Borrower and Lender Resilience

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1 Introduction

In the decade since the Global Financial Crisis, the official sector has enacted a slew of new regulations to prevent a replay of the crisis. These changes have been relatively bank-centric. The biggest innovations include a revision of bank capital rules, new liquidity regulations and widespread adoption of stress tests to assess bank resilience. Given the prominent role that banking weakness played during the crisis these efforts are welcome.

However, there is still a debate over which pre-crisis features of the advanced economies were critically responsible for depth and duration of crisis. One way to frame the question is to ask whether insuring the resilience of lenders is sufficient to prevent a replay. A great deal of empirical work, e.g. Mian and Sufi (2018) and Jorda Schularick Taylor (2017), has argued that household leverage was a critical amplifying factor in downturn after the crisis. There has been much less regulatory attention directed at building borrower resilience.

In this paper, we propose a simple model that allows us to assess how borrower and lender fragility interact to amplify shocks and to study potential regulatory responses. To our knowledge, while both sources of risk have been widely studied individually, there is not a canonical model that focuses on the interaction of the two. By including both channels, we are able to explore the conditions under which a focus on lender resilience alone is or not sufficient to deliver stability.

The model is set in three periods and is populated by two types of agents, banks and consumers. At date zero, the consumers receive idiosyncratic income draws and decide whether to save or borrow in order to smooth consumption. Savers make a deposit their savings in a bank. Borrowers take out a one period loan from a bank. At date one, there is production. Households draw different levels of labor productivity and supply labor in order to produce. We consider two versions of the labor market. A flexible wage version, in which all households are fully employed. A version with downward wage rigidity, in which unemployment is possible. The latter case is the one we are most interested in, but the full employment version is a useful benchmark. Whether unemployment arises depends on an aggregate shock that shifts consumers' demand by changing future incomes. The central bank can respond to these shifts in consumers' demand by lowering the interest rate, but its interest rate choices are constrained by the zero lower bound.

The interesting interactions in the model arise through the feedback between production, default, and lending. Suppose there is a reduction in output. This causes a reduction in household incomes. When incomes are low, borrowers can adjust in two ways. Some will opt to cut back spending to avoid defaulting. If enough of the households make this choice, then aggregate demand can fall sufficiently to push the economy against the zero lower bound and cause a reduction in output, amplifying the initial reduction in income. Alternatively, faced with low income, some households will choose to default on their loans. These defaults create non-performing loans for the banks which reduces banks' net worth. The model features a standard form of financial accelerator mechanism, so when banks' net worth falls, so does the supply of credit. This leads to an increase in loan rates on new loans, impairing households' ability to smooth consumption between period 1 and 2. This kind of credit crunch can also cause a contraction in the demand and amplify the initial output fall.

We then study how various policies might alter these outcomes. The critical object in the model is the distribution of borrowing and lending choices made at date zero because those choices determine whether production reaches full employment at date 1. Private agents choices can be inefficient in two respects. One factor is that individuals do not internalize how their defaults could impact the lending capacity of the banks. The second deviation between social and private choices comes because households do not recognize that by delevering they can lower overall employment. A planner would worry about both of these considerations and potentially seek to reduce initial borrowing levels.

The optimal policy in general depends on the full distribution of all agents borrowing and lending choices. A useful benchmark to consider is when the aggregate shock has a two point distribution. In particular, suppose that if the shock takes a high value, then incomes in period 2 will large enough to support full employment at date 1. Alternatively, when the shock takes its lower value there is underemployment. In this scenario we would know exactly the extent of the underemployment and can contemplate how much less borrowing at date zero would have been needed to prevent the underemployment.

In this very special case, the only distortion to correct is to reduce the date zero borrowing by just enough to compensate for lower income at date 1. This can be done in either of two ways, a limit can be imposed to constrain households' ability to borrow at date 0 or a policy can limit the ability of banks to lend at date 0. Either approach can be calibrated to fix the distortion. For a more general distribution of shocks, however, this equivalence will not hold so that borrower and lender targeted interventions will lead to different outcomes.

We then go on to consider a richer version of the baseline model in which there is a second asset to which the banking system is exposed. We model the asset as a tree that pays a dividend each period and value of the tree at date 1 is uncertain. The presence of the tree creates the risk that shocks to its value can create losses for the banks that impair their ability to lend. This second source of risk complicates the macroprudential problem because the regulator now needs additional tools to limit both risks. If borrowers were identical this would be relatively straightforward because a combination of an overall capital ratio and risk-weighted capital ratio would force the bank to differentially account for the risks of lending to fund trees or households.

However, because debt positions of households will differ, the risk associated with household lending now depends on individual borrower characteristics. This dependence means that a single risk weight in a capital requirement will not fully account for the distribution of risk in the economy. This creates the possibility of feedback between the conditions of borrowers and lenders. For instance, if the banks suffer losses on their investments, this can cause an increase in the cost of loans for the households that is unrelated to the borrowers' risk. Faced with higher interest rates, some of the households in turn may be cutback lending and if enough do that it can depress aggregate activity more than otherwise would have occurred. To manage this kind of amplification explicit borrower resilience tools must be considered. We discuss the efficacy of different combinations of tools.

The remainder of the paper is laid out as follows. In section 2, we review related empirical and theoretical research. The relevant empirical analysis consists of cross-country studies that characterize the types of tools that are now being used by macroprudential regulators. This practice is evolving quickly, however, it is still possible to document some patterns, particularly for largest advanced economies. For these countries, we will show that there has been substantial progress on monitoring and assessing the resilience of lenders since the global financial crisis. There has been less progress (and inclination) to address and deal with borrower resilience.

There is a large theoretical literature that has developed to assess macroprudential regulation. We are unaware of any papers that take up our challenge of nesting stability risks from both borrowers and lenders in a single framework. So we review the key papers that look at the two types of risks separately. We also briefly describe some papers that aktry to characterize the effectiveness of competing policy in different models.

In section 3, we describe the baseline environment that we will study. Almost all of the ingredients are familiar, so the novel aspect of our setup comes from combining relatively standard assumptions rather than from exploring unfamiliar ones.

In section 4, we solve for the equilibrium in the model. Here is it helpful to study the goods market and credit market separately and then solve for the prices that clear both markets simultaneously. Generically, the privately optimal choices that the agents will make will inefficient.

In section 5, we explore whether a social planner can use regulations to improve upon the private outcomes. We start with the special case mentioned above, in which the aggregate shock takes either a high or low value and show that in this case, either a lender-targeted or borrower-targeted regulation can deliver constrained efficient allocations. For a general set of shocks this would not be the case.

In section 6, omitted in this draft, we extend the model to allow banks to invest in loans and another asset. The richer setup allows for additional interactions between borrower and lender resilience that can further amplify shocks. These interactions substantially complicate the policymaker's problem and mitigating amplification requires the use of both borrower and lender focused tools.

Section 7 contains some brief conclusions that focus on the implications of the analysis for the toolkit that macroprudential regulators need. The need to use of borrower specific tools raises both practical considerations about which ones that regulators ought to focus on, as well political economy issues. In many countries, there is little experience using such tools and the acceptance by both the public and the politicians to start using them is untested.

2 Related Literature

Broadly, our paper touches on four ares of research, each of which has grown substantially in the wake of the global financial crisis. We note the connections to some of the most prominent papers, but do not claim that our review is by any means comprehensive.

