Contagion and Volatility with Imperfect Credit Markets

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Abstract

This paper interprets contagion effects as an increase in the volatility of aggregate shocks impinging on the domestic economy. The implications of this approach are analyzed in a model with two types of credit market imperfections: domestic banks borrow at a premium on world capital markets, and domestic producers (whose demand for credit results from working capital needs) borrow at a premium from domestic banks. Higher volatility of producers' productivity shocks increases both domestic and foreign financial spreads and the producers' cost of capital, resulting in lower employment and higher incidence of default. Welfare effects are nonlinearly related to the degree of international financial integration.

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SUMMARY

This paper analyses the aggregate effects and transmission process of contagious shocks. Specifically, this paper interprets contagion effects as an increase (triggered by events occurring elsewhere) in the volatility of aggregate shocks impinging on the domestic economy. The implications of this approach are analyzed in a model with two types of credit market imperfections: domestic banks borrow at a premium on world capital markets, and domestic producers (whose demand for credit results from working capital needs) borrow at a premium from domestic banks—which possess comparative advantage in monitoring the behavior of domestic agents. Financial intermediation spreads are shown to be determined by a markup that compensates for the expected cost of contract enforcement and state verification and for the expected revenue lost in adverse states of nature.

The analysis shows that higher volatility of producers' productivity shocks increases both financial spreads and the producers' cost of capital, resulting in lower employment and a higher incidence of default. Simple empirical calculations, based on data for a small group of Asian and Latin American countries, before and after the Mexican peso crisis suggest that (as predicted by the model) increased volatility tends to be associated with reductions in economic activity. It is also shown that, in this setting, the welfare effects of volatility are nonlinear. Higher volatility does not impose any welfare cost for countries characterized by relatively low volatility and efficient financial intermediation. The adverse welfare effects are large (small) for countries that are at the threshold of full integration with international capital markets (close to financial autarky), that is, countries characterized by a relatively low (high) probability of default.
I. INTRODUCTION

The Mexican peso crisis of December 1994 brought turmoil to financial and foreign exchange markets worldwide, but nowhere more dramatically than in Argentina. Between December 1994 and March 1995, prices of Argentine bonds and stocks traded on domestic and international markets fell abruptly. The central bank lost about a third of its liquid international reserves, and the interest rate spread between U.S. dollar-denominated bonds issued by Argentina and U.S. Treasury bills increased sharply, to more than 700 basis points during 1995 (Figure 1), whereas the spread between domestic and U.S. interest rates widened to around 900 basis points during the same period. Turbulences in financial markets escalated very quickly into a full-blown economic crisis: bank deposits and bank credit dropped dramatically, and domestic interest rates (on both peso- and dollar-denominated loans) increased sharply—from about 10 percent to more than 40 percent and 30 percent per annum, respectively, between December 1994 and March 1995. Real GDP fell by almost 5 percent for 1995 as a whole, and unemployment increased sharply (Figure 2). Bank closures and restructuring operations led to a consolidation of the banking system.

Several observers have argued that the collapse of the Mexican peso in December 1994, and the ensuing sharp swing in investors’ sentiment toward emerging markets (the so-called Tequila effect), triggered Argentina’s economic crisis. Various papers have recently attempted to model contagious shocks of this type. Uribe (1996), for instance, formalizes the Tequila effect as a situation in which domestic agents learn at a given moment in time that, at some point in the future, foreign investors will liquidate their holdings of domestic assets—in effect imposing a binding borrowing constraint on domestic agents. Goldfajn and Valdés (1997) highlight the role of liquidity factors in the spread of exchange market pressures across countries, and show how currency crises can have contagious effects of the type observed in the aftermath of the peso crisis. Agénor (1997) formalizes a contagious shock as a temporary increase in the autonomous component of the risk premium (reflecting “country risk” factors or exogenous elements in market perceptions) that domestic borrowers face on world capital markets. The real effects of such a shock in Agénor’s model are captured by linking the financial sector and the supply side via firms’ working capital needs—namely, the need to finance labor costs prior to the sale of output. The model’s predictions, under the assumption that the shock was perceived to be of a sufficiently

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2There appears to be some agreement that this shift in market sentiment was not entirely warranted by fundamentals. On the one hand, the real exchange rate had indeed appreciated substantially since the introduction of the Convertibility Plan in April 1991, and the current account deficit (as a share of output) was increasing. On the other, however, inflation was low and falling, output and exports were growing at a relatively high rate (with real GDP growing by more than 7 percent a year between 1991 and 1994), and ample liquid reserves appeared to be available to defend the fixed parity between the U.S. dollar and the peso.
Figure 1
Argentina and Mexico: Secondary Market Yield Spreads on U.S. Dollar-Denominated Eurobonds *
(In basis points)

Sources: Bloomberg and Reuters.
Figure 2
Argentina and Mexico: Industrial Output
(December 1994 = 100)

Sources: FIEL and IFS.
long duration, replicate some of the main features of Argentina's economic downturn.  

The analysis presented in the present paper departs from existing studies in two important ways. First, we model not only distortions on world capital markets but also domestic credit market imperfections. We do so by considering a two-level financial intermediation process: domestic banks borrow at a premium on world capital markets, and domestic agents (which consist only of producers) borrow also at a premium from domestic banks. The reason why modeling domestic capital market imperfections is important is well illustrated by the data shown in Figure 3: not only did the "foreign" financial intermediation spread increased sharply in the immediate aftermath of the peso crisis in Argentina (as well as Mexico), but so did the spread between domestic bank lending and deposit rates. As it turns out, the model is capable of accounting for this fact by showing how financial intermediation spreads are related to default probabilities and underlying shocks. Second, instead of modeling contagion effects as a deterministic (and temporary) shift in an exogenous component of the risk premium, we focus, in a stochastic setting, on the case where contagion takes the form of an increase in the volatility of aggregate shocks impinging on the domestic economy—that is, a mean-preserving increase in the range of values that such shocks may take. To the extent that such increases translate into a rise in the probability of default of domestic producers on their loan commitments, domestic and foreign interest rate spreads will tend to rise, leading to a drop in expected output. We are thus able to identify factors that may have contributed to propagating and magnifying an initial exogenous shock. Our analysis also helps clarify the effects of changes in the expected cost of enforcement of loan contracts, both at the domestic and international levels. 

The predictions of our framework are not only consistent with the observed increase in financial intermediation spreads and a contraction in activity in Argentina and Mexico (as discussed earlier) but also with higher volatility of output. Our calculations, as discussed below, also support this prediction.

