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Debt Reduction and New Loans: A Contracting Perspective

Prepared by John A. Carlson, Aasim M. Husain, and Jeffrey A. Zimmerman*

Authorized for distribution by Charles Adams

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Abstract

International debt contracts can incorporate—at least implicitly—contingencies governing debt reduction. This paper examines a series of debt contracts that allow for the possibility of rescheduling, forgiveness, and rescheduling with forgiveness. The contract with both rescheduling and forgiveness permits a higher credit ceiling than other types of debt contracts, and contains features found in the HIPC and other recent debt reduction initiatives. If an adverse state of nature occurs, some of the debt is forgiven, a portion is rescheduled, and the remainder is repaid. At the same time, the debtor country is a net recipient of new loans.

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Authors' E-Mail Addresses:carlson@mgmt.purdue.edu; ahusain@imf.org; jazimm@aol.com

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*Carlson: Department of Economics, Purdue University; Husain: Asia and Pacific Department, IMF; Zimmerman: Department of Economics, Methodist College. The authors are grateful to Harold Cole and Charles Adams for helpful comments.

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SUMMARY

Economic arguments in favor of debt reduction have generally been related to the "debt overhang" hypothesis, which holds that as the external obligations of a sovereign debtor become large, the debtor country's incentive to invest weakens since a significant share of future investment returns accrues to creditors in the form of debt repayments. In such a situation, debt reduction could potentially increase future repayments by improving the debtor's incentive to invest.

However, empirical investigations have, for the most part, failed to establish the presence of large investment disincentive effects, even for the most heavily indebted countries. Moreover, the debt overhang hypothesis implies that the appropriate debt relief strategy should involve either debt reduction or new (net) lending, but generally not both. In practice, though, actual debt relief initiatives—including the recent heavily indebted poor countries (HIPC) initiative—have called for both debt reduction and new net loans.

In view of these difficulties with the debt overhang hypothesis, this paper draws on an argument in favor of debt reduction that has received only modest attention—that debt reduction may have been part, at least implicitly, of the original loan contracts and should therefore be implemented. The paper analyzes a series of debt contracts in which contingencies for debt rescheduling and forgiveness are part of the contractual agreement.

The analysis indicates that the contract that allows the highest debt ceiling—and hence the highest welfare—is the one with contingencies for both rescheduling and forgiveness. Under this contract, if the debtor country experiences an adverse output shock, part of the debt coming due is repaid, a portion is forgiven, and the remainder is rescheduled. At the same time, there is net new lending to the debtor country.

I. INTRODUCTION

The debt crisis of the 1980s has, by now, largely been resolved for the middle income debtor countries. Many of these heavily indebted countries (HICs)—mainly in Latin America—have undergone debt reduction programs and have subsequently been able to obtain new funds from international capital markets. However, a large number of heavily indebted poor countries (HIPCs)—many of which are in subsaharan Africa—remain saddled with high levels of external debt and continue to face difficulties in borrowing from abroad.

The recent HIPC initiative, organized principally by the international financial institutions but seeking the participation of both official and commercial creditors, aims to reduce the external debt burden of the HIPCs so that new lending can become viable.[1] Under the initiative, external debt obligations of eligible HIPCs would be reduced to sustainable levels, in exchange for the adoption of programs of economic adjustment and reform.

While economic arguments for debt reduction—usually based on the debt overhang hypothesis which suggests that debt reduction could improve the debtor country's incentive to invest—have been proposed in the literature,[2] they have generally found limited support, either on theoretical grounds or empirically.[3] Moreover, even if investment disincentives arising from the large stock of debt are sufficiently strong to warrant debt reduction on efficiency grounds, models of debt overhang generally imply that both debt reduction and new (net) lending should not take place simultaneously.[4] In such a case, new loans would only raise

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[1]A detailed review of the HIPC initiative is contained in Boote and Thugge (1997). In some cases, debt could be reduced by as much as 80 percent.

[2]Sachs (1988), Krugman (1989), and Froot (1989), for example, present models capturing the investment disincentive effects of a high level of external debt.

[3]Empirical investigations such as Claessens (1990), Cohen (1993), and Warner (1994) have uncovered little, if any, evidence to suggest that such a disincentive effect was sizable in the HICs during the 1980s. On a theoretical level, Husain (1997) shows that the conditions required for welfare-enhancing debt forgiveness are unlikely to be fulfilled.

[4]Exceptions are Diwan and Kletzer (1992) and Diwan and Spiegel (1994), who present models with multiple creditors facing a menu of debt relief options in which some creditors choose debt reduction while others choose new lending in equilibrium. However, these models rely on heterogeneity among creditors that cannot be arbitraged away on secondary markets for sovereign debt claims.

external indebtedness and add to the disincentive effect, thereby offsetting the benefits of debt reduction.[5]

Given the success of debt relief programs for middle income countries and the importance of new lending alongside debt reduction in these programs, observed practice would appear to be out of line with theory. The theory suggests that since the necessary conditions for efficiency-enhancing debt reduction are rarely met, debt-reduction programs should generally not take place, at least on economic grounds. The theory also indicates that in the rare cases that the conditions for welfare-improving debt reduction are met, debt relief and new (net) lending should not be offered at the same time. In practice, however, debt and liquidity relief have gone hand-in-hand.

