Monetary Policy and Sovereign Risk in Emerging Economies (NK-Default)*

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Abstract

This paper develops a New Keynesian model with sovereign default risk (NK-Default). We focus on the interaction between monetary policy, conducted according to an interest rate rule that targets inflation, and external defaultable debt issued by the fiscal government. Monetary policy and default risk interact as they both affect domestic consumption and production. We find that default risk amplifies monetary frictions and results in more aggressive monetary policy response to shocks. These monetary frictions in turn discipline sovereign borrowing, slowing down debt accumulation and lowering sovereign spreads. Our framework replicates the positive comovements of spreads with domestic nominal rates and inflation, a salient feature of emerging markets data, and can rationalize the experience of Brazil during the 2015 downturn, with high inflation, nominal rates, and spreads. A counterfactual experiment shows that, by raising the domestic rate, the Brazilian central bank not only reduced inflation but also alleviated the debt crisis.

Keywords: monetary policy, inflation, sovereign default, interest rates
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1 Introduction

Inflation and sovereign risk are important markers for the credibility of governments in emerging markets. Since the early 2000s, following the steps of advanced economies, many central banks in emerging markets have achieved independence from the central government and have increased their credibility by conquering their historically high inflation. Monetary policy in these markets now largely consists of setting nominal interest rates to target inflation, and the toolkit for central banks is the New Keynesian monetary model with pricing frictions. These models analyze the transmission of interest rate policies to inflation and output but are silent on the interactions with sovereign risk, because these models have been formulated for advanced open economies, where sovereign risk is not a primary concern.[1] This paper develops a New Keynesian model with sovereign default to analyze the interactions between monetary policy and sovereign default risk. In our framework, the efficacy of interest rates rules in managing inflation depends on sovereign risk.

In our integrated framework, default risk shapes monetary distortions and affects monetary policy, which is conducted according to a nominal interest rate rule that targets inflation. High default risk leads to low domestic consumption and production and is associated with contractionary monetary distortions. These effects amplify the volatility of inflation and alter the properties of monetary policy. Monetary policy also affects default risk because the added monetary distortions curb government borrowing and lead to lower sovereign spreads. Our *NK-Default* framework combines the workhorse New Keynesian monetary model of Galí and Monacelli (2005) with a standard sovereign default model and can deliver the positive correlations of sovereign spreads with inflation and nominal rates, which we document is a hallmark of emerging market data.

The small open economy model we consider consists of households, firms, a monetary authority, and a fiscal government that borrows internationally. Households value consumption of domestic and imported goods. They supply labor to intermediate goods firms that produce domestic varieties. The intermediate goods firms are subject to productivity shocks and face frictions in setting their prices, in the tradition of Rotemberg (1982). Final goods firms are competitive and use intermediate goods varieties to produce domestic output, which is both consumed by domestic households and exported to the rest of the world.

The government borrows from the rest of the world by issuing long-term bonds denominated in foreign currency and transfers the proceeds from these operations to households. The government lacks commitment to repay its debt and can choose to default. Default is associated with a decline in productivity, which reduces consumption and production. The price of bonds compensates risk-neutral lenders for the risk of default. In this environment, the *default risk*

[1] For example, the influential paper by Galí and Monacelli (2005) analyzes monetary policy in the context of perfect financial markets.

frictions lead to time-varying sovereign spreads and excessive default events.

The monetary authority sets nominal interest rates in local currency using an interest rate rule to target domestic inflation. In our model, monetary policy and firms' pricing frictions can generate *monetary frictions*, which distort the efficient allocation of inputs for production. These monetary frictions are state dependent and can be measured with a *monetary wedge*, recovered from the markup in the New Keynesian Phillips Curve and which reflects the dynamics of inflation. As in standard New Keynesian models, high nominal rates trigger contractionary monetary frictions as they depress production and increase the monetary wedge. In our model, the severity of the monetary frictions also depends on default risk and the level of government debt. We show that high default risk worsens monetary frictions and generates a decline in production. An expectation of default in the future lowers current consumption through the intertemporal consumption-smoothing channel, as future consumption is low in default. Low consumption demand depresses production and increases the monetary wedge.

We consider a Markov problem for the government. The government internalizes that its borrowing and default decisions affect the private economy and the monetary policy response, as well as shape future default. Importantly, the additional costs from high default risk imposed by monetary frictions reduce sovereign borrowing incentives and discipline the government's default risk. Lowering debt is useful in our model because the government overborrows and experiences excessive costly default.[2]

We establish that monetary policy interacts with sovereign risk both theoretically, in a simplified version of our model, and quantitatively, in a model parameterized to Brazil. We also find that these interactions are robust to alternative interest rate rules and to the currency denomination of sovereign debt. Our theoretical results are derived in an example with perfectly rigid prices. We show that increases in default risk, arising from high borrowing or low expected output, increase monetary wedges because high default risk decreases domestic consumption and exports. These forces also result in *borrowing wedges from monetary frictions* in the optimal borrowing conditions for the government and reduce incentives to accumulate debt. The borrowing wedges in our economy with pricing frictions induce lower debt and lower default risk, compared with an otherwise equivalent economy without pricing frictions.

We parameterize our quantitative dynamic model with default and monetary frictions by setting parameters controlling default, the interest rate rule, and the volatility of productivity shocks such that our model replicates the volatility of sovereign spreads, inflation, and output in Brazil. The model can closely match the target moments and contains additional implications that are consistent with the data. Our model delivers a strong positive comovement of spreads

[2] Hatchondo et al. (2016) find that sovereign default models with long-term defaultable bonds give the government incentives to overborrow and dilute existing bondholders, and that such dilution incentives are large and important for explaining the sizable spreads in emerging economies.

with nominal interest rates and inflation, a salient feature in emerging markets.[3] The model is further consistent with the evidence on comovements with spreads: a negative correlation with output, and positive ones with the trade balance and the nominal exchange rate, with magnitudes similar to the data. Our model also generates comovements of these variables with output present in emerging markets: output is negatively correlated with inflation, nominal rates, the trade balance, and the nominal exchange rate.

To study the mechanisms generating these results and to measure the interactions between monetary and default risk frictions, we compare our model with two reference models: an *NK-Reference* monetary model without sovereign default risk, similar to Galí and Monacelli (2005), and a real *Default-Reference* model in the tradition of the sovereign default literature, as in Chatterjee and Eyigungor (2012), but with two goods and production. The amplifying effects of sovereign risk on monetary policy lead to a higher volatility of inflation and nominal rates in our benchmark NK-Default model, relative to the NK-Reference model without default risk. For example, nominal rates are almost twice as volatile because of default risk. The disciplining effects of monetary frictions, however, lower equilibrium spreads. The mean spread in our benchmark model is about 50 basis points lower than in the Default-Reference model without pricing frictions. These reference models also fail to deliver the empirically strong positive comovement of spreads with inflation and nominal rates.

We illustrate the interactions between default risk and monetary frictions by comparing monetary wedges and policy rules across models. We find that the monetary wedge in our NK-Default model varies significantly with the level of debt and default risk, whereas in the NK-Reference model it does not vary with debt. In our model, when default risk is high and rising with debt, the monetary wedge increases rapidly because rising default risk depresses domestic consumption, leading to a decline in production. When default risk is low, in contrast, the monetary wedge decreases with debt. Bond price schedules constrain the ability to roll over the debt and increase the need to produce more to export and pay off the debt. The increase in exports leads to a decline in the monetary wedge. The behavior of the monetary wedge in the NK-Reference model as a function of debt differs sharply. Without default risk, the monetary wedge is completely insensitive to the level of debt because default risk is zero and borrowing is always ample. These different dynamics of monetary wedges across models lead to a higher volatility of inflation, and hence nominal rates, in our benchmark NK-Default model.

Monetary frictions in turn also affect borrowing incentives and default risk. We find that the lower mean spread in our NK-Default model relative to the Default-Reference model is driven by a distinct accumulation of debt: the monetary frictions in the NK-Default model slow down debt accumulation. Slower debt accumulation makes bond price schedules looser when debt is long term because the spreads reflect the default probabilities over the horizon of the bond.

[3] We document that the positive correlations of spreads with inflation and nominal rates is a robust feature in 10 emerging markets with central banks that target inflation.

We also measure these mechanisms in an event analysis. We compare the time paths predicted by our model with the event in Brazil around the 2015 recession and evaluate a counterfactual monetary policy scenario. During the event, output fell in Brazil by about 6%, inflation and nominal interest rates increased by about 4%, and spreads increased by about 3%. We apply our model to this event by feeding into the model a sequence of productivity shocks such that it reproduces the dynamics of output. Our model delivers time paths for inflation, nominal rates, spreads, and exchange rates that resemble those in Brazil. A decline in productivity in our model leads to an increase in the probability of default and hence an increase in sovereign spreads. Inflation rises because lower productivity increases the unit costs of production for firms and because the rise in default risk also increases monetary frictions. This increase in inflation causes a depreciation of nominal exchange rates. Nominal interest rates increase as the monetary authority tightens, in response to the high inflation. We then perform a counterfactual experiment that considers looser monetary policy during the event. In the counterfactual, inflation increases more, output decreases less, and spreads increase substantially more. We conclude that the increase in nominal rates in Brazil during the event not only controlled inflation but also moderated the debt crisis.

We evaluate the robustness of our results in three extended models. In the first extension, we change the currency denomination of sovereign debt to local currency. We find that the amplifying effects of default risk on monetary policy are robust. The volatility of inflation and nominal rates in this model with default risk is quite similar to that in our benchmark model and higher than in the NK-Reference model. Sovereign spreads, however, are lower in this model relative to the benchmark and much lower than in the Default-Reference model without pricing frictions. The disciplining benefits of monetary frictions are amplified with local currency because high depreciation rates in recessions make local currency debt a better hedge. The second and third extensions consider alternative interest rate rules. We evaluate a rule that places more weight on inflation deviations as well as a rule that adds an output gap term. Our results on the interactions between monetary policy and sovereign risk are unaltered. These rules produce lower volatility of inflation and nominal rates, and spreads that are comparable to the benchmark.

Finally, we evaluate the welfare implications of the benchmark model, reference models, and extended models. Our NK-Default model has two sources of inefficiencies arising from monetary and default risk frictions. A comparison of welfare across these models is shaped by how the details of the sovereign debt market and monetary rules interact with these two frictions. Across the models with default risk, we find that welfare is higher with monetary frictions than without monetary frictions in two specifications: when the interest rate rule weights inflation heavily and when debt is denominated in local currency. The combination of lower spreads and low volatility of inflation in these monetary economies dominates the higher spreads and nil volatility of inflation in the economy without monetary frictions. Welfare in the benchmark parameterization of the interest rate rule and in the case with an output gap is lower than in the model without monetary frictions. Welfare is the highest in the NK-Reference model because here borrowing is ample, default risk is nil, and these properties more than compensate for the monetary frictions.

Related Literature Our project builds on insights from two distinct literatures on emerging markets business cycle: the work on New Keynesian monetary policy in small open economies, following Galí and Monacelli (2005), and the literature on fundamental sovereign default risk, following Eaton and Gersovitz (1981).

We follow the quantitative sovereign default models of Aguiar and Gopinath (2006) and Arellano (2008) with long-term debt, as in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). Similarly, our domestic monetary environment is close to the reference model of Galí and Monacelli (2005) and abstracts from the many extensions considered in the (medium-scale) open economy DSGE literature, as in Christiano et al. (2011). One methodological difference between our project and these projects is that we use global methods rather than local approximations around the steady state. Furthermore, we focus on a simple interest rate rule that captures features of the inflation-targeting regime in emerging markets and do not address optimal monetary policy, along the lines of Schmitt-Grohé and Uribe (2007) or Corsetti et al. (2010).

The literature on sovereign default has recently turned to questions raised by nominal rigidities. Several papers have considered environments with defaultable sovereign debt and downward rigidity of nominal wages. Na et al. (2018) first introduced this friction in a model of sovereign debt and emphasize that exchange rate pegs are costly because they prevent devaluations that would adjust real wages to their efficient level. Optimal policy in their environment delivers the joint incidence of devaluations and defaults. Bianchi and Mondragon (2018) find that downward ridigity of nominal wages also increases the incidence of self-fulfilling debt crises for economies that lack monetary independence. Bianchi et al. (2018) show that this environment produces the procyclical fiscal policies present in emerging markets, which results from a tradeoff between the ability of fiscal stimulus to stimulate demand but possibly increase default risk. Our project shares the emphasis in these papers on the interaction between sovereign risk and monetary frictions but differs in important ways. First, price frictions in our model arise from optimal price setting by firms, as in the standard textbook New Keynesian model, whereas these papers impose directly that nominal wages are downwardly rigid. Second, our modeling of monetary policy focuses on a positive theory that resembles the practice of many emerging markets central banks, which set interest rates to target inflation.[4]

[4]The statutory objectives given by the legislature to central banks in inflation-targeting emerging markets center

A large literature, following Calvo (1988), studies the incentives of governments to default on debt that is denominated in local currency with inflation and how these incentives increase the probabilities of self-fulfilling crises. Aguiar et al. (2013) analyze the trade-offs generated by monetary policy credibility in a dynamic continuous-time model of self-fulfilling default. They show that strong monetary policy credibility helps suppress self-fulfilling debt crises but hinders the benefits of state-contingent payments induced by inflation. Concerning the multiplicity of equilibria and the role inflation can play in selecting among them, Corsetti and Dedola (2016) focus on unconventional monetary policy whereas Bacchetta et al. (2018) analyze how interest rate rules can be used to prevent the self-fulfilling crises in the environment of Lorenzoni and Werning (2019). Hur et al. (2018) and Sunder-Plassmann (2018) also study the interaction between inflation and defaultable debt denominated in local currency. The former considers exogenous inflation, for given covariance structures with fundamentals, whereas the latter builds on a cash-and-credit model with a constant money supply. Nuno and Thomas (2019) build a continuous-time model with local currency debt and default and a discretionary choice of inflation, whereas Engel and Park (2019) analyze how default and inflation incentives shape the composition of sovereign debt between local and foreign currency. In contrast with these papers, we emphasize the joint dynamics of endogenous inflation and country risk, with a monetary authority that uses interest rate rules, hence abstracting from the incentives of using monetary policy to inflate away the debt.

Finally, our model's implications for the terms of trade, nominal and real exchange rates, and centralized borrowing raise a natural comparison with the work on capital controls and exchange rates in small open economies, such as Farhi and Werning (2012), Fanelli (2017), and Devereux et al. (2019).

2 Model

We consider a small open economy composed of households, final good producers, intermediate goods firms, a monetary authority, and a fiscal government. There are three types of goods: imported, domestic intermediate varieties, and final domestic goods. The final good is produced using all varieties of differentiated intermediate goods and consumed by both domestic and foreign households. Each intermediate good variety is produced with labor.

Foreign demand for domestic goods (export demand) is given by

$$X_t = \left(\frac{P_t^d}{\varepsilon_t P_t^*} \right)^{-\rho} \xi,$$

on controlling inflation. For example, the only objective for the Monetary Policy Committee of the Central Bank of Brazil is the achievement of the inflation targets set by the National Monetary Council.

where P_t^* is the price of foreign goods in foreign currency, ξ is the level of overall foreign demand, and ρ is the trade elasticity. In the previous equation, P_t^d is the price of domestic goods in local currency and ε_t is the nominal exchange rate, with an increase in ε_t representing a depreciation of the home currency. We assume that the law of one price holds, so we can write the price of the foreign good in local currency as

$$P_t^f = \varepsilon_t P_t^*.$$

The terms of trade e_t is equal to

$$e_t = \frac{P_t^f}{P_t^d} = \frac{\varepsilon_t P_t^*}{P_t^d}. \tag{1}$$

The foreign demand for domestic goods is a function of the terms of trade and the level of overall foreign demand ξ:

$$X_t = e_t^\rho \xi. \tag{2}$$

We normalize the foreign price P_t^* to one in all periods, which implies zero inflation abroad.

2.1 Households

Households consume domestic goods C_t and foreign goods C_t^f and supply labor N_t. Their preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t,\ C_t^f, N_t), \tag{3}$$

where the per-period utility function is given by

$$u(C_t,\ C_t^f, N_t) = \log\left[H(C_t, C_t^f)\right] - \frac{N_t^{1+1/\zeta}}{1+1/\zeta}.$$

and $H\left(C_t, C_t^f\right)$ is the CES composite

$$H(C_t, C_t^f) = \left(\theta C_t^{\frac{\rho-1}{\rho}} + (1-\theta)(C_t^f)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}.$$

The parameter θ controls the share of imports in consumption, and ρ is again the trade elasticity.

Taking prices as given, the households choose consumption, labor supply, and holdings of domestic bonds B_t^d. These domestic bonds are denominated in local currency and can only be traded by domestic households. Households own intermediate goods firms and receive their profits Ψ_t. They also earn labor income and receive government transfers T_t. Their budget constraint is given by

$$P_t^d C_t + (1+\tau_f) P_t^f C_t^f + q_t^d B_{t+1}^d \leq W_t N_t + B_t^d + \Psi_t + T_t$$

where q_t^d is the nominal prices of domestic discount bonds and τ_f is a constant consumption tax that households pay on imports. It is convenient to write the budget constraint in real terms, in domestic good units, deflating by the price index P_t^d:

$$C_t + (1+\tau_f)e_t C_t^f + q_t^d b_{t+1}^d \leq w_t N_t + \frac{b_t^d}{\pi_t} + \psi_t + t_t. \tag{4}$$

where real domestic bonds are $b_{t+1} = B_{t+1}^d / P_t^d$, the real wage is $w_t = W_t / P_t^d$, real profits and transfers are $\psi_t = \Psi_t / P_t^d$, $t_t = T_t / P_t^d$, and the gross domestic goods inflation, hereafter *inflation*, is $\pi_t = P_t^d / P_{t-1}^d$. We can characterize the representative consumer's choices with the following optimality conditions:

$$-\frac{u_{N,t}}{u_{c,t}} = \frac{W_t}{P_t^d} = w_t, \tag{5}$$

$$\frac{u_{c^f,t}}{u_{c,t}} = (1+\tau_f)e_t, \tag{6}$$

$$q_t^d = \beta \mathbb{E}_t \left[\frac{u_{c,t+1}}{u_{c,t}} \frac{1}{\pi_{t+1}} \right]. \tag{7}$$

The domestic nominal interest rate is the yield of the discount bond price $i_t \equiv 1/q_t^d$.

2.2 Final Goods Producers

The final good is produced using a measure of differentiated varieties, intermediate goods y_{it}, $i \in [0,1]$ under perfect competition,

$$Y_t = \left[\int_0^1 y_{it}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{8}$$

where η is the elasticity of substitution between intermediate goods. Let the prices of intermediate goods be $\{p_{it}\}$. The profit maximization problem of the final good producer is

$$\max P_t^d \left[\int_0^1 y_{it}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} - \int_0^1 p_{it} y_{it} di,$$

inducing a demand function,

$$y_{it} = \left(\frac{p_{it}}{P_t^d} \right)^{-\eta} Y_t, \tag{9}$$

and a domestic price index,

$$P_t^d = \left[\int_0^1 p_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}. \tag{10}$$

2.3 Intermediate Goods Producers

Each differentiated intermediate good is produced with labor n_{it}, using a constant returns to scale production function, subject to aggregate productivity shocks z_t:

$$y_{it} = z_t n_{it}. \tag{11}$$

Intermediate goods firms are monopolistically competitive and set the prices for their products, taking as given the demand system (9). These firms, however, face price-setting frictions in that they have to pay a quadratic adjustment cost when they increase their prices away from the target inflation rate $\overline{\pi}$, as in Rotemberg (1982). Taking as given the wage rate W_t and the final good price P_t^d, an intermediate firm i chooses labor and its price to maximize the present discounted value of profits,

$$\max_{\{p_{it},n_{it}\}} \mathbb{E}_0 \sum_t Q_{t,0} \left\{ p_{it}y_{it} - (1-\tau)W_t n_{it} - \frac{\varphi}{2}\left(\frac{p_{it}}{p_{it-1}} - \overline{\pi}\right)^2 P_t^d Y_t \right\},$$

subject to the production function, where $Q_{t,0}$ is the stochastic discount factor of households, denominated in units of domestic goods, and τ is a labor subsidy.[5]

Using the households' stochastic discount factor and the production function, this problem is

$$\max_{\{p_{it}\}} \mathbb{E}_0 \sum_t \beta^t \frac{u_{c,t}}{u_{c,0}} \frac{P_0^d}{P_t^d} \left\{ p_{it}y_{it} - \frac{(1-\tau)W_t}{z_t} y_{it} - \frac{\varphi}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - \overline{\pi}\right)^2 P_t^d Y_t \right\}.$$

The first-order condition for each firm, after imposing symmetry across all firms ($p_{it} = P_t^d$), results in

$$(1-\tau)\frac{w_t}{z_t} = \frac{\eta-1}{\eta} + \frac{1}{\eta}\left\{ \varphi\left(\pi_t - \overline{\pi}\right)\pi_t - \mathbb{E}_t\left[\beta \frac{u_{c,t+1}}{u_{c,t}} \frac{Y_{t+1}}{Y_t} \varphi\left(\pi_{t+1} - \overline{\pi}\right)\pi_{t+1}\right]\right\}. \tag{12}$$

This equation is a standard New Keynesian Phillips Curve (NKPC) that relates inflation to a measure of contemporaneous unit cost, $(1-\tau)w_t/z_t$, and expected inflation.

2.4 The Monetary Authority

The monetary authority conducts policy using a nominal interest rate rule. In the baseline model, the nominal rate i responds to the deviation of inflation from target, π_t relative to $\overline{\pi}$, and to

[5] We follow the standard practice in the New Keynesian literature by introducing a constant subsidy designed to alleviate inefficiencies induced by market power.

monetary shocks m_t such that

$$i_t = \bar{i}\left(\frac{\pi_t}{\overline{\pi}}\right)^{\alpha_p} m_t, \tag{13}$$

where the intercept $\bar{i}$ satisfies the steady-state condition $\bar{i} = \overline{\pi}/\beta$. In our model the interest rate rule targets domestic goods inflation, which Galí and Monacelli (2005) find most closely approximates the optimal policy in their setting.[6]

2.5 Government and External Debt

The fiscal authority, the government, engages in international borrowing using long-term bonds denominated in foreign currency. To keep long-term debt tractable, we consider random maturity bonds, as in Hatchondo and Martinez (2009). The bond is a perpetuity that specifies a price q_t and a quantity ℓ_t such that the government receives $q_t\ell_t$ units of foreign goods in period t. The following period, a fraction δ of the debt matures and, conditional on not defaulting, the government's debt is the sum of the outstanding debt and the new issuance ℓ_t such that $B_{t+1} = (1-\delta)B_t + \ell_t$. Each unit of debt calls for a payment of $r^* + \delta$ every period. We normalize the debt service payment of the bond to $r^* + \delta$ so that the default-free bond price for this instrument equals 1.

As in standard New Keynesian models, we let the fiscal government subsidize employment and tax foreign consumption at time-invariant rates τ and τ_f to correct the markup in goods markets and allow for a static optimal tariff on exports in steady state.

The government transfers T_t, the net receipts from its operations, to households. Letting B_t denote the outstanding foreign currency debt of the government, the budget constraint in local currency is

$$T_t + \tau W_t N_t = \varepsilon_t \left[q_t(B_{t+1} - (1-\delta)B_t) - (r^* + \delta)B_t\right] + \tau_f P_t^f C_t^f, \tag{14}$$

where the net capital inflow from debt operations is multiplied by the nominal exchange rate ε_t to convert it to domestic currency. Using the definition of the terms of trade (1), the government budget constraint in terms of domestic goods is

$$t_t + \tau w_t N_t = e_t[q_t(B_{t+1} - (1-\delta)B_t) - (r^* + \delta)B_t] + \tau_f e_t C_t^f. \tag{15}$$

Every period the government experiences an enforcement shock ν_t and decides whether to default D_t on its outstanding debt. Whenever it chooses not to default, it can pick the level of debt next period B_{t+1}. Default has the benefit that it eliminates all the debt obligations, but it is costly in terms of utility, productivity, and financial market access. If the government

[6] We also analyze, as an extension of our model, interest rate rules that contain an output gap term in addition to inflation and find that the results from that economy are very similar. We abstract from the case with a consumer price inflation target, for computational limitations, as it would require keeping track of the previous period's terms of trade as an additional state variable.

defaults, $D_t = 1$, the economy suffers a one-time utility cost ν_t and a reduction in productivity to $z_t^d(z_t) \leq z_t$. In addition, a defaulting country is excluded from international financial markets for a random length of time. With probability ι the economy reenters financial markets with zero debt obligations.