One set of papers that share similarities to ours are those that focus on aggregate demand externalities. There is a long literature in macroeconomics dating back to

Blanchard and Kiyotaki (1987) that have studied economies that have this property. The more recent variants have focused on how the presence of non-contingent debt can amplify income shocks. Many of these papers are empirical and focus on the role of household borrowing in exacerbating recessions. For instance, Jordà et al. (2017) show, using data from 17 advanced economies over the last 150 years, that high ratios of credit to GDP go hand in hand with a higher risk of deep recessions. Mian and Sufi, in a series of papers, find complementary evidence for this across regions of the US following the global financial crisis. Their recent survey of the literature (Mian and Sufi (2018)) summarizes this research.

There also have been several theoretical papers that explore this channel. Korinek and Simsek (2016) investigate how the effective lower bound on interest rates can be a problem. In their model, when borrowers need to de-lever following an adverse shock, the reduction in credit leads interest rates to fall. However, if the rate runs into the lower bound, the demand for credit is lower than what is needed for the economy to operate at the efficient level. As we explain below, our model builds on their mechanism by adding bank frictions and the possibility of default to their set up.

Our paper is also closely related to Guerrieri and Lorenzoni (2017) who study a Bewley-style model in which heterogeneous households take on different debt levels. If borrowing constraints tighten so that the borrowers need to de-lever, their spending declines may not be offset by spending increases by savers. This discrepancy can lead to a recession, and the recession will deeper if the zero lower bound constrains the interest rate adjustment.

Another branch of the literature studies how financial frictions can amplify shocks because of credit supply disruptions. Again there were many models that had this feature dating back to at least the 1980s (e.g. Greenwald Stiglitz and Weiss (1984)). The more recent contributions have mostly been empirical and emphasize trying to specifically identify how a shock to a particular lender or market is transmitted. For instance, Benmelech et al (2017) studies how disruptions to the commercial paper market during the crisis starved some non-bank lenders of an important funding source. They establish that in areas where these firms were most active auto-financing became harder to obtain and auto-spending dropped. Ivashina and Scharfstein (2010) and Chodorow-Reich (2014) both use the failure of Lehman Brothers to investigate loan supply effects. Ivashina and Scharfstein note that other financial institutions differed in the extent to which they were in lending syndicates with Lehman. By virtue of its failure, Lehman dropped out of all syndicates and that meant some other intermediaries had to pick up the loans that Lehman had been expected to make. They document that the ones with larger exposures to Lehman cut back on new lending. Chodorow-Reich matches borrowers with banks, and shows that banks with higher Lehman co-syndication cut back on lending to small and medium-sized borrowers who in turn reduced employment.

The third literature on which we touch considers how macroprudential regulation can be best deployed in different circumstances. Korinek and Simsek investigate this question in their model and find that policies that are targeted at reducing leverage can be welfare improving. Farhi and Werning (2016) propose a complete markets framework in which restrictions on prices induces inefficiencies. The prices that allow for the efficient allocation may not be in the feasible set, which leads to wedges relative to the efficient allocation. With Pigouvian taxes or quantity restrictions a planner can channel demand to achieve an allocation closer to the efficient one.

The paper nicely distinguishes between aggregate demand externalities and pecuniary externalities. The first arise in complete markets due to nominal rigidities, the latter in incomplete markets since changes in prices alter the insurance properties of the asset span. Both types of externalities are introduced in a general framework which allows to characterize inefficiencies and optimal Pigouvian taxes in a wide range of settings.

The central difference of our paper relative to previous work is the simultaneous inclusion of both an aggregate demand externality and the potential for credit supply disruptions. By treating both of them we are able to ask several questions that previously could not be considered. Most importantly we can determine the conditions under which a single policy can address both potential inefficiencies. However, we can also explore the efficacy of different tools for dealing with each distortion.

Finally, we take some guidance from the emerging research on how macroprudential tools are used in practice. Akinci Olmstead-Ramsey (2018) provide a nice empirical summary of the evolution of policy choices in 57 countries between 2000 and 2013. One of their key findings is that macroprudential policies have been used much more actively since the crisis.

Edge and Liang (2019a, 2019b) dig deeper into the tools available to FSCs and the governance arrangements within them. They note that there has been an explosion of multi-agency committees since the crisis, with number of countries with such bodies rising from 12 as of 2008 to 47 by 2017. However, the reach some disturbing conclusions. One finding in Edge and Liang (2019a) is that "countries are placing a relatively low weight on the ability of policy institutions to take action and a high weight on political economy considerations in developing their financial stability governance structures." A second concern is their observation that in many countries the FSCs appear to be structured and operating in ways that are best explained as being more about the optics of being able to say the authorities have responded to the last crisis than about taking actions to prevent another one.

One piece of support for the cynical interpretation is to notice the types of powers that have been granted and deployed in advanced countries. Table 1 uses data from the IMF Macroprudential Survey to determine, whether in each country, the relevant macroprudential body has used either borrower specific or lender specific tools. Microprudential regulators have always had a host of tools that can be used to constrain lenders. With the creation of the countercyclical capital buffer and the advent of stress tests, macroprudential tools have also been widely made available. As indicated in the table, virtually every country has taken some sort of action to enhance the resilience of lenders to shocks. In contrast, borrower specific tools have hardly been used. We define these tools to be ones where the characteristics of the individual borrower are used in affecting credit availability. Examples would be the loan to income ratio or a debt service to income ratio, because an individual's income would be used in applying the tool. These kinds of tools have much less familiarity in most countries, but can serve as a powerful brake on credit extension. We will see in our model that this is also the case.

3 The Model

The model is set in three periods $t = 0, 1, 2$. There is a unit measure of consumers and a unit measure of bankers. Consumers are heterogeneous: depending on the income shocks they receive they choose whether to save or borrow. Consumers are indexed by $i \in [0, 1]$. Bankers are all identical. Each of them runs a bank, which acts as an intermediary between saving consumers and borrowing consumers.

The model features three types of frictions. First, consumer debt is non-state-contingent and is subject to default. Second, bankers are subject to a moral hazard friction, which makes their intermediation capacity sensitive to their net worth. Third,

Table 1: IMF Macroprudential Survey

Country	Borrower tool used	Lender tool used
Australia	No	Yes
Austria	No	Yes
Belgium	No	Yes
Canada	Yes	Yes
Denmark	No	Yes
Finland	No	Yes
Germany	No	No
Ireland	Yes	Yes
Israel	Yes	Yes
Italy	No	Yes
Japan	No	Yes
Korea	Yes	Yes
Luxembourg	No	Yes
Netherlands	Yes	Yes
New Zealand	No	Yes
Norway	Yes	Yes
Spain	No	Yes
Sweden	No	Yes
Switzerland	Yes	Yes
United Kingdom	Yes	Yes
United States	No	Yes

there are nominal rigidities which can make output fall below full employment.

Now to a detailed description of the environment.

Preferences. Consumer preferences are represented by the utility function

$$E\left[u\left(c_{i0}\right)+u\left(c_{i1}\right)+c_{i2}\right],$$

where c_{it} is consumption in period t and $u(c)=c^{1-\gamma}/(1-\gamma)$. Consumption in period 2 must be non-negative, $c_{i2}\geq 0$. The linearity of the utility function in period 2 helps to simplify some derivations. Bankers only consume in period 2, and have linear preferences.

Production. In each period t, consumers receive the random income flow y_{it}. The incomes y_{i0} and y_{i2} are endowments of consumption goods, drawn from continuous distributions with cumulative distribution functions, respectively, $F_0\left(y_{i0}\right)$ and $F_2\left(y_{i2}|\theta\right)$. The variable θ is an aggregate shock, the only source of aggregate uncertainty in the model.

