_1)], \\
  u'(c_1) &= R_{L2} + \lambda^{sp} (1 - p'(c_1)).
\end{align*}
\]

By comparing (6) and (14) and noting that \( p'(c_1) > 0 \), it is clear that there is a wedge between the decentralized and the social planner allocation: the social planner saves more than private agents whenever the borrowing constraint is expected to bind in period 1 with positive probability (i.e., whenever \( \lambda^{sp} > 0 \)). This reflects the fact that the social planner internalizes the endogeneity of next period’s asset price to this period’s aggregate saving. As a consequence, when the constraint never binds, the allocation of resources in the economy is efficient (ignoring the other frictions in the model). However, when there is a positive probability that the constraint binds in period 1, the allocation is not efficient. Consumption and borrowing in the decentralized equilibrium are excessive relative to the allocation chosen by the social planner (i.e., there is overborrowing in the parlance of the literature). As a result, the crisis probability is also higher in the decentralized equilibrium relative to the social planner equilibrium.

\footnote{See Korinek (2010), Bianchi (2011), Jeanne and Korinek (2010a,b), Benigno, Chen, Otrok, Rebucci, and Young (2013) for a more detailed discussion.}
A Pigouvian Tax on Borrowing

In this set-up, Jeanne and Korinek (2010a) show that efficiency can be restored in the decentralized economy by imposing a Pigouvian tax on borrowing in period 0, namely \( b_1(1 - \tau) \), which is rebated with transfers (\( TR \)) in a lump-sum fashion. The optimal tax is given by:

\[
\tau = E_0 \left[ \frac{\lambda^{sp} p'(c_1)}{u'(c_1)} \right],
\]

(15)

This equation states that whenever the borrowing constraint binds in period 1 with positive probability, the policy-maker imposes a positive tax on borrowing in period 0, prompting private agents to issue less debt in period 0 than under decentralized equilibrium. This is because both the shadow value of the collateral constraint (\( \lambda^{sp} \)) and the derivative \( p'_1(c_1) \) are positive.

An Interest Rate Policy

A Pigouvian tax on borrowing may be difficult to implement. But the constrained efficient allocation can also be decentralized with the interest rate. The policy-maker can equally curtail households’ borrowing by increasing lending interest rates. For instance, the policy-maker (e.g., a central bank in this specific case) can increase the interest rate at the beginning of period 0, affecting banks marginal cost and, therefore, consumers’ borrowing and consumption decisions.

This increase in interest rates —if rebated with lump sum transfers (\( TR \))— has the same effect of the Pigouvian tax analyzed above. To see this, assume for simplicity that the central bank can affect the interest rate by an additive factor \( \psi \), so that the marginal cost for banks would be given by \( R^* + \psi \) (see Stein, 2012). The consumers’ maximization problem becomes:

\[
\max_{b_1, b_2} \left\{ u(b_1) + E_0 \left[ u(e + b_2 + \pi_1 - b_1, M(R^* + \psi) + TR) + y - b_2 R_{L2} - \lambda^{sp}(b_2 - p_1) \right] \right\}.
\]

By equalizing the first order condition with respect to \( b_1 \) of the decentralized equilibrium and the social planner equilibrium, we can derive the level of \( \psi \) which closes the wedge:

\[
\begin{align*}
\left\{ & u'(c_0) = R_{L1} E_0 [u'(c_1) + \lambda^{sp} p'(c_1)], \\
& u'(c_0) = M(R^* + \psi) u'(c_1),
\end{align*}
\]

Solving for \( \psi \) yields:

\[
\psi = E_0 \left[ \frac{\lambda^{sp} p'(c_1)}{u'(c_1)} \right] R^*.
\]

(16)
Notice that as long as the shadow value of the collateral constraint ($\lambda^{sp}$) is different from zero, $\psi$ is positive and can be interpreted as a prudential “markup” factor on the risk-free interest rate. This, in turn, implies that whenever the constraint is binding with positive probability, the central bank would raise interest rates so that households consume less and issue less debt in period 0, reducing the probability of hitting the constraint in case of an adverse shock in period 1.

In summary, when the borrowing constraint is the only friction in the economy and the policy rate is the only policy instrument, a social planner can achieve constrained efficiency by increasing interest rates in period 0. This allocation is isomorphic to the one obtained with the Pigouvian tax on debt analyzed in the previous section. As we shall see below, however, this is not always the case. When both the financial and the macroeconomic frictions are present it will depend on the sign of the shock hitting the economy.

### B Addressing Monopolistic Competition and Interest Rate Stickiness

As mentioned in the previous sections, our model embeds two macroeconomic distortions. The first distortion is the presence of market power in loan markets. The second distortion is staggered adjustment of lending rates. In this section we discuss how policy can address them.

Monopolistic competition in the banking sector implies an inefficiently low level of consumption, because lending interest rates are, on average, higher than under perfect competition. As it is standard in the New Keynesian literature, this inefficiency could be eliminated in the decentralized economy through the suitable choice of a subsidy to interest rate repayments such that:

$$R_{L1} = M (1 - \eta_t) R^*_f.$$  \hfill (17)

Hence, the optimal allocation can be attained if $M (1 - \eta_t) = 1$ or, equivalently, by setting $\eta = \zeta$.

Staggered interest rate setting implies an inefficient level of borrowing, consumption, and net worth because the economy’s aggregate lending rate will generally differ from the one prevailing under flexible rates.

One way to address the consequences of interest rate stickiness is as follows. Assume that the central bank can affect the interest rate by an additive factor $\psi$. Thus, the marginal cost of funds for banks —conditional on a shock to the risk free interest rate— would be given by $R^* + \nu + \psi$.

Then, the central bank could set:

$$\psi : R_{L1} = M (R^* + \nu),$$
which is the efficient (i.e., without distortions) level of the lending interest rate. Solving this equality yields:
\[ \psi = \frac{1 - \mu}{\mu} \nu. \] \hfill (18)

Hence, in response to a positive shock to the risk-free rate (\(\nu > 0\)), the central bank would raise interest rates above the competitive equilibrium level by the factor \(\psi > 0\); in contrast, in response to a negative shock to the risk free rate (\(\nu < 0\)), the central bank would lower interest rates below the competitive equilibrium level by the factor \(\psi < 0\).

To see why this would be a constrained-efficient decentralized equilibrium, note the following: in response to such a policy intervention, banks that can adjust their interest rates would do so and make an optimal decision. In contrast, banks that are not allowed to change their interest rates would not be optimizing anyway. But consumers would face the same aggregate interest rate prevailing without sticky rates (that is, as in the undistorted economy) and hence make optimal decisions.

C Addressing Both Frictions with Two Instruments

We now analyze how to implement the constrained-efficient allocation in the decentralized economy when both frictions are present. We focus on the case in which the subsidy to interest rate repayments, \(\eta\) in equation (17), is always in place —so as to remove the distortion generated by monopolistic competition— and the policy-maker has two policy instruments at disposal. Specifically, we consider a policy-maker who maximizes the expected utility of consumers (5), subject to their budget constraints (3) and borrowing constraints (4). The two instruments to address the two distortions in the economy are (1) the interest rate wedge (\(\psi\)) to address the distortion generated by staggered interest rate setting and (2) a prudential tax on debt (\(\tau\)) to address the distortion generated by the pecuniary externality.

When two instruments are available, the policy-maker can address the macroeconomic and financial stabilization problems separately. This is regardless of whether a single policy authority is in charge of both monetary and financial-stability policy (e.g., a central bank) or whether one authority is in charge of monetary policy and the other is in charge of macroprudential policy. In other words, in our set-up, there are no incentives for a central bank and a financial stability authority to deviate from a coordinated equilibrium.

With these considerations in mind, we consider first a positive shock to the risk-free interest rate. The dashed line of Figure 5 displays the model equilibrium when the policy-maker restores efficiency. Figure 5 also displays two additional allocations: the competitive equilibrium in which a
positive shock hits the economy and interest rates are flexible (triangles line); and the competitive equilibrium in which a positive shock hits the economy and interest rates are sticky (squares line). Note that, unlike in the decentralized equilibria discussed in the previous section, here a subsidy (η) is also removing the distortionary effect of the markup and restores the level of interest rates that would prevail in a perfectly competitive banking sector. For this reason, borrowing under flexible interest rates in Figure 5 (triangles line) is now larger than borrowing in Figure 3 (asterisks line).\footnote{Results reported in Figure 5 are robust to the case in which the distortions generated by monopolistic competition are not removed.}

The policy-maker undertakes two independent policy actions, one to address the distortion generated by staggered interest rate setting (the macroeconomic friction) and another one to address the distortion generated by the occasionally binding borrowing constraint (the financial friction). Consider first the macroeconomic friction and then the financial friction.\footnote{Note here that changing the order of the policy actions would not alter the results.} The policy-maker first raises interest rates by a factor ψ > 0 to restore the aggregate lending rate that would prevail un-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5}
\caption{Efficient Allocation with Both Frictions: Positive Shock to the Interest Rate. On the horizontal axis is the maximum size of the endowment shock (ε). The thin lines with triangle and square markers display the equilibrium after a positive shock hits the risk-free rate under flex-rates and sticky rates, respectively; the dashed line displays the efficient allocation with two policy instruments. A subsidy is in place to remove the distortionary effect of monopolistic competition.}
\end{figure}
der flex rates, moving the economy from the sticky-rates competitive equilibrium (squares line) to the flex-rates competitive equilibrium (triangles line). Then, the policy-maker imposes a distortionary tax on debt ($\tau$) to restore the efficient level of borrowing, moving the economy to the constrained efficient equilibrium (dashed line).

As shown in equation 15, the optimal level of $\tau$ is zero when the constraint never binds and positive when the constraint is expected to bind with positive probability in period 1. Figure 5 shows that when $\varepsilon \leq \varepsilon^b$ the triangles line and dashed line coincide. However, when $\varepsilon > \varepsilon^b$ the tax on borrowing is positive, borrowing in period 0 is lower than in the flex-rates competitive equilibrium (upper-right panel of Figure 5), while consumption in period 1 is larger (lower-right panel of Figure 5). That is, whenever the collateral constraint is expected to bind with a positive probability, the policy-maker forces private agents to borrow less in period 0 —therefore increasing their net worth next period— and to consume more in period 1, thereby reducing the probability of a financial crisis.

Note here that, like before, in the flex-rate equilibrium the net worth (and the crisis probability) is lower (higher) than in the sticky-rate one because of the higher debt repayment in period 1. With flexible interest rates, borrowing is lower but it is more costly to service, so net worth in period 1 is lower and the probability of a crisis is higher. Adding the tax on debt curtails borrowing without increasing debt service costs. As the maximum size of the endowment shock increases, the optimal level of debt falls. And above a certain threshold, the crisis probability falls even below the one in the sticky-rate equilibrium.

Consider now a negative shock to the risk-free interest rate (Figure 6). To address the pecuniary externality, the policy-maker can impose a tax on debt whenever there is a positive probability that the constraint will bind in period 1 regardless of the sign of the shock. To address the interest rate rigidity, when a negative shock hits the economy, the policy-maker can lower interest rates by $\psi$. In this case —and differently from a positive shock to the risk-free rate— achieving the flex-rate equilibrium already reduces the probability of a crisis, as the economy moves from the sticky-rates competitive equilibrium (squares line) to the flex-rates competitive equilibrium (triangles line). This is because the higher borrowing than in the sticky-rate case is more than compensated by the lower interest payment. So, when the policy maker uses also the tax on debt, the probability of a crisis decreases even more relative to the sticky-rate case, and it is always below it.

In summary, with two instruments, such as a tax on borrowing and the monetary policy interest rate, a policy maker can address both the financial and the macroeconomic friction, thereby achieving constrained efficiency, independently of the sign of the shock hitting the economy.
Figure 6 Efficient Allocation with Both Frictions: Negative Shock to the Interest Rate. On the horizontal axis is the maximum size of the endowment shock ($\varepsilon$). The thin lines with triangle and square markers display the equilibrium after a negative shock hits the risk-free rate under flex-rates and sticky rates, respectively; the dashed line displays the efficient allocation with two policy instruments. A subsidy is in place to remove the distortionary effect of monopolistic competition.

D The Trade-off: Addressing Both Frictions with One Instrument

Let us now consider the case in which both frictions are present in the model but the interest rate is the only instrument at the policy-maker’s disposal.

Before proceeding it is useful to recall that, in our model, the financial friction results in more borrowing than socially desirable in period 0 when the collateral constraint has a positive probability to bind in period 1, regardless of the sign of the shock. In contrast, the macroeconomic friction generates either more or less borrowing than socially desirable depending on whether the economy is hit by a positive or a negative shock. It is thus evident that, if the policy-maker has only one instrument, she/he may face a trade off in the face of negative shocks when the economy requires interventions in opposite direction.

Consider a positive shock to the risk-free interest rate. As we showed before, both the macroeconomic and the financial friction result in higher borrowing in period 0 relative to the socially efficient allocation. To address the macroeconomic friction, the policy-maker can raise interest rates by the factor $\psi = (1 - \mu) \nu / \mu > 0$, as implied by equation (18); and, to address the financial
friction, she/he can further raise interest rates by the factor \( \psi = \mathbb{E}_0 \left[ (R^* \lambda^p p'(c_1))/\mu'(c_1) \right] > 0 \), as implied by equation (16). Therefore, when a positive shock hits the economy, a single instrument can restore efficiency.

However, when a negative shock hits the economy, the macroeconomic friction and the financial friction require opposite actions on the interest rate. The macroeconomic friction requires a decrease in interest rates: given that interest rates fall by less than in the flexible rate case, the social planner intervenes to lower interest rates by the factor \( \psi = - (1 - \mu) \nu / \mu < 0 \). In contrast, the financial friction requires an increase in interest rates independently of the sign of the shock. Hence, if the interest rate is the only instrument, the social planner would try to lower interest rates to address the macroeconomic friction and, at the same time, to raise the interest rate to address the financial friction.

Summarizing, when both macroeconomic and financial frictions are present, if the policy interest rate is the only available instrument, a policy maker that aims to achieve both macroeconomic and financial stability faces a policy trade-off. In particular, the trade-off emerges when the economy is hit by negative interest rate shocks, because addressing both frictions requires interventions of opposite sign on the policy instrument.

Note here that this result is consistent with the findings of Kashyap and Stein (2012), who raise the issue of a potential conflicts between price stability and financial stability when the policy rate is the only policy instrument. Formally, they show that the introduction of a second instrument (interest payments on reserves, in their model) can resolve that trade-off. Our analysis above not only corroborates their result in a different setting, but also shows that trade-off emerges depending on the sign of the shock.

V IMPLICATIONS FOR US MONETARY POLICY AND FINANCIAL STABILITY

In this section we look at the U.S. recent experience through the lens of our model in a qualitative way. Specifically, the theoretical results in the previous section have implications for the debate on the role of U.S. monetary policy in the run-up to the Great Recession.

Under former Chairman Alan Greenspan, the Federal Reserve lowered its benchmark rate from 6.5 percent to about 2 percent in 2000-01 as a response to the burst of the dot-com bubble. It further lowered interest rates to 1 percent in 2002-03 in response to a deflationary scare, and finally started a long sequence of tightening actions that, during the 2004-06 period, brought the Federal fund rate back to 5 percent (see Figure 1).
Against this background, Taylor (2007) put forth the idea that the Federal Reserve helped inflate U.S. housing prices by keeping rates too low for too long after 2002. His main argument departed from the observation that the policy rate was well below what implied by a standard Taylor rule, a good approximation to the conduct of monetary policy in the previous several years (Figure 1). As a consequence, “those low interest rates were not only unusually low but they logically were a factor in the housing boom and therefore ultimately the bust.”\(^{16}\) Therefore, according to this view, higher interest rates would have reduced both the probability and the severity of the bust that led to the Great Recession.

In this section we evaluate this claim against the qualitative predictions of our model. In particular, we will show that Taylor’s argument can be rationalized within the logic of our model only if we make the following auxiliary assumptions: the policy authority is responsible for financial stability—in addition to the traditional objective of price stability—and it has only one instrument at its disposal. However, Taylor’s argument is no longer valid within the logic of our model if the policy authority has two instruments to address the macroeconomic and the financial friction or, as we showed in the previous section, when there are two different policy authorities for macroeconomic and financial stability with one instrument each. In the latter case, which is the institutional set-up prevailing in the United States, in response to a negative aggregate demand shock, the “optimal” response of the central bank is to slash interest rates without concern for financial stability, which is addressed with the second instrument (or by the other authority).

As we discuss below, the evidence suggests that the U.S. regulators were at best ineffective in curbing the continued expansion of subprime mortgage lending well past the point at which prime lending had started to decline. We conclude from this analysis that Taylor’s claim that U.S. monetary policy is to blame for the Great Recession is not justified within the logic of our model, given the regulatory regime prevailing in the United States and the evidence we report on its inability to curb subprime lending while monetary policy was tightening its stance during the 2004-06 period.

To assess Taylor’s contention through the lenses of the model, consider a negative shock hitting the economy, such as the one that occurred in March 2000 when the dot-com bubble burst. Set the beginning of period 0 as the year 2000 and assume that the economy comes back to its pre-shock level of activity after four years, namely at the beginning of 2004—consistent with the fact that the policy rate was raised for the first time in July 2004. Therefore, each time period in our model corresponds to about 4 years in the data.

Figure 7 reports the qualitative behavior of the lending interest rate as implied by our model when a negative shock hits the economy. We consider two policy regimes. First, the policy-maker has just one instrument to address both frictions (Panel a). Second, the policy-maker has two separate instruments to address macroeconomic and financial friction (Panel b). Notice that each panel of Figure 7 reports the behavior of two interest rates. The solid line is the lending rate that would prevail when there is no interest rate stickiness and no policy action is undertaken (i.e., in the decentralized economy when interest rates are fully flexible) and will serve as a benchmark. The dashed line is the lending interest rate that would prevail when interest rate stickiness is present under the two policy regimes analyzed.

As we discussed in the previous section, if the interest rate is the only policy instrument, there is a trade-off between macroeconomic and financial stability conditional on a negative shock to our model economy. To achieve the efficient allocation, the policy-maker ought to move interest rates in opposite directions. On the one hand, she would have to lower interest rates to restore the aggregate lending rate that would prevail in the absence of interest rate stickiness. On the other hand, she/he would have to raise them to contain the excess borrowing generated by the pecuniary externality. As a result, interest rates in this environment would be set higher than the level...
predicated by focusing only on macroeconomic stability. As illustrated by the left-hand panel of Figure 7, assuming that the weight attached to macroeconomic and financial stability is the same, average lending interest rates would fall by less than in the flex-rates case. Therefore, under this regime, our model is consistent with Taylor’s argument in the sense that it suggests keeping interest rates higher than the flex-rate case to avoid excessive borrowing and large asset price increases, and to reduce the probability of a crisis if the economy is hit by a negative shock in the future.

The results are different when the policy-maker has two separate instruments to address financial and macroeconomic friction. As noted above, this is equivalent to the case in which there are two separate and independent policy authorities, such as a central bank with the objective of price stability and a financial regulator with the objective of financial stability. As we discussed above, in this case, the policy-maker can achieve efficiency with two independent policy actions, regardless of the sign of the shock. Therefore, once the excess borrowing generated by the financial friction is addressed with a macro-prudential tool, it is optimal for the central bank to lower interest rates in order to address interest rate stickiness and restore the flex-rates allocation. As a matter of fact, the right-hand panel of Figure 7 displays how the average lending rate under this regime (dashed line) is effectively equal to the one prevailing under the flexible interest rates (solid line).

In the United States, institutional responsibility for financial stability is shared among a multiplicity of agencies. Therefore, for Taylor’s contention to be justified within our model, we would have to observe an effective regulatory clampdown on mortgage lending during the period in which monetary policy was unusually lax by the standard of the Taylor rule. As we shall see below, the regulatory effort to contain mortgage lending during the period 2003-06 was at best ineffective, if not absent altogether. The evidence we report, therefore, provides support for the idea that regulation (or, more exactly, the lack thereof) was a key factor in determining the magnitude of the boom-bust cycle experienced by the U.S. housing market rather than monetary policy per se.

Since the Glass-Steagall Act of 1932, U.S. depository institutions (e.g., banks, thrifts, credit unions, savings and loans, etc.) have been regulated by different federal agencies. In contrast, non-depository mortgage originators have enjoyed much more freedom even when they were subsidiaries of bank holding companies (see Engel and McCoy, 2011, Demyanyk and Loutskina, 2012). Moreover, as it is well known, the rise of securitization was accompanied by a shift in

---

17 For instance, the Office of the Comptroller of the Currency is in charge of nationally chartered banks and their subsidiaries. The Federal Reserve covers affiliates of nationally chartered banks. The Office of Thrift Supervision oversees savings institutions. The Federal Deposit Insurance Corporation insures deposits of both state-chartered and nationally chartered banks.
the structure of the mortgage industry from an originate-and-hold model to an originate-and-distribute model. Thus, well before the crisis, financial intermediation theory pointed out the risks associated with this shift, with securitization potentially leading to a reduction of financial intermediaries’ incentives to carefully screen borrowers (see Diamond and Rajan, 2001; Petersen and Rajan, 2002).

Figure 8 provides a picture of the evolution of the U.S. mortgage market and monetary policy over the 2000-07 period. Broadly speaking, the picture shows that, after the Federal Reserve started to tighten its monetary policy stance and the prime segment of the mortgage market turned around, the subprime segment of the market continued to boom, with increased perceived risk of loans portfolios and declining lending standards. Despite this evidence, the first restrictive regulatory action was undertaken only in late 2006, after almost two years of steady increases in the federal funds rate.

The upper-left panel of Figure 8 (Panel a) reports the evolution of the federal funds rate (annual average) together with mortgage originations by category over the period 2001-2007. While prime mortgage originations started to fall in 2003, non-prime mortgage originations continued to increase in 2004 and 2005. As a matter of fact, the share of non-prime mortgage over total mortgage originations went from about 20 percent in 2001 to more than 50 percent in 2006, experiencing the largest increase in 2004, while the Federal Reserve was already tightening its monetary policy stance. A similar pattern emerges by looking at the issuance of mortgage backed securities (MBS). The upper-right panel of Figure 8 (Panel b) shows how the share of private label MBS sharply increased in the 2003-06 period.

The lower-left panel of Figure 8 (Panel c) reports the federal funds rate together with the share of mortgage originations with a Loan-to-Value (LTV) ratio greater than 90 percent. Note here that, while the use of countercyclical LTV ratios has been suggested —and in some emerging market economies has already been adopted— as a macro-prudential policy tool, the share of high LTV ratio mortgages in the U.S. spiked in 2005, two years after the beginning of the monetary policy tightening.

Finally, the lower-right panel of Figure 8 (Panel d) reports additional evidence on the fact that, while loan quality was relatively stable or improving from 2000 to 2003, it deteriorated sharply

---

18 By prime loans we refer to loans that conform to Government Sponsored Enterprises (GSE) guidelines; by non-prime loans we refer to Alt-A, Home Equity, FHA/VA, and subprime mortgages.

19 MBS which are issued or guaranteed by a government sponsored enterprise (GSE) such as Fannie Mae or Freddie Mac are referred to as “agency MBS.” Some private institutions, such as subsidiaries of investment banks, banks, financial institutions, non-bank mortgage lenders and home builders, also issue mortgage securities, the so-called “private label” MBS.
Figure 8 Monetary Policy and the U.S. Housing Sector. The figure provides a picture of the evolution of the U.S. mortgage market and monetary policy over the 2000-07 period.

from 2004 to 2007. The Office of the Comptroller of the Currency publishes an annual underwriting survey to identify trends in lending standards and credit risk for the most common types of commercial and retail credit offered by national banks. Using data from the 2009 survey, which covered 52 banks engaged in residential real estate lending, Panel (d) reports the evolution of changes in underwriting standards (dash-dotted line) and the perceived level of credit risk (dashed line) in residential real estate loan portfolios.20 The figure shows that, while the level of perceived risk was sharply increasing starting from 2004, banks started easing their lending standards from 2003 and did even more so in the 2004-05 period.

20 Net percentage calculated by subtracting the percent of banks tightening from the percent of banks easing. Negative values, therefore, indicate easing.
Despite this evidence, U.S. regulators did not take action while monetary policy was being tightened. On the contrary, for instance, the SEC proposed in 2004 a system of voluntary regulation under the Consolidated Supervised Entities program, allowing investment banks to hold less capital in reserve and increase leverage that might have contributed to fueling the demand for mortgage-backed securities (vertical line in our charts under label SEC).

When regulators finally decided to act, it was too late. It was not until September 2006 that regulators agreed on new guidelines (vertical line under label FDIC 1) aimed at tightening “non-traditional” mortgage lending practices. Note however that, even if it served as a signal to the mortgage market of changing direction of regulatory policy, the new underwriting criteria did not apply to subprime loans, whose standards were discussed in a subsequent regulatory action which was introduced in June 2007 (vertical line under label FDIC 2). By that time, more than 30 subprime lenders had gone bankrupt and many more followed suit.

In summary, the evidence above suggests that Taylor’s contention that excessively lax monetary policy might have contributed to the occurrence and the severity of the great recession does not appear justified within the logic of our model. Indeed, in the context of a framework in which the regulatory and monetary policy functions are assigned to different agencies that can rely on different instruments, the evidence above suggests that monetary policy was appropriately targeting macroeconomic stability. The regulatory function of the system, instead, was at best ineffective in addressing the financial imbalance that continued to grow in the subprime mortgage market while monetary policy was tightened in 2004-05. With the fall in interest rates after the burst of the dot-com bubble and with house prices at bubble-inflated levels, the mortgage industry found creative ways to expand lending and make large profits. Government regulators maintained a hands-off approach for too long: even though the variables plotted are equilibrium outcomes, Figure 8 shows that policy measures aimed at tightening a largely unregulated sector of the U.S. mortgage market kicked in much later than the tightening of monetary policy enacted by the Federal Reserve.

VI CONCLUSIONS

In this paper, we develop a model featuring both a macroeconomic and a financial friction that speaks to the interaction between monetary and macro-prudential policy and to the role of U.S. monetary policy in the run up to the Great Recession.

There are two main results. First, we show that real interest rate rigidities have a different impact on financial stability (defined and measured as the probability that a borrowing constraint binds), depending on the sign of the shock hitting the economy. In response to positive shocks to
the risk-free interest rate, real interest rate rigidity acts as an automatic macro-prudential stabilizer. This is because higher debt today associated with lower interest rates (relative to the flexible interest rate case) is offset by lower interest repayments, resulting in higher net worth and lower probability of a crisis in the future. In contrast, when the risk-free rate is hit by a negative shock, real interest rate rigidity leads to a relatively higher crisis probability through the same mechanisms working in reverse (borrowing and consumption are relatively lower today, but they are offset by relatively higher debt service tomorrow, resulting in lower future net-worth and higher crisis probability).

Second, we show that, when the interest rate is the only policy instrument to address both the macroeconomic and the financial friction, and a shock that lowers interest rates hits the economy, a policy trade-off emerges. This is because the two frictions require interventions of opposite direction on the same instrument. Other instruments, however, may be at the policy-maker’s disposal in order to achieve and maintain financial stability. Our model shows that, when two instruments are available, this trade-off disappears and efficiency can be restored.

Our analysis has interesting implications regarding the role of U.S. monetary policy in the run-up to the Great Recession. In a series of recent papers Taylor (2007, 2010) suggested that higher interest rates in the 2002-2006 period would have reduced both the likelihood and the severity of the Great Recession. Our findings above support this argument only if we make the auxiliary assumption that the policy authority seeks to address all distortions in the model with a single instrument, namely the policy interest rate. In contrast, when the policy authority has two different instruments, interest rates can be lowered as much as needed in response to a contractionary shock without concerns for financial stability. This is consistent with the view of Bernanke (2010) that additional policy tools, to limit dangerous expansions in leverage, were needed to prevent the global financial crisis.
References


A Appendix. Numerical Solution

First order conditions. We solve for the equilibrium going backward, as in Jeanne and Korinek (2010a). In period 1, the consumers maximize their utility subject to the budget constraint in (3) and the collateral constraint in (4). The problem for the representative consumer therefore is:

$$V_1 = \max_{b_2, \theta_2} \left\{ u \left( e + b_2 + (\theta_1 - \theta_2)p_1 + \pi_1 - b_1R_L \right) + \theta_2y + \pi_2 - b_2R_L - \lambda(b_2 - \theta p_1) \right\},$$

where notice that the net worth $e - b_1R_L$ is taken as given. The first order conditions read:

$$\begin{align*}
FOC(b_2) : & \quad u'(c_1) = R_L + \lambda, \\
FOC(\theta_2) : & \quad p_1 = y/u'(c_1).
\end{align*}$$

In period 0, consumers solve the following problem:

$$\max_{b_1} \{ u(b_1) + \mathbb{E}_0[V_1] \},$$

where we make use of the fact that, in equilibrium, $\theta_t = 1$. The maximization yields:

$$u'(c_0) = R_L \mathbb{E}_0[u'(c_1)].$$

Preliminaries. The first order conditions of the competitive equilibrium (CE) therefore are:

$$\begin{align*}
FOC(b_1) : & \quad u'(c_0) = R_L \mathbb{E}_0[u'(c_1)], \\
FOC(b_2) : & \quad u'(c_1) = R_L + \lambda, \\
FOC(\theta_2) : & \quad p_1 = y/u'(c_1).
\end{align*}$$

When the economy is not constrained ($\lambda = 0$) the model has the following close form solution:

$$\begin{align*}
u'(c_1) = R_L, \\
u'(c_0) = \mathbb{E}_0[R_L R_L], \\
p_1 = \frac{y}{R_L}.\end{align*}$$

Moreover, by definition, the collateral constraint must hold when the economy is not constrained$^{21}$:

$$\frac{b_2^*}{c_1^* + b_1^*R_L} - e \leq p_1^*,$$

which we can rewrite as:

$$e \geq e^b = c_1^* + b_1^*R_L - \frac{y}{R_L}.$$  

That is, whenever the endowment is above a certain threshold ($e \geq e^b$) the economy is not constrained. On the other hand, when the economy is constrained ($e < e^b$) the collateral constraint is

---

$^{21}$Note here that we are assuming that profits are realized at the end of the period so that they have no effect on the borrowing constraint.
binding and consumers would like to borrow \( b_2 > p_1 \). Given that this is not possible, consumers will borrow as much as they can, trying to maximize their consumption in period 1. In this case, the collateral constraint will bind with equality \( b_2 = p_1 \), so that:

\[
c_1 + b_1 R_{L1} - e = \frac{y}{u'(c_1)},
\]

and using the fact that the utility function is in CES form:

\[
c_1 + b_1 R_{L1} - e = y c_1^p.
\]

Therefore, depending whether the constraint is binding or not, we can express borrowing in period 0 as:

\[
b_1 = \begin{cases} (R_{L2} R_{L1})^{-\frac{1}{\rho}} & e \geq e^b \\ \frac{y c_1^p - c_1 + \varepsilon}{R_{L1}} & e < e^b \end{cases}
\]

Finally, we assume that the endowment is stochastic and follows a uniform distribution \( e \sim U(\bar{e} - \varepsilon, \bar{e} + \varepsilon) \).

**Assumption on parameter values.** To be able to solve the model we need to make assumptions on the value of two parameters: \( y \) and \( \bar{e} \). In particular, we will consider values such that 1) the economy may be constrained for sufficiently large negative shocks but 2) would not be constrained in the absence of uncertainty.

First, we want a condition that is necessary and sufficient for the economy to be constrained with some probability, when \( e \sim U(\bar{e} - \varepsilon, \bar{e} + \varepsilon) \). Let’s reason the other way round: we already showed that the economy is indeed unconstrained in period 1 if and only if:

\[
e \geq e^b = c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}}.
\]

When \( e \) is stochastic, for the economy to be unconstrained, the above inequality must hold for all possible realizations of \( e \) (in particular the adverse realizations). In other words it must be the case that:

\[
e - \varepsilon \geq c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}},
\]

\[
\bar{e} \geq c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}} + \varepsilon.
\]

Therefore, when \( \bar{e} < c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}} + \varepsilon \) there exists a non-zero probability that the constraint binds.

Second, we want a condition that is necessary and sufficient for the economy to be unconstrained when there is no uncertainty around the realizations of \( e \) (i.e., \( \varepsilon = 0 \) and \( \bar{e} = e \)). When \( \varepsilon = 0 \), the constraint is not binding in period 1 if and only if \( e = \bar{e} \geq e^b \), that is:

\[
\bar{e} \geq c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}}.
\]

Therefore, with no uncertainty, when \( \bar{e} \geq c_1^* + b_1^* R_{L1} - \frac{y}{R_{L2}} \) the constraint never binds.
Summarizing we choose an \( \bar{\epsilon} \) such that would not be constrained in the absence of uncertainty but it economy may be constrained for sufficiently large negative shocks:

\[
(R_{L2})^{-\frac{1}{\beta}} + (R_{L2}R_{L1})^{-\frac{1}{\beta}} R_{L1} - \frac{y}{R_{L2}} \leq \bar{\epsilon} < (R_{L2})^{-\frac{1}{\beta}} + (R_{L2}R_{L1})^{-\frac{1}{\beta}} R_{L1} - \frac{y}{R_{L2}} + \epsilon.
\]

This implies that there will be a threshold for the size of the shock (\( \epsilon^b \)) above which the collateral constraint will start to be binding with positive probability. Specifically, the collateral constraint would be binding for realizations of \( \epsilon \) in the interval \( [\bar{\epsilon} - \epsilon, \bar{\epsilon} - \epsilon^b] \). The level of \( \epsilon^b \) can be easily computed as:

\[
\epsilon^b = \bar{\epsilon} - \epsilon^b = \bar{\epsilon} - c^*_1 - b^*_1 R_{L1} + \frac{y}{R_{L2}}.
\]

**Competitive equilibrium.** We will find numerical values for consumption at time 1 (\( c_1 \)) by using is the Euler equation \( FOC(b_1) \), which gives us an optimal relation between consumption in period 0 and consumption in period 1.\(^{22}\) In order to be able to solve this equation we need 1) to find an expression for borrowing as a function of consumption for both constrained and unconstrained states, as we already did in equation (A.2); and 2) to weight those states for their probability. Combining \( FOC(b_1) \), the budget constraint, and the expression for \( b_1 \) derived earlier in equation (A.2) we get the following system of equations:

\[
\begin{cases}
  b^{-\rho}_1 = R_{L1}E_0\left[c^{-\rho}_1\right], \\
  b_1 = \begin{cases}
    (R_{L2}R_{L1})^{-\frac{1}{\beta}} & e \geq \epsilon^b, \\
    \frac{y(c^*_1 - c_1) + e}{R_{L1}} & e < \epsilon^b.
  \end{cases}
\end{cases}
\]

By plugging the second equation in the first one we can write:

\[
Pr(e < \epsilon^b) \cdot \left[ b^{-\rho}_1 \right]^{\text{binding}} + Pr(e \geq \epsilon^b) \cdot \left[ b^{-\rho}_1 \right]^{\text{non-binding}} = R_{L1}E_0\left[c^{-\rho}_1\right].
\]

Now, by effectively substituting for \( b_1 \), The LHS of the previous equation can be expressed as

\(^{22}\)Rember that \( c_0 = b_1 \) from the budget constraint.
follows:\textsuperscript{23}
\[ b_1^{-\rho} = \frac{1}{2\varepsilon} \int_{-\varepsilon}^{\varepsilon} \left( \frac{yc_1^\rho - c_1 + e}{R_{L1}} \right)^{-\rho} de + \frac{1}{2\varepsilon} \int_{-\varepsilon}^{\varepsilon} R_{L2}R_{L1} de = \]
\[ = \frac{1}{2\varepsilon} \int_{-\varepsilon}^{\varepsilon} \left( \frac{yc_1^\rho - c_1 + e}{R_{L1}} \right)^{-\rho} de + \frac{RL_2R_{L1}}{2\varepsilon} \left[ e \right]_{-\varepsilon}^{\varepsilon} + \frac{RL_2R_{L1}}{2\varepsilon} \left[ e + \varepsilon^b \right] \]
\[ = \frac{RL_1^\rho}{2\varepsilon(1-\rho)} \left[ \left( yc_1^\rho - c_1 + \varepsilon - \varepsilon^b \right)^{-\rho + 1} - \left( yc_1^\rho - c_1 + \varepsilon - \varepsilon \right)^{\rho + 1} \right] + \frac{RL_2R_{L1}}{2\varepsilon} \left[ e + \varepsilon^b \right] \]

By equalizing LHS to RHS numerically, we obtain the competitive equilibrium level of consumption at time 1, where remember that:
\[ \text{LHS} = \frac{RL_1^\rho}{2\varepsilon(1-\rho)} \left[ \left( yc_1^\rho - c_1 + \varepsilon - \varepsilon^b \right)^{\rho + 1} - \left( yc_1^\rho - c_1 + \varepsilon - \varepsilon \right)^{\rho + 1} \right] + \frac{RL_2R_{L1}}{2\varepsilon} \left[ e + \varepsilon^b \right] \]
\[ \text{RHS} = R_{L1}E_0 \left[ c_1^{-\rho \cdot \frac{1}{\rho}} \right]. \]

Finally, one can also derive the level of optimal debt at time 0, by using again \( FOC(b_1) \):
\[ b_1 = E_0 \left[ \left( R_{L1}c_1^{-\rho} \right)^{-\frac{1}{\rho \cdot \rho}} \right]. \]

\textbf{Social planner.} The social planner problem is solved with the same strategy. The first order conditions are:
\[ \begin{cases} 
FOC(b_1) : u'(c_0) = R_{L1}E_0[u'(c_1) + \lambda p'(c_1)], \\
FOC(b_2) : u'(c_1) = R_{L2} + \lambda (1 - p'(c_1)), \\
FOC(\theta_2) : p_1 = \frac{\gamma}{u'(c_1)}. 
\end{cases} \]

First we have to find an expression for \( p'(c_1) \). From \( FOC(\theta_2) \) we get:
\[ p(c_1) = \frac{y}{u'(c_1)} = yc_1^\rho, \]
and computing the derivative:
\[ p'(c_1) = \frac{\partial (yc_1^\rho)}{\partial c_1} = \rho yc_1^{\rho - 1}. \]
Notice that the \( p'(c_1) \) is positive and decreasing. Notice also that, by definition, the Lagrange multiplier (\( \lambda \)) is positive only when the constraint is binding. By looking at \( FOC(b_1) \) of the social planner problem, we can state that the planner limits over-borrowing. In fact, \( u'(c_1)^{SP} > \]

\textsuperscript{23}If \( X \) is uniformly distributed with \( U(a,b) \), then the \( n^{th} \) moment of \( X \) is given by \( E[X^n] = \frac{1}{n-a} \int_a^b x^n dx. \)
$u'(c_1)^{CE}$ which implies that consumption and, therefore, borrowing at time 1 are lower relative to the competitive equilibrium. On the other hand, the planner increases consumption in period 1: given that $p'(c_1) > 0$ from $FOC(b_2)$ we see that $u'(c_1)^{SP} < u'(c_1)^{CE}$.

We also need a value of $\lambda$. Notice that the Lagrange multiplier of the social planner is numerically different from the one of the competitive equilibrium problem. In fact, from $FOC(b_2)$ we get

$$\lambda = \frac{c_1 - \rho}{1 + y}.$$ 

Combining these two results we can compute:

$$\lambda p'(c_1) = \begin{cases} 0 & e \geq e^b, \\ \frac{\rho y}{1 + y} \left( c_1^{-1} - R_{L2} c_1^{\rho - 1} \right) & e < e^b. \end{cases}$$

We can now solve for the level of $c_1$. The $FOC(b_1)$ can be written:

$$b_1^{-\rho} = R_{L1} \bar{E}_0 \left[ c_1^{-\rho} + \lambda p'(c_1) \right].$$

The LHS has already been computed before. The RHS is:

$$\frac{R_{L1}}{2\epsilon} \int_{\bar{\epsilon} - \epsilon^b}^{\bar{\epsilon} + \epsilon} \left( c_1^{-\rho} + \frac{\rho y}{1 + y} \left( c_1^{-1} - R_{L2} c_1^{\rho - 1} \right) \right) de + \frac{R_{L1}}{2\epsilon} \int_{\bar{\epsilon} - \epsilon^b}^{\epsilon} c_1^{-\rho} \bar{e} d\bar{e},$$

$$= \frac{R_{L1}}{2\epsilon} \left[ \left( c_1^{-\rho} + \frac{\rho y}{1 + y} \left( c_1^{-1} - R_{L2} c_1^{\rho - 1} \right) \right) \left( \epsilon - \epsilon^b + \epsilon \right) + c_1^{-\rho} \left( \epsilon + \epsilon^b \right) \right],$$

$$= \frac{R_{L1}}{2\epsilon} \left[ \left( c_1^{-\rho} + \rho y \left( c_1^{-1} - R_{L2} c_1^{\rho - 1} \right) \right) \left( \epsilon - \epsilon^b \right) + 2c_1^{-\rho} \epsilon \right]$$

As we have done above, by equalizing LHS to RHS numerically, we obtain the competitive equilibrium level of consumption at time 1, where remember that:

$$\text{LHS} = \frac{R_{L1}}{2\epsilon(1 - \rho)} \left[ (\rho y \epsilon + c_1 + \bar{\epsilon} - \epsilon^b)^{\rho + 1} - (\rho y \epsilon - c_1 + 1 - \epsilon - \rho)^{\rho + 1} \right] + \frac{R_{L2} R_{L1}}{2\epsilon} \left[ \epsilon + \epsilon^b \right]$$

$$\text{RHS} = \frac{R_{L1}}{2\epsilon} \left[ \left( \rho y \left( c_1^{-1} - R_{L2} c_1^{\rho - 1} \right) \right) \left( \epsilon - \epsilon^b \right) + 2c_1^{-\rho} \epsilon \right].$$

Finally, one can derive the optimal expression for borrowing at time 1 from the social planner $FOC(b_1)$:

$$b_1 = \left( R_{L1} \bar{E}_0 \left[ c_1^{-\rho} + \lambda p'(c_1) \right] \right)^{-\frac{1}{\rho}}.$$ 

**Crisis Probability.** The crisis probability is defined as the probability of the constraint to be
binding:

\[ \Pr [b_2 > p_1] = \frac{1}{2\varepsilon} \int_{\tilde{\varepsilon} - \varepsilon}^{\tilde{\varepsilon} - \varepsilon^b} \, de = \frac{1}{2\varepsilon} \left( \varepsilon - \varepsilon^b \right). \]

which, using the optimality conditions and the budget constraint, can be written as

\[ \Pr \left[ (c_1 - (e - b_1 R_{L1}) > \frac{y}{u'(c_1)} \right]. \]

Now, knowing that \( e = \tilde{\varepsilon} + \varepsilon \) and that \( \varepsilon \sim U(-\varepsilon, \varepsilon) \), we can write

\[ \Pr \left[ \tilde{\varepsilon} < c_1 - \tilde{\varepsilon} + b_1 R_{L1} - \frac{y}{u'(c_1)} \right]. \]

In particular, the probability of the constraint to be binding is given by:

\[ \Pr \left[ -\varepsilon \leq \tilde{\varepsilon} < x \right] = \frac{x - (-\varepsilon)}{2\varepsilon} = \frac{c_1 - \tilde{\varepsilon} + b_1 R_{L1} - y/u'(c_1) + \varepsilon}{2\varepsilon}. \]