The government's objective is to maximize the present discounted value of the flow utility derived from consumption and labor by the representative household, $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_g^t u(C_t, C_t^f, N_t)$. The government's discount factor β_g can differ from the households' β. The government borrows from competitive international lenders that discount the future at a foreign currency rate r^*. The bond price is such that they break even in expectation, thus receiving compensation for any expected losses from default:

$$q_t = \frac{1}{1+r^*} \mathbb{E}_t \left[(1 - D_{t+1})(r^* + \delta + (1-\delta) q_{t+1}) \right]. \tag{16}$$

In states where the government does not default, $D_{t+1} = 0$, each unit of the discount bond makes a payment $r^* + \delta$, and the fraction that does not mature, $1 - \delta$, has market value $(1-\delta)q_{t+1}$. If the country does default in a state next period, the associated payoff for lenders is zero. We define the government *spread* as the difference in the yield-to-maturity of the bond and the international rate r^*, which equals

$$\text{spread}_t = (r^* + \delta) \left(\frac{1}{q_t} - 1 \right).$$

3 Equilibrium

We consider a Markov equilibrium where the government takes into account that its default and borrowing policies affect the allocations of the private equilibrium and the monetary authority's response. In the beginning of the period, the aggregate state of this economy includes the productivity, monetary, and enforcement shocks $s = \{z, m, \nu\}$ as well as the government debt B. The government chooses its policies, whether to default D, and how much to borrow B'. The private and monetary equilibrium depends on both the state $\{s, B\}$ and the government's choices because they affect government transfers $t(S)$. Let $S = \{s, B, D, B'\}$ be the end of the period state that is relevant for the private equilibrium.

Definition 1. *Private and Monetary Equilibrium. Given state $\{S\}$, the government policy functions for default $D'(s', B') = H_D(s', B')$, borrowing $B''(s', B') = H_B(s', B')$, and transfer function $t(S)$ consistent with the government budget constraint, the symmetric private and monetary equilibrium consists of*

- *Households' policies for domestic goods consumption $C(S)$, foreign goods consumption $C^f(S)$, labor $N(S)$, and domestic debt $B^d(S)$,*

- *Intermediate and final goods firms' policies for labor $n(S)$, prices $p^d(S)$, and final domestic goods' output $Y(S)$ and exports $X(S)$,*

- *The wage rate $w(S)$, domestic nominal interest rate $i(S)$, aggregate domestic price $P^d(S)$, inflation $\pi(S)$, and the terms of trade $e(S)$*

such that: (i) *the policies for households satisfy their budget constraint (4) and optimality conditions (5), (6), (7);* (ii) *the policies of intermediate and final goods firms satisfy their optimization problem (8), (9), (11), and (12);* (iii) *export demand (2) is satisfied;* (iv) *the nominal interest rate satisfies the monetary authority's interest rate rule (13); and* (v) *labor, domestic goods, and domestic bond markets clear, and the balance of payments condition is satisfied.*

The labor market clears so that labor demanded by firms equals labor supplied by households $n = N$. Domestic bonds are in zero net supply in the economy, reflected in the market clearing condition $B^d = 0$. The resource constraint for domestic goods requires that domestic final goods' output equals domestic consumption and exports net of the adjustment costs,

$$C(S) + X(S) + \frac{\varphi}{2}(\pi - \overline{\pi})^2 Y(S) = Y(S) \tag{17}$$

where aggregate output $Y(S) = z\, N(S)$.

The balance of payments condition requires that net exports equal net capital outflows, which here equal the government transfer, net of the labor subsidy,

$$X(S) - e(S)C^f(S)(1+\tau_f) = t(S) - \tau w(S)N(S). \tag{18}$$

The Monetary Wedge. The presence of price rigidities leads to inefficient use of labor, as monopolistic firms set time-varying markups. We will make use of a *monetary wedge* to measure these distortions in production, defined as

$$1 + \text{monetary wedge} \equiv \frac{z}{w(S)} = \frac{z u_C}{u_N}. \tag{19}$$

This wedge captures deviations from production efficiency and depends on the dynamics of current and future inflation, as seen in the NKPC equation (12).

3.1 Government Recursive Formulation

We now describe the recursive problem of the government, which borrows in international financial markets and can default. The government chooses its policies internalizing that they affect the private and monetary equilibrium.

The government trades long-term discount bonds denominated in foreign currency with international lenders and can default on its debt. The government starts with debt B and decides on default D and new borrowing B', which carries price $q(s, B')$. The government internalizes that its choices of borrowing and default alter the private equilibrium. The bond price is an endogenous function that depends on the amount of borrowing B' and the shocks s, in a way that compensates lenders for default risk. These risk-neutral lenders discount the future at the international interest rate r^*. The break-even condition for them implies that the bond price schedule satisfies

$$q(s, B') = \frac{1}{1+r^*}\mathbb{E}_{s'|s}\left[(1 - H_D(s', B'))(r^* + \delta + (1-\delta)q(s, H_B(s', B'))\right], \tag{20}$$

where $H_D(s', B')$ and $H_B(s', B')$ are the default and borrowing policy functions of the government.

As is standard in New Keynesian models, we set the labor subsidy $(1-\tau) = \frac{\eta-1}{\eta}$ to offset the market power of firms in the steady state and set $(1+\tau_f) = \frac{\rho}{\rho-1}$ to be equal to the static optimal tariff.[7]

By consolidating the equilibrium conditions and the government budget constraint, the private and monetary allocations can be summarized with the decision rules for domestic and foreign consumption $\{C(S), C^f(S)\}$, labor $N(S)$, inflation $\pi(S)$, the nominal interest rate $i(S)$, and the terms of trade $e(S)$, which satisfy the following system of dynamic equations:

$$C(S) + e(S)^\rho \xi = \left[1 - \frac{\varphi}{2}\left(\pi(S) - \overline{\pi}\right)^2\right] zN(S) \tag{21}$$

$$e(S)^\rho \xi = e(S)[C^f(S) + (r^* + \delta)B - q(s, B')(B' - (1-\delta)B)] \tag{22}$$

$$\frac{u_{c^f}(S)}{u_c(S)} = \frac{\rho}{\rho - 1} e(S) \tag{23}$$

$$u_c(S) = \beta\, i(S) M(s, B') \tag{24}$$

$$i(S) = \bar{i}\left(\frac{\pi(S)}{\overline{\pi}}\right)^{\rho_p} m \quad \text{with} \quad \bar{i} = \overline{\pi}/\beta \tag{25}$$

$$\frac{1}{z}\frac{u_n(S)}{u_c(S)} = 1 + \frac{1}{\eta - 1}\varphi\left(\pi(S) - \overline{\pi}\right)\pi(S) - \frac{1}{u_c(S) zN(S)} F(s, B'), \tag{26}$$

where $q(s, B')$ satisfies (20) and the functions $M(s, B')$ and $F(s, B')$ are the expectations in the

[7] By setting this tariff, we neutralize the potential incentive of the government to use debt to exert market power with respect to the downward-sloping demand for the country's exports.

households' Euler condition and the firms' pricing condition (NKPC), respectively, given by

$$M(s, B') = \mathbb{E}_{s'|s} \frac{u_c(S')}{\pi(S')} \tag{27}$$

$$F(s, B') = \frac{\beta}{\eta - 1} \mathbb{E}_{s'|s} \left[z' N(S') u_c(S') \varphi \left(\pi(S') - \overline{\pi} \right) \pi(S') \right], \tag{28}$$

where the future state is denoted by $S' = (s', H_D(s', B'), H_B(s', B'))$.

These equilibrium conditions are analogous to those arising from the standard New Keynesian small open economy in Galí and Monacelli (2005). The difference in our model is that the government understands that its choice of borrowing B' and default D, elements of S, affect the equilibrium. The equilibrium depends on government choices because current *and* future allocations and prices, as characterized by the system of equations (21) to (26), depend on B' and D.

Moreover, the government's choices determine next period's state variables, which means that future allocations and prices also depend on the government's current choices. These future effects are encoded in the functions $q(s, B')$, $M(s, B')$, and $F(s, B')$, which are the bond price function, the households' expected marginal utility function, and the firms' expected inflation function, respectively. These functions are the marginal changes associated with a change in the B' choice, taking as given future government policies $H_D(s', B')$ and $H_B(s', B')$.

We can now set up the recursive problem of the government, following the quantitative sovereign default literature. The government can choose to default in any period. Let $V(s, B)$ be the value with the option to default, with $s = \{z, m, \nu\}$. After default, the debt B is eliminated, productivity is reduced to $z^d(z)$, and the government suffers the default cost ν. The value of the option to default is then

$$V(s, B) = \max_{D \in \{0,1\}} \left\{ (1 - D) W(z, m, B) + D \left[W^d(z^d, m) - \nu \right] \right\}, \tag{29}$$

where $D = 1$ in default and 0 otherwise, $W(z, m, B)$ is the payoff from repaying debt, and $W^d(z^d, m) - \nu$ is the payoff from defaulting. Specifically, the value of repaying is

$$W(z, m, B) = \max_{B'} \left\{ u(C, C^f, N) + \beta_g \mathbb{E}_{s'|s} V(s', B') \right\} \tag{30}$$

subject to the private and monetary equilibrium, which is characterized by conditions (21) through (26) and the break-even condition for the bond price schedule (20).

After default, with probability ι, the government regains access to the international financial markets with zero debt. The defaulting value W^d net of the enforcement cost is given by

$$W^d(z^d, m) = \left\{ u(C, C^f, N) + \beta_g \mathbb{E}_{s'|s} \left[\iota V(s', 0) + (1-\iota) W^d(z^{d\prime}, m') \right] \right\} \tag{31}$$

subject to the private and monetary equilibrium characterized by conditions (21) through (26) with $B = B' = 0$, productivity $z = z^d(z)$, and where the expectations in the functions (27) and (28) are taken also over the probability to regain access to international financial markets ι.

It is convenient to write the default decision of the government as a cutoff rule based on the default cost ν. Given that default costs are i.i.d., the default decision $D(s, B)$ can be characterized by a cutoff default cost $\nu^*(z, m, B)$ at which the repayment value is equal to the default value such that

$$\nu^*(z, m, B) = W^d(z^d, m) - W(z, m, B), \tag{32}$$

and the sovereign is indifferent between the two options. Then $D(s, B) = 1$, whenever $\nu \leq \nu^*(z, m, B)$ and zero otherwise. Let Φ be the cumulative distribution of ν. The default probability is therefore given by $\Phi(\nu^*(z, m, B))$.

We now define the recursive equilibrium for the economy.

Definition 2. *Equilibrium. Given the aggregate state $\{s, B\}$, a recursive equilibrium consists of government policies for default $D(s, B)$ and borrowing $B'(s, B)$, and government value functions $V(s, B)$, $W(z, m, B)$, and $W^d(z^d, m)$ such that*

- *Taking as given future policy and value functions $H_D(s', B')$, $H_B(s', B')$, $V(s', B')$, and $W(z', m', B')$, government policies for default and borrowing $D(s, B)$ and $B'(s, B)$ solve the government's optimization problem.*

- *Government policies and values are consistent with future policies and values.*

3.2 Optimal Government Borrowing

We characterize the optimal borrowing decision arising from the government's problem. The government chooses its borrowing taking into account the effect that it has on the private equilibrium, both contemporaneously and in the future. We manipulate the government's problem and derive its optimality condition for borrowing to illustrate the forces at play. In this derivation we have assumed that the functions in the government problem are differentiable.[8] Optimal borrowing satisfies the following Euler equation:

$$u_{C^f} \left[q + \frac{dq}{dB'} \left(B' - (1-\delta) B \right) \right] (1 - \tau_m^X) - \tau_m^C = \beta_g \mathbb{E} \left\{ (1 - D') u'_{C^f} \left[r^* + \delta + (1-\delta) q' \right] \left(1 - \tau_m^{X\prime} \right) \right\} \tag{33}$$

[8]We do not require this assumption for the computation of the model, nor do we employ the Euler equation derived in this section.

where τ_m^X and τ_m^C are *borrowing wedges from monetary frictions* that reflect the constraints of the domestic Euler (24) and the firms' NKPC condition (26). Let κ be the Lagrange multiplier on the domestic Euler condition (24) and γ be the Lagrange multiplier on the NKPC condition (26). The borrowing wedges τ_m^X and τ_m^C are functions of these multipliers and satisfy

$$\tau_m^X = G^X u_C\, \kappa + G^X \left(1 + \frac{1}{\eta - 1}\varphi(\pi - \bar{\pi})\pi\right)\gamma$$
$$\tau_m^C = \beta i \frac{\partial M}{\partial B'}\kappa + \frac{1}{u_C Y}\,\frac{\partial F}{\partial B'}\gamma,$$

where the function G^X depends on the allocations $\{C(S), C^f(S), N(S), \pi(S), i(S), e(S)\}$ and is positive.[9] The functions M and F are the future expected marginal utility of domestic consumption and future expected inflation given in (27) and (28). Appendix A contains explicit derivations.

Note that without monetary frictions, when both domestic Euler (24) and NKPC (26) conditions are slack $\kappa = \gamma = 0$, the borrowing wedges are zero, $\tau_m^X = \tau_m^C = 0$. The borrowing wedge τ_m^C depends on $\partial M(s, B')/\partial B'$ and $\partial F(s, B')/\partial B'$, which are the derivatives of the expected marginal utility and firms' expected inflation with respect to B', respectively. These derivatives are positive in our quantitative model because higher debt increases marginal utility and inflation.

Positive borrowing wedges reduce the incentive to borrow, as captured in equation (33). As we will see, contractionary monetary frictions tend to increase borrowing wedges because they raise the multiplier of the domestic Euler κ and increase the marginal utility of consumption, raising the first term in τ_m^X and τ_m^C. The contribution of the second terms in these wedges, arising from NKPC constraint γ, is more complicated. Contractionary monetary frictions tend to decrease γ and inflation at the same time, which leads to an ambiguous effect on τ_m^X and a negative effect on τ_m^C. For example, an increase in nominal rates from the monetary shock raises κ, which leads to a decline in consumption and an increase in the current marginal utility of consumption. As a result, the wage falls, which generates a lower γ and lower inflation. When the κ effect dominates, raising nominal rates increases the borrowing wedges and lowers the benefit of borrowing.

These effects can be derived cleanly in the extreme case of perfectly rigid inflation. As we

[9] The function $G^X(S) = \left(\frac{1}{\rho}\frac{1}{\rho^{\frac{1}{\rho-1}}e^{1-\rho}u_C C C^f + u_C C + u_{C^f} C^f} + \frac{\rho-1}{\rho}\right)$, where the allocations e, C^f, C are functions of S.

show in Appendix A, in this case $\kappa = \left(1 - \frac{u_N}{zu_C}\right)\frac{1}{G}$, and borrowing wedges equal

$$\tau_m^X = \left(1 - \frac{u_N}{zu_C}\right)\frac{u_C G^X}{G}, \tag{34}$$

$$\tau_m^C = \left(1 - \frac{u_N}{zu_C}\right)\frac{\partial M}{\partial B'}\frac{\beta i}{G}, \tag{35}$$

where the function G depends on the allocations and is always positive.[10] Expressions (34) and (35) make it explicit that the borrowing wedges are positive if the monetary wedge, defined in (19), is larger than 0 and if the expected marginal utility of domestic consumption increases with B', $\partial M/\partial B' \geq 0$.

We can now turn to the trade-offs for borrowing in equation (33). The government's borrowing incentives are affected by three major forces. The first is the standard force to smooth imported consumption. It is useful to compare our model's Euler equation (33) with an undistorted Euler equation that arises in the standard Galí and Monacelli (2005) model without sovereign borrowing. This undistorted international Euler equation with long-term bonds is

$$q\, u_{C^f} = \beta \mathbb{E}\left[u'_{C^f}(r^* + \delta + (1-\delta)q')\right]. \tag{36}$$

Optimal borrowing in this set up smooths the marginal utility of foreign consumption against shocks and is used to achieve the right tilting of consumption over time given q and β. This force also shapes borrowing in our model.

The second force that affects the government's borrowing incentive is the endogenous bond price schedule q and the legacy debt $(1-\delta)B$. Because of default risk, bond prices decrease with borrowing to reflect higher default risk, $\frac{\partial q}{\partial B'} \leq 0$. Also, a higher legacy debt $(1-\delta)B$ increases borrowing incentives because lower prices dilute this debt, $-\frac{\partial q}{\partial B'}(1-\delta)B \geq 0$. This time inconsistency in debt issuances and dilution incentives have been studied in the sovereign default literature in Chatterjee and Eyigungor (2012) and Hatchondo et al. (2016). Such dilution incentives are one potential source of overborrowing in our model. Also, the government may discount future heavier than the households $\beta_g \leq \beta$, for example due to the political turnover explored in Cuadra and Sapriza (2008) and Hatchondo et al. (2009). Lower β_g also induces excessive borrowing, as discussed by Aguiar et al. (2019).

The third force that affects government choices works through the borrowing wedges and is unique to our model with sovereign risk and monetary frictions. Positive borrowing wedges τ_m^X and τ_m^C weaken the marginal benefit from borrowing B' and are associated with contractionary monetary frictions and large monetary wedges.

The borrowing wedge τ_m^X arises because B' affects *current allocations* through capital flows. A

[10] The function $G(S) = \left[\frac{1}{\rho}\frac{\rho\frac{1}{\rho-1}e^{1-\rho}C^f+1}{\rho\frac{1}{\rho-1}e^{1-\rho}u_C C C^f + u_C C + u_{C^f}C^f} + \frac{\rho-1}{\rho}\right]$, where the allocations e, C^f, and C are functions of S.

positive τ_m^X, from contractionary monetary frictions, lowers the valuation for the marginal utility of foreign consumption and tends to decrease capital flows and imported consumption C^f. This effect is useful because a decline in C^f exerts depreciation pressures on the terms of trade, which boosts exports X. Boosting exports ameliorates the contractionary monetary frictions. The future borrowing wedge $\tau_m^{X'}$ also reflects that capital flows the next period will depend on B' and will alter monetary frictions in the future.

The borrowing wedge τ_m^C arises because B' affects *future allocations*, as it determines the future state of debt, and the government exploits these dependencies to alleviate frictions today. High B' tends to decrease future domestic consumption and hence increase the marginal utility of expected consumption. With contractionary monetary frictions, a decrease in borrowing B' is useful because it increases expected consumption and, through the domestic Euler condition, increases current domestic consumption C as well and ameliorates the friction.

These monetary frictions lower the benefit of borrowing and help to discipline the government's incentive to overborrow. The frictions themselves are in turn shaped by the government's debt and default risk. We explore these interactions in more detail in the following section.

4 Monetary Policy and Sovereign Risk in a Simple Example

We develop a simple example to analyze theoretically the interaction between monetary frictions and default risk. We show two main results. First, we establish that default risk amplifies monetary frictions and leads to a larger monetary wedge. Second, we show that monetary frictions discipline the government's borrowing incentives and reduce default risk, compared with a model without monetary frictions.

We consider a simplified version of our model with two periods. In the first period, prices are perfectly rigid, at a fixed level. In the second period firms can freely choose their prices without any cost. The nominal interest rate in period 1 is set at i. We assume the government starts with no debt and the only shocks are default costs ν, which hit the economy in the second period and shape the default decision. Productivity in the first period is 1 and in the second period is z, which we assume is known in period 1.

The government issues one-period bonds B in international markets in period 1. In period 2, the government observes the default cost ν after which it decides whether to repay or default. The default cost ν is drawn from a normal distribution with mean $\overline{\nu}$ and standard deviation σ_ν. Let the cdf and pdf of ν be Φ and ϕ, respectively. Default in period 2 induces a productivity loss $z_d \leq z$. We assume that preferences are separable between domestic and foreign goods and are linear with respect to foreign goods consumption such that

$$u(C, C^f, N) = \log C + C^f - \frac{N^{1+\frac{1}{\zeta}}}{1 + 1/\zeta}. \tag{37}$$

We start by analyzing the behavior in the second period. Households choose consumption and labor supply, taking as given the terms of trade and wage, as well as the government's policies. Firms choose labor demand, taking as given the wage. The resulting allocation depends on B and the default decision of the government. Conditional on repaying, the private equilibrium $\{C_2(B), C_2^f(B), N_2(B), e_2(B)\}$ satisfies the following conditions:

$$C_2 + e_2^\rho = zN_2, \qquad e_2^\rho = e_2\left(C_2^f + B\right), \qquad C_2 = \frac{\rho}{\rho-1}e_2, \qquad N_2^{\frac{1}{\zeta}}C_2 = z.$$

These conditions are the domestic resources constraint, the balance of payments, the relative demand, and the efficient labor market condition, respectively, jointly arising from the behavior of households and firms under flexible prices. Note that in this example, the terms of trade e depend only on the level of domestic goods consumption C_2. The private equilibrium allocation in default $\{C_{2d}, C_{2d}^f, N_{2d}, e_{2d}\}$ satisfies the same conditions, with $B = 0$ and productivity z_d.

The government makes its default decision to maximize utility, $\max\left\{W_2(B), W_2^d - \nu\right\}$, where $W_2(B) = u\left(C_2(B), C_2^f(B), N_2(B)\right)$ and $W_2^d = u\left(C_{2d}, C_{2d}^f, N_{2d}\right)$. We show that for any given debt level B, there exists a cutoff $\nu^*(B)$ such that the government defaults if and only if default costs are low enough, $\nu \leq \nu^*(B)$. We can characterize the default cost cutoff ν^* as

$$\nu^*(B) = \mathcal{U}(C_{2d}, z_d) - \mathcal{U}(C_2, z) + B$$

with $\mathcal{U}(C_2, z) \equiv u(C_2, [C_2(\rho-1)/\rho]^{\rho-1}, (z/C_2)^\zeta)$, and consumption in the second period, with and without default, is implicitly determined by $C_2 + [C_2(\rho-1)/\rho]^\rho = z\,(z/C_2)^\zeta$ and $C_{2d} + [C_{2d}(\rho-1)/\rho]^\rho = z_d\,(z_d/C_{2d})^\zeta$.

The default probability is given by $\Phi(\nu^*(B))$. The bond price schedule compensates international lenders for default risk,

$$q(B) = \frac{1}{1+r^*}[1 - \Phi(\nu^*(B))], \tag{38}$$

and the sensitivity of this price to borrowing is

$$\frac{\partial q(B)}{\partial B} = -\frac{1}{1+r^*}\phi(\nu^*(B)). \tag{39}$$

Lemma 1. *Default risk $\Phi(\nu^*)$ increases with debt B, decreases with second-period productivity z, and decreases with the mean default cost $\overline{\nu}$.*

See Appendix B for the proof. These results characterize the probability of default in the second period $\Phi(\nu^*)$. This default probability matters for the allocations in period 1, including the monetary frictions, to which we now turn.

We consider a first-period problem with pricing frictions. To that end, we assume that future inflation is $\tilde{\pi}$. Note that this level of inflation does not generate any resource losses or distortions in the second period and is consistent with the firms' optimization problem for period 2. In fact, firms in period 2 are indifferent toward inflation because it does not alter their period 2 profits. The private equilibrium in period 1 $\{C_1(B), C_1^f(B), N_1(B), e_1(B)\}$ depends on the government's borrowing B and satisfies the resource constraint, the relative demand condition, the following balance of payments condition

$$e_1^{\rho} = e_1 \left[C_1^f - q(B)B \right], \tag{40}$$

and the domestic Euler condition

$$\frac{1}{C_1} = \frac{\beta i}{\tilde{\pi}} \left[\frac{1 - \Phi(\nu^*(B))}{C_2} + \frac{\Phi(\nu^*(B))}{C_{2d}} \right]. \tag{41}$$

With pricing frictions and an arbitrarily given i in the first period, the efficient labor market condition $u_C/u_N = 1$ is not satisfied in general. Instead the equilibrium is shaped by the domestic Euler condition (41), which links the monetary policy i and future default risk $\Phi(\nu^*(B))$ to current domestic consumption. In the following proposition, we show that high default risk amplifies monetary frictions and increases the monetary wedge.