This paper attempts to bridge the gap between key features of actual debt relief initiatives and theories that seek to explain them by drawing on an argument in favor of debt reduction that has so far received considerably less attention. This argument, advanced by Grossman and van Huyck (1988) and Calvo (1989), suggests that debt relief may have been part of the original (implicit) debt contract, and therefore should be implemented.[6]

The analysis below begins with a very simple debt contract structure, and solves for the interest rate and credit ceiling that would prevail. Successively more complex contracts, which allow for different types of contract enforcement mechanisms, are then analyzed. These include contracts with partial debt forgiveness in bad states of nature and contracts under which unpaid balances are rescheduled. In addition, contracts with both forgiveness and rescheduling are examined.

The principal results of the model are that the inclusion of the possibility of either rescheduling or (partial) forgiveness in the (implicit) loan contract, by raising the borrower's debt ceiling, improves the welfare of the debtor without hurting the creditor. Whether the contract with rescheduling or the one with forgiveness is preferred depends on the severity of shocks to the borrower's output, with rescheduling contracts favored when shocks are of higher intensity. Contracts which allow for *both* rescheduling and forgiveness, however, are preferable—from a welfare standpoint—to contracts with only one of these features, regardless of the severity of output shocks. Finally, in two cases—that is, forgiveness and rescheduling with forgiveness—the equilibrium involves both new lending and debt reduction, at least in some states of nature. Even in the pure rescheduling (without forgiveness) case,

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[5]Husain (1993) shows that, depending on the conditions prevailing in the debtor country, either debt reduction or new (net) lending—but generally not both—is welfare improving.

[6]In a related paper, Detragiache (1992) asks why sovereign loans are often not repaid in full. The assumption that contracts are perfectly enforceable is dropped and the relative advantages of floating- versus fixed-interest rate debt are examined.

new lending takes place along with debt roll-over in some states. These results, then, provide theoretical support for what has been observed in the HIPC, and earlier, debt relief initiatives.

The remainder of this paper is organized as follows: Section II describes the framework and outlines a very simple two-period debt contract. A contract with partial debt forgiveness in some states is contained in Section III. Section IV extends the basic model to a multi-period setting, and analyzes a contract that allows the possibility of debt rescheduling. Contracts with both rescheduling and forgiveness—also in a multiperiod setting—are explored in Section V. Section VI concludes.

II. Framework

Consider a simple two-period framework with two agents—a debtor and a creditor.[7] In the first period, the creditor lends an amount L, which the debtor consumes. At the start of the second period, the debtor country's output—which is uncertain at the time the loan is made—is observed. The debtor's output takes the value y in the good state of nature, which occurs with probability $1-\pi$, where $0<\pi<1$, and $(1-\theta)y$ with probability π, where $0<\theta<1$.

In keeping with the literature on sovereign debt, it is assumed that the creditor may not file bankruptcy proceedings against the debtor in the event of a default. Instead, the mechanism for ensuring at least some repayment from the debtor is the creditor's access to a penalty technology that allows it to impose a cost on the debtor equivalent to a fraction λ, where $0<\lambda<1$, of its second period output.[8] While the imposition of such penalties may not benefit the creditor as much as it hurts the debtor, it is assumed that the debtor agrees to give up the entire share λ of its output to avoid the penalty.[9] In order to generate borrowing in this

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[7]The single creditor assumption is for expository ease. Throughout the paper, the creditor is assumed to behave competitively.

[8]Several interpretations of the parameter λ have been offered in the literature. For example, Froot (1989) interprets λ as the creditor's ability to extract resources from the debtor country. Bulow and Rogoff (1989) derive λ as the result of a game between the debtor and the creditor in which both bargain over the amount of the repayment. Diwan (1990) studies a model in which λ depends on the openness of the debtor country's trade regime.

[9]Fernandez and Rosenthal (1990) assess which agent—debtor or creditor—gets the efficiency gains under alternate bargaining formulations of debt forgiveness. In two of the three variants of the model they analyze, they find that the solution gives all of the bargaining power to the creditor.

model, it is assumed that the debtor discounts the future at a higher rate than the creditor. Both agents are assumed to be risk-neutral.[10]

The simplest possible loan contract in this framework is one in which the lender makes a safe loan (L^S) that satisfies:

$$L^S = \frac{(1-\theta)\lambda y}{1+\rho}, \tag{1}$$

where ρ is the creditor's discount rate or, since the creditor is assumed to behave competitively, the world (riskless) interest rate. Regardless of which state of nature occurs, the creditor is repaid fully. The interest rate charged in such a loan contract is simply the safe rate of return.