Incomes in period 1, y_{i1}, are endogenous and are determined as follows. Each consumer has one unit of labor in period 1 and receives an idiosyncratic labor-efficiency shock ω_{1i}, with cumulative distribution function $F_1\left(\omega_{1i}\right)$. Consumers are employed by competitive firms that transform labor-efficiency units into consumption goods one to one. We will consider two versions of our economy. In the flexible wage version of the economy, the wage adjusts so that all consumers are fully employed and total output Y_1 is equal to $Y^*=\int\omega dF_1\left(\omega\right)$. In this case, individual incomes are simply $y_{i1}=\omega_{i1}$. In the downward-rigid wage version of the economy, output Y_1 can be lower than Y^*. As in Werning(2015), in this case, each consumer is rationed in her labor supply choice and can only supply Y_1/Y^* units of labor. Individual incomes are

$$y_{i1}=\omega_{it}\frac{Y_1}{Y^*}.$$

Summing up the production side: periods 0 and 2 are simple endowment economies, period 1 is a production economy which, in the case of wage rigidity, can display unemployed labor. We have simplified periods 0 and 2 because the interesting action in terms of endogenous output determination and aggregate demand externalities will take place in period 1.

Asset trading and budget constraints. In period 0, consumers observe their current income y_{i0} and trade one-period claims to be paid at date $t = 1$. The consumer budget constraint is

$$P_0 \left(a_{i1} \right) a_{i1} + c_{i0} = y_{i0},$$

where a_{1i} denotes the consumer's net position in one-period claims, which can be positive or negative. When $a_{1i} > 0$ the consumer is a net saver, when $a_{1i} < 0$ a net borrower. The fact that P_0 is a function allows the interest rate to be different for saving and borrowing positions and, in the case of borrowing positions, to vary depending on the amount borrowed, reflecting default risk. The function P_0 will be determined in equilibrium by competition among banks.[1]

Banks enter period 0 with a given initial endowment N_0,[2] they sell safe one-period claims D_1 (deposits) at the price q_0, and buy a portfolio of one-period loans. The loan portfolio is described by the cumulated mass function $\Phi_0(b)$, that, for any loan size $b \in [0, \infty)$, denotes the mass of loans of size less than or equal than b made by the bank. The bank's budget constraint is then

$$\int_0^\infty P_0 \left(-b \right) b d\Phi_0 \left(b \right) = N_0 + q_0 D_1.$$

In period 1, consumers learn the values of their incomes y_{i1} and y_{i2}. At the beginning of period 1, if the consumer has a loan to repay, $a_{i1} = -b < 0$, the consumer can choose between repayment and default. If the consumer repays, the budget constraint is

$$P_1 \left(a_{i2} \right) a_{i2} + c_{i1} = a_{i1} + y_{i1}.$$

If the consumer chooses default, she makes no payment, is excluded from financial markets and receives the fixed consumption flows $c_{i1} = \underline{c}$ and $c_{i2} = 0$. The bank observes the consumer's income realization and, prior to the consumer's default decision, can make a take-it-or-leave it offer to write off the consumer's debt to a lower value $\tilde{b} \leq b$.

To compute the net worth of banks at date $t = 1$ we need to determine the payoff of the loans issued at $t = 0$. We assume each bank pools together a diversified portofolio of loans of size b, so as to perfectly diversify idiosyncratic risk. This means that the

[1] We are implicitly assuming that a consumer cannot take both a lending and borrowing position. This assumption is just for notational convenience as it would never be optimal to do both at the same time in our environment.

[2] In extensions, we endogenize N_0.

payoff of loans of size b at $t = 1$ is deterministic and given by the function

$$\rho(b, \theta)$$

which is equal to the expected repayment by consumers who borrowed b and depends on the aggregate shock θ. The function ρ is an endogenous object, to be determined in equilibrium. The net worth of banks at date 1 is then given by

$$N_1 = \int_0^\infty \rho(b, \theta)\, d\Phi_0(b) - D_1.$$

In period 1, because the consumer's current and future income are observed, banks only make loans that are repaid with probability 1. The bank's budget constraint is then

$$p_1 L_2 = N_1 + q_1 D_2, \tag{1}$$

where p_1 is the price of a one-period loan.

In period 2, we close the model. Consumers repay their loans and consume $c_{i2} = a_{i2} + y_{i2}$. For simplicity, we assume that there is no option to default at $t = 2$. The non-negativity constraint $c_{i2} \geq 0$ implies that consumers face the natural borrowing limit $a_{i2} \geq -y_{i2}$ when choosing their asset position a_{i2} in period 1 .[3]

In period 2, bankers consume their final wealth

$$N_2 = L_2 - D_2.$$

Bankers' moral hazard. We assume that in period 1 bankers can choose between two levels of effort: shirking or not shirking. If they do not shirk they make loans as described above. If they shirk with probability $1 - \pi$ they make loans as described above, but with probability π they make all their loans to a bad pool of borrowers, who never repay their loans. In the latter case, all the loans made by the bank are valueless. Shirking bankers cannot recognize ex ante whether they are making loans to regular consumers or to the bad pool of borrowers. We assume that the bad borrowers are a separate group of agents, who play no other role in the model. In equilibrium, the incentives of the bank will be setup so that these agents get no loans. The cost of

[3] Adding the possibility of default in $t = 2$ would simply make this borrowing limit tighter. Given that the values of y_{i2} play no other role in the model, we can assume that any ingredient limiting the consumers' capacity to borrow at $t = 1$ is embedded in y_{i2} .

effort for a bank is proportional to the loans made and is δL_2. To ensure banks do not shirk, we need to check the incentive compatibility constraint

$$L_2 - D_2 - \delta L_2 \geq (1 - \pi)(L_2 - D_2).$$

The role of this constraint is to limit the capacity of bankers to intermediate in period 1.

In period 1, there is no default risk, so the only reason why p_1 and q_1 can differ is banks are constrained in their intermediation between savers and borrowers. This leads us to define the intermediation spread, the difference between the interest rates for borrowers and savers:

$$\frac{1}{p_1} - \frac{1}{q_1}.$$

This spread will be zero when banks are unconstrained and positive otherwise.

Equilibrium. For ease of exposition, we described the economy directly in terms of real variables and relative prices. The only role of nominal variables in the model is to allow for downward rigid nominal wages, which introduces the possibility of unemployment. In the appendix, we provide the details on how nominal wage rigidity works, making explicit assumptions on how nominal wages and prices are determined and how the central bank sets the nominal interest rate. Here we define an equilibrium directly in terms of real allocations and relative prices.

Let the distribution of consumer claims a_{i1} at the end of period 1 be represented by the CDF $\Psi(a_1)$.

An equilibrium with flexible prices is given by a vector of prices

$$\{q_0, P_0(.), (q_1(\theta), p_1(\theta))_\theta\},$$

and allocations such that all agents solve their individual optimization problems and all markets clear. In particular, labor market clearing in period 1 requires

$$Y_1 = Y^*,$$

in all states θ.[4] Credit market equilibrium requires:

[4]For ease of notation, the dependence of equilibrium variables on θ is left implicit when no confusion is possible.

i. Market clearing for loans at date 0:

$$\Phi_0(b) = \Psi(0) - \Psi(-b) \text{ for all } b;$$

ii. Market clearing for loans at date 1 in all states θ:

$$L_2 = \int_0^1 a_{i2}^- di,$$

where a_{i2}^- denotes the negative part of a_{i2};

iii. Market clearing for deposits:

$$D_t = \int_0^1 a_{it}^+ di,$$

with $t = 1$ and with $t = 2$ for all states θ .