Proposition 1. *An increase in default risk $\Phi(\nu^*)$ increases the monetary wedge $u_{C_1}/u_{N_1} - 1$.*

See Appendix B for the proof. A higher default risk increases the future marginal utility of consumption since the right-hand-side expectation in the domestic Euler equation places more weight on default states in which consumption is low. We show that consumption is lower in default, $C_{2d} \leq C_2$, because of lower productivity, $z_d \leq z$. To satisfy the domestic Euler equation (41), the current marginal utility of consumption has to increase, and hence domestic consumption in period 1, C_1, declines. The lower domestic demand C_1 leads to a real appreciation, e_1 decreases, through the relative demand condition, which further reduces export demand $X_1 = e_1^{\rho}$. Labor is lower because of lower demand in both domestic and export markets. Hence, the monetary wedge increases, owing to lower labor supply and lower domestic consumption.[11]

The government borrows B to maximize utility,

$$\max_B u\left(C_1(B), C_1^f(B), N_1(B)\right) + \beta_g \left\{ [1 - \Phi(\nu^*(B))] W_2(B) + \int_{-\infty}^{\nu^*(B)} (W_2^d - \nu) d\Phi(\nu) \right\},$$

taking as given the private equilibrium and the bond price schedule. Using the expressions for the bond price schedules (38) and (39), we can show that the optimal borrowing $B = B^*$ satisfies

[11] In the next section, we show that in our full model high default risk continues to be associated with an increase in the monetary wedge but with an exchange rate depreciation.

the Euler condition

$$h(\nu^*(B))B + \frac{\tau_m^C(B)}{1-\Phi(\nu^*(B))} = 1 - \beta_g(1+r^*), \tag{42}$$

where $h(\nu)$ is the hazard function $h(\nu) = \phi(\nu)/(1-\Phi(\nu))$. In this example, we have only one borrowing wedge $\tau_m^C(B)$ because of the linearity in preferences over imported goods. We can derive a closed-form solution for the borrowing wedge for any B:

$$\tau_m^C(B) = \left(1 - \frac{u_{N_1}(B)}{u_{C_1}(B)}\right) \frac{[1+(\rho-1)e_1(B)^{\rho-1}]\beta i \phi(\nu^*(B))}{u_{C_1}(B)\tilde{\pi}}(u_{C_{2d}} - u_{C_2}).$$

The monetary friction $\tau_m^C \geq 0$ if and only if the monetary wedge is larger than 0.

To analyze the role of monetary frictions for default, we compare the risk of default in our model with the risk of default in a *flexible* price model without price frictions in either period. These two models have identical equilibria in the second period, given the same level of debt B. In the flexible price model, the private equilibrium in the first period, conditional on not defaulting, $\left\{C_1^{\text{flex}}(B), C_1^{f,\text{flex}}(B), N_1^{\text{flex}}(B), e_1^{\text{flex}}(B)\right\}$ is characterized by conditions similar to those in our model except that the domestic Euler condition (41) is replaced with the efficient labor market condition $u_{N_1^{\text{flex}}} = u_{C_1^{\text{flex}}}$. Note that the default risk cutoff $\nu^*(B)$ and the probability of default $\Phi(\nu^*(B))$ depend on future default decisions and are independent of whether there are pricing frictions in the first period. Let the optimal borrowing and the associated default risk be B^*_{flex} and $\Phi^*_{\text{flex}} = \Phi(\nu^*(B^*_{\text{flex}}))$, respectively. Also define the real interest rate in this economy r^{flex}, based on the domestic Euler condition with C_1^{flex} and Φ^{flex}, which satisfy

$$\frac{1}{C_1^{\text{flex}}} = \beta r^{\text{flex}} \left[\frac{1-\Phi^{\text{flex}}}{C_2} + \frac{\Phi^{\text{flex}}}{C_{2d}}\right]. \tag{43}$$

Real interest rates in the flexible price economy depend on default risk. High default risk Φ^{flex} tends to decrease real rates because domestic consumption in default is low. Optimal borrowing $B = B^*_{\text{flex}}$ in the flexible price model satisfies the following Euler equation without borrowing wedges:

$$h(\nu^*(B))B = 1 - \beta_g(1+r^*). \tag{44}$$

We restrict attention to the case in which the default probability in the flexible model is less than one-half, $\Phi(B^*_{\text{flex}}) < 1/2$.

We compare the equilibrium default risk in our baseline economy with monetary frictions to the default risk in the flexible price model by analyzing Euler equations (42) and (44) with and without borrowing wedges. We focus on the relation between monetary policy, monetary wedges, and default risk. The following assumption sets monetary policy so that the real rate in the economy with price frictions is higher than in the flexible price model.

Assumption 1. $\frac{i}{\tilde{\pi}} \geq r^{flex}$.

When the real interest rate $i/\tilde{\pi}$ equals the real interest rate in the flexible price model r^{flex}, the allocation with price frictions is the same as the flexible one and the monetary wedge equals 0. Under Assumption 1 real interest rates are higher than in the flexible price model and if the government were to choose B^*_{flex}, domestic consumption and labor would be lower than in the flexible economy and the monetary wedge would be higher than 0. When $B \geq B^*_{\text{flex}}$, default risk increases and, according to Proposition 1, such high borrowing induces an even higher monetary wedge. Hence, whenever $B \geq B^*_{\text{flex}}$, the monetary wedge is larger than 0 and the borrowing wedge $\tau^C_m(B) \geq 0$. These forces are summarized in the next lemma.

Lemma 2. *The borrowing wedge $\tau^C_m(B) \geq 0$ for all $B \geq B^*_{flex}$.*

Comparing the Euler equation with price frictions (42) to the one for the flexible price model (44), and noting that the default risk function Φ and its derivatives are the same, the optimal borrowing B^* will be lower than B^*_{flex} as long as $\tau^C_m(B)$ is larger than zero. Hence, as long as the monetary wedge is higher than 0, borrowing and default risk are lower with price frictions than without, $B^* \leq B^*_{\text{flex}}$ and $\Phi^* \leq \Phi^*_{\text{flex}}$.

Proposition 2. *Default risk is lower with price frictions, $\Phi^* \leq \Phi^*_{flex}$.*

See Appendix B for the proof. These results show that default risk worsens monetary frictions and makes monetary policy contractionary, but such forces lower default risk because they reduce the government's incentive to borrow. We have shown these results theoretically in this simple example, but as we show in the next section, these forces are present in our general model as well.

Before moving forward, we discuss the role of the assumptions made for this simple example and how these assumptions compare with our general model. First, the main assumption in this analysis is that monetary policy sets nominal rates higher than in the flexible economy. It turns out that this high nominal rate can be welfare enhancing for households despite generating monetary wedges because it lowers default risk, as shown in Proposition 2. Default risk is excessive for households because they discount the future at a lower rate than the government, $\beta \geq \beta_g$.[12] Nominal rates are high here as an assumption, whereas in the general model, they arise in regions of the state space with high default risk even though nominal rates always respond to inflation. As we will see these elevated nominal rates will also reduce the incentives of the government to borrow. Second, we assume in this example that in the second period firms can freely adjust their prices costlessly, whereas in our general model, firms always face price adjustment costs. This assumption simplifies the comparison of our model with the flexible price

[12] In general, households welfare is hump-shaped as a function of $i/\tilde{\pi}$, reaching its maximum at a level that exceeds r^{flex}.

model, since both face the same default risk and bond price schedule in period 1. The results that high default risk increases the monetary wedge and that default risk is lower with price frictions, however, are also present in the general model on average, although across the state space we will find some interesting non-monotonicities. Third, we have assumed that preferences are linear in imported goods consumption C_f. This assumption eliminates the borrowing wedge τ_m^X from the simple model by eliminating the dependency of the terms of trade on imported goods consumption. In the general model, we find that the effects arising from τ_m^X effects are minor.

5 Quantitative Analysis

We now document key patterns of inflation, nominal rates, and spreads in emerging market data and conduct the quantitative analysis of our model. We describe the parameterization of the model, discuss decision rules and impulse responses, and compare the model's implications with the data and reference models.

5.1 Inflation, Nominal Rates, Output, and Spreads in the Data

	Means		Correlation with Spread		
	CPI Inflation	Govt Spread	CPI Inflation	Domestic Rate	Output
Brazil	5.9	2.6	59	59	−62
Chile	3.0	1.4	30	39	−49
Colombia	5.2	3.2	74	76	−60
Indonesia	6.6	2.8	17	75	−62
Korea	2.6	1.1	44	74	−30
Mexico	4.3	2.3	48	27	−54
Peru	2.8	3.0	50	55	−33
Philippines	3.9	2.9	17	82	−26
Poland	3.0	1.7	59	52	−11
South Africa	5.8	1.9	54	20	−49
Mean	**4.4**	**2.4**	**45**	**58**	−38

Table 1: Emerging Market Inflation Targeters, Key Statistics

Many emerging markets have adopted inflation-targeting regimes as their monetary policy since the early 2000s. This effort has been largely successful in bringing inflation down to single digits.[13] We collect data on inflation, spreads, output, and domestic nominal rates for

[13]See Roger (2009) and Ha et al. (2019) for more details on the implementation and performance of inflation

10 emerging markets that are inflation targeters. The sample of emerging markets consists of those in the JP-Morgan Emerging Market Bond Index. Table 1 reports key statistics on the joint behavior of inflation, spreads, output, and domestic nominal rates. The data start in 2004, by which point all countries considered had adopted inflation targeting for their monetary policy, and run through 2017. Data are quarterly. Because of data availability, we focus on inflation based on the consumer price index (CPI) and compute it as the log difference in the index relative to four quarters prior.[14] The spreads are EMBI-based and are measured as the difference in yields between foreign currency government bonds of these emerging markets and a U.S. government bond of similar maturity. Domestic nominal rates are short-term rates in local currency from either interbank markets or government instruments, the shortest maturity available. Output is the four-quarter difference in log gross domestic product. We highlight several salient features of the data that will inform our quantitative work.

Inflation is low for these inflation-targeting emerging markets. These single-digit inflation patterns contrast sharply with the historical experience of these countries, which have featured several episodes of hyperinflation. Table 1 also shows average EMBI spreads for these emerging markets, which continue to be sizable.

We also report correlations of spreads with inflation, domestic rates, and output. As documented in many studies, spreads are negatively correlated with output for this sample, with an average correlation of -38%. Correlations of spreads with nominal rates are strongly positive, on average 58%. Note that the underlying bonds from which spreads and nominal rates are constructed are in different currencies, and hence these comovements reflect positive correlations between inflation and default risk. The correlation between spreads and inflation is positive, with a sample average of 45%.

5.2 Price Indices and Exchange Rates

It is useful to define relations between the terms of trade, exchange rates, and consumer and producer price indices in our model. Recall that π corresponds to domestic goods inflation, and e is the relative price of foreign goods, the terms of trade. We can derive the CPI as the price of the bundle of domestic and imported consumption goods,

$$P^{\text{CPI}} = \left[\theta^{\rho} {P^{d}}^{1-\rho} + (1-\theta)^{\rho} {P^{f}}^{1-\rho}\right]^{\frac{1}{1-\rho}} = P^{d}\left[\theta^{\rho} + (1-\theta)^{\rho} e^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

targeting in emerging markets.

[14]We have confirmed that the main moments for domestic goods (producer price index) inflation are very similar to the CPI ones, for the countries where both are available.

and the resulting CPI inflation,

$$\pi^{\text{CPI}} = \frac{P^{\text{CPI}}}{P^{\text{CPI}}_{-1}} = \pi \frac{\left[\theta^{\rho} + (1-\theta)^{\rho} e^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[\theta^{\rho} + (1-\theta)^{\rho} e_{-1}^{1-\rho}\right]^{\frac{1}{1-\rho}}},$$

where the subscript $_{-1}$ denotes the previous period's value. The rate of depreciation in the nominal exchange rate is

$$\frac{\varepsilon}{\varepsilon_{-1}} = \frac{e}{e_{-1}} \frac{P^{d}}{P^{d}_{-1}} = \frac{e}{e_{-1}} \pi, \tag{45}$$

which depends on inflation and the depreciation in the terms of trade.

5.3 Parameterization

We assume that productivity shocks z_t follow an AR(1) process $\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_t$ with $\varepsilon_t \sim \mathcal{N}(0,1)$. Following Chatterjee and Eyigungor (2012), we assume that while in default, productivity suffers a convex penalty $z^d(z) = z - \max\left\{0, \lambda_0 z + \lambda_1 z^2\right\}$ with $\lambda_0 \leq 0 \leq \lambda_1$. We abstract from monetary shocks m for the benchmark model parameterization, but incorporate them in a later section to analyze the monetary transmission with counterfactuals.

The model also contains enforcement shocks ν that control the relative values of repayment and defaulting. We integrate these shocks into our computational technique following Dvorkin et al. (2018) and Gordon (2019). This computational technique consists of augmenting the model with taste shocks in the discrete choice tradition. The taste shocks slightly perturb the borrowing B' and the default-repayment choices and help with numerical stability and robust convergence in models with long-term defaultable debt. Appendix F details the structure of these shocks and their numerical properties. The shocks to the default-repayment decisions map into the model's enforcement shocks ν as a logistic distribution with location 0 and scale 1. The parameter ϱ_D controls the relative importance of the enforcement shocks for the default decision.

We choose the parameters in the model based on other studies and as part of a moment-matching exercise to replicate properties of the data of Brazil. The first set of parameters that are assigned directly include the Frisch elasticity ζ, the share of domestic goods in consumption θ, the trade elasticity ρ, the international interest rate r^*, varieties' elasticity and markups η, the persistence of the producivity shock ρ_z, the probability of return to financial markets after default ι, and the Rotemberg adjustment cost φ. For the Frisch elasticity, we choose a value of 0.33 following Galí and Monacelli (2005). This is a conservative value in line with the open economy New Keynesian literature. The trade elasticity ρ is set at 5, as in Devereux et al. (2019). This number is within the range of estimates in the trade elasticity literature. We set θ to get Brazil's imports as a share of consumption of 15%, which implies $\theta = 0.6225$ given the value of the trade elasticity. The international risk-free rate is 2% annually, consistent with U.S. Treasury yields.

Assigned Parameters	
Share domestic in consumption	$\theta = 0.62$
Frisch elasticity	$\zeta = 0.33$
Persistence of productivity	$\rho_z = 0.9$
Trade elasticity	$\rho = 5$
Export demand level	$\xi = 1$
Varieties elasticity	$\eta = 6$
Interest rate rule intercept	$\bar{i} = \overline{\pi}/\beta$
International rate	$r^* = 0.5\%$
Market reentry probability	$\iota = 4.17\%$
Price adjustment cost	$\varphi = 58$
Parameters from Moment Matching	
Private discount factor	$\beta = 0.9866$
Government discount factor	$\beta_g = 0.9766$
Inflation target	$\overline{\pi} = 1.015$
Interest rate rule	$\alpha_P = 1.4$
Std of productivity shock	$\sigma_z = 0.95\%$
Productivity in default	$\lambda_0 = -0.17$
	$\lambda_1 = 0.19$
Enforcement shock	$\varrho_D = 1e^{-4}$

Table 2: Parameter Values

	Data (%)	NK-Default
Mean inflation	5.9	5.9
Mean domestic rate	11.2	11.1
Volatility of inflation	1.8	1.8
Volatility of output	1.9	1.9
Volatility of consumption	1.8	2.0
Mean spread	2.6	2.6
Volatility of spread	0.9	0.9
Output, spread correlation	-62	-60

Table 3: Model Fit

For our quarterly model, we set $r^* = 0.5\%$. The elasticity of substitution between varieties η is 6, standard in the literature, inducing a 20% markup. We target an average length of market exclusion of roughly six years, which is an average duration of sovereign defaults based on Cruces and Trebesch (2013). Given that we are considering a short horizon of the data, it is difficult to precisely estimate the persistence in the productivity process. Instead, we set the persistence parameter ρ_z to a reference value of 0.9, comparable with many international real business cycle studies. We set the Rotemberg adjustment cost using the well-known first-order equivalence between Calvo and Rotemberg pricing frictions: our varieties' elasticity of $\eta = 6$ and a Calvo frequency of price changes of roughly once per year (once every fourth quarter) imply a value for φ of 58.[15] Finally, we normalize the level of export demand ξ to 1.

The second set of parameters are chosen to match a set of moments of Brazil. These 8 parameters are the discount factor of the private sector β and of the government β_g, the inflation target $\overline{\pi}$, the interest rate rule coefficient α_P, the volatility of the productivity innovations σ_z, the parameters of the default cost function $\{\lambda_0, \lambda_1\}$, the parameter governing the importance of the enforcement shock ϱ_D. The moments we target are the mean and volatilities for CPI inflation and spreads, mean nominal rates, the volatility of output and consumption, and the correlation between the spread and output. Most parameters affect all moments, yet some moments are more informative of certain parameters. The average CPI inflation rate in the data is the most informative on $\overline{\pi}$. The weight on inflation in the interest rate rule α_P heavily affects the volatility of CPI inflation. The volatility of productivity shocks is the main driver of that of output. As in standard sovereign default models, the productivity default cost parameters, the borrower's discount β_g, and the enforcement shock parameter ϱ_D are crucial for the dynamics of spreads and the volatility of consumption. The discount factor of the private sector β controls the average interest rate. In Table 2 we collect the values of all the parameters.

Table 3 contains the results of the moment-matching exercise. CPI inflation, interest rates, and spreads are reported annualized. The model matches quite closely the moments in the data. In the model and data, CPI inflation is about 5.9%, spreads are 2.6%, and nominal rates are about 11%. The volatilities for CPI inflation, output, and consumption are a bit under 2%, and the volatility of spreads is just under 1%. Output is negatively correlated with spreads, with a correlation close to -60.

Finally, computing our model with discrete choice methods introduces an additional parameter that controls the shock to the taste for borrowing ϱ_B. We set this parameter to $1e^{-6}$, which is the smallest value that guarantees fast, near monotone convergence for a wide range of parameter values while at the same time keeping choice probabilities quite tight.[16]

[15] See, for example, Miao and Ngo (2018) for the mapping between the Calvo and Rotemberg parameters.

[16] As illustrated in Appendix F, over most of the state space, about 70% of the probability mass over B' is concentrated over a couple of neighboring grid points.

5.4 Reference Models

To better understand the interactions between monetary frictions and default risk, we compare our findings with two reference models. The first reference model is a version of the Galí and Monacelli (2005) model, with nominal rigidities and perfect financial markets. The second reference model is our sovereign default model without monetary frictions.

The reference model with nominal rigidities and perfect financial markets is labeled **NK-Reference**. The equilibrium of this model is characterized by conditions (21–26), an exogenous debt-elastic bond price schedule to close the model, as in Schmitt-Grohé and Uribe (2003), and international borrowing as in equation (36), without sovereign debt and default. The debt-elastic bond price schedule we use for this model is $q^*(B)^{-1} = \beta + \Gamma\left[\exp(B - \overline{B}) - 1\right]$. We set $\Gamma = 1e^{-5}$, which gives a very loose borrowing schedule, and $\overline{B}$ to give the same average debt level as our baseline. We solve the NK-Reference model with a first-order log-linear approximation of the equilibrium conditions, for the same parameters as the baseline. We focus on this version of this reference model, with an undistorted Euler equation (36), for a more direct comparison with the literature.[17]

The reference model with sovereign default and without monetary frictions is labeled **Default-Reference**. This is a real sovereign default model with flexible prices. The allocations of this model can be implemented in a monetary model, when monetary policy pursues a "strict inflation target" policy. Under such policy, inflation is always at target and domestic nominal rates equal the real interest rates in the flexible price economy plus the inflation target. The equilibrium of this model is characterized by conditions (21–23) and an efficient labor allocation $u_n/u_c = z$. Nominal interest rates are equal to the real interest rates implied by the domestic Euler equation (24) with $\pi_t = 1$ always. We compute the Default-Reference model with the same global methods and for the same parameters as for the baseline NK-Default.

5.5 Default Risk and Monetary Frictions

Before analyzing the model-generated time series, we illustrate the main mechanisms relating default risk to monetary distortions. We show that default risk has important effects on the monetary wedge. High default risk is associated with large monetary wedges and nominal rates that remain high and exacerbate such wedges. Moreover, we find that these dependencies of nominal rates on default risk are an additional source of volatility for monetary policy.

We present policy rules as a function of government debt B relative to mean exports for the median level of productivity and focus on the behavior conditional on not defaulting. We start

[17] In Appendix E, we show that a version of our sovereign default model parameterized to have very loose borrowing schedules and zero sovereign spreads in equilibrium, where borrowing is governed by equation (33), displays very similar properties to this NK-Reference model.

with measures of default risk, then move on to real allocations and prices, and conclude by relating the monetary wedge to debt and default risk.

In Figure 1, panel (a) plots the one-period-ahead default probability (dashed red line) and spreads (solid blue line) as a function of debt. Default probabilities increase with current debt B because debt due next period $B' = H_B(s, B)$ increases with B, which makes default more likely. We emphasize two regions: a high default zone, for B roughly above 0.5, and a low default zone, for lower levels of debt. As is typical in sovereign default models, in the high default zone, the probability of default sharply increases in the current debt level. The figure also plots spreads which with long-term debt reflects not only one-period-ahead default probabilities but also the default risk at all horizons, increasing with debt.

Panels (b) through (f) in Figure 1 display key variables of the private and monetary equilibrium. We describe these policy rules in the high default zone first, then turn to the low default zone.

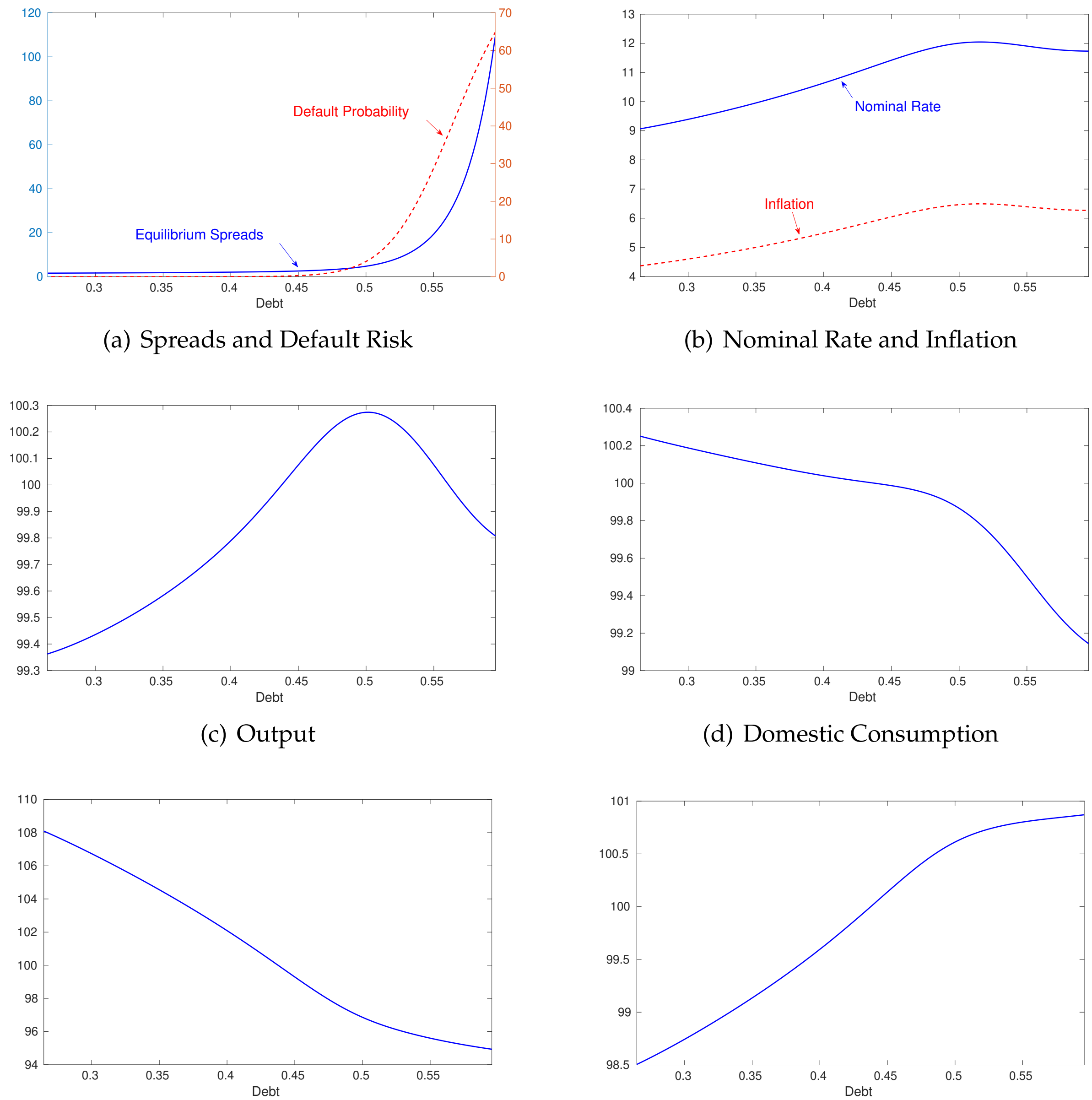
High Default Zone Policies. The behavior of variables in the zone of high default risk is largely driven by expectations of allocations and prices during actual default events. The two intertemporal conditions in the private and monetary equilibrium, the domestic Euler equation and the New Keynesian Phillips Curve, link current allocations, prior to default, to expected future allocations in default.

