Demand Imbalances, Global Inflation and Monetary Policy Coordination*

Javier Bianchi
Federal Reserve Bank of Minneapolis

Louphou Coulibaly
Federal Reserve Bank of Minneapolis, University of Wisconsin-Madison & NBER

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Abstract

This paper studies the scope for international monetary cooperation in an environment with global demand imbalances and financial spillovers. We show that the absence of international monetary policy cooperation may lead to over- or under-tightening depending on two sufficient statistics: differences in labor intensity across sectors and the sign of the output gap in the competitive equilibrium.

Keywords: Global inflation, international monetary policy spillovers, coordination

JEL Classifications: E21, E23, E43, E44, E52, E62, F32

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Central banks nearly everywhere feel accused of being on the back foot. The present danger, however, is not so much that current and planned moves will fail eventually to quell inflation. It is that they collectively go too far and drive the world economy into an unnecessarily harsh contraction...by simultaneously all going in the same direction, they risk reinforcing each other's policy impacts without taking that feedback loop into account.

Maurice Obstfeld "Uncoordinated monetary policies risk a historic global slowdown," blog post, Peterson Institute, 09/12/2022

1 Introduction

Following a prolonged phase of expansionary monetary policy, central banks around the world have begun a tightening cycle in an effort to quell inflation and cool down the economy. Concerns, however, have emerged about the fast pace of collective interest rate hikes and the extent to which it can trigger an unnecessarily sharp contraction in global economic activity (Obstfeld, 2022).[1]

Does a cooperative monetary policy indeed call for a more expansionary monetary policy? Or is it possible that countries under-tighten relative to the socially optimal level? More generally, what are the gains from international monetary policy coordination?

A traditional argument for the desirability of monetary policy coordination is that it prevents strategic terms of trade manipulation. According to this argument, every central bank has incentives to lower domestic output to move the terms of trade in its favor. Following this logic, central banks find it individually optimal to keep a tighter monetary policy relative to what is optimal for the world economy.[2]

On the other hand, policy discussions on international monetary policy cooperation tend to orbit around questions of macro-stabilization. For example, a widespread concern expressed by policymakers is that increases in foreign policy rates can lead to pressures on domestic inflation and the value of a country's currency. It is unclear, however, why under a flexible exchange rate regime, an appropriate response of the domestic central

[1]See the Peterson Institute blog post by Maurice Obstfeld "Uncoordinated Monetary Policies Risk a Historic Global Slowdown", from September 12, 2022, which has spearheaded extensive policy discussions.

[2]This argument is at the heart of the New Open Economy Macroeconomics (NOEM) framework in which countries have monopoly power over the good they produce. See Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2005) for early contributions and Corsetti, Dedola and Leduc (2010) for a review of this literature.

bank would be unable to insulate the economy from foreign monetary policy shocks and why a cooperative arrangement would achieve a superior collective outcome.

In this paper, we take a different perspective, focusing on a financial channel of international monetary policy spillovers. At the center of our model is the idea that foreign monetary policies affect the world real interest rate and, through this channel, the degree of domestic demand imbalances. Our analysis builds on Bianchi and Coulibaly (2021), who show that international monetary policy spillovers operate through an aggregate demand externality in the presence of an occasionally binding zero lower bound constraint, and Fornaro and Romei (2022), who study policy cooperation in a related framework with costly inflation, which we generalize, in particular, by allowing for the possibility of overheating and non-tradable inflation.

We demonstrate that the Nash equilibrium may feature over- or under-tightening relative to the cooperative solution, depending on the state of demand imbalances and differences in labor intensities across sectors. Under the assumptions that non-tradables are more labor intensive than tradables, we obtain the result that a coordinated monetary policy calls for more expansionary policy when the economy is facing a recession, echoing the results in Fornaro and Romei (2022). However, if the economy is overheating, we show that coordinated monetary policy calls for a more contractionary policy. Intuitively, when the output gap is positive, a higher real interest rate helps cool down the economy. Individual central banks do not internalize that raising the nominal rate leads to an increase in the world real rate and improves the output-inflation tradeoff abroad. Therefore, they under-tighten relative to the Nash equilibrium. In addition, we find that the Nash equilibrium may also feature under-tightening if the economy is in a recession, but tradables are more labor intensive. Finally, if labor intensities are the same across sectors, we find that there is no scope for monetary policy cooperation regardless of the sign of the output gap. A key contribution of our paper is to establish analytically that whether cooperation calls for lower or higher rates can be framed entirely in terms of the sign of the output gap and the differences in labor intensity across the tradable and non-tradable sectors.

Related Literature. There is a vast literature on international monetary policy coordination going back to Hamada (1976), Oudiz and Sachs (1984) and Canzoneri and Henderson (1991). Two pioneer papers adopting a microfounded approach to cooperative monetary policy are Obstfeld and Rogoff (2002) and Corsetti and Pesenti (2005) (see also, e.g., Benigno and Benigno, 2006; Devereux and Engel, 2003; Egorov and Mukhin, 2020; and Bodenstein, Corsetti and Guerrieri, 2020). A key theme in this literature is that individual countries have incentives to lower their own production to shift terms of trade in their favor at the expense of other countries. According to this optimal tariff argument, central banks generally over-tighten monetary policy relative to the socially optimal level. We highlight instead a financial channel, which generates the possibility of under-tightening.

In Bianchi and Coulibaly (2021), we show that the extent to which a foreign monetary policy shock is welfare improving for the domestic economy depends on whether it increases the vulnerability to a zero lower bound. Moreover, we show that capital controls can insulate the economy from changes in foreign monetary policy. The setup we consider in this paper also builds on Fornaro and Romei (2022). They show how an increase in the preference for tradables, acts as a cost-push shock, leading in equilibrium to inflation and a negative output gap. Crucially, they argue that a cooperative monetary policy would implement higher output levels relative to the Nash equilibrium because countries do not internalize that letting domestic inflation rise induces positive spillover effects to the rest of the world. Our analysis shows that there is also a possibility of under-tightening. More generally, we establish analytically that, independently of the shocks, whether cooperation calls for lower or higher rates depend on the degree of slack in the economy and differences in labor intensities across sectors. [3,4]

The key mechanism at play in our model is also related to the literature on aggregate demand externalities and pecuniary externalities.[5] In Farhi and Werning (2016) and Schmitt-Grohé and Uribe (2016), nominal rigidities and constraints on monetary policy generate aggregate demand externalities, leading to a welfare scope for capital controls. In this paper, we consider a model in which monetary policy faces no constraints, but inflation is costly, and sectorial shocks break divine coincidence. In turn, as individual countries seek to deal with the domestic output-inflation tradeoff, this generates a pecuniary externality through the world real interest rate. In this literature, there is scope for individual countries to use capital controls in the presence of constraints on monetary policy. In this paper, we

[3] Our setup differs by considering an elastic labor supply, binding upward and downward rigid wage rigidity, and decreasing returns to scale in labor in the non-tradable sector, which in turn allow for the possibility of overheating and non-tradable inflation. In their setup wages are perfectly sticky in the first period, but given that households have a fixed endowment of labor and there is no disutility cost from working, the possibility of overheating does not emerge. Indeed, they only consider a shock that leads to involuntary unemployment while non-tradable inflation is fixed because the production function is linear in employment.

[4] Acharya and Bengui (2018), Eggertsson, Mehrotra, Singh and Summers (2016), Caballero, Farhi and Gourinchas (2021), and Fornaro and Romei (2019) study the propagation of liquidity traps across countries, but do not consider the scope for monetary policy cooperation. For the empirical literature on international monetary policy spillovers, see, for example, Rey (2013) and Kalemli-Ozcan (2019).

[5] See, for example, Bianchi (2011), Farhi and Werning (2016), Schmitt-Grohé and Uribe (2016), Fornaro and Romei (2019), Bianchi and Coulibaly (2021), Bengui and Coulibaly (2022).

consider a model in which monetary policy faces no constraints, but inflation is costly, and sectorial shocks break divine coincidence.

Finally, there has been an active recent literature on sectorial reallocation and the connection with the rise of inflation following the Covid-19 pandemic.[6] Relative to this literature, our paper provides a normative analysis of the importance of global monetary policy coordination.

2 Model

Time is discrete and infinite. We consider a world economy composed of a continuum of identical small open economies $k \in [0,1]$ with two types of goods: tradables and non-tradables. The environment is deterministic and features nominal rigidities.

2.1 Households

Each economy k is populated by a continuum of identical households of measure one.[7] Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\log c_t - n_t + \frac{\chi}{2} \left(\pi_t - \bar{\pi} \right)^2 \right], \tag{1}$$

Households have log utility over the consumption good c_t which is a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a Cobb-Douglass aggregator:

$$c_t = \left(c_t^T \right)^{\phi_t^T} \left(c_t^N \right)^{\phi_t^N}$$

where $\phi_t^T, \phi_t^N \in (0,1)$ captures the preference for tradables and non-tradables respectively, with $\phi_t^N = 1 - \phi_t^T$. In addition, they have a linear disutility from working $n_t = n_t^T + n_t^N$ hours where n_t^T denotes hours worked in the tradable sector and n_t^N denotes aggregate hours worked in the non-tradable sector. Finally, households are assumed to face a utility cost of inflation which is quadratic on deviations of inflation $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$ from a target $\bar{\pi}$. The parameter $\beta \in (0,1)$ represents the discount factor and $\chi > 0$ is the disutility from inflation.

[6] See, for example, Rubbo (2020), Guerrieri, Lorenzoni, Straub and Werning (2021), di Giovanni, Kalemli-Özcan, Silva and Yildirim (2022, 2023), and Baqaee and Farhi (2022).

[7] The notation does not index variables in each country by k to avoid clutter.

We denote by P_t^N and P_t^T respectively, the price of non-tradables and tradables (in terms of the domestic currency). Given the assumption on unitary elasticity of substitution between tradables and non-tradables, the consumer price index P_t satisfies

$$P_t = \frac{1}{(\phi_t^T)^{\phi_t^T}(\phi_t^N)^{\phi_t^N}}(P_t^T)^{\phi_t^T}(P_t^N)^{\phi_t^N}. \tag{2}$$

We assume that the law of one price holds for the tradable good; that is for any country pair k and j, $P_{kt}^T = e_{kt}^j P_{jt}^T$, where e_{kt}^j is the nominal exchange rate defined as the price of the country j currency in terms of the domestic currency, and P_{jt}^T is the price of the tradable good denominated in term of country j's currency.

We will assume that in the initial period, wages are sticky at $\bar{W}$, and for $t > 0$, wages become flexible. In particular, we assume that households are off their labor supply in the initial period and their hours are determined by firms' labor demand, as we will see below.

Each period households receive their labor income, $W_t(n_t^T + n_t^N)$, where $n_t^T + n_t^N$ denotes the hours they work in each sector. In addition, households collect firms' profits, φ_t, and trade two types of one-period non-state-contingent bonds in credit markets: a real international bond that pays a constant net return of R units of tradables, and a nominal domestic bond that pays $\tilde{R}_t$ in units of the domestic currency. The budget constraint of the representative household is therefore given by

$$P_t^T c_t^T + P_t^N c_t^N + \frac{b_{t+1}}{R_t} + \frac{P_t^T b_{t+1}^*}{R_t^*} = W_t(n_t^T + n_t^N) + \varphi_t + b_t + P_t^T b_t^*. \tag{3}$$

where b_t and b_t^* denote respectively the amount of nominal bond and real bond assumed in period $t-1$ and due in period t. The left-hand side represents total expenditures in tradable and non-tradable goods and purchases of bonds, while the right-hand side represents total income, including the returns from bond issuance.

The households' problem consists of choosing sequences of consumption $\{c_t^N, c_t^T\}_{t=0}^{\infty}$, asset positions $\{b_{t+1}, b_{t+1}^*\}_{t=0}^{\infty}$, and hours $\{n_t^T, n_t^N\}_{t=0}^{\infty}$, to maximize the expected present discounted value of utility (1), subject to (3) and taking as given profits $\{\varphi_t\}$, and prices $\{W_t, P_t^N, P_t^T, R_t, R_t^*\}_{t=0}^{\infty}$.

The optimality condition with respect to c_t^T and c_t^N equate the marginal rate of substitution between the two goods to the relative price. Owing to the homotheticity of preference and unitary elasticity of substitution, this implies that the ratio of expenditures on tradable consumption relative to expenditures on non-tradable consumption equals the relative preference for the two goods:

$$\frac{P_t^N c_t^N}{P_t^T c_t^T} = \frac{\phi_t^N}{\phi_t^T}. \tag{4}$$

For $t \geq 1$, households are on their labor supply. Given that disutility of working is linear in the sum of total hours, optimality implies that

$$\frac{c_t^N}{\phi_t^N} = \frac{W_t}{P_t^N}, \qquad \frac{c_t^T}{\phi_t^T} = \frac{W_t}{P_t^T} \tag{5}$$

Finally, the first-order conditions for the nominal and real bond holdings yield the following Euler equation and interest parity condition:

$$\frac{\phi_t^T}{c_t^T} = \beta R_t^* \frac{\phi_{t+1}^T}{c_{t+1}^T} \tag{6}$$

$$R_t^* = R_t \frac{P_t^T}{P_{t+1}^T}, \tag{7}$$

The Euler equation (6) equates the marginal cost of purchasing a real bond with its marginal benefit. Condition (7) is a non-arbitrage condition that relates the return on real international bonds to the return on nominal domestic bonds and the expected inflation in the tradable sector.