In rigid wage version of the economy, the equilibrium definition is the same, with one difference. We introduce a zero lower bound constraint on the riskless interest rate, which puts an upper bound on the bond price, $q_1 \leq 1$, and replace the condition $Y_1 = Y^*$, with the two inequalities

$$q_1 \leq 1, \quad Y_1 \leq Y^*,$$

with at least one of them holding as an equality for every θ. In the appendix, we show that this equilibrium definition comes from the assumption of downard rigid nominal wages, non-walrasian labor market clearing, and the assumption that the central bank's sets the nominal interest rate with the objective to achieve full employment, subject to the zero lower bound. The credit market clearing conditions (i)-(iii) remain the same in the two versions of our economy.

4 Output and Credit in Equilibrium

In this section we characterize the equilibrium of our economy in a lassez-faire benchmark with no regulatory intervention in financial markets.

In this section, we focus on how output and the intermediation spread $1/p_1 - 1/q_1$ are jointly determined in period 1. The distribution of consumers' claims a_{i1}, described by the CDF $\Psi(a_1)$, is the crucial state variable that we take as given. At the end of the section, we show how this state variable is determined in equilibrium and how its shape determines equilibrium outcomes.

Given the distribution of a_{i1} and a realization of the aggregate shock θ, we want to determine the equilibrium levels of Y_1, p_1, and q_1. We will provide a graphical representation of how the equilibrium is determined. This representation relies on analyzing first how the equilibrium is determined in the goods market and in the credit market.

4.1 The consumption function and goods market equilibrium

Let us begin by deriving individual consumption functions in period 1.

Consider a consumer entering the period with assets a_1 who learns that her income flows are y_1, y_2. For ease of notation, we drop the i subscript in this section.The consumer optimization problem, conditional on no default, is

$$\max_{c_1, c_2, a_2} u(c_1) + c_2$$

subject to

$$q_1 a_2^+ - p_1 a_2^- + c_1 = a_1 + y_1,$$

$$c_2 = a_2 + y_2 \geq 0,$$

where a_2^+ and a_2^- denote the positive and negative parts of a_2. This problem yields four different cases:

(i) The consumer is a net saver, $a_2 > 0$, and the Euler equation $q_1 u'(c_1) = 1$ holds.

(ii) The consumer chooses a zero position $a_2 = 0$, and the following inequalities hold

$$q_1 u'(c_1) \geq 1 \geq p_1 u'(c_1).$$

(iii) The consumer is an unconstrained borrower $0 > a_2 > -y_2$ and the Euler equation holds at the price p_1: $p_1 u'(c_1) = 1$.

(iv) The consumer is a constrained borrower $a_2 = -y_2$ and the following inequality holds

$$p_1 u'(c_1) \geq 1.$$

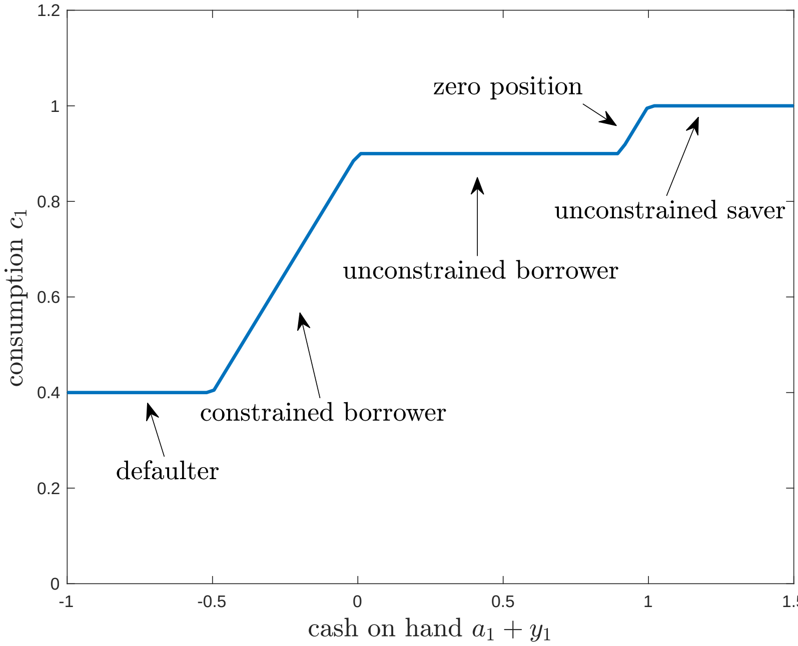

Figure 1: Consumption function

Consider next the possibility of default and debt renegotiation. For the rest of the analysis we focus on economies that, in equilibrium, satisfy two properties: $p_1 u'(\underline{c}) > 1$ and $y_1 + p_1 y_2 - \underline{c} > 0$ for all y_1, y_2. If these properties are satisfied, it is easy to show that the possibility of default and renegotiation leads to the following outcome: if

$$a_1 + y_1 + p_1 y_2 \geq \underline{c}, \tag{2}$$

there is no default; otherwise, the bank offers to write down the debt to a value that satisfies

$$\tilde{a}_1 + y_1 + p_1 y_2 = \underline{c},$$

the consumer accepts and consumes $c_1 = \underline{c}$ and $c_2 = 0$, the bank recovers $-\tilde{a}_1 < -a_1$. The repayment for the bank can be written compactly as

$$\min\{-a_1, y_1 + p_1 y_2 - \underline{c}\} \tag{3}$$

which is the payoff of a debt claim backed by the value $y_1 + p_1 y_2 - \underline{c}$, under limited liability.

Combining all the cases considered above, we have a full characterization of the individual consumption function

$$c_1 = C\left(a_1 + y_1, y_2, p_2, q_2\right).$$

In Figure 1 we plot the relation between cash on hand $a_1 + y_1$ and consumption c_1 that comes from the characterization above. The following observations are useful for the analysis that follows: consumers with sufficiently high levels of cash on hand $a_1 + y_1$ have a zero marginal propensity to consume (MPC), constrained borrowers have an MPC equal to one, defaulting borrowers have a zero MPC. Notice that the zero MPC of defaulters does not imply that their default has no effect on output. Defaults produce losses on banks' balance sheets that affect credit supply, which, as we shall see shortly, can act to amplify a reduction in aggregate output.

Aggregating individual consumption choices, we obtain the following equilibrium condition in the goods market:

$$Y_1 = \int\int\int \left[C\left(a_1 + \omega Y_1, y_2, p_1, q_1\right)\right] dF_1\left(\omega\right) dF_2\left(y_2|\theta\right) d\Psi\left(a_1\right). \tag{4}$$

The following result shows how the output level Y_1 changes as p_1 and q_1 vary. It will be useful for our graphical representation below of the equilibrium in period 1.

Proposition 1. *Given p_1 and q_1 there is a unique output level Y_1 that satisfies the good market clearing condition (4). This level is increasing in both p_1 and q_1.*

The intuition for this proposition is straightforward. Higher values of p_1 correspond to lower interest rates for borrowers, higher values of q_1 correspond to lower interest rates for savers, both shift upwards the consumption function.

Combining the last result with the assumption that the central bank objective is to get the economy as close as possible to Y^*, we have that output in this economy is determined as follows: if there is a $q_1 \leq 1$ such that (4) is satisfied at $Y_1 = Y_1^*$, the equilibrium output is Y^*; otherwise, the central bank sets $q_1 = 1$ and equilibrium output is determined by (4). To complete the analysis we need to determine p_2, which we do next.

