

Limited energy supply, sunspots, and monetary policy*

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Abstract

A common assumption in macroeconomics is that energy prices are determined in a world-wide, rather frictionless market. The recent European experience, instead, has raised the possibility of shortages in the quantity of energy available. Such limits to the supply of energy could mean that the local price of energy is affected by domestic economic activity. In a simple open-economy New Keynesian setting, the paper explores conditions under which energy shortages can raise the risk of self-fulfilling fluctuations. A firmer focus of the central bank on input prices (or on headline consumer prices) removes such risks.

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1 Introduction

A common assumption in macroeconomics is that energy prices are determined in a world-wide, rather frictionless market. From the perspective of a small open economy, then, the supply of energy is abundant. This means that domestic economic activity does not affect the domestic-currency price of energy other than through the exchange rate. Abundance, though, may not appear to adequately describe the situation in Europe today. Instead, a shortage of energy may render the domestic price of energy endogenous to domestic economic activity, with potentially profound implications for stabilization policy.

This paper studies such a scenario of shortage, and if risks to macroeconomic stability emerge. Toward this end, we look at a small open economy that imports energy from the rest of the world. Goods are produced using labor and energy; both goods and energy feature in households' consumption baskets. Goods prices are rigid. Energy prices are not. This core of the model is as in [Blanchard and Galí \(2009\)](#). International trade need not be balanced immediately. To this, we add household heterogeneity as in [Bilbiie \(2021\)](#). Namely, a share of households consume all their wage income. For the remaining households, changes in idiosyncratic risk matter. There is one important change relative to the literature: energy is not abundantly available at an exogenous *price*. Rather, we treat the *quantity* of energy available to households and firms as fixed and the price as endogenous: energy is scarce.

We ask in theory under which considerations the scarcity of energy exposes the economy to the risk of self-fulfilling fluctuations. In a calibrated version of the model we then explore how likely such a scenario is. The theory that we explore is as follows: A situation of self-fulfilling expectations could arise from a feedback loop between energy prices and economic activity. Namely, suppose that households and firms hold the non-fundamental belief that energy prices will be high. Under these beliefs, firms face high marginal costs. Inflation rises. Since goods prices are rigid, firms cannot pass all the costs on to domestic and foreign consumers. Higher energy prices mean higher external demand, the strength of which depends on the trading partner's propensity to demand exports when energy revenues rise. What is more, the fall in domestic demand does not fully reflect the rise

in energy prices. Aggregate demand (domestic plus external) can, therefore, rise. The ensuing rise in labor demand in turn has two effects. It raises the incomes of those domestic households that have a high marginal propensity to consume and it reduces idiosyncratic risk. Both of this supports aggregate demand even though energy prices are high. To the extent that the real wage rises, it raises marginal costs further, validating the initial beliefs. To rule out such a feedback loop, the central bank would need to reduce domestic demand sufficiently much to reduce total aggregate demand. We find that this may require a notably stronger response to inflation than conventional estimates suggest, or a response to input prices.

At the core of the policy implications lies that, in order to interrupt the energy-price-activity feedback loop, monetary policy has to lean sufficiently strongly against rising input costs. It can do so directly (*raising* rates when energy prices *rise* or when *nominal wages* rise) or it can do so indirectly through its response to inflation and economic activity. A focus on headline inflation is more conducive to cutting the feedback loop than a monetary response focused on core inflation. The reason is that core inflation does reflect the rise in energy prices to a lesser extent. The feedback loop comes with rising production and employment, but a fall in value added (GDP). A monetary response that focuses on stabilizing GDP would, thus, further fuel the feedback loop, whereas a monetary response to the level of production or employment works against the loop.

The remainder of the paper is structured as follows. We review the literature next. Section 2 presents the model. Section 3 provides pencil-and-paper results for a special case, so as to provide intuition for the possibility that the feedback loop arises. Section 4 calibrates the model economy to a stylized euro area and provides quantitative results. The same section also provides sensitivity analysis. A final section concludes.

Related literature Our paper emphasizes that an environment of scarce energy may make it notably harder for the central bank to anchor inflation expectations and economic activity. The key finding is that, in such an environment, there is a case for focusing on headline inflation instead of core inflation or, more generally, to engineer tighter monetary policy in the face of what looks like a cost-push shock.

There is, of course, a vast literature on energy and the macro-economy, to which we cannot do full justice here. Closest in terms of modeling are [Blanchard and Galí \(2009\)](#). They and a related paper, [Blanchard and Riggi \(2013\)](#), point to the structural features that shape the response to fundamental energy-supply shocks; namely, the share of energy in production and consumption, the monetary response, and the extent of real wage rigidities. [Nakov and Pescatori \(2009\)](#) focus on the energy elasticity of output. [Olivi, Sterk and Xhani \(2022\)](#) and [Känzig \(2022\)](#) have analyzed the distributional effects of changes in energy prices. All these papers consider an environment of abundant energy supply, which rules out the energy-price-activity feedback loop that we study. Other papers, like us, work with exogenous energy supply, for example, [Datta et al. \(2021\)](#). Differences in the calibration explain why a feedback loop does not emerge in their work. Our calibration in large measure relies on [Bachmann et al. \(2022\)](#) who are motivated by the current situation and estimate the effect that an exogenous cut to natural-gas supply from Russia has for the German economy, abstracting from nominal rigidities. [Pieroni \(2023\)](#) provides an assessment of a European scenario in a heterogeneous-household New Keynesian model. [Lorenzoni and Werning \(2023\)](#) study wage-price spirals in a New Keynesian model with a fixed input factor. What sets us apart from all these papers is that we study how limits to energy supply (or a fixed input factor) may translate into *self-fulfilling* energy-price-activity loops.

In our calibrated model, an increase of energy prices by 20 percent is related to a fall in GDP of 0.5 percent. This is broadly in line with empirical estimates in the literature; for example, the effect of inventory-demand shocks on global activity in [Baumeister and Hamilton \(2019\)](#), the SVAR-based findings in [Blanchard and Galí \(2009\)](#) and [Blanchard and Riggi \(2013\)](#), and the oil-supply news shocks identified by [Känzig \(2021\)](#). Needless to say, though, that these authors understand fluctuations as originating from exogenous fundamental shocks rather than the sunspots that drive prices in our environment.

We propose a novel mechanism that can generate an energy-price-activity feedback loop. This loop opens the economy to sunspot equilibria even if with flexible energy supply there would be none. This novel mechanism differentiates the current paper from other

work that also questions the Taylor principle. [Bilbiie \(2008\)](#) and [Galí, López-Salido and Vallés \(2004\)](#) derive the failure of the Taylor principle in a closed economy with limited asset-market participation. [Bilbiie \(2021\)](#) adds to this precautionary savings. Our paper shares with the aforementioned what technically lies at the heart of the indeterminacy: an inversion of the IS curve, that is, the relationship between aggregate output and the *ex-ante* real interest rate.¹ As a result, our pencil and paper solutions for the determinacy regions are almost nested by those in the aforementioned papers. Almost, because in our framework also the Phillips curve relationship is affected by the energy shortage (owing to the fact that higher prices for imported energy make households poorer and there is a wealth effect on labor supply). [Holden \(2022\)](#) shows that if monetary policy not only responds to inflation but also one-to-one to movements in the real rate of interest, the Taylor principle is restored in a wide range of environments. The same is true here. In the energy-price-economic-activity loop, high energy prices come with low real real incomes received by *savers*. This raises the real rate. Leaning against both inflation and the real rate, therefore, means a stronger monetary response against the loop.

Our results appear to run counter to the conventional wisdom about the best monetary response to energy-price shocks. In positive contributions, [Carlstrom, Fuerst and Ghironi \(2006\)](#) show that, in their setting, a central bank that reacts more than one-to-one to contemporaneous inflation of *any* arbitrary subset of goods in the economy (e.g., to core inflation only) ensures determinacy; compare also [Airaud and Zanna \(2012\)](#). In our setting, instead, it matters which price index the central bank targets. The reason is that different price indexes reflect energy-price changes differently, and that these price changes shape external demand. A monetary response to energy-price changes, as would help ensure determinacy in our setting, appears to run counter to the normative implications of a long stream of literature that finds that central banks should best focus on the inflation rate of those goods and services that have rigid prices rather than of those goods or services that have flexible prices. [Aoki \(2001\)](#) formalizes the notion that policy should react to

¹[Branch and McGough \(2009\)](#) and [Ilabaca and Milani \(2021\)](#) derive a break-down of the Taylor principle from adaptive expectations, [Ascari and Ropele \(2009\)](#) from trend inflation. [Llosa and Tuesta \(2009\)](#) focus on a cost channel of monetary policy and [Kurozumi \(2006\)](#) focuses on non-separability of consumption and real money balances.

inflation in goods with rigid prices for the closed economy, [Bodenstein, Erceg and Guerrieri \(2008\)](#) for the open economy with an energy-supply shock. Contributions for multi-sector models, such as [Eusepi, Hobijn and Tambalotti \(2011\)](#) and [Rubbo \(2022\)](#) come to similar conclusions. Whereas this literature focuses on welfare-maximizing monetary policy, we focus on belief formation. We find that unless the targeting policies mentioned above are implemented in a sufficiently strict manner, a further response to energy prices may help prevent non-fundamental fluctuations.

2 Model

We look at an infinite-horizon model. Time t is discrete and marked by $t = 0, 1, \dots$. There are two countries, Home and Foreign. The Home economy imports energy from a generic energy-exporting country (Foreign) in exchange for goods that are produced domestically. International trade need not be balanced immediately as Foreign can accumulate net foreign assets in the form of domestic-currency bonds. Energy is used in two ways: it is consumed by households directly and it serves as an input factor for the production of consumption goods. This setting is as in [Blanchard and Galí \(2009\)](#). There are two types of households: savers and spenders. Savers seek to smooth consumption over time. Their consumption demand is affected by their permanent income and changes in idiosyncratic risk. Spenders, instead, have a marginal propensity to consume of unity. This setting on the household side follows [Bilbiie \(2021\)](#); see Appendix A for details.

2.1 Households

There is a unit mass of infinitely-lived households, each of which has the same preferences. The notation below anticipates that, in equilibrium, the setting gives rise to two representative types of households: a representative hand-to-mouth household and a representative saver household, marked by subscripts H and S , respectively. Hand-to-mouth households cannot participate in financial markets. They consume all of their income. Savers, instead, optimize intertemporally. They can save in liquid, risk-free nom-

inal bonds, the rate of return on which the central bank controls. The bonds are traded between savers and with Foreign. We follow Bilbiie (2021) in modeling the two agent economy with precautionary savings and liquidity. Households transition between the two states. The constant probability to *stay*, respectively, hand-to-mouth or a saver is h and s . The mass of hand-to-mouth households, thus is given by $\lambda = (1 - s)/(2 - s - h)$. A household i , with $i \in \{H, S\}$, consumes a basket of goods, $C_{i,t}$. The household works and, potentially, saves in domestic-currency bonds. The household maximizes expected life-time utility

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{i,t}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{i,t}^{1+\varphi}}{1+\varphi} \right] \right\},$$

with parameters $\beta \in (0, 1)$, $\sigma > 0$, $\chi > 0$, and $\varphi \geq 0$. \mathbb{E}_t marks expectations conditional on period- t information. $N_{i,t}$ marks hours worked.

The household's consumption basket is comprised of the consumption of energy, $C_{i,E,t}$, that is imported from Foreign, and non-energy goods, $C_{i,G,t}$, that are produced in Home. Consumption preferences are described by the CES aggregator

$$C_{i,t} = \left[\gamma^{\frac{1}{\eta}} (C_{i,E,t} - \bar{e})^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{i,G,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}.$$

Above, $\bar{e} \geq 0$ marks the subsistence level for the consumption of energy. $\gamma \in (0, 1)$ measures the weight of energy in the consumption basket and $\eta > 0$ measures the consumer's elasticity of substitution between energy and goods. Marking the respective prices by $P_{E,t}$ and $P_{G,t}$, the household's allocation of consumption obeys

$$C_{i,E,t} - \bar{e} = \gamma \left(\frac{P_{E,t}}{P_t} \right)^{-\eta} C_{i,t}, \quad \text{and} \quad C_{i,G,t} = (1-\gamma) \left(\frac{P_{G,t}}{P_t} \right)^{-\eta} C_{i,t}.$$

P_t marks the price of a marginal unit of consumption of the basket (that is, beyond subsistence), with

$$P_t = \left[\gamma P_{E,t}^{1-\eta} + (1-\gamma) P_{G,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \quad (1)$$

For a hand-to-mouth household ($i = H$), the budget constraint is given by

$$P_{E,t}C_{H,E,t} + P_{G,t}C_{H,G,t} = W_t N_{H,t} + \frac{\tau^d}{\lambda} P_t (D_t - T_t^y) + \frac{1-s}{\lambda} R_{t-1} B_{t-1}. \quad (2)$$

The household spends its income on energy and non-energy consumption. P_t marks the price for a marginal consumption expenditure. The household derives income from supplying labor in the competitive labor market in Home, W_t being the nominal wage. In addition, the right-hand side anticipates that the government may engage in redistribution between savers and hand-to-mouth households. D_t are the real profits accruing to firms in Home. T_t^y marks the real per-capita tax burden that the government imposes on households. We assume that a share $\tau^d \in [0, 1]$ of both the firm's profits and the tax burden accrues to the mass of hand-to-mouth households. Finally, a share of today's hand-to-mouth households has been a saver last period, saving in liquid bonds. Since these households are excluded from the asset market, they consume their savings immediately when becoming constrained. The notation here anticipates equilibrium asset flows. So as to support the allocations here and keep the setup of two representative households, there is an implicit assumption on redistribution across households of the same type. The reader may think of there being a lump-sum tax on those households that switch from being savers to being hand-to-mouth, with the proceeds accruing to those households that stay hand-to-mouth. The size of the tax is such that it ensures that all households of the same type start the period with the same asset position.

For a saver household ($i = S$), the budget constraint is given by

$$\begin{aligned} P_{E,t}C_{S,E,t} + P_{G,t}C_{S,G,t} + \frac{B_t}{1-\lambda} \\ = W_t N_{S,t} + \frac{1-\tau^d}{1-\lambda} P_t (D_t - T_t^y) - \frac{P_t T_t^e}{1-\lambda} + \frac{s}{1-\lambda} R_{t-1} B_{t-1}. \end{aligned} \quad (3)$$

This reflects that savers, in equilibrium, can accumulate savings or liabilities against Foreign. B_t marks the nominal expenditure for the bonds that underpin these savings. On the income side, savers work and receive their share of profits net of the tax burden. Additionally, savers potentially need to pay taxes T_t^e to finance energy-price subsidies. The

final term marks the income that accrues to the saver household, in equilibrium, from the bond position.² The setup assumes that bonds are traded among saver households, while firms are not.