2.2 Firms

There is a continuum of firms producing tradable goods and non-tradable goods, both of measure one. Each firm produces the final good using a production function that uses labor as the sole input. We assume that the production functions have decreasing returns $y_t^N = (h_t^N)^{\alpha^N}$ and $y_t^T = (h_t^T)^{\alpha^T}$. Profits for tradables are given by $P^T (h^T)^\alpha - W_t h$ and the expression for firms' profits producing non-tradables is analogous. Firms' optimality conditions then imply that

$$P_t^N \alpha^N (h_t^N)^{\alpha^N - 1} = W_t. \tag{8}$$

$$P_t^T \alpha^T (h_t^T)^{\alpha^T - 1} = W_t. \tag{9}$$

That is, firms equate the value of the marginal product of labor to the nominal wage in the two sectors. Notice that by combining these two equations, we obtain that

$$\frac{h^N}{y^N} = \frac{\alpha^N}{W_t}, \quad \frac{h^T}{y^T} = \frac{\alpha^T}{W_t}$$

and so non-tradables feature higher labor intensity if $\alpha^N > \alpha^T$ and viceversa.

As we mentioned before, we assume that the nominal wage is fully rigid at date $t = 0$ and that hours are determined by labor demand. That is, given prices, hours in equilibrium in each sector will be determined at $t = 0$ by (8) and (9) evaluated at $W_0 = \bar{W}$.

2.3 Monetary Policy

The policy instrument is the sequence of nominal interest rates $\{R_t\}$. Because of the assumption that prices are flexible for $t > 0$, we assume monetary policy targets inflation equal to $\bar{\pi}$ for $t > 0$. This implies that the nominal rate follows $R_t = R_t^*(1 + \bar{\pi})$ for $t > 0$. For $t = 0$, we will evaluate the optimal monetary policy, comparing the cooperative and non-cooperative outcomes.

2.4 Market Clearing Conditions

Market clearing for labor requires the labor demand by firms in each sector equals the units of labor services that the household sells to each firm $n_t^T = h_t^T$ and $n_t^N = h_t^N$. Market clearing for the non-tradable good requires that output be equal to the demand for non-tradables in each country:

$$c_t^N = (h_t^N)^{\alpha^N}. \tag{10}$$

We assume without loss of generality that the bond denominated in domestic currency is traded only domestically. Market clearing therefore implies

$$b_{t+1} = 0. \tag{11}$$

Combining the budget constraints of households, firms, and the central bank, as well as market clearing conditions, we arrive at the resource constraint for tradables, or the balance of payment condition:

$$c_t^T - (h_t^T)^{\alpha^T} = b_t^* - \frac{b_{t+1}^*}{R_t^*}, \tag{12}$$

which says that the trade balance in each country must be financed with net bond issuances.

Finally, at the world level, real bonds are in zero net supply. Therefore, aggregating across countries, we have

$$\int b^*_{kt+1} dk = 0. \tag{13}$$

2.5 Competitive Equilibrium

We now define a competitive equilibrium in the global economy. With full capital mobility, the real rate must be the same across countries, and the market for the real bond must clear globally. In addition, we must have that agents in each of the k economies optimize and markets for non-tradable goods clear in each country. We use the following notation: a variable of the form $\{x_t\}$ denotes a vector of values, one for each country k. An equilibrium is then defined as follows.

Definition 1 (Competitive Equilibrium)**.** Given initial positions for each country b^*_{k0} and a sticky wage $\bar{W}$, an equilibrium is a sequence of government policies $\{R_t\}_{t=0}^{\infty}$ for each country, world real rate $\{R^*_t\}_{t=0}^{\infty}$, and prices $\{P^T_t, P^N_t, W_t\}_{t=0}^{\infty}$ and allocations $\{c^T_t, c^N_t, h^T_t, h^N_t, b_{t+1}, b^*_{t+1}\}_{t=0}^{\infty}$ in each country such that

- (i) Households optimize, and hence the following conditions hold: (4), (6), (7) for all $t \geq 0$ and (5) holds for all $t \geq 1$
- (ii) Firms optimize, implying (8) and (9) hold for all $t \geq 0$;
- (iii) The law of one price holds for tradables: $P^T_{kt} = e^j_t P^T_{jt}$ for any country-pair k and j,
- (iv) The market for non-tradables (10) and domestic bonds (11) clears; moreover, the labor market clears for $t \geq 1$.
- (v) Globally, the market for the real bond clears: implying (13)

Notice that by combining firms' optimality condition with households, optimality condition (4), we arrive at an equation determining the demand for hours in the non-tradable sector in each country as a function of the price of tradables P^T_t, the nominal wage W_t and the level of tradable consumption c^T_t:

$$h^N_t = \frac{\phi^N_t}{\phi^T_t} \frac{\alpha^N}{W_t} P^T_t c^T_t.$$

On the other hand, equilibrium hours in the tradable sector are independent of consumption of domestic households, a distinction that will play a central role in the analysis.

We assume that countries have symmetric initial net foreign asset positions at $t = 0$, and so we can restrict the analysis to symmetric competitive equilibrium.

2.6 Efficient Allocation

We conclude the description of the model by presenting the first-best allocation. We consider a benevolent social planner of the world economy who chooses allocations subject to a resource constraint. Because preferences are symmetric and all countries are identical, each country consumes its tradable output that is, there are no movement in the net foreign asset position of the countries. Using the resource constraint for non-tradables (10), the planner's problem can be written as

$$\max_{h_t^N, h_t^T} \sum_{t=0}^{\infty} \sum_{i \in \{T,N\}} \beta^t \left[\phi_t^i \alpha^i \log(h_t^i) - h_t^i \right]. \tag{14}$$

The first-best allocation equates the value of one additional employed unit of labor in each sector to the marginal cost of leisure:

$$\frac{\alpha^N \phi_t^N}{h_t^N} = 1, \qquad \frac{\alpha^T \phi_t^T}{h_t^T} = 1. \tag{15}$$

Given a linear disutility from labor, this implies that hours in each sector are proportional to the weight of each good in the consumption aggregate. We summarize these results in the following Lemma:

Lemma 1 (First-best)**.** *The first-best allocation features* $h_t^N = \alpha^N \phi_t^N$ *and* $h_t^T = \alpha^T \phi_t^T$.

Proof. In Appendix A.1 □

It should be clear that the first-best allocation coincides with the allocations in a competitive equilibrium with flexible prices. This can be seen by noting that if the nominal wage could be adjusted, we would have from households' labor supply decision, $c_t^i / \phi^i = W_t / P_t^i$ for $i \in \{T, N\}$, which combined with firms' demand for labor would yield (15).

Output gaps. To describe the policy trade-off that the central banks face, we introduce a measure of "output gaps", which are defined as the gap between the current and the efficient level of employment described in Lemma 1

$$\tilde{h}_t^N \equiv \frac{h_t^N}{\alpha^N \phi_t^N} - 1, \qquad \tilde{h}_t^T \equiv \frac{h_t^T}{\alpha^T \phi_t^T} - 1 \tag{16}$$

Nominal rigidities will distort these ratios away from one, which would correspond to the first-best allocations. In our case, the assumption that prices are flexible and that labor is perfectly mobile across sectors, implies that output gaps are equated across sectors.

Lemma 2. *In any competitive equilibrium, the output gaps in the two sectors are equated. That is, the output gaps for tradables and non-tradables are equalized* $\tilde{h}_0^T = \tilde{h}_0^N = \tilde{h}_0$.

Proof. In Appendix A.2 □

We note that this result applies to a competitive equilibrium given any monetary policy. Depending on aggregate conditions and the stance of monetary policy, this could imply a situation with a positive or negative output gap (i.e., $\tilde{h}_0 > 0$ or $\tilde{h}_0 < 0$).

2.7 Experiment

Assume that all countries start at the stationary equilibrium where allocations and relative prices are constant. At the steady state, allocations are efficient and symmetric, and the inflation rate coincides with the central bank's target $\bar{\pi}$, which we assume to be zero for simplicity. We consider values for P_{-1} and $\bar{W}$ such that in the absence of any shocks, the central bank is able to implement the efficient allocation where all output gaps equal zero. Without loss of generality we normalize $\bar{W} = 1$ which implies

$$P_{-1} = \prod_{i \in \{T,N\}} (\alpha^i \phi^i)^{-\alpha^i \phi^i}$$

Moreover, by symmetry we have that $c^T = (h^T)^{\alpha^T}$ for all countries and $R^* = 1/\beta$.

We then assume that at $t = 0$, there is a shock. In particular, we will focus us on an increase in the demand for non-tradables ϕ_0^N and an increase in $\bar{\pi}$. We see in particular the former as a way to capture the rebalancing of consumption towards consumption of services post-Covid and the latter as capturing the tolerance for higher inflation.

3 Optimal Policy in a Nash Equilibrium

This section studies non-cooperative monetary policy. We model the non-cooperative game as a Nash equilibrium where central banks choose their monetary policy to maximize their own welfare, taking as given monetary policy abroad.

3.1 Optimal Monetary Policy for a Single Country

We first study the individual problem of a central bank that takes as given R^* and policies conducted in other countries. Recall that because prices are flexible for $t \geq 1$, we can focus on a situation where the government implements a flexible price allocation with $\pi_t = \bar{\pi}$ for all $t \geq 1$.[8] Starting from $t \geq 1$, the lifetime welfare for a government that starts with net saving b_1^* is given by the following lemma

Lemma 3. *Consider a government that starts with b_1^* at $t = 1$. Then, the value is given by*

$$V_1(b_1^*) = \frac{1}{1-\beta} \left\{ \phi^T \log \left[(h_1^T)^{\alpha^T} + (1-\beta) b_1^* \right] - h_1^T + \phi^N \alpha^N \left[\log(\alpha^N \phi^N) - 1 \right] \right\} \tag{17}$$

where h_1^T is uniquely determined by

$$\frac{\alpha^T \phi^T}{h_1^T} = 1 + (1-\beta) \frac{b_1^*}{(h_1^T)^{\alpha^T}}. \tag{18}$$

Proof. In Appendix A.3 □

Given that wages are flexible for $t \geq 1$, hours worked in each sector are determined by the equalization of the value of one additional employed unit of labor in that sector to the marginal cost of leisure. (18) reflects the idea that hours worked in the tradable sector for $t \geq 1$ is decreasing in the level of saving at $t = 0$.[9]

We now turn to the problem at $t = 0$. The objective of the government is to choose the domestic nominal interest rate R_0, to maximize the utility flow in period 0 plus the continuation value (17) characterized above subject to households' budget constraint (3), optimality conditions (4), (6), (7), firms' optimality conditions (8)-(9) and market clearing constraints (10)-(12). The key difference relative to the problem for the planner under coordination is that the individual central bank takes as given R_0^* and can choose allocations

[8] Notice there are no time inconsistency issues associated with the optimal monetary policy.

[9] It is worth noting that the marginal value of hours worked in the non-tradable sector is independent of the level of external indebtedness due to separability of preferences.

that lead to deviate from a zero trade balance (even though in the Nash equilibrium, with identical countries, the trade balance would be zero).[10]

We denote by $z_0 \equiv \frac{c_0^T}{(h_0^T)^{\alpha^T}}$ the trade balance of the country at date 0. Demand imbalances are captured by a value of z_0 different from unity. In particular, a value of z_0 is above unity signals an excess demand for tradable goods relative to the domestic supply of tradables, and the country runs trade deficit. When z_0 rises above unity, the country runs a trade deficit. Replacing constraints (8)-(10) and (12) in the objective, and using the fact that given an allocation (7) can be used to back out the policy rate of the central bank, we arrive to the following problem for the domestic central bank, which consists in choosing $\{h_0^N, h_0^T, z_0, \pi_0\}$ to maximize

$$\sum_{i\in\{T,N\}} \left[\alpha^i \phi_0^i \log(h_0^i) - h_0^i\right] - \frac{\chi}{2}(\pi_0 - \bar{\pi})^2 + \phi_0^T \log(z_0) + \beta V_1 \left(R_0^*(h_0^T)^{\alpha^T}(1 - z_0)\right) \quad (19)$$

subject to

$$\pi_0 = \kappa_0 \prod_{i\in\{T,N\}} \left(\frac{h_0^i}{\alpha^i \phi_0^i}\right)^{(1-\alpha^i)\phi_0^i} - 1 \quad (20)$$

$$h_0^N = \frac{\alpha^N \phi_0^N}{\alpha^T \phi_0^T} h_0^T z_0 \quad (21)$$

$$\frac{1}{z_0} = \beta R_0^* \frac{(h_0^T)^{\alpha^T}}{\alpha^T \phi_0^T} \left(\mathcal{H}_1^T(R_0^*(h_0^T)^{\alpha^T}(1 - z_0))\right)^{1-\alpha^T} \quad (22)$$

where $\mathcal{H}_1^T(R_0^*(h_0^T)^{\alpha^T}(1 - z_0))$ is defined in (18) and $\kappa_0 \equiv \prod_i [(\alpha^i \phi_0^i)^{-\alpha^i \phi_0^i} / (\alpha^i \phi^i)^{-\alpha^i \phi^i}]$.

The restriction (20) combines the ratio of CPIs (2) at dates 0 and 1 with (8) and (9) to relate the dynamics of the aggregate price index and output in both sectors and can be interpreted as the static Phillips curve. The logic here is as follows: a positive demand shock for tradables, that pushes upward pressure on the price of tradables, increases tradable output on the one hand and increases demand for non-tradables via the expenditure switching channel on the other hand. The latter leads to a rise in the price and then output in the non-tradable sector.