III. Forgiveness

Consider next a loan under which the borrower makes a larger payment in the good state than in the bad state. Note that the creditor can collect λy and $(1-\theta)\lambda y$ in the good and bad states, respectively. Hence, it would be willing to loan an amount L^D, where

$$L^D = \frac{(1-\pi)\lambda y+\pi(1-\theta)\lambda y}{1+\rho} = \frac{(1-\pi\theta)}{1+\rho}\lambda y . \tag{2}$$

In order to collect the full amount λy in the good state, the creditor must set the contractual interest rate (r^D) so that

$$(1+r^D)L^D = \lambda y \Rightarrow (1+r^D) = 1+\frac{\rho+\pi\theta}{1-\pi\theta} . \tag{3}$$

The greater the probability of a negative supply shock (higher π) or the greater the severity of the shock when it occurs (higher θ), the smaller will be the amount loaned and the higher the risk premium in the interest rate charged. An increase in ρ, the lender's risk-free rate, will cause the lender to raise the interest rate and lower the amount that can be borrowed. Furthermore, the stronger is the lender's ability to inflict a credible default penalty (higher λ), the more the lender will be willing to lend. The interest rate, however, is unaffected by the size

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[10]Endowing the debtor with concave preferences would also generate borrowing, but would not change in spirit the analytics discussed below. The assumption of risk-neutrality allows for a more tractable exposition.

of the default penalty. If the bad state of nature occurs, the repayment falls short of the contractual amount by an amount F^D, where

$$F^D = (1+r^D)L^D - (1-\theta)\lambda y = \theta\lambda y \qquad (4)$$

may be interpreted as the portion of the loan coming due that is forgiven.[11]

Since the borrower discounts the future at a higher rate than the lender, and given that the lender behaves competitively, the borrower will prefer a contract that allows it to borrow more. In this case, the debtor prefers to pay a risk premium on the loan in exchange for a higher credit ceiling. The gain in initial loan size is greater than the present value to the borrower of expected future repayments.[12]

IV. Rescheduling

Now suppose that the lender, in addition to exacting a repayment up to the penalty costs, can exclude the borrower from the loan market if it does not reschedule the unpaid portion of the initial loan.[13] In other words, suppose that at the end of a period the debtor is confronted with a choice of rescheduling an unpaid balance on its current loan and receiving a new loan or defaulting on an unpaid balance and not receiving a new loan.

The lender is assumed to be able to extract repayments only one period at a time. So the borrower, in agreeing to reschedule and receive a new loan, is committing only to one additional (third) period of repayments. At that time the borrower can again decide whether to

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[11]Since the model ends after the second period, it makes no difference whether the creditor forgives unpaid debt in the second period or retains claims when the period ends. However, in the multiperiod models analyzed below, this distinction will become important.

[12]Assuming the debtor discounts the future at the rate δ, where $\delta > \rho$, its gain in net present value terms is greater under the loan $\{L^D, r^D\}$ than under the safe loan $\{L^D, \rho\}$.

[13]Eaton and Gersovitz (1981) analyze a model in which the debtor repays debt in order to preserve its reputation as a good borrower and to maintain access to international credit markets. However, Bulow and Rogoff (1989a) point out that as long as the debtor is able to hold assets abroad, a pure reputational equilibrium will unravel. Hence, penalties are also needed—along with reputational incentives—to generate sovereign borrowing and repayment. Indeed, models analyzed by Cole, Dow, and English (1995) and Cole and Kehoe (1997) involve both reputational considerations—stemming from the threat of exclusion—and default penalties.

reschedule and receive a new loan or to default on any unpaid balances and be excluded from the loan market for one period.[14]

To extend the simple two-period framework to many periods, suppose that in each period a new loan may be made and unpaid balances on prior loans may be rescheduled. Since repayments will be larger in good states than in bad states, the mix between new loans and rescheduled balances will be state-dependent. It turns out, however, that total debt—which includes new loans as well as rescheduled loans—depends on the expected value of future output, not on the actual value of current output. Hence, total debt can be shown not to be state-dependent.

Let R_b denote the amount of rescheduled debt and L_b the amount of new loans in the bad state. Define α_b as the ratio of rescheduled to total debt when the bad state occurs:

$$\alpha_b = \frac{R_b}{L_b + R_b} \quad . \tag{5}$$

Similarly, in the good state, the ratio of rescheduled to total debt is defined by:

$$\alpha_g = \frac{R_g}{L_g + R_g} \quad , \tag{6}$$

where R_g and L_g denote rescheduled loans and new loans, respectively, in the good state.

In order to calculate the expected present discounted value to the lender of a new loan in period t, given that the bad state has occurred, assume for now that rescheduling will take place. (The condition for this will be developed below.) When a new loan is extended in a bad state it will have a claim on a fraction $(1-\alpha_b)$ of total repayments at time $t+1$ while the remaining α_b share will go to pay loans rescheduled at time t.[15] Thus, recalling that total expected repayments in any period are $(1-\pi\theta)\lambda y$, the present value to the lender of the new loan at t from the expected payment at $t+1$ is

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[14]The framework can readily be extended to consider the case in which the creditor is able to exclude the borrower from loan markets for n periods, where $n>1$. Under such a formulation, rescheduling would entail the debtor agreeing to repay unpaid balances over n additional periods. While the magnitude of the equilibrium amount of debt that is rescheduled would clearly be different, the nature of the results would be similar to the case $n=1$ discussed here.