The remainder of the paper proceeds as follows. The process of domestic financial intermediation (which involves producers and commercial banks) is described in Section II. Financial intermediation on world capital markets (involving domestic

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3In addition to these studies, Kaufman (1996) uses the Stiglitz-Weiss model of credit rationing (see Jaffee and Stiglitz, 1990) to argue that the credit crunch in Argentina resulted from an increase in the share of illiquid borrowers induced by the rise in interest rates, and increased incidence of adverse selection problems. Essentially, banks faced greater difficulties screening out between "safe" and "risky" borrowers, because those borrowers most willing to pay a higher interest rate on loans were precisely those for which the potential risk of default had increased. Catão (1997, p. 6) estimates that problem loans had already exceeded 10 percent of the loan portfolio of all financial institutions by end-1994.

4In contrast to some of the existing studies, our model is static and partial equilibrium in nature. In particular, we do not model explicitly consumption decisions or central bank regulations. However, some of these features could be added at the cost of greater complexity, without adding much insight.
Figure 3
Argentina and Mexico: Domestic Interest Rate Spread

Sources: IFS.
1/ Lending rate minus deposit rate.
2/ Average cost of funds minus the deposit rate.
banks and foreign lenders) is discussed in Section III. Section IV studies the effects of a contagious shock, modeled as an increase in the volatility of the aggregate stochastic shock faced by domestic producers. Section V examines the welfare implications of this shock, by focusing on changes in the expected producers’ surplus. Finally, Section VI considers some possible extensions of the analysis, discusses alternative factors that may affect the transmission process of contagious shocks (notably the role of asymmetric information), and offers some final remarks.

II. OUTPUT AND THE CREDIT MARKET

We consider an economy in which two categories of domestic agents operate: producers and commercial banks. Both categories of agents are risk neutral—allowing us to use expected income as a measure of welfare—and behave identically. The “representative” bank borrows on world capital markets, facing a gross expected cost of funds equal to \(1 + \bar{r}_f\), and lends to domestic agents at the contractual gross interest rate \(1 + r_L\). Domestic banks have comparative advantage in enforcing repayment of their loans to domestic producers, and are therefore not subject to direct competition from foreign lenders. Thus, producers borrow only from domestic banks, and not directly from foreign lenders.

To simplify exposition, we refrain from modeling domestic saving, and assume that the domestic saving is zero at the international interest rates facing the country.\(^5\)

Output of producer \(h\) is a function of labor employed and a composite productivity shock:

\[
y_h = n_h^\beta (1 + \delta_0 + \delta + \varepsilon_h). \quad \beta < 1
\]

(1)

In the above equation, \(n_h\) is employment, and \(\delta\) is an aggregate shock with zero mean and a density function \(g(\delta)\) defined over the interval \((-\delta_m, \delta_m)\), with \(\delta_m > 0.\(^6\) \varepsilon_h\) is a producer-specific, idiosyncratic shock with zero mean and a density function \(f(\varepsilon_h)\) defined over the interval \((-\varepsilon_m, \varepsilon_m)\), where \(\varepsilon_m > 0\). Because \(\delta\) and \(\varepsilon_h\) have zero mean, expected productivity is \(1 + \delta_0\). The aggregate shock will be viewed as capturing the productivity effects of random shifts in external factors.

\(^5\)Domestic saving would become relevant in financial autarky, where all the financing is supported by domestic saving. It can be verified that adding the domestic saving schedule would not change the key results. This is because the real interest rate that prevails in autarky is high relative to the international one, implying that with integrated financial markets the marginal financing is done via foreign saving, and financial autarky may be associated with relatively small investment.

\(^6\)Although we use, throughout the paper, the expression “aggregate productivity shock” to refer to \(\delta\), a more general interpretation is to think of \(\delta\) as a composite (or reduced-form) shock to output. See the discussion in the concluding section.
The process of domestic financial intermediation can be characterized as follows. Producers must finance their entire working capital needs (which consist only of labor costs) prior to the sale of output. They cannot issue claims on their capital stock to finance these needs, and therefore borrow from domestic banks. Total production costs faced by producer $h$ are thus equal to the wage bill plus interest payments made on bank loans needed to pay labor in advance, $(1 + r_L)\omega h$, where $w$ is the going wage (assumed constant). If producer $h$ chooses to default on part or all of his repayment obligations (after all shocks are realized), domestic banks have the capacity to force him (through appropriate legal actions) to pay a fraction $0 \leq \kappa \leq 1$ of his realized output. Enforcing repayment involves a cost $C'$ (measured in units of output) to the bank.\footnote{The enforcement cost can be related, in particular, to the idea of costly state verification (see Townsend, 1979). That is, it costs $C$ to verify the realization of $\varepsilon_h$ and to force the producer to repay accordingly. Although $C$ is modeled as a fixed monitoring and enforcement cost per loan, the analysis can be extended to allow for a variable cost, proportional to the size of the loan, without changing the key results derived below. $\kappa$ could also be made endogenous. For earlier models of imperfect creditworthiness with costly state verification in a related context, see Aizenman et al. (1996), Bernanke and Gertler (1989), Boot et al. (1991), Calvo and Kaminsky (1991), and Eaton (1986).}

In line with the willingness-to-pay approach to debt contracts, producer $h$ will choose to default if the following constraint is satisfied\footnote{As is usual in the literature, repayment obligations are assumed not to be contingent on the state of nature. Loan agreements in practice typically specify a single contractual rate, which lenders can adjust only if the borrower violates some specific terms of the contract.}

$$
\kappa n_h^\beta(1 + \delta_0 + \delta + \varepsilon_h) < (1 + r_L)\omega h. \tag{2}
$$

If the probability of default is zero, the contractual lending rate is equal to the bank’s expected cost of borrowing on world capital markets ($r_L = \tilde{r}_b^f$) and producer $h$’s expected profits are given by

$$
n_h^\beta(1 + \delta_0) - (1 + \tilde{r}_b^f)\omega h. \tag{3}
$$

From the first-order condition for profit maximization, optimal employment $n_0$ is thus given by

$$
\beta n_0^{\beta-1}(1 + \delta_0) = (1 + \tilde{r}_b^f)w.
$$

Given that employment is set optimally, equation (2) implies that for the probability of default to be zero over the whole range of realizations of $\varepsilon_h$ and $\delta$ requires that, setting $\varepsilon_h = -\varepsilon_m$ and $\delta = -\delta_m$:

$$
\kappa n_0^\beta(1 + \delta_0 - \delta_m - \varepsilon_m) > (1 + \tilde{r}_b^f)\omega n_0,
$$
or equivalently, using the first-order condition given above:

$$
\kappa(1 + \delta_0 - \delta_m - \varepsilon_m) > \beta(1 + \delta_0),
$$

which can be rearranged to give

$$
\delta_m + \varepsilon_m < (1 - \frac{\beta}{\kappa})(1 + \delta_0).
$$

To make the problem nontrivial, we assume that $\delta_m + \varepsilon_m$ is sufficiently large to ensure that inequality (4) is reversed.\(^9\) Thus, ex ante, some producers are always expected to default on their loan obligations. At the same time, we assume for expositional simplicity that no producer will default if the economy is in the “best” (aggregate) state of nature (that is, if $\delta = \delta_m$, all producers would opt to repay fully their loan obligations).