The restriction (21) emerges from combining household optimal mix for tradable and non-tradable consumption and the firms' optimality condition for employment.[11] This condition reflects that the ratio of hours employed in each sector is determined by the

[10] Owing to the possibility of non-zero trade balance, the output gaps need not to be equated across sectors from the individual central bank's perspective. However, they are equated in the Nash equilibrium as shown in Lemma 2.

[11] In particular, it uses (4), (8), (9) and (12).

relative preference of households for the two goods as well as the labor intensity in each sector. Absent demand imbalances $z_0 = 1$, (21) states that the ratio of hours worked h_0^N / h_0^T corresponds to the ratio of hours worked in the efficient allocation $(\alpha^N \phi_0^N)/(\alpha^T \phi_0^T)$. Households finance trade surplus $z_0 < 1$ by working relatively more hours in the tradable sector. When the economy runs a trade deficit $z_0 > 1$, households work relatively fewer hours in the tradable sector. We denote by η_0 the multiplier on (21) where here a multiplier $\eta_0 > 0$ would reflect the fact that the central bank perceives that being able to reallocate hours towards the non-tradable sector would improve welfare (the opposite happens when $\eta_0 < 0$).[12] As we will see, this implementability constraint and the associated multiplier will play a crucial role in the analysis.

Finally, the central bank is subject to a dynamic implementability constraint associated with domestic households' borrowing choices (22). An individual central bank can in principle use monetary policy to change its net position in the global market for real assets. By doing so, it affects the amount of tradable resources and thus the optimal mix of hours worked in the two sectors, as reflected in equation (21). We let μ_0 denote the multiplier on (22) where a multiplier $\mu_0 > 0$ would reflect the fact that the central bank perceives that borrowing from abroad would improve welfare. The optimality condition for domestic demand imbalances z_0 requires that:

$$\frac{\mu_0}{z_0} = \delta^N h_0^N \eta_0 \tag{23}$$

where $\frac{1}{\delta^N} \equiv 1 + \frac{1-\alpha^T}{1+\alpha^T \tilde{h}_1^T} \frac{(1-\beta)\phi_0^T}{\beta \phi^T}$. Remarkably, condition (23) reveals that the Lagrange multipliers on (21) and (22) have the same sign. In other words, when the central bank would rather reallocate more hours towards non-tradable, it is also the case that it would like households to borrow more. Using this condition, we arrive at the following optimality conditions for h_0^N and h_0^T,

$$\frac{\phi_0^N \alpha^N}{h_0^N} - 1 = \frac{\phi_0^N}{h_0^N}(1-\alpha^N)\chi\left(1+\pi_0\right)\left(\pi_0 - \bar{\pi}\right) + \eta_0 \tag{24}$$

$$\frac{1}{z_0}\frac{\phi_0^T \alpha^T}{h_0^T} - 1 = \frac{\phi_0^T}{h_0^T}(1-\alpha^T)\chi\left(1+\pi_0\right)\left(\pi_0 - \bar{\pi}\right) - \left[1 - \alpha^T \frac{z_0 - 1 + \delta^N}{z_0}\right] \frac{h_0^N}{h_0^T}\eta_0 \tag{25}$$

Condition (24) equates the marginal utility benefits from raising one more unit of non-tradable employment and the associated increase in output net of leisure value of time with the marginal utility costs. The latter is given by the costs of higher inflation, and the

[12] We present more formally the Lagrangian associated with the global planner problem in Appendix A.4.

effect on the implementability constraint (21). The inflation cost emerges because raising employment in the non-tradable sector requires a higher price level to stimulate firms to hire more workers, a standard Phillips curve relationship. Condition (25) is the analogous condition for tradable employment. Notice that the last terms in the two conditions have the opposite sign as increasing h_0^T and h_0^N have the opposite effects on constraint (35).

Combining these conditions we arrive to

$$\left[\frac{\sum_i (1-\alpha^i)\phi_0^i}{\alpha^T \phi_0^T} z_0 + (1-z_0-\delta^N)(1-\alpha^N)\frac{\phi_0^N}{\phi_0^T}\right]\eta_0 = \left(\frac{1-\alpha^T}{\alpha^T} z_0 - \frac{1-\alpha^N}{\alpha^N}\right)\left(\frac{\phi_0^N \alpha^N}{h_0^N} - 1\right) \quad (26)$$

We can infer from this condition that, starting zero net position in the global market $z_0 = 1$, the sign of the multiplier η_0 depends on two sufficient statistics: the difference in labor intensity across sectors $(\alpha^N - \alpha^T)$ and the sign of the output gap $\tilde{h}_0$. When the two sectors are equally labor-intensive $\alpha^N = \alpha^T$, households' composition of consumption between tradable and non-tradable is socially optimal. An individual central bank does not perceive any additional benefit from reallocating consumption across sectors.

When the non-tradable sector is more labor intensive than the tradable sector (i.e., $\alpha^N > \alpha^T$), an individual central bank internalizes that a reallocation of consumption from the tradable sector toward the non-tradable sector would help mitigate the recession and improve social welfare by $\eta_0 > 0$. Private agents do not internalize this aggregate demand externality and over-borrow. By the same token, if the non-tradable sector is less labor-intensive than the tradable sector and there overheating in the labor market, private agents do not internalize that running a larger trade deficit helps reallocate consumption away from the tradable sector and improves welfare by $\eta_0 < 0$; and thus under-borrow. We summarize these findings in the proposition below.

Proposition 1 (Demand Imbalances)**.** *Consider R_0^* such that private borrowing implies $z_0 = 1$. Suppose $\alpha^N > \alpha^T$, then from the domestic central bank's perspective, private agents over-borrow (i.e. $\mu_0 > 0$) if and only if $\tilde{h}_0 > 0$ and under-borrow (i.e. $\mu_0 < 0$) if and only if $\tilde{h}_0 < 0$. The relationship is reversed when $\alpha^T > \alpha^N$.*

Proof. In Appendix A.5 □

As we will see later, in the constrained-efficient allocation in which monetary policy is chosen under cooperation, the world real rate is such that private borrowing implies $z_0 = 1$. Proposition 1 thus describes the forces at play that lead individual central banks to deviate from the cooperative solution. In particular, a domestic central bank would find it optimal to loosen its monetary policy in an effort to run a trade surplus ($z_0 > 1$) and reallocate of consumption away from the non-tradable sector when the non-tradable good is more labor intensive $\alpha^N > \alpha^T$ and the labor market is overheated $\tilde{h}_0 > 0$.

3.2 Nash Equilibrium

We saw that an individual central bank has incentives to generate demand imbalances when the labor intensity differs across sectors. However, by symmetry in any competitive equilibrium where all central banks are optimizing, there are no capital flows and exchange rates are constant. Combining (24)-(25) and using $z_0 = 1$ in the competitive equilibrium in the global economy, we arrive to

$$\left(\frac{\phi_0^N \alpha^N}{h_0^N} - 1\right)\left[h_0^T + (1-\alpha^T\delta^N)h_0^N\right] = \left[\sum_i (1-\alpha^T\delta^i)(1-\alpha^i)\phi_0^i\right]\chi\left(1+\pi_0\right)\left(\pi_0 - \bar{\pi}\right) \quad (27)$$

with $\delta^T = 0$ and $\delta^N \equiv \left[1 + (1-\alpha^T)\frac{(1-\beta)\phi_0^T}{\beta\phi^T}\right]^{-1}$. This condition reveals how depending on the nature of the shock, only one of the two scenarios can emerge: either the world economy is overheating, $\tilde{h}_0 > 0$, and inflation is below target or there is a recession $\tilde{h}_0 < 0$ and inflation is above target. To understand the intuition, consider the possibility that in a coordinated equilibrium, there is a negative output gap in the tradable sector (and the non-tradable) and inflation is below the target. In that case, by lowering the nominal interest rate and allowing for higher prices, the central bank can narrow the output gap and the inflation gap. By the same token, if is a positive output gap and inflation is above the target, it would be optimal to raise the policy rate, as this would help lower inflation and take output closer to the efficient level. It is also clear from these conditions that if the inflation cost is zero $\chi = 0$, central banks can implement the first-best allocation for any shocks. The following proposition provides a characterization of employment and inflation in the each country in the competitive (Nash) equilibrium.

Proposition 2. *In the Nash equilibrium, employment in each country is uniquely determined by*

$$\tilde{h}_0^T = \tilde{h}_0^N = -\frac{\sum_i (1-\alpha^T\delta^i)(1-\alpha^i)\phi_0^i}{\sum_i (1-\alpha^T\delta^i)\alpha^i\phi_0^i}\chi\left[1+\pi_0(\tilde{h}_0^N)\right]\left[\pi_0(\tilde{h}_0^N) - \bar{\pi}\right], \quad (28)$$

and where the inflation rate of the consumer price index $\pi_0(\tilde{h}_0^N)$ satisfies

$$\pi_0(\tilde{h}_0^N) = \kappa_0\left(1+\tilde{h}_0^N\right)^{\sum_i (1-\alpha^i)\phi_0^i} - 1. \quad (29)$$

Proof. In Appendix A.6 □

Condition (28) describes the output-inflation tradeoff that central banks face in this environment. It states that in the Nash equilibrium, central banks trade-offs the goals of eliminating output gaps and driving inflation back to its steady-state level; and where the product of the relative weight of inflation costs in welfare χ and the slope of the Phillips curve determines the relative weights given to inflation. The nominal interest rate that implements the desired level of employment is described in the next lemma.

Lemma 4. *In any competitive equilibrium in the global economy, the nominal interest rate set is uniquely determined by*

$$R_0 = \frac{\kappa_0}{\beta}\left(1+\tilde{h}_0\right)^{-\sum_i \alpha^i \phi_0^i} \tag{30}$$

Proof. In Appendix A.7 □

This expression describes a negative relationship between the nominal interest rate R_0 and the output gap $\tilde{h}_0$. Intuitively, a higher nominal rate lowers the price of tradables by (7). Faced with a lower price of tradables, firms in the tradable sector reduce production, while households on the other hand shift demand away from the non-tradable goods which in turn pushes downward on prices in the non-tradable sector and leads to a fall in non-tradable output. Output and prices in both sector therefore fall following an interest hike.

4 Over-tightening or Under-tightening?

Having characterized monetary policy in the Nash equilibrium, we now turn to analyze the question of whether coordination calls for tighter or looser monetary policy relative to the Nash equilibrium. We will argue that the source of inefficiency arises from the inability of individual central banks to internalize the financial channel of international spillovers.

4.1 The Financial Channel of International Spillovers

In a competitive equilibrium in the global economy, individual central banks take as given the world real rate when setting monetary policy. A question that emerges is, What are the effects of a change in the world real rate on welfare? Letting V_0 denote the welfare of households in a given small open economy, the next proposition characterizes these effects.

Proposition 3 (Financial Channel of International Spillovers)**.** *Consider small changes* $\{dR_0^*\}$ *in the world real rate. Starting from a symmetric equilibrium with no demand imbalances, the effect on households' welfare in a generic country is given by*

$$dV_0 = \left[\alpha^N \phi_0^N \, \tilde{h}_0^N + (1-\alpha^N)\phi_0^N \chi(1+\pi_0)(\pi_0 - \bar{\pi})\right] \delta^N \frac{dR_0^*}{R_0^*} \tag{31}$$

Proof. In Appendix A.8 □

The proposition underscores that the effects of changes in the world real rate on welfare is determined by the output gap and the deviation of the inflation rate from its target. If both gaps were zero, changes in the world real rate would have no effects on welfare. Because the country has a zero net position in the global market, a marginal change in the world real rate does not affect the country's resource constraint. When these gaps are different from zero, changes in the world real rate have in general effects on domestic welfare. In particular, when the economy is overheated, a marginal increase in the world real rate $dR_0^* > 0$ leads households to substitute consumption intertemporally toward the future. Because $\tilde{h}_0 > 0$ the reduction in demand for both tradables and non-tradables improves welfare by bringing output closer to the efficient level. On the other hand, the fall in demand for goods pushes prices down which improves welfare if inflation is above the target by bringing inflation closer to the target and reduces welfare otherwise.

In the Nash equilibrium, however, the two gaps always have opposite signs (28) and the net effect of a marginal increase in the world real depends on the difference in labor intensity across sectors.

Corollary 1. *Starting from a Nash equilibrium, the effect of small changes* dR_0^* *in the world real rate on households' welfare in a generic country is given by*

$$dV_0 = \frac{(1-\alpha^T\delta^N)\phi_0^N\phi_0^T}{\sum_i (1-\alpha^T\delta^i)\,(1-\alpha^i)\,\phi_0^i}\,(\alpha^N - \alpha^T)\tilde{h}_0\,\frac{dR_0^*}{R_0^*} \tag{32}$$

Proof. The proof substitutes (28) into (31). □

Consider that countries are overheating in the Nash equilibrium $\tilde{h}_0 > 0$, which by (28) implies a negative inflation gap $\pi_0 < \bar{\pi}$. Following a marginal increase in the world real rate, holding constant the price of both tradable and non-tradable goods, the fall in the consumption of tradables translate one-to-one into a fall in the demand for non-tradables bringing output in both sectors closer to the efficient level by the same amount. In equilibrium, prices fall in both sectors pushing inflation further away from its target.