[15]This assumes equal seniority between new and rescheduled claims.

$$\frac{(1-\alpha_b)(1-\pi\theta)\lambda y}{1+\rho} \quad . \tag{7}$$

Assuming that rescheduling will take place again in period $t+1$, the new loan of period t will yield further expected repayments in period $t+2$. The expected share of these repayments accruing to holders of claims rescheduled in the previous period (period $t+1$), which includes the holder of the new loan in period t, is a^*, where

$$a^* = (1-\pi)\alpha_g + \pi\alpha_b \quad . \tag{8}$$

Since the claimant of the period t loan has a share of $(1-\alpha_b)$ in the loans rescheduled at period $t+1$, its expected share in total repayments in period $t+2$ is $(1-\alpha_b)a^*$. At time $t+3$, the expected share falls to $(1-\alpha_b)(a^*)^2$, and so on. Thus, the expected present discounted value of new lending in the bad state, which must equal the amount of new loans (given competitiveness among lenders), is:

$$L_b = \frac{(1-\alpha_b)(1-\pi\theta)\lambda y}{1+\rho} \{\sum_{j=0}^{\infty} [\frac{\alpha^*}{1+\rho}]^j\} = \frac{(1-\alpha_b)(1-\pi\theta)\lambda y}{1+\rho-\alpha^*} \quad . \tag{9}$$

Note from (5) and (9) that total debt in the bad state can be written:

$$L_b + R_b = \frac{(1-\pi\theta)\lambda y}{1+\rho-\alpha^*} \quad . \tag{10}$$

Similarly, the size of new loans in the good state will be:

$$L_g = \frac{(1-\alpha_g)(1-\pi\theta)\lambda y}{1+\rho-\alpha^*} \quad , \tag{11}$$

and from (6) and (11), total debt in the good state can be shown to be:

$$L_g + R_g = \frac{(1-\pi\theta)\lambda y}{1+\rho-\alpha^*} = D^R \ . \tag{12}$$

A comparison of (10) and (12) reveals that total debt will be the same whichever state occurs. This is denoted D^R to indicate total debt with rescheduling (but without any forgiveness). For later reference, equation (10) may be rewritten, making use of the definitions of a^*, α_b, and α_g, and the fact that total debt remains constant across states, as follows:

$$(1+\rho)(L_b + R_b) - (1-\pi)R_g - \pi R_b = (1-\pi\theta)\lambda y \ . \tag{13}$$

Since total debt stays constant and more will be repaid in the good state than in the bad state, less needs to be rescheduled in the good state. Formally, this can be shown by subtracting state dependent repayments from total debt.

$$R_b = \frac{(1-\pi\theta)\lambda y}{1+\rho-\alpha^*} - (1-\theta)\lambda y \tag{14}$$

$$R_g = \frac{(1-\pi\theta)\lambda y}{1+\rho-\alpha^*} - \lambda y \tag{15}$$

Hence, new loans will be smaller in the bad state than in the good state $(L_b<L_g)$. Assume for now that (15) is positive, and subtract it from (14) to obtain R_b-R_g=$\theta\lambda y$.

Next, the question of whether the borrower is willing to reschedule is addressed. The answer depends on the size of new loans relative to the present value of having to make payments on total debt, both new and rescheduled. In the model, the expected payment at the end of the period is $(1-\pi\theta)\lambda y$, and the borrower's rate of time preference is δ. The new loan in the bad state, which is lower than the new loan in the good state, must be at least as great as the present value to the borrower of committing to an additional payment on the total debt. This amounts to the condition that:

$$L_b \le \frac{(1-\pi\theta)\lambda y}{1+\delta} \ . \tag{16}$$

If (16) does not hold, the borrower would prefer to default on the unpaid balance on prior loans and be excluded from the loan market for a period. It will be in the lenders' interest, therefore, not to let this condition be violated. At the same time, the borrower, with a rate of time preference greater than the safe rate of interest, will want to borrow and reschedule all it can. Under those circumstances, (16) will hold with equality, which is what will be implied in subsequent references to *equation* (16).