The contractual interest rate charged by the representative bank to any given domestic producer $r_L$ is determined by the condition that expected gross repayment from borrower $h$ (evaluated over the whole range of variation of $\varepsilon_h$ and $\delta$, and thus taking into account partial repayment and enforcement costs in adverse states of nature) be equal to the gross expected value of the loan contracted on world capital markets by the bank. In turn, this value equals the size of the loan to producer $h$ times the expected cost of funds faced by domestic banks abroad, $1 + \hat{r}_b^f$:

$$
(1 + \hat{r}_b^f)\omega n_h = (1 + r_L)\omega n_h \int_{\delta_m}^{\delta^*} g(\delta) d\delta + \int_{-\delta_m}^{\delta^*} \Phi(\delta) g(\delta) d\delta,
$$

where\(^10\)

$$
\Phi(\delta) = (1 + r_L)\omega n_h \int_{\varepsilon_m}^{\varepsilon^*} f(\varepsilon_h) d\varepsilon_h + \int_{\varepsilon_m}^{\varepsilon^*} (\kappa y_h - C) f(\varepsilon_h) d\varepsilon_h,
$$

and $\delta^*$ defined as

$$
\delta^* = (1 + r_L)\omega n_h / \kappa n_h^\beta - 1 - \delta_0 + \varepsilon_m,
$$

and $\varepsilon^*$ defined as

$$
\varepsilon^* = (1 + r_L)\omega n_h / \kappa n_h^\beta - 1 - \delta_0 - \delta.
$$

In the above expressions, $\delta^*$ is the threshold value of the aggregate productivity shock below which realizations of $\delta$ (satisfying therefore the condition $\delta \leq \delta^*$) may

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\(^9\)Note that if the bargaining power of creditors is sufficiently low, so that $\beta / \kappa > 1$, the probability of default is always positive and condition (4) is always violated.

\(^10\)Note that $\Phi(\cdot)$ is a function of $\delta$ through the effect of that variable on $y_h$, as shown in equation (1).
induce some producers to default. \( \varepsilon^* \) is the threshold value of the idiosyncratic shock to productivity associated with partial default, for a given realization of \( \delta \). The term \( \Phi(\delta) \) measures the expected repayment per producer, conditional on a given realization of the aggregate shock \( \delta \); it can be verified that \( \partial \Phi(\delta) / \partial \delta > 0 \). The second term of equation (5) captures the fact that, even for realized values of \( \delta \) satisfying the necessary condition \( \delta \leq \delta^* \), values of \( \varepsilon_h \) that are greater than \( \varepsilon^* \) ensure that the cost of default exceeds contractual repayment obligations.

We assume in what follows that each bank deals with a large number of small independent producers, such that the law of large numbers applies.\(^{11}\)

Equation (5) can be rearranged to give\(^{12}\)

\[
(1 + r_L)wn_h = (1 + \tilde{r}_L^f)wn_h + \Gamma,
\]

where

\[
\Gamma = \int_{-\delta_0}^{\delta^*} \int_{-\varepsilon_m}^{\varepsilon^*} \left[ \{ \kappa n_h^0 (\varepsilon^* - \varepsilon_h) + C \} f(\varepsilon_h) d\varepsilon_h \right] g(\delta) d\delta.
\]

Equation (8) shows that the contractual lending rate charged by the representative bank on loans to domestic producers exceeds the bank’s expected borrowing cost by a margin that compensates for the expected loss in revenue incurred in states of nature in which partial default occurs, as given by the expression

\[
\int_{-\delta_0}^{\delta^*} \int_{-\varepsilon_m}^{\varepsilon^*} \left\{ \kappa n_h^0 (\varepsilon^* - \varepsilon_h) \right\} f(\varepsilon_h) d\varepsilon_h \right\} g(\delta) d\delta,
\]

adjusted for the expected enforcement cost

\[
C \int_{-\delta_0}^{\delta^*} \left\{ f(\varepsilon_h) d\varepsilon_h \right\} g(\delta) d\delta.
\]

Figure 4 defines the region in which default will occur. The figure is cast in terms of \( \varepsilon_h \) and \( 1 + \delta_0 + \delta \) for convenience. Although \( 1 + \delta_0 + \delta \) can take realized values over the range given by the segment \( AC \) (with mean value of \( 1 + \delta_0 \)), default can occur only for values on the segment \( BC \)—with point \( B \) corresponding to the lowest possible realization of \( \varepsilon_h, -\varepsilon_m \). For any realized value of \( 1 + \delta_0 + \delta \) in that range, the threshold value of the idiosyncratic shock (as defined by (7)) is given by the negatively-sloped segment \( BD \). The dotted triangle \( V \), defined by \( BCD \), defines the set of realizations of \( 1 + \delta_0 + \delta \) and \( \varepsilon_h \) for which producers will choose to default on their repayment obligations.

\(^{11}\)That is, banks diversify away the i.i.d. risk.

\(^{12}\)In deriving equations (8), as well as (15) and (17) below, we use the general result

\[
Z \int_{x_m}^{x^*} q(x) dx = Z - Z \int_{-x_m}^{x^*} q(x) dx,
\]

where \( x = \delta, \varepsilon_h, q = f, g. \)
Figure 4
Determination of the Default Region
Equation (8) can be rewritten as
\[ r_L - \tilde{r}_b^f = \Gamma/wn_h. \]  

Let \( 1 + \tilde{r}_P \) denote the expected gross cost of funds by domestic producer \( h \). This cost is determined from the condition that
\[ (1 + \tilde{r}_P)wn_h = (1 + r_L)wn_h \int_{\delta^*}^{\delta_m} g(\delta) d\delta \]
\[ + \int_{-\delta_m}^{\delta^*} \left\{ (1 + r_L)wn_h \int_{\varepsilon_m}^{\varepsilon^*} f(\varepsilon_h) d\varepsilon_h + \int_{-\varepsilon_m}^{\varepsilon^*} \kappa_y f(\varepsilon_h) d\varepsilon_h \right\} g(\delta) d\delta, \]
which differs from the right-hand side expression in equation (5) essentially because producers, in forming their expectations, do not internalize the enforcement cost incurred by lenders in case of default.