However, the fall in the price of non-tradables, in constrat to the change in demand for non-tradables, may be larger or smaller than the fall in the price of tradables depending on the labor intensity. Because the change in the price is inversely proportional to the labor intensity, the price of non-tradables fall less than the price of tradables when the former is more labor intensive $\alpha^N > \alpha^T$. As a result, the inflation cost associated with the increase in the world real rate is lower than the benefits from reducing the output gaps, and therefore improves welfare. The opposite happens when the tradable good is more labor intensive as price of non-tradables fall more than the price of tradables.

The corollary below summarizes the effects of a marginal increase in the world rate on welfare conditional on the two sufficient statistics: the output gap in the competitive equilibrium and the labor intensity across sectors.

Corollary 2. *Suppose $\alpha^N > \alpha^T$. A marginal increase in the world real rate increases welfare, i.e. $dV_0/dR_0^* > 0$, if and only if there is a positive output gap in the Nash equilibrium. The relationship is reversed when $\alpha^T > \alpha^N$.*

Proof. The proof follows directly from Corollary 1. □

The implication of our findings is that, starting from a Nash equilibrium in which labor markets are overheated and non-tradable goods are more labor intensive than tradables, at the margin, a collective tightening of monetary policy is welfare-improving in all countries. To shed light on this result and determine the extent to which countries under-tighten in this scenario we characterize in the next section the cooperative solution.

4.2 Cooperative Monetary Policy

The cooperative solution is the solution a global planning problem that chooses a sequence of nominal interest rates, allocations, and prices for each country to maximize total welfare. Given that all countries are identical, the optimal monetary policy problem can be reduced to maximizing an arbitrary country's welfare under financial autarchy.

The global planner chooses a sequence of nominal interest rates, prices, and allocations to maximize (1) subject to firms' and households' optimality conditions (4) and (8)-(9) and market clearing constraints for labor and consumption (10)-(12). Notice that because there is a single tradable good and countries are identical, we must have that $(h_t^T)^{\alpha^T} = c_t^T$. The problem is thus static.[13] Replacing constraints in the objective, we arrive at the following problem:

$$\max_{h_0^T, h_0^N} \sum_{i \in \{T,N\}} \left[\alpha^i \phi_0^i \log(h_0^i) - h_0^i\right] - \frac{\chi}{2}(\pi_0 - \bar{\pi})^2 \tag{33}$$

subject to

$$\pi_0 = \kappa_0 \prod_{i \in \{T,N\}} \left(\frac{h_0^i}{\alpha^i \phi_0^i}\right)^{(1-\alpha^i)\phi_0^i} - 1 \tag{34}$$

$$h_0^N = \frac{\alpha^N \phi_0^N}{\alpha^T \phi_0^T} h_0^T \tag{35}$$

One of the key difference between the optimization problem of individual central banks and the global planner is that the global planner understand that, even though an individual central bank can in principle change its net position in the global market for real asset, in any competitive equilibrium in the global economy there is no demand imbalances, i.e. $z_0 = 1$. In particular, the restriction (35) on the optimal mix for tradable and non-tradable consumption faced by the global planner is analoguous to the one faced by the individual central bank, with the exception that $z_0 = 1$.

We denote by η_0 the multiplier on (35) where here again a multiplier $\eta_0 > 0$ reflects the fact that the planner perceives that being able to reallocate hours towards the non-tradable sector would improve welfare and the opposite happens when $\eta_0 < 0$.[14] The optimality conditions with respect to h_0^T and h_0^N can be combined to obtain

$$\frac{\sum_i (1-\alpha^i)\phi_0^i}{\alpha^T \phi_0^T} \eta_0 = \left(\frac{1-\alpha^T}{\alpha^T} - \frac{1-\alpha^N}{\alpha^N}\right)\left(\frac{\phi_0^N \alpha^N}{h_0^N} - 1\right) \tag{36}$$

The difference between the cooperative solution relative to Nash equilibrium follows from examining (36) with the corresponding optimality condition for individual central banks (26) absent demand imbalances $z_0 = 1$. When the two sectors are equally labor-intensive $\alpha^N = \alpha^T$, both the global planner and individual central banks in the competitive equilibrium do not perceive any additional benefit from reallocating consumption across sectors. Away from this special case, that is for $\alpha^N \neq \alpha^T$, the global planner perceives a larger benefit from reallocating consumption across sectors relative to individual planners. This difference in the value of reallocating consumption can be intuited from Corollary 1. That

[13] Recall also that prices are flexible for $t \geq 1$.

[14] We present more formally the Lagrangian associated with the global planner problem in Appendix A.10.

is, when the labor intensity differs across sectors, the general equilibrium response of the world real rate to central banks actions has implications for the allocation of consumption across sectors and welfare that are not internalized by individual central banks.

The next proposition provides a characterization of employment and inflation under the optimal cooperative monetary policy.

Proposition 4 (Optimal monetary policy under cooperation)**.** *Consider the optimal monetary policy under cooperation. We have that employment is determined by*

$$\tilde{h}_0^T = \tilde{h}_0^N = -\frac{\sum_i (1-\alpha^i)\phi_0^i}{\sum_i \alpha^i \phi_0^i}\chi \left[1 + \pi_0(\tilde{h}_0^N)\right]\left[\pi_0(\tilde{h}_0^N) - \bar{\pi}\right], \tag{37}$$

where the inflation rate of the consumer price index $\pi_0(\tilde{h}_0^N)$ *satisfies* (29).

Proof. In Appendix A.9 □

The monetary policy target under cooperation (37) shares with its counterpart in the Nash equilibrium the feature that either the world economy is overheating and inflation is below target, or there is a global recession and inflation is above target. However, due to the differences in the perceived benefit from reallocating consumption across sectors, the global planner put a different relative weight on the two objectives (eliminating output gaps and driving inflation back to its steady-state level) compared to the one in the Nash equilibrium (28).

4.3 Comparison of Policy Rates

We now turn to comparing the nominal interest rates in the Nash equilibrium and in the cooperative equilibrium. In light of Proposition 2, 4 and Lemma 4, we have the following Proposition.

Proposition 5. *Suppose* $\alpha^N > \alpha^T$*. We have under-tightening, i.e.* $R_0^{NE} < R_0^{GP}$*, if and only if there is a positive output gap in the Nash equilibrium. The relationship is reversed when* $\alpha^T > \alpha^N$*.*

Proof. In Appendix A.11 □

The proposition establishes that under the assumption that $\alpha^N > \alpha^T$, when economies are experiencing overheating cooperation calls for higher interest rates. This result can be intuited from the observation that during overheating countries perceive a benefit from shifting relatively more consumption (and employment) towards the sector with lower labor intensity as this would help bring employment back to its efficient level for the same inflation. By encouraging domestic households to save more abroad, an individual central bank can alter aggregate demand for consumption and thus non-tradable employment, while keeping the same level of tradable output. This implies that, in the competitive equilibrium, countries will tend to loosen more monetary policy in an attempt to have a relatively less appreciated currency and generate a trade surplus. At the world level, however, not all countries can run a trade surplus and a loosened monetary policy brings more overheating. Under cooperation, countries internalize this general equilibrium effect and would set higher interest rates.

On the other hand, when economies are experiencing a recession under the assumption that $\alpha^N > \alpha^T$, cooperation calls for lower interest rates, as in Fornaro and Romei (2022).

Illustration. Figure 1 illustrates the workings of monetary policy in the Nash equilibrium and under cooperation following a reallocation shock toward non-tradable goods (i.e., ϕ_0^N increases). The dashed (NE) and solid (GP) downward-sloping line represents the output-inflation tradeoff of central banks respectively in the Nash equilibrium (28) and under cooperation (37). These curves illustrate the optimal output gap given arbitrary levels of inflation. The upward sloping line (PC) represents the short-run Philips curve, given by (29). This curve illustrates the level of inflation for arbitrary levels of the output gap. The monetary policy response in each equilibrium, for given parameters, is represented by the intersection between the downward sloping curve and the upward sloping curve corresponding to the output-inflation trade-off in that equilibrium.

The figure illustrates these two curves starting at the steady state (solid lines) and how they shift in response to a shock (dashed lines). The left panel considers an increase in ϕ_0^N. Notice that when starting at steady state, the curves intersect at $(\bar{\pi}, 0)$. That is, the first-best allocation is feasible both in the Nash equilibrium and under cooperation.

In response to an increase in ϕ_0^N, the Philips curve shifts down. The output-inflation curves rotates counter-clockwise and intersects the Phillips (PC) curve in the right bottom of (0,$\bar{\pi}'$) where the economy experiences more overheating. In particular, to the intersection of the curves at point E. Because the non-tradable good is more labor intensive than the tradable good, the global planner internalizes that a higher world real rate would help cool down the economy at the expense of a (relative) smaller welfare costs of inflation. As a result, in the cooperative solution, the global planner would choose higher nominal interest rate and the intersection of the two solid curves would occur at point G where the

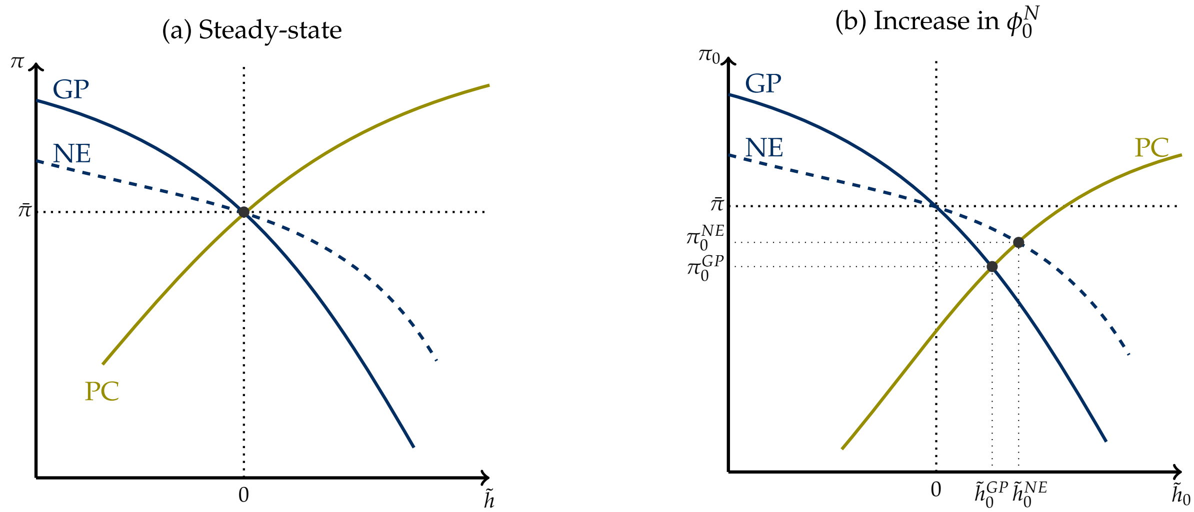

Figure 1: Policy response in the Nash equilibrium vs under cooperation for $\alpha^N > \alpha^T$
(NE) corresponds to the output-inflation tradeoff (28) in the Nash equilibrium. (GP) corresponds to the output-inflation tradeoff (28) under cooperation. (PC) is the Philips curve and corresponds to (29).

economy experiences less overheating relative to the Nash equilibrium.

4.4 Extension with Imperfect Labor Mobility

Our baseline model assumes perfect labor mobility across sectors. In this section, we introduce frictions in labor mobility across the two sectors by assuming that aggregate hours worked is a composite of hours worked in the tradable sector and in the non-tradable sector according to the following CES aggregator:

$$n_t = \left[\left(n_t^T\right)^{1+\frac{1}{\eta}} + \left(n_t^N\right)^{1+\frac{1}{\eta}} \right]^{\frac{\eta}{\eta+1}}, \tag{38}$$

where η measures the degree of labor mobility: that is how easy it is for a household to substitute hours worked in the tradable sector for hours worked in the tradable sector. When $\eta \to \infty$, there is perfect labor mobility and the aggregate hours worked reduce to $n_t = n_t^T + n_t^N$ as in Section 2. For $\eta = 0$ labor is perfectly immobile across sectors. In order to simplify the exposition, we adopt the linear-quadratic approach, commonly used in the New-Keynesian literature (see for example Clarida, Gali and Gertler, 2002 among others).

Nash equilibrium. We derive the loss function of the policy problem of a single country from the second-order approximation to households' utility function (1),[15]

$$\frac{1}{2}\left[\alpha^T\phi_0^T\left(\tilde{h}_0^T\right)^2+\alpha^N\phi_0^N\left(\tilde{h}_0^N\right)^2+\chi\left(\pi_0-\bar{\pi}\right)^2+\left(\frac{\phi_0^T}{\hat{\delta}^N}+\frac{1}{\eta\sum_i(\alpha^i\phi_0^i)^{-1}}\right)\left(\tilde{z}_0\right)^2\right]. \tag{39}$$

where $\hat{\delta}^N$ is defined in (A.35) with $\hat{\delta}^N=\delta^N$ when $\eta\to\infty$. The loss function (39) represents the difference between the households' welfare under the efficient allocation (the maximum welfare achievable) and households' welfare under the current market-determined levels of consumption and leisure. The first two terms in (39) reflect the inefficient use of labor in both tradable and non-tradable sectors stemming from nominal rigidities. The third term is the welfare cost from deviations of inflation from the target, while the last term reflects welfare losses from demand imbalances.