To solve explicitly for the amount of debt rescheduled in each state, first substitute for L_b from equation (16) into (13) and rearrange terms:

$$R_b = (1-\pi\theta)\lambda y[\frac{\delta-\rho}{\rho(1+\delta)}] - [\frac{1-\pi}{\rho}](R_b - R_g) \ . \tag{17}$$

Recalling that R_b-R_g=$\theta\lambda y$, and substituting into (17),

$$R_b = \frac{(\delta-\rho)(1-\pi\theta)\lambda y}{\rho(1+\delta)} - \frac{(1-\pi)\theta\lambda y}{\rho} \tag{18}$$

and

$$R_g = \frac{(\delta-\rho)(1-\pi\theta)\lambda y}{\rho(1+\delta)} - \frac{(1+\rho-\pi)\theta\lambda y}{\rho} \tag{19}$$

In order for R_g to be positive (some rescheduling even in the good state), it is necessary that

$$\theta < \frac{\delta-\rho}{(1+\rho)(1+\delta-\pi)} \ . \tag{20}$$

Assuming that (20) holds, total debt may be obtained by summing equations (16) and (17):

$$D^R = L_b + R_b = \frac{[\delta-\theta(1+\delta-\pi)]\lambda y}{\rho(1+\delta)} \ . \tag{21}$$

To find the rate of interest that will prevail on these loans, consider the following:

$$(1+r^R)D^R - (1-\theta)\lambda y = R_b \ . \tag{22}$$

This says that given an interest rate r^R, the unpaid balance if the bad state occurs and repayment is $(1-\theta)\lambda y$, will be R_b. This amount must then be rescheduled. Substituting for D^R from (21) and for R_b from (18), one obtains $r^R=\rho$.

Thus, although the total loans coming due each period exceed the repayments that can be extracted from the borrower in either state, the power to exclude the borrower from the loan market can be used to induce rescheduling if the new loan does not fall short of the expected present value of committing to make another payment on the total loan. If rescheduling can be enforced by not allowing total loans to become too large, this becomes a safe loan and there is no risk premium. While payments are deferred, every loan is eventually paid in full.

Finally, it may be noted that if (16) holds and the debtor is willing to reschedule, then condition (20) is sufficient to ensure that total debt with rescheduling and no forgiveness (D^R in equation (21)) exceeds L^D the loan with partial forgiveness in the bad state but no rescheduling. Thus, at an initial period, with the same expected future payments, the borrower prefers the contract with rescheduling because it means a higher initial loan and hence prefers at the time of the initial loan that the lenders have the power to exclude the borrower for not rescheduling unpaid balances.[16]

V. Forgiveness and Rescheduling

The solution to the pure rescheduling contract indicates that if the borrower can be counted on to reschedule the unpaid balance rather than be excluded from loans in the next period, then loans will be issued at the safe rate of interest. Earlier, in analyzing contracts without exclusion, it was shown that the borrower prefers to pay an interest rate higher than the safe rate in order to receive a higher initial loan. These two results suggest exploring a contract in which there is both forgiveness in the bad state and rescheduling.

Consider a contract under which total debt (D^F) is chosen so that, if the good state were to occur, the unpaid balance (R_g) would be sufficiently low that the debtor would prefer to reschedule and receive new loans ($L_g = D^F - R_g$) over default. If the bad state were to occur, the creditor would forgive part of the unpaid balance so that only R_g in unpaid loans would remain. Hence, under this contract $R_g = R_b$, and $L_g = L_b$. By construction, then, the debtor chooses to reschedule in both states. The analysis below solves for levels of total debt, rescheduled debt, new loans, and the amount of debt forgiven in the bad state. In addition, the interest rate prevailing on such a contract is obtained, and it is shown that this contract allows

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[16]In the appendix, we show that the results on the interest rate under pure rescheduling go through whether or not (20) holds. The appendix also shows that, for sufficiently large θ, loans will be larger under forgiveness without rescheduling as derived in the last section than under rescheduling without forgiveness.

higher borrowing—and, hence, higher welfare—than the contracts analyzed in previous sections.

In any period (except the initial period), let the share of rescheduled obligations in total debt be denoted by α, where

$$\alpha = \frac{R^F}{L^F + R^F} \tag{23}$$

and L^F and R^F are the values of new loans and rescheduled loans, respectively. The share of expected payments in each period going to holders of rescheduled debt is also α. As before, the expected payments in each period are $(1-\pi\theta)\lambda y$ and the creditors' discount rate is ρ. Thus, the expected present discounted value of a loan—which is equivalent to the amount lent given the assumption of competitive lenders—is

$$L^F = \frac{(1-\alpha)(1-\pi\theta)\lambda y}{(1+\rho)} + \frac{\alpha(1-\alpha)(1-\pi\theta)\lambda y}{(1+\rho)^2} + \ldots = \frac{(1-\alpha)(1-\pi\theta)\lambda y}{(1+\rho)} \{\sum_{j=0}^{\infty} [\frac{\alpha}{1+\rho}]^j\} \tag{24}$$

or

$$L^F = \frac{(1-\alpha)(1-\pi\theta)\lambda y}{(1+\rho-\alpha)} . \tag{25}$$

Substituting for α from (23) in (25) and solving for L^F:

$$L^F = \frac{(1-\pi\theta)\lambda y}{(1+\rho-\alpha)} - \frac{\rho}{1+\rho} R^F . \tag{26}$$