Using (5) and (8), it follows that
\[ (1 + \tilde{r}_P)wn_h = (1 + \tilde{r}_b^f)wn_h + C \Pr(d/p), \]
where
\[ \Pr(d/p) = \int_{-\delta_m}^{\delta^*} \left\{ \int_{-\varepsilon_m}^{\varepsilon^*} f(\varepsilon_h) d\varepsilon_h \right\} g(\delta) d\delta, \]
is the probability that any given producer will default. Thus
\[ \tilde{r}_P - \tilde{r}_b^f = C \Pr(d/p)/wn_h. \]

Equation (12) shows that, in the absence of enforcement costs \( (C = 0) \), or with a zero probability of default \( \Pr(d/p) = 0 \), the producers' expected lending rate is simply equal to the representative bank's expected cost of funds.

Producer \( h \)'s decision problem is to choose employment \( n_h \) that maximizes expected profits, which are given by an expression similar to (3) with the contractual lending rate—which was shown to be equal to the expected cost of funds, \( \tilde{r}_b^f \), in the absence of default risk—replaced by the expected cost of borrowing from domestic banks, \( \tilde{r}_P \):
\[ n_h^2 (1 + \delta_0) - (1 + \tilde{r}_P) wn_h. \]
Substituting (10) in the above expression implies that expected profits can be written as
\[ n_h^d (1 + \delta_0) - (1 + r_L) \omega n_h \int_{\delta_m}^{\delta^*} g(\delta) d\delta \]
\[-\int_{-\delta_m}^{\delta^*} \left\{ (1 + r_L) \omega n_h \int_{-\epsilon_m}^{\epsilon_m} f(\epsilon_h) d\epsilon_h + \int_{-\epsilon_m}^{\epsilon^*} \kappa n_h \frac{f(\epsilon_h) d\epsilon_h}{\kappa n_h} \right\} g(\delta) d\delta,\]
or more compactly, using (11):
\[ n_h^d (1 + \delta_0) - (1 + \bar{r}_b^f) \omega n_h - C \Pr(d/p). \]

Deriving the above expression with respect to \( n_h \) and setting the result to zero gives
\[ \beta n_h^{\beta - 1} (1 + \delta_0) - \left\{ w(1 + \bar{r}_b^f) + C(1 - \beta) \frac{w(1 + r_L)}{\kappa n_h^{\beta}} \int_{-\delta_m}^{\delta^*} f(\epsilon^*) g(\delta) d\delta \right\} = 0, \]
which determines the optimal demand for labor.\(^{13}\)

We will assume that the shocks \( \epsilon_h \) and \( \delta \) follow a uniform distribution, so that
\[ f(\epsilon_h) = 1/2\epsilon_m, \quad g(\delta) = 1/2\delta_m, \quad \text{and} \quad \Pr(z > x) = (z_m - x)/2z_m, \quad \text{with} \quad z = \epsilon_h, \delta. \]
From (8) and (13), we can thus establish the following proposition.\(^{14}\)

**Proposition 1** Higher volatility and lower expected productivity reduce employment and increase the bank's contractual lending rate:
\[ \frac{\partial n_h}{\partial \delta_m} < 0, \quad \frac{\partial n_h}{\partial \delta_m} < 0, \quad \frac{\partial n_h}{\partial \delta_0} > 0, \quad \frac{\partial r_L}{\partial \delta_m} > 0, \quad \frac{\partial r_L}{\partial \epsilon_m} > 0, \quad \frac{\partial r_L}{\partial \delta_0} < 0. \]

Figure 5 illustrates graphically these results. The downward-sloping curve NN represents the combinations of employment and the contractual lending rate implied by the first-order condition (13), whereas the convex curve BB represents the combinations of \( n_h \) and \( r_L \) associated with zero expected profits by the representative bank, as implied by (5). The intersection of these two curves gives the pair \((r_L, n_h)\)

\(^{13}\)Note that \( r_L \) is taken as given by each producer in determining optimal employment. The reason is that we assume the existence of a large group of ex ante homogeneous producers, all of them are charged the same interest rate by lenders. As shown earlier, \( r_L \) is determined in equilibrium by a break-even condition that internalizes all the information about the distribution of shocks—including idiosyncratic shocks.

\(^{14}\)An Appendix providing formal derivations of these results is available upon request.
Figure 5
Optimal employment and domestic borrowers interest rate.
Simulated for $C=0.15$, $\beta=0.5$, $w=0.5$, $\bar{r}_b^f = 0$, $k=0.6$

\[ \delta_m = \delta_0 = 0.1 \]

\[ \delta_m = \epsilon_m = 0.2, \quad \delta_0 = 0.1 \]
\[ \delta_m = 0.2, \quad \epsilon_m = 0.3, \quad \delta_0 = 0.1 \]
\[ \delta_m = 0.3, \quad \epsilon_m = 0.2, \quad \delta_0 = 0.1 \]

\[ \delta_m = \epsilon_m = 0.2, \delta_0 = 0 \]
\[ \delta_m = \epsilon_m = 0.3, \delta_0 = 0 \]
\[ \delta_m = \epsilon_m = 0.2, \delta_0 = 0.1, \bar{r}_b^f = 0.1 \]
consistent with (expected) profit maximization by producer $h$ and zero expected profits by the representative bank on its loan to producer $h$. Simulation 1 is the benchmark case. Simulation 2 (3) correspond to an increase in the standard deviation of the micro (macro) shock by 50 percent relative to the benchmark case. Simulation 4 corresponds to a drop in expected productivity by 10 percent relative to the benchmark case. Note that a similar adjustment occurs in all these cases—a significant increase in the financial intermediation spread (an increase in $r_L$ relative to $\bar{r}_f$), and a drop in employment. Simulation 5 traces the adjustment to a combination of the above 3 shocks, showing a profound drop in employment and a sharp increase in the financial intermediation spread. Finally, simulation 6 shows the adjustment to a rise in the bank’s expected real cost of funds, from 0 to 10 percent. Again, the results are an increase in $r_L$ and a drop in employment.

III. THE WORLD CAPITAL MARKET

Domestic banks have access to world capital markets and borrow, at the contractual interest rate $r^*_i$, $wn_h$ per domestic producer. If domestic banks default partially, repaying less than $wn_h(1 + r^*_i)$, foreign banks have the ability to force domestic banks to pay a fraction $\kappa_b$ of the realized revenue $\Phi(\delta)$. Enforcing that repayment involves real costs $C_b$ to the foreign bank.

In this setting, a domestic bank lending to producer $h$ will default if and only if

$$\kappa_b \Phi(\delta) < wn_h(1 + r^*_i),$$

where $\Phi(\delta)$ is as given in equation (5).

Let $\delta^*$ denote the threshold value of the aggregate shock below which domestic banks will choose to default. It is defined implicitly by the condition $\kappa_b \Phi(\delta^*) = wn_h(1 + r^*_i)$.