Because for a given output gap in the tradable sector, a negative demand imbalance can help reduce the output gap in the non-tradable sector $\tilde{h}_0^N=\tilde{h}_0^T+\tilde{z}_0$, a domestic central bank may wish to exploit a certain degree of demand imbalances to reduce welfare loss from nominal rigidities. In particular, a trade deficit during recessions can help alleviate the under-provision of non-tradable goods. From households' borrowing choices (22), the equilibrium demand imbalances is determined by,

$$\tilde{z}_0=-\hat{\delta}^N\left[(r_0^*-r_0^{*n})+\alpha^T\tilde{h}_0^T\right], \tag{40}$$

which says that the domestic country runs a trade deficit when the world real rate falls below its natural level r_0^{*n} (that is, the world interest rate would prevail absent nominal frictions) or when there is an under-provision of the tradable good. Using (40), it is possible to rewrite the loss function (39) conveniently as

$$\sum_i\frac{\alpha^i\phi_0^i}{2}\left[(1-\alpha^T\delta^i)\tilde{h}_0^T-\hat{\delta}^i(r_0^*-r_0^{*n})\right]^2+\left(\frac{\phi_0^T}{\hat{\delta}^N}+\frac{1}{\eta\sum_i(\alpha^i\phi_0^i)^{-1}}\right)(\tilde{z}_0)^2+\frac{\chi}{2}(\pi_0-\bar{\pi})^2 \tag{41}$$

with $\hat{\delta}^T=0$ and where the world real rate $r_0^*=\log(R_0^*)$ is taken as given by the central bank. It is worth noting from the loss function (41) that a higher world real rate helps cool down the economy when the labor market is overheated. During recessions, on the other hand, a lower world real rate stimulates the economy. Central banks, however, do not internalize the effects of their decision on the equilibrium world interest rate. The optimal policy problem of an individual central bank consists in choosing the domestic inflation

[15] See Appendix B for more details.

rate and the output gap in the tradable sector $\{\pi_0, \tilde{h}_0^T\}$ to minimize (41) subject to (40) and the following linearized Phillips curve,[16]

$$\pi_0 = \log(\kappa_0) + \sum_i (1-\alpha^i)\phi_0^i \left[(1-\alpha^T\hat{\delta}^i)\tilde{h}_0^T - \hat{\delta}^i(r_0^* - r_0^{*n})\right]. \tag{42}$$

The optimal policy rule or "target criterion" trades off the goal of closing the output gap with the goal of driving inflation back to its target, and is given by

$$\sum_i (1-\alpha^T\delta^i)\alpha^i\phi_0^i\,(\tilde{h}_0^T + \tilde{z}_0) + \left(\phi_0^T + \frac{\hat{\delta}^N}{\eta\sum_i(\alpha^i\phi_0^i)^{-1}}\right)\alpha^T\tilde{z}_0 = -\sum_i (1-\alpha^T\delta^i)(1-\alpha^i)\phi_0^i\,\chi\pi_0$$

Given that in any competitive equilibrium, the global market for real assets clears, i.e $\int_0^1 \tilde{z}_{0,k}dk = 0$, we arrive at the following target criterion in the Nash equilibrium[17]

$$\tilde{h}_0^T = -\Psi\pi_0, \quad \text{with } \Psi \equiv \frac{\sum_i(1-\alpha^T\delta^i)(1-\alpha^i)\phi_0^i}{\sum_i(1-\alpha^T\delta^i)\alpha^i\phi_0^i}\chi. \tag{43}$$

Point E in the left panel of Figure 2 represents the output gap and inflation in the Nash equilibrium assuming that non-tradable goods are more labor intensive $\alpha^N > \alpha^T$. At point E, the solid upward-sloping curve (PC) representing the Phillips curve is tangent to the indifference curve (IC) and the economy is overheating. Point I in this panel corresponds to the point where output gaps are closed and inflation is at his target $(0, \bar{\pi})$. In the right panel, we plot the linear approximation of the relationship between the domestic nominal interest rate and the output gap,

$$r_0 = r_0^n + \left[1-(1-\alpha^N\delta^N)\phi_0^N\right](r_0^* - r_0^{*n}) - \left[(1-\alpha^T\delta^N)\alpha^N\phi_0^N\right]\tilde{h}_0^T. \tag{44}$$

where r_0^n is the natural level of the nominal interest rate.

Cooperative equilibrium and undertightening. In contrast to individual central banks, the global planner internalizes how an increase in the world real rate helps reduce the loss from nominal rigidities (41) when the labor market is overheated at the expense of a steeper slope of the Phillips curve (42). Substituting for $r_0^* - r_0^{*n} = -\alpha^T\tilde{h}_0^T$ into (41) and

[16]The linearized Phillips curve (42) combines the log-linearized equations (20) and (22).

[17]Note that (43) corresponds to the linearized version of the optimal policy target (28) described in Proposition 2 for $\eta \to \infty$.

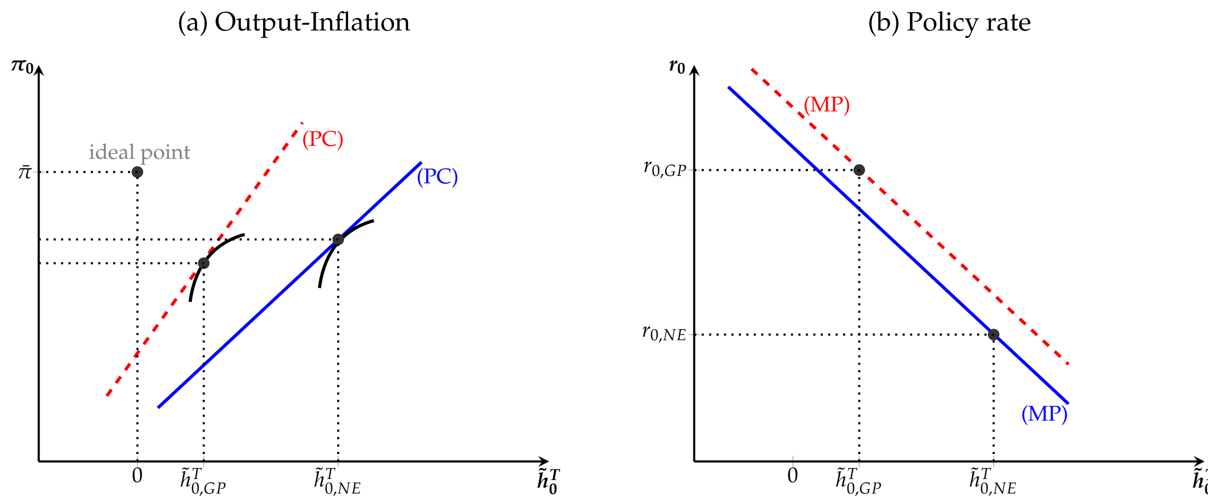

Figure 2: Monetary policy: competitive equilibrium vs. cooperation for $\alpha^N > \alpha^T$

(42), the optimality conditions yield

$$\tilde{h}^T_0 = -\underbrace{\left[1-(\alpha^N-\alpha^T)\Delta\right]}_{\substack{\text{net benefit of} \\ \text{an increase in } r^*_0}}\Psi\pi_0, \quad \text{with } \Delta \equiv \frac{\alpha^N\delta^N}{\sum_i \alpha^i\phi^i_0}\cdot\frac{\phi^N_0\phi^T_0}{\sum_i\left(1-\alpha^i\hat{\delta}^i\right)\left(1-\alpha^i\right)\phi^i_0} \tag{45}$$

Therefore, if the non-tradable good is more labor intensive ($\alpha^N > \alpha^T$) and the labor market is overheated, the welfare benefit of the reduction in output gaps induced by an increase in the world interest rate r^*_0 outweighs the welfare cost of inflation.[18] Because countries fail to internalize how the increase in r^*_0 alleviates the overheating in the labor market, they put less weight on closing output gaps and under-tighten, as illustrated in Figure 2.

5 Prudential Undertightening

Until now, we considered an economy that faces a sudden shock that creates an output-inflation tradeoff at $t = 0$. We saw in the previous section how this may lead to a situation of under-tightening relative to the optimal cooperation outcome. In this section, we expand the environment by assuming that countries are able to perfectly anticipate the shock at date $t = -1$. To simplify the notations, we denote variables at date $t = -1$ without a subscript, e.g. $x = x_{-1}$.

[18]In particular, with a constant return to scale production technology in the non-tradable sector ($\alpha^N = 1$) a marginal increase in r^*_0 raises non-tradable output without incurring any inflation cost as the price of the non-tradable good fully inherits the nominal wage rigidity.

Given that the problem of the global planner is static, its optimization problem at date $t=-1$ is identical to the optimization problem at date $t=0$, (33) (absent shocks). The optimal monetary policy under cooperation is thus characterized by equations (37) and (29) in Proposition 4, which absent shocks imply zero output gap and no inflation gap,

$$\tilde{h} = \pi = 0. \tag{46}$$

Therefore, when economies are not currently hit by a global cost-push shock that leads to a non-trivial output-inflation tradeoff, a global planner would set monetary policy to replicate flexible wage allocation and close both the output gap and the inflation gap. We argue that at the prevailing world real rate consistent with the global planner solution, individual central banks perceive that changes in domestic demand imbalances can help mitigate the effects of an anticipated global shock, creating an incentive to act prudentially.

Let $V_0(b_0^*)$ denote the value of a central bank that starts with a net asset position b_0^* at date 0. The central bank at date $t=-1$ chooses output, inflation, and the degree of demand imbalances $\{h^T, h^N, \pi, z\}$ to maximize the utility plus the continuation value $V_0(b_0^*)$ which solves the date-0 problem (19). The problem of an individual central bank is given by

$$\max_{h^T,h^N,z} \sum_{i\in\{T,N\}} \left[\alpha^i \phi^i \log(h^i) - h^i\right] + \phi^T \log(z) - \frac{\chi}{2}(\pi)^2 + \beta V_0\left(b_0^*\right)$$

subject to

$$\frac{b_0^*}{R^*} = (1-z)(h^T)^{\alpha^T} \tag{47}$$

$$\pi = \prod_{i\in\{T,N\}} \left(\frac{h^i}{\alpha^i \phi^i}\right)^{(1-\alpha^i)\phi^i} - 1 \tag{48}$$

$$h^N = \frac{\alpha^N \phi^N}{\alpha^T \phi^T} h^T z \tag{49}$$

$$\frac{1}{z} = \beta R^* \frac{\phi_0^T}{\phi^T} \frac{(h^T)^{\alpha^T}}{C_0^T(b_0^*)} \tag{50}$$

The restriction (47) is the balance of payment identity equation which says that the trade balance must be financed with net bond issuances. The optimal mix for hours worked in the tradable and non-tradable sectors has a multiplier η, and the dynamic implementability constraint (50) has the multiplier μ. Similar to the central bank problem at date 0, $\eta > 0$ and $\mu > 0$ respectively capture that reallocating hours towards the non-tradable sector and borrowing from abroad would improve welfare. The central bank's optimal choice for demand imbalances is given by

$$\left[\delta^N + (1-\delta^N)\gamma_0\right]\frac{\mu}{z} = \delta^N h_0^N \eta - (1-\delta^N)\frac{\phi^T}{\phi_0^T}\frac{\mu_0}{z_0} \tag{51}$$

with $\gamma_0 \equiv \frac{db_1^*}{db_0^*}\frac{R^* c^T}{R_0^* c_0^T} > 0$. Condition (51) states that the central bank would like households to borrow less when reallocating more hours towards tradable today improves welfare or when anticipated shocks would require more borrowing next period. In the latter case, by encouraging saving in the current period, the central brings more resources to the future which helps reduce the welfare cost of a future trade deficit. As shown in Section 3, whether individual central banks have incentives borrow more from the rest of the world when a global shock hits depends on the output gap in the competitive equilibrium and the difference in labor intensity. In particular, we have in any competitive equilibrium,

$$\frac{\mu_0}{\delta^N} = h_0^N \eta_0 = -\frac{(\alpha^N - \alpha^T)\phi_0^N \phi_0^T}{\sum_i (1-\alpha^T \delta^i)(1-\alpha^i)\phi_0^i}\tilde{h}_0^N.$$

As a result, when the labor intensity differs across the tradable and the non-tradable sectors, central banks in a competitive equilibrium sets monetary policy to trade-off the cost of opening up the output and the inflation gaps in the current period with the benefit of reallocating consumption across sectors next period. The target criterion is given by

$$\tilde{h}^N + \frac{\sum_i (1-\alpha^T \delta_0^i)(1-\alpha^i)\phi^i}{\sum_i (1-\alpha^T \delta_0^i)\alpha^i \phi^i}\chi\left[1+\pi(\tilde{h}^N)\right]\pi(\tilde{h}^N) = -\frac{(1-\delta^N)\delta_0^N \phi^T \alpha^T}{\sum_i (1-\alpha^T \delta_0^i)\alpha^i \phi^i}\cdot\frac{h_0^N}{\phi_0^T}\eta_0 \tag{52}$$

with $\delta_0^i = \delta^i / [\delta^N + (1-\delta^N)\gamma_0]$ and where $\pi(\tilde{h}^N)$ satisfies (29) with $\kappa_0 = 1$. Therefore, when the non-tradable sector is more labor intensive $\alpha^N > \alpha^T$, central banks lean with the wind by loosening monetary policy to engineer a boom today when they anticipate that an adverse shock will drive the economy into a recession. Central banks under-tighten relative to the cooperative solution. If the non-tradable sector is more labor intensive and central banks anticipate that a global shock next period will lead to an overheated labor market, they over-tighten and drive the economy into an unnecessary recession. These findings are summarized in the next Proposition.