Notice that the higher is the amount rescheduled each period (R^F), the greater is total debt since L^F falls by less than the increase in R^F. To ensure that the borrower will be willing to reschedule, new loans must satisfy equation (16). Thus, equating (26) to (16) yields:

$$R^F = \frac{(\delta-\rho)(1-\pi\theta)\lambda y}{\rho(1+\delta)} . \tag{27}$$

Total outstanding debt, which includes both new debt and rescheduled debt, may be calculated by adding equations (26) and (27):

$$D^F = L^F + R^F = \frac{\delta(1-\pi\theta)\lambda y}{\rho(1+\delta)} \quad . \qquad \textbf{(28)}$$

Moreover, the proportion of total debt that is rescheduled—from (23), (27) and (28)—is:

$$\alpha = \frac{\delta-\rho}{\delta} \quad . \qquad \textbf{(29)}$$

Hence, the proportion of rescheduled debt depends on the borrower's rate of time preference relative to the safe rate of interest.

The next step is to derive the contractual interest rate (r^F). Note that the interest rate must be chosen so that the total amount due equals the payment if the good state of nature occurs plus the amount that may be rescheduled. Thus,

$$(1+r^F)\, D^F = \lambda y + R^F \qquad \textbf{(30)}$$

or, substituting for D^F from (28) and for R^F from (27):

$$1+r^F = \frac{\delta(1+\delta) + (\delta-\rho)(1-\pi\theta)}{\delta(1-\pi\theta)} = 1 + \frac{\rho(\delta+\pi\theta)}{\delta(1-\pi\theta)} \quad . \qquad \textbf{(31)}$$

The risk premium is therefore positively related to the severity of the shock as well as the likelihood of its occurrence. In contrast to the risk premium with forgiveness but without rescheduling (equation (3)), the risk premium when both forgiveness and rescheduling are possible also depends on the debtor's discount rate (δ). Indeed, the risk premium declines as the excess of δ over ρ becomes larger, because that gives rise to a greater proportion of total debt that may be rescheduled from one period to the next.

The models imply that total debt with forgiveness and rescheduling (D^F) is unambiguously larger than debt under rescheduling without forgiveness. At the time the debt is initially incurred, the expected future payments are the same whichever contract is in place, so the borrower prefers the contract with forgiveness and rescheduling because it allows more initial consumption, despite the higher contractual rate of interest.

VI. Conclusion

The analysis presented above suggests that if the possibility of forgiveness or rescheduling is incorporated—either explicitly or implicitly—in sovereign loan contracts, the amount creditors are willing to lend to a sovereign borrower rises. Whether the credit ceiling—and, therefore, the debtor's welfare—is higher under the rescheduling contract or the forgiveness contract depends on the intensity of output shocks in the debtor country. However, contracts with both rescheduling and forgiveness allow higher borrowing than either the pure rescheduling or the pure forgiveness contracts, regardless of the intensity of output shocks.

The model also indicates that the preferred contract—one which incorporates both rescheduling and forgiveness—carries a risk premium. To the extent that past loans to sovereign countries carried such a premium, however small, it could be argued that the original contracts at least implicitly included the possibility of forgiveness and rescheduling of a portion of the obligations. Hence, debt reduction initiatives could be justified on the grounds that they simply involve implementation of contingencies in the original contracts.

A striking, and perhaps more important, feature of the loan contract with both rescheduling and forgiveness is its similarity with the terms of the HIPC initiative. Under the HIPC initiative, outstanding external debt obligations of the HIPCs are to be written off, as is the case when the bad state of nature occurs in the rescheduling with forgiveness contract analyzed above. In addition, the time period over which remaining obligations of the HIPCs are to be repaid would be extended, as indeed is the case in the contract analyzed in Section V. Moreover, debt reduction under the HIPC initiative is designed to facilitate renewed lending to debtor countries, which is precisely the equilibrium outcome in the rescheduling with forgiveness outcome presented above. Hence, the rescheduling with forgiveness contract appears to provide a firm theoretical underpinning for the HIPC initiative.

One simplification in this model is that the original loan is used by the debtor country to expand consumption possibilities. If, instead, debt were used to finance productive investment, future repayment prospects would undoubtedly be improved. This would, in turn, raise the debtor's present debt ceiling. In this context, policy adjustments to improve the investment climate may be a means by which debtor countries can credibly commit to higher investment levels. Indeed, such policy adjustments feature prominently in the HIPC and other debt reduction initiatives.

The Pure Rescheduling Case When the Constraint on θ Does Not Hold

If inequality (20) in Section IV does not hold, some aspects of the analysis of rescheduling without forgiveness need to be reworked. Defining θ^* such that

$$\theta^* = \frac{\delta-\rho}{(1+\delta-\pi)(1+\rho)} , \tag{32}$$

observe that for $\theta \geq \theta^*$, there will be no rescheduling in the good state ($R_g=0$).