Let $r^*_0$ be the foreign lender's cost of funds, which is equal to the risk-free interest rate in the absence of transactions costs. Under risk neutrality, the interest rate $r^*_0$ charged by foreign lenders to domestic banks is again determined by the condition that expected gross repayment be equal to the gross value of the loan, measured at the risk-free rate. Expected repayment accounts for the possibility that, in adverse states of nature, the lender may be able to appropriate only a fraction of realized output, subject to enforcement costs:

$$(1 + r^*_0)wn_h = (1 + r^*_i)wn_h \int_{\delta^*}^{\delta_m} g(\delta)d\delta + \int_{-\delta_m}^{\delta^*} [\kappa_b \Phi(\delta) - C_b]g(\delta)d\delta,$$  \hspace{1cm} (14)
which can be rewritten as

\[(1 + r_0^f)w_nh = (1 + r_b^*)w_nh - \Gamma_b, \tag{15}\]

where

\[\Gamma_b = \int_{-\delta_m}^{\delta_m} \{\kappa_b[\Phi(\delta) - \Phi(\delta)] + C_b\} g(\delta)d\delta.\]

From the point of view of the domestic bank, the expected cost of funds \(\bar{r}_b^f\) is determined by the condition

\[(1 + \bar{r}_b^f)w_nh = (1 + r_b^*)w_nh \int_{-\delta_m}^{\delta_m} g(\delta)d\delta + \int_{-\delta_m}^{\delta_m} \kappa_b\Phi(\delta)g(\delta)d\delta, \tag{16}\]

which shows that the expected cost of capital is the sum of the expected interest repayment in relatively good states of nature (the first term on the right-hand side of the equation) plus the expected repayment in adverse states of nature, when partial default occurs (the second term on the right-hand side). Again, the expression on the right-hand side of (16) differs from the expression on the right-hand side of (14) because domestic banks do not internalize the enforcement costs that foreign lenders must incur in case of partial default.

Combining equations (14) and (16) yields

\[(1 + \bar{r}_b^f)w_nh = (1 + r_b^*)w_nh + C_b \Pr(d/b),\]

where \(\Pr(d/b) = \int_{-\delta_m}^{\delta_m} g(\delta)d\delta\) is the probability that the representative domestic bank will default. Alternatively, using (16):

\[(1 + \bar{r}_b^f)w_nh = (1 + r_b^*)w_nh - \int_{-\delta_m}^{\delta_m} \kappa_b[\Phi(\delta) - \Phi(\delta)]g(\delta)d\delta. \tag{17}\]

From the above results, we have

\[r_b^* - \bar{r}_b^f = \frac{\int_{-\delta_m}^{\delta_m} \kappa_b[\Phi(\delta) - \Phi(\delta)]g(\delta)d\delta}{w_nh}, \quad \bar{r}_b^f - r_0^f = \frac{C_b \Pr(d/b)}{w_nh}, \quad r_b^* - r_0^f = \frac{\Gamma_b}{w_nh},\]

which, together with (12), can be summarized in the following propositions:

**Proposition 2** The expected cost of funds for domestic banks on world capital markets, and for producers on the domestic capital market, can be written as a markup over the world safe interest rate:

\[\bar{r}_b^f = r_0^f + \frac{C_b}{w_nh} \Pr(d/b), \quad \bar{r}_p = r_0^f + \frac{C}{w_nh} \Pr(d/p) + \frac{C_b}{w_nh} \Pr(d/b).\]
The markup adjusts the lender's cost of capital by the expected cost of contract enforcement and state verification.

**Proposition 3** The domestic and foreign financial intermediation spreads, defined as the difference between the relevant contractual interest rate and the relevant expected cost of funds, are equal to the sum of the expected contract enforcement costs plus the expected revenue lost in adverse states of nature, when partial default takes place:

\[ r_b^* - r_0^f = \frac{\Gamma_b}{wn_h}, \quad r_L - \hat{r}_b^f = \frac{\Gamma}{wn_h}, \]

where \( \Gamma \) and \( \Gamma_b \) are defined in equations (8) and (15).

**IV. VOLATILITY AND CONTAGION**

Having established the mechanism through which financial spreads, employment and output are determined, we are now in a position to study the adjustment process to a rise in the volatility of macroeconomic shocks. A key feature of a debt contract in our framework is the nonlinear dependency of repayment capacity on the aggregate shock \( \delta \). To illustrate this point, recall that domestic banks will default on their foreign debt if \( \kappa_b \Phi(\delta) < (1 + r_b^*)wn_h \). This condition is plotted in Figure 6, where curve \( RR \) draws the debt repayment schedule if banks default, \( \kappa_b \Phi(\delta) \), whereas curve \( DD \) plots the repayment due, \( (1 + r_b^*)wn_h \). The bold kinked curve depicts actual repayment. *Ceteris paribus*, higher volatility does not affect repayment in good states of nature, but increases the incidence and severity of partial default in adverse states of nature. In terms of Figure 6, higher volatility adds segment \( \Delta \) to the range of default. The partial equilibrium effect is to increase the probability of default by (approximately) \( \Delta/2\delta_m \), thereby reducing expected repayment and raising the expected cost of funds. There is also a general equilibrium effect, which results from the fact that foreign lenders increase the interest rate charged to domestic banks to compensate for lower expected repayment and for the higher expected cost of contract enforcement. This adjustment leads to an upward shift in curve \( DD \) (as shown in the figure), further increasing the incidence of default. The general equilibrium effect therefore magnifies the increase in the probability of default, because it leads also to a rise in the expected cost of funds and a rise in financial intermediation spreads.

A similar analysis applies for the impact of higher volatility on domestic financial intermediation. Thus, by showing how interest rate spreads are related to default probabilities and changes in volatility of underlying shocks, our analysis is capable of explaining not only the increase in "foreign" financial intermediation spreads recorded
Figure 6
Effect of an Increase in Volatility
by Argentina in the immediate aftermath of the peso crisis, but also the sharp increase in the spread between the country's bank lending and deposit rates (see Figure 3).

A key empirical prediction of our analysis is an inverse relationship between volatility and economic activity. Figure 7 illustrates a simple way of testing this proposition, using data for a group of Asian and Latin American countries for which monthly data on output (measured by either the industrial output index or the manufacturing production index) were readily available. The variable measured on the horizontal axis is the ratio of the standard deviations of post- and pre-Mexican peso crisis of the cyclical component of output, calculated in each case by taking the difference between actual output and its trend level, computed with the Hodrick-Prescott filter. The variable measured on the vertical axis is the ratio of post- and pre-Mexican peso crisis of the average rate of growth of the trend component of output. The figure shows indeed that higher volatility tends to be associated with a slowdown in economic activity. Of course, we do not view the evidence presented in Figure 7 as compelling or definitive; more sophisticated econometric tests would be needed to assert in a rigorous manner the links between volatility and economic activity. Nevertheless, it is encouraging to find that simple calculations are not at variance with one of the main predictions of our analytical framework.