As in the simple example in Section 4, greater default risk is associated with a decline in domestic consumption and output because of the increasing monetary frictions. These forces drive the policies in this zone, as default events continue to be associated with declines in consumption and output. When the risk of default increases, the domestic Euler equation for households calls for a reduction in current consumption and hence output, given the expectations of low future consumption in default and an unresponsive nominal rate.

Inflation and nominal rates are fairly flat as a function of debt in this zone. Although output and consumption fall, firms do not reduce contemporaneous inflation because of high expected inflation next period, in case of default. Default events are associated with high inflation because of the high marginal cost arising from low productivity. Nominal rates do not respond very much to debt and remain elevated because contemporaneous inflation does not fall.

Foreign consumption declines with debt even though the economy borrows fairly aggressively at increasingly high interest rates. The terms of trade are fairly flat as domestic and imported consumption fall with debt at comparable rates.

Low Default Zone Policies. When default risk is low, debt affects allocations and prices mainly through the incentive to pay the debt without taking large loans at increasingly high interest rates. Foreign consumption declines sharply in this zone to service the debt. The decrease in foreign consumption leads to a depreciation in the terms of trade, which boosts exports and production. Firms increase inflation in response to the rising export demand. Nominal interest rates rise in response to the higher inflation, which depresses domestic consumption.



(a) Spreads and Default Risk

(b) Nominal Rate and Inflation

(c) Output

(d) Domestic Consumption

(e) Imported Consumption

(f) Terms of Trade

Figure 1: Policy Rules

The Monetary Wedge. We now turn to describing the monetary wedge, our measure of monetary distortions. To highlight the interactions between the monetary wedge and default risk, in Figure 2 we plot both the monetary wedge (left axis) and the one-period-ahead default probability (right axis) as a function of debt.

In the high default zone, the monetary wedge increases rapidly with debt because both consumption and output fall: rising default risk sharply depresses domestic consumption, leading to inefficiently low demand and a substantially depressed level of activity. In the low default zone, the monetary wedge is decreasing in the level of debt because the increase in output dominates the decline in domestic consumption. The economy produces more to export and service the debt, owing to the tightening bond price schedule.

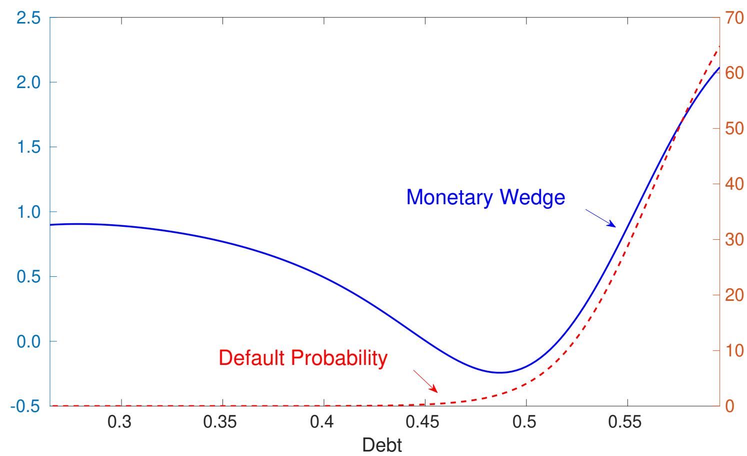


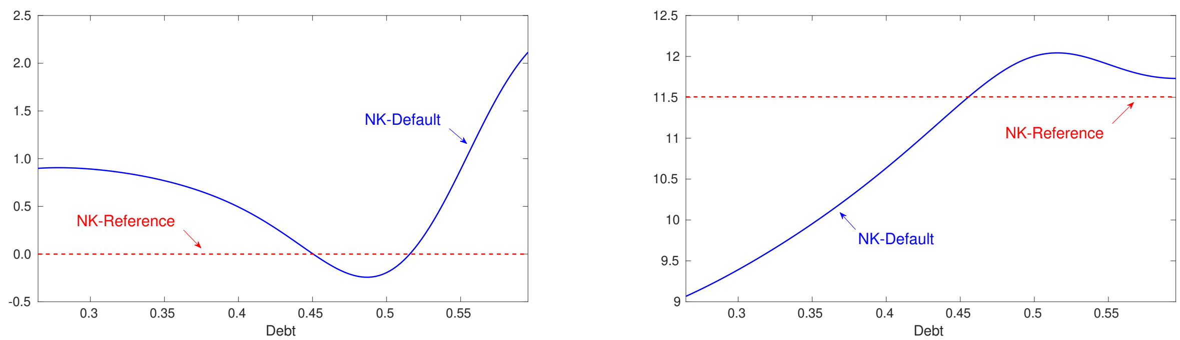
Figure 2: The Monetary Wedge and the Probability of Default

Despite the monetary wedge rising rapidly with debt in the high default zone, nominal rates do not fall to accommodate this rise. The reason is that firms do not reduce inflation despite the sharp reduction in demand from high default risk, as seen in panel (b) of Figure 1. In this zone, inflation remains fairly flat and is not a good proxy for the monetary wedge. In contrast, in the low default zone inflation and nominal rates are responsive to debt and increase when monetary wedges fall. Inflation in this zone is a good proxy for the monetary distortion.

The disconnect between current inflation and monetary distortions in the high default zone can be understood using the New Keynesian Phillips Curve (12) and the monetary wedge (19),

$$\frac{1}{\text{1+monetary wedge}} = 1 + \frac{\varphi}{\eta - 1}\left\{(\pi - \overline{\pi})\,\pi - \beta\mathbb{E}\left[\frac{Y'\,u_c'}{Y\,u_c}\,(\pi' - \overline{\pi})\,\pi'\right]\right\}.$$

The monetary wedge will be different from 0 whenever inflation either today or next period is away from target. In the low default zone, future inflation is close to target and the expectation term on the right side is small. Then, current inflation is tightly connected to the monetary wedge, and when the monetary authority responds to deviations of inflation from target, it is in effect responding to the monetary wedge. In this zone, the monetary authority is leaning against the friction. In the high default zone, in contrast, the expectation on the right side is positive and reflects the low consumption and high inflation during default. These future inflation and consumption expectations create a disconnect between current inflation and the monetary wedge. Firms are not lowering prices because they risk price increases in the future if the government defaults. Unresponsive inflation implies that the monetary authority does not lean against the friction in this zone.



(a) The Monetary Wedge

(b) Nominal Interest Rate

Figure 3: The Monetary Wedge and Nominal Rates

Figure 3 compares the consequences of debt in the NK-Default baseline model with the NK-Reference model. Panel (a) compares monetary wedges, and panel (b) shows domestic nominal interest rates. In the reference model, debt does not distort the level of activity; the monetary wedge is flat at zero. The NK-Reference model has very loose borrowing schedule, which allows foreign consumption to be insensitive to debt. Such ample borrowing possibilities disconnect domestic allocations from the indebtedness of the economy. Moreover, absent default, the intertemporal channels in the high default zone of the NK-Default model are not operative.

Panel (b) of Figure 3 shows that nominal rates in the NK-Reference model do not vary with debt, in contrast to our baseline model. The responsiveness of nominal rates to debt is an additional source of volatility for nominal rates in our baseline model.

5.6 Monetary Frictions Discipline Borrowing

We have shown in Section 4 that the presence of monetary frictions disincentivizes borrowing. We illustrate this in the context of our quantitative results by comparing borrowing and spreads in our model, NK-Default, to the Default-Reference model that has flexible prices.

Figure 4 compares the pattern of debt accumulation in the two models: both are simulated starting with zero debt, keeping productivity at median throughout. In both models, the government accumulates debt and settles at a level of debt that is similar. However, in the NK-Default model, this accumulation is slower than in the Default-Reference model because of monetary frictions. Along the transition, borrowing wedges are positive and associated with large monetary wedges, disincentivizing borrowing, as seen in equation (33). Lenders offer a more favorable bond price schedule since slower debt accumulation results in less debt dilution.

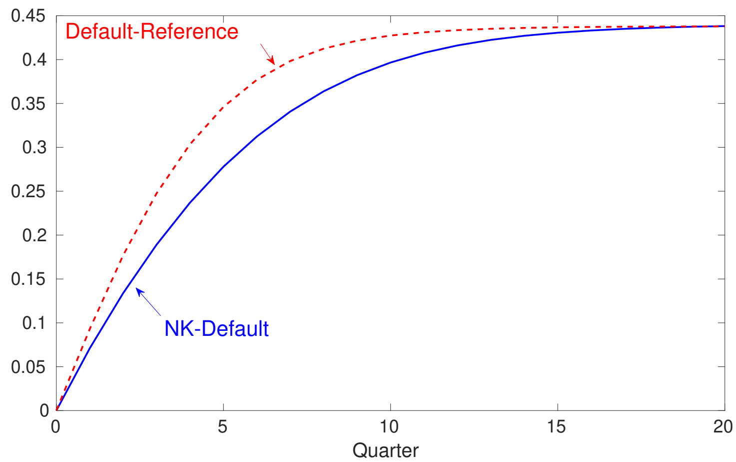


Figure 4: Debt Accumulation

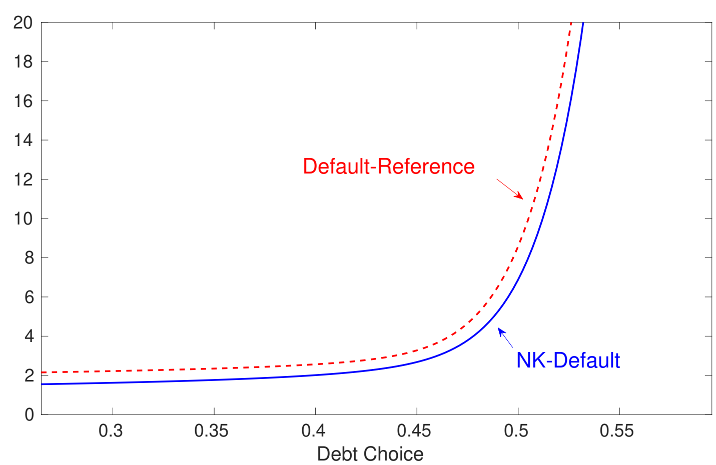


Figure 5: Spread Curves

Figure 5 plots spread schedules in the two models, NK-Default (solid blue line) and Default-Reference (dashed red line) for arbitrary B'. Spreads are lower for each level of B' in the NK-Default model because of lower default risk and lower debt dilution. These schedules imply that, as shown in Figure 4, the government pays lower spreads throughout this simulated path of the NK-Default model.

5.7 Impulse Response Functions

In Section 5.5 we described the policy rules as a function of debt. Here, we analyze responses of the main variables of interest to productivity shocks z. We construct the impulse response functions (IRFs) in our nonlinear model following Koop et al. (1996). We simulate a panel of $50,000$ units for $5,000$ periods. For the first 4,950 periods, the shocks follow their underlying Markov chain so that the cross-sectional distribution converges to the ergodic distribution of the model. In period 4,951, the *impact period*, normalized to 0 in the plots, we reduce the shock realizations by 1.3%, about half of the standard deviation of the shock. From period 4,952 onward, shocks resume their Markov chain processes. The impulse responses plot the average across the time series.[18] We also contrast the IRFs of our baseline model to those from the two reference models in Section 5.5, NK-Reference and Default-Reference.

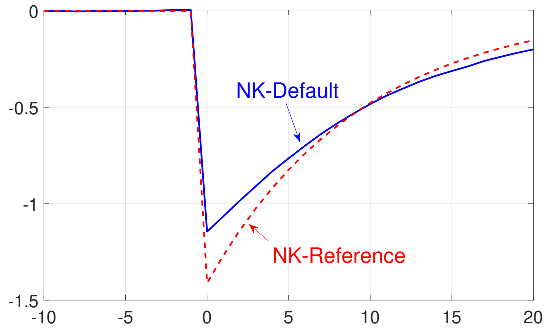
Figure 6 plots the responses of output, domestic consumption, imports, terms of trade, inflation, the nominal interest rate, spreads, and debt. The blue lines are the IRFs of our baseline NK-Default model; the dashed red lines are the IRFs of the NK-Reference model. First we describe the responses of our baseline model.

Declines in productivity lead to a decline in output of about 1.14%, which is quite similar to the decline in the shock. Consumption declines a bit more; consumption of domestic and foreign goods falls by 1.33% and 1.25%, respectively. The terms of trade appreciates mildly, especially over the medium run because domestic goods consumption recovers more slowly than foreign goods consumption.

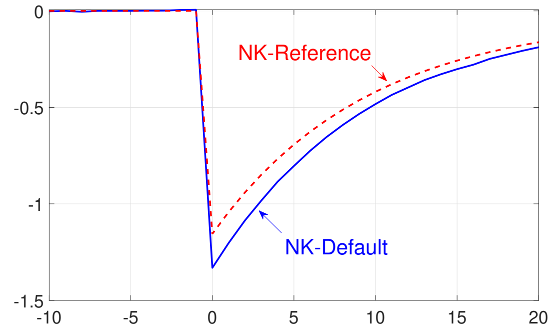
As is typical for New Keynesian models, inflation rises with low productivity, about 1.2% on impact, since low productivity raises the unit cost for firms. Nominal interest rates rise in response to the elevated inflation, about 1.6%. Nominal rates respond more than inflation because the coefficient in the interest rate rule $\alpha_P > 1$.[19] The figure also shows the responses of government spreads and debt. As is typical in sovereign default models, low productivity tightens the bond price schedule because default is more likely in recessions, and with persistent shocks, low productivity makes future recessions more likely. The tight bond price schedule leads to higher spreads and a reduction in debt. Spreads rise about 0.8%, and debt contracts

[18] The impulse responses are computed over all units, including those with defaults. Discarding defaults from the cross-sectional average does not alter the properties of the IRFs.

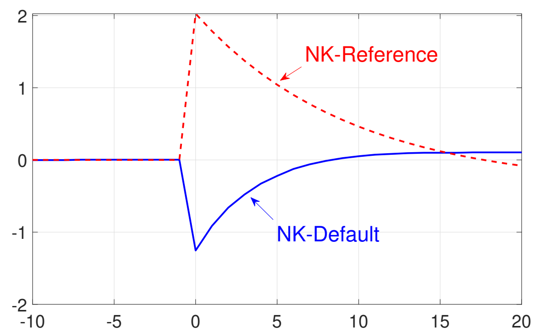
[19] In our numerical explorations, we find that our model also requires the coefficient on inflation in the interest rate rule to be greater than 1, as in Galí and Monacelli (2005).



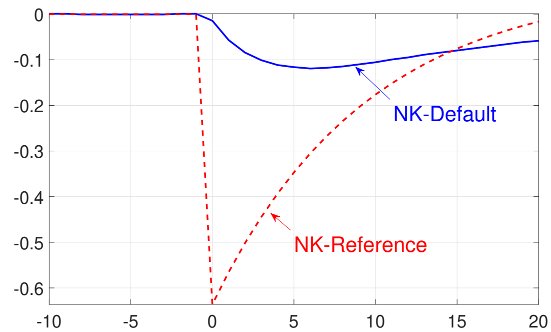
(a) Output



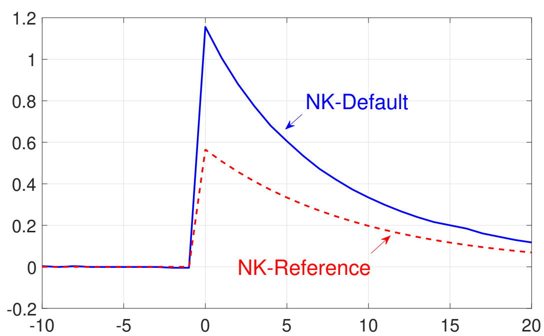
(b) Domestic Consumption (C)



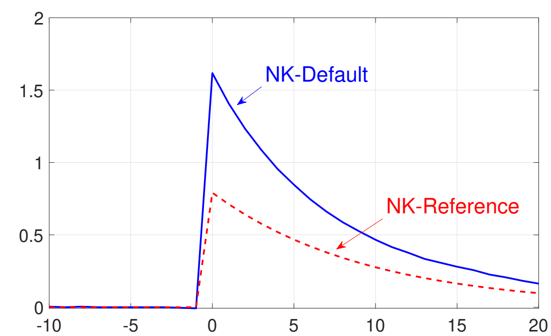
(c) Imported Consumption (C^f)



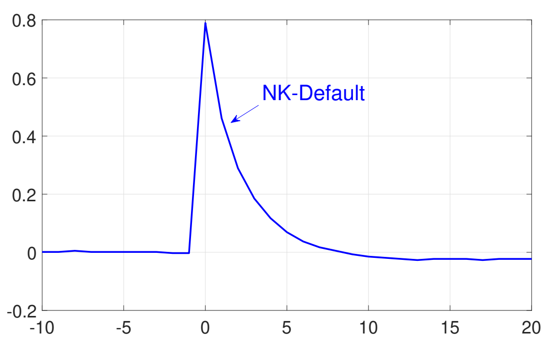
(d) Terms of Trade (e)



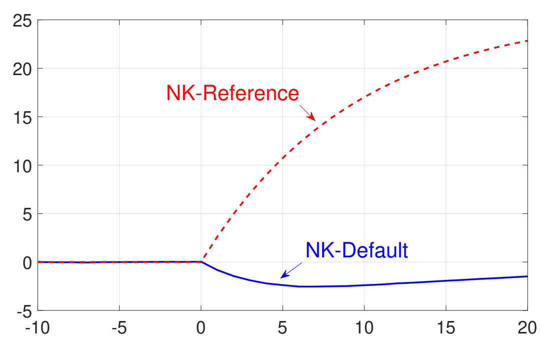
(e) Inflation (π)



(f) Nominal Interest Rate (i)

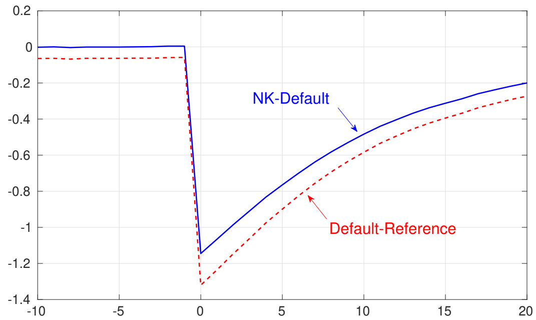


(g) Spread

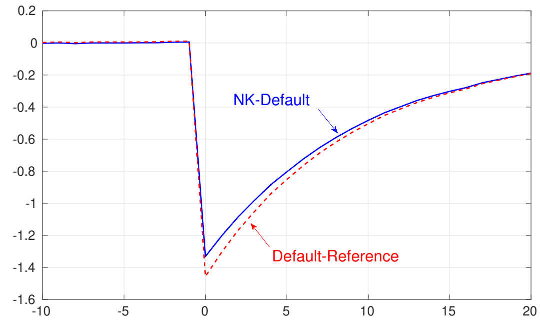


(h) Debt (B)

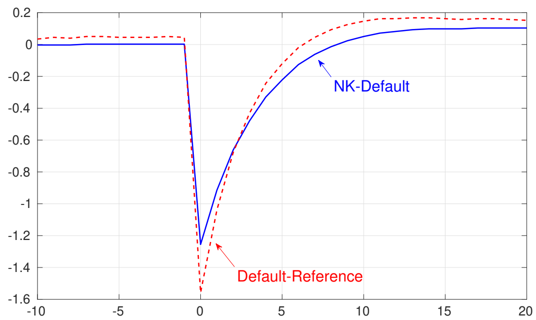
Figure 6: Impulse Responses: Compared with NK-Reference



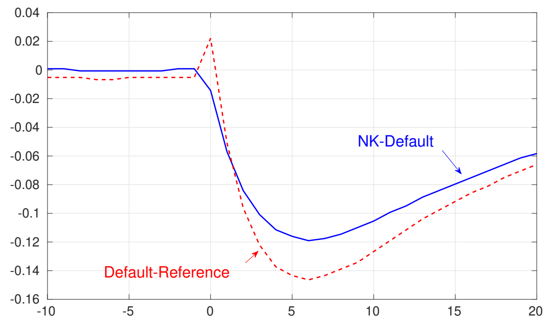
(a) Output



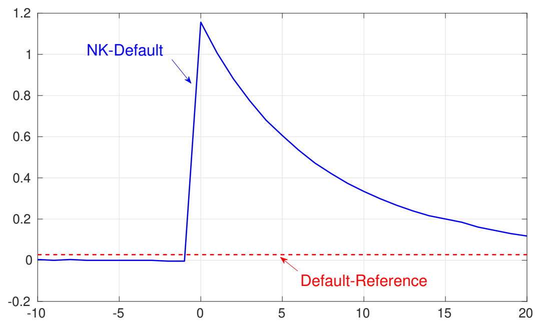
(b) Domestic Consumption (C)



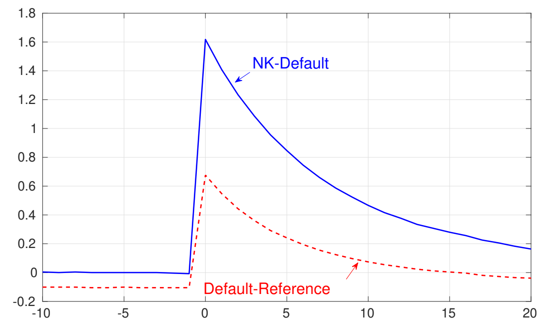
(c) Imported Consumption (C^f)



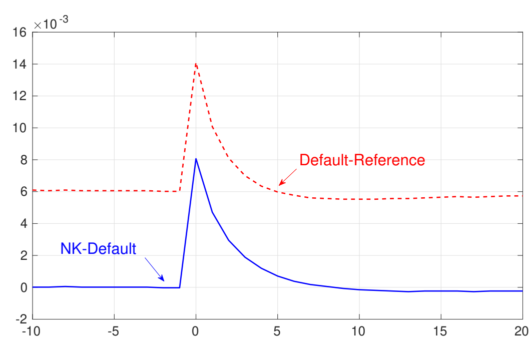
(d) Terms of Trade (e)



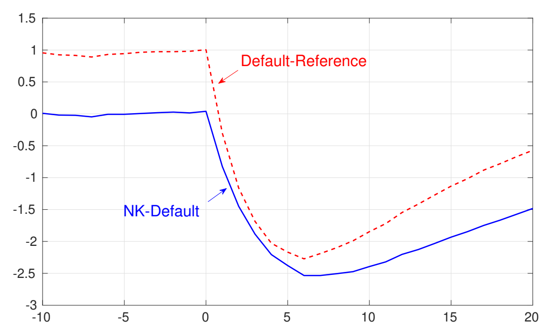
(e) Inflation (π)



(f) Nominal Interest Rate (i)



(g) Spread



(h) Debt (B)

Figure 7: Impulse Responses: Compared with Default-Reference

slowly by about 3%. These dynamics illustrate that productivity shocks lead to strong, positive comovements of spreads with inflation and nominal rates.

The responses of the NK-Reference model are also shown in Figure 6. Output and domestic consumption responses are similar to those of our baseline. Foreign consumption, in contrast, expands during the downturn. The increase in imported consumption reflects the ample possibilities for insurance with external borrowing. Recall that preferences are non-separable between domestic and foreign goods, and hence smoothing the marginal utility of foreign goods requires an increase in foreign consumption, given the drop in domestic consumption. Inflation and nominal rates also rise in the NK-Reference model because of the higher unit cost for production, but their responses are much more muted, about half of the responses in the NK-Default baseline. Terms of trade appreciate sharply in the NK-Reference model because of uncovered interest parity forces: high nominal domestic rates and unchanged foreign interest rates force an expected depreciation, which in turn implies that the exchange rate appreciates on impact. The appreciation depresses exports and output, consistent with the more muted rise in inflation. Borrowing expands significantly to support the consumption of foreign goods and the spreads are always zero, reflecting the perfect financial markets.