4.2 Credit market

Let us first derive the demand for loans. Define the individual net saving function

$$S\left(a_1+y_1,y_2,p_1,q_1\right)=a_1+y_1-C\left(a_1+y_1,y_2,p_2,q_2\right)$$

if the no default condition (2) is satisfied and let $S=-p_1y_2$ otherwise. The aggregate demand for loans in period 1 is then

$$\frac{1}{p_1}\int\int\int S^{-}\left(a_1+\omega Y_1,y_2,p_1,q_1\right)d\Psi\left(a_1\right)dF_1\left(\omega\right)dF_2\left(y_2|\theta\right).$$

Turning to credit supply, let us derive the representative bank's net worth. The expected payoff of a loan of size b is given by

$$R\left(b,Y_1,p_1,\theta\right)=\int\int\min\{b,\omega Y_1+p_1y_2-\underline{c}\}dF_1\left(\omega\right)dF_2\left(y_2|\theta\right)$$

given the repayments in equation (3). The bank's net worth at the beginning of the period is then

$$N_1=\int_0^{\infty}R\left(b,Y_1,p_1,\theta\right)d\Phi_0\left(b\right)-D_1.$$

In the appendix, we show that the bank's incentive compatibility constraint can be rewritten as

$$D_2\leq\phi L_2,\tag{5}$$

for some coefficient $\phi\in(0,1)$. So the bank's problem is to maximize L_2-D_2 subject to the budget constraint (1) and the incentive compatibility constraint (5). This leads to two cases:

(i) No intermediation spread. The loan price satisfies $p_1=q_1$, the bank is unconstrained and loan supply is any

$$L_2\in[0,\frac{1}{\left(1-\phi\right)q_1}N_1].$$

(ii) Constrained credit supply. The loan price satisfies $\phi q_1<p_1<q_1$ the bank is constrained and the loan supply is

$$L_2=\frac{1}{p_1-\phi q_1}N_1.$$

In both cases the bank's expected profits are

$$\frac{1-\phi}{p_1-\phi q_1}N_1. \tag{6}$$

Notice that in equilibrium we can never have $p_1 > q_1$ or loan supply would be 0 and we cannot have $p_1 \leq \phi q_1$ or there would be an unbounded supply of loans.

Combining the derivations above, equilibrium in the credit market requires that the following two conditions be satisfied

$$\frac{p_1}{p_1-\phi q_1}N_1 \geq \int\int\int S^{-}\left(a_1+\omega Y_1, y_2, p_1, q_1\right) d\Psi\left(a_1\right) dF_1\left(\omega\right) dF_2\left(y_2|\theta\right), \tag{7}$$

and

$$p_1 \leq q_1 \tag{8}$$

with at least one strict equality.

Figure 2 shows how the equilibrium is determined in the credit market for a given initial distribution of a_1 and for given values of Y_1 and q_1. The figure is based on the numerical example described below. For ease of interpretation, we plot credit demand and credit supply against the interest rate on loans $1/p_1 - 1$. It is easy to prove that credit demand is decreasing in the interest rate. For credit supply, on the other hand, two opposite forces are at work. For a given level of banks' net worth, an increase in the interest rate leads to an increase in credit supply, simply because $p_1/\left(p_1-\phi q_1\right)$ is decreasing in p_1. However, an increase in the interest rate also affects default decisions and, through that channel, it decreases banks' net worth, as higher interest rates make it harder for borrowers to rollover their debt and thus lead to larger debt write offs, reducing ρ_1. In the parametrization we have chosen, the first effect dominates so credit supply is upward sloping.

The figure also shows how the equilibrium level of p_1 changes with Y_1, which will be needed for the graphical characterization below.

4.3 The output-credit loop

The conditions derived above for equilibrium in the goods and in the credit market can be combined graphically as in Figure 3, where we focus on the case in which the credit market equilibrium condition yields an increasing relation between Y_1 and p_1. The point where the good market equilibrium curve and the credit market equilibrium

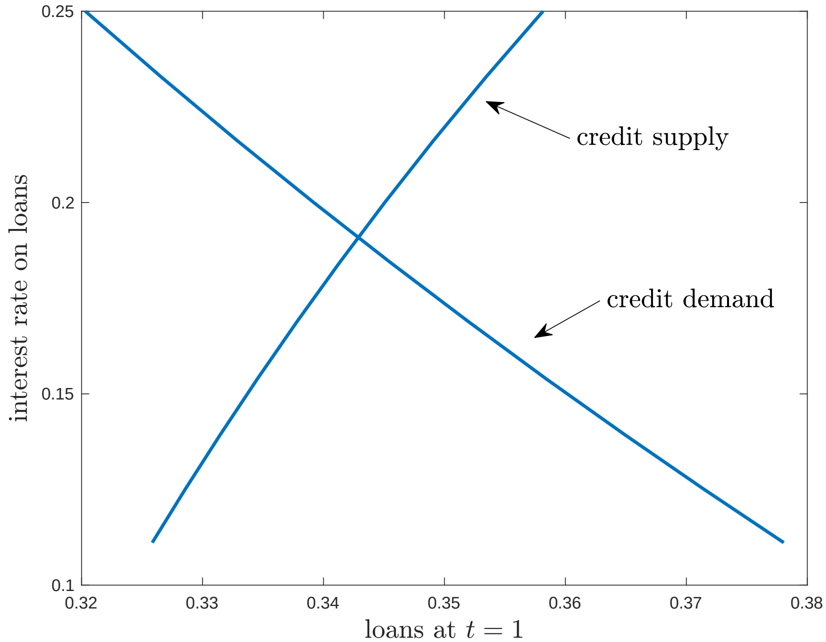

Figure 2: Credit market equilibrium in period 1.*Note:* Interest rate on loans is $1/p_1 - 1$.

curve intersect gives an equilibrium of our economy. Notice that here we are implicitly focusing on a case in which the aggregate shock θ is such that in equilibrium the ZLB is binding, so $q_1 = 1$ and output is below Y^*.

In the case depicted in Figure 3 the credit market amplifies changes in output coming from shocks on the goods' market. That is, a shift in the blue line (due to forces to be discussed) will produce a more than proportional increase in equilibrium output. The mechanism is the following: an increase in Y_1 reduces defaults, increasing the value of the legacy loan portfolio in the banks' balance sheets increasing the banks net worth and credit supply. The increase in credit supply reduces the intermediation spread (increases p_1), which makes it easier for borrowers to rollover their debt and avoid default, thus reinforcing the initial effect. This self-reinforcing effect will play an important role in the analysis of regulation below.