V. WELFARE EFFECTS

We turn now to an evaluation of the impact of a contagious shock (or, more precisely here, an increase in volatility) on welfare, as approximated by the expected producers’ surplus, $S^p_e$. Suppose that there are $N$ identical producers in the economy. Based on our discussion in the previous section, we can infer that the expected producer’s surplus (at the optimal level of employment) is, taking into account the expected cost of borrowing by producers, as given in equation (12):

$$S^p_e = N \left[ n_0^0 (1 + \delta_0) - (1 + \tilde{r}_b) w n_0 - CPr(d/p) \right]. \quad (18)$$

Consequently, the effect of higher volatility on the expected producers' surplus is given by

$$\frac{dS^p_e}{d\delta_m} = \frac{\partial S^p_e}{\partial n_0} \cdot \frac{\partial n_0}{\partial \delta_m} + \frac{\partial S^p_e}{\partial \tilde{r}_b} \cdot \frac{\partial \tilde{r}_b}{\partial \delta_m} + \frac{\partial S^p_e}{\partial Pr(d/p)} \cdot \frac{\partial Pr(d/p)}{\partial \delta_m}. \quad (19)$$

By virtue of the envelope theorem, the first term on the right-hand side of the above expression is zero (recall that each producer $k$ sets employment so as to

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15 A similar analysis would apply to labor, where the ultimate welfare effect of the drop in employment is the drop in employment times the difference between the producers' real wage and the supply price of labor.
Figure 7
Changes in Output Volatility and Trend Output Growth
Before and after the Mexican Peso Crisis

Notes: Output is measured using the industrial production index for Argentina, Brazil, Mexico, India, and Malaysia; and the manufacturing production index for Chile, Peru, the Philippines, and Venezuela. Trend and cyclical components of output are obtained by applying the HP filter to the deseasonalized output series, themselves obtained by applying the X-11 procedure with multiplicative seasonal factors. The rate of growth of trend output is calculated on a monthly basis. The ratio of standard errors of the cyclical component of output is calculated by dividing the standard error of the series estimated for the period January 1995-June 1995 by the standard error estimated for the period March 1991-December 1994. A similar method is used to calculate the ratio of the rates of growth of trend output.

Source: Authors' calculations, based on data from International Financial Statistics.
maximize $S^e_P$). Applying Proposition 2 to substitute for $\partial \tilde{n}_b^f / \partial \delta_m$, it follows that
\[
\frac{dS^e_P}{d\delta_m} = -N \left[ C \frac{\partial \text{Pr}(d/p)}{\partial \delta_m} + C_b \frac{\partial \text{Pr}(d/b)}{\partial \delta_m} \right].
\] (20)

The above equation can be reduced to a simple form for the case where the repayment associated with partial default is approximated by a linear function (see the Appendix for details). In the range of partial default, in which $\text{Pr}(d/p)$ and $\text{Pr}(d/b)$ are both positive, it can be shown that
\[
\frac{dS^e_P}{d\delta_m} = -N \left\{ C \frac{1 - \text{Pr}(d/p)}{2 \delta_m - \frac{1}{\kappa \text{Pr}(d/p)}} + \Omega C_b \frac{1 - \text{Pr}(d/b)}{2 \delta_m - \frac{1}{\kappa_b \text{Pr}(d/b)}} \right\},
\] (21)
where $B$ is a constant term measuring the partial effect of $\delta$ in the linear approximation to $\Phi(\cdot)$, and $\Omega$ is defined by
\[
\Omega = \frac{2 \delta_m [1 - \text{Pr}(d/p)]}{2 \delta_m [1 - \text{Pr}(d/p)] - \frac{C}{\kappa B}}.
\]

Let $\text{Pr}$ refer to the probability of default of either domestic producers or domestic banks and suppose that (everything else equal) a highly integrated capital market is characterized by a low incidence of default. The term $1 - \text{Pr}$, the probability of repayment, may thus be viewed as a measure of the country’s integration with the global financial market. It can be verified that $1 - \text{Pr}$ depends positively on creditor’s bargaining power (coefficients $\kappa$ and $\kappa_b$), and negatively on the cost of state verification and contract enforcement, $C$ and $C_b$ (see the Appendix for further discussion).
Applying (21), it follows that the impact of volatility is large for countries that are on the verge of full integration with global financial markets, because for these countries the expression $1 - \text{Pr}$ is maximized. These countries are in a precarious state—for low volatility the marginal effect of more turbulent markets is zero, but for volatility above a threshold, this effect can be profound. This may explain why countries like Argentina in the early 1990s are the most exposed to volatility. The above equation also implies that higher volatility matters very little for highly risky countries where the probability of full repayment is low—that is, where $1 - \text{Pr}$ is close to zero. Such countries operate, to begin with, on the relatively inelastic portion of the supply of funds, hence higher volatility has little effect at the margin.

This, in turn, implies a nonlinear association between volatility and the expected producers’ surplus, as illustrated in Figure 8 (based on (18)). For small enough volatility (assuming a high enough expected productivity), the probability of default is zero. In these circumstances higher volatility does not have any impact on welfare.
Figure 8
Volatility and the Expected Producers' Surplus
Once a threshold is reached (point $A$), higher volatility increases the probability of default, leading to a welfare loss proportional to the cost of intermediation times $\Delta Pr$. This nonlinearity may explain why contagious shocks may have highly heterogeneous effects across countries. Suppose that a crisis like the Mexican peso collapse increases financial markets’ perception of volatility in developing countries (or emerging markets) in general. The adverse, domestic effects of this perception will differ across countries, even if the perceived increase in volatility is identical across countries. For countries that are viewed as relatively safe, $Pr = 0$, and the effect is nil. For countries that were viewed to begin with as mildly risky ventures ($Pr > 0$ but close to point $A$), the effect will be large. By contrast, this adverse effect tends to be smaller for countries whose degree of financial openness is relatively small, because for these countries the probability of default is large.

For a given probability of default, the adverse effect of higher volatility tends to be magnified for countries where the cost of contract enforcement is large. In terms of Figure 8, a larger cost of financial intermediation ($C$ or $C_h$) is associated with an inward shift of the downward-sloping portion of the curve from the solid portion that starts at point $A$, to the broken portion that starts at point $A'$. If the cost of financial intermediation is large enough, the welfare effect of higher volatility would be traced by the dotted curve that starts at point $A'$. In these circumstances, volatility may lead to a situation akin to credit rationing, where producers are not able to obtain bank financing for their working capital.