The labor-supply first-order decision for both types of household is given by

$$\frac{W_t}{P_t} = \chi C_{i,t}^\sigma N_{i,t}^\varphi. \quad (4)$$

Hand-to-mouth households do not participate in financial markets by assumption. Savers, in addition, have an intertemporal consumption decision. Their consumption Euler equation is given by

$$C_{S,t}^{-\sigma} = \mathbb{E}_t \left[\beta \left(s C_{S,t+1}^{-\sigma} + (1-s) C_{H,t+1}^{-\sigma} \right) \frac{R_t}{\Pi_{t+1}} \right]. \quad (5)$$

This reflects that a period- t saver remains a saver with probability s . Otherwise, the saver becomes hand-to-mouth in $t+1$. Since beginning-of-period redistribution across households of the same type is lump-sum, it does not affect returns at the margin.

2.2 Firms

The setup of production follows the usual New-Keynesian structure. There is a unit mass of producers of differentiated goods, indexed by $j \in [0, 1]$ and subject to price adjustment costs. Each differentiated good, $y_{G,t}(j)$ is produced using labor and energy. A representative competitive retailer buys the differentiated goods and assembles them into the consumption good that consumers purchase (at competitive price $P_{G,t}$).

²As for the hand-to-mouth households, this accounts for lump-sum redistribution at the beginning of the period from asset-owning saver households to those that have been hand-to-mouth before and do not bring assets into the period.

2.2.1 Retailer

The representative retailer transforms the differentiated inputs into the (non-energy) consumption good according to production function

$$Y_{G,t} = \left[\int_0^1 y_{G,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Here $\varepsilon > 0$ is the elasticity of substitution between the different differentiated inputs. The retailer takes prices $P_{G,t}(j)$ of inputs and $P_{G,t}$ of outputs as given. Profit maximization leads to the conventional Dixit-Stiglitz demand function

$$y_{G,t}(j) = \left(\frac{P_{G,t}(j)}{P_{G,t}} \right)^{-\varepsilon} Y_{G,t}, \quad (6)$$

with $P_{G,t}(j)$ being the price of intermediate good j and $P_{G,t} = \left[\int_0^1 P_{G,t}(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)}$ the producer-price index.

2.2.2 Producers of differentiated goods

The firms that produce differentiated goods are not traded. We assume that they discount future profits as would a perpetual family of savers. The stochastic discount factor for firms, $\mathcal{F}_{t,t+k}$, thus, is defined as

$$\mathcal{F}_{t,t+k} := \beta^k \left(\frac{C_{S,t+k}}{C_{S,t}} \right)^{-\sigma}.$$

The producer sets its price $P_{G,t}(j)$ so as to maximize

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k} \frac{1}{P_{t+k}} \left[P_{G,t+k}(j)(1 + \tau^y)y_{G,t+k}(j) - W_{t+k}N_{t+k}(j) - P_{E,t+k}E_{t+k}(j) - \frac{\psi}{2}P_{G,t+k}Y_{G,t+k} \left(\frac{P_{G,t+k}(j)}{P_{G,t+k-1}(j)} - 1 \right)^2 \right] \right\}.$$

Here, $\tau^y \geq 0$ is a constant subsidy to production and $\psi > 0$ indexes the price-adjustment costs. N_t and E_t mark labor and energy input into production, respectively. Maximization

is subject to demand function (6) and the production function

$$y_{G,t}(j) = \left[\alpha E_t(j)^{\frac{\theta-1}{\theta}} + (1-\alpha)N_t(j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}.$$

$\alpha \in (0, 1)$ marks the input share of energy in production and $\theta > 0$ marks the elasticity of substitution of energy and hours worked in production.

Symmetry means that all differentiated goods producers set the same price, produce the same amount of output, and face the same marginal costs. The first-order conditions deliver a rather standard New Keynesian Phillips curve:

$$\begin{aligned} \psi \Pi_{G,t}(\Pi_{G,t} - 1) &= (1 + \tau^y)(1 - \varepsilon) + \varepsilon \Lambda_t \left(\frac{P_{G,t}}{P_t} \right)^{-1} \\ &+ \psi \mathbb{E}_t \left[\mathcal{F}_{t,t+1} \Pi_{G,t+1}(\Pi_{G,t+1} - 1) \frac{Y_{G,t+1}}{Y_{G,t}} \frac{P_{G,t+1}/P_{t+1}}{P_{G,t}/P_t} \right], \end{aligned} \quad (7)$$

Above, $\Pi_{G,t} = P_{G,t}/P_{G,t-1}$ is producer-price inflation (here commensurate with core inflation), and Λ_t marks real marginal costs, real in terms of the consumption aggregate. The optimal factor input shares obey

$$\frac{W_t}{P_{E,t}} = \frac{1-\alpha}{\alpha} \left(\frac{E_t}{N_t} \right)^{1/\theta}. \quad (8)$$

Real marginal costs are given by³

$$\Lambda_t = \left[\alpha^\theta \left(\frac{P_{E,t}}{P_t} \right)^{1-\theta} + (1-\alpha)^\theta \left(\frac{W_t}{P_t} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (9)$$

Last, the firm sector's real profits, in equilibrium are given by

$$D_t = (1 + \tau^y) \frac{P_{G,t}}{P_t} Y_{G,t} - \frac{W_t}{P_t} N_t - \frac{P_{E,t}}{P_t} E_t - \frac{\psi}{2} \frac{P_{G,t}}{P_t} Y_{G,t} (\Pi_{G,t} - 1)^2.$$

³Alternatively, (8) and (9) can be replaced by $\frac{W_t}{P_t} = \Lambda_t(1-\alpha) \left(\frac{Y_{G,t}}{N_t} \right)^{\frac{1}{\theta}}$, and $\frac{P_{E,t}}{P_t} = \Lambda_t \alpha \left(\frac{Y_{G,t}}{E_t} \right)^{\frac{1}{\theta}}$.

2.3 Fiscal and monetary policy

Fiscal policy runs a balanced budget and does not issue debt. Taxes T_t^y on households finance the production subsidy to firms so that

$$P_t T_t^y = \tau^y P_{G,t} Y_{G,t}.$$

A share τ^d of these taxes is collected from hand-to-mouth households, the remaining share from savers. Similarly, after redistributing firms' profits, a share τ^d of these accrues to hand-to-mouth households. The remaining share of profits remains with savers.

It is important to note that, in the households' and firms' problems above $P_{E,t}$ is the price for energy that households and firms pay. This price for energy may differ from the price that Foreign receives. Namely, there are energy subsidies. Fiscal policy subsidizes the energy price that households and firms pay, $P_{E,t}$, whenever it differs from the steady state energy price, P_E . The subsidized energy price is given by

$$\frac{P_{E,t}}{P_t} = \frac{\tilde{P}_{E,t}}{P_t} \times \left(\frac{\tilde{P}_{E,t}/P_t}{P_E/P} \right)^{-\tau^e},$$

where $\tilde{P}_{E,t}$ denotes the energy price paid to the energy suppliers. Here $\tau^e \geq 0$ marks the percentage subsidy on the energy price. Energy subsidies are financed through non-distortionary taxation of unconstrained households. The level of taxes, paid by savers, is then given by $P_t T_t^e = (\tilde{P}_{E,t} - P_{E,t}) \xi_E$, where $\xi_E = (C_{E,t} + E_t)$ denotes the total amount of energy supplied and consumed.

The central bank controls the gross nominal interest rate R_t . In the baseline, the Taylor rule responds to core inflation

$$R_t = R \cdot (\Pi_{G,t})^{\phi_\Pi}, \text{ where } \phi_\Pi \geq 0. \quad (10)$$

Below, we will also consider a response to other concepts of "inflation." Recall that the so-called "Taylor-principle" asserts that $\phi_\Pi > 1$ would ensure a unique bounded equilibrium irrespective of what inflation index the central bank responds to.

2.4 Energy supply and international trade

In keeping with the change in the energy-supply paradigm that serves as a motivation of the paper, the *quantity* of energy, ξ_E , that is supplied by Foreign is fixed. This quantity is sold to Home at the currently-prevailing, unsubsidized price of energy, $\tilde{P}_{E,t}$. Allowing for the accumulation of net foreign assets, Foreign's budget (expressed in Home's currency) is thus

$$P_{G,t}X_{G,t} - [B_t - R_{t-1}B_{t-1}] = \tilde{P}_{E,t}\xi_E. \quad (11)$$

On the right-hand side, there are the revenues from the energy that Home imports. These revenues can be used for importing goods (Home's exports to Foreign, $X_{G,t}$) or for building up the net foreign asset position. We do not model the Foreign economy in any more detail. Letting $b_t := B_t/P_t$ mark real debt, we parameterize Foreign's export demand as

$$\frac{X_{G,t}}{X_G} = \left(\frac{Y_t^*}{Y^*}\right)^{\mu_1} \times \exp\left(-\mu_2 \frac{b_{t-1}}{Y^*}\right),$$

Here, $Y_t^* = \tilde{P}_{E,t}/P_{G,t} \times \xi_E$ marks the real energy revenue of the Foreign economy. Parameters $\mu_1 \geq 0$ and $\mu_2 \geq 0$ measure, respectively, Foreign's contemporaneous marginal propensity to consume out of energy revenues and its marginal propensity to consume out of savings. For $\mu_1 = 1$, and $\mu_2 = 0$, we recover the case of balanced trade.

2.5 Market clearing

In equilibrium, all markets clear. The labor market clears if $N_t = \lambda N_{H,t} + (1 - \lambda)N_{S,t}$. Next, energy demand by households and firms needs to be in line with the energy supply, that is, $\xi_E = \lambda C_{H,E,t} + (1 - \lambda)C_{S,E,t} + E_t$. Finally, the non-energy goods market clears is

$$Y_{G,t} = \lambda C_{H,G,t} + (1 - \lambda)C_{S,G,t} + X_{G,t} + \frac{\psi}{2}(\Pi_{G,t} - 1)^2 Y_{G,t}.$$

The first term two terms are aggregate domestic demand for the good for consumption, the third term are exports, and the final term are price adjustment costs.

2.6 Additional definitions: aggregates and inflation

For future use, here we define a few further concepts. Define aggregate consumption of (non-energy) goods and energy as

$$C_{G,t} = \lambda C_{H,G,t} + (1 - \lambda) C_{S,G,t} \quad \text{and} \quad C_{E,t} = \lambda C_{H,E,t} + (1 - \lambda) C_{S,E,t}.$$

Next, the left-hand side of the below is the nominal gross domestic product, with GDP_t being a measure of real gross domestic product

$$P_t \text{GDP}_t = P_{G,t} C_{G,t} + \tilde{P}_{E,t} C_{E,t} + P_{G,t} X_{G,t} - \tilde{P}_{E,t} \xi_E. \quad (12)$$

So far, we have defined the core (consumer-price) inflation rate $\Pi_{G,t}$. A measure of headline consumer-price inflation is given by $\Pi_t := P_t/P_{t-1}$.⁴

Later, we will also look at input-price inflation. For this, we define an index of the change in nominal marginal costs as

$$\Pi_{\text{nm},t} = \frac{[\alpha^\theta P_{E,t}^{1-\theta} + (1 - \alpha)^\theta W_t^{1-\theta}]^{\frac{1}{1-\theta}}}{[\alpha^\theta P_{E,t-1}^{1-\theta} + (1 - \alpha)^\theta W_{t-1}^{1-\theta}]^{\frac{1}{1-\theta}}}, \quad (13)$$

compare (9).

3 Pencil-and-paper intuition

This paper's main result is that limits to energy supply can give rise to a self-fulfilling energy-price-activity feedback loop. And that this asks for a stronger monetary response than absent the loop. This section provides approximate closed-form pencil-and-paper solutions for a special case of the model. The derivations provide the source of the indeterminacy and the factors that drive the indeterminacy. Indeterminacy here arises from a source of demand for goods that is not as interest-sensitive as domestic absorption. For the latter source to matter, the share of energy in production needs to be large

⁴This captures only the change in costs of marginal consumption (beyond the subsistence level). Results are very similar for a measure of CPI inflation that includes the costs for subsistence consumption.

enough, energy demand has to be sufficiently inelastic, the prices of non-energy goods have to be sufficiently rigid, and households need to be sufficiently unwilling to substitute consumption over time.

3.1 Parametric assumptions

For this section, we suppose that energy is used in production only. That is, we look at the case $\gamma \rightarrow 0$ and $\bar{e} \rightarrow 0$. This also means that core and headline consumer price inflation are identical, that is, $\Pi_{G,t} = \Pi_t$. In addition, we focus on the representative-household version of the model. More precisely, we consider $s = 1$ and $\lambda = 0$ such that there are savers only (and, for example, aggregate consumption is given by savers' consumption). We assume balanced trade, that is, $\mu_1 = 1$. We make a few more assumptions that are not essential but that simplify the exposition of results. Namely, production subsidies are used to render the steady state efficient and energy supply is fixed at $\xi_E = 1$. Energy prices are not subsidized, $\tau^e = 0$. Next, we assume that the scaling parameter of the disutility of work, χ , is such that in the steady state the labor supply equals unity. Last, we look at the limit $\beta \rightarrow 1$.

3.2 Steady state and first-order dynamics

Under these assumptions, the steady state and the first-order dynamics are as follows.

3.2.1 Steady state

Letting variables without time index denote their steady state values, then, steady state inflation is given by $\Pi = 1$, the steady state gross nominal interest is given by $R = 1/\beta$, and steady state hours worked by $N = 1$. Steady state production is given by $Y_G = 1$, steady state marginal costs by $\Lambda = 1$, and steady state consumption in Home is given by $C = (1 - \alpha)$. Let $q_t := P_{E,t}/P_t$ be the real price of energy and let $w_t := W_t/P_t$ be the real wage. In the steady state, $q = \alpha$, and $w = (1 - \alpha)$. α , thus, marks the equilibrium share of energy in production.