Proposition 6. *Suppose $\alpha^N > \alpha^T$. We have under-tightening, i.e. $R^{NE} < R^{GP}$, if and only if when central banks anticipate a negative output gap in the Nash equilibrium next period. The relationship is reversed when $\alpha^T > \alpha^N$.*

Proof. In Appendix A.13 □

6 Conclusion

We studied the scope for international monetary cooperation in an environment with global demand imbalances and ask whether the Nash equilibrium features under- or over-tightening relative to the optimal cooperative solution. We show that the answer depends on the sign of the output gap and the difference in labor intensity across sectors. Under the assumption that labor intensity is higher in the non-tradable sector, cooperation calls for lower interest rates when the economy is in a recession and for higher interest rates when the economy is overheating.

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APPENDIX

A Proofs

A.1 Proof of Lemma 1

The proof follows directly from rearranging (15). □

A.2 Proof of Lemma 2

Using (8), (9), and that $c_t^T = (h_t^T)^{\alpha^T}$ we obtain

$$\frac{P_t^N}{P_t^T} = \frac{\alpha^T (h^T)^{\alpha^T - 1}}{\alpha^N (h^N)^{\alpha_N - 1}}. \tag{A.1}$$

Combining this with (4)

$$\frac{\phi^T \alpha^T}{h_t^T} = \frac{\phi^N \alpha^N}{h_t^N}. \tag{A.2}$$

Rearranging and using Lemma 1, we obtain $\tilde{h}_t^T = \tilde{h}_t^N$. □

A.3 Proof of Lemma 3

Because for $t \geq 1$ the economy is in a stationary equilibrium, the continuation value is

$$V(b_1^*) = \frac{1}{1-\beta}\left[\phi^T \log(c_1^T) + \phi^N \log(c_1^N) - (h_1^T + h_1^N)\right] \tag{A.3}$$

and since wages are flexible for $t \geq 1$, (5) holds. Combining it with (8) and (9) yields

$$c_1^i (h_1^i)^{1-\alpha^i} = \phi^i \alpha^i \tag{A.4}$$

for any $i \in \{T, N\}$. Market clearing for tradables (12) requires

$$c_1^T = (h_1^T)^{\alpha^T} + (1-\beta) b_1^* \tag{A.5}$$

where we use $R^* = 1/\beta$. Using (10) we arrive at $c_1^N = h_1^N = \alpha^N \phi^N$. Substituting it into (A.3) along with (A.5) we arrive at (17). Equation (18) is obtained by plugging (A.4) for $i = T$ into (A.5). □

A.4 Single Central Bank Problem

The Lagrangian associated with (19) can be written as follows:

$$\mathcal{L} = \sum_{i\in\{T,N\}} \left[\alpha^i \phi_0^i \log(h_0^i) - h_0^i\right] - \frac{\chi}{2}\left[\kappa_0 \prod_{i\in\{T,N\}} \left(\frac{h_0^i}{\alpha^i\phi^i}\right)^{(1-\alpha^i)\phi_0^i} - (1+\bar{\pi}_0)\right]^2 + \phi_0^T \log(z_0)$$
$$+ \beta V\left(R_0^*(h_0^T)^{\alpha^T}(1-z_0)\right) + \eta_0\left[\frac{\alpha^N\phi_0^N}{\alpha^T\phi_0^T} h_0^T z_0 - h_0^N\right] + \mu_0\left[\frac{1}{z_0} - \beta R_0^* \frac{(h_0^T)^{\alpha^T}}{\alpha^T\phi_0^T}(\mathcal{H}_1^T)^{1-\alpha^T}\right]$$

with $\mathcal{H}_1^T \equiv \mathcal{H}_1^T\left(R_0^*(h_0^T)^{\alpha^T}(1-z_0)\right)$ for simplicity. □

A.5 Proof of Proposition 1

The proof follows directly from (23) and (26). Consider R_0^* such as $z_0 = 1$. Then, combining (23) and (26) yields

$$\left[\frac{\sum_i (1-\alpha^i)(1-\alpha^i\delta^i)\phi_0^i}{\alpha^T\phi_0^T}\right]\frac{\mu_0}{\delta^N} = -\left[\frac{1-\alpha^T}{\alpha^T} - \frac{1-\alpha^N}{\alpha^N}\right]\phi_0^N\alpha^N\tilde{h}_0^N \tag{A.6}$$

Suppose that $\alpha^N > \alpha^T$. Then $\mu_0 < 0$ if and only $\tilde{h}_0 > 0$. Suppose that $\alpha^N < \alpha^T$, Then $\mu_0 < 0$ if and only $\tilde{h}_0 < 0$. □

A.6 Proof of Proposition 2

The proof follows directly from (36) and (35). Rearranging (36), we get

$$-\frac{\phi_0^T\alpha^T}{h_0^T}\tilde{h}_0^T\left[1+(1-\alpha^T\delta^N)\frac{h_0^N}{h_0^T}\right] = \left[\frac{\phi_0^T}{h_0^T}(1-\alpha^T)+\frac{\phi_0^N}{h_0^T}(1-\alpha^N)(1-\alpha^T\delta^N)\right]\chi\,(1+\pi_0)\,(\pi_0-\bar{\pi})$$

where we use (16). Then, using (35) we arrive at

$$\tilde{h}_0 = -\frac{(1-\alpha^T)\phi_0^T + (1-\alpha^T\delta^N)(1-\alpha^N)\phi_0^N}{\alpha^T\phi_0^T + (1-\alpha^T\delta^N)\alpha^N\phi_0^N}\chi\,(1+\pi_0)\,(\pi_0-\bar{\pi})$$

Equation (29) is obtained by plugging directly (21) into (20) with $z_0 = 1$. □

A.7 Proof of Lemma 4

Combining (2) and (4) with market clearing $c_t^T = (h_t^T)^{\alpha^T}$ and $c_t^N = (h_t^N)^{\alpha^N}$, we get

$$\frac{P_1}{P_0} = \frac{(\phi_0^T)^{\phi_0^T}(\phi_0^N)^{\phi_0^N}}{(\phi^T)^{\phi^T}(\phi^N)^{\phi^N}} \left(\frac{\phi_0^N}{\phi_0^T} \frac{\left(h_0^T\right)^{\alpha^T}}{\left(h_0^N\right)^{\alpha^N}} \right)^{-\phi_0^N} \left(\frac{\phi^N}{\phi^T} \frac{\left(h_1^T\right)^{\alpha^T}}{\left(h_1^N\right)^{\alpha^N}} \right)^{\phi^N} \frac{P_1^T}{P_0^T}$$

Because wages are flexible for $t \geq 1$, $h_1^T = \alpha^T \phi^T$, $h_1^N = \alpha^N \phi^N$ and monetary policy stabilizes prices $P_1 = P_0$ for $t \geq 1$, we arrive at

$$\frac{P_1^T}{P_0^T} = \left(\frac{\left(h_0^T\right)^{\alpha^T}}{\left(h_0^N\right)^{\alpha^N}} \right)^{\phi_0^N} \frac{\phi^T}{\phi_0^T} \frac{\left(\alpha^N \phi^N\right)^{\alpha^N \phi^N}}{\left(\alpha^T \phi^T\right)^{\alpha^T (1-\phi^T)}} \tag{A.7}$$

Next, plugging (35) into (A.7) we arrive at

$$\frac{P_1^T}{P_0^T} = \left(\frac{h_0^T}{\alpha^T \phi_0^T} \right)^{\left(\alpha^T - \alpha^N\right)\phi_0^N} \left[\frac{\left(\alpha^T \phi^T\right)^{\alpha^T \phi^T} \left(\alpha^N \phi^N\right)^{\alpha^N \phi^N}}{\left(\alpha^N \phi_0^N\right)^{\alpha^N \phi_0^N} \left(\alpha^T \phi^T\right)^{\alpha^T \phi_0^T}} \right] \frac{\phi^T}{\phi_0^T} \left(\alpha^T \phi^T\right)^{-\alpha^T} \tag{A.8}$$

Using market clearing for tradables and plugging $h_1^T = \alpha^T \phi^T$ into (6), we obtain

$$R_0^* = \frac{1}{\beta} \frac{\phi_0^T}{\phi^T} (\alpha^T \phi^T)^{\alpha^T} \left(h_0^T\right)^{-\alpha^T} \tag{A.9}$$

Finally, substituting (A.8) and (A.9) into the interest parity condition (7), we get

$$R_0 = (1 + \tilde{h}_0)^{-(\alpha^N \phi_0^N + \alpha^T \phi_0^T)} \frac{1}{\beta} \left[\prod_{i \in \{T,N\}} \frac{\left(\alpha^i \phi^i\right)^{\alpha^i \phi^i}}{\left(\alpha^i \phi_0^i\right)^{\alpha^i \phi_0^i}} \right]$$

where the last equality uses (16). □

A.8 Proof of Proposition 3

We substitute (20) and (21) into (19) to express the single central bank problem as follows:

$$V_0 = \min_{h_0^T, z_0} \left[\sum_i \alpha^i \phi_0^i \right] \log(h_0^T) - \left[1 + \frac{\alpha^N \phi_0^N}{\alpha^T \phi_0^T} z_0 \right] h_0^T + \alpha^N \phi_0^N \log\left(\frac{\alpha^N \phi_0^N}{\alpha^T \phi_0^T} z_0 \right) + \phi_0^T \log(z_0)$$
$$- \frac{\chi}{2} \left[\kappa_0 \left(\frac{h_0^T}{\alpha^T \phi_0^T} \right)^{\sum_i (1-\alpha^i)\phi_0^i} z_0^{(1-\alpha^N)\phi_0^N} - (1 + \bar{\pi}) \right]^2 + \beta V_1 \left(R_0^* (h_0^T)^{\alpha^T} (1 - z_0) \right)$$

subject to

$$\frac{1}{z_0} = \beta R_0^* \frac{(h_0^T)^{\alpha^T}}{\alpha^T \phi_0^T} \left(\mathcal{H}_1^T (R_0^* (h_0^T)^{\alpha^T} (1 - z_0)) \right)^{1-\alpha^T}$$

We determine $\partial V_0 / \partial R_0^*$ by applying the envelope theorem to the central bank problem,

$$\frac{dV_0}{dR_0^*} = \beta (h_0^T)^{\alpha^T} (1 - z_0) \frac{dV_1(b_1^*)}{db_1^*} + \left[(1 - z_0)(h_0^T)^{\alpha^T} \frac{1 - \alpha^T}{\mathcal{H}_1^T} \frac{d\mathcal{H}_1^T(b_1^*)}{db_1^*} + \frac{1}{R_0^*} \right] \frac{1}{z_0} \mu_0 \quad \text{(A.10)}$$

Starting from a symmetric equilibrium with $z_0 = 1$, (A.10) reduces to

$$\frac{dV_0}{dR_0^*} = \frac{\mu_0}{R_0^*}. \quad \text{(A.11)}$$

Applying the envelope theorem to changes in z_0, we have

$$\frac{dV_0}{dz_0} = \left[\left(-h_0^N + \alpha^N \phi_0^N \right) - (1 - \alpha^N) \phi_0^N \chi (\pi_0 - \bar{\pi}) \right] + \frac{1}{\delta^N} \mu_0 = 0, \quad \text{(A.12)}$$

where we use $z_0 = 1$. Substituting (A.12) into (A.11) , we arrive to

$$dV_0 = \left[\alpha^N \phi_0^N \tilde{h}_0^T + (1 - \alpha^N) \phi_0^N \chi (1 + \pi_0)(\pi_0 - \bar{\pi}) \right] \delta^N \frac{dR_0^*}{R_0^*}.$$

A.9 Proof of Proposition 4

We combine the optimality conditions for h_0^T and h_0^N of the global planner problem (33)

$$\frac{\phi_0^N \alpha^N}{h_0^N} - 1 = \frac{\phi_0^N}{h_0^N} (1 - \alpha^N) \chi (1 + \pi_0)(\pi_0 - \bar{\pi}) + \eta_0$$
$$\frac{\phi_0^T \alpha^T}{h_0^T} - 1 = \frac{\phi_0^T}{h_0^T} (1 - \alpha^T) \chi (1 + \pi_0)(\pi_0 - \bar{\pi}) - \frac{h_0^N}{h_0^T} \eta_0$$

to arrive to

$$\left(\frac{\phi_0^N \alpha^N}{h_0^N} - 1 \right) \left(h_0^T + h_0^N \right) = \left[\sum_i (1 - \alpha^i) \phi_0^i \right] \chi (1 + \pi_0)(\pi_0 - \bar{\pi})$$

we then use (35) to substitute for h_0^N and obtain

$$\left[\alpha^T \phi_0^T + \alpha^N \phi_0^N \right] \tilde{h}_0^T = - \left[(1 - \alpha^T) \phi_0^T + (1 - \alpha^N) \phi_0^N \right] \chi (1 + \pi_0) (\pi_0 - \bar{\pi})$$

which corresponds to (37). Equation (29) is obtained by plugging directly (35) into (34). □

A.10 Global Planner Problem

After substituting the restriction (34) in the objective, the Lagrangian associated with the global planner problem (33) is given by

$$\mathcal{L} = \sum_{i\in\{T,N\}}\left[\alpha^i\phi_0^i\log(h_0^i)-h_0^i\right]-\frac{\chi}{2}\left[\kappa_0\prod_{i\in\{T,N\}}\left(\frac{h_0^i}{\alpha^i\phi_0^i}\right)^{(1-\alpha^i)\phi_0^i}-(1+\bar{\pi}_0)\right]+\eta_0\left[\frac{\alpha^N\phi_0^N}{\alpha^T\phi_0^T}h_0^T-h_0^N\right]$$