These IRF comparisons highlight the role of default risk in monetary policy. Default risk and tight borrowing conditions raise spreads and induce additional volatility in inflation and aggregate consumption, mainly driven by imports. These larger swings in inflation call for more aggressive monetary policy with stronger responses of nominal rates.

We also compare the IRFs in the NK-Default baseline to the Default-Reference model. Figure 7 plots the IRFs for the same variables, normalizing each panel to the level in period -1 in the NK-Default model.[20] The responses of output and domestic consumption in the Default-Reference model are quite similar to those in our baseline. The decline in foreign goods consumption is somewhat larger because default risk is higher in this model and borrowing conditions are tighter. Spreads are higher on average in the Default-Reference model and increase on impact by about the same amount as in our baseline. Debt is higher on average and also decreases in the downturn. Domestic goods inflation is zero by construction and nominal interest rates equal the domestic real interest rate implied by the domestic Euler equation. Domestic interest rates rise in the recession because consumption is expected to grow after the downturn; their response, however, is more modest than in our baseline, with a response on impact of about one-third of the baseline.

These IRF comparisons highlight the role of monetary frictions in default risk. Monetary frictions lead to lower spreads and debt on average, more volatile nominal rates, and less volatile imports.

[20]The unconditional means across these models differ mainly because the higher default probability in the Default-Reference model leads to a larger fraction of observations with low productivity due to the default cost.

5.8 Second Moments

Table 4 reports key variables of interest for Brazilian data, as well as for our baseline NK-Default model and two reference models. We report the first and second moments for the series of CPI inflation, nominal interest rates, spreads, and output that we introduced in Section 5.1. We also report second moments for the trade balance, measured as a ratio relative to output, and nominal depreciation rates of the trade-weighted exchange rate. The model statistics are computed over simulations of 50,000 periods in length, excluding periods in default and the 20 periods following the return to market access.[21]

Overall, the moments in the baseline model resemble the Brazilian data. The mean CPI inflation, nominal interest rate, and spread as well as the volatility of inflation, output, and spreads are targets in our moment-matching exercise. The model delivers a volatility of the nominal rate that is close to the data, whereas it underestimates the volatility of the trade balance and misses the high volatility in nominal exchange rates. Our model shares the result from Galí and Monacelli (2005) that exchange rates are about 30% more volatile than output, far from the relative volatility in the data, which reflects the common disconnect between exchange rates and fundamentals in international business cycle theory.

The model delivers the positive correlation of inflation and nominal rates with spreads in the data. The magnitudes of these correlations, of about 60%, also resemble those in the data. The positive correlations of spreads with inflation and nominal rates arise in our model because across both state variables, namely, productivity z and debt B, inflation and spreads comove positively. As seen in the impulse response functions, productivity fluctuations lead to both: countercyclical inflation in the presence of pricing frictions and countercyclical spreads in the presence of default risk. Debt dynamics also lead to a positive correlation because, as seen in the policy rules section, high (low) default zones are associated with high (low) spreads and high (low) inflation.

The model also features the countercyclicality of inflation, nominal rates, nominal depreciations, spreads, and the trade balance present in the Brazilian data. As explained with the IRFs, inflation and nominal rates tend to rise with low productivity. Nominal exchange rates depreciate because their dynamics are mainly driven by inflation. Spreads are countercyclical because recessions are associated with high default risk. The trade balance is also countercyclical because the high spreads in recessions lead to a reduction in international borrowing. These properties induce the positive correlations of spreads with the trade balance and the nominal depreciation rates, that are present in the data. These findings are consistent with Na et al. (2018), who find in their work that default risk is correlated with depreciation rates in emerging

[21]This selection criterion ensures that statistics are not driven by the low levels of debt that the economy holds upon return to market access. Expanding our sample to include all periods with no default does not significantly alter any of the second moments.

Mean	Data (%)	NK-Default	NK-Reference	Default-Reference
CPI inflation	5.9	5.9	6.1	0.0
Domestic rate	11.2	11.1	11.5	5.3
Spread	2.6	2.6	—	3.2
Standard Deviation				
CPI inflation	1.8	1.8	1.0	0.6
Domestic rate	2.2	2.5	1.3	1.8
Spread	0.9	0.9	—	0.8
Output	1.9	1.9	2.4	2.1
Consumption aggregate	1.8	2.0	2.0	2.2
Trade balance	0.9	0.3	1.9	0.5
Nominal depreciation rate	14.7	2.4	2.0	1.9
Correlation with Spread				
CPI inflation	59	60	—	−1
Domestic rate	59	64	—	18
Output	−62	−60	—	−42
Trade balance	61	35	—	33
Nominal depreciation rate	51	45	—	−1
Correlation with Output				
CPI inflation	−16	−88	−81	7
Domestic rate	−23	−96	−98	−60
Trade balance	−77	−18	62	−23
Nominal depreciation rate	−18	−62	−28	7

Table 4: Moments: Data, NK-Default, and Reference Models

countries.

Table 4 also reports the moments from the NK-Reference model that are silent on default risk. Average CPI inflation and the nominal rate are the same as in the baseline, largely reflecting the parameters $\overline{\pi}$ and $\bar{i}$, which are the same across models. The volatilities of the nominal interest rate and CPI inflation are, however, only about half of those in the NK-Default baseline. This comparison shows that in an environment with default risk, a central bank targeting inflation must implement a more volatile interest rate policy than it would absent default risk. Default risk makes inflation more volatile because it affects the monetary distortion, as discussed in Section 5.5 and illustrated in the IRFs. The end result is that a larger response of the nominal interest rate is needed to keep inflation close to target.

As in our baseline model, the NK-Reference model also features countercyclical inflation, domestic rates, and nominal depreciation, yet the cyclicality of the trade balance is positive. With well-functioning financial markets, the country increases borrowing in recessions to smooth consumption.

The final column of Table 4 lists moments for the Default-Reference model. This is a real version of our model, with zero domestic goods price inflation. CPI inflation fluctuates only because of changes in the terms of trade, and nominal rates are equal to real rates implied by consumption dynamics. The means and volatilities of CPI inflation and nominal rates are substantially lower than in the baseline model, whereas output and consumption volatility are comparable. Mean spreads, however, are higher by about 0.5% in the Default-Reference model compared with the baseline, reflecting the analysis in Section 5.6 on the disciplining role of monetary frictions on borrowing. Another manifestation of the greater propensity to borrow in the Default-Reference model is its more volatile trade balance, with a standard deviation almost twice that of the baseline.

Absent nominal frictions, the Default-Reference model fails to generate the strong positive comovement of spreads with CPI inflation or the nominal depreciation rate, and they are essentially uncorrelated. The model does exhibit a positive but quantitatively modest positive correlation of spreads with the domestic rate, less than a third of the value in the baseline. Finally, CPI inflation and the nominal depreciation rate are essentially acyclical, in contrast with the strong countercyclical pattern in NK-Default, whereas domestic rates and the trade balance cyclical patterns are negative, as in the baseline model.

5.9 Brazil Event and Counterfactual

Now we perform an event analysis and compare our model with the Brazilian 2015 recession. We find that our model produces similar time paths as in the data, with increases in nominal rates, inflation, and spreads. We also conduct a counterfactual experiment where the nominal rate is kept low during the recession. In this counterfactual, the recession would be milder but inflation and spreads would have risen even more.

For the event analysis, we feed in a path of productivity shocks such that the time path of output in the model replicates the one in the data. We fix the initial level of debt to its mean in the limiting distribution. We then compare the predictions of the model for CPI inflation, the nominal interest rate, spreads, and nominal exchange rates with the data.

The dashed blue lines with circle markers in Figure 8 represent the series in the data. Brazil experienced a recession from 2014 to late 2016, with output contracting from 3% above trend to 3% below trend, a 6% decline in total. It then recovers starting in 2016Q3. During this period, CPI inflation increases by 4%, the nominal rate increases by 2%, and spreads rise from about 2% to 5%. When output recovers after 2017, inflation, the nominal rate, and spreads all fall.

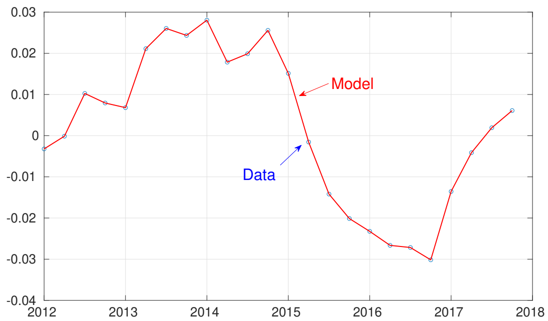
The solid red lines in Figure 8 are the corresponding series in the model. To match the dynamics of output, the model requires that the underlying productivity shock first decreases from 2014 to late 2016 and then recovers. This implies that during the recession the unit cost of production increases, leading to an increase in inflation. Monetary policy responds to this high inflation with hikes in the nominal rate. The recession also drives up sovereign spreads. Quantitatively, the model matches the rise in inflation and spreads during the recession, around 4% for inflation and 3% for spreads. After the recovery from 2016Q3 onward, the model also reproduces the inflation decrease of about 4% and the drop in spreads of about 3%.

In terms of exchange rates, the model delivers a depreciation in the nominal exchange rate of about 20% during the recession, starting from 2015Q1 onward. In the data, the overall depreciation in the nominal exchange rate during this period is also about 20%, but with much higher volatility that we miss, as discussed in Section 5.8. Although our model matches the overall depreciation, it fails to replicate the large depreciation in Brazil in 2016.

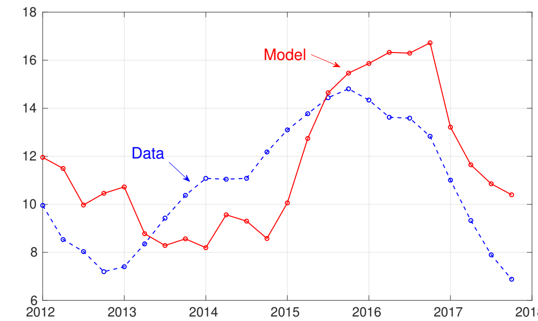
To evaluate the effects of the nominal rate hikes on the Brazilian economy during this event, we conduct a counterfactual experiment with a dovish central bank. In this alternative scenario, instead of tightening in response to inflation higher than its target, the central bank keeps a low nominal interest rate, similar to its 2015 level following the start of the recession.

We implement these counterfactual interest rates through the use of the monetary shock m to the interest rate rule (13). In the parameterization for the main quantitative results, we abstracted from these shocks. We now allow for low probability, i.i.d. m shocks and compute the model over a wide range of values for m. In the counterfactual, we feed in the appropriate level for m such that the nominal rate remains at its 2015 level. We also confirm that the main quantitative properties of our model are unaltered with these small variance monetary shocks and illustrate the transmission of these monetary shocks by constructing the impulse response functions displayed in Appendix C.

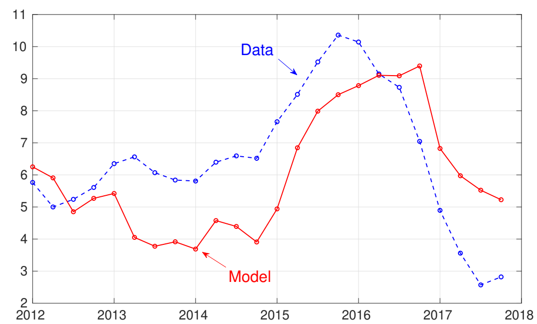
The counterfactual series are plotted in black lines in Figure 9 and the baseline results are again in solid red lines. The expansionary monetary shocks in the counterfactual induce



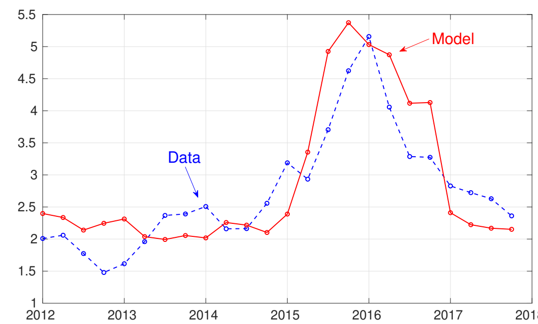
(a) Output



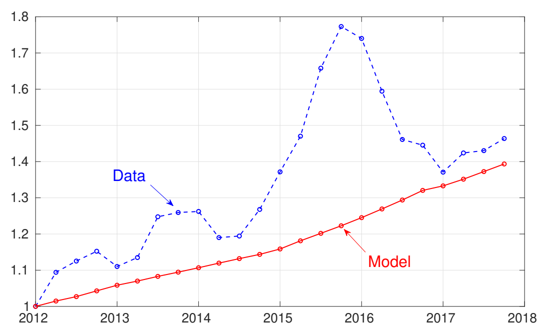
(b) Nominal Interest Rate



(c) Inflation



(d) Sovereign Spread



(e) Nominal Exchange Rate

Figure 8: Event Analysis for Brazil

(a) Output

(b) Nominal Interest Rate

(c) Inflation

(d) Sovereign Spread

(e) Nominal Exchange Rate

Figure 9: Counterfactual Experiment

lower nominal interest rates and thus stimulate consumption and output. This higher demand increases the unit cost of production, which in turn generates high inflation. In late 2016, the inflation rate in the counterfactual scenario would be about 2% higher than in the benchmark case. Lower nominal interest rates also lead to higher spreads, since low rates reduce monetary frictions and incentivize the government to increase borrowing. This counterfactual highlights the disciplining role of monetary frictions for sovereign borrowing and default risk.

In summary, our model closely matches the patterns of inflation, nominal rates, and spreads during the Brazilian downturn of 2015. The counterfactual analysis highlights the role of monetary frictions in limiting borrowing and moderating crisis events. Had Brazil's central bank deviated from its pursuit of price stability, the recession would have been milder but at a cost of much higher inflation and a deeper debt crisis.

6 Extensions, Robustness, and Welfare

In this section we study the robustness of our findings in environments that extend the baseline model. We analyze two main variations. The first extension considers an environment with government debt denominated in local currency. The second extension considers alternative specifications for the interest rate rule. Here we perform a comparative static exercise over the weight on inflation and also extend the interest rate rule to respond to output.

6.1 Local Currency Government Debt

Governments in emerging markets increasingly borrow in local currency, as documented in Du and Schreger (2016) and Ottonello and Perez (2019). Here we explore the implications of sovereign debt denominated in local currency for our NK-Default model. Domestic monetary policy in this framework, of course, directly affects the real liabilities of the central government because domestic inflation alters the real value of the debt. In our model, however, inflating away the debt is not a consideration for monetary policy because the nominal interest rate rule responds only to inflation deviations from target. We find that in our environment, the denomination of government debt has only minor effects on the dynamics of inflation, spreads, or nominal interest rates.

To analyze the case with local currency government debt, we modify the government's budget constraint (14) in the baseline model. The nominal government budget constraint in local currency is

$$T_t + \tau W_t N_t = q_t \left(B^{\text{lc}}_{t+1} - (1-\delta) B^{\text{lc}}_t \right) - (r^* + \delta) B^{\text{lc}}_t + \tau_f P^f_t C^f_t, \tag{46}$$

where nominal local currency government debt is B^{lc}_{t+1}.

We deflate this budget constraint by the price of domestic goods P_t^d and combine it with the budget constraint of households to obtain the balance of payments condition,

$$e_t^{\rho}\xi = e_t C_t^f + (r^* + \delta)\frac{B_t}{\pi_t} - q_t\left(B_{t+1} - (1-\delta)\frac{B_t}{\pi_t}\right), \tag{47}$$

where real government debt, in terms of domestic goods, is $B_t = B_t^{\mathrm{lc}}/P_{t-1}^d$. This expression makes it explicit that inflation π_t affects the real value of government debt.[22]

The private and monetary equilibrium of the model with local currency debt consists of equations (21–28) with the balance of payments condition (22) replaced by (47). The bond pricing condition is also modified, as international lenders arbitrage the foreign currency risk-free return r^* with the foreign currency return on the local currency government debt. This arbitrage implies that lenders need to be compensated not only for default risk, but also for the expected nominal exchange rate depreciation (45). The bond price for local currency bonds is

$$q_t = \frac{1}{1+r^*}\mathbb{E}\left[\frac{e_t}{e_{t+1}}\frac{1}{\pi_{t+1}}(1-D_{t+1})(r^* + \delta + (1-\delta)q_{t+1})\right]. \tag{48}$$

For our definition of sovereign spreads with local currency debt, we follow Du and Schreger (2016) and measure them with the *local currency credit spread*. This credit spread is the difference in yield-to-maturity between defaultable and default-free bonds, both for instruments that are denominated in the same currency and have equal duration.[23] We compute the model with local currency government debt using the same parameter values as in the baseline model.

The second column of Table 5 reports the results for the model with local currency government debt and shows that the properties of this version of the model are very similar to those in the baseline model. The only significant difference is that mean spreads are lower when debt is denominated in local currency, which we elaborate on below. Importantly, the standard deviations of nominal rates and inflation, as well as their correlations with spreads, are unchanged relative to the baseline. In Appendix D, we also show that the impulse response to productivity shocks and the behavior of the monetary wedge and nominal rate as functions of debt are similar to the ones in the baseline model.

The similar volatilities in nominal rates across debt denomination highlights the robustness of our first finding: default risk amplifies monetary frictions. The volatility in nominal interest rates of 2.5 is larger than the volatility of 1.3 in the NK-Reference model with no default in Table 4.[24]

[22] With abuse of notation B_t in this extension refers to debt in domestic goods, whereas in the baseline model, it refers to debt in foreign currency, which is equivalent to foreign goods.

[23] In our model, we construct the price in dollars for default-free bonds in local currency as $q_t^* = \frac{1}{1+r^*}\mathbb{E}\left[\frac{e_t}{e_{t+1}}\frac{1}{\pi_{t+1}}(r^* + \delta + (1-\delta)q_{t+1}^*)\right]$.

[24] For simplicity, we focus only on the comparisons with the NK-Reference model with foreign currency because

Mean	Benchmark	Local currency	Rule with larger α_P	Rule with output gap
CPI inflation	5.9	5.9	6.0	5.7
Domestic iate	11.2	11.3	11.2	11.0
Spread	2.6	1.9	2.9	2.7
Standard Deviation				
CPI inflation	1.8	1.9	0.9	1.5
Domestic rate	2.5	2.5	1.6	2.2
Spread	0.9	0.4	0.9	0.9
Output	1.9	2.0	2.0	2.0
Consumption aggregate	2.0	2.0	2.1	2.0
Trade balance	0.3	0.4	0.4	0.4
Nominal depreciation rate	2.4	2.5	1.8	2.3
Correlation with Spread				
CPI inflation	60	57	45	72
Domestic rate	64	61	62	76
Output	−60	−57	−63	−79
Trade balance	35	26	34	29
Nominal depreciation rate	45	41	20	45
Correlation with Output				
CPI inflation	−88	−86	−61	−84
Domestic rate	−96	−95	−91	−90
Trade balance	−18	5	−21	−12
Nominal depreciation rate	−62	−58	−22	−48

Table 5: Moments: Extended Models

Our second main finding, that monetary frictions discipline government default risk, is also robust across debt denomination. The local currency debt specification also delivers a lower mean spread of 1.9 relative to the mean spread of 3.2 in the Default-Reference model with flexible prices in Table 4. In fact, with local currency, spreads are even lower than in the baseline model because the inflation dynamics induce state-contingent debt repayments that are a good hedge for the sovereign: high inflation in recessions means that the real burden of debt falls when income is low. These desirable properties of local currency debt alleviate financial frictions and default risk, leading to a lower average spread.

Average credit spreads are also lower with local currency debt because here lenders price the product of nominal devaluations $\frac{e}{e'}\frac{1}{\pi'}$ and future default risk, encoded in future prices q', which implies that the covariance between these two variables alters the bond price. In our model, this covariance is positive: high prices are associated with low nominal devaluation rates, which boosts the average price of local currency bonds (or lowers the mean credit spread). We find, nevertheless, that this effect is quantitatively small.

6.2 Variants on Interest Rate Rules

We now turn to our second extension of the baseline model that evaluates variations in the interest rate rule. We consider two variations: one that increases the weight on inflation deviations and another that adds an output gap term.

Larger Weight on Inflation We have seen that our model generates sizable volatility in inflation and nominal rates relative to both reference models: NK-Reference without default and Default-Reference without nominal rigidities. As in standard New Keynesian models, the volatilities of these variables are affected by the weight on inflation deviations in the interest rate rule. We consider a comparative static exercise that increases the weight on inflation α_P to 2.5, from 1.4 in the benchmark. All other parameters remain unchanged in this experiment.

In Table 5, we report the first and second moments for this comparative static exercise. Higher α_P lowers the volatility of inflation, nominal interest rates, and nominal devaluations. Spreads in this model, however, increase modestly on average and display similar variability as in the benchmark. All other volatilities and correlations are similar to the benchmark, including the positive correlations of spreads with nominal rates and inflation. This comparative static exercise maintains the same two-way interactions of monetary policy and default risk in the benchmark: default risks amplify monetary frictions, as seen by the higher volatility in nominal rates of 1.6 relative to the NK-Reference of 1.3, and monetary frictions continue to discipline default risk, as seen by the lower mean spread of 2.9% relative to the Default-Reference of 3.2%.[25]

results from an NK-Reference model with local currency debt are almost identical.

[25]The difference in the volatility of nominal rates arising from default becomes larger when we recompute the

Weight on Output Gap The monetary policy rule in the baseline model has nominal rates responding only to inflation deviations. In this subsection, we expand the rule to include an output gap term. We modified the baseline model by replacing the interest rate rule in equation (13) with

$$i = \bar{i}\left(\frac{\pi_t}{\overline{\pi}}\right)^{\alpha_P}\left(\frac{Y_t}{Y_t^{\text{flex}}}\right)^{\alpha_Y} m_t. \tag{49}$$

The output gap is defined as the ratio of output in the model Y_t relative to output in the model with flexible prices Y_t^{flex}. Flexible output is defined as the average output in the Default-Reference model conditional on the realization of the shock z_t and also on whether or not the economy is in default.[26] We set the weight on the output gap to 0.5, which is the value in the rule of Taylor (1993) and maintain all other parameters as in the benchmark model.

In Table 5 we report results from our model with the extended interest rate rule that includes the output gap and shows that the properties of our model change very modestly. The mean and volatility of spreads remain practically unchanged, the volatilities of inflation and nominal rates are a bit lower, and the correlations of spreads with nominal rates and inflation continue to be positive. In Appendix D we present impulse response functions to productivity shocks for this model as well as monetary wedges as a function of debt and show that they behave similarly to the benchmark model.

The moments and functions are similar across these two rules because here, as in many New Keynesian models, inflation deviations are good indicators of output gaps, and hence a rule with only inflation deviations behaves similarly to a rule that also contains output gaps. More relevant for us, this robustness exercise shows that adding an output gap term to the interest rate rule basically does not alter the interactions between monetary policy and default risk: default risk amplifies monetary frictions, and monetary frictions discipline default risk.

6.3 Welfare

The paper has established that monetary policy interacts with sovereign default risk. We have shown that in our benchmark model, as well as in the extensions presented in Table 5, interest rate rules affects not only inflation but also the properties of sovereign spreads. Here we discuss how welfare varies across these economies and focus on household welfare, as defined in (3). The benchmark economy has two sources of inefficiency: pricing frictions and lack of commitment to repay its debt, or default risk. Monetary policy affects both frictions, and therefore welfare comparisons across monetary rules depend on how these rules interact with the two frictions.

NK-Reference model with a larger weight on inflation of 2.5, where the volatility of nominal rates is 0.9%.

[26] We average output across debt to smooth any potential differences in output arising from varying tightness in financial frictions across debt levels because such tightness is not the same across models. Results nevertheless are similar if we condition flexible output on all states and choices $S = \{z, B, B', D\}$.

Across model economies, the volatility of inflation and the mean spread are measures of the costs of price frictions and default risk.