VI. SUMMARY AND CONCLUSIONS

The purpose of this paper has been to analyze the aggregate effects and transmission process of contagious shocks. In contrast to the existing literature, our model does so by capturing imperfections on both world capital markets and domestic credit markets. Specifically, we assume a two-level financial intermediation process with risk-neutral lenders: domestic banks borrow at a premium on world capital markets, and domestic producers borrow also at a premium from domestic banks. In addition, we offer a different interpretation of contagion effects: in our analysis contagion takes the form of a rise in volatility of aggregate shocks impinging on the domestic economy—more specifically, a mean-preserving increase in the range of values that such shocks may take.

Our analysis shows that both foreign and domestic interest rate spreads are determined by a markup that compensates for the expected cost of contract.

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16 As discussed by Catão (1997) and Powell et al. (1997), severe limitations to the seizure of collateral property still prevails in Argentina. Judicial actions take time, have uncertain outcomes, and are relatively costly—thereby affecting lending rates by raising the potential cost of default.
enforcement and state verification, and for the expected revenue lost in adverse states of nature. Higher volatility raises financial intermediation spreads as well as the producers’ cost of funds, resulting in lower employment and higher incidence of default. In addition, our analysis shows that the welfare effects of an increase in volatility are highly nonlinear. Higher volatility does not impose any welfare cost on countries characterized by relatively low volatility and efficient financial intermediation. The adverse welfare effects are large (small) for countries that are at the threshold of full integration with international capital markets (close to financial autarky), that is, countries characterized by relatively low (high) probability of default. A general implication of our analysis is thus that increased integration with world financial markets may be accompanied with a potential cost resulting from greater exposure to volatility—an implication also emphasized by Calvo (1997), in a different setting. Simple empirical calculations, based on data relative to the pre- and post-Mexican peso crisis periods for a small group of Asian and Latin American countries, suggest that (as predicted by our analysis), increased volatility tends to be associated with reductions in economic activity.

Our model can be readily extended to account for other relevant factors. For instance, if lenders on both domestic and world financial markets are risk averse, the greater perceived volatility will induce a further increase in interest rate spreads to account for a higher risk premium, magnifying therefore the effect of an increase in the probability of default and the welfare cost of volatility. The pricing of risk by foreign could be formulated so as to capture the existence of implicit insurance from the country’s central bank. The existence of bail-out options would naturally affect the foreign intermediation spread, particularly in the presence of uncertainty regarding the extent and timing of domestic public assistance.

We also refrain from modeling the precise mechanism leading to an increase in the volatility of shocks. The reason for this is our contention that any external event that leads to an increase in the volatility of shocks will trigger the type of adjustment process modeled in our paper. This does not preclude, of course, the existence of alternative channels that may intensify the transmission mechanism. For instance, it is sensible to assume that information asymmetries may lead foreign lenders to increase the real interest rate—or the risk premium, as discussed by Agénor (1997)—that they would demand from a country like Argentina following the Mexican peso crisis. This, in turn, will lead to higher incidence of default in Argentina. Our model can be used to account for this sequence. Specifically, it can be shown that the ultimate welfare cost of asymmetric information will depend positively on the cost of financial intermediation.

An extended version of our model would show that even if the “trigger mechanism” is the existence of asymmetric information, as discussed above, it would lead to higher variances of domestic shocks, inducing the adjustment mechanism
described in the present paper. This will be the case if there are complementarities among producers, either on the supply of the demand side.\textsuperscript{17} While we refrain from modeling these interesting channels of transmission, one may view our model as a reduced form of a more complex economy, characterized by the above complementarities. In such an environment, the higher cost of credit would lead both to a drop in the expected productivity as well as to a higher volatility of the productivity shocks, magnifying the ultimate adverse output effects.

Our model can be extended to account for several mitigating factors that reduce the adverse effects of volatility and interest rate shocks, like the use of collateral, and the “rolling over” and the partial refinancing of debt obligations. It is worth noting, however, that efficacy of these mitigating effects would be ultimately determined by the efficiency of the financial intermediation. For instance, if the legal system is associated with time consuming and costly contract enforcement, liquidating a collateral would be more costly, reducing its potential benefits. Furthermore, if the market value of the collateral drops in “bad” times (or if the collateral is more valuable to the producer compared to the lender), it would help little to alleviate the costs of volatility.

Notwithstanding these extensions, the present framework (despite being static and partial equilibrium in nature) offers a particularly useful setting for interpreting some of the events that occurred in Argentina and Mexico in early 1995. In particular, it helps to understand how changes in the volatility of aggregate shocks may have played a role in the transmission and magnification of an initial adverse shock on world capital markets.\textsuperscript{18} It also highlights the role of domestic factors (such as the cost of contract enforcement) in this process. This prediction is consistent with the econometric results of Powell et al. (1997), which emphasize the role of default risk and the lack of legal security for debt contracts in the determination of bank lending rates in Argentina. It is also broadly consistent with the analysis of contagion effects by Sachs et al. (1996), whose study focuses on the evolution of 20 emerging market economies.

\textsuperscript{17}An example of supply complementarity is the case where producers use nontraded inputs provided by other producers. Default by some producers due to higher cost of credit will trigger default by other producers due to the increase in the price of needed inputs, implying that the increase in the cost of credit may trigger higher volatility of domestic productivity shocks of domestic producers. Similar examples of demand complementarities will arise if producers have some market power—as is the case in a world of monopolistic competition.

\textsuperscript{18}In Argentina, various other factors have also played a role in this process. An increase in the perceived risk of confiscation of bank deposits—as occurred in December 1989, when the government, in an effort to reduce inflation, forced the conversion of time deposits and public sector debt into U.S. dollar-denominated government (BÖNEX) securities—and the fact that bank deposits were not insured certainly played a role of in the bank run and the credit crunch that took place in early 1995 (see Catão, 1997). There may also have been increased doubts about the sustainability of full convertibility of current and capital account transactions, as well as perceived constraints on the lender-of-last-resort function of the central bank, under the quasi-currency board in place since the Convertibility Plan was introduced in early 1991.
economies in the aftermath of the peso crisis. They emphasized the role of domestic imbalances in countries that suffered the most from speculative attacks, and identified as important factors not only overvalued exchange rates, low foreign exchange reserves, but also banking system fragility.
Appendix

This Appendix derives the probability of default for the case where the partial repayment function can be approximated by a linear curve. For simplicity of exposition we do it for the case where employment $n_h$ is constant, and focuses on intermediation between foreign lenders and domestic banks.\textsuperscript{19}