A.11 Proof of Proposition 5

Let define

$$F_{NE}(\tilde{h}_0) \equiv \tilde{h}_0 + \frac{(1-\alpha^T)\phi_0^T+(1-\alpha^T\xi)(1-\alpha^N)\phi_0^N}{\alpha^T\phi_0^T+(1-\alpha^T\xi)\alpha^N\phi_0^N}\chi\prod_i\frac{(\alpha^i\phi_0^i)^{-\alpha^i\phi_0^i}}{(\alpha^i\phi^i)^{-\alpha^i\phi^i}}$$
$$\times(1+\tilde{h}_0)^{\sum_i(1-\alpha^i)\phi_0^i}\left[\frac{\prod_i(\phi_0^i)^{(1-\alpha^i)\phi_0^i}}{\prod_i(\phi^i)^{(1-\alpha^i)\phi^i}}(1+\tilde{h}_0)^{\sum_i(1-\alpha^i)\phi_0^i}-(1+\bar{\pi})\right] \quad \text{(A.13)}$$

and

$$F_{GP}(\tilde{h}_0) \equiv \tilde{h}_0 + \frac{(1-\alpha^T)\phi_0^T+(1-\alpha^N)\phi_0^N}{\alpha^T\phi_0^T+\alpha^N\phi_0^N}\chi\prod_i\frac{(\alpha^i\phi_0^i)^{-\alpha^i\phi_0^i}}{(\alpha^i\phi^i)^{-\alpha^i\phi^i}}$$
$$\times(1+\tilde{h}_0)^{\sum_i(1-\alpha^i)\phi_0^i}\left[\frac{\prod_i(\phi_0^i)^{(1-\alpha^i)\phi_0^i}}{\prod_i(\phi^i)^{(1-\alpha^i)\phi^i}}(1+\tilde{h}_0)^{\sum_i(1-\alpha^i)\phi_0^i}-(1+\bar{\pi})\right] \quad \text{(A.14)}$$

Notice that $F_{NE}(\tilde{h}_0)=0$ is the implicit function that determines the output gap in the Nash equilibrium. It combines (28) and (29). Similarly $F_{GP}(\tilde{h}_0)=0$ is the one that determines the output gap under cooperation by combining (37) and (29). We denote by $\tilde{h}_0^{GP}$ the solution to $F_{GP}(\tilde{h}_0)=0$ and $\tilde{h}_0^{NE}$ the solution to $F_{NE}(\tilde{h}_0)=0$. We have that

$$F_{GP}(\tilde{h}_0^{NE}) = \underbrace{F_{NE}(\tilde{h}_0^{NE})}_{=0}+\frac{\left(\alpha^N-\alpha^T\right)\alpha^T\phi_0^T\phi_0^N\xi}{[(1-\alpha^T)\phi_0^T+(1-\alpha^T\xi)(1-\alpha^N)\phi_0^N][\alpha^T\phi_0^T+\alpha^N\phi_0^N]}\tilde{h}_0^{NE} \quad \text{(A.15)}$$

Moreover, we have

$$F'_{GP}(\tilde{h}_0) = 1+\frac{\left(\sum_i(1-\alpha^i)\phi_0^i\right)^2}{\sum_i\alpha^i\phi_0^i}\frac{\chi}{1+\tilde{h}_0}\left(1+\pi_0\right)\left(1+2\pi_0-\bar{\pi}\right)>0.$$

Now, suppose $\alpha^N>\alpha^T$. If $\tilde{h}_0^{NE}>0$, then from (A.15) we have that $F_{GP}(\tilde{h}_0^{NE})>0$. Further, because $F'_{GP}(\tilde{h}_0)>0$, it follows that $h_0^{NE}>h_0^{GP}$. By (30) in Proposition 4, this implies that

$R_0^{NE} < R_0^{GP}$. If $\tilde{h}_0^{NE} < 0$, then from (A.15) we have that $F_{GP}(\tilde{h}_0^{NE}) < 0$ and thus $h_0^{NE} < h_0^{GP}$. By (30), this implies that $R_0^{NE} > R_0^{GP}$. The opposite happens if $\alpha^T > \alpha^N$. □

A.12 Optimal Policy with Anticipated Shocks

Optimal Policy for a Single Country

We start by rewriting the problem of the central bank in period 0

$$V_0(b_0^*) = \max_{h^T,h^N,z} \sum_{i\in\{T,N\}} \alpha^N \phi_0^N \log(h_0^N) - (h_0^T + h_0^N) - \frac{\chi}{2}(\pi_0 - \bar{\pi})^2 + \beta V_1(b_1^*)$$

subject to

$$\frac{b_1^*}{R_0^*} = (1 - z_0)(h_0^T)^{\alpha^T} + b_0^* \tag{A.16}$$

$$\pi_0 = \kappa_0 \prod_{i\in\{T,N\}} \left(\frac{h_0^i}{\alpha^i \phi_0^i}\right)^{(1-\alpha^i)\phi_0^i} - 1 \tag{A.17}$$

$$h_0^N = \frac{\alpha^N \phi_0^N}{\alpha^T \phi_0^T} h_0^T z_0 \tag{A.18}$$

$$\frac{1}{z_0} = \beta R_0^* \frac{\phi^T}{\phi_0^T} \left[\frac{\left(\mathcal{H}_1^T(b_1^*)\right)^{1-\alpha^T}}{\alpha^T \phi^T}\right] (h_0^T)^{\alpha^T} \tag{A.19}$$

where $\mathcal{H}_1^T(b_1^*)$ is defined in (18). The first order condition for z_0 is given by

$$\frac{\mu_0}{z_0} = \delta^N h_0^N \eta_0 \tag{A.20}$$

where η_0 and μ_0 are the Lagrange multipliers associated with (A.18) and (A.19) respectively. By the envelope theorem, we have

$$\begin{aligned} V_0'(b_0^*) &= \beta R_0^* V_1'(b_1^*) - R_0^*(1-\alpha^T)\frac{1}{h_1^T}\frac{\partial \mathcal{H}_1^T}{\partial b_1^*}\frac{\mu_0}{z_0} \\ &= \frac{\phi_0^T}{c_0^T} + \frac{1}{c_0^T}\left(\frac{1}{\delta^N} - 1\right)\frac{\mu_0}{z_0} \end{aligned} \tag{A.21}$$

where we use (17) and (18).

We now turn to the problem of the individual central bank at date $t = -1$. It solves

$$\max_{h^T, h^N, z} \sum_{i \in \{T,N\}} \left[\alpha^i \phi^i \log(h^i) - h^i \right] + \phi^T \log(z) - \frac{\chi}{2} (\pi)^2 + \beta V_0 \left(b_0^* \right) \tag{A.22}$$

subject to

$$\frac{b_0^*}{R^*} = (1 - z)(h^T)^{\alpha^T} \tag{A.23}$$

$$\pi = \prod_{i \in \{T,N\}} \left(\frac{h^i}{\alpha^i \phi^i} \right)^{(1-\alpha^i)\phi^i} - 1 \tag{A.24}$$

$$h^N = \frac{\alpha^N \phi^N}{\alpha^T \phi^T} h^T z \tag{A.25}$$

$$\frac{1}{z} = \beta R^* (h^T)^{\alpha^T} \frac{\phi_0^T}{\phi^T} \frac{1}{\mathcal{C}_0^T (b_0^*)} \tag{A.26}$$

We letting η and μ denote the Lagrange multipliers associated respectively with (A.25) and (A.26). Substituting (A.23) into both (A.22) (A.26), and substituting (A.24) into (A.22), the first-order condition with respect to z yields

$$\frac{\phi^T}{z} - \beta R^* (h^T)^{\alpha^T} V'(b_0^*) - \left(1 - \frac{z}{c_0^T} \frac{d\mathcal{C}_0^T (b_0^*)}{dz} \right) \frac{\mu}{(z)^2} + \frac{h_0^N}{z} \eta = 0$$

Plugging (A.21) and using (A.19) we arrive at

$$- \left(\frac{1}{\delta^N} - 1 \right) \frac{\phi^T}{\phi_0^T} \frac{\mu_0}{z_0} - \left[1 + \frac{db_1^*}{db_0^*} \frac{R^* c^T}{R_0^* c_0^T} \left(\frac{1}{\delta^N} - 1 \right) \right] \frac{\mu}{z} + h_0^N \eta = 0 \tag{A.27}$$

which corresponds to (51). The first-order conditions for h^N and h^T are given by

$$[h_0^N] :: \quad \frac{\alpha^N \phi^N}{h^N} - 1 = \frac{1}{h^N} (1 - \alpha^N) \phi^N \chi \left(1 + \pi \right) \pi + \eta \tag{A.28}$$

$$[h_0^T] :: \quad \frac{\alpha^T \phi^T}{h^T} - 1 = \frac{1}{h_0^T} (1 - \alpha^T) \phi_0^T \chi \left(1 + \pi \right) \pi - \frac{h^N}{h^T} \eta + \frac{\alpha^T}{h^T} \left[1 - \frac{b_0^*}{c_0^T} \frac{d\mathcal{C}_0^T (b_0^*)}{db_0^*} \right] \frac{\mu}{z_0} - \frac{\alpha^T b_0^*}{h^T} \beta V_1 (b_0^*). \tag{A.29}$$

Nash Equilibrium

In the Nash equilibrium, there is no demand imbalance $z = z_0 = 1$ which implies $b_0^* = b_1^* = 0$. By (A.27) we have $\left[\delta^N + \gamma_0 (1 - \delta^N) \right] \mu = \delta^N h_0^N \eta - (1 - \delta^N) \frac{\phi^T}{\phi_0^T} \delta^N h_0^N \eta_0$. Plugging it into (A.29) and combining (A.28) and (A.29), to eliminate η we arrive at

$$\tilde{h}\sum_i \left(1-\alpha^T\delta_0^i\right)\alpha^i\phi^i = -\sum_i \left(1-\alpha^T\delta_0^i\right)\left(1-\alpha^i\right)\phi^i\chi\left(1+\pi\right)\pi + \alpha^T\delta_0^N\left(1-\delta^N\right)\frac{\phi^T}{\phi_0^T}h_0^N\eta_0$$

where we use $\mu_0 = \delta^N h_0^N \eta_0$ from (A.20) and $\delta_0^i \equiv \frac{\delta^i}{\delta^N+(1-\delta^N)\gamma_0}$. □

A.13 Proof of Proposition 6

We denote variables under cooperation and in the Nash equilibrium with superscripts GP and NE, respectively. Under cooperation, $\tilde{h}^{GP}=0$ and by (30) $R^{GP}=\frac{1}{\beta}$. In the Nash equilibrium, $\tilde{h}^{NE}=0$ satisfies $F_{NE}(\tilde{h}^{NE})=0$ with

$$F_{NE}(\tilde{h}^{NE};\tilde{h}_0) \equiv \tilde{h}^{NE} + \Psi\chi\left(1+\tilde{h}^{NE}\right)^{\sum_i(1-\alpha^i)\phi_0^i}\left[(1+\tilde{h}^{NE})^{\sum_i(1-\alpha^i)\phi_0^i}-1\right] - \Psi\frac{\delta^N(1-\delta^N)\alpha^T\phi^T}{(\sum_i(1-\alpha^T\delta^i)\alpha^i\phi^i)^2}\cdot(\alpha^N-\alpha^T)\tilde{h}_0 \quad \text{(A.30)}$$

with $\Psi \equiv \frac{\sum_i(1-\alpha^T\delta^i)(1-\alpha^i)\phi^i}{\sum_i(1-\alpha^T\delta^i)\alpha^i\phi^i}$. Further, note that $\frac{\partial F_{NE}(\tilde{h}^{NE};\tilde{h}_0)}{\partial\tilde{h}^{NE}}>0$. Suppose $\alpha^N>\alpha^T$.

If $\tilde{h}_0>0$, then $F_{NE}(0;\tilde{h}_0)>0$. Because $F_{NE}(\tilde{h}^{NE};\tilde{h}_0)$ is increasing in $\tilde{h}^{NE}$, this implies that $\tilde{h}^{NE}>0$. Using (30) we arrive at $R_0^{NE}<R_0^{GP}=\beta^{-1}$. If $\tilde{h}_0<0$, then $\tilde{h}^{NE}<0$ by $F_{NE}(0;\tilde{h}_0)<0$ and because $\partial F_{NE}(\tilde{h}^{NE};\tilde{h}_0)$ is increasing in $\tilde{h}^{NE}$. Using (30), this implies that $R_0^{NE}>R_0^{GP}=\beta^{-1}$. □

B Linear-Quadratic Approximation

B.1 Derivation of the Loss Function

The goal is to write the objective function of the domestic central bank (19) in terms of the squared output gap, squared inflation, squared terms of trade, and demand imbalance. The loss function is defined as the gap relative to the efficient outcome $V_0 - V^{max}$, where V^{max} is the maximized welfare, defined as welfare when consumption and leisure take on their efficient values. We start by describing the efficient allocation.