4.4 A comparative statics exercise

Figure 4 shows the result of a simple comparative statics exercise in which we vary the shape of the distribution of asset positions Ψ at the beginning of period 1 and look at equilibrium outcomes. Of course, the distribution Ψ is endogenous and we will shortly

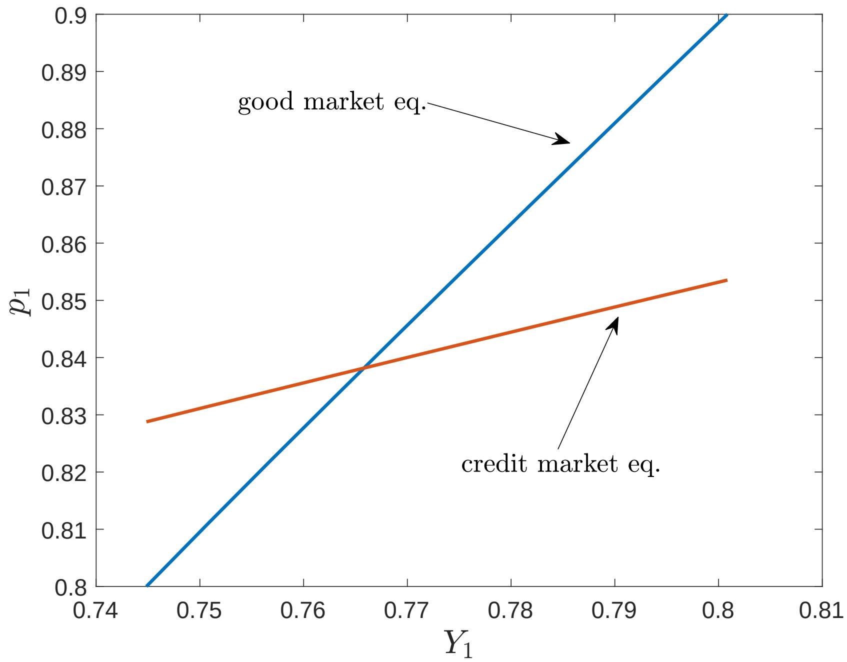

Figure 3: Goods and credit market equilibrium.

discussed how it is determined in equilibrium and how policy can affect its shape. But it is useful to first visualize the general equilibrium effects of a change in Ψ in a simple numerical example, building on the graphic representation just derived.

To plot Figure 4 we start from a log normal distribution Ψ and shows the effects of a reduction in the variance of the distribution Ψ while keeping the mean $\int a_1 d\Psi(a_1)$ unchanged.[5] The idea is that this comparative statics inform us on the general equilibrium effects of a policy that discourages borrowing in period 0, shifting negative values of a_1 to the right and positive values to the left. Keeping the mean unchanged means keeping constant the total net worth of the household sector. The dotted lines correspond to the good market and credit market equilibrium conditions after the change in Ψ. The effect of reduced borrowing is twofold. The good market equilibrium condition shifts to the right, as less indebted consumers spend more, while savers with smaller savings don't reduce consumption much. This is a consequence of the lower MPC of savers with respect to constrained borrowers.[6] On the other hand, the credit market

[5]The parameters used for this exercise are reported and discussed in the appendix.

[6]Notice that the consumption function is not everywhere concave, due to the presence of defaulters and agents with zero positions. So this result is not general and depends on the properties of the numerical example used.

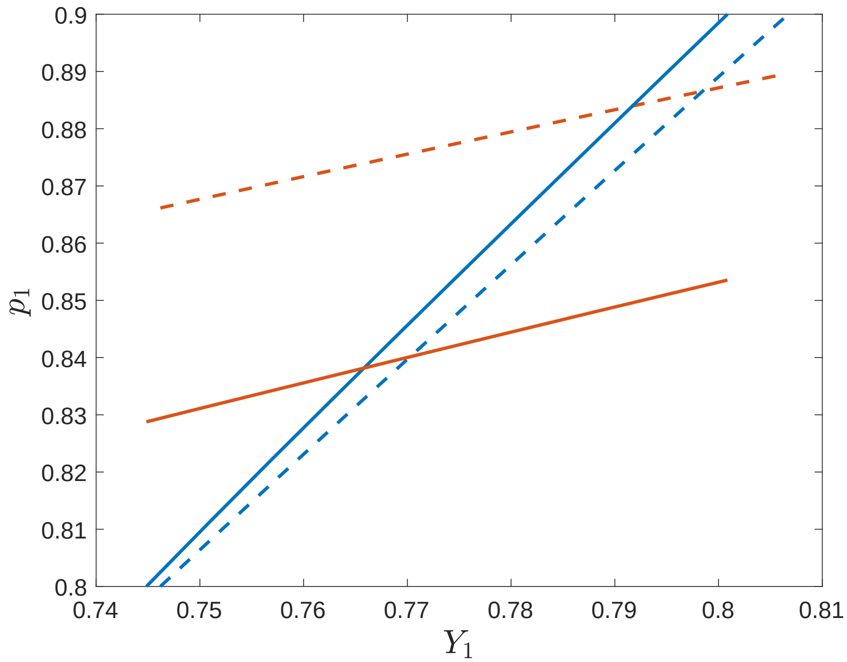

Figure 4: Effects of a reduction in borrowing ex ante.

equilbrium condition shifts upwards. This is due to the fact that a reduction of the right tail of the distribution of a_1 implies fewer defaults and higher bank net worth, leading to an increase in credit supply and to an increase in p_1. These two shifts produce an increase in equilibrium output and a reduction in the intermediation spread. These increases are larger due to the output-credit loop discussed above. We will see shortly what are the welfare implications of the effects just described.

4.5 Borrowing and Lending Ex Ante

We now go back to period $t = 0$ and complete the characterization of an equilibrium with no government intervention by looking at the decisions that determine the distribution of a_1, which was so far taken as given in the analysis. This characterization sets the stage for the study of optimal regulation in the next section.

Let the value function

$$V\left(a_1; Y_1, p_1, q_1, \theta\right)$$

denote the expected utility of an individual consumer conditional on the aggregate state θ, before the consumer learns the realizations of the individual shocks ω_1 and y_2.

A semicolon separates the variables that are chosen by the individual agent from those that she takes as given. When consumers choose their financial position a_1 at date 0 their first-order condition can then be written as:[7]

$$\left(P_0\left(a_1\right)+P_0'\left(a_1\right)a_1\right)u'\left(c_0\right)=E\left[\frac{\partial V\left(a_1,Y_1,p_1,q_1,\theta\right)}{\partial a_1}\right].$$

Borrowing consumers (with $a_1<0$) know that by borrowing more (reducing a_1) they benefit from selling the marginal claim at the price $P_0(a_1)$, but also increase their borrowing cost on the infra-marginal claims issued, as $P'(a_1)>0$ (if the probability of default is non-zero). This explains why they perceive a marginal cost of borrowing equal to $P_0+P_0'a_1>P_0$.

Consider now the banks. The value function of the representative bank conditional on the state θ was derived above, in equation (6), and we can write it as

$$V_B\left(\Phi_0,D_1;Y_1,p_1,q_1,\theta\right)=\frac{1-\phi}{p_1-\phi q_1}\left[\int R\left(b,Y_1,p_1,\theta\right)d\Phi_0\left(b\right)-D_1\right].$$

The optimality of the bank's portfolio Φ_0 at date 0 can then be derived pointwise and yields

$$\lambda_0P_0\left(-b\right)b=E\left[\frac{1-\phi}{p_1-\phi q_1}R\left(b,Y_1,p_1,\theta\right)\right].$$

where λ_0 is the Lagrange multiplier on the bank's date 0 budget constraint.

Combining these conditions, we see that the competitive equilibrium is characterized by the condition

$$u'(c_{i0})E\left[\frac{1-\phi}{p_1-\phi q_1}\frac{\partial R\left(-a_{i1},Y_1,p_1,\theta\right)}{\partial b}\right]=\lambda_0E\left[\frac{\partial V\left(a_{i1},Y_1,p_1,q_1,\theta\right)}{\partial a_1}\right].\tag{9}$$

The slope of the P_0 function fully reflects the slope of the repayment function R and it induces the consumer to correctly internalize the cost that her borrowing decision has on the lender's balance sheet. So a competitive market for loans leads to full internalization of the private costs of default. This however does not necessarily imply that the equilibrium is constrained efficient, because the consumer and the lender fail to internalize the general equilibrium consequences of their choices, as we shall see in the next section.