In order to solve for the equilibrium levels of total debt, rescheduled debt, and new loans in each period, the first step is to derive a new rescheduling condition for the debtor. The debtor will agree to reschedule in the bad state only if the additional liquidity is at least equal to the expected value of the repayment:

$$L_b \geq \frac{(1-\pi)(1+r^R)(L_b+R_b) + \pi(1-\theta)\lambda y}{1+\delta} . \tag{33}$$

Since the debtor will want to borrow as much as possible, this will hold with equality.

The next step is to calculate the expected present discounted value of new loans. In the good state, there is no rescheduling, so in the following period there is a probability $1-\pi$ of receiving full repayment, and a probability π of receiving partial payment and rescheduling the unpaid balance. In the event of partial payment, the rescheduled loan will have a share α_b of expected payments in the subsequent period. With a discount rate of ρ, the lender's expected present discounted value given that the good state has occurred in the present period, is:[17]

$$L_g = \frac{(1-\pi)(1+r^R)L_g}{1+\rho} + \frac{\pi(1-\theta)\lambda y}{1+\rho} + \frac{(\pi\alpha_b)(1-\pi)(1+r^R)L_g}{(1+\rho)^2} + \frac{(\pi\alpha_b)\pi(1-\theta)\lambda y}{(1+\rho)^2} + \ldots \tag{34}$$

or

$$L_g = \frac{(1-\pi)(1+r^R)L_g}{1+\rho} \{\sum_{j=0}^{\infty}[\frac{\pi\alpha_b}{1+\rho}]^j\} + \frac{\pi(1-\theta)\lambda y}{1+\rho} \{\sum_{j=0}^{\infty}[\frac{\pi\alpha_b}{1+\rho}]^j\} , \tag{35}$$

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[17]Note that once a good state occurs in the future, all debt obligations are repaid, so no further repayments will accrue to holders of preexisting debt.

which simplifies to:

$$L_g = \frac{(1-\pi)(1+r^R)L_g + \pi(1-\theta)\lambda y}{1+\rho-\pi\alpha_b} \quad . \tag{36}$$

Similarly, if the bad state has occurred in the present period and a new loan L_b is lent, there is a probability 1-π of full repayment of the new loan in the next period and a probability π of a partial repayment that must be shared with holders of rescheduled debt carried over from the present period. By a manipulation similar to the one used to calculate L_g, the expected present discounted value to the lender of a new loan in the bad state may be shown to be:

$$L_b = \frac{(1-\pi)(1+r^R)L_b + \pi(1-\alpha_b)(1-\theta)\lambda y}{1+\rho-\pi\alpha_b} \quad . \tag{37}$$

After substituting for α_b in equations (36) and (37):

$$L_g = L_b + R_b \quad . \tag{38}$$

In other words, total debt stays constant from one period to the next, since $R_g=0$. Furthermore, it may be seen that:

$$L_b = \frac{[(1-\pi)(1+r^R) - (1+\rho) + \pi]R_b + \pi(1-\theta)\lambda y}{(1+\rho) - (1-\pi)(1+r^R)} \quad . \tag{39}$$

Using equations (39) and (33) to eliminate L_b:

$$\{(\rho-r^R)(1+\delta) + \pi[(1-\pi)(1+r^R) + r^R(1+\delta)]\}\ R_b = (\delta-\rho)\pi(1-\theta)\lambda y \quad . \tag{40}$$

This provides one equation in R_b and r^R.

A second equation can be obtained by noting that the amount rescheduled in the bad state is simply the total amount due minus the amount that may be extracted:

$$R_b = (1+r^R)(L_b+R_b) - (1-\theta)\lambda y. \tag{41}$$

Substituting for L_b from equation (39) and collecting terms:

$$[(1+\delta)r^R + (1-\pi)(1+r^R)]\ R_b = (\delta - r^R)(1-\theta)\lambda y \quad . \tag{42}$$

It can be verified that the solutions for R_b and r^R from equations (40) and (41) are:

$$r^R = \rho \tag{43}$$

and

$$R_b = \frac{(\delta-\rho)(1-\theta)\lambda y}{(1+\delta)\rho + (1-\pi)(1+\rho)} \quad . \tag{44}$$

The amount of new loans in the bad state (L_b) can be found by substituting into equation (41):

$$L_b = \frac{(1+\delta-\pi)(1-\theta)\lambda y}{(1+\delta)\rho + (1-\pi)(1+\rho)} \quad . \tag{45}$$

Total debt (D^R), therefore, is:

$$D^R{}_{\theta \geq \theta^*} = L_b + R_b = g(\theta)\lambda y \quad , \tag{46}$$

where

$$g(\theta) = \frac{(1+\delta-\pi)(1-\theta)}{(1+\delta)\rho + (1-\pi)(1+\rho)} \quad . \tag{47}$$

If $\theta < \theta^*$, rescheduling occurs in both states and the overall level of debt, equation (21) in Section IV, can be written as:

$$D^R_{\theta<\theta^*} = L_b + R_b = f(\theta)\lambda y \quad , \tag{48}$$

where

$$f(\theta) = \frac{[\delta - \theta(1+\delta-\pi)]}{(1+\delta)\rho} \quad . \tag{49}$$