3.2.2 Linearized equilibrium dynamics

Let a hat mark percentage deviations of a variable from the steady state outlined above. The following system of seven equations in seven unknowns describes the evolution of the economy up to a first-order approximation around the steady state. The consumption Euler equation (after substituting the central bank's Taylor rule) gives $-\sigma\widehat{C}_t = -\sigma\mathbb{E}_t\widehat{C}_{t+1} + \left[\phi_\Pi\widehat{\Pi}_{G,t} - \mathbb{E}_t\widehat{\Pi}_{G,t+1}\right]$. The household's labor supply first-order condition gives $\widehat{w}_t = \varphi\widehat{N}_t + \sigma\widehat{C}_t$. The Phillips curve gives $\psi\widehat{\Pi}_{G,t} = \psi\beta\mathbb{E}_t\widehat{\Pi}_{G,t+1} + \epsilon\widehat{\Lambda}_t$. The firms' first-order conditions for factor inputs give $\widehat{w}_t = \widehat{\Lambda}_t + \frac{1}{\theta}[\widehat{Y}_{G,t} - \widehat{N}_t]$, and $\widehat{q}_t = \widehat{\Lambda}_t + \frac{1}{\theta}\widehat{Y}_{G,t}$, where we have already used that energy is in fixed supply.

Goods-market and energy-market clearing imply $\widehat{Y}_{G,t} = (1 - \alpha)\widehat{C}_t + \alpha\widehat{q}_t$. Last, the production function implies $\widehat{Y}_{G,t} = (1 - \alpha)\widehat{N}_t$.

Consolidating the IS equation and the goods-market clearing condition, we have

$$\widehat{Y}_{G,t} - \alpha\widehat{q}_t = \mathbb{E}_t\widehat{Y}_{G,t+1} - \alpha\mathbb{E}_t\widehat{q}_{t+1} - \frac{(1 - \alpha)}{\sigma} \left[\phi_\Pi\widehat{\Pi}_{G,t} - \mathbb{E}_t\widehat{\Pi}_{G,t+1} \right].$$

Combining labor demand and supply as well as the goods-market clearing condition, one can further show that marginal costs are given by

$$\widehat{\Lambda}_t = \left[\frac{1}{1 - \alpha} [\varphi + \sigma + 1/\theta] - \frac{1}{\theta} \right] \widehat{Y}_{G,t} - \frac{\sigma\alpha}{1 - \alpha} \widehat{q}_t.$$

Since at the same time, the energy-demand equation of firms gives

$$\widehat{\Lambda}_t = -\frac{1}{\theta}\widehat{Y}_{G,t} + \widehat{q}_t,$$

we have that the equilibrium price of energy is given by

$$\widehat{q}_t = \frac{\varphi + \sigma + \frac{1}{\theta}}{1 - \alpha + \sigma\alpha} \widehat{Y}_{G,t}.$$

That is, the energy price is the more elastic to output, the less substitutable energy is as an input (the smaller θ) and the less elastic labor supply is (the larger φ). With this, real

marginal costs are given by

$$\widehat{\Lambda}_t = \frac{\sigma + \varphi + \frac{\alpha}{\theta}(1 - \sigma)}{1 - \alpha + \sigma\alpha} \widehat{Y}_{G,t}. \quad (14)$$

3.3 Two-equation system of equations and main insights

Combining all this, we can write the model as a system of two equations in output $\widehat{Y}_{G,t}$ and inflation $\widehat{\Pi}_{G,t}$. The IS-equation is given by

$$\widehat{Y}_{G,t} = \mathbb{E}_t \widehat{Y}_{G,t+1} - \frac{1}{\widetilde{\sigma}} \left[\phi_{\Pi} \widehat{\Pi}_{G,t} - \mathbb{E}_t \widehat{\Pi}_{G,t+1} \right], \quad (15)$$

with $\widetilde{\sigma} := \frac{\sigma}{1-\alpha} \frac{1-\alpha-\alpha\varphi-\alpha/\theta}{1-\alpha+\sigma\alpha}$. That is, relative to the textbook New-Keynesian model, what changes is the interest-sensitivity of aggregate demand, $\widetilde{\sigma}$. If $\alpha[\varphi + 1/\theta]$ becomes sufficiently large, the sensitivity changes sign.

The Phillips curve in turn is given by

$$\widehat{\Pi}_{G,t} = \beta \mathbb{E}_t \widehat{\Pi}_{G,t+1} + \widetilde{\kappa} \widehat{Y}_{G,t}, \quad (16)$$

with $\widetilde{\kappa} := \frac{\epsilon}{\psi} \frac{\sigma + \varphi + \frac{\alpha}{\theta}(1 - \sigma)}{1 - \alpha + \sigma\alpha}$.

The two equations (15) and (16) summarize the evolution of output and inflation. This means that the analysis of (in)determinacy can conceptually follow standard lines. This gives us the following proposition.

Proposition 1 (Determinacy). *Consider the model of Section 2. Apply the parametric assumptions listed in Section 3.2.1. In addition, let $\beta \rightarrow 1$. Then the following two cases summarize the conditions for determinacy.*

- 1) “Conventional.” *If $\widetilde{\sigma}$ and $\widetilde{\kappa}$ have the same sign, there is local determinacy if and only if $\phi_{\Pi} > 1$.*
- 2) “Unconventional.” *If $\widetilde{\sigma} < 0$ and $\widetilde{\kappa} > 0$, there is local determinacy if and only if*

$$\phi_{\Pi} > \max \left(1, -4 \frac{\widetilde{\sigma}}{\widetilde{\kappa}} - 1 \right). \quad (17)$$

Proof. The proof is provided in Appendix B and is entirely standard. It follows the well-known lines of proof of (in)determinacy for the textbook three-equation New Keynesian model, such as the one in Woodford (2003, p. 670 ff). $\tilde{\kappa}$ can be negative only when $\tilde{\sigma}$ is negative as well. This explains why there are two cases only and not a third case (no $\tilde{\sigma} > 0$ and $\tilde{\kappa} < 0$). \square

The proposition shows that obeying the standard Taylor principle ($\phi_{\Pi} > 1$) may not be sufficient to ensure determinacy. In particular, it will not be sufficient if $\tilde{\sigma} < 0$ and $\tilde{\kappa} > 0$, and if $|\tilde{\sigma}/\tilde{\kappa}|$ is large. The following corollary clarifies the conditions under which this can be the case.

Corollary 1. *Consider the same conditions as in Proposition 1. Suppose further that $\alpha = \theta$, that is, that the weight of energy in production equals the elasticity of substitution between energy and labor. Then the lower bound on ϕ_{Π} that ensures determinacy will be higher than suggested by the Taylor principle if and only if*

$$1 > \frac{1}{2} \frac{\epsilon/\psi}{\sigma} \frac{1-\alpha}{\alpha},$$

that is, if the Phillips curve absent energy-price feedback is sufficiently flat (low ϵ/ψ), if households are sufficiently unwilling to substitute intertemporally (high σ), and if energy inputs are a sufficiently important cost factor in production (high α).

Proof. This follows directly from Proposition 1. If $\alpha = \theta$, we have that $\tilde{\sigma} < 0$ and $\tilde{\kappa} > 0$. This means that what is applicable is case 2) of the proposition. The result here emerges after substituting for $\tilde{\sigma}$ and $\tilde{\kappa}$ in inequality (17) (and using $\alpha = \theta$). \square

In sum, in an environment with limits to energy supply, a central bank that seeks to uniquely anchor expectations may need to react more strongly to inflation than otherwise. Appendix C provides analytical results also for the heterogeneous-household version. The extent to which these concerns might matter in practice is a quantitative question, however. We turn to such a quantitative assessment next.

4 Implications for monetary policy

This section first calibrates the full model to a stylized euro area. In the baseline, limits to energy supply are important: the calibrated baseline shows indeterminacy although with potentially abundant supply the equilibrium would be determinate. That is, the energy-price-activity feedback loop emerges. With the calibrated model baseline at hand, we explore how different policy choices by the central bank and the fiscal authority shape the (in)determinacy.

4.1 Calibration

The model is calibrated to a stylized euro area. One period is taken to be a quarter. We jointly calibrate several parameters to meet energy-related ratios in the national accounts. The calibration focuses only on the portion of energy that is imported. All the remaining parameters are taken from the literature.

4.1.1 Calibrated parameters

Table 1 gives the calibrated parameters for the model's baseline. We set time preferences to $\beta = 0.995$, in line with a two-percent annualized real rate of interest in the steady state. The parameter of constant relative risk aversion is set to $\sigma = 3$. This implies a realistic intertemporal elasticity of substitution of consumption of $1/3$ (Best et al., 2020; Havránek, 2015), also in line with the recent heterogeneous agent literature (Bayer, Born and Luetticke, 2022). We set the scaling parameter in the disutility of work to $\chi = 0.8998$ such that steady state labor supply is normalized to unity. We calibrate $\varphi = 2$ so as to have a Frisch elasticity of labor supply of 0.5, which is within the range of values regularly used in the literature (Chetty et al., 2011).

In line with the euro-area estimates in Slacalek, Tristani and Violante (2020), we consider a share of hand-to-mouth households of 22 percent ($\lambda = 0.22$). We set the probability of becoming constrained as in Bilbiie, Primiceri and Tambalotti (2022), to $1 - s = 1 - 0.98$. The probability of staying constrained, h , follows from λ and s . Together with our assumptions on taxation and allocation of profits, to be discussed below, this parameteri-

Table 1 Parameters of the baseline calibration

Parameter	Value	Description
<u>Households</u>		
β	0.995	Discount factor
σ	3	Inverse of intertemporal elasticity of substitution
χ	0.8998	Disutility of labor supply
φ	2	Inverse of Frisch elasticity of labor supply
λ	0.22	Share of hand-to-mouth households
s	0.98	Probability of staying unconstrained
\bar{e}	0.05	Subsistence level of energy consumption
γ	0.099361	Share of energy expenditures in consumption expenditures
η	0.1	Elasticity of substitution energy/goods in consumption
<u>Firms</u>		
ε	11	Elasticity of substitution different varieties of differentiated goods
ψ	500	Price adjustment costs
α	0.05	Production cost share of energy
θ	0.04	Elasticity of substitution between energy and labor in production
<u>Energy supply</u>		
ξ_E	1.19	Steady state energy supply
μ_1	0.5	Foreign's MPC out of energy revenues
μ_2	0.02	Foreign's MPC out of savings
<u>Government</u>		
τ^y	0.1	Production subsidy
τ^d	0	Profit redistribution
τ^e	0.25	Energy price subsidy
ϕ_Π	1.5	Response to inflation

Notes: Parameters of the baseline calibration. See the text for details.

zation would imply reasonable values for the amplification of the effect of monetary policy shocks relative to the full-insurance benchmark and for the size of the indirect effects of that policy in a model without energy; see [Bilbiie \(2020\)](#) for details.

Turning to the energy-related part of preferences, we set the subsistence level of energy consumption to $\bar{e} = 0.05$ such that subsistence energy expenditures account for only 0.3 percent of GDP and roughly 25 percent of households' energy expenditures, following [Fried, Novan and Peterman \(2022\)](#). We calibrate $\gamma = 0.099361$ with a view towards capturing the share of expenditures for raw energy imports in households' consumption expenditures; this share is roughly one percent, see [Bachmann et al. \(2022\)](#) for Germany or compare [Känzig \(2022\)](#) for the UK. The elasticity of substitution between energy and

the other goods in consumption is $\eta = 0.1$, a value that we take from the literature as well (Bachmann et al., 2022).

Turning to the production sector next, we set the own-price elasticity of demand to $\varepsilon = 11$, a conventional value that implies a 10 percent price markup. Price adjustment costs are calibrated to match a slope of the paper-and-pencil Phillips curve of 0.05, in line with Hazell et al. (2022). This gives $\psi = 500$.

Parameter α , that governs the energy intensity of production, is set to $\alpha = 0.05$. Here we follow Bachmann et al. (2022) and Fried, Novan and Peterman (2022) with an eye toward matching the share of energy expenses in production costs. We set the elasticity of substitution between energy and labor in production to $\theta = 0.04$, a value that is in line with the range of estimates reported in Bachmann et al. (2022). The elasticity of substitution within production is a crucial parameter, of course. The parameter here is to be understood as a short-run elasticity.

We target a steady state in which we normalize the total supply of energy, ξ_E , so that firms' energy usage (a fraction of the supply) takes on a value of unity. This is just a normalization. It makes sure that output is unity in steady state and that we can directly interpret α as the energy cost share in production.

The parameters that relate to the government's policies are as follows. We set $\tau^y = 0.1$ so that—in the steady state—the production subsidy undoes the distortion of production associated with firms' market power. With a steady state of zero net foreign assets, that we impose at the same time, this means that in steady state all households have the same level of income. Further, we assume that savers receive all the profits in the economy ($\tau^d = 0$) and bear the burden of taxation. This follows Bilbiie (2020). We set the energy subsidy to the middle of the range of what the literature has considered ($\tau^e = 0.25$); see, for example, Auclert et al. (2023). Next, and unless specified otherwise, we look at a monetary response to inflation of $\phi_{\Pi} = 1.5$, a conventional value.

Last, what remains to calibrate are the two parameters μ_1 and μ_2 . These determine Foreign's marginal propensity to consume Home's goods out of energy revenue and out of net foreign assets. We follow the literature by calibrating the debt elasticity to a value

that—for the parameters considered here—stabilizes net foreign assets at a long-run value of zero, $\mu_2 = 0.02$. We, furthermore, set $\mu_1 = 0.5$, which means that for an additional euro in energy revenue, Foreign orders an additional half a euro’s worth of goods in Home. We choose this parameter having in mind as trading partner an emerging-market economy that does not run a sovereign wealth fund.

4.1.2 Implied steady state

Table 2 reports on the steady state associated with this parametrization. Importantly, we consider a steady state where both household types are symmetric. Therefore, Table 2 only reports aggregates. The steady state would be identical if we would not consider household heterogeneity.

Table 2 Steady state under baseline parametrization

Variable	Value	Description	Variable	Value	Description
<u>Households</u>			<u>Prices</u>		
C	1.0556	Consumption	$\Pi_G = \Pi$	1	Inflation
C_E	0.19	Energy cons.	P_E/P	0.055705	Real energy price
C_G	0.9405	Goods cons.	P_G/P	1.1141	Real goods price
N	1	Labor supply	W/P	1.0584	Real wage
<u>Production</u>			R	1.005	Gross nom. rate
Y_G	1	Output			
E	1	Energy in prod.			
D	0.11141	Profits			
Λ	1.1141	Real marginal costs			
<u>Implied ratios</u>					
$\frac{P_E C_E}{P_E C_E + P_G C_G}$	0.01	Expenditure share of raw energy in consumption expenditures			
$\frac{P_E E}{P_G Y_G}$	0.05	Energy input in production bill over value of output			
$\frac{P_E \xi_E}{P_G Y_G}$	0.0595	Economy-wide expenditure on energy over value of output			

Notes: steady state that corresponds to the baseline parameters.