[7]Recall that we are leaving implicit the dependence of Y_1,p_1,q_1 on θ.

5 Optimal Financial Regulation

We now consider the problem of a benevolent government with a limited set of policy instruments who considers intervening to limit the potential damage from a negative shock leading to a contraction in output and an increase in the intermediation spread, as in the example depicted in Figure 3.

To study ex-ante prudential policies, we restrict the government to intervene at date 0. In particular, we consider a government who only uses the following tools:

- A constraint on banks' leverage at $t = 0$, of the type

$$q_0 D_1 \leq \zeta N_0$$

  for some ζ;

- A non-linear tax on individual asset positions $\tau(a_{i1})$;

- A lump-sum tax/transfer on all consumers—which cannot be conditioned on any individual variable—and a lump-sum tax/transfer on the representative bank at date 0.

We assume that the tax $\tau(a_{i1})$ is levied on consumers. The lump-sum tax/transfer on consumers and banks allows us to abstract from distributional concerns. The welfare weight on the bank can be chosen so that this transfer is zero at the optimal policy.

The government's objective is to maximizes the social welfare function

$$\int \xi_i U_i di + \xi_B U_B$$

where U_i is the expected utility of consumer i and U_B the expected utility of the banker and ξ_i and ξ_B are general welfare weights, that allow us to fully characterize the Pareto frontier.

To characterize optimal policy, we consider first the problem of a planner who can choose directly the distribution of consumption at date 0, $\{c_{i0}\}$, and the distribution of asset positions $\{a_{i1}\}$ subject to resource constraints and a set of incentive compatibility constraints derived in the appendix, which are necessary conditions for an equilibrium with taxes in period 0. We then show that the planner optimum can be implemented with the tools available.

In particular, we consider the problem of a planner who can choose two functions: a transfer $T(y_{i0})$ to the consumer at date 0 and an asset position $a_{i1} = A(y_{i0})$, both contingent on the consumer's income y_{i0}. We assume that the Pareto weights ξ_i only depend on the income y_{i0} and are denoted by $\xi(y_{i0})$. The incentive compatibility conditions are derived by assuming that y_{i0} is not observed by the planner: the consumer reports $\hat{y}_0 \in [\underline{y}, \overline{y}]$ and consume $y_0 + T(\hat{y}_0)$ in period 0 and enter period 1 with an asset position $A(\hat{y}_0)$. Incentive compatibility requires consumers to report truthfully $\hat{y}_0 = y_0$. The underlying idea here is that the planner can impose taxes to induce agents to borrow more or less, but cannot improve on the market's capacity to identify agents with a more or less urgent need to borrow.

In the appendix, we formulate the planner's problem formally and show that the planner's choice of $A(.)$ maximizes:

$$\int_{\underline{y}}^{\overline{y}} E\left[V\left(A\left(y_0\right), Y_1, p_1, q_1, \theta\right)\right] \mu\left(y_0\right) dF_0\left(y_0\right) + \tag{10}$$
$$+ \xi_B E\left[\frac{1-\phi}{p_1 - \phi q_1}\left[\int_{A(y_0)<0} R\left(-A\left(y_0\right), Y_1, p_1, \theta\right) dF_0\left(y_0\right) - \int_{A(y_0)\geq 0} A\left(y\right) dF_0\left(y_0\right)\right]\right],$$

where the function $\mu(y_0)$ is a Lagrange multiplier that comes from the optimal choice of the transfer $T(y_0)$. Notice that in the problem above we have left implicit the fact that Y_1, p_1, q_1 depend on the shock θ and on the function $A(.)$ as shown in Section 4.

After some algebra (derived in the appendix), we can write the first order condition that characterizes the optimal choice of $A(y_0)$ as follows:

$$E\left[u'\left(c_1\right)\iota | y_0\right]\mu\left(y_0\right) - \xi_B E\left[\frac{1-\phi}{p_1-\phi q_1}\frac{\partial R\left(-A\left(y_0\right), Y_1, p_1, q_1, \theta\right)}{\partial b}\right] + \tag{11}$$
$$+ E\left\{E\left[u'\left(\tilde{c}_1\right)\mu\left(\tilde{y}_0\right)|\theta\right]\frac{dY_1\left(\theta, A(.)\right)}{dA\left(y_0\right)}\right\} + E\left\{E\left[\left(u'\left(\tilde{c}_1\right)\mu\left(\tilde{y}_0\right) - \xi_B\frac{1-\phi}{p_1-\phi q_1}\right)\tilde{a}_2^{-}|\theta\right]\frac{dp_1\left(\theta, A(.)\right)}{dA\left(y_0\right)}\right\}$$
$$- E\left\{E\left[\left(u'\left(\tilde{c}_1\right)\mu\left(\tilde{y}_0\right) - \xi_B\frac{1-\phi}{p_1-\phi q_1}\right)\tilde{a}_2^{+}|\theta\right]\frac{dq_1\left(\theta, A(.)\right)}{dA\left(y_0\right)}\right\},$$

where ι is a random variable that denotes debt repayment and where, for clarity, we now make explicit the dependence of Y_1, p_1, q_1 on θ and $A(.)$.

It is useful to give a detailed interpretation of the terms in equation (11). The source of inefficiency in our economy is due to the fact that individual borrowers and lenders at date 0 make their decisions without taking into account the effect of these decisions on the equilibrium values of Y_1, p_1 and q_1, and, due to incomplete markets and to nominal rigidities, these variables have non-zero effects on the efficiency of the allocation at date 1. That is, there are both pecuniary and aggregate demand externalities at work in our economy. The first line of equation (11) captures marginal effects that are included in private calculations, while the second and third line of (11) captures aggregate demand and pecuniary externalities.

Let us abstract for a moment from these externalities and consider a planner who can only modify the choices of a small group of consumers and banks (a group of measure zero), so that it cannot affect the equilibrium values of Y_1, p_1, q_1. For such a planner, with an appropriate choice of Pareto weights, the competitive equilibrium allocation with no taxes maximizes social welfare as shown in the following proposition.

Proposition 2. *If the values of Y_1, q_1, p_1 conditional on θ are fixed, the competitive equilibrium allocation maximizes the social welfare function with Pareto weights $\xi_i = 1/u'(c_{i0})$ for the consumers and $\xi_b = 1/\lambda_0$ for the banker.*

It is useful to sketch the proof of this result. If the values of Y_1, p_1, q_1 are taken as given, the planner's optimality boils down to the first line of (11)

6 Conclusions

The fact that lender-only regulations are not sufficient to correct all the potential distortions that arise in our model is hardly surprising. Essentially this result arises because we have an aggregate demand externality that comes because borrowers sometimes have debt levels that can constrain spending when aggregate conditions are bad. The entire distribution of borrowers' loan choices matter for determining whether this problem exists. This is why policies that ignore the heterogeneity of borrowers typically are insufficient for dealing with the externality.

Recognizing that successful mitigation requires regulation that depends on borrowers' incomes raises a number of tricky issues. The most basic challenge is that in some countries, including the U.S., no regulator has the clear authority to implement rules that can constrain borrowers' access to credit. If there is no entity within the official sector that can act, then that precludes a first-best regulatory response.