	Welfare (%)
NK-Default	
Benchmark	$-.019$
Rule with higher α_P	$+.009$
Rule with output gap	$-.007$
Local currency debt	$+.018$
NK-Reference (no default risk)	$+.189$
Default-Reference (no pricing frictions)	.000

Table 6: Welfare, Relative to Default-Reference

In Table 6 we compare welfare across models with different monetary and financial environments. We report gains and losses in consumption equivalence terms relative to the Default-Reference model, which has no pricing frictions but features default risk.[27] We evaluate welfare at the mean level of productivity and zero debt. As is standard in the business cycle literature, including monetary economies, welfare differences are small across models. Nevertheless, we find that, for some monetary rules, welfare is higher with pricing frictions rather than without.

The top part of the table reports welfare for the NK-Default model, our benchmark together with the alternative monetary rules considered in the previous section. It shows that welfare in our benchmark specification is lower than in the Default-Reference model. Consumption equivalent welfare is .019% lower in our benchmark model because, although our model has lower spreads, it has higher inflation variability. This welfare ranking depends crucially on the coefficient on inflation in the interest rate rule α_P. With a larger α_P coefficient, welfare becomes higher in our NK-Default model than in the Default-Reference model, even though this latter model does not suffer from pricing frictions: for $\alpha_P = 2.5$, consumption equivalent welfare is .009% higher. The ranking arises because the disciplining benefits of monetary frictions on borrowing compensate for the costs arising from inflation volatility. We find that welfare in our benchmark model is actually non-monotonic with respect to the coefficient α_P, reaching an interior maximum at $\alpha_P = 2.5$. The rule with an output gap term performs better than the benchmark but worse than the Default-Reference model and the high α_P case because inflation remains quite volatile.

[27] Consumption equivalence is computed from household welfare V^h using log utility as $C^E = \exp[(1-\beta)V^h]$.

The table also illustrates the importance of the denomination of sovereign debt for the welfare effects of monetary policy. Welfare is higher when sovereign debt is denominated in local currency in our environment with pricing frictions than in the Default-Reference model. Consumption equivalent welfare is .018% higher with pricing frictions under the benchmark interest rate rule parameterization. Interestingly, our results show that local currency debt is superior to foreign currency debt by about 0.037% of consumption. The benefits of local currency debt come from lower spreads and the insurance-like properties discussed in Section 6.1.

Turning to the NK-Reference model without default risk but subject to pricing frictions, we find that welfare is 0.189% higher than in the Default-Reference model. This finding reflects loose borrowing opportunities and the lack of equilibrium default costs. This comparison suggests that the welfare losses derived from default risk are larger than the welfare losses derived from pricing frictions.

Finally, we compare these welfare results with those in Galí and Monacelli (2005). A main finding in that paper is that the optimal monetary policy completely stabilizes domestic inflation and replicates the flexible price model. These results do not hold in our model when the government lacks commitment to repay its debt. With default risk, welfare can be higher with rules that do not completely stabilize inflation, as seen by the higher welfare in the model with a higher α_P coefficient, relative to the Default-Reference model with no price frictions. Nevertheless, our results resemble those in Galí and Monacelli (2005) in that an interest rate rule that targets domestic inflation in useful for welfare. In that paper, such an interest rate rule achieves welfare levels close to the optimal policy because they work well to stabilize domestic inflation. In our framework with default risk, the same interest rate rule is useful to alleviate both frictions because it stabilizes inflation and disciplines default risk.

7 Conclusion

We have developed a framework that combines two important aspects of current policy in emerging markets: sovereign risk in government debt and inflation-targeting as monetary policy. Our work embeds sovereign risk in a New Keynesian model, which is the workhorse framework used for central bank policy. We find that the monetary transmission is altered when economies face sovereign default risk. Monetary policy that targets inflation requires larger fluctuations in nominal rates when the risk of default is a concern. Monetary frictions also discipline sovereign risk and slow down debt accumulation. These results are robust to salient extensions of the benchmark environment, including debt denominated in local currency and rich interest rate rules. Quantitatively, the model replicates key moments in emerging market data, including the comovements of spreads with domestic nominal rates and inflation, and it provides insight into the conduct of monetary policy during Brazil's 2015 recession. These results show that the current standard paradigm in central banks is incomplete and points toward incorporating sovereign risk.

In the last decade, many emerging markets have been successful in bringing down inflation and giving central banks the independence to maintain stable prices. Monetary theory has been an important pillar in guiding such processes, but such theory was developed for countries with deep and well-functioning financial markets. The growing literature on the interaction between monetary theory and financial frictions has identified that some of the lessons are modified, yet many open questions remain. An important question relates to the optimality of interest rate rules and on whether these rules should depend not only on inflation but also on financial conditions. Aoki et al. (2018) show that a combination of capital controls and monetary policy works best for economies with pricing frictions and financial frictions arising in banks with balance sheet concerns. Arellano et al. (2019) show that in economies with sovereign risk, an interest rate rule that also responds to sovereign spreads goes a long way toward preventing overborrowing while maintaining the benefits of low inflation. Low inflation, however, has not been uniformly achieved in emerging markets, especially those with chronic financial crises, such as in Argentina and Turkey. An important open question involves the links between financial imperfections and the inability of governments to achieve a successful monetary policy.

References

Aguiar, M., M. Amador, E. Farhi, and G. Gopinath (2013, October). Crisis and commitment: Inflation credibility and the vulnerability to sovereign debt crises. Working Paper 19516, National Bureau of Economic Research.

Aguiar, M., M. Amador, and S. Fourakis (2019). On the welfare losses from external sovereign borrowing.

Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics 69*(1), 64–83.

Aoki, K., G. Benigno, and N. Kiyotaki (2018). Monetary and financial policies in emerging markets. Princeton university working paper.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review 98*(3), 690–712.

Arellano, C., Y. Bai, and G. Mihalache (2019). Optimal simple monetary rules with sovereign risk. University of rochester mimeo.

Bacchetta, P., E. Perazzi, and E. van Wincoop (2018). Self-fulfilling debt crises: What can monetary policy do? *Journal of International Economics 110*, 119 – 134.

Bianchi, J. and J. Mondragon (2018, December). Monetary Independence and Rollover Crises. Working paper, Federal Reserve Bank of Minneapolis.

Bianchi, J., P. Ottonello, and I. Presno (2018, March). Fiscal policy, sovereign risk and unemployment.

Calvo, G. A. (1988). Servicing the public debt: The role of expectations. *The American Economic Review 78*(4), 647–661.

Chatterjee, S. and B. Eyigungor (2012). Maturity, indebtedness, and default risk. *American Economic Review 102*(6), 2674–2699.

Christiano, L. J., M. Trabandt, and K. Walentin (2011). Introducing financial frictions and unemployment into a small open economy model. *Journal of Economic Dynamics and Control 35*(12), 1999 – 2041. Frontiers in Structural Macroeconomic Modeling.

Corsetti, G. and L. Dedola (2016). The mystery of the printing press: Monetary policy and self-fulfilling debt crises. *Journal of the European Economic Association 14*(6), 1329–1371.

Corsetti, G., L. Dedola, and S. Leduc (2010). Chapter 16 - optimal monetary policy in open economies. Volume 3 of *Handbook of Monetary Economics*, pp. 861 – 933. Elsevier.

Cruces, J. J. and C. Trebesch (2013, July). Sovereign defaults: The price of haircuts. *American Economic Journal: Macroeconomics 5*(3), 85–117.

Cuadra, G. and H. Sapriza (2008). Sovereign default, interest rates and political uncertainty in emerging markets. *Journal of International Economics 76*(1), 78 – 88.

Devereux, M. B., E. R. Young, and C. Yu (2019). Capital controls and monetary policy in sudden-stop economies. *Journal of Monetary Economics 103*, 52 – 74.

Du, W. and J. Schreger (2016). Local currency sovereign risk. *Journal of Finance 71*(3), 1027–1069.

Dvorkin, M., J. M. Sanchez, H. Sapriza, and E. Yurdagul (2018, June). Sovereign Debt Restructurings. Working Papers 2018-13, Federal Reserve Bank of St. Louis.

Eaton, J. and M. Gersovitz (1981). Debt with Potential Repudiation: Theoretical and Empirical Analysis. *Review of Economic Studies 48*(2), 289–309.

Engel, C. and J. Park (2019, May). Debauchery and original sin: The currency composition of sovereign debt. University of wisconsin working paper.

Fanelli, S. (2017). Monetary policy, capital controls, and international portfolios.

Farhi, E. and I. Werning (2012, June). Dealing with the trilemma: Optimal capital controls with fixed exchange rates. Working Paper 18199, National Bureau of Economic Research.

Galí, J. and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies 72*(3), 707–734.

Gordon, G. (2019, September). Efficient Computation with Taste Shocks. Working Paper 19-15, Federal Reserve Bank of Richmond.

Ha, J., M. A. Kose, and F. Ohnsorge (2019). *Inflation in Emerging and Developing Economies*.

Hatchondo, J. C. and L. Martinez (2009). Long-duration bonds and sovereign defaults. *Journal of International Economics 79*(1), 117 – 125.

Hatchondo, J. C., L. Martinez, and H. Sapriza (2009, November). Heterogeneous Borrowers In Quantitative Models Of Sovereign Default. *International Economic Review 50*(4), 1129–1151.

Hatchondo, J. C., L. Martinez, and C. Sosa-Padilla (2016). Debt dilution and sovereign default risk. *Journal of Political Economy 124*(5), 1383–1422.

Hur, S., I. O. Kondo, and F. Perri (2018, December). Real Interest Rates, Inflation, and Default. Staff Report 574, Federal Reserve Bank of Minneapolis.

Koop, G., M. H. Pesaran, and S. M. Potter (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics 74*(1), 119–147.

Lorenzoni, G. and I. Werning (2019). Slow moving debt crises. *American Economic Review*.

Miao, J. and P. Ngo (2018). Does calvo meet rotemberg at the zero lower bound? Boston university working paper.

Na, S., S. Schmitt-Grohé, M. Uribe, and V. Yue (2018, July). The twin ds: Optimal default and devaluation. *American Economic Review 108*(7), 1773–1819.

Nuno, G. and C. Thomas (2019, February). Monetary policy and sovereign debt vulnerability. Technical report, Bank of Spain.

Ottonello, P. and D. Perez (2019). The currency composition of sovereign debt. *American Economic Journal: Macroeconomics*.

Roger, S. (2009). Inflation targeting at 20 - achievements and challenges.

Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy 90*(6), 1187–1211.

Schmitt-Grohé, S. and M. Uribe (2003). Closing small open economy models. *Journal of International Economics 61*(1), 163–185.

Schmitt-Grohé, S. and M. Uribe (2007). Optimal simple and implementable monetary and fiscal rules. *Journal of Monetary Economics 54*(6), 1702 – 1725.

Sunder-Plassmann, L. (2018). Inflation, default and sovereign debt: The role of denomination and ownership.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195 – 214.

ONLINE APPENDIX TO "MONETARY POLICY AND SOVEREIGN RISK IN EMERGING ECONOMIES (NK-DEFAULT)"

BY CRISTINA ARELLANO, YAN BAI, AND GABRIEL MIHALACHE

A Deriving Government's Optimal Borrowing Equation

In this appendix, we derive the government's optimal borrowing equations in Section 3.2 for both the baseline model and the model with perfect rigid prices. To illustrate the government's incentive for borrowings, we assume all the policy functions are differentiable with respect to the state B and the first order conditions are sufficient for the government's optimization problem.

Baseline Model For any given state (s, B) with $s = (z, m)$ and default cost shock ν, the government chooses to default if and only if $W^d(z^d, m) - \nu \geq W(s, B)$. Clearly there exists a cutoff level of $\nu^*(s, B) = W^d(z^d, m) - W(s, B)$, the government defaults if $\nu \leq \nu^*(s, B)$. The default probability can be defined as $\Phi_d(s, B') = \Phi(\nu^*(s, B'))$ where $\Phi(.)$ is the distribution function of ν.

Conditional on not defaulting, the government chooses $\{C, N, C^f, \pi, b'\}$ to solve the following problem,

$$W(s, B) = \max \ u(C, C^f, N) + \beta_g E_{s'|s} \left\{ \int_{\nu^*(s,B')} W(s', B') d\Phi(\nu) + \int^{\nu^*(s,B')} [W^d(s^d, m) - \nu] d\Phi(\nu) \right\} \tag{50}$$

subject to the private equilibrium constraints

$$C+[C^f+(\delta+r^*)B-q(s,B')\left(B'-(1-\delta)B\right)]^{\frac{\rho}{\rho-1}}=\left[1-\frac{\varphi}{2}\left(\pi-\bar{\pi}\right)^2\right]zN \qquad (\lambda) \tag{51}$$

$$\frac{u_{C^f}}{u_C}=\frac{\rho}{\rho-1}[C^f+(\delta+r^*)B-q(s,B')\left(B'-(1-\delta)B\right)]^{\frac{1}{\rho-1}} \qquad (\lambda_e) \tag{52}$$

$$\beta R\left(\frac{\pi}{\bar{\pi}}\right)^{\rho_p}mM(s,B')=u_C \qquad (\kappa) \tag{53}$$

$$\frac{1}{z}\frac{u_N}{u_C}+\frac{1}{u_C zN}F(s,B')=1+\frac{1}{\eta-1}\varphi\left(\pi-\bar{\pi}\right)\pi \qquad (\gamma) \tag{54}$$

and the functions $M(s,B')$ and $F(s,B')$ as defined in (27) and (28). Let λ, λ_e, κ, and γ be the Lagrange multipliers on the resources constraint (51), the relative demand condition (52), domestic Euler condition (53), and the NKPC condition (54), respectively. Note that we have substitute the terms of trade e and nominal interest rate i using the balance of payment condition (22) and the monetary rule (25). Positive κ is associated with the left hand side less than the right hand of (53),.

The first order conditions over C, C^f, N, π, and B' are

$$u_C-\lambda-\lambda_e\frac{u_{CC^f}u_C-u_{CC}u_{C^f}}{(u_C)^2}+\kappa u_{CC}+\gamma\frac{u_{CC}}{u_C}\left[\frac{u_n}{zu_C}+\frac{1}{zNu_C}F(s,B')\right]=0,$$

$$u_{C^f}-\lambda\frac{\rho}{\rho-1}e+\lambda_e\frac{1}{\rho-1}\frac{\rho}{\rho-1}e^{2-\rho}-\lambda_e\frac{u_{C^fC^f}u_C-u_{CC^f}u_{C^f}}{(u_C)^2}+\kappa u_{CC^f}+\gamma\frac{u_{CC^f}}{u_C^2}\left[\frac{u_n}{z}+\frac{1}{zN}F(s,B')\right]=0$$

$$-u_N+\lambda\left[1-\frac{\varphi}{2}\left(\pi-\bar{\pi}\right)^2\right]z-\gamma\left(\frac{u_{NN}}{zu_C}-\frac{F(s,B')}{N^2zu_C}\right)=0$$

$$-\lambda\varphi(\pi-\bar{\pi})zN-\kappa\rho_p\frac{u_C}{\pi}+\gamma\frac{1}{\eta-1}\varphi(2\pi-\bar{\pi})=0$$

$$\begin{aligned}&\left[q+\frac{dq}{dB'}\left(B'-(1-\delta)B\right)\right]\left\{\lambda\frac{\rho}{\rho-1}e-\lambda_e\frac{1}{\rho-1}\frac{\rho}{\rho-1}e^{2-\rho}\right\}-\beta i\frac{\partial M}{\partial B'}\kappa-\frac{1}{u_C zN}\frac{\partial F}{\partial B'}\gamma\\&=\beta_g E(1-\Phi(s',B'))(\delta+r+(1-\delta)q(s',B''))\left\{\lambda'\frac{\rho}{\rho-1}e'-\lambda_e'\frac{1}{\rho-1}\frac{\rho}{\rho-1}(e')^{2-\rho}\right\}\end{aligned} \tag{55}$$

We can replace the multipliers λ and λ_e with allocations

$$\lambda=u_C-u_C\left[\frac{1}{\rho}\frac{\rho^{\frac{1}{\rho-1}}e^{1-\rho}C^f+1}{\rho^{\frac{1}{\rho-1}}e^{1-\rho}u_CCC^f+u_CC+u_{C^f}C^f}+\frac{\rho-1}{\rho}\right]\left[u_C\kappa+\left(1+\frac{1}{\eta-1}\varphi(\pi-\bar{\pi})\pi\right)\gamma\right]$$

$$\lambda\frac{\rho}{\rho-1}e-\lambda_e\frac{1}{\rho-1}\frac{\rho}{\rho-1}e^{2-\rho}$$
$$=u_{C^f}-u_{C^f}\left(\frac{1}{\rho}\frac{1}{\rho\frac{1}{\rho-1}e^{1-\rho}u_CCC^f+u_CC+u_{C^f}C^f}+\frac{\rho-1}{\rho}\right)\left[u_C\kappa+\left(1+\frac{1}{\eta-1}\varphi(\pi-\bar{\pi})\pi\right)\gamma\right].$$

Let

$$G^X=\left(\frac{1}{\rho}\frac{1}{\rho\frac{1}{\rho-1}e^{1-\rho}u_CCC^f+u_CC+u_{C^f}C^f}+\frac{\rho-1}{\rho}\right)$$

It is easy to see that G^X is positive.

We can then define the borrowing wedges as

$$\tau_m^X=G^Xu_C\kappa+G^X\left(1+\frac{1}{\eta-1}\varphi(\pi-\bar{\pi})\pi\right)\gamma$$
$$\tau_m^C=\beta i\frac{\partial M}{\partial B'}\kappa+\frac{1}{u_CY}\frac{\partial F}{\partial B'}\gamma.$$

we can rewrite the government's Euler equation (55) as

$$u_{C^f}\left[q+\frac{dq}{dB'}\left(B'-(1-\delta)B\right)\right](1-\tau_m^X)-\tau_m^C\quad=\beta_gE(1-\Phi(s',B'))(\delta+r+(1-\delta)q(s',B''))(1-\tau_m^{X\prime}).\tag{56}$$

Model with perfectly rigid prices In this model, firms do not choose prices the NKPC condition 54 drops. The inflation rate remains constant at $\tilde{\pi}$. Conditional on not defaulting, the government chooses $\{C,N,C^f,b'\}$ to maximize the repaying value (50) subject to three constraints (51), (52), and (53)

$$C+[C^f+(\delta+r^*)B-q(s,B')\left(B'-(1-\delta)B\right)]^{\frac{\rho}{\rho-1}}=zN\qquad(\lambda)\tag{57}$$
$$\frac{u_{C^f}}{u_C}=\frac{\rho}{\rho-1}[C^f+(\delta+r^*)B-q(s,B')\left(B'-(1-\delta)B\right)]^{\frac{1}{\rho-1}}\qquad(\lambda_e)\tag{58}$$
$$\beta iM(s,B')=u_C\qquad(\kappa)\tag{59}$$

with the nominal rate $i=Rm$. The expected marginal utility function $M(s,B')$ is given by $M(s,B')=\frac{1}{\tilde{\pi}}\mathbb{E}_{s'|s}u_c(S')$. We can write the first order conditions as

$$u_C-\lambda-\lambda_e\frac{u_{CC^f}u_C-u_{CC}u_{C^f}}{(u_C)^2}+\kappa u_{CC}=0,$$

$$u_{C^f}-\lambda\frac{\rho}{\rho-1}e+\lambda_e\frac{1}{\rho-1}\frac{\rho}{\rho-1}e^{2-\rho}-\lambda_e\frac{u_{C^fC^f}u_C-u_{CC^f}u_{C^f}}{(u_C)^2}+\kappa u_{CC^f}=0$$

$$\lambda = u_N/z$$

$$\left[q + \frac{dq}{dB'}\left(B' - (1-\delta)B\right)\right]\left\{\lambda\frac{\rho}{\rho-1}e - \lambda_e\frac{1}{\rho-1}\frac{\rho}{\rho-1}e^{2-\rho}\right\} - \beta i\frac{\partial M}{\partial B'}\kappa$$
$$= \beta_g E(1-\Phi(s',B'))(\delta + r + (1-\delta)q(s',B''))\left\{\lambda'\frac{\rho}{\rho-1}e' - \lambda_e'\frac{1}{\rho-1}\frac{\rho}{\rho-1}(e')^{2-\rho}\right\}.$$

In this case, we can see explicitly that the multiplier κ is directly related to the monetary wedge zu_C/u_N and is given by

$$\kappa = \frac{1}{G}\left[1 - \frac{u_N}{zu_C}\right]$$

with G function defined as

$$G = \frac{1}{\rho}\frac{\rho\frac{1}{\rho-1}e^{1-\rho}C^f + 1}{\rho\frac{1}{\rho-1}e^{1-\rho}u_C CC^f + u_C C + u_{C^f}C^f} + \frac{\rho-1}{\rho}.$$

The borrowing wedges becomes

$$\tau_m^X = \left[1 - \frac{u_N}{zu_C}\right]\frac{u_C G^X}{G}, \qquad \tau_m^C = \left[1 - \frac{u_N}{zu_C}\right]\frac{\partial M}{\partial B'}\frac{\beta i}{G}.$$

B Proofs

B.1 Proof of Lemma 1

Proof. Conditional on not defaulting, the second period's consumption $C_2(z)$ solves

$$C_2 + [C_2(\rho-1)/\rho]^\rho = z\,(z/C_2)^\zeta. \tag{60}$$

Using implicit function theorem, we get derivate of C_2 over z

$$\frac{\partial C_2(z)}{\partial z} = \frac{(1+\zeta)z^\zeta C_2^{-\zeta}}{\zeta z^{1+\zeta}C_2^{-\zeta-1} + 1 + \rho C_2^{\rho-1}\left[(\rho-1)/\rho\right]^\rho} \geq 0. \tag{61}$$

The consumption under default satisfies equation (60) with the productivity given by z_{2d}. Let's define the utility function $\mathcal{U}(C_2, z)$ as

$$\begin{aligned}\mathcal{U}(C,z) &\equiv u(C, [C(\rho-1)/\rho]^{\rho-1}, (z/C)^\zeta) \\ &= \log(C) + [C(\rho-1)/\rho]^{\rho-1} - \chi\frac{(z/C)^{\frac{\zeta}{1+\zeta}}}{1+1/\zeta}.\end{aligned}$$

It is easy to show that the repaying value $W_2(B)$ is linear in B, $W_2(B) = \mathcal{U}(C_2, z) - B$. The defaulting value is given by $\mathcal{U}(C_{2d}, z_d) - \nu$ for any default cost ν. Assume the labor elasticity ζ is small enough that $\partial\mathcal{U}/\partial z \geq 0$.

The default cutoff $\nu^*(B; z)$ equalize the defaulting value and the repaying value, i.e.,

$$\mathcal{U}(C_2(z), z) - B = \mathcal{U}(C_{2d}, z_d) - \nu^*.$$

We can simply write

$$\nu^*(B; z) = \mathcal{U}(C_{2d}(z_d), z_d) - \mathcal{U}(C_2(z), z) + B.$$

We can take the derivatives of ν^*,

$$\frac{\partial \nu^*(B; z)}{\partial z} = -\frac{\partial \mathcal{U}}{\partial z} \leq 0.$$

The default probability is given by $\Phi(\nu^*(B; z))$ and it depends on the mean of the default cost $\bar{\nu}$. The derivatives are given by

$$\begin{aligned}
\frac{\partial \Phi(\nu^*(B; z))}{\partial B} &= \phi(\nu^*(B; z))\frac{\partial \nu^*(B; z)}{\partial B} = \phi(\nu^*(B; z)) \geq 0 \\
\frac{\partial \Phi(\nu^*(B; z))}{\partial z} &= \phi(\nu^*(B; z))\frac{\partial \nu^*(B; z)}{\partial z} = -\phi(\nu^*(B; z))\frac{\partial \mathcal{U}}{\partial z} \leq 0 \\
\frac{\partial \Phi(\nu^*(B; z))}{\partial \bar{\nu}} &= -\sigma_\nu \phi(\nu^*(B; z)) \leq 0.
\end{aligned}$$

Hence the default probability $\Phi(\nu^*)$ increases with debt B, decreases with z, and decreases with the mean default cost $\bar{\nu}$. □

B.2 Proof of Proposition 1

Proof. We first characterize the private equilibrium for any government choice of borrowing B in the first period,

$$C_1 + e_1^\rho = N_1, \tag{62}$$

$$C_1 = \frac{\rho}{\rho - 1} e_1, \tag{63}$$

$$\frac{1}{C_1} = \frac{\beta i}{\tilde{\pi}} \left[\frac{1 - \Phi(\nu^*(B))}{C_2} + \frac{\Phi(\nu^*(B))}{C_{2d}} \right], \tag{64}$$

$$e_1^\rho = e_1 \left[C_1^f - (1 - \Phi(B))B \right]. \tag{65}$$

The monetary wedge in the first period is defined as $1/\left[N_1^{1/\zeta} C_1\right]$.