In the steady state, one percent of consumption goes toward energy directly, as targeted. Similarly, the cost-share of energy in output is five percent, again as targeted. These two targets taken together mean that the cost of energy imports runs to the equivalent of roughly six percent of the value of economy-wide production.

4.2 Results

With the calibrated model baseline at hand, this section analyzes how central-bank policy affects (in)determinacy in an environment with limits to energy supply. Throughout, we focus on the first-order dynamics of the model, after linearizing around a zero-inflation steady state. We first document that the calibrated baseline allows for multiple equilibria. This leads us to zoom in on the energy-price-activity feedback loop. Thereafter, we discuss which modifications to the monetary-policy reaction function may ensure determinacy. We look at a stronger response to core inflation, a focus on *headline* instead of core consumer prices, a response to the level of output, or an explicit focus on stabilizing marginal costs. Not all of the parameters chosen here are uncontroversial. The end of this section provides extensive sensitivity analysis. This includes a discussion of the quantitative role of household heterogeneity. Household heterogeneity heavily shapes the dynamics in response to the sunspot shocks, whereas in the current setting it hardly affects the transmission of fundamental energy-supply shocks.

4.2.1 Indeterminacy with a response to core inflation

The standard Taylor principle suggests that a more than one-for-one response of interest rates to inflation ensures determinacy. With household heterogeneity, the cutoff can be different even absent the energy-price feedback loop (Bilbiie, 2021). Indeed, this is the case here, too. Namely, for potentially abundant energy supply the baseline would have the determinacy cutoff at $\phi_{\Pi} = 1.39$. This is the value to which all the following results should be compared.

A common prescription for the optimal response of monetary policy to relative price changes is that the central bank should focus on the inflation rates of those goods or services that have sticky prices; see, for example, Aoki (2001). In the current context, this would mean to focus on the inflation rate associated with non-energy goods, “core inflation.” This is why a response to core inflation is our baseline and why we first discuss the scope for indeterminacy when the central bank responds to core inflation $\Pi_{G,t}$.

It turns out that for core inflation the energy-price feedback loop shifts the determinacy

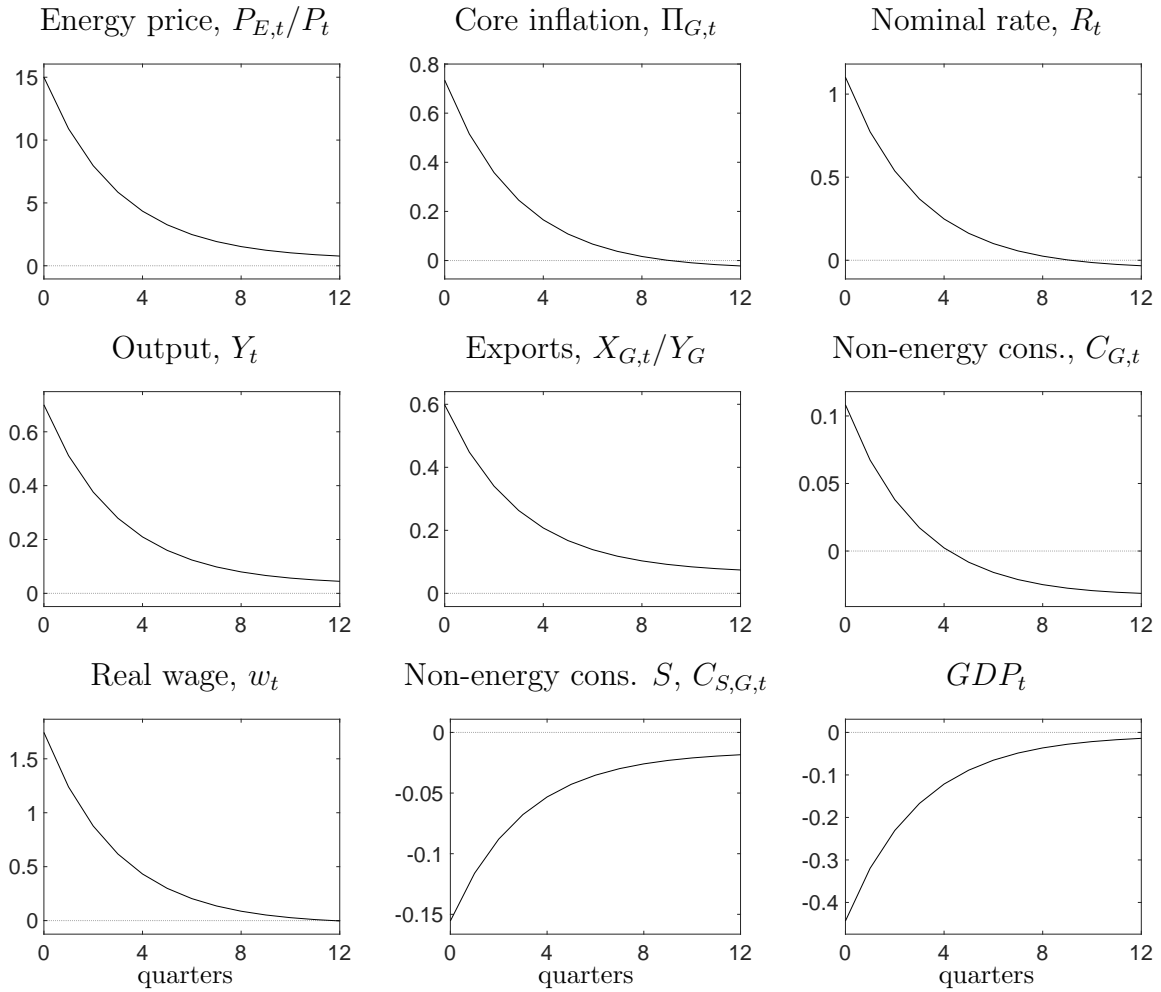
cutoff substantially. Whereas one might think that a response to core inflation in the usual range should suffice to anchor expectations, this is not the case in the scarce-energy environment that we map out here. Instead, there would be determinacy only for a much stronger response to inflation; namely, whenever $\phi_{\Pi} > 23.84$. In order to anchor expectations about inflation and economy activity, the central bank would, thus, need to lean notably more aggressively against core inflation.⁵ The sensitivity analysis presented later shows that this rise in the cutoff owes almost exclusively to the *interaction* of scarce energy supply and heterogeneity.

In the baseline model, at $\phi_{\Pi} = 1.5$, exactly one explosive root is missing to satisfy the Blanchard-Kahn conditions: there is exactly one degree of indeterminacy and room for a possible sunspot shock. So as to see the mechanism at work more clearly, Figure 1 plots impulse responses to this “energy-price sunspot” shock. The impulse responses are computed following the methodology of Bianchi and Nicolò (2021). In computing the sunspot shock, we assume that there is no contemporaneous correlation of innovations to the sunspot and innovations to any potential fundamental shock. Theory then uniquely pins down the sunspot shock’s persistence. In a linearized model like ours, it does not pin down the size of the shock. We anchor the shock’s size such that the shock raises import prices for energy by 20 percent on impact.

Under the sunspot beliefs of 20 percent higher energy prices and with a 25 percent energy subsidy, energy prices for domestic consumers rise by 15 percent (top row, left panel). On the back of this, core inflation rises by about a percentage point (top row, center panel) reflecting that firms face higher production costs. In line with the Taylor rule, the central bank raises the nominal interest rate more than one for one with inflation (top row, right panel). A higher real interest rate means that savers’ consumption falls (bottom row, center panel). Nevertheless, under this sunspot belief of higher energy prices, the volume of production in Home rises by about 0.7 percent (second row, left panel). This is so for

⁵There also is another segment that implies determinacy, namely, $\phi_{\Pi} \in [0, 0.98]$. The pencil-and-paper results did not have this segment by virtue of the assumption in Section 3 that there is no heterogeneity and that $\beta \rightarrow 1$, that is, the assumption that the Phillips curve is vertical in the long run. In the following, we do not focus on this determinacy region since it arises through threatening explosive paths (passive policy) rather than through active policy.

Figure 1 Sunspot shock amid targeting core inflation



Notes: Impulse response to a sunspot shock that raises energy prices by 20 percent on impact. The central bank responds to core inflation, with response parameter $\phi_{\Pi} = 1.5$. Scaling: all response are scaled to give percentage deviations from steady state. The exception is the response of exports (percent of steady state production). Also, interest rates and inflation rates are in annualized percentage points.

two reasons. First, under the sunspot belief of higher energy prices, Foreign’s revenue rises. In our calibration, Foreign immediately uses half of the rise in revenue for spending on exports (second row, center panel). A further effect, directly linked to the heterogeneity of households is that the domestic demand for the consumption goods as a whole does not fall until about a year after the shock (second row, right panel). This reflects that while savers retrench their consumption demand as interest rates rise and their income falls, the hand-to-mouth households’ budget is supported by the labor-market effects in this scenario. Namely, labor demand rises. This is the case because, on the one hand, a higher energy price means that firms substitute toward labor. On the other, also the higher level of production comes with higher labor demand. All this raises the wage. The

real wage rises by 1.5 percent (bottom row, left panel). Next to this, the energy subsidy relaxes the burden of high energy prices on hand-to-mouth households' consumption. It is important to note, however, that in all these dynamics, GDP, as opposed to the level of production, falls (bottom row, right panel). The reason is simple: the trade balance deteriorates on the back of more expensive energy imports.

Heterogeneity plays a crucial role for the existence and the size of these effects. There are two channels through which household heterogeneity affects the scope for the energy-price-activity feedback loop. First, a two-agent "TANK" channel, that arises from the hand-to-mouth dimension of the model. Namely, when savers and firms coordinate on non-fundamental beliefs of high energy prices, not only does output rise but also will wages rise at the expense of profits. Therefore, hand-to-mouth households' income rises. Their consumption, thus, increases. This strengthens the increase in aggregate demand, which supports the self-fulfilling beliefs. Second, a risk-related "THANK" channel driven by precautionary motives. A household that is unconstrained today (a saver) faces the risk of becoming constrained (hand-to-mouth) tomorrow. The sunspot shock persistently raises consumption by hand-to-mouth households but reduces consumption by savers. In other words, the sunspot shock reduces consumption risk for savers. Therefore, even though the incomes of savers fall, they do not reduce consumption as much as the savers' fall in permanent income would suggest.

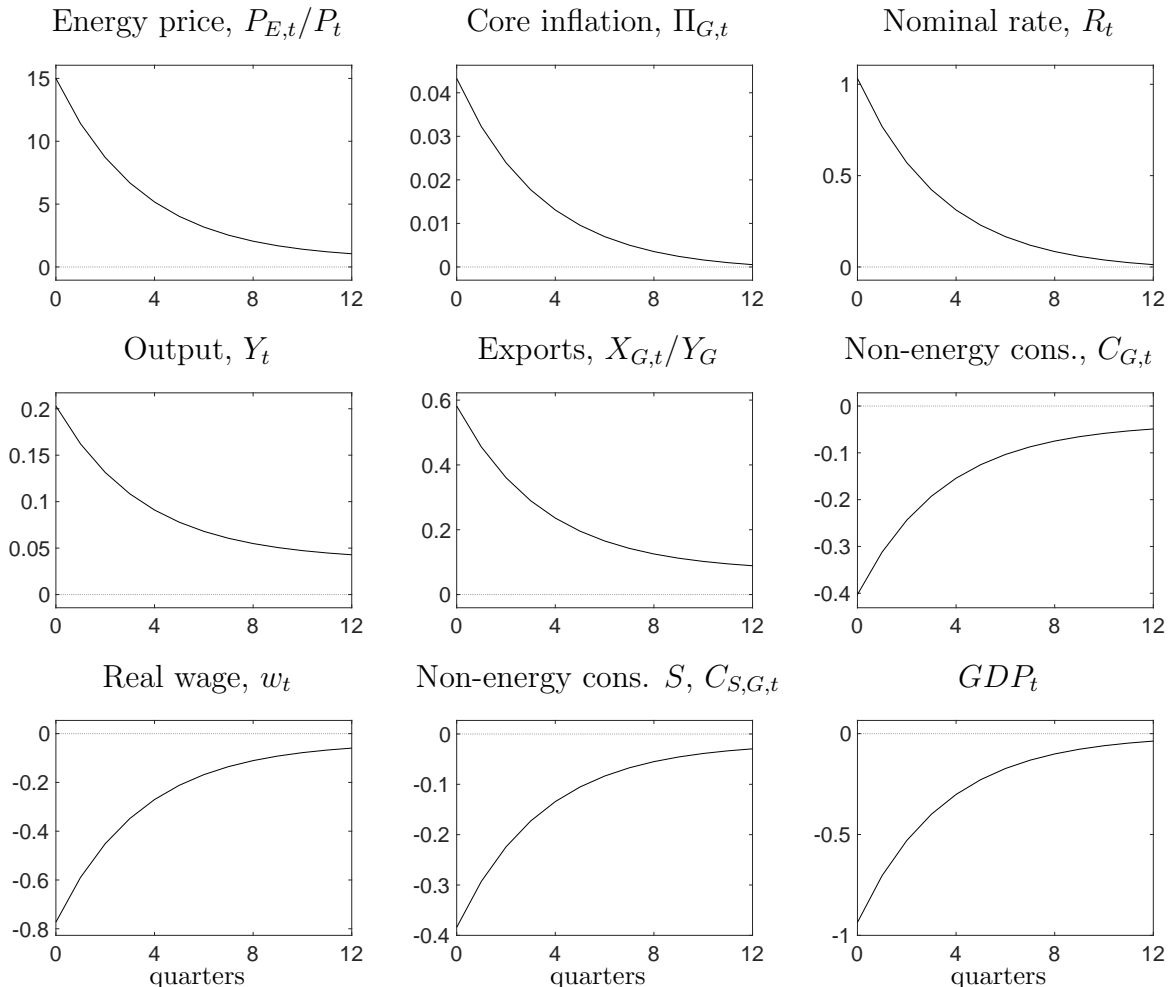
4.2.2 Fundamental energy supply-shock and heterogeneity

So as to further highlight the role of heterogeneity, Figure 2 zooms in on a scenario where monetary policy responds strongly enough to inflation to rule out sunspot beliefs. Namely, monetary policy reacts with $\phi_{\Pi} = 23.84$, that is, notably more strongly than in the baseline. The figure plots the economy's response to a persistent negative fundamental energy-supply shock (a different shock than above). To make the exercises roughly comparable, we assume that energy supply no longer is constant but is governed by

$$\ln(\xi_{E,t}/\xi_E) = \rho \ln(\xi_{E,t}/\xi_E) + \epsilon_t,$$

where ϵ_t is white noise and for the figure $\rho = 0.75$, which leads to a persistence of the energy-price response that is comparable to the one in Figure 1. The figure shows the responses to a fall in energy supply that leads to a 20 percent increase in the price of energy imports. We choose this size of the shock to ensure that the price response of energy in the two figures is comparable in size (top rows, left panels).