From the definitions of $g(\theta)$ and $f(\theta)$, it may be seen that $f'(\theta)<g'(\theta)$ for all $\theta\in [0,1]$. Also,

$$f(0) = \frac{\delta}{\rho(1+\delta)} > g(0) = \frac{(1+\delta-\pi)}{(1+\delta)\rho + (1-\pi)(1+\rho)} > 0 \tag{50}$$

and

$$g(1) = 0 > f(1) = \frac{\pi-1}{\rho(1+\delta)} \quad . \tag{51}$$

These relationships are illustrated in Figure 1. The relevant debt ceiling when there is rescheduling but not forgiveness is $f(\theta)\lambda y$ for $\theta<\theta^*$ and $g(\theta)\lambda y$ for $\theta\geq\theta^*$. This is shown by the heavy line in Figure 1.

The level of total debt in the case of rescheduling and forgiveness can now be compared with the case of rescheduling without forgiveness, both when the constraint (20) on θ holds and when it does not. To do so, consider equation (28) in Section V:

$$D^F = L^F + R^F = h(\theta)\lambda y \quad , \tag{52}$$

where

$$h(\theta) = \frac{\delta(1-\pi\theta)}{\rho(1+\delta)} \quad . \tag{53}$$

Note that $h(0)=f(0)$ and $h(1)>0$. So the debt ceiling with rescheduling and forgiveness exceeds the debt ceiling for pure rescheduling for all $\theta>0$. This is shown by the line $h(\theta)$ in Figure 1.

Finally, consider the debt ceiling (level of debt) when there is forgiveness without rescheduling. From equation (2), the size of debt in each period is:

$$L^D = k(\theta)\lambda y \quad , \tag{54}$$

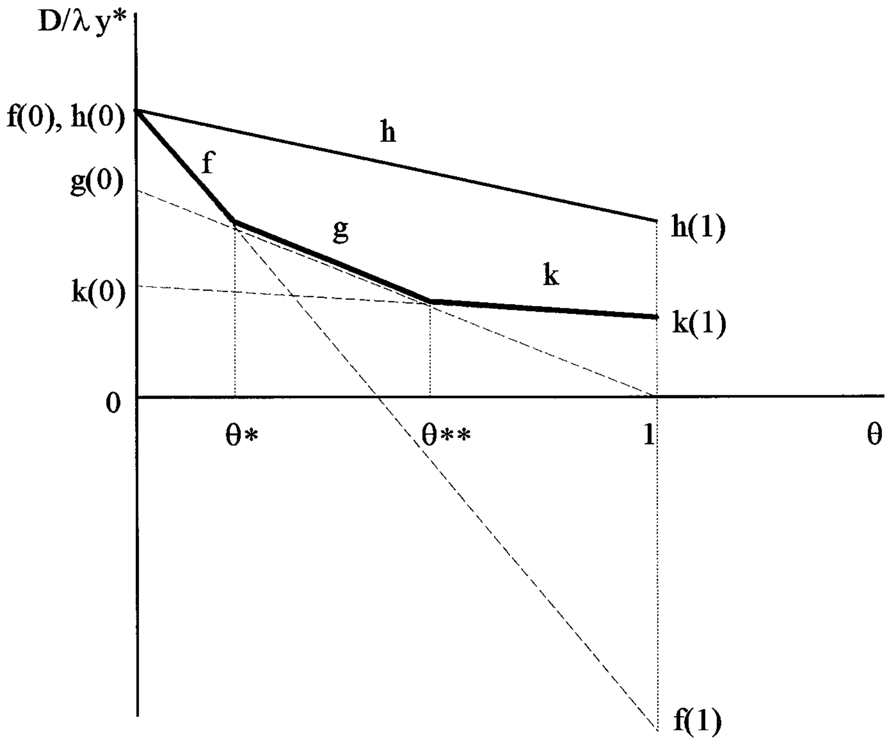
where

$$k(\theta) = \frac{1-\pi\theta}{1+\rho} \quad . \tag{55}$$

One can readily show that $k(0)<g(0)$ and that $h(1)>k(1)>g(1)=0$, so that $k(\theta)$ is everywhere below $h(\theta)$ and crosses $g(\theta)$ from below. Denoting as θ^{**} the level of θ where f crosses g, note that for $\theta>\theta^{**}$, total debt is larger for forgiveness without rescheduling than for rescheduling without forgiveness.

Thus, if output shocks are severe in intensity (θ close to unity), the pure forgiveness contract is preferable to the one with only rescheduling. If, however, output shocks are less severe, the pure rescheduling contract yields a higher credit ceiling. If output shocks are of low intensity (θ close to zero), the preferable pure rescheduling contract involves rescheduling only if the bad state occurs. Finally, the rescheduling with forgiveness contract—because it allows a higher debt ceiling—dominates all other contracts regardless of the intensity of output shocks.

Figure 1
The Debt Ceiling under Alternate Contracts



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