Log-differentiating the private equilibrium in the first period, we have

$$\frac{C_1}{N_1} d \log C_1 + \rho \frac{e_1^{\rho}}{N_1} d \log e_1 = d \log N_1$$
$$d \log C_1 = d \log e_1$$
$$d \log C_1 = -\frac{C_1 \beta \; i \; \Phi(\nu^*)}{\tilde{\pi}} \left(\frac{1}{C_{2d}} - \frac{1}{C_2} \right) d \log \Phi(\nu^*)$$
$$d \log \text{monetary wedge} = -\left(\frac{1}{\zeta} d \log N_1 + d \log C_1 \right).$$

Hence

$$d \log N_1 = \left(\frac{C_1}{N_1} + \rho \frac{e_1^{\rho}}{N_1} \right) d \log C_1$$

Since $C_{2d} \leq C_2$, we have consumption decreases when there is a larger default risk or higher $\bar{\nu}$,

$$\frac{d \log \text{monetary wedge}}{d \log \Phi} = -\left[\frac{1}{\zeta} \left(\frac{C_1}{N_1} + \rho \frac{e_1^{\rho}}{N_1} \right) + 1 \right] \frac{d \log C_1}{d \log \Phi(\nu^*)}$$

$$\frac{d \log \text{monetary wedge}}{d \log \Phi} = \left[\frac{1}{\zeta} \left(\frac{C_1}{N_1} + \rho \frac{e_1^{\rho}}{N_1} \right) + 1 \right] \frac{C_1 \beta \; i \; \Phi(\nu^*)}{\tilde{\pi}} \left(\frac{1}{C_{2d}} - \frac{1}{C_2} \right) \geq 0$$

Higher default rate raises monetary wedge.

□

B.3 Proof of Lemma 2

Proof. For the model with perfectly price frictions, We can solve the allocations of $C_1(B)$ and $N_1(B)$ from the system of equation (62)-(64)

$$C_1(B) = \frac{1}{\beta(i/\tilde{\pi})} \frac{1}{\frac{1-\Phi(\nu^*(B))}{C_2} + \frac{\Phi(\nu^*(B))}{C_{2d}}}, \tag{66}$$
$$N_1(B) = C_1(B) + \left(\frac{\rho - 1}{\rho} \right)^{\rho} C_1(B)^{\rho}. \tag{67}$$

Similarly, the equilibrium allocations in the flexible price model satisfies domestic Euler equation and the resource constraint under the optimal choice of B_{flex},

$$C_1^{\text{flex}} = \frac{1}{\beta r^{\text{flex}}} \frac{1}{\frac{1-\Phi(\nu^*(B_{\text{flex}}^*))}{C_2} + \frac{\Phi(\nu^*(B_{\text{flex}}^*))}{C_{2d}}}, \tag{68}$$
$$N_1^{\text{flex}} = C_1^{\text{flex}} + \left(\frac{\rho - 1}{\rho} \right)^{\rho} (C_1^{\text{flex}})^{\rho}. \tag{69}$$

We now show that when $B \geq B^*_{\text{flex}}$, $C_1(B) \leq C_1^{\text{flex}}$ for two reasons. First, according to Proposition 1, when $B \geq B^*_{\text{flex}}$, default risk is higher, $\Phi(\nu^*(B)) \geq \Phi(\nu^*(B^*_{\text{flex}}))$. This together with $C_{2d} \leq C_2$ due to default punishment implies the future marginal utility is higher in the model with price frictions,

$$\frac{1-\Phi(\nu^*(B))}{C_2} + \frac{\Phi(\nu^*(B))}{C_{2d}} \geq \frac{1-\Phi(\nu^*(B^*_{\text{flex}}))}{C_2} + \frac{\Phi(\nu^*(B^*_{\text{flex}}))}{C_{2d}}.$$

Second, according to Assumption 1, the real interest rate is higher in the model with perfectly rigid prices, $i/\bar{\pi} \geq r^{\text{flex}}$. Higher default risk together with the high real interest rate implies $C_1(B) \leq C_1^{\text{flex}}$, which can be seen from the comparison of equation (66) and (68).

Comparing (67) and (69), we can see that $N_1(B) \leq N_1^{\text{flex}}$ because $C_1(B) \leq C_1^{\text{flex}}$. It is therefore $N_1(B)^{1/\zeta} C_1(B) \leq (N_1^{\text{flex}})^{1/\zeta} C_1^{\text{flex}} = 1$, and the monetary wedge in the model with price frictions, monetary wedge ≥ 1, which implies $\tau_m^C(B) \geq 0$. □

B.4 Proof of Proposition 2

Proof. We first derive the government's Euler equations under the baseline model and under the flexible price model, (42) and (44). Under the flexible price model, the government chooses B, C_1, C_1^f, and N_1 to maximize

$$\max u(C_1, C_1^f, N_1) + \beta \max\left\{W_2(B), W_2^d - \nu\right\}$$

subject to the resources constraint

$$C_1 + \left(C_1^f - \frac{1}{1+r^*}[1-\Phi(\nu^*(B))]B\right)^{\frac{\rho}{\rho-1}} = N_1.$$

The first order condition on B gives arise (44).

In our baseline model, the government chooses B, C_1, C_1^f, and N_1 to maximize

$$\max u\left(C_1(B), C_1^f(B), N_1(B)\right) + \beta_g \left\{[1-\Phi(\nu^*(B))]W_2(B) + \int_{-\infty}^{\nu^*(B)} (W_2^d - \nu) d\Phi(\nu)\right\},$$

subject to

$$C_1 + \left[C_1^f - \frac{1}{1+r^*}[1-\Phi(\nu^*(B))]B\right]^{\frac{\rho}{\rho-1}} = N_1 \qquad (\lambda)$$

$$C_1 = \frac{\rho}{\rho-1}\left[C_1^f - \frac{1}{1+r^*}[1-\Phi(\nu^*(B))]B\right]^{\frac{1}{\rho-1}} \qquad (\lambda_e)$$

$$\frac{1}{C_1} = \beta i M(B) \qquad (\kappa)$$

where the expected future marginal utility function $M(B)$ is given by

$$M(B) = \frac{1}{\tilde{\pi}} \left\{ [1 - \Phi(\nu^*(B))] \frac{1}{C_2} + \Phi(\nu^*(B)) \frac{1}{C_{2d}} \right\}.$$

The derivative $\partial M/\partial B$ is given by

$$\frac{\partial M}{\partial B} = \frac{\phi(\nu^*(B))}{\tilde{\pi}} \left[\frac{1}{C_{2d}} - \frac{1}{C_2} \right] \tag{70}$$

where we apply the derivative of $\partial \nu^*/\partial B = 1$. Note that $\partial M/\partial B \geq 0$ since default lowers consumption, $C_{2d} \leq C_2$.

Let λ, λ_e, and κ be the multiplier on the budget constraint, relative demand condition, and domestic Euler condition, respectilvely. We can write the first order conditions on C_1, C_1^f, N_1, and B' as

$$u_C - \lambda - \lambda_e - \kappa \frac{1}{C^2} = 0 \tag{71}$$

$$1 - \lambda \frac{\rho}{\rho - 1} e + \lambda_e \frac{\rho}{\rho - 1} \frac{1}{\rho - 1} e^{2-\rho} = 0 \tag{72}$$

$$u_N = \lambda z \tag{73}$$

$$\left[q + \frac{\partial q}{\partial B} B \right] \left(\lambda \frac{\rho}{\rho - 1} e - \lambda_e \frac{\rho}{\rho - 1} \frac{1}{\rho - 1} e^{2-\rho} \right) - \beta i \frac{\partial M}{\partial B} \kappa = \beta [1 - \Phi(\nu^*(B))]. \tag{74}$$

According to the FOC over C^f (72), $\lambda \frac{\rho}{\rho-1} e - \lambda_e \frac{\rho}{\rho-1} \frac{1}{\rho-1} e^{2-\rho} = 1$. Define

$$\tau_m^C(B) = \left(1 - \frac{u_{N_1}(B)}{u_{C_1}(B)} \right) \frac{[1 + (\rho - 1) e_1(B)^{\rho-1}] \beta i \phi(\nu^*(B))}{u_{C_1}(B) \tilde{\pi}} (u_{C_{2d}} - u_{C_2}),$$

we can show equation (42) holds.

We now prove that default probability is lower in the baseline model, $\Phi^* \leq \Phi^*_{\text{flex}}$. We prove by contradiction. Suppose $\Phi^* > \Phi^*_{\text{flex}}$ because $B^* > B^*_{\text{flex}}$. We show that if this is the case, the Euler equation (42) of the baseline model does not hold, which violates the optimization of B^*.

Suppose $B^* > B^*_{\text{flex}}$, according to Lemma 2, $\tau_m^C(B^*) > 0$. Together with the property of increasing hazard function $h(\nu)$, we can show the following inequalities hold,

$$h(\nu^*(B^*)) B^* + \frac{\tau_m^C(B^*)}{1 - \Phi(\nu^*(B^*))} > h(\nu^*(B^*)) B^* \geq h(\nu^*(B^*_{\text{flex}})) B^*_{\text{flex}} = 1 - \beta_g (1 + r^*).$$

We reach a contradiction on the optimality of B^*. In equilibriu, it has to be the case that $B^* \leq B^*_{\text{flex}}$ and $\Phi^* \leq \Phi^*_{\text{flex}}$.

$\square$

C Impulse Response Functions to Monetary Shock

In the event analysis we perform a counterfactual to Brazil by altering the path of nominal rates using monetary shocks m. In this appendix, we present detailed impulse response functions for high monetary shocks m for aggregate output, domestic consumption, imports, inflation, the nominal interest rate, terms of trade, debt, and spreads for our benchmark model augmented to have monetary shocks such that $\log m \sim \mathcal{N}(0, 0.001)$. Figure 10 plots the responses to a 1% in m shock. The solid blue lines are for the benchmark NK-Default model while the red dashed lines are the NK-Reference model. In our model tight monetary policy depresses output, domestic and imported consumption, inflation, and leads to a decline in borrowing and sovereign spreads.

D Extensions and Robustness

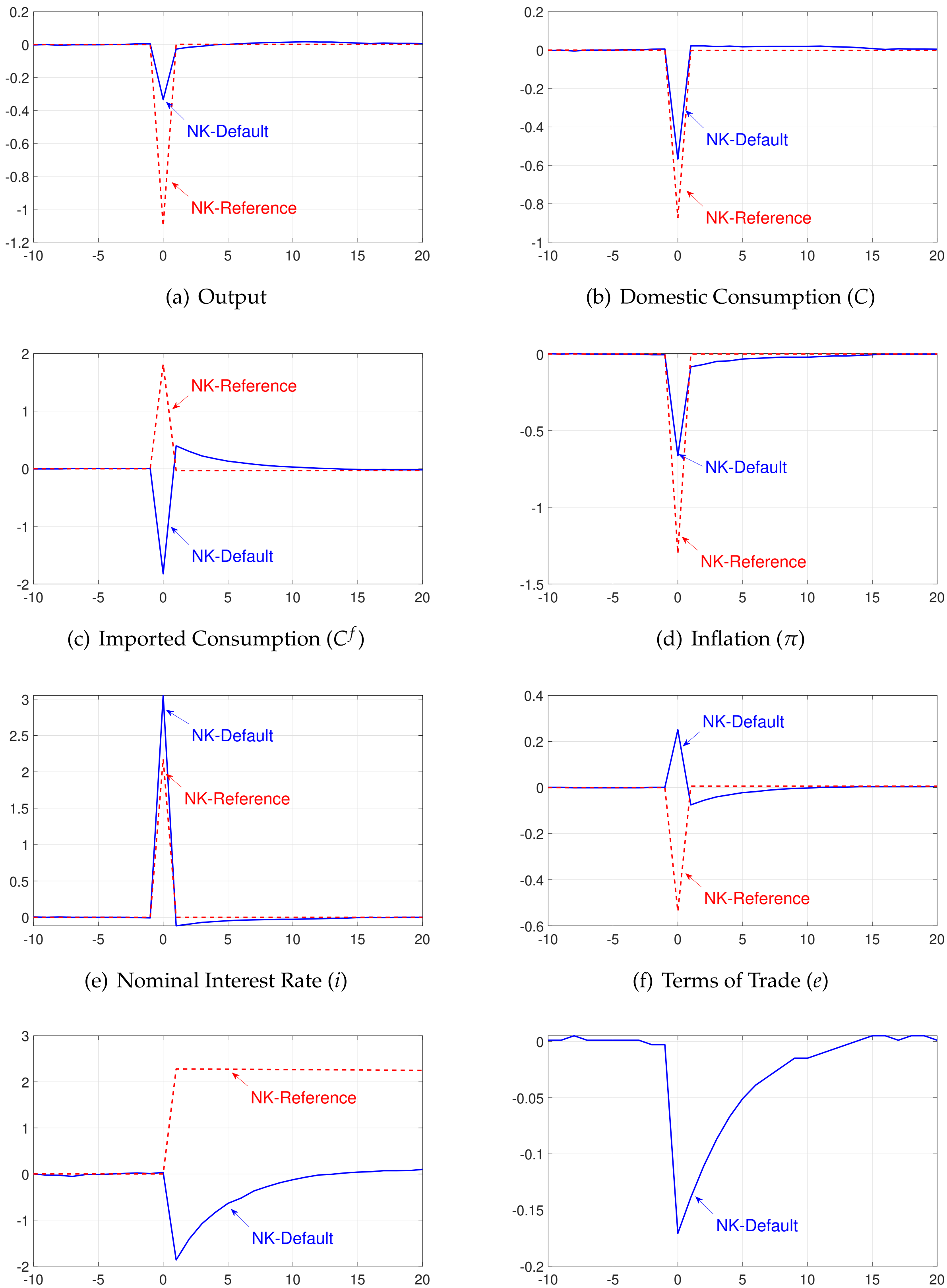
This appendix contains additional plots for the extensions and robustness exercises in Section 6. We report Impulse Response Functions to low productivity realizations, mirroring Figure 6, for the 3 extensions considered: local currency government debt in Figure 11, an interest rate rule with a higher weight on inflation in Figure 12, and the interest rate rule with an output gap term in Figure 13. In all 3 figures, the solid blue line is our baseline NK-Default model while the dashed red line is the extended model.

Figure 14 plots the labor wedge (left panels) and the nominal interest rate (right panels) against the level of debt for the benchmark model, in solid blue, and extended models, in dashed red. This Figure shows that the analysis in Section 5.5 applies to the extended models, as the behavior of the labor wedge and nominal rates in the two High and Low Default Zones, respectively, is similar to the baseline model.

E NK-Reference with Global Methods

In the main text, we use as the NK-Reference model a version of Galí and Monacelli (2005) solved using local methods with very loose borrowing constraints. This appendix contains results from a version of our baseline model reparameterized to have loose borrowing constraints, no default risk in equilibrium, and solved using global methods. We show that despite seemingly different international borrowing Euler equations (33) and (36), arising from households borrowing in the NK-Reference model while the government borrowing in our model, the results from these are very similar.

We reparameterize the NK-Default model to match key properties of the NK-Reference environment: spreads with zero mean and zero volatility, and consumption volatility in line with the data. To implement these alternative targets we start with the baseline parameter



(a) Output

(b) Domestic Consumption (C)

(c) Imported Consumption (C^f)

(d) Inflation (π)

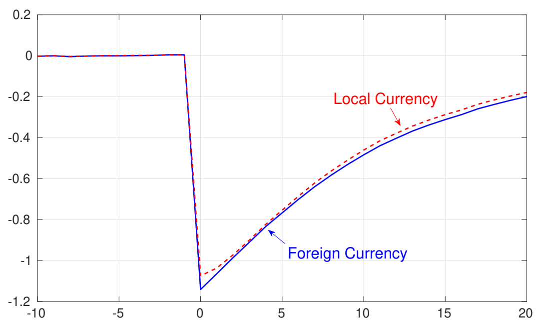
(e) Nominal Interest Rate (i)

(f) Terms of Trade (e)

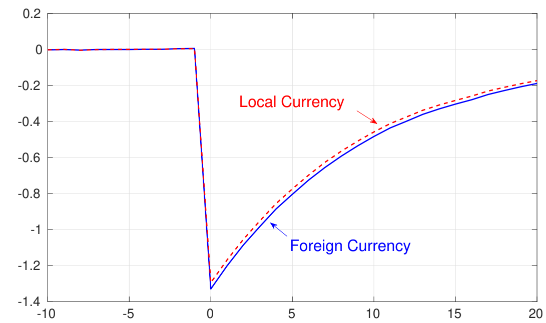
(g) Debt (B)

(h) Spread

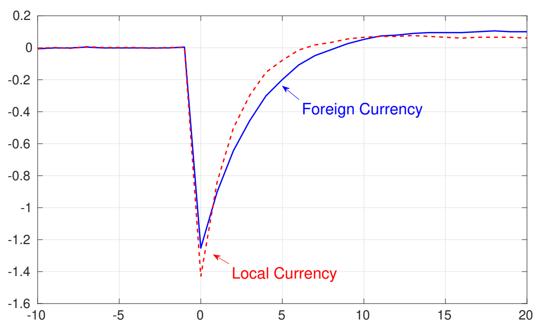
Figure 10: Impulse Responses to Monetary Shock



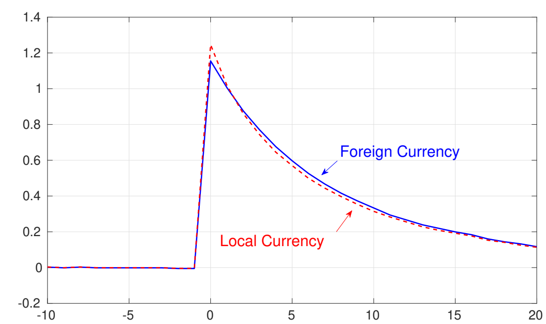
(a) Output



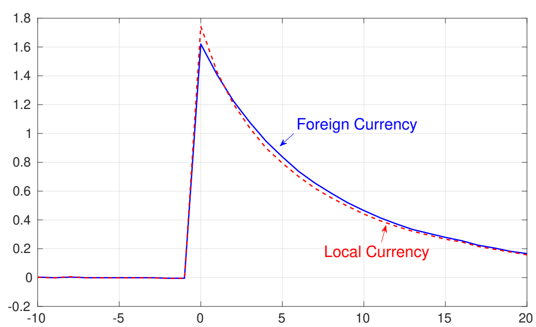
(b) Domestic Consumption (C)



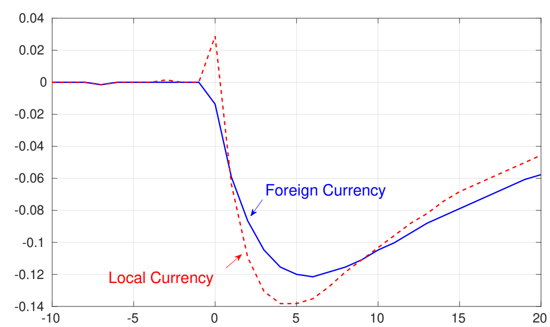
(c) Imported Consumption (C^f)



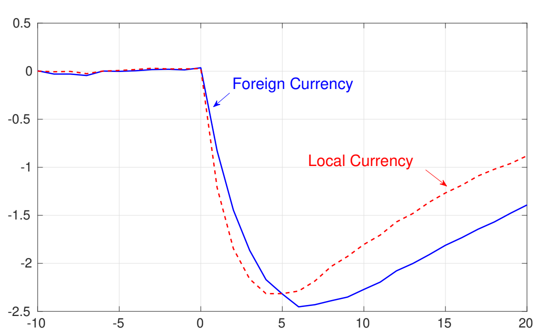
(d) Inflation (π)



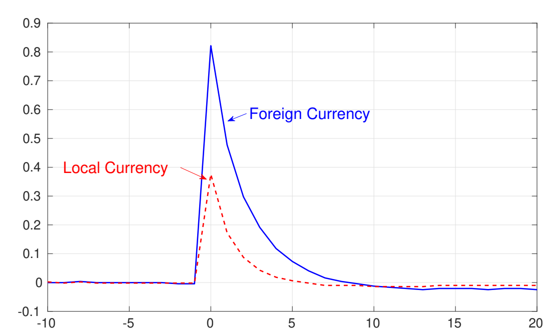
(e) Nominal Interest Rate (i)



(f) Terms of Trade (e)

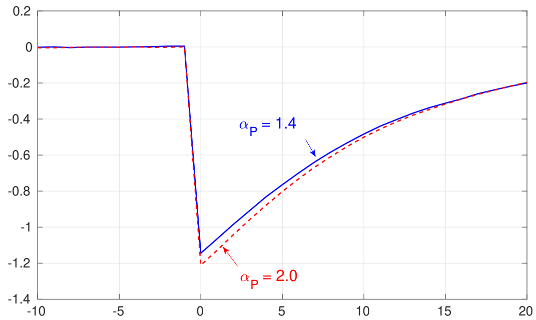


(g) Debt (B)

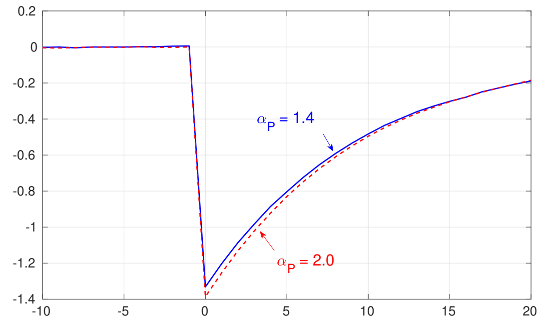


(h) Spread

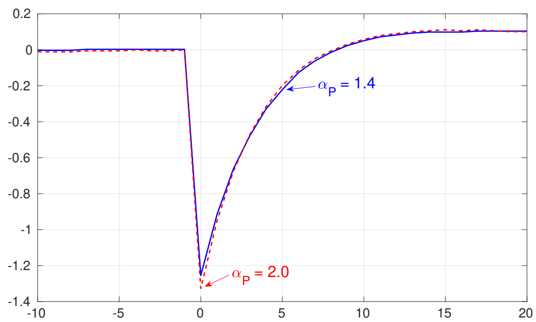
Figure 11: Local Currency: Impulse Responses to Productivity Shock



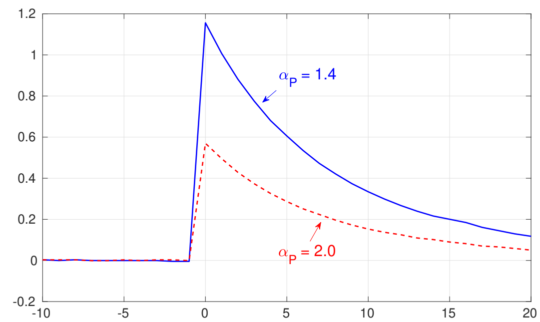
(a) Output



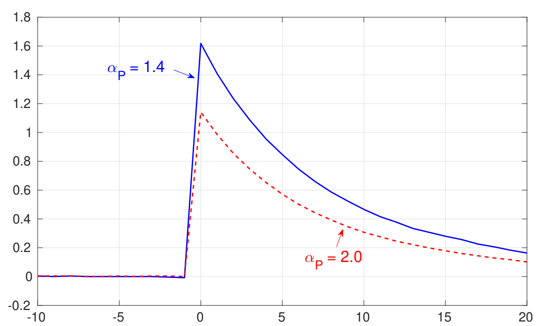
(b) Domestic Consumption (C)



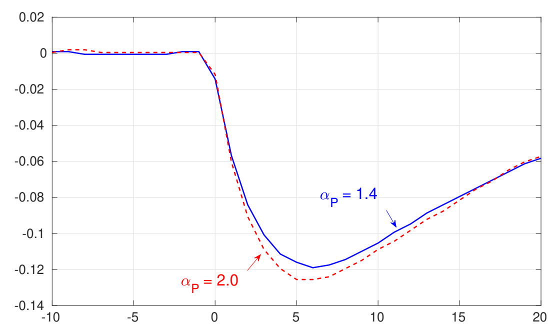
(c) Imported Consumption (C^f)