Figure 2 Energy-supply shock under hawkish policy



Notes: Same as Figure 1 but now the source of shock is a persistent cut in energy supply. Responses are calibrated to match a 20 percent increase in the relative energy price. The central bank responds to core inflation, with response parameter $\phi_\Pi = 23.84$. The scaling of the responses is as in Figure 1.

As in Figure 1, also this cut in energy supply raises the price of energy. Even though the quantity of energy supplied falls, energy expenditures as a whole rise, leading to rising energy revenue for Foreign. In line with this, export demand rises by about as much as under the sunspot dynamics (second row, center panel). What differs, instead, is the strength of the monetary response and the distributional feedback. Now the interest rate

(top row, right panel) rises by much more than core inflation (top row, center panel). This means that the real interest rate rises notably more strongly than in Figure 1. What this means is that non-energy consumption falls sharply (second row, right panel), curbing the rise in aggregate output that follows the energy shock (compare second row, left panel and Figure 1). Very importantly, with the strong response of the central bank to inflation, the role of heterogeneity for all this is muted: savers' and hand-to-mouth households' consumption move in lockstep. This can be seen clearly from the fact that the savers' and aggregate consumption are almost perfectly aligned (second row, right panel; and bottom row, center panel). This is owed to the fact that the wage falls (bottom row, left panel; instead of rising with the sunspot dynamics). Hours worked still rise, but consumption now falls sufficiently much to allow for a fall in the real wage. Both savers' and hand-to-mouth households' incomes, therefore, fall. In line with the dampened activity, GDP falls more strongly than in the other figure (bottom row, right panel).

4.2.3 Responding to alternative measures of inflation, prices, and activity

Comparing the scenarios in Figures 1 and 2 suggests that a stronger focus of the central bank on inflation may help anchor expectations and ensure macroeconomic stability. This section looks into the implications for policy in somewhat more detail. First, it discusses targeting headline instead of core inflation. Second, it discusses targeting input prices rather than core or headline consumer prices. Last, it discusses targeting measures of economic activity or real rates alongside measures of inflation.

Targeting energy prices. Common wisdom suggests that central banks best “see through” fluctuations of energy prices and rather focus on stabilizing core inflation. As we showed above, however, under certain conditions this may invite self-fulfilling cyclical fluctuations. The energy-price-activity feedback loop entails a rise in energy prices, which in turn translates into external demand, which translates into income for hand-to-mouth households and supports consumption. Determinacy can be restored by leaning precisely against this rise in energy prices. If the central bank were to continue to react to core inflation with a weight of $\phi_{\Pi} = 1.5$ but would, in addition, put a (seemingly small)

additional weight on energy-price inflation, $P_{E,t}/P_{E,t-1}$, determinacy would be ensured. Indeed, a weight in the Taylor rule of 0.03 would suffice.

Targeting headline inflation. Similarly, core inflation accounts for energy prices only indirectly through the rise in input costs. Headline consumer price inflation instead puts more weight on energy prices since households also consume energy directly. Therefore, if the central bank responds to headline inflation, Π_t , (instead of core inflation) indeterminacy should be less of a concern. This is true, but even then the Taylor rule of the baseline would not ensure determinacy. Instead, determinacy would prevail only if $\phi_{\Pi} > 2.66$. In this sense, *not* seeing through fluctuations in energy prices helps avoid the energy-price-activity feedback loop. However, the strength of the monetary response would still be considerably above “standard” estimates.

Targeting input prices rather than consumer prices. The energy-price-activity loop is at work if prices are rigid, so that firms do not fully pass rising costs on to domestic or foreign consumers. This suggests that the central bank might as well try to respond to those rising nominal marginal costs directly; namely to input price inflation, (13). Indeed, with such a focus, the baseline response of $\phi_{\Pi} = 1.5$ ensures determinacy. What is more, any response with $\phi_{\Pi} > 1$ would. Leaning against the rise in input prices means both that the central bank leans against the energy-price fluctuations and that it leans against the wage increases that are part of the “second-round effect.”

Response to economic activity. The energy-price-activity loop sees higher energy prices go in hand with higher output (and employment) but lower GDP (since a larger share of value added accrues to Foreign). Consider a central bank that responds to core inflation with the calibrated weight of $\phi_{\Pi} = 1.5$. Next to this, however, let the central bank respond to a measure of economic activity. We discuss next that the choice of what measure of activity to respond to matters.

In a scenario that could give rise to the energy-price-activity loop, a central bank that responds to the level of production (or employment) implicitly leans against the rise in

energy prices. The central bank needs to be committed to *raising* the real rate sufficiently much so as to lean against the non-fundamental beliefs (engineering a deep-enough recession when households anticipate rising foreign demand). In our setting, this requires a rather strong response to output; a coefficient on output of at least 0.9. Note that this is notably stronger than under both the rules of Taylor (1993) and Taylor (1999) (which have output coefficients of 0.125 and 0.25, respectively).

Alternatively, the central bank may respond to GDP. Since the energy-price sunspot shock lets GDP fall, however, leaning against movements in GDP *stimulates* economic activity (and the energy price) still further. A response to GDP, therefore, further exacerbates the risks to macroeconomic stability.⁶

Real rate rules. Holden (2022) recently proposed that, in a wide range of environments, if the central bank responds to a market measure of the real rate of interest and not only to inflation, the Taylor principle is restored. The same is true in the current setting. Namely, if instead of applying (10), the central bank follows the Taylor rule

$$R_t = r_t (\Pi_{G,t})^{\phi_\pi}, \text{ with } \phi_\pi \geq 0 \text{ and with } r_t := R_t / [\mathbb{E}_t \Pi_{G,t+1}], \quad (18)$$

determinacy is restored for any $\phi_\pi > 1$.⁷ The economics are as follows: In the course of the energy-price-economic-activity feedback loop, the real incomes received by savers fall. Savers will, thus, be inclined to borrow, which puts upward pressure on the real rate of interest. A central bank that follows rule (18), therefore, leans more firmly against the feedback loop than under the more conventional rule (10). The main conclusion, therefore, stands: if there is the possibility of an energy-price-economic-activity feedback loop as envisioned in this paper, to avoid self-fulfilling fluctuations the central bank may have to be willing to *raise* interest rates firmly enough in an already recessionary episode.

⁶It should be noted that an “extreme” response to GDP (above a response coefficient of 7.51) would once more ensure determinacy. The mechanism by which the sunspot would be ruled out would seem to be entirely different, though. Namely, in this case any expectation of a non-fundamental rise in energy prices/a boom in output/a rise in inflation would be invalidated by the central bank as it engineers a larger boom/price drift still – up to the point where the paths of output or inflation would be explosive.

⁷As explained in Holden (2022), this rule implies a difference equation in $\Pi_{G,t}$ only, which has a unique stable solution if and only if $\phi > 1$. Removing the nominal indeterminacy removes the real indeterminacy.

4.2.4 Sensitivity analysis

Above, we have argued that an insufficient response of the central bank to fluctuations in the price of energy imports can expose the economy to the risk of sunspot-driven fluctuations. This section probes the results from several angles. All tables referred to here are in Appendix E.

Foreign’s marginal propensity to demand exports. Essential for the energy-price-activity feedback loop is that higher prices for the energy that Foreign supplies induce Foreign to demand more of Home’s goods. Therefore, the higher Foreign’s marginal propensity to consume, μ_1 , the more likely is the feedback loop; see Table E.1 in the appendix. *Vice versa*, for $\mu_1 \leq 0.3$ the baseline calibration induces a unique equilibrium.

The elasticity of energy supply. The more elastic the energy price to domestic economic activity, the more likely is the feedback loop. The elasticity of energy supply, therefore, is important. We looked into this, letting energy supply be given by $\xi_{E,t} = \left(\frac{P_{E,t}/P_t}{P_E/P}\right)^\delta \xi_E$. δ is a measure of the price elasticity of supply. For $\delta = 0$, our previous results emerge. The feedback loop breaks for any $\delta \geq 0.02$.

Household heterogeneity. We have run all the exercises also for a representative-agent version of the model (“RANK,” with $s = 1$, $\lambda = 0$). With otherwise the baseline calibration, in a RANK setting there is determinacy. Moving closer to balanced trade ($\mu_1 = 1$, as in the proposition) also in the RANK setting the feedback loop arises.

Ownership of energy. The feedback loop requires that incomes are redistributed to economic agents who have a higher marginal propensity to consume than the savers. Rather than distributing the energy-related income to Foreign, we have also looked at a closed economy, where all the energy is owned by domestic households. We assumed that all households own the energy in equal proportion. This retains the symmetry of the steady state. Then, the baseline calibration comes with determinacy.

Sensitivity with respect to other parameters. Appendix E reports on the sensitivity of our results to the parametrization in other dimensions. The larger the energy-dependence of production is (for a given weight of raw energy in consumption baskets), the larger the risk of indeterminacy; see Table E.2. Related, for a given size of energy imports, the more of these imports are consumed directly by the household sector, the less of a concern is indeterminacy; see Table E.3.

Next, the more elastically households or firms can respond to energy-price fluctuations, the lower the risk of indeterminacy. Tables E.4 through E.6 illustrate this for \bar{e} , η , and θ . The cutoff for determinacy rises in \bar{e} , and falls in both η and θ .

Starting from the baseline calibration, the less willing households are to substitute intertemporally, the larger the range of policy responses for which there is indeterminacy; see Table E.7. Next, the less elastic labor supply is (the larger φ), the more does the rise in energy prices translate into a rise in wages when output rises. The wage increase strengthens the rise in hand-to-mouth households' income and accelerates the feedback loop. Hence, the determinacy threshold increases in φ , see Table E.8 in the appendix.

Last, the more rigid goods prices are (the larger ψ), the more does the average markup fall with rising demand. The central bank, then, needs to respond aggressively to core inflation to tame self-fulfilling expectations; Table E.9. The more of the profits the hand-to-mouth households receive, instead, the less does their income rise after an energy-price increase. Higher redistribution, thus, weakens the feedback loop, Table E.10.

5 Conclusions

The current paper has worked with a New Keynesian business-cycle model with energy imports and heterogeneous households. Energy supply was supplied inelastically by the foreign economy. And the price at which energy traded internationally moved flexibly to align domestic demand with fixed foreign supply.

We showed that this environment could give rise to an energy-price-activity feedback loop, provided energy prices are sufficiently elastic to domestic demand and that foreign demand responds sufficiently strongly to rising energy revenues. In this loop, beliefs about high

energy prices go in hand with high external demand. This demand would support labor income, so that domestic demand would fall little, even if the central bank raised interest rates in a conventional way with inflation. The outcome is a high energy price, high core inflation, high interest rates *and* high economic activity as measured by employment, all at the same time; but a fall in gross domestic product.

The channel provides a rationale, why—in a scarce-energy situation—the central bank may precisely *not* follow the common wisdom to “see through shocks” or to disregard movements in energy prices even if they are flexible. Instead, it may rather want to *raise* interest rates if energy prices rise—even if this is recessionary. In the same vein, the central bank may want to precisely focus on headline inflation instead of core, may precisely overweight the energy price in inflation considerations, or may more actively seek to curb economic activity even if the gross domestic product has fallen already.

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A Further details on the model

A.1 Households

There is mass of infinitely-lived households, each of which with the same preferences. Households can be of two types, or rather, live on two distinct islands: hand-to-mouth, H , or saver, S . Importantly, households living on the same island pool their resources and jointly decide about their consumption, savings and labor supply. Therefore, in equilibrium, two representative households arise, where we focus on a stationary equilibrium in which the respective household sizes do not change.

Hand-to-mouth households cannot participate in financial markets in the period in which they are constrained. They consume all of their income. Savers, instead, optimize intertemporally. They can save in liquid, risk-free nominal bonds, the rate of return on which the central bank controls. The bonds are in zero net supply globally, yet, the Home economy can hold gross positions against Foreign.

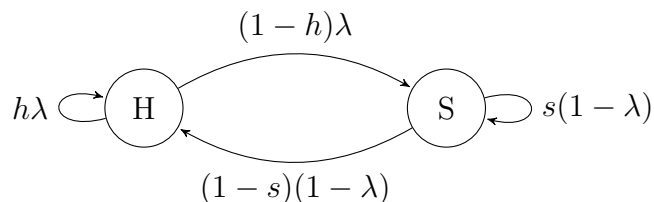
Yet, there are flows of *individual* households across the types—generating idiosyncratic risk. At the end of period t , there are thus flows of households across islands. The probability to stay, respectively, hand-to-mouth or a saver is h and s . Stationarity implies $\lambda = (1 - s)/(2 - s - h)$, where λ denotes the share of hand-to-mouth households.

Aggregation For all household-specific variables with index i , the corresponding aggregate is defined as the weighted sum over the household-specific measures, for instance, aggregate consumption is given by

$$C_t = (1 - \lambda)C_{S,t} + \lambda C_{H,t}.$$

Asset flows across islands Figure A.1 illustrates the flows of households across “islands”.

Figure A.1 Flows across islands between period t and $t + 1$



Crucially, a household that is a saver in t can trade in financial markets. *Liquid* assets bought by a saver in t are carried over by her to the island H if she becomes constrained at the beginning of period $t + 1$. Hence, we need to distinguish two concepts. Between the end-of-period t (when the current savers decided about their financial market positions) and the beginning-of-period $t + 1$, only flows across islands takes place. Let (i) Z_{it+1}

denote the end-of-period t bond holdings and (ii) B_{it+1} denote the beginning-of-period $t + 1$ bond holdings, where the asset is in the respective period held on island $i \in \{H, S\}$. The asset flows are then given by

$$\begin{aligned}(1 - \lambda)B_{S,t+1} &= s(1 - \lambda)Z_{S,t+1} + (1 - h)\lambda Z_{H,t+1}, \\ \lambda B_{H,t+1} &= (1 - s)(1 - \lambda)Z_{S,t+1} + h\lambda Z_{H,t+1},\end{aligned}$$

i.e., beginning-of-period assets of savers $B_{S,t+1}$ consist of the end-of-period assets of remaining savers and of the end-of-period assets of new savers—previously hand-to-mouth. Given $\lambda = (1 - s)/(2 - s - h)$, we can rewrite asset flows as

$$B_{S,t+1} = sZ_{S,t+1} + (1 - s)Z_{H,t+1} \quad \text{and} \quad B_{H,t+1} = (1 - h)Z_{S,t+1} + hZ_{H,t+1}$$

Since hand-to-mouth households are not allowed to participate in financial markets, their end-of-period bond holdings will be zero in equilibrium, i.e., $Z_{H,t+1} = 0$. Hence,

$$B_{S,t+1} = sZ_{S,t+1} \quad \text{and} \quad B_{H,t+1} = (1 - h)Z_{S,t+1}$$

Anticipating aggregation of household variables, we can write aggregate holdings as

$$B_{t+1} = (1 - \lambda)Z_{S,t+1} + \lambda Z_{H,t+1} = (1 - \lambda)Z_{S,t+1}.$$

Therefore, island-wide beginning-of-period and end-of-period holdings are given by

$$\begin{aligned}B_{S,t+1} &= \frac{s}{1 - \lambda}B_{t+1} & \text{and} & & B_{H,t+1} &= \frac{1 - s}{\lambda}B_{t+1}, \\ Z_{S,t+1} &= \frac{1}{1 - \lambda}B_{t+1} & \text{and} & & Z_{H,t+1} &= 0,\end{aligned}$$

which we used in Section 2 already in the budget constraints of both household types.

B Proof of the proposition and corollary

B.1 Proof of the Proposition

This appendix provides the proof to the proposition in the main text. The proof is straightforward and the steps well-known in the New Keynesian literature.

The model is given by equations (15) and (16), repeated here for convenience.

$$\widehat{Y}_{G,t} = \mathbb{E}_t \widehat{Y}_{G,t+1} - \frac{1}{\tilde{\sigma}} \left[\phi_{\Pi} \widehat{\Pi}_t - \mathbb{E}_t \widehat{\Pi}_{t+1} \right], \quad \text{with } \tilde{\sigma} := \frac{\sigma}{1-\alpha} \frac{1-\alpha [1+\varphi+1/\theta]}{1-\alpha+\sigma\alpha}, \quad (19)$$

$$\widehat{\Pi}_t = \beta \mathbb{E}_t \widehat{\Pi}_{t+1} + \tilde{\kappa} \widehat{Y}_{G,t}, \quad \text{with } \tilde{\kappa} := \frac{\epsilon}{\psi} \frac{\sigma + \varphi + \alpha \frac{1}{\theta} (1-\sigma)}{1-\alpha+\sigma\alpha}. \quad (20)$$

The proposition states the importance of the signs of $\tilde{\sigma}$ and $\tilde{\kappa}$ which are determined by

$$\begin{aligned} \text{sgn } \tilde{\sigma} &= \text{sgn} \frac{\sigma}{1-\alpha} \frac{1-\alpha [1+\varphi+\frac{1}{\theta}]}{1-\alpha+\sigma\alpha} = \text{sgn} \left(1-\alpha \left[1+\varphi+\frac{1}{\theta} \right] \right), \\ \text{sgn } \tilde{\kappa} &= \text{sgn} \frac{\epsilon}{\psi} \frac{\sigma + \varphi + \alpha \frac{1}{\theta} (1-\sigma)}{1-\alpha+\sigma\alpha} = \text{sgn} \left(\sigma + \varphi + \alpha \frac{1}{\theta} (1-\sigma) \right), \end{aligned}$$

or

$$\begin{aligned} \tilde{\sigma} > 0 &\iff 1 - \frac{\alpha}{\theta} > \alpha(1+\varphi), \\ \tilde{\kappa} > 0 &\iff 1 - \alpha \frac{1}{\theta} > -\frac{1}{\sigma} \left(\varphi + \alpha \frac{1}{\theta} \right), \end{aligned}$$

where $\alpha(1+\varphi) > 0$ and $-\frac{1}{\sigma} \left(\varphi + \frac{\alpha}{\theta} \right) < 0$. Hence, whenever $\tilde{\sigma} > 0$, also $\tilde{\kappa} > 0$. For $\tilde{\sigma} < 0$, we can still have either $\tilde{\kappa} > 0$ or $\tilde{\kappa} < 0$.

Intuition for the sign $\tilde{\kappa}$

The sign of $\tilde{\kappa}$ is determined by the sign of the term $\frac{\sigma+\varphi+\alpha(1/\theta)(1-\sigma)}{1-\alpha+\sigma\alpha}$, which gives the elasticity of marginal costs with respect to output. The denominator is positive. We, thus, focus on the numerator $(\sigma + \varphi + \alpha(1/\theta)(1-\sigma))$, the sign of which is ambiguous. The first two summands in the numerator are conventional. If energy prices were to move in lock-step with wages, these terms would give the entire effect of output on marginal costs. The terms are the wealth effect on labor supply and the compensation for the disutility of work that would come with rising output alone (rising employment). Both of these terms are unambiguously positive. What could make the sign of $\tilde{\kappa}$ switch, instead, is the term $\alpha(1/\theta)(1-\sigma)$. This term captures the effect on marginal costs of the excess sensitivity of the energy price with respect to output (the rise of energy prices in excess of movements in the wage). The energy price is given by $\widehat{q}_t = \widehat{w}_t + \frac{1}{1-\alpha} \widehat{Y}_{G,t}$. Real marginal costs are given by $\widehat{\Lambda}_t = (1-\alpha)\widehat{w}_t + \alpha\widehat{q}_t$ and, hence, by $\widehat{\Lambda}_t = \widehat{w}_t + \frac{\alpha/\theta}{1-\alpha} \widehat{Y}_{G,t}$. The less substitutable

the production factors are (the smaller θ), the larger is the excess sensitivity of energy prices with respect to output. In turn, the larger the share of energy is in production (the larger α), the more this matters for marginal costs. This excess sensitivity has two countervailing effects on marginal costs: a direct effect and an indirect effect that runs through the wage. The direct effect means higher output comes with higher marginal costs still. The indirect effect works in the opposite direction. Namely, for a given level of output, a rise in energy costs reduces household consumption. Through the wealth effect on labor supply, this reduces wages and marginal costs. The larger σ is, the larger is the wealth effect on labor supply and, thus, the larger is this opposite force on marginal costs. For a strong-enough wealth effect on labor supply (large-enough σ) and strong-enough excess sensitivity of energy prices (large enough α/θ), the sign of the slope of the Phillips curve could, thus, invert.

Intuition for the sign of $\tilde{\sigma}$

The intuition for the sign of $\tilde{\sigma}$ is more straightforward. The sign of $\tilde{\sigma}$ is determined by the sign of $A := 1 - \alpha \left[1 + \varphi + \frac{1}{\theta} \right]$. The term reflects the comovement of aggregate consumption with economic activity. Namely, by market clearing $\widehat{Y}_{G,t} = (1 - \alpha)\widehat{c}_t + \alpha\widehat{q}_t$, so that $\widehat{c}_t = \frac{1}{1-\alpha} \left[\widehat{Y}_{G,t} - \alpha\widehat{q}_t \right]$. Since $\widehat{q}_t = \frac{\sigma + \varphi + \frac{1}{\theta}}{1 - \alpha + \sigma\alpha} \widehat{Y}_{G,t}$, combining terms, we have that $\widehat{c}_t = \frac{1 - \alpha \left[1 + \varphi + \frac{1}{\theta} \right]}{(1 - \alpha)(1 - \alpha + \sigma\alpha)} \widehat{Y}_{G,t}$. α is the share of energy in production and, therefore, the share of output exported in steady state. If energy prices were constant altogether and all factor inputs linear in output, $A = 1 - \alpha$. The term in the squared bracket in term A reflects that input prices move disproportionately with output. The reason is that households' disutility of labor increases in output. If energy prices were to move one-to-one with wages $A = 1 - \alpha(1 + \varphi)$. Term $\frac{1}{\theta}$ in the square bracket in turn captures once again the excess sensitivity of energy prices to movements in output.

Thus, the IS curve inverts if $\alpha \left[1 + \varphi + \frac{1}{\theta} \right] > 1$, or, in words, if energy is an important input factor (high α), labor supply of households is sufficiently inelastic (high φ), and if energy is hard to substitute (high $1/\theta$). Suppose that firms seek to produce an additional unit of output $Y_{G,t}$. Since energy is in fixed supply, labor supply needs to expand. At the same time, both energy and labor are needed in production due to the low elasticity of substitution. The lower the elasticity of substitution (low θ) the more does the energy price increase. Similarly, when households' labor supply is inelastic ($\varphi > 0$), the wage increases and firms would prefer to use energy instead of labor: again, the energy price rises. Up to first order, firms' profits (here labelled Γ_t) are given by

$$\Gamma_t = (1 - \alpha)\widehat{N}_t - (1 - \alpha)(\widehat{w}_t + \widehat{N}_t) - \alpha\widehat{q}_t = -(1 - \alpha)\widehat{w}_t - \alpha\widehat{q}_t.$$

Incomes accruing to domestic households are given by profits and wages, up to first order

these incomes evolve as

$$\Gamma_t + \widehat{w}_t + \widehat{N}_t = \alpha[\widehat{w}_t - \widehat{q}_t] + \frac{1}{1-\alpha}\widehat{Y}_{G,t}.$$

This means that if energy prices rise sufficiently steeply when output rises, the incomes of domestic households can fall even though wages may still rise. In turn, an increase in the real rate of interest could *crowd in* domestic consumption by depressing energy costs (and, thus, the share of output transferred abroad).

Determinacy cutoffs

Write the model in [Blanchard and Kahn \(1980\)](#) form:

$$\begin{bmatrix} 1 & 1/\tilde{\sigma} \\ 0 & \beta \end{bmatrix} \mathbb{E}_t \begin{bmatrix} \widehat{Y}_{G,t+1} \\ \widehat{\Pi}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \phi_{\Pi}/\tilde{\sigma} \\ -\tilde{\kappa} & 1 \end{bmatrix} \begin{bmatrix} \widehat{Y}_{G,t} \\ \widehat{\Pi}_t \end{bmatrix}$$

or, alternatively

$$\mathbb{E}_t \begin{bmatrix} \widehat{Y}_{G,t+1} \\ \widehat{\Pi}_{t+1} \end{bmatrix} = \underbrace{\frac{1}{\beta} \begin{bmatrix} \beta + \frac{\tilde{\kappa}}{\tilde{\sigma}} & \beta\phi_{\Pi}/\tilde{\sigma} - \frac{1}{\tilde{\sigma}} \\ -\tilde{\kappa} & 1 \end{bmatrix}}_{:=A} \begin{bmatrix} \widehat{Y}_{G,t} \\ \widehat{\Pi}_t \end{bmatrix}$$

There are two nonpredetermined variables. So there will always be bounded equilibria. There is a locally unique bounded equilibrium iff either (cf. [Woodford, 2003](#), p. 670):

- Case a): $\det(A) > 1$, $\det(A) - \text{tr}(A) > -1$ and $\det(A) + \text{tr}(A) > -1$, or
- Case b): $\det(A) < 1$, $\det(A) - \text{tr}(A) < -1$ and $\det(A) + \text{tr}(A) < -1$.

Here, $\det(A) = \left[\frac{1}{\beta} + \frac{\phi_{\Pi} \tilde{\kappa}}{\beta \tilde{\sigma}} \right]$ and $\text{tr}(A) = \left[1 + \frac{1}{\beta} + \frac{1}{\beta} \frac{\tilde{\kappa}}{\tilde{\sigma}} \right]$.

Proof of the proposition's item 1). Suppose that $\tilde{\sigma} > 0$ and $\tilde{\kappa} > 0$. Then the determinacy conditions are as in the standard closed-economy New Keynesian model. More in detail, $\det(A) > 1$ and $\text{tr}(A) > 0$, so that Case a) applies. The condition that may bind is $\det(A) - \text{tr}(A) > -1$, which leads to the conventional determinacy condition $\phi_{\Pi} > 1$.

Proof of the proposition's item 1) c'td. Suppose that $\tilde{\sigma} < 0$ and $\tilde{\kappa} < 0$. Then the determinacy conditions are as in the standard closed-economy New Keynesian model. Again, in this case $\det(A) > 1$ for any $\phi_{\Pi} > 0$. Thus, we need to check Case a) again. $\text{tr}(A) > 0$, so that $\det(A) + \text{tr}(A) > -1$ always. So, what we need for determinacy is $\det(A) - \text{tr}(A) > -1$. Or, equivalently $\left[\frac{1}{\beta} + \frac{\phi_{\Pi} \tilde{\kappa}}{\beta \tilde{\sigma}} \right] - \left[1 + \frac{1}{\beta} + \frac{1}{\beta} \frac{\tilde{\kappa}}{\tilde{\sigma}} \right] = \frac{1}{\beta \tilde{\sigma}} [\phi_{\Pi} - 1] - 1 > -1$, or, once more, $\phi_{\Pi} > 1$.

Proof of the proposition's item 2). By assumption for this case, $\tilde{\sigma} < 0, \tilde{\kappa} > 0$. This is when non-standard determinacy regions arise. In this case, *two* determinacy regions can arise.

Focus on the set of conditions for case a) first. $\det(A) = \left[\frac{1}{\beta} + \frac{\phi_{\Pi} \tilde{\kappa}}{\beta \tilde{\sigma}} \right] > 1$ can be achieved by setting $\phi_{\Pi} < \frac{\tilde{\sigma}}{\tilde{\kappa}}(\beta - 1)$, where $\frac{\tilde{\sigma}}{\tilde{\kappa}}(\beta - 1) > 0$ since $\frac{\tilde{\sigma}}{\tilde{\kappa}} < 0$. The second condition can be achieved by setting $\phi_{\Pi} < 1$. Finally, the third condition can be achieved by setting $\phi_{\Pi} < -2(1 + \beta)\frac{\tilde{\sigma}}{\tilde{\kappa}} - 1$. Hence, in sum, this determinacy region exists if there is a $\phi_{\Pi} \geq 0$ such that

$$\phi_{\Pi} < \min \left(\frac{\tilde{\sigma}}{\tilde{\kappa}}(\beta - 1), 1, -2(1 + \beta)\frac{\tilde{\sigma}}{\tilde{\kappa}} - 1 \right)$$

Note that, for $\beta \rightarrow 1$, this determinacy region disappears.

Focus on the set of conditions for case b) next. $\det(A) < 1$ can be achieved for $\phi_{\Pi} \geq 0$ since $\frac{\tilde{\sigma}}{\tilde{\kappa}} < 0$. For $\det(A) - \text{tr}(A) < -1$, we need $\frac{1}{\beta}\frac{\tilde{\kappa}}{\tilde{\sigma}}[\phi_{\Pi} - 1] - 1 < -1$, meaning $\phi_{\Pi} > 1$. For $\det(A) + \text{tr}(A) < -1$, we need $1 + \frac{2}{\beta} + \frac{1}{\beta}\frac{\tilde{\kappa}}{\tilde{\sigma}}(\phi_{\Pi} + 1) < -1$, meaning $\phi_{\Pi} > -2(1 + \beta)\frac{\tilde{\sigma}}{\tilde{\kappa}} - 1$. So that both $\det(A) \pm \text{tr}(A) < -1$, therefore we need

$$\phi_{\Pi} > \max \left(1, -2(1 + \beta)\frac{\tilde{\sigma}}{\tilde{\kappa}} - 1 \right),$$

or for $\beta \rightarrow 1$, $\phi_{\Pi} > \max(1, -4\tilde{\sigma}/\tilde{\kappa} - 1)$. This is the cutoff mentioned in Proposition 1, equation (17). \square

B.2 Proof of the Corollary

Consider case 2) of Proposition 1, that is, $\tilde{\sigma} < 0$ and $\tilde{\kappa} > 0$. For those restrictions to be satisfied, we require that

$$\begin{aligned} \text{if } 1 - \alpha(1 + \varphi) \leq 0 & \quad \text{then } \alpha \frac{\sigma - 1}{\varphi + \sigma} < \theta \\ \text{if } 1 - \alpha(1 + \varphi) > 0 & \quad \text{then } \alpha \frac{\sigma - 1}{\varphi + \sigma} < \theta < \frac{\alpha}{1 - \alpha(1 + \varphi)}. \end{aligned} \tag{21}$$

The lower bound on θ comes from $\tilde{\kappa} > 0$. The lower bound puts limits on the strength of the wealth effect on labor supply, discussed on top of page 40. For $\sigma \leq 1$, the lower bound never longer binds ($\theta > 0$ in any case). If the share of energy is large enough and labor supply sufficiently inelastic (the top case), labor and energy could be infinitely substitutable; and, nevertheless, the IS curve inverts, compare the text on the bottom of page 40. Otherwise, if the share of energy in production, α , is smaller or the labor supply elasticity takes on intermediate values, the elasticity of substitution between energy and labor must not be too large to see the IS curve invert. The reason is that, then, the effect of demand energy prices must be sufficiently to generate inversion. Corollary 1 looks at the case $\theta = \alpha$. For this case, the inequalities above are satisfied for any $\alpha > 0, \sigma > 0$,

and $\varphi > 0$.

The determinacy threshold for case two is tighter than the Taylor principle would require if $\bar{\phi}_{\Pi} > 1$, where $\bar{\phi}_{\Pi}$ is defined as the threshold

$$\begin{aligned}\bar{\phi}_{\Pi} &:= -2(1 + \beta) \frac{\tilde{\sigma}}{\tilde{\kappa}} - 1 \\ &= 2(1 + \beta) \frac{\sigma}{\epsilon/\psi} \frac{\alpha}{1 - \alpha} \frac{1 + \varphi + \frac{1}{\theta} - \frac{1}{\alpha}}{\varphi + \sigma + (1 - \sigma) \frac{\alpha}{\theta}} - 1.\end{aligned}$$

Hence, for a tighter determinacy criterion than the Taylor principle ($\bar{\phi}_{\Pi} > 1$), thus, we have

$$(1 + \beta) \frac{\sigma}{\epsilon/\psi} \frac{\alpha}{1 - \alpha} \frac{1 + \varphi + \frac{1}{\theta} - \frac{1}{\alpha}}{\varphi + \sigma + (1 - \sigma) \frac{\alpha}{\theta}} > 1. \quad (22)$$

Given the parameter restrictions on θ in (21), we have that

$$1 + \varphi + \frac{1}{\theta} - \frac{1}{\alpha} > 0 \quad \text{and} \quad \varphi + \sigma + (1 - \sigma) \frac{\alpha}{\theta} > 0,$$

such that we can rearrange condition (22) to

$$1 > \frac{1}{1 + \beta} \frac{\epsilon/\psi}{\sigma} \frac{1 - \alpha}{\alpha} \frac{\varphi + \sigma + (1 - \sigma) \frac{\alpha}{\theta}}{1 + \varphi + \frac{1}{\theta} - \frac{1}{\alpha}} =: \Omega, \quad (23)$$

where each single fraction is strictly positive. It may be useful to characterize the properties of the right-hand side.

With this, we have the following corollary, which nests corollary B.1 as a special case when $\alpha = \theta$.

Corollary B.1. *Consider the same conditions as in Proposition 1 in the main text. Suppose that the parameter restrictions in (21) hold. Given any θ within that range, the lower bound on ϕ_{Π} that ensures indeterminacy will be higher than suggested by the Taylor principle if and only if*

$$1 > \frac{1}{2} \frac{\epsilon/\psi}{\sigma} \frac{1 - \alpha}{\alpha} \frac{\varphi + \sigma + (1 - \sigma) \frac{\alpha}{\theta}}{1 + \varphi + \frac{1}{\theta} - \frac{1}{\alpha}}.$$

As to the qualitative statements in Corollary 1 in the main text, they emerge when applying the restriction $\alpha = \theta$ to (23). In that case, the right-hand side gives

$$\Omega = \frac{1}{1 + \beta} \frac{\epsilon/\psi}{\sigma} \frac{1 - \alpha}{\alpha},$$

making the sign of the respective derivatives obvious.

C Pencil-and-paper T(H)ANK model

This section provides the two-agent counterpart to Appendix B. Other than allowing for household heterogeneity, the parametric assumptions are identical to those in said appendix. Namely, we focus on an environment in which households consume goods but do not directly consume energy. For tractability, we also focus on a version in which only the savers receive profit income and in which they bear the burden of all taxation. That is, we assume that $\tau^d = 0$.

C.1 Equilibrium conditions

The non-linear equilibrium conditions in this case are, on the household side,

$$\begin{aligned} C_{S,t}^{-\sigma} &= \mathbb{E}_t \left[\beta \left(s C_{S,t+1}^{-\sigma} + (1-s) C_{H,t+1}^{-\sigma} \right) \frac{R_t}{\Pi_{t+1}} \right], & \text{and} & & C_{H,t} &= \frac{W_t}{P_t} N_{H,t}, \\ \frac{W_t}{P_t} &= \chi C_{S,t}^\sigma N_{S,t}^\varphi, & \text{and} & & \frac{W_t}{P_t} &= \chi C_{H,t}^\sigma N_{H,t}^\varphi. \end{aligned}$$

The supply side is given by

$$\begin{aligned} Y_{G,t} &= \left[\alpha E_t^{\frac{\theta-1}{\theta}} + (1-\alpha) N_t^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \\ \psi \Pi_t (\Pi_t - 1) &= (1 + \tau^y)(1 - \varepsilon) + \varepsilon \Lambda_t + \psi \mathbb{E}_t \left[\mathcal{F}_{t,t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{G,t+1}}{Y_{G,t}} \right], \end{aligned}$$

and

$$\frac{W_t}{P_t} = \Lambda_t (1 - \alpha) \left(\frac{Y_{G,t}}{N_t} \right)^{\frac{1}{\theta}} \quad \text{and} \quad \frac{P_{E,t}}{P_t} = \Lambda_t \alpha \left(\frac{Y_{G,t}}{E_t} \right)^{\frac{1}{\theta}}.$$

Aggregates and market clearing are given by

$$\begin{aligned} N_t &= (1 - \lambda) N_{S,t} + \lambda N_{H,t}, & \text{and} & & C_t &= (1 - \lambda) C_{S,t} + \lambda C_{H,t}, \\ \xi_E &= E_t, & \text{and} & & Y_{G,t} &= C_t + \frac{P_{E,t}}{P_t} \xi_E. \end{aligned}$$

Finally, monetary policy is given by

$$R_t = R \Pi_t^{\phi_\Pi}.$$

C.2 Steady state

As before, we consider $\xi_E = 1$ and choose χ such that $N = 1$. We linearize around a steady state in which both household types are symmetric in the sense that they have the same levels of income, consumption, and hours worked in the steady state. This steady

state satisfies $Y_G = 1$, $\Lambda = 1$ (via production subsidy τ^y), $w = 1 - \alpha$ (with $w_t := W_t/P_t$), $q = \alpha$ (with $q_t := P_{E,t}/P_t$), $C = C_S = C_H = 1 - \alpha$, $N = N_S = N_H = 1$, $R = 1/\beta$, $\Pi = 1$.

C.3 Log-linearized dynamics

Exploiting $E_t = \xi_E = 1$ and log-linearizing the model yields, on the household side,

$$\begin{aligned} \widehat{C}_{S,t} &= \mathbb{E}_t \left[s\widehat{C}_{S,t+1} + (1-s)\widehat{C}_{H,t+1} - \sigma^{-1} \left(\widehat{R}_t - \widehat{\Pi}_{t+1} \right) \right], & \text{and} & \quad \widehat{C}_{H,t} = \widehat{w}_t + \widehat{N}_{H,t}, \\ \widehat{w}_t &= \sigma\widehat{C}_{S,t} + \varphi\widehat{N}_{S,t}, & \text{and} & \quad \widehat{w}_t = \sigma\widehat{C}_{H,t} + \varphi\widehat{N}_{H,t}. \end{aligned}$$

The supply side is given by

$$\begin{aligned} \widehat{Y}_{G,t} &= (1-\alpha)\widehat{N}_t, & \text{and} & \quad \widehat{\Pi}_t = \beta\mathbb{E}_t \left[\widehat{\Pi}_{t+1} \right] + \frac{\varepsilon}{\psi}\widehat{\Lambda}_t, \\ \widehat{w}_t &= \widehat{\Lambda}_t + \frac{1}{\theta} \left(\widehat{Y}_{G,t} - \widehat{N}_t \right) & \text{and} & \quad \widehat{q}_t = \widehat{\Lambda}_t + \frac{1}{\theta}\widehat{Y}_{G,t}. \end{aligned}$$

Aggregates and market clearing are given by

$$\begin{aligned} \widehat{N}_t &= (1-\lambda)\widehat{N}_{S,t} + \lambda\widehat{N}_{H,t}, & \text{and} & \quad \widehat{C}_t = (1-\lambda)\widehat{C}_{S,t} + \lambda\widehat{C}_{H,t}, \\ 0 &= \widehat{E}_t, & \text{and} & \quad \widehat{Y}_{G,t} = (1-\alpha)\widehat{C}_t + \alpha\widehat{q}_t. \end{aligned}$$

Finally, monetary policy is given by

$$\widehat{R}_t = \phi_{\Pi}\widehat{\Pi}_t.$$

C.4 Two-equation representation

Combining households' labor supply schedules with firms' marginal costs, the production function and goods market clearing yields the same relation between marginal costs and production as in the main text, (14), repeated here for convenience

$$\widehat{\Lambda}_t = \frac{\sigma + \varphi + \frac{\alpha}{\theta}(1-\sigma)}{1-\alpha + \sigma\alpha} \widehat{Y}_{G,t}.$$

The Phillips curve of this economy is then given by

$$\widehat{\Pi}_t = \beta\mathbb{E}_t \widehat{\Pi}_{t+1} + \widetilde{\kappa}\widehat{Y}_{G,t} \quad \text{with} \quad \widetilde{\kappa} := \frac{\varepsilon}{\psi} \frac{\sigma + \varphi + \frac{\alpha}{\theta}(1-\sigma)}{1-\alpha + \sigma\alpha}.$$

Notably, the Phillips curve here is identical to (16) in the main text: The Phillips curve, thus, is not affected by household heterogeneity.

Combining firms' marginal costs, the production function and hand-to-mouths' labor

supply and budget constraint, we obtain hand-to-mouth households' consumption as

$$\widehat{C}_{H,t} = \Gamma_{CH} \widehat{Y}_{G,t} \quad \text{with} \quad \Gamma_{CH} := \frac{1 + \varphi}{\sigma + \varphi} \left(\frac{\varphi + \frac{\alpha}{\theta} + \sigma \left[1 - \frac{\alpha}{\theta}\right]}{1 - \alpha + \sigma\alpha} + \frac{1}{\theta} \left(1 - \frac{1}{1 - \alpha}\right) \right).$$

Goods market clearing together with marginal costs yields aggregate consumption as

$$\widehat{C}_t = \Gamma_C \widehat{Y}_{G,t} \quad \text{with} \quad \Gamma_C := \frac{1 - \alpha - \alpha\varphi - \frac{\alpha}{\theta}}{(1 - \alpha)(1 - \alpha + \sigma\alpha)}.$$

Aggregate consumption can be rearranged to obtain savers' consumption as

$$\widehat{C}_{S,t} = \Gamma_{CS} \widehat{Y}_{G,t} \quad \text{with} \quad \Gamma_{CS} := \frac{1}{1 - \lambda} (\Gamma_C - \lambda \Gamma_{CH}).$$

Thus, combining all consumption dynamics with the savers' Euler equation and the policy rule, we obtain the aggregate IS curve as

$$\widehat{Y}_{G,t} = \widetilde{\delta} \mathbb{E}_t \widehat{Y}_{G,t+1} - \frac{1}{\widetilde{\sigma}} \left(\phi_{\Pi} \widehat{\Pi}_t - \mathbb{E}_t \widehat{\Pi}_{t+1} \right),$$

with

$$\widetilde{\sigma} := \sigma \Gamma_{CS}, \quad \text{and} \quad \widetilde{\delta} := \frac{s \Gamma_{CS} + (1 - s) \Gamma_{CH}}{\Gamma_{CS}}.$$

Hence, while the Phillips curve is independent of household heterogeneity, the aggregate IS curve is not. Both representations nest [Bilbiie \(2020\)](#), with discounting or compounding in the IS curve. In the absence of a precautionary savings motive, that is, if savers always stay savers ($s = 1$), $\widetilde{\delta} = 1$. Note that for $s = 1$ and $\lambda = 0$, we obtain exactly the same IS curve results as in the representative agent paper-and-pencil representation.

C.5 Determinacy conditions

Given the two-equation representation above (IS curve and Phillips curve), we can derive conditions similar to the ones derived in [Appendix B](#). In the following, we assume that parameters are such that $\widetilde{\delta} > 0$, as is the case in the calibrated baseline in the main text. Furthermore, so as to facilitate the expressions, let $\beta \rightarrow 1$. Then, there again are two cases.

Same signs. Suppose $\widetilde{\sigma}$ and $\widetilde{\kappa}$ have the same sign. Following the same steps as [Appendix B](#), the determinacy criterion is given by

$$\phi_{\Pi} > \max \left\{ 1, (\widetilde{\delta} - 1) \frac{\widetilde{\sigma}}{\widetilde{\kappa}} \right\},$$

where, for reasonable calibrations, the first term is the binding constraint. Then, for $\tilde{\delta} = 1$ or if $\beta \rightarrow 1$, the cutoff is unity, so that the standard Taylor principle applies. Whenever there is discounting $\tilde{\delta} < 1$, indeed, the cutoff would be lower still than the Taylor principle commands. The qualitative implication of $\tilde{\delta}$ on the determinacy threshold is as in [Bilbiie \(2021\)](#).

Opposite signs. Suppose $\tilde{\sigma} < 0$ and $\tilde{\kappa} > 0$. Following the same steps as in Appendix B, we obtain one active and one passive region. The determinacy criterion for the “**active**” region is given by

$$\phi_{\Pi} > \max \left\{ 1, -2(1 + \tilde{\delta}) \frac{\tilde{\sigma}}{\tilde{\kappa}} - 1 \right\}.$$

Note that this is structurally similar to the determinacy criterion (17) in the main text, with the difference that the terms governing $\tilde{\delta}$ and $\tilde{\sigma}$ differ. All else equal, discounting shifts the cutoff lower, while lower $\tilde{\sigma}$ shifts the cutoff outward.

The determinacy criterion for the “**passive**” region is given by

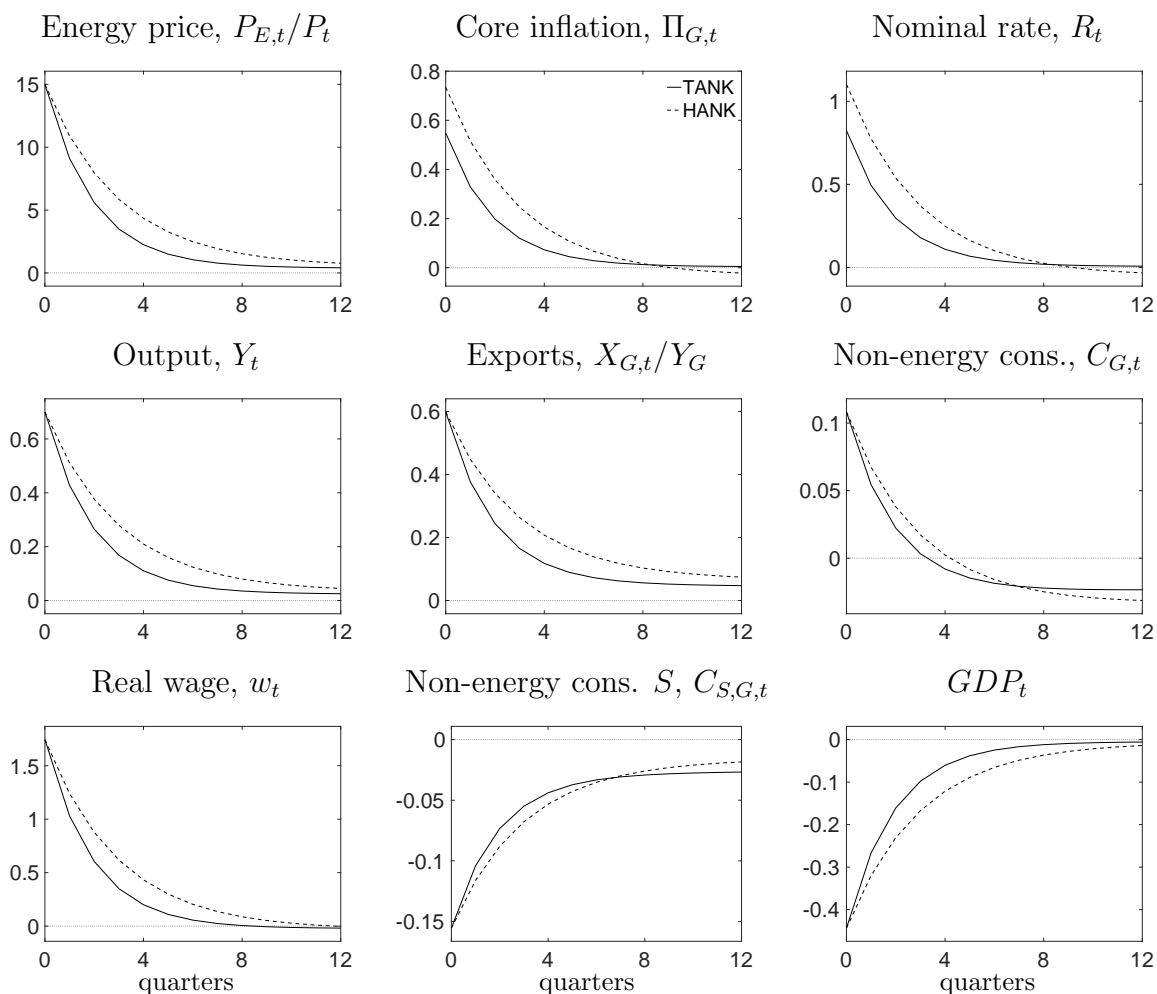
$$0 \leq \phi_{\Pi} < \min \left\{ 1, -2(1 + \tilde{\delta}) \frac{\tilde{\sigma}}{\tilde{\kappa}} - 1, (\tilde{\delta} - 1) \frac{\tilde{\sigma}}{\tilde{\kappa}} \right\}.$$

D Role of hand-to-mouth vs. idiosyncratic risk

The paper considers a model with heterogeneous households. This appendix seeks to ascertain which dimension of heterogeneity matters most for the results: idiosyncratic risk or the fact that there are hand-to-mouth households. Toward this end, we consider the model in the calibration of Section 4.1. This baseline has both hand-to-mouth households and idiosyncratic risk for the savers. We label this baseline the “THANK” variant henceforth. We seek to see what the role of idiosyncratic risk is by shutting down that risk in the THANK model. That is, we set $s = h = 1$, and label this version the “TANK” variant. As discussed in appendix C, the THANK variant displays discounting in the IS curve. The determinacy cutoff, therefore, is lower. This is also quantitatively true in the present calibration.

Figure D.1 shows the impulse response to a sunspot shock for the two model variants. The size of the shock in each variant is scaled such that the unsubsidized energy price on impact rises by 20 percent. The black solid lines refer to the responses in the TANK variant. The dashed lines refer to the responses in the THANK variant. Idiosyncratic risk implies that the sunspot shock, that moves economic activity, shows more persistence. As a result, the impulse responses in the THANK economy are more persistent as well (dashed lines). Similarly, the impact response of both, inflation and the nominal rate are somewhat larger.

Figure D.1 Sunspot shock amid targeting core inflation, TANK vs THANK



Notes: Impulse response to a sunspot shock that raises energy prices by 20 percent on impact. The central bank responds to core inflation, with response parameter $\phi_{\Pi} = 1.5$. Scaling: all response are scaled to give percentage deviations from steady state. The exception is the response of exports (percent of steady state production). Also, interest rates and inflation rates are in annualized percentage points. Responses in the TANK variant are shown as solid lines, responses in the THANK variant as dashed lines.

E Heterogeneous-household model:

Sensitivity with respect to parameter choices

This appendix collects results on the sensitivity of the determinacy cutoffs for different parametrizations of the baseline heterogeneous household model.

In the following tables, each row represents one variation of the parameter mentioned in the caption. First, the new steady state is computed. Next, the determinacy regions for core-inflation targeting are computed: (a) the first column shows the upper bound ($\phi_{\Pi} <$) of a potential passive determinacy region, (b) the second column shows the lower bound ($\phi_{\Pi} >$) of a potential active determinacy region.

If a determinacy region does not exist at all, the table shows a dot for “not available.”

Table E.1 Sensitivity: Foreign’s MPC, μ_1

	Steady State							Determinacy	
	ξ_E	$\frac{P_E}{P}$	$\frac{P_G}{P}$	C	C_E	C_G	W	$\phi_{\Pi} <$	$\phi_{\Pi} >$
0.3	1.19	0.06	1.11	1.06	0.19	0.94	1.06	.	0.98
0.4	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.97	4.4
0.5	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.98	23.84
0.6	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.99	47.47
0.7	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.99	76.8

Notes: see explanation in the introduction of this section.

Table E.2 Sensitivity: targeted steady state energy share of production, $\frac{P_E E}{P_G Y_G}$

	Steady State							Determinacy	
	ξ_E	$\frac{P_E}{P}$	$\frac{P_G}{P}$	C	C_E	C_G	W	$\phi_{\Pi} <$	$\phi_{\Pi} >$
0.03	1.32	0.04	1.21	1.18	0.32	0.96	1.18	.	1.25
0.035	1.28	0.04	1.18	1.14	0.28	0.96	1.14	.	1.22
0.04	1.24	0.05	1.15	1.1	0.24	0.95	1.11	0.99	1.97
0.045	1.21	0.05	1.13	1.08	0.21	0.95	1.08	0.99	12.4
0.05	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.98	23.84
0.055	1.17	0.06	1.1	1.04	0.17	0.94	1.04	0.96	36.4
0.06	1.16	0.07	1.09	1.02	0.16	0.93	1.02	0.88	50.2

Notes: see explanation in the introduction of this section.

Table E.3 Sensitivity: targeted steady state energy share of consumption, $\frac{P_E C_E}{P_E C_E + P_G C_G}$

	Steady State							Determinacy	
	ξ_E	$\frac{P_E}{P}$	$\frac{P_G}{P}$	C	C_E	C_G	W	$\phi_{\Pi} <$	$\phi_{\Pi} >$
0.005	1.1	0.05	1.04	0.98	0.1	0.95	0.98	0.95	39.26
0.01	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.98	23.84
0.015	1.29	0.06	1.19	1.13	0.29	0.94	1.13	0.99	14.12
0.02	1.38	0.06	1.27	1.2	0.38	0.93	1.21	0.99	7.42
0.025	1.48	0.07	1.35	1.28	0.47	0.93	1.28	0.99	2.52
0.03	1.57	0.07	1.43	1.35	0.57	0.92	1.36	.	1.11

Notes: see explanation in the introduction of this section.

Table E.4 Sensitivity: subsistence level of energy consumption, \bar{e}

	Steady State							Determinacy	
	ξ_E	$\frac{P_E}{P}$	$\frac{P_G}{P}$	C	C_E	C_G	W	$\phi_{\Pi} <$	$\phi_{\Pi} >$
0	1.19	0.06	1.15	1.1	0.19	0.94	1.1	0.99	15.26
0.02	1.19	0.06	1.14	1.08	0.19	0.94	1.08	0.99	18.43
0.04	1.19	0.06	1.12	1.06	0.19	0.94	1.07	0.98	21.94
0.05	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.98	23.84
0.06	1.19	0.06	1.11	1.05	0.19	0.94	1.05	0.98	25.86
0.08	1.19	0.05	1.09	1.03	0.19	0.94	1.04	0.97	30.27
0.1	1.19	0.05	1.07	1.01	0.19	0.94	1.02	0.96	35.28

Notes: see explanation in the introduction of this section.

Table E.5 Sensitivity: households' elasticity of subs. between energy and goods, η

	Steady State							Determinacy	
	ξ_E	$\frac{P_E}{P}$	$\frac{P_G}{P}$	C	C_E	C_G	W	$\phi_{\Pi} <$	$\phi_{\Pi} >$
0.06	1.19	0.06	1.12	1.06	0.19	0.94	1.07	0.96	34.21
0.08	1.19	0.06	1.12	1.06	0.19	0.94	1.06	0.98	28.7
0.1	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.98	23.84
0.2	1.19	0.05	1.09	1.04	0.19	0.94	1.04	0.99	6.18
0.5	1.19	0.05	1.05	1	0.19	0.94	1	.	1.32

Notes: see explanation in the introduction of this section.

Table E.6 Sensitivity: firms' elasticity of subs. between energy and labor, θ

	Steady State							Determinacy	
	ξ_E	$\frac{P_E}{P}$	$\frac{P_G}{P}$	C	C_E	C_G	W	$\phi_{\Pi} <$	$\phi_{\Pi} >$
0.03	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.9	42.41
0.04	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.98	23.84
0.05	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.99	12.05
0.06	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.99	3.89
0.07	1.19	0.06	1.11	1.06	0.19	0.94	1.06	.	1

Notes: see explanation in the introduction of this section.

Table E.7 Sensitivity: risk aversion/inverse IES, σ

	Steady State							Determinacy	
	ξ_E	$\frac{P_E}{P}$	$\frac{P_G}{P}$	C	C_E	C_G	W	$\phi_{\Pi} <$	$\phi_{\Pi} >$
1	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.66	12.24
2	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.93	20.6
3	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.98	23.84
4	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.96	24.79

Notes: see explanation in the introduction of this section.

Table E.8 Sensitivity: inverse Frisch elasticity, φ

	Steady State							Determinacy	
	ξ_E	$\frac{P_E}{P}$	$\frac{P_G}{P}$	C	C_E	C_G	W	$\phi_{\Pi} <$	$\phi_{\Pi} >$
2	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.98	23.84
3	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.99	48.37
4	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.99	65.65

Notes: see explanation in the introduction of this section.

Table E.9 Sensitivity: price adjustment costs, ψ

	Steady State							Determinacy	
	ξ_E	$\frac{P_E}{P}$	$\frac{P_G}{P}$	C	C_E	C_G	W	$\phi_{\Pi} <$	$\phi_{\Pi} >$
1	1.19	0.06	1.11	1.06	0.19	0.94	1.06	.	1
100	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.21	3.97
250	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.53	11.42
500	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.98	23.84
750	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.97	36.26
1000	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.97	48.68

Notes: see explanation in the introduction of this section.

Table E.10 Sensitivity: profit redistribution, τ^d

	Steady State							Determinacy	
	ξ_E	$\frac{P_E}{P}$	$\frac{P_G}{P}$	C	C_E	C_G	W	$\phi_{\Pi} <$	$\phi_{\Pi} >$
0	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.98	23.84
0.05	1.19	0.06	1.11	1.06	0.19	0.94	1.06	0.72	9.81
0.1	1.19	0.06	1.11	1.06	0.19	0.94	1.06	.	0.99
0.15	1.19	0.06	1.11	1.06	0.19	0.94	1.06	.	0.99
0.2	1.19	0.06	1.11	1.06	0.19	0.94	1.06	.	0.98

Notes: see explanation in the introduction of this section.