A second consideration is that even if a macroprudential regulator can potentially make rules that change the access to credit based on borrowers' income, doing so raises some political economy challenges. These kinds of restrictions are potentially much more invasive and visible to the public than lender-oriented regulations. Particularly, in cases where low-income and/or younger households or small businesses will be clearly affected, the political push-back is likely to be non-trivial. Moreover, if the policies are in place and succeed in dampening shocks, explaining the success of the policy requires the public and the legislature understanding potential counter-factual analyses. That may make it difficult to sustain support over time.

A third difficulty in deploying such regulations is the limited available experience that can be used for calibrating them. Among the largest, advanced economies these tools have hardly been deployed; we do not have the benefit of having watched them operate over the course of several business cycles. This is likely to lead to further hesitation in using these kinds of tools.

One more hopeful note comes from the experience in the U.K. Starting in 2014, there have been borrower-specific loan restrictions with respect to mortgage credit. These rules require lenders to initiate no more than 15 percent of the flow of new loans to borrowers with loan to income ratios above 4.5. Borrowers are also subject to an affordability test, where they must be able to continue to service their debts even if interest rates rise by 300 basis points within the first 5 years of the loan. These regulations have not generated much controversy, and do seem to have limited the amount of highly-indebted borrowers.

While it is dangerous to extrapolate from a single example (with a short history) there are several aspects of the U.K. situation that might be informative for other countries. One factor is that housing price booms and busts have been ubiquitous in the U.K. So the idea that restrictions might be needed to address this does not seem to have been perceived as an over-reach or power grab by regulators. In some countries, this kind of presumption might be a problem.

Another consideration is that the regulations are not designed to bind on any *specific* individual. Lenders can give credit to some high loan to income borrowers. Also, borrowers can comply with the restriction by taking out a smaller loan. Indeed, since the policy has been in place, the percentage of new loans above 4.5 has been flat, while the percentage of loans to people borrowing between 4 and 4.5 of their income has increased. This ought to be stability enhancing if the 4.5 threshold is properly calibrated.

While it is too early to tell whether the U.K. policies will prevent amplification of future shock, the mere fact that the policy has been adopted shows that implementing such policies is at least possible.

7 Appendix

7.1 Banks' moral hazard problem

Recall that the banker's incentive compatibility constraint is

$$L_2 - D_2 - \delta L_2 \geq (1 - \pi)(L_2 - D_2),$$

which can be rewritten as

$$(\pi - \delta) L_2 \geq \pi D_2.$$

Defining

$$\phi = \frac{\pi - \delta}{\pi}$$

the last inequality yields (5).

7.2 Planner's problem

Define the expected payoff of consumers as a function of their individual asset position a and of the function $A(.)$ as follows

$$v(a, A) = E[V(a, Y_1, p_1, q_1, \theta)]$$

where Y_1, p_1, q_1 depend on θ and on the distribution

$$\Psi(a) = Pr[A(y) \leq a],$$

as per the general equilibrium analysis in Section 4. Similarly define the expected payoff of the banker

$$v_B(A) = E\left[\frac{1 - \phi}{p_1 - \phi q_1}\left(\int R(b, Y_1, p_1, \theta) L(b)\, db - d\right)\right].$$

Incentive compatibility for consumers requires that consumers truthfully reveal y, that is,

$$u(y + T(y)) + v(A(y), A) \geq u(y + T(\tilde{y})) + V(A(\tilde{y}), A)$$

for all $\tilde{y} \in [\underline{y}, \overline{y}]$. Defining

$$U(y) = \max_{\tilde{y}} u(y + T(\tilde{y})) + v(A(\tilde{y}), A),$$

the envelope theorem gives

$$U'(y) = u'(y + T(y)).$$

So a necessary condition following from the incentive compatibility constraint is

$$u(y + T(y)) + V(A(y)) = \underline{U} + \int_{\underline{y}}^{y} u'(\tilde{y} + T(\tilde{y}))\, d\tilde{y}, \tag{12}$$

where $\underline{U} = U(\underline{y})$.

The planner's problem we want to analyze is the problem of choosing the functions $T(y), A(y)$ to maximize

$$\int \xi(y)\left(u(y + T(y)) + v(A(y), A)\right) dF(y) + \xi_B v_B(A)$$

subject to the resource constraint

$$\int T(y)\, dF(y) = N_0$$

and the incentive compatibility constraint (12).

Normalize the weights $\xi(y)$ so that

$$\int_{\underline{y}}^{\overline{y}} \xi(y) f(y)\, dy = 1$$

and define

$$G(y) = \int_{\underline{y}}^{y} \xi(\tilde{y}) f(\tilde{y})\, d\tilde{y}$$

and $g(y) = G'(y)$. Integrating by parts implies that

$$\left[U(y)(1 - G(y))\right]_{\underline{y}}^{\overline{y}} = \int_{\underline{y}}^{\overline{y}} (1 - G(y))\, U'(y)\, dy - \int_{\underline{y}}^{\overline{y}} U(y)\, g(y)\, dy$$

which yields

$$\int_{\underline{y}}^{\overline{y}} \xi(y) U(y) dF(y) = \int_{\underline{y}}^{\overline{y}} U(y) g(y) dy = \underline{U} + \int_{\underline{y}}^{\overline{y}} (1 - G(y)) U'(y) dy.$$

We can then write the Lagrangean for the planner's problem as follows

$$\begin{aligned}
&\underline{U} + \int_{\underline{y}}^{\overline{y}} u'(y + T(y)) (1 - G(y)) dy + \xi_B v_B(A) + \\
&\quad + \int_{\underline{y}}^{\overline{y}} \left(u(y + T(y)) + v(A(y), A) - U(\underline{y}) - \int_{\underline{y}}^{y} u'(\tilde{y} + T(\tilde{y})) d\tilde{y} \right) \mu(y) f(y) dy + \\
&\quad + \lambda_0 \left(\int T(y) dF(y) - N_0 \right).
\end{aligned}$$

where $\mu(y)$ is the Lagrange multiplier on constraint (12).

Define the function

$$M(y) = \int_{\underline{y}}^{y} \mu(\tilde{y}) f(\tilde{y}) d\tilde{y}.$$

The Lagrangean can then be rewritten, integrating by parts, as

$$\begin{aligned}
&(1 - M(\bar{y})) \underline{U} + \int_{\underline{y}}^{\overline{y}} u'(y + T(y)) (M(y) - M(\bar{y}) + 1 - G(y)) dy + \xi_B v_B(A) + \\
&\quad + \int_{\underline{y}}^{\overline{y}} u(y + T(y)) + v(A(y), A) \mu(y) f(y) dy + \\
&\quad + \lambda_0 \left(\int T(y) dF(y) - N_0 \right).
\end{aligned}$$

Optimality for $\underline{U}$ requires that the Lagrange multipliers $\mu(y)$ be normalized so that

$$M(\bar{y}) = \int_{\underline{y}}^{\overline{y}} \mu(y) f(y) dy = 1.$$

Then the Lagrangean becomes

$$\int_{\underline{y}}^{\overline{y}} u'\left(y+T\left(y\right)\right)\left(M\left(y\right)-G\left(y\right)\right)dy+\xi_{B}v_{B}\left(A\right)+$$
$$+\int_{\underline{y}}^{\overline{y}} u\left(y+T\left(y\right)\right)+v\left(A\left(y\right),A\right)\mu\left(y\right)f\left(y\right)dy+$$
$$+\lambda_{0}\left(\int T\left(y\right)dF\left(y\right)-N_{0}\right).$$

Collecting the elements that depend on A gives expression (10) in the text

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