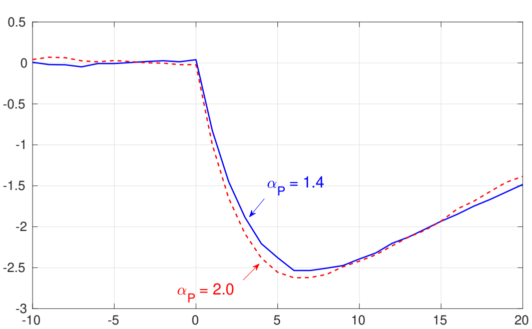
(d) Inflation (π)



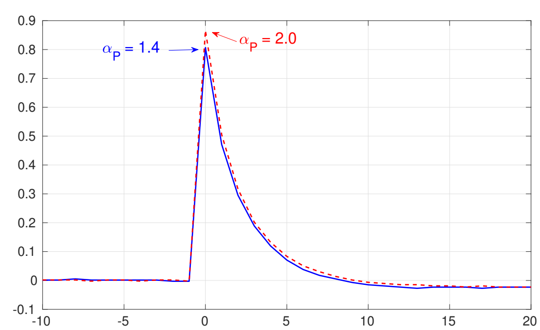
(e) Nominal Interest Rate (i)



(f) Terms of Trade (e)

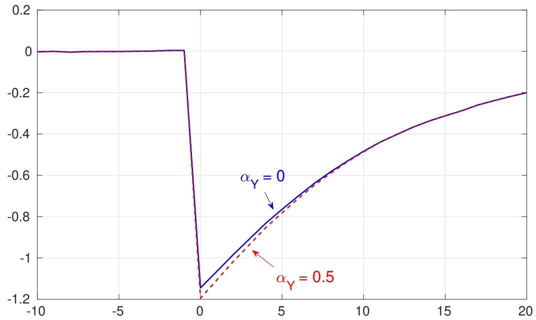


(g) Debt (B)

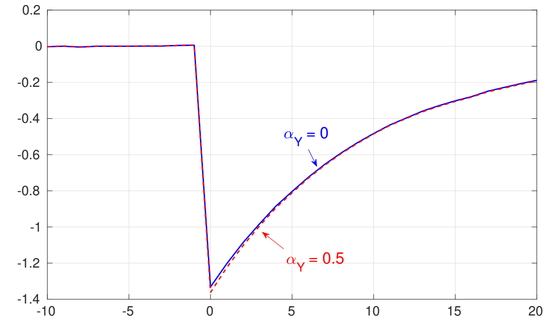


(h) Spread

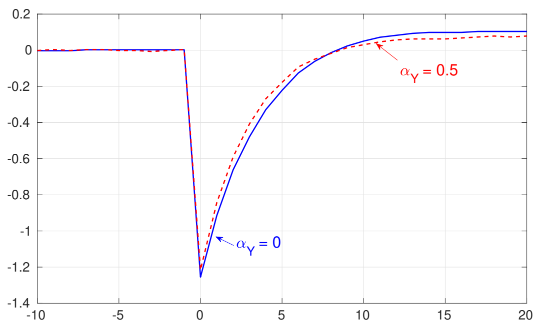
Figure 12: Rule with Higher α_P: Impulse Responses to Productivity Shock



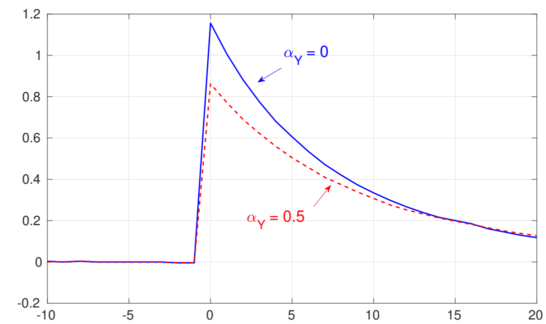
(a) Output



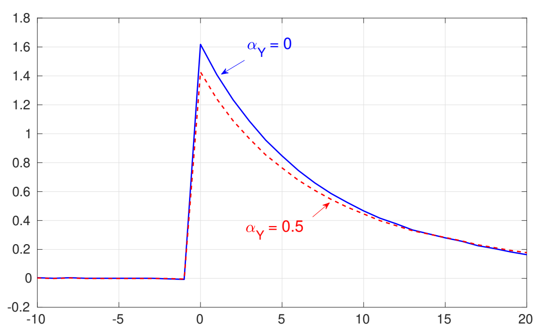
(b) Domestic Consumption (C)



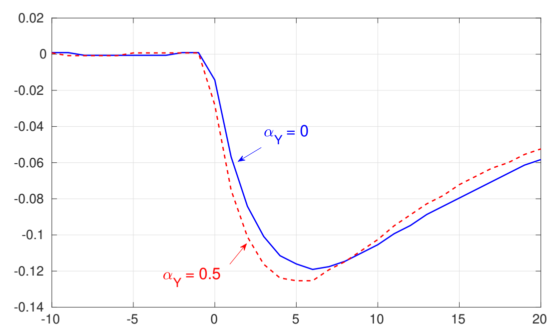
(c) Imported Consumption (C^f)



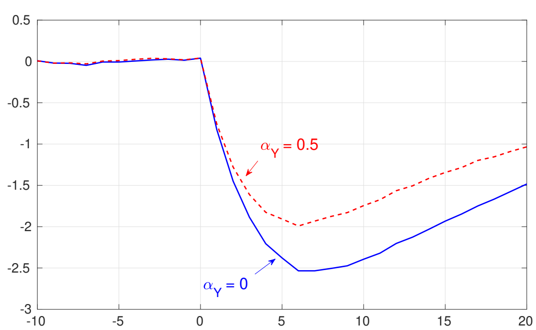
(d) Inflation (π)



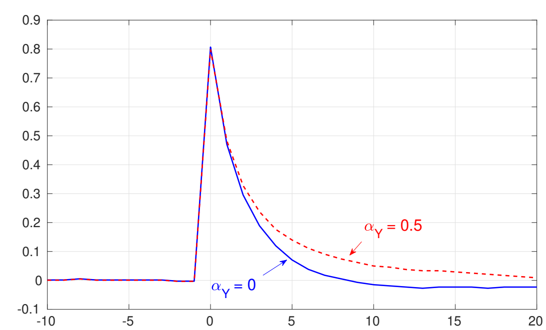
(e) Nominal Interest Rate (i)



(f) Terms of Trade (e)

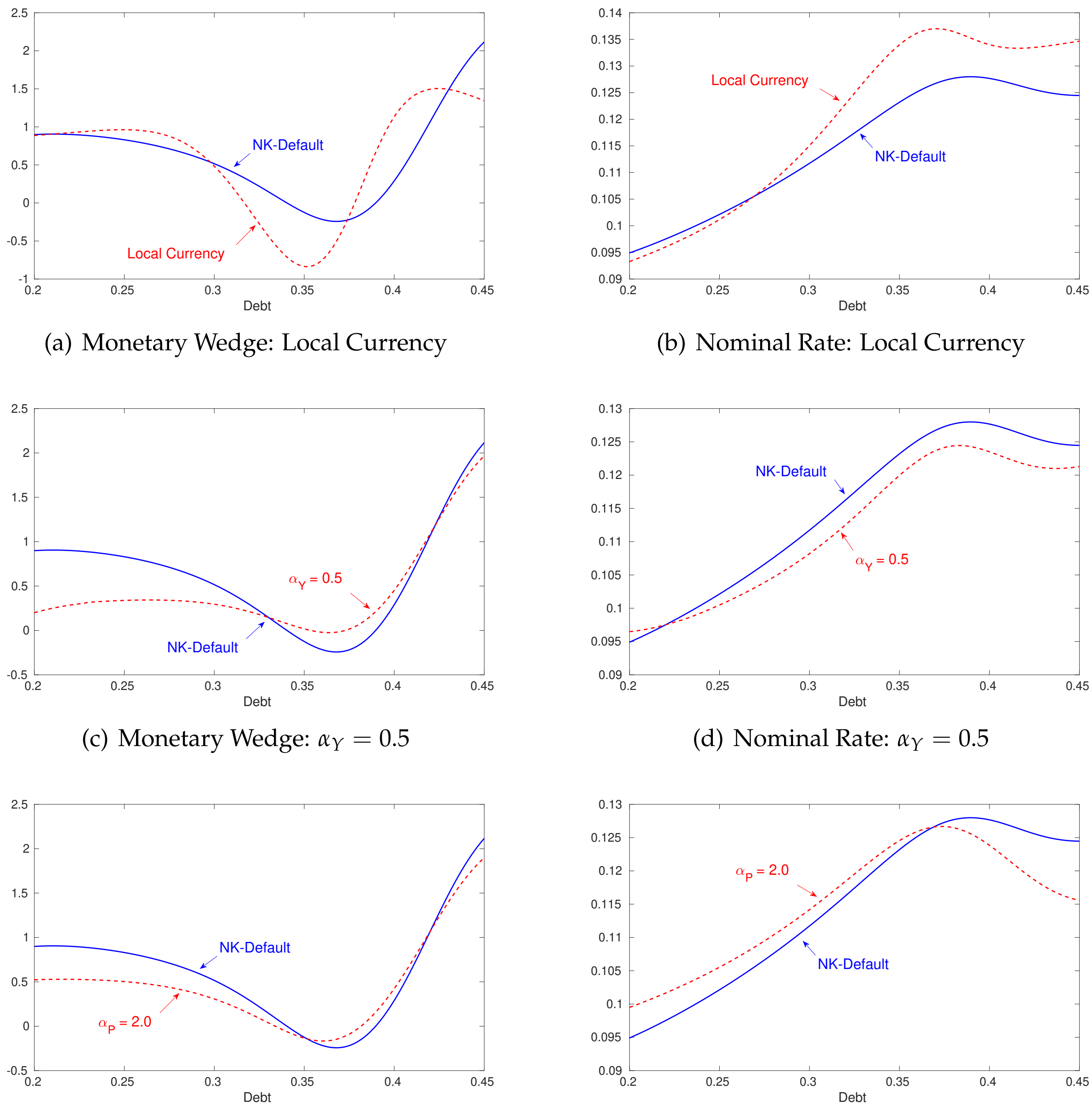


(g) Debt (B)



(h) Spread

Figure 13: Rule of Output Gap: Impulse Responses to Productivity Shock



(a) Monetary Wedge: Local Currency

(b) Nominal Rate: Local Currency

(c) Monetary Wedge: $\alpha_Y = 0.5$

(d) Nominal Rate: $\alpha_Y = 0.5$

(e) Monetary Wedge: $\alpha_P = 2.0$

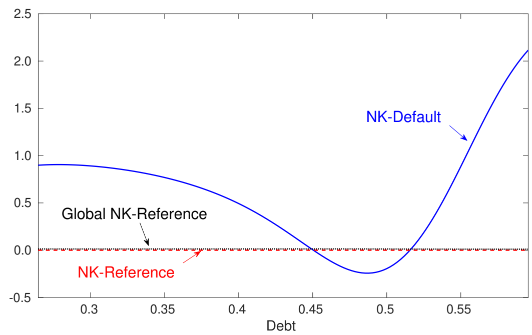
(f) Nominal Rate: $\alpha_P = 2.0$

Figure 14: Robustness: Monetary Wedges and Nominal Rates

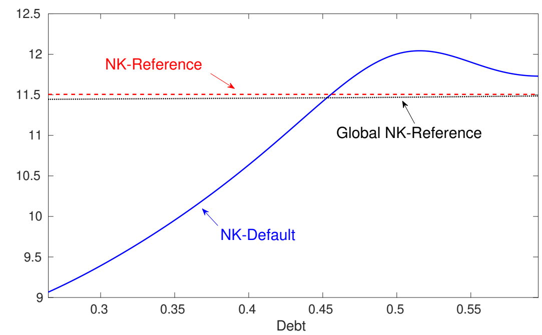
values but set $\lambda_0 = -0.145$, $\varrho_D \to 0$, and $\beta_g = \beta$. These parameters result in ample borrowing opportunities and spreads with mean and volatility less than 1 basis point. Table 7 compares the second moments of NK-Reference and the reparameterized NK-Default model, which we label Global NK-Reference. Their behavior is very similar: the volatility of CPI inflation and the domestic nominal rate are close and much lower than in the baseline model, while the trade balance to output ratio is strongly procyclical and 4 times more volatile than in baseline, a tell-tale sign of the greatly relaxed financial frictions. In Figure (15), we also show that the monetary wedge and the nominal interest rate function from the Global NK-Reference model is very similar to the NK-Reference model, in contrast to the resulting functions with default risk.

Mean	NK-Reference	Global NK-Reference
CPI Inflation	6.1	6.1
Domestic Rate	11.5	11.5
Spread	—	0.0
Standard Deviation		
CPI Inflation	1.0	1.1
Domestic Rate	1.3	1.2
Spread	—	0.0
Output	2.4	2.4
Consumption Aggregate	2.0	1.6
Trade Balance	1.8	1.3
Nominal Depreciation Rate	2.0	2.2
Correlation with Output		
CPI Inflation	−81	−36
Domestic Rate	−98	−78
Trade Balance	62	78
Nominal Depreciation Rate	−28	−1

Table 7: NK-Reference Results Using Global Methods



(a) The Monetary Wedge



(b) Nominal Interest Rate (i)

Figure 15: Debt and the Monetary Wedge, Reparameterized NK-Default

F Numerical Implementation

F.1 Computation with Taste Shocks

We compute the model using discrete choice methods, following Dvorkin et al. (2018) and Gordon (2019), who adapt tools frequently used in structural applied work for the study of sovereign default with long-term debt. Chatterjee and Eyigungor (2012) follow a related strategy, also perturbing the borrowing choice (B'), in order to address the convergence problems inherent in models with long-term debt.

We restrict the choice of B' to be in a discrete set and associate each option with an iid taste shock distributed Gumbel (Extreme Value Type I). The government's problem becomes

$$\mathcal{W}\left(s, B, \langle\epsilon_{B'}\rangle\right) = \max_{B'} \left\{\mathcal{J}(s, B, B') + \varrho_B\, \epsilon_{B'}\right\} \tag{75}$$

with

$$\mathcal{J}(s, B, B') \equiv u\left[C(s, B, B'), C^f(s, B, B'), N(s, B, B')\right] + \beta_g \mathbb{E}_{s'|s} V(s', B') \tag{76}$$

and where ϱ_B is a constant governing the relative importance of the taste shocks for the choice of B' and $\langle\epsilon_{B'}\rangle$ is a vector of taste shocks, one for each possible value of B' on the grid. As $\varrho_B \to 0$ we recover the unperturbed initial problem, with poor numerical convergence properties, while as $\varrho_B \to +\infty$ the taste shocks dominate and the choice of B' become uniform iid. Ex-ante, before taste shocks are realized, the choice probabilities are given by

$$\Pr\left(B' = x | s, B\right) = \frac{\exp\left[\mathcal{J}(s, B, x)/\varrho_B\right]}{\sum_{\tilde{x}} \exp\left[\mathcal{J}(s, B, \tilde{x})/\varrho_B\right]} = \frac{\exp\left[\left(\mathcal{J}(s, B, x) - \overline{\mathcal{J}}(s, B)\right)/\varrho_B\right]}{\sum_{\tilde{x}} \exp\left[\left(\mathcal{J}(s, B, \tilde{x}) - \overline{\mathcal{J}}(s, B)\right)/\varrho_B\right]} \tag{77}$$

with $\overline{\mathcal{J}}(s,B) = \max_{B'} \mathcal{J}(s,B,B')$ and government's value is

$$W(s,B) = \mathbb{E}_{\langle \epsilon_{B'} \rangle} \left\{ \mathcal{W}\left(s,B,\langle \epsilon_{B'} \rangle\right) \right\} = \overline{\mathcal{J}}(s,B) + \varrho_B \log \left\{ \sum_{B'} \frac{\mathcal{J}(s,B,B') - \overline{\mathcal{J}}(s,B)}{\varrho_B} \right\}. \tag{78}$$

$\overline{\mathcal{J}}(s,B)$ is the value the government would achieve if all the taste shock would be zero (or if the problem were unperturbed) while $W(s,B)$ is the expected value before the realization of the taste shocks. Panel (a) of Figure 16 plots an example of choice probabilities, associated with state $\langle z=1, B=0.25 \rangle$. The probability mass is tightly centered around the B' that maximizes $\mathcal{J}(s,B,B')$.

Additionally, we perturb the default decision in a similar fashion. At the start of each period, the government observes default decision taste shocks and decides accordingly:

$$V(s,B) = \mathbb{E}_{\epsilon_{\text{Repay}},\epsilon_{\text{Default}}} \max \left\{ W(s,B) + \varrho_D\, \epsilon_{\text{Repay}},\; W_d(s) + \varrho_D\, \epsilon_{\text{Default}} \right\} \tag{79}$$

As a consequence, if state $\langle s,B \rangle$ is realized, the government chooses default with probability

$$\Pr(D=1|s,B) = \frac{\exp\left[W_d(s)/\varrho_D\right]}{\exp\left[W_d(s)/\varrho_D\right] + \exp\left[W(s,B)/\varrho_D\right]} \tag{80}$$

For values of ϱ_D greater than zero, the default probability is everywhere nondegenerate, although often numerically indistinguishable from zero or one. This induces bond price schedules that are smooth in the borrowing choice B', further aiding numerical convergence. Panel (b) of Figure 16 plots "borrowing Laffer curves" ($q(s,B')B'$) for various levels of the productivity shock.

In the model augmented with taste shocks, the expression for the bond price schedule becomes

$$q(s,B') = \frac{1}{1+r^*} \mathbb{E}_{s'|s} \Pr(D=0|s',B') \left\{ r^* + \delta + (1-\delta) \sum_{B''} \Pr(B''|s',B') q(s',B'') \right\} \tag{81}$$

The expectation functions $M(s,B')$ and $F(s,B')$ are updated analogously.

Note that the enforcement shocks ν in the model map into the default-repayment taste shocks as follows

$$\begin{aligned} V(s,B) &= \mathbb{E}_{\epsilon_{\text{Repay}},\epsilon_{\text{Default}}} \max \left\{ W(s,B) + \varrho_D\, \epsilon_{\text{Repay}},\; W_d(s) + \varrho_D\, \epsilon_{\text{Default}} \right\} \\ &= \mathbb{E}_{\epsilon_{\text{Repay}},\epsilon_{\text{Default}}} \max\{ W(s,B),\; W_d(s) + \varrho_D\, \underbrace{(\epsilon_{\text{Default}} - \epsilon_{\text{Repay}})}_{\nu} \}. \end{aligned}$$

The ϵ terms are iid Gumbel (Extreme Value Type I) with location parameter given by minus the Euler-Mascheroni constant and scale parameter 1 and, as a result, their difference ν follows the Logistic distribution with location parameter 0 and scale 1. The parameter ϱ_D controls the relative importance of the enforcement shock for the default decision.

F.2 Algorithm

The model is subject to an AR(1) productivity shock z, which we discretize over a grid with $\#z = 21$ points spanning ± 3 standard deviations of the unconditional distribution. We also allow for a zero-probability shock to the interest rate rule m, which we discretize over $\#m = 7$ points spanning $\pm 1.5\%$. The m shock is iid, with $\Pr(m = 1) = 1$ and $\Pr(m \neq 1) = 0$. We use these zero probability shock to study the consequences of unexpected monetary tightening in the quantitative analysis. The B grid consists of $\#B = 250$ points equally spaced over $[0, 0.5]$.

The algorithm proceeds as follows

1. We start with initial guesses for the value functions V_0, W_0^d and the bond price schedule q_0, together with guesses for the F_0 and M_0 functions and the default and borrowing policies. We assume the probability of default is 1 and $B' = B$ with probability one, everywhere in the state space.

2. We solve for for the Private and Monetary Equilibrium everywhere in the state space, for arbitrary B'. We restrict attention to B' values that do not induce "too large" capital inflows or outflows, for which a PME might not exist and confirm that this restriction does not bind in equilibrium: $|-(r^* + \delta)B + q(s, B')(B' - \delta B)| < 0.1$.

   We solve the PME via root-finding using Powell's hybrid method, on a system of two equations in two unknowns, C^f and N:

   (a) Use current guess of $\langle C^f, N \rangle$ and the capital inflow $-\lambda B + q(s, B')(B' - \delta B)$, we compute the terms of trade e from the Balance of Payments condition.

   (b) We compute the implied level of exports X associated with the Terms of Trade e.

   (c) Given C^f and e we can recover domestic consumption C from the relative consumption condition.

   (d) Given C and the government's borrowing choice B' we compute the domestic nominal rate i from the domestic Euler equation.

   (e) Given i we use the interest rate rule to compute the level of PPI inflation π.

   (f) We use these quantities to compute equation residuals for the New Keynesian Philips Curve and the domestic resource constraint.

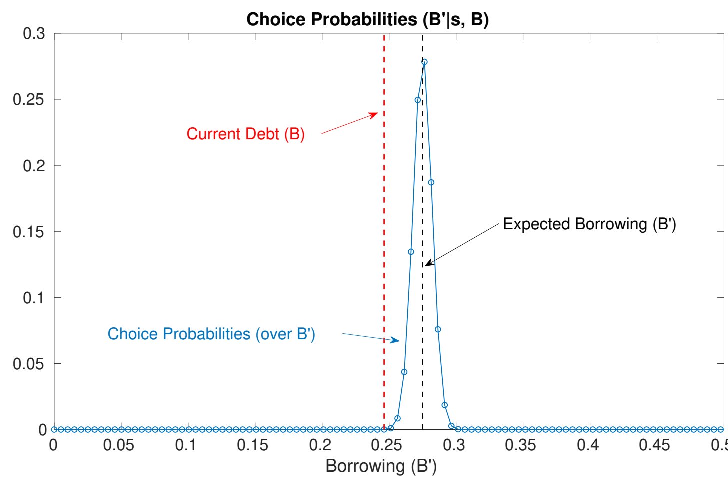
   The solution to the PME yields policy functions $C(s, B, B')$, $C^f(s, B, B')$, $N(s, B, B')$, $\pi(s, B, B')$, $i(s, B, B')$, $e(s, B, B')$.

3. We solve the PME in default similarly. In particular, in default trade is balanced and the capital inflow term is zero, and productivity is penalized $h(z) < z$. The solution constitutes policy functions in default: $C_d(s)$, $C_d^f(s)$, $N_d(s)$, $\pi_d(s)$, $i_d(s)$, $e_d(s)$.

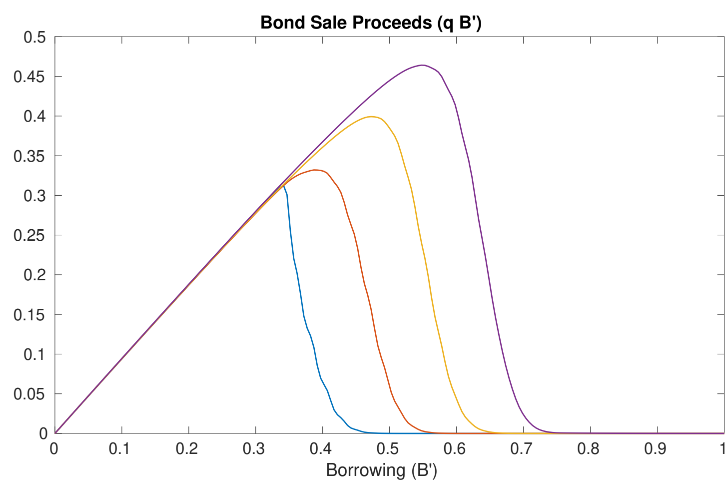
4. Using PME results, we compute the value of the government in each state (V) and in default (W_d) and derive choice probabilities for the B' policy and default probabilities.

5. Given borrowing and default policies (probabilities) we update the bond price schedule q and the expectation functions M and F.

6. We check for the convergence of the bond price schedule, value functions, and expectation functions. We stop if values are closer than $1e^{-7}$ and prices closer than $1e^{-5}$ in the sup norm, otherwise we fully update and iterate.



(a) Choice Probabilities ($\varrho_B = 1.0e^{-5}$)



(b) Bond Price Schedules ($\varrho_D = 1.0e^{-4}$)

Figure 16: Computation with Taste Shocks