Efficient Allocation. The socially optimal allocation solves

$$V^{\text{fb}} = \max_{h_t^N, h_t^T} \sum_{t=0}^{\infty} \beta^t \left\{ \phi_t^T \alpha^T \log(h_t^T) + \phi_t^N \alpha^N \log(h_t^N) - \left[(h_t^T)^{1+\frac{1}{\eta}} + (h_t^N)^{1+\frac{1}{\eta}} \right]^{\frac{\eta}{\eta+1}} \right\}$$

The optimality conditions require

$$\frac{\phi_t^T \alpha^T}{h_t^T} = \left(\frac{h_t^T}{h_t} \right)^{\frac{1}{\eta}} \quad \text{and} \quad \frac{\phi_t^N \alpha^N}{h_t^N} = \left(\frac{h_t^N}{h_t} \right)^{\frac{1}{\eta}}$$

Using $h_t = \left[(h_t^T)^{1+\frac{1}{\eta}} + (h_t^N)^{1+\frac{1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$ we arrive at

$$h_t^{T,\text{fb}} = \left(\alpha^T \phi_t^T \right)^{\frac{\eta}{\eta+1}} \left[\alpha^T \phi_t^T + \alpha^N \phi_t^N \right]^{\frac{1}{\eta+1}}, \tag{A.31}$$

$$h_t^{N,\text{fb}} = \left(\alpha^N \phi_t^N \right)^{\frac{\eta}{\eta+1}} \left[\alpha^T \phi_t^T + \alpha^N \phi_t^N \right]^{\frac{1}{\eta+1}}. \tag{A.32}$$

Loss function. We now turn to the second-order approximation of the objective function of the domestic central bank, given by

$$V_0 = \phi_0^T \alpha^T \log(h_0^T) + \phi_0^N \alpha^N \log(h_0^N) - \left[(h_0^T)^{1+\frac{1}{\eta}} + (h_0^N)^{1+\frac{1}{\eta}} \right]^{\frac{\eta}{\eta+1}} - \frac{\chi}{2} (\pi_0 - \bar{\pi})^2 + \phi_0^T \log(z_0) + \beta V_1 \left(R_0^* (h_0^T)(1 - z_0) \right) \tag{A.33}$$

The continuation value V_1 (derived in Appendix B.2) satisfies

$$V_1 = \frac{1}{1-\beta} \Bigg[V^{\text{fb}} + \phi^T \log(z_1) + \alpha^T \phi^T \left(\frac{z_1 - 1}{z_1} + \frac{1}{\eta+1} \frac{\sum_i \alpha^i \phi^i}{\alpha^T \phi^T} \log \left(1 - \frac{\alpha^T \phi^T}{\sum_i \alpha^i \phi^i} \frac{z_1 - 1}{z_1} \right) - \frac{\eta}{\eta+1} \log(z_1) \right) \Bigg] \tag{A.34}$$

where $z_1 = 1 + (1-\beta)R_0^* \left(\frac{h_0^T}{\mathcal{H}_1^T(z_1)}\right)^{\alpha^T}(1-z_0)$ and $\mathcal{H}_1^T(z_1) = \left(\frac{\alpha^T\phi^T}{z_1}\right)^{\frac{\eta}{\eta+1}}\left[\frac{\alpha^T\phi^T}{z_1} + \alpha^N\phi^N\right]^{\frac{1}{\eta+1}}$.
A second-order Taylor expansion to the welfare (A.33) yields

$$V_0 = V^{\text{fb}} - \frac{1}{2}\left[\alpha^T\phi_0^T(\tilde{h}_0^T)^2 + \alpha^N\phi_0^N(\tilde{h}_0^N)^2\right] - \frac{1}{2\eta}\frac{\left(\alpha^T\phi_0^T\right)\left(\alpha^N\phi_0^N\right)}{\alpha^T\phi_0^T + \alpha^N\phi_0^N}\left(\tilde{h}_0^T - \tilde{h}_0^N\right)^2$$
$$- \frac{\chi}{2}(\pi_0 - \bar{\pi})^2 + \phi_0^T\tilde{z}_0 - \phi_0^T\left[\tilde{z}_0 + \frac{1}{2}\frac{1}{\hat{\delta}^N}\left(\tilde{z}_0\right)^2\right] + o||s||^3$$

where $+o(||s||^3)$ indicate the 3rd and higher order terms left out, s is a notation for the log of the shocks, and

$$\frac{1}{\hat{\delta}^N} \equiv 1 + \frac{1-\beta}{\beta}\frac{\phi_0^T}{\phi^T}\left[1 - \alpha^T\left(1 - \frac{1}{\eta+1}\frac{\alpha^N\phi^N}{\sum_i \alpha^i\phi^i}\right)\right] > 1. \tag{A.35}$$

Denoting by $\mathbb{L} \equiv V^{\text{fb}} - V$ the loss function, we arrive at

$$\mathbb{L} = \frac{1}{2}\left[\alpha^T\phi_0^T(\tilde{h}_0^T)^2 + \alpha^N\phi_0^N(\tilde{h}_0^N)^2 + \chi(\pi_0 - \bar{\pi})^2 + \left(\frac{\phi_0^T}{\hat{\delta}^N} + \frac{1}{\eta\sum_i(\alpha^i\phi_0^i)^{-1}}\right)\left(\tilde{z}_0\right)^2\right] + o||s||^3$$

□

B.2 Derivation of the Continuation Value

Because for $t \geq 1$ the economy is in a stationary equilibrium, the continuation value is

$$V(b_1^*) = \frac{1}{1-\beta}\left\{\phi^T\log(c_1^T) + \phi^N\log(c_1^N) - \left[(h_t^T)^{1+\frac{1}{\eta}} + (h_t^N)^{1+\frac{1}{\eta}}\right]^{\frac{\eta}{\eta+1}}\right\} \tag{A.36}$$

and since wages are flexible for $t \geq 1$, we have

$$\frac{W_1^T}{P_1^T} = \frac{c_1^T}{\phi_1^T}\left(\frac{h_1^T}{h_1}\right)^{\frac{1}{\eta}}, \quad \text{and} \quad \frac{W_1^N}{P_1^N} = \frac{c_1^N}{\phi_1^N}\left(\frac{h_1^N}{h_1}\right)^{\frac{1}{\eta}}$$

Combining it with (8) and (9) yields

$$\frac{c_1^i}{(h_1^i)^{\alpha^i}}h_1^i = \phi^i\alpha^i\left(\frac{h_1^i}{h_1}\right)^{-\frac{1}{\eta}}$$

Denoting by $z_1 \equiv \frac{c_1^T}{(h_1^T)^{\alpha^T}}$ the demand imbalance at $t = 1$, we arrive at

$$h_1^T = \left(\frac{\alpha^T \phi^T}{z_1}\right)^{\frac{\eta}{\eta+1}} \left[\frac{\alpha^T \phi^T}{z_1} + \alpha^N \phi^N\right]^{\frac{1}{\eta+1}}, \tag{A.37}$$

$$h_1^N = \left(\alpha^N \phi^N\right)^{\frac{\eta}{\eta+1}} \left[\frac{\alpha^T \phi^T}{z_1} + \alpha^N \phi^N\right]^{\frac{1}{\eta+1}}. \tag{A.38}$$

for any $i \in \{T, N\}$, where using market clearing for tradables (12), we get

$$z_1 = 1 + (1-\beta)\frac{b_1^*}{(h_1^T)^{\alpha^T}} \tag{A.39}$$

Substituting (A.37), (A.38) into (A.36) along with market clearing $c_1^N = (h_1^N)^{\alpha^T}$ and $c_1^T = z_1 (h_1^T)^{\alpha^T}$, we arrive at (A.34). □

B.3 Linearized Model

For convenience, we present the problem of the central bank here

$$\max_{\{h_0^T, h_0^N, \pi_0, z_0\}} V_0\left(h_0^T, h_0^N, \pi_0, z_0\right)$$

subject to

$$\pi_0 = \kappa_0 \prod_{i \in \{T,N\}} \left(\frac{h_0^i}{h^{i,\text{fb}}}\right)^{(1-\alpha^i)\phi_0^i} - 1 \tag{A.40}$$

$$h_0^N = \left(\frac{\alpha^N \phi_0^N}{\alpha^T \phi_0^T}\right)^{\frac{\eta}{\eta+1}} h_0^T z_0 \tag{A.41}$$

$$\frac{1}{z_0} = \beta R_0^* \frac{(h_0^T)^{\alpha^T}}{\phi_0^T} \frac{1}{C_1^T(b_1^*)} \tag{A.42}$$

where V_0 is given by (A.33) and $h^{T,\text{fb}}$ and $h^{N,\text{fb}}$ are respectively given by (A.31) and (A.32).

The goal here is to linearize (A.40)-(A.42). For simplicity, we omit the term "$o||s||^2$" which indicates that 2nd order and higher terms are left out when using the 1st order approximations.

Relative consumption. The first order Taylor expansion of (A.41) is given by

$$\tilde{h}_0^N = \tilde{h}_0^T + \tilde{z}_0 \tag{A.43}$$

Aggregate demand. Using $c_1^T = z_1(h_1^T)^{\alpha^T}$, the first order approximation of (A.42) yields

$$\tilde{z}_0 = -(r_0^* - r_0^{*n}) - \alpha^T \tilde{h}_0^T + \tilde{z}_1 + \alpha^T \tilde{h}_1^T \tag{A.44}$$

with $r_0^{*n} = r^* + (1 - \frac{\alpha^T\eta}{\eta+1}) \log\left[\frac{\phi_0^T}{\phi^T}\right] + \frac{1}{\eta} \log\left[\frac{\sum_i \alpha^i \phi^i}{\sum_i \alpha^i \phi_0^i}\right]$. Then, plugging $b_1^* = R_0^*(h_0^T)^{\alpha^T}(1 - z_0)$ into (A.39) we obtain, up to a first-order, that (A.37) and (A.39) become

$$\tilde{h}_1^T = -\left[1 - \frac{1}{\eta+1}\frac{\alpha^N\phi^N}{\alpha^T\phi^T + \alpha^N\phi^N}\right]\tilde{z}_1 \tag{A.45}$$

$$\tilde{z}_1 = -(1-\beta)\frac{\beta\phi_0^T}{\phi^T}\tilde{z}_0 \tag{A.46}$$

Finally, substituting (A.45) and (A.46) we arrive at

$$\tilde{z}_0 = -\hat{\delta}^N\left[(r_0^* - r_0^{*n}) + \alpha^T \tilde{h}_0^T\right]. \tag{A.47}$$

Phillips curve. Linearizing (A.40) around the efficient allocation, we get

$$\pi_0 = \log(\kappa_0) + (1-\alpha^N)\phi_0^N \tilde{h}_0^N + (1-\alpha^T)\phi_0^T \tilde{h}_0^T \tag{A.48}$$

Using (A.43) and substituting (A.47) into (A.48), we get

$$\pi_0 = \log(\kappa_0) - (1-\alpha^N)\phi_0^N \hat{\delta}^N (r_0^* - r_0^{*n}) + \sum_i (1 - \alpha^T\hat{\delta}^i)(1-\alpha^i)\phi_0^i\, \tilde{h}_0^T.$$

B.4 Nominal interest rate

We start by deriving the non-linear relationship between the nominal interest rate and output gap for a single central bank. Using (2) and (4) and market clearing $c_t^N = (h_t^N)^{\alpha^N}$ and $c_t^T = (h_t^T)^{\alpha^T} z_t$, we can express the inflation rate as

$$\frac{P_1}{P_0} = \frac{\phi_0^T}{\phi^T}\left(\frac{(h_0^T)^{\alpha^T}}{(h_0^N)^{\alpha^N}} z_0\right)^{-\phi_0^N}\left(\frac{(h_1^T)^{\alpha^T}}{(h_1^N)^{\alpha^N}} z_1\right)^{\phi^N}\frac{P_1^T}{P_0^T} \tag{A.49}$$

Using $P_1 = P_0$, and substituting (A.41), (A.37) and (A.38) into (A.49) we get

$$\frac{P_1^T}{P_0^T} = (\kappa_0)^{\frac{\eta}{\eta+1}}\left(\frac{\phi^T}{\phi_0^T}\right)^{1-\frac{\alpha^T\eta}{\eta+1}} \frac{(z_0)^{\left(1-\alpha^N\right)\phi_0^N}}{(z_1)^{\left(1-\frac{\eta}{\eta+1}\alpha^T\right)\phi^N}} \frac{\left[\left(\alpha_0^T\phi_0^T\right)^{-\frac{\eta}{\eta+1}} h_0^T\right]^{\left(\alpha^T-\alpha^N\right)\phi_0^N}}{\left[\frac{\alpha^T\phi^T}{z_1} + \alpha^N\phi^N\right]^{\frac{1}{\eta+1}\left(\alpha^T-\alpha^N\right)\phi^N}} \tag{A.50}$$

The nominal interest rate is given by the interest parity condition, $R_0 = R_0^* \frac{P_1^T}{P_0^T}$ where $\frac{P_1^T}{P_0^T}$ satisfies (A.50). Linearizing this expression yields

$$\begin{aligned} r_0 - r_0^n = r_0^* - r_0^{*n} + (\alpha^T - \alpha^N)\phi_0^N \tilde{h}_0^T \\ + (1-\alpha^N)\phi_0^N \tilde{z}_0 - \left[1 - \alpha^T\left(1 - \frac{1}{\eta+1}\frac{\alpha^N\phi^N}{\alpha^T\phi^T + \alpha^N\phi^N}\right)\right]\phi^N \tilde{z}_1 \end{aligned} \tag{A.51}$$

with $r_0^n = r^* + \frac{\eta}{\eta+1}\log(\kappa_0)$. Finally, substituting (A.46) and (A.47) into (A.51), we obtain

$$r_0 = r_0^n + \left[1 - \left(1 - \alpha^N \hat{\delta}^N\right)\phi_0^N\right](r_0^* - r_0^{*n}) - \left(1 - \alpha^T \hat{\delta}^N\right)\alpha^N \phi_0^N \, \tilde{h}_0^T.$$