Suppose that in the relevant region domestic banks’ revenue is given by the linear approximation

$$\Phi(\delta) = A + B\delta,$$

with $A, B > 0$. The threshold value $\hat{\delta}$ of the aggregate shock $\delta$ that makes banks indifferent between partial default and repayment is thus

$$\kappa_b(A + B\hat{\delta}) = (1 + r_b^*)wn_h, \quad \text{(A1)}$$

or equivalently

$$\hat{\delta} = B^{-1} \left\{ \frac{(1 + r_b^*)wn_h}{\kappa_b} - A \right\},$$

and the probability of bank default is

$$\Pr(d/b) = \frac{\hat{\delta} + \delta_m}{2\delta_m}. \quad \text{(A2)}$$

Using (14), we infer that, with $g(\delta) = 1/2\delta_m$:

$$(1 + r_0^f)wn_h = (1 + r_b^*)wn_h - \int_{-\delta_m}^{\hat{\delta}} \frac{\kappa_b(\Phi(\hat{\delta}) - \Phi(\delta) + C_b)}{2\delta_m} d\delta,$$

that is

$$(1 + r_0^f)wn_h = (1 + r_b^*)wn_h - \int_{-\delta_m}^{\hat{\delta}} \frac{\kappa_b B(\hat{\delta} - \delta) + C_b}{2\delta_m} d\delta. \quad \text{(A3)}$$

Solving (A3) yields

$$(1 + r_0^f)wn_h = (1 + r_b^*)wn_h - C_b \frac{\hat{\delta} + \delta_m}{2\delta_m} + \frac{\kappa_b B (\hat{\delta} - \delta)^2}{2} \bigg|_{-\delta_m}^{\hat{\delta}},$$

\textsuperscript{19}Recall that in the expression determining the welfare effects of higher volatility (equation (19)), changes in employment are of secondary importance by virtue of the envelope theorem.
that is, using (A2):

\[(1 + \tau_0^*)\omega_n = (1 + r_b^*)\omega_n - Pr(d/b)C_b - \kappa_B\delta_m Pr(d/b)^2. \quad (A4)\]

Applying (A1) and (A2), we can solve for the contractual interest rate facing domestic banks, \(r_b^*\), in terms of \(Pr(d/b)\). Substituting the result in (A4) yields a quadratic equation for the probability of default:

\[\kappa_b B\delta_m Pr(d/b)^2 + (C_b - 2\kappa_B\delta_m) Pr(d/b) + \omega_n (1 + \tau_0^*) - \kappa_B A + \kappa_B\delta_m = 0\]

Applying the implicit function theorem to this equation yields

\[
\frac{\partial Pr(d/b)}{\partial \delta_m} = \frac{1 - Pr(d/b)}{2\delta_m - \frac{C_b}{\kappa_b B[1 - Pr(d/b)]}}, \quad (A5)
\]

which can be combined with (20) to give (21).

Note also that, for \(\delta_m\) given:

\[
\frac{\partial Pr(d/b)}{\partial C_b} = -\frac{Pr(d/b)}{2B\kappa_B\delta_m [1 - Pr(d/b)] - C_b}.
\]

If creditors' capacity to enforce partial repayment is small (that is, if \(\kappa\) or \(\kappa_b\) is small), or if the cost of financial intermediation is large enough, there is no internal solution—that is, the value of \(Pr\) that solves the quadratic equation given above is outside the [0,1] interval. Furthermore, for certain parameter values we may observe multiple equilibria, as is the case where there are two values of \(Pr\), satisfying \(0 < Pr < 1\), corresponding to low or high interest rate rates. Henceforth we assume that the model's parameters are such that an internal equilibrium exits. Specifically, we assume that creditors' bargaining power is large enough, and that the cost of financial intermediation is small enough to ensure that the probability of default is zero in the absence of aggregate volatility \((\Delta\delta = 0)\), and the probability of default is positive for large enough volatility. We also assume that, in the presence of multiple equilibria, the market chooses the equilibrium associated with the lower interest rate. This is also the equilibrium associated with the lower probability of default and the higher welfare level. It can be shown that these assumptions imply that, in an internal equilibrium satisfying \(0 < Pr < 1\), (A5) is positive.

A similar analysis applies for the impact of higher volatility on the producer's probability of default. The main difference between analyzing the partial effects
\( \partial \operatorname{Pr}(d/b)/\partial \delta_m \) and \( \partial \operatorname{Pr}(d/p)/\partial \delta_m \) is that, as can be inferred from Proposition 2, higher volatility increases the cost of funds for domestic banks by

\[
\frac{\partial r^f_b}{\partial \delta_m} = \left( \frac{C_b}{w_{n_h}} \right) \frac{\partial \operatorname{Pr}(d/b)}{\partial \delta_m},
\]

whereas higher volatility does not affect the domestic banks' expected cost of funds on world capital markets (which is equal to the safe interest rate \( r^f_0 \)). Adjusting for this effect, and assuming that the default repayment \( \Phi(\cdot) \) is linear, it follows that

\[
\frac{\partial \operatorname{Pr}(d/p)}{\partial \delta_m} = \frac{\left[ 1 - \operatorname{Pr}(d/p) \right]^2 + \left( \frac{C_b}{\kappa_B} \right) \frac{\partial \operatorname{Pr}(d/b)}{\partial \delta_m}}{2\delta_m[1 - \operatorname{Pr}(d/p)] - \frac{C}{\kappa_B}},
\]

implying that

\[
C_b \frac{\partial \operatorname{Pr}(d/p)}{\partial \delta_m} + C_b \frac{\partial \operatorname{Pr}(d/b)}{\partial \delta_m} = C \frac{\left[ 1 - \operatorname{Pr}(d/p) \right]^2 + \left( \frac{C_b}{\kappa_B} \right) \frac{\partial \operatorname{Pr}(d/b)}{\partial \delta_m}}{2\delta_m[1 - \operatorname{Pr}(d/p)] - \frac{C}{\kappa_B}} + C_b \frac{\partial \operatorname{Pr}(d/b)}{\partial \delta_m},
\]

which can be rearranged to give

\[
C \frac{\left[ 1 - \operatorname{Pr}(d/p) \right]^2}{2\delta_m[1 - \operatorname{Pr}(d/p)] - \frac{C}{\kappa_B}} + C_b \Omega \frac{\partial \operatorname{Pr}(d/b)}{\partial \delta_m},
\]

where \( \Omega \) is defined by

\[
\Omega = \frac{2\delta_m[1 - \operatorname{Pr}(d/p)]}{2\delta_m[1 - \operatorname{Pr}(d/p)] - \frac{C}{\kappa_B}} > 1.
\]

Using (A5) this expression becomes

\[
C \frac{1 - \operatorname{Pr}(d/p)}{\kappa_B[1 - \operatorname{Pr}(d/p)]} + C_b \Omega \frac{1 - \operatorname{Pr}(d/b)}{\kappa_B[1 - \operatorname{Pr}(d/b)]},
\]

which can be substituted in (20) to give